Product Differentiation and the Gains from Trade under Bertrand Duopoly

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Abstract

In the literature on the welfare effects of free trade under imperfect competition, one important case seems to have been overlooked and that is the Bertrand duopoly model with differentiated products. Although many authors have analysed the welfare effects of free trade under Cournot duopoly, and demonstrated the possibility of losses from trade, there has been no thorough analysis of the welfare effects of free trade under Bertrand duopoly. This paper presents a thorough analysis of the welfare effects of free trade under Bertrand duopoly with differentiated products, and it is shown that there are always gains from trade.

Keywords: Gains from Trade, Bertrand Oligopoly.

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1. Introduction

In the literature on the welfare effects of free trade under imperfect competition, one important case seems to have been overlooked and that is the Bertrand duopoly model with differentiated products. Although many authors have analysed the welfare effects of free trade under Cournot duopoly, and demonstrated the possibility of losses from trade, there has been no thorough analysis of the welfare effects of free trade under Bertrand duopoly with differentiated products. The reason for this oversight maybe that many authors believed that the effect of free trade in a Bertrand duopoly model would be similar to that in a Cournot duopoly model, but it will turn out that there is a significant difference between the two models.[1] This paper presents a thorough analysis of the welfare effects of free trade under Bertrand duopoly with differentiated products, and it is shown that there are always gains from free trade. The result holds regardless of differences in demand parameters or marginal costs so it is as general as possible given the assumptions of linear demand functions and constant marginal costs.

The welfare effects of free trade under Cournot duopoly were first analysed, and the possibility of losses from trade was first demonstrated, by Brander (1981) for the case of segmented markets and by Markusen (1981) for the case of integrated markets. Brander (1981) shows that intra-industry trade may occur in homogeneous products under Cournot duopoly when markets are segmented even in the presence of transport costs. Assuming two identical countries and linear demand functions, he shows that there are gains from multilateral free trade if transport costs are sufficiently low, but there are losses from trade if transport costs are close to the prohibitive level. Brander and Krugman (1983) extend the analysis to allow general demand functions, and they also show that there will always be gains from multilateral free trade when there is free entry of firms. For the case of integrated markets, Markusen (1981) shows that multilateral free trade between two identical countries each with a single firm will have a pro-competitive effect and increase the output of both firms even though no trade will actually occur. When countries differ in terms of market size, the country with the small market will always gain from trade but the country with the large market may lose from trade if the output of its firm falls as a result of free trade. A sufficient condition for the country with the large market to gain is that trade leads to an expansion of the output of its firm.[2]

These articles concentrated upon the case of multilateral free trade whereas Collie (1996) analyses the welfare effects of unilateral free trade under Cournot duopoly. Assuming homogeneous products and linear demand functions, Collie (1996) shows that a country will only gain from unilateral free trade if the foreign firm has a significant cost advantage so there will be losses from trade if both firms have the same marginal costs.[3] Also, he shows that a sufficient condition for a country to gain from unilateral free trade is that its firm is so uncompetitive that it ceases production under free trade. Cordella (1993) analyses a Cournot oligopoly with many firms in each country assuming linear demand functions and zero marginal costs. He shows that a country will only gain from unilateral free trade if the number of foreign firms is much larger than the number of domestic firms.

The conclusion from this brief survey of the literature is that losses from trade under Cournot duopoly are quite possible especially under unilateral free trade or with differences between the countries in terms of cost or demand functions. In all cases when there are losses from trade, the increase in consumer surplus as a result of competition from the foreign firm is outweighed by the reduction in the profits of the home firm.

As all the analysis of the welfare effects of free trade under imperfect competition assumes that the market structure is Cournot oligopoly, one might wonder what would happen under Bertrand oligopoly. With homogeneous products, it is well known that if there was a Bertrand duopoly rather than a Cournot duopoly in the Brander (1981) or Brander and

Krugman (1983) models then the result would be that each firm would undercut its competitor so no intra-industry trade would occur.[4] Thus, free trade has a pro-competitive effect leading to lower prices and so there are clearly gains from trade with homogeneous products.

As Brander and Krugman (1983) noted, the addition of product differentiation would result in the occurrence of international trade under Bertrand duopoly.[5] The objective of this paper is to analyse the welfare effects of free trade under Bertrand duopoly with differentiated products, and to prove that there are <u>always</u> gains from trade.[6] A two country Bertrand duopoly model, with linear demands and constant marginal costs, will be presented. Since the possibility of losses from trade under Cournot oligopoly occurred when there were differences between the two countries, the model will allow for differences between the two countries in terms of demand and cost functions. Firstly, it will be shown that there are gains from unilateral free trade, where the foreign firm can sell in the home market but the home firm cannot sell in the foreign market. Although there will always be a reduction in the profits of the home firm this will be outweighed by the increase in consumer surplus as a result of a lower price for the product of the home firm and the availability of the differentiated product of the foreign firm. Secondly, as the only difference in terms of welfare between multilateral free trade and unilateral free trade is the profits that the home firm earns from its exports to the foreign market, which must be positive, it is noted that there must be gains from multilateral free trade if there are gains from unilateral free trade.

Finally, the special case of multilateral free trade between two identical countries in the presence of transport costs will be considered. This case is the Bertrand duopoly analogue of the Brander (1981) and Brander and Krugman (1983) models of intra-industry trade. It is shown that although welfare as a function of the transport cost is U-shaped when trade occurs, as it is under Cournot duopoly, there are always gains from trade under Bertrand duopoly even when the transport cost is high. The reason is that under Bertrand duopoly, in contrast to Cournot duopoly, free trade may have a pro-competitive effect even when no trade occurs.

2. The Model

Assume that there are two countries, a home and a foreign country and that each country has a single imperfectly competitive firm that produces a differentiated good, with the home firm labelled as firm one and the foreign firm labelled as firm two. For completeness, there is also a perfectly competitive industry in both countries producing a homogeneous good using constant returns to scale technology. This good is traded freely between the two countries and acts as the numeraire good. Under autarky, each imperfectly competitive firm faces no competition in its domestic market and so can act as a monopolist. Under unilateral free trade, assuming that markets are segmented, the two firms compete in a Bertrand duopoly in the home market but the foreign firm does not face any competition and so can act as a monopolist in the foreign market. With segmented markets and constant marginal costs, the Bertrand duopoly in the home market can be analysed independently of the foreign market so the analysis will focus on the home market. The home firm has constant marginal cost c_1, sets price p_1, and sells output y_1 in the home market while the foreign firm has constant marginal cost c_2, sets price p_2, and sells output y_2 in the home market. Consumption of the home firm's differentiated product in the home market is equal to the sales of the home firm in the home market, y_1; consumption of the foreign firm's differentiated product in the home market is equal to the sales of the foreign firm in the home market, y_2; and consumption of the numeraire good is z. It is assumed that there is a representative consumer in the home country with quasi-linear preferences that can be represented by a quadratic utility function, as in Vives (1985):

$$U(\mathbf{y},z)=\sum_{i=1}^{2}\alpha_i y_i-\frac{1}{2}\sum_{i=1}^{2}\beta_i y_i^2-\gamma\, y_1 y_2+z \qquad \alpha_i,\beta_i,\gamma>0;\quad \beta_i>\gamma \tag{1}$$

where $0<\gamma^2/\beta_1\beta_2<1$ is a measure of the degree of product substitutability ranging from zero when the products are independent to one when the products are perfect substitutes. Also, it is assumed that $\alpha_i>c_i$ otherwise the *i*th firm will not produce any output even if it has a monopoly. As the demand parameters may differ for the products of the two firms and the firms may have different marginal costs, the model is as general as possible given the assumed functional forms, i.e. quadratic utility function and constant marginal costs. Without these assumptions, the explicit comparison of welfare under autarky and free trade would not be possible unless some other tractable functional forms were used.

It is straightforward to show that the utility function (1) yields the following inverse and direct demand functions:

$$\begin{aligned} p_i &= \alpha_i-\beta_i y_i-\gamma\; y_j \\ y_i &= \frac{1}{R}\Big[\big(\alpha_i\beta_j-\alpha_j\gamma\big)-\beta_j\, p_i+\gamma\; p_j\Big] \end{aligned} \qquad i,j=1,2\quad i\neq j \tag{2}$$

where $R=\beta_1\beta_2-\gamma^2>0$. Since the utility function is quadratic, these functions are linear in prices. Moreover, since preferences are assumed to be quasi-linear, consumer surplus will be a valid measure of consumer welfare. In this case, as in Vives (1985), consumer surplus is:

$$CS=U-p_1y_1-p_2y_2-z=\frac{1}{2}\beta_1 y_1^2+\frac{1}{2}\beta_2 y_2^2+\gamma\; y_1 y_2 \tag{3}$$

The profit functions of the home firm and the foreign firm, respectively, from sales in the home country market are:

$$\pi_1 = (p_1 - c_1)\, y_1 \qquad\qquad \pi_2 = (p_2 - c_2)\, y_2 \tag{4}$$

The welfare of the home country under autarky and unilateral free trade is given by the sum of consumer surplus and the profits of the home firm from its domestic market:

$$W_1 = CS + \pi_1 = \frac{1}{2}\beta_1 y_1^2 + \frac{1}{2}\beta_2 y_2^2 + \gamma\, y_1 y_2 + (p_1 - c_1)\, y_1 \tag{5}$$

Under autarky, the home firm has a monopoly in the home market and faces the inverse demand function: $p_1 = \alpha_1 - \beta_1 y_1$, which is obtained by setting the output of the foreign firm equal to zero in the inverse demand function (2). It is straightforward to show that the monopoly price is $p_1^A = (\alpha_1 + c_1)/2$, the monopoly output is $y_1^A = (\alpha_1 - c_1)/2\beta_1$, and monopoly profits are $\pi_1^A = (\alpha_1 - c_1)^2/4\beta_1$. Substituting $y_1 = y_1^A$ and $y_2 = 0$ into (3) yields consumer surplus under autarky: $CS^A = (\alpha_1 - c_1)^2/8\beta_1$. Therefore, since the welfare of the home country is equal to the sum of consumer surplus and the profits of the home firm, welfare under autarky can be shown to be:

$$W_1^A = 3\,(\alpha_1 - c_1)^2/8\beta_1 \tag{6}$$

This provides the benchmark for the welfare analysis of the gains from unilateral and multilateral free trade in the next section.

3. The Bertrand Equilibrium and the Gains from Trade

Under unilateral free trade, with Bertrand competition, both firms are able to supply the market in the home country and each firm sets its own price to maximise its profits given the price set by the other firm. To show that there are gains from trade for all parameter values, a thorough analysis of the Bertrand equilibrium is required allowing for the possibility of boundary solutions where the sales of one firm are equal to zero.

The first step is to derive the Bertrand duopoly best-reply functions of the two firms, which are shown as the bold lines in figure one. The $y_1 = 0$ and $y_2 = 0$ lines in figure one can be derived by setting the direct demand functions in (2) equal to zero and solving for the prices where sales are equal to zero. Sales of the domestic firm are equal to zero below the $y_1 = 0$ line and sales of the foreign firm are equal to zero above the $y_2 = 0$ line. In the region between these two lines there will be an interior solution, where both firms have positive sales in the home market, and the first order condition for profit maximisation by the *i*th firm is:

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1}{R}\left[\alpha_i \beta_j - \alpha_j \gamma - 2\beta_j p_i + \gamma p_j + \beta_j c_i\right] = 0 \tag{7}$$

Solving for p_i yields the best-reply function for the *i*th firm:

$$r_i\left(p_j\right) = \frac{\alpha_i + c_i}{2} - \frac{\gamma}{2\beta_j}\left(\alpha_j - p_j\right) = p_i^M - \frac{\gamma}{2\beta_j}\left(\alpha_j - p_j\right) \tag{8}$$

It is never profitable for the firms to set a price below marginal cost so the best-reply function of the home firm is vertical at c_1 when the foreign firm sets a price below b_2 in figure one, and the best-reply function of the foreign firm is horizontal at c_2 when the domestic firm sets a price below b_1.

If the foreign firm sets a price between d_2 and e_2 then the home firm can increase its profits by raising its price above that given by the dashed line (8) until it reaches the $y_2 = 0$ line, as the sales of the foreign firm are equal to zero until this point, but beyond this point the profits of the home firm will decrease. Hence, the best-reply function of the home firm is given by the $y_2 = 0$ line in figure one. If the foreign firm sets a price above e_2 then the sales of the foreign firm are equal to zero even if the home firm sets its monopoly price so the best-reply function of the home firm is given by the vertical line at the monopoly price. A similar analysis applies to the derivation of the best-reply function of the foreign firm in figure one. Thus, allowing for the possibility of boundary solutions, the best-reply function of the *i*th firm $r_i(p_j)$ is defined as:

$$r_i(p_j) = \begin{cases} c_i & \text{if } p_j \le b_j \equiv \alpha_j - \dfrac{\beta_j}{\gamma}(\alpha_i - c_i) \\ p_i^M - \dfrac{\gamma}{2\beta_j}(\alpha_j - p_j) & \text{if } b_j < p_j < d_j \equiv \alpha_j - \dfrac{\beta_j \gamma}{S}(\alpha_i - c_i) \\ \alpha_{\mathrm{i}} - \dfrac{\beta_i}{\gamma}(\alpha_j - p_j) & \text{if } d_j \le p_j < e_j \equiv \alpha_j - \dfrac{\gamma}{2\beta_i}(\alpha_i - c_i) \\ p_i^M & \text{if } p_j \ge e_j \end{cases} \tag{9}$$

where $S = 2\beta_1\beta_2 - \gamma^2$ Having derived the best-reply functions shown in figure one for the two firms, the Bertrand equilibrium can now be derived and the welfare effects of unilateral free trade analysed.

There are five possible cases to be considered that depend upon the relative costs of the two firms. Firstly, consider the case when $c_1 < d_1$ and $c_2 < d_2$ so that the Bertrand equilibrium is an interior solution where both firms have positive sales in the Bertrand equilibrium; such as *E* in figure one. Using the best-reply functions (8) to solve for the Bertrand equilibrium prices and sales yields:

$$\begin{aligned} p_i &= c_i + \frac{1}{T}\left[S(\alpha_i - c_i) - \beta_i\gamma(\alpha_j - c_j)\right] \\ y_i &= \frac{\beta_j}{RT}\left[S(\alpha_i - c_i) - \beta_i\gamma(\alpha_j - c_j)\right] \end{aligned} \tag{10}$$

where $T = 4\beta_1\beta_2 - \gamma^2 > 0$. Substituting the Bertrand equilibrium prices and sales into (5) yields the welfare of the home country under unilateral free trade:

$$W_1^F = \frac{\beta_2}{2RT}\left[\left(3\beta_1\beta_2 - 2\gamma^2\right)\left(\alpha_1 - c_1\right)^2 - 2\beta_1\gamma\left(\alpha_1 - c_1\right)\left(\alpha_2 - c_2\right) + \beta_1^2\left(\alpha_2 - c_2\right)^2\right] \qquad (11)$$

The gains from trade are given by subtracting welfare under autarky (6) from welfare under unilateral free trade (11), which yields:

$$G\left(c_1, c_2\right) = \frac{1}{8\beta_1 RT}\left[M\left(\alpha_1 - c_1\right)^2 - 8\beta_1^2\beta_2\gamma\left(\alpha_1 - c_1\right)\left(\alpha_2 - c_2\right) + 4\beta_1^3\beta_2\left(\alpha_2 - c_2\right)^2\right] \qquad (12)$$

where $M = \left(7\beta_1\beta_2 - 3\gamma^2\right)\gamma^2 > 0$. Obviously, this is a quadratic form in $\left(\alpha_1 - c_1\right)$ and $\left(\alpha_2 - c_2\right)$ that has a stationary point at $\left(c_1 = \alpha_1, c_2 = \alpha_2\right)$ where its value is equal to zero. This stationary point will be a unique minimum if the quadratic form is strictly convex in costs. To show that it is indeed strictly convex in costs, twice differentiating $G\left(c_1, c_2\right)$ yields the Hessian matrix:

$$H = \frac{1}{4\beta_1 RT}\begin{bmatrix} M & -4\beta_1^2\beta_2\gamma \\ -4\beta_1^2\beta_2\gamma & 4\beta_1^3\beta_2 \end{bmatrix} \qquad (13)$$

and the Hessian determinant is $\left|H\right| = 3\beta_1\beta_2\gamma^2 / 4RT^2 > 0$. Since both pure second derivatives are always positive and the Hessian determinant is positive, $G\left(c_1, c_2\right)$ is always strictly convex in costs so it has a unique minimum. The minimum value of $G\left(c_1, c_2\right)$ is equal to zero and occurs at $\left(c_1 = \alpha_1, c_2 = \alpha_2\right)$, therefore $G\left(c_1, c_2\right)$ is strictly positive for $\left(c_1 < \alpha_1, c_2 < \alpha_2\right)$ and there are gains from trade.

Secondly, consider the trivial case when $c_2 \geq e_2$ so that the Bertrand equilibrium is a boundary solution where the foreign firm has sales of zero and the home firm can set the monopoly price. Here, the home firm sets the monopoly price and sells the monopoly output so the situation is exactly the same as under autarky with no gains or losses from unilateral free trade.[7]

Thirdly, consider the case when $d_2 \le c_2 < e_2$ so that the Bertrand equilibrium is a boundary solution where the foreign firm has sales of zero and the home firm sets a price lower than the monopoly price; such as equilibrium *B* in figure two. The foreign firm sets its price equal to its marginal cost, $p_2 = c_2$, so from (9) and (2) the price and sales of the home firm are:

$$p_1^B = b_1 = \alpha_1 - \frac{\beta_1}{\gamma}(\alpha_2 - c_2) \qquad y_1^B = \frac{1}{\gamma}(\alpha_2 - c_2) \tag{14}$$

The situation facing the home firm in this Bertrand equilibrium is shown in figure three, where the profit maximising price is p_1^B and sales are y_1^B at the kink in the demand curve. The kink occurs because if the firm reduces its price then it faces the monopoly demand curve as the sales of the foreign firm will be equal to zero, but if it increases its price then it will face the more elastic duopoly demand curve (2) as the sales of the foreign firm will be positive. As the home firm sets a lower price and has higher sales than under autarky, there are clearly gains from free trade given by the shaded area in figure three.[8] This does not happen under Cournot duopoly, where free trade only has an effect if trade actually occurs, because the home firm faces the monopoly demand curve if the foreign sets its output equal to zero whereas the home firm faces a kinked demand curve under Bertrand duopoly when the sales of the foreign firm are equal to zero.

Fourthly, consider the case when $c_1 \ge e_1$ so that the Bertrand equilibrium is a boundary solution where the home firm has sales of zero and the foreign firm can set the monopoly price and sell the monopoly output, $y_2 = (\alpha_2 - c_2)/2\beta_2$. Substituting these sales into (5) yields the welfare of the home country under unilateral free trade: $W_1^F = (\alpha_2 - c_2)^2 / 8\beta_2$. To compare this with welfare under autarky (6), note that $c_1 \geq e_1$ implies that $\gamma(\alpha_2 - c_2) \geq 2\beta_2(\alpha_1 - c_1)$, which yields the following inequality when both sides are squared:

$$W_1^F = \frac{1}{8\beta_2}(\alpha_2 - c_2)^2 \geq \frac{4\beta_1\beta_2}{3\gamma^2}\frac{3}{8\beta_1}(\alpha_1 - c_1)^2 > \frac{3}{8\beta_1}(\alpha_1 - c_1)^2 = W_1^A \tag{15}$$

There are gains from trade even though the monopoly of the home firm under autarky has been replaced by the monopoly of the foreign firm under unilateral free trade. The reason is that if the home firm has zero sales under free trade then the relative cost advantage of the foreign firm has to be so large that the gain in consumer surplus outweighs the loss of the home firm's monopoly profits.[9]

Finally, consider the case when $d_1 \leq c_1 < e_1$ so that the Bertrand equilibrium is a boundary solution where the home firm has sales of zero and the foreign firm sets a price lower than its monopoly price. The home firm sets its price equal to its marginal cost, $p_1 = c_1$, so from (9) and (2) the price and sales of the foreign firm are:

$$p_2 = \alpha_2 - \frac{\beta_2}{\gamma}(\alpha_1 - c_1) \qquad y_2 = \frac{1}{\gamma}(\alpha_1 - c_1) \tag{16}$$

Substituting these sales into (5) yields the welfare of the home country under free trade, and comparing with welfare under autarky (6) gives:

$$W_1^F = \frac{\beta_2}{2\gamma^2}(\alpha_1 - c_1)^2 > \frac{3}{8\beta_1}(\alpha_1 - c_1)^2 = W_1^A \tag{17}$$

As in the previous case, the loss of the home firm's monopoly profits is outweighed by the gain in consumer surplus due to the relative cost advantage of the foreign firm, and there are gains from trade.

The results from all five cases, lead to the following proposition:

Proposition 1: *Under Bertrand duopoly, with linear demand and constant marginal costs, there are always gains from unilateral free trade.*

In all cases except the trivial case, when free trade has absolutely no effect, the gains from unilateral free trade are strictly positive. This is a very strong result and it should be stressed that it holds for all demand and cost parameters given the functional forms employed. In contrast, Collie (1996) shows that there will only be gains from unilateral free trade under Cournot duopoly if the foreign firm has a significant cost advantage so there will be losses from trade if the firms have the same marginal costs.

This proposition can easily be extended to the case of multilateral free trade. With multilateral free trade, since markets are segmented, the outcome in the home market would be the same as under unilateral free trade, but the home firm would earn additional profits from its exports to the foreign market, since it will only export to the foreign market if it is profitable. This increases the welfare of the home country under free trade so if there are gains from unilateral free trade then there must be gains from multilateral free trade. This leads to the following proposition:

Proposition 2: *Under Bertrand duopoly, with linear demand and constant marginal costs, there are always gains from multilateral free trade.*

If the home firm has strictly positive sales in the foreign market then it will make strictly positive profits and there will certainly be gains from multilateral free trade even in the trivial case when the home firm can act as a monopolist in the home market under free trade. Again this is a strong result, and contrasts with the situation under Cournot duopoly where there may be losses from multilateral free trade if there are asymmetries between the countries in terms of demand and cost parameters.

4. The Brander-Krugman Case: Symmetry and Transport Costs

One special case worthy of attention is the Bertrand duopoly analogue of the Brander (1981) and Brander and Krugman (1983) models. They consider multilateral free trade between two identical countries under Cournot duopoly with homogeneous products in the presence of transport costs. Their results were that intra-industry trade will occur when markets are segmented, and that there will be losses from free trade when transport costs are high. They showed that welfare under free trade as a function of the transport cost was U-shaped, and that a small reduction in transport costs below the prohibitive level would reduce free trade welfare below autarky welfare. This happens because, as a result of competition from the foreign firm, the output of the home firm falls reducing its profits and this reduction in profits outweighs any increase in consumer surplus. Although proposition two implies that there will undoubtedly be gains from trade if the market structure in these models is changed from Cournot to Bertrand duopoly, it is worth looking at this special case to see exactly what happens as the Brander (1981) and Brander and Krugman (1983) models are so well known.

In this special case, the two countries are identical in terms of demand parameters ($\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$), but the products are differentiated so $0 < \gamma < \beta$. The firms both have the same marginal cost of production, c, but there is a transport cost of k per unit when products are traded between the two countries. Thus, when the two firms compete in the home market, the home firm has marginal cost $c_1 = c$, and the foreign firm has marginal cost $c_2 = c + k$, and vice-versa when they compete in the foreign market.

With multilateral free trade, the welfare of the home country consists of consumer surplus in the home country plus the profits of the home firm from the domestic market and the profits from the export market in the foreign country. However, using the symmetry of the model, the profits of the home firm from exports to the foreign country will be equal to the profits of the foreign firm from exports to the home country. Hence, the total profits of the home firm can be written as $\pi_1 = (p_1 - c) y_1 + (p_2 - c - k) y_2$ and the welfare of the home country under multilateral free trade is equal to:

$$W_1^F = \frac{1}{2}\beta\, y_1^2 + \frac{1}{2}\beta\, y_2^2 + \gamma\; y_1 y_2 + (p_1 - c) y_1 + (p_2 - c - k) y_2 \tag{18}$$

As in the previous section, the model can be solved for the Bertrand equilibrium then welfare under free trade can be derived as a function of the transport cost, k, which is shown in figure four. For $k \geq k_M \equiv (2\beta - \gamma)(\alpha - c)/2\beta$, which is equivalent to $c_2 \geq e_2$ in the previous section, the transport cost is so high that there is no trade and free trade has no effect on the home firm and it sets its monopoly price. Hence, welfare under free trade is the same as under autarky.

For $k_T \leq k < k_M$, where $k_T \equiv (\beta - \gamma)(2\beta + \gamma)(\alpha - c)/S$, which is equivalent to $d_2 \leq c_2 < e_2$ in the previous section, again there is no trade but the home firm sets a price that is below its monopoly price as a result of competition from the foreign firm. The situation is the same as that shown in figures two and three in the previous section. Noting that $y_2 = 0$, welfare can be shown to be:

$$W_1^F = \frac{1}{2}\beta_1 y_1^2 + (p_1 - c_1) y_1 = \frac{1}{\gamma}(\alpha - c)(\alpha - c - k) - \frac{\beta}{2\gamma^2}(\alpha - c - k)^2 \tag{19}$$

Differentiating (19) with respect to the transport cost yields:

$$\frac{\partial W_1^F}{\partial k} = (p_1 - c_1)\frac{\partial y_1}{\partial k} = \frac{1}{\gamma^2}\left[\beta(\alpha - c - k) - \gamma(\alpha - c)\right] < 0 \tag{20}$$

which is clearly negative for $k_T < k < k_M$ so welfare under free trade is downward sloping as shown in figure four. Welfare increases as the transport cost decreases because the home firm reduces its price and its sales increase, $\partial y_1/\partial k < 0$. Hence, free trade has a pro-competitive effect even though no trade actually occurs, and there are unambiguous gains from trade.

For $k < k_T$, which is equivalent to $c_2 < d_2$ and $c_1 < d_1$ in the previous section, there is an interior solution where the two firms supply both markets, and it can be shown that welfare under free trade is quadratic in the transport cost:

$$W_1^F = \frac{\beta}{2RT^2}\left[N(\alpha-c)^2 - N(\alpha-c)k + \left(12\beta^4 - 9\beta^2\gamma^2 + 2\gamma^4\right)k^2\right] \tag{21}$$

where $N \equiv 2(\beta-\gamma)(3\beta-2\gamma)(2\beta+\gamma)^2 > 0$. As in Brander (1981) and Brander and Krugman (1983), welfare as a function of the transport cost is U-shaped when intra-industry trade occurs. Initially, at $k = k_T$, imports are equal to zero, $y_2 = 0$, so the effect on consumer surplus of a decrease in transport costs is through the reduction in the home firm's price but this is offset by the loss of the home firm's profit as a result of the reduction in its price. Thus, the overall effect on welfare as a result of a decrease in the transport cost is due to the loss of the home firm's profits as a result of the reduction in its sales. When the transport cost is low, the effect on consumer surplus and the profits of the home firm from exports will outweigh the loss of its profits in the home market so welfare will rise as the transport cost decreases giving the U-shaped curve in figure four. Thus, whether market structure is Cournot or Bertrand duopoly, when trade initially occurs the sales of the home firm fall thereby reducing welfare, and ensuring that welfare as a function of the transport cost is U-shaped.

However, although welfare is U-shaped under Bertrand duopoly, it increases from the autarky level before trade actually occurs so welfare at k_T is higher than under autarky and there are not necessarily losses from trade. In fact, for $k < k_T$, it can be shown that the minimum level of welfare is:

$$W_1^{\text{Min}} = \frac{\beta\left(9\beta^2 - 4\gamma^2\right)}{2\left(12\beta^4 - 9\beta^2\gamma^2 + 2\gamma^4\right)}\left(\alpha - c\right)^2 \tag{22}$$

This is the minimum of the U-shaped part of the curve, and comparing it with welfare under autarky (6) yields:

$$W_1^{\text{Min}} - W_1^A = \frac{\gamma^2\left(11\beta^2 - 6\gamma^2\right)}{8\beta\left(12\beta^4 - 9\beta^2\gamma^2 + 2\gamma^4\right)}\left(\alpha - c\right)^2 > 0 \tag{23}$$

Thus, when intra-industry trade occurs, the minimum level of welfare under free trade is higher than the level of welfare under autarky so there are always gains from trade whatever the level of transport costs. This contrasts with the results of Brander (1981) and Brander and Krugman (1983) where there are always losses if the transport cost is high. The reason for the difference under Bertrand duopoly is that free trade has a pro-competitive effect on the home firm when transport costs are so high that trade does not occur, and this means that the home firm is producing more output when trade starts than under autarky so although welfare falls it does not fall below autarky welfare.

5. Conclusions

This paper has analysed the welfare effects of free trade under Bertrand duopoly with differentiated products, and proved the very strong result that there are always gains from trade. This result was demonstrated for unilateral and multilateral free trade, and it holds for all demand and cost parameters given the functional forms employed. The special case of the Bertrand duopoly analogue of the Brander (1981) model was presented, and it was shown that there were no losses from trade even when the transport cost was high. These results are very significant as they contrast strongly with the many results for Cournot duopoly that demonstrate the possibility of losses from trade.

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Notes

[1] It should be stressed that the reason for this difference between the two duopoly models is not the well-known difference that outputs are strategic substitutes under Cournot duopoly and prices are strategic complements under Bertrand duopoly.

[2] This result is generalised for many market structures by Helpman and Krugman (1985), but it does not really help to show that there are always gains from trade under Bertrand duopoly as the output of the home firm may very well fall especially under unilateral free trade. Also, it does not hold for segmented markets.

[3] Collie (1996) also shows that there are always gains from unilateral free trade if demand functions are iso-elastic regardless of the costs of the two firms.

[4] In each market, the foreign firm incurs a transport cost, k, in addition to the marginal cost of production, c, so the home firm has a cost advantage and can undercut the foreign firm by setting a price fractionally below $c+k$, assuming that this is less than the monopoly price. Thus, the home firm will supply all of the domestic demand but at a lower price than under autarky.

[5] In Brander and Krugman (1983), it is stated that 'If price is the strategy variable, reciprocal dumping does not arise in the homogeneous product case. However, a slight amount of product differentiation will restore the reciprocal dumping result, in which case the intra-industry trade motives described here augment the usual product differentiation motives for intra-industry trade'. It should be noted that they do not conjecture about the welfare effects of free trade under Bertrand oligopoly.

[6] Recently Bernhofen (2001) has introduced product differentiation into Cournot and Bertrand oligopoly models of intra-industry trade between two identical countries. Although he looks at the effect of trade on profits and consumer surplus under Cournot oligopoly, he says nothing about the welfare effects of trade under Bertrand oligopoly. His main concerns are the effect of product differentiation on the volume of trade, and the effect of trade liberalisation on profits as in Anderson *et al* (1989).

[7] This trivial case corresponds to what happens in Brander (1981) if transport costs are prohibitive, or in the neoclassical model if the free trade price ratio is equal to the autarky price ratio. There is no trade and free trade has no effect on welfare.

[8] Note that the price set by the home firm is increasing and its sales are decreasing in the costs of the foreign firm. A reduction in the costs of the foreign firm, say due to a lower tariff or transport

costs, would lower the price set by the home firm and increase its sales. Thus, the welfare of the home country is decreasing in the costs of the foreign firm.

[9] Collie (1996) obtains a similar result under Cournot oligopoly with homogeneous products, and he shows that this result holds for general demand functions as well as for linear demand.

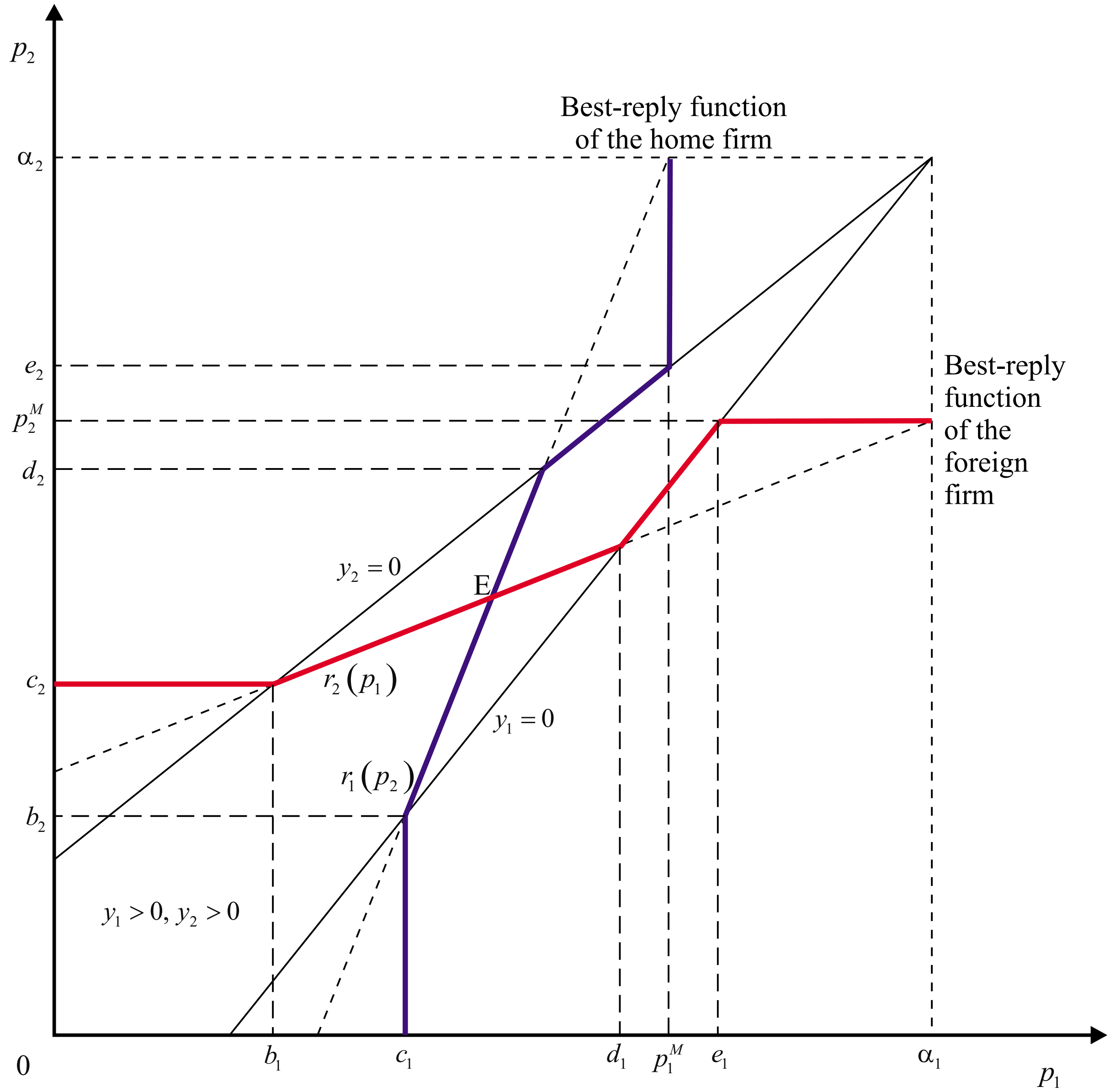

Figure One: Bertrand duopoly best-reply functions

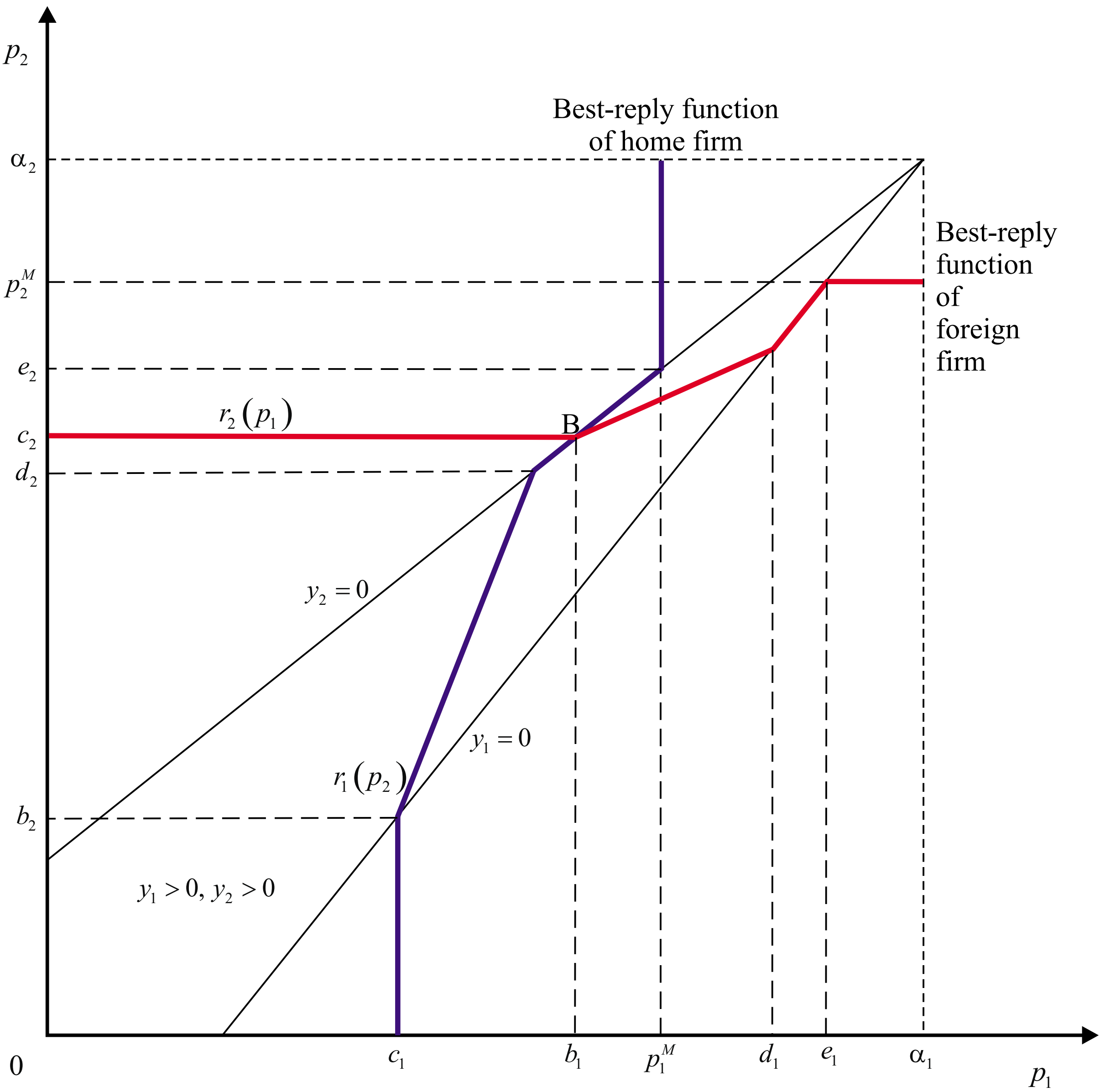

Figure Two: Boundary solution Bertrand equilibrium

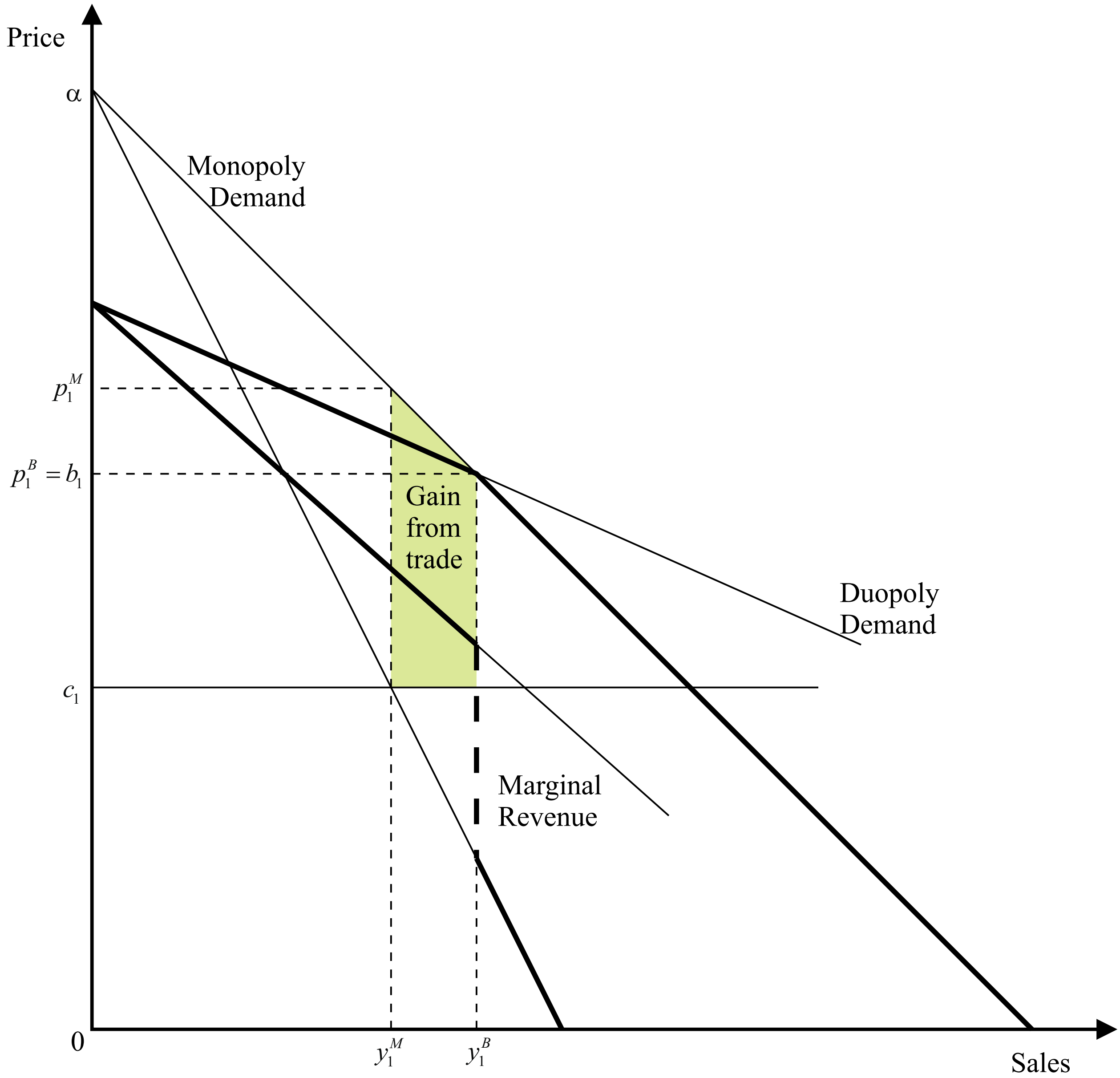

Figure Three: Boundary solution Equilibrium for home firm

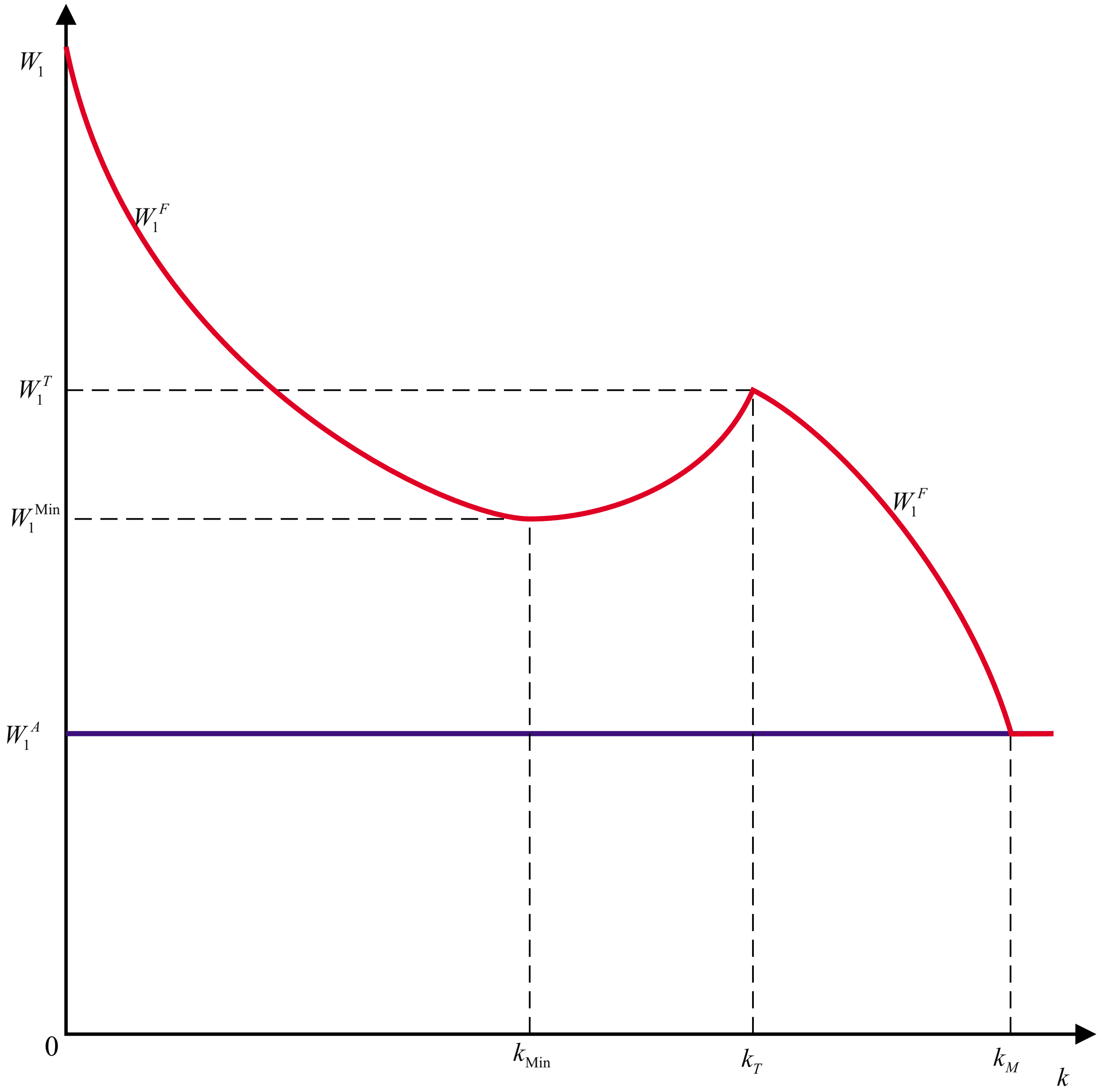

Figure Four: Welfare as a function of transport costs