An Axiomatization of Linear Cumulative Prospect Theory with Applications to Portfolio Selection and Insurance Demand

by Ulrich Schmidt[a] and Horst Zank (YE)[b,*]

[a]Institut für Volkswirtschaftslehre, Christian-Albrechts-Universität zu Kiel, Germany.

[b]School of Economic Studies, The University of Manchester, United Kingdom.

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Abstract. The present paper combines loss attitudes and linear utility by providing an axiomatic analysis of corresponding preferences in a cumulative prospect theory (CPT) framework. (CPT) is one of the most promising alternatives to expected utility theory since it incorporates loss aversion, and linear utility for money receives increasing attention since it is often concluded in empirical research, and employed in theoretical applications. Rabin (2000) emphasizes the importance of linear utility, and highlights loss aversion as an explanatory feature for the disparity of significant small-scale risk aversion and reasonable large-scale risk aversion. In a sense we derive a two-sided variant of Yaari's dual theory, i.e. nonlinear probability weights in the presence of linear utility. The first important difference is that utility may have a kink at the status quo, which allows for the exhibition of loss aversion. Also, we may have different probability weighting functions for gains than for losses. The central condition of our model is termed independence of common increments. The applications of our model to portfolio selection and

*Corresponding author: Horst Zank, School of Economic Studies, The University of Manchester, Oxford Road, Manchester, M13 9PL, UK, tel.: +44 161 2754872, fax: +44 161 2754812, e-mail: horst.zank@man.ac.uk

insurance demand show that CPT with linear utility has more realistic implications than the dual theory since it implies only a weakened variant of plunging.

Keywords: cumulative prospect theory, insurance demand, linear utility, loss aversion, portfolio selection, risk aversion.

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1 Introduction

Empirical research has shown that expected utility (EU) fails to provide a good description of individual behavior under risk. Examples are the famous paradoxes of Allais (1953) and Ellsberg (1961). This evidence has motivated the development of alternative theories (the so-called non-expected utility theories), which allow for the exhibition of "paradoxical behavior." Building upon its predecessor prospect theory (Kahneman and Tversky, 1979), cumulative prospect theory (CPT) has nowadays become the most prominent of these alternatives.

Recently, a new criticism of EU has been put forward by Rabin (2000) and Rabin and Thaler (2001). Following earlier work by Hansson (1988), these authors show that reasonable degrees of risk aversion over small and modest stakes imply unreasonable high degrees of risk aversion over large stakes in the EU framework. For instance an EU-maximizer who initially rejects a 50-50 bet of loosing \$10 and winning \$11 regardless of the current wealth would also reject any 50-50 bet of losing \$100 and winning \$$x$ for any large value of x. Since this high degree of risk aversion seems to be irrational, Rabin (2000) concluded that EU is only a good representation of risk neutral behavior, which necessarily means that utility has to be linear. Neilson (2001) has shown that this criticism on EU carries over to rank-dependent utility which is further prominent alternative to EU and a precursor of CPT. More precisely, in the rank-dependent utility framework the utility function should also be linear because concave utility implies, as for EU, unreasonable high degrees of risk aversion over large stakes.

Considering the importance of linear utility, the goal of the present paper is to investigate linear utility for decision under risk in a CPT framework by providing an axiomatic analysis of corresponding preferences. Rabin (2000) points out that a model incorporating loss attitudes reconciles significant degrees of risk aversion for small-scale outcomes and reasonable degrees of risk aversion for large-scale outcomes. CPT, as a direct generalization of expected utility and of rank dependent utility, takes into account attitudes towards losses. The characteristic features of CPT are rank-dependence, reference-dependence, and sign-dependence. Rank-dependence resolves the paradoxes of Allais (1953) and Ellsberg (1961), and has played an important role in the axiomatizations of rank-dependent expected utility (Quiggin 1981, Wakker 1989, Schmeidler 1989). By transforming cumulative instead of single probabilities rank-dependence enables the incorporation of nonlinear perception of probabilities into decision under risk, without implying violations of stochastic dominance as in the original prospect theory model.

Experimental findings suggest that decision makers perceive outcomes as differences to their status quo rather than absolute wealth levels (Markovitz 1952, Edwards 1954, Yaari 1965, Kahneman and Tversky 1979, Tversky and Kahneman 1991, Harless and Camerer 1994). Reference-dependence means that losses (i.e., negative deviations from the status quo) are perceived differently than gains. A loss seems to have a greater impact than the corresponding gain, which has motivated Kahneman and Tversky (1979) to propose the hypothesis of loss aversion (the aversion of a loss weights significantly more than the attraction by a corresponding gain). Loss aversion has been proved to be fruitful in explaining paradoxical phenomena as the equity premium puzzle (Benartzi and Thaler 1995, Gneezy and Potters 1997), the overtime premium puzzle (Dunn 1996), the status quo bias (Samuelson and Zeckhauser 1988), and the endowment effect (Thaler 1980). The latter explains the observed disparity between the willingness-to-pay and willingness-to-accept (Kachelmeier and Shehata 1992). The status quo is usually assumed to be the current wealth position, however, it may be influenced by other factors. For example, based on empirical evidence by Green (1963), Swalm (1966), and Halter and Dean (1971), the status quo was assumed to be a target return in the models of Fishburn (1977) and Holthausen (1981).

There is also empirical evidence which suggests that the nonlinear perception of probabilities, as explained above, is influenced by whether gains or losses are considered (Edwards 1955, Slovic and Lichtenstein 1968, Einhorn and Hogarth 1986, Currim and Sarin 1989, Tversky and Kahneman 1992). This feature, referred to as sign-dependence, leads to different decision weights for gains than for losses.

Because CPT combines these three desirable features, it is currently the most used model for decision under risk in empirical research. It was first proposed by Starmer and Sugden (1989). Later, axiomatizations of CPT have been provided by Luce and Fishburn (1991), Tversky and Kahneman (1992), Wakker and Tversky (1993), Chateauneuf and Wakker (1999), and Schmidt (2000). This paper provides a new axiomatization of CPT with a piecewise linear utility function. More precisely, utility is linear for gains and linear for losses with a possible kink at the status quo. If loss aversion is satisfied, utility is steeper in the domain of gains than in the domain of losses.

Linear utility has a long tradition in theoretical and empirical research. An axiomatic foundation of subjective expected utility with linear utility was provided by de Finetti (1931). Preston and Baratta (1948) used a linear utility model in order to estimate probability distortions. Edwards (1955) reports about a series of experiments which support our model. He finds evidence for sign-dependent probability distortions and also for linear utility. Many studies observed linear utility for losses (Hershey and Schoemaker 1980, Schneider and Lopes 1986, Cohen, Jaffray, and Said 1987, Weber and Bottom 1989, Lopes and Oden 1999). Moreover, for small stakes it seems to be commonly agreed that utility is linear (Lopes 1995, Fox, Rogers, and Tversky 1996, Kilka and Weber 1998).

Handa (1977) axiomatized a model of subjective expected value, which was implicitly used by Preston and Baratta (1948) and already discussed in Edwards (1955). In that model the value of a lottery is given by the sum of distorted probabilities multiplied with their corresponding outcomes. As in the original prospect theory, not cumulative but single probabilities are distorted and, therefore, violations of stochastic dominance are implied. This was first pointed out by Fishburn (1978). One model that combines linear utility and distorted probabilities without violating stochastic dominance is the dual theory (DT) of Yaari (1987). Similarly to our model, cumulative probabilities are distorted, however, the resulting probability weights are not sign-dependent. Moreover, reference-dependence and, therefore, also loss aversion are not permitted under DT. A second axiomatization of DT was offered by Safra and Segal (1998). As Yaari (1987) they exclude sign- and reference-dependence by considering only positive consequences. Their essential assumptions are constant proportional and constant absolute risk aversion jointly unified under the name constant risk aversion. Moreover, they use additional preference conditions which impose strong restrictions on the range of probability weighting functions.

We are convinced that CPT with linear utility is not only useful as a descriptive model in empirical research but may also generate new insights in theoretical applications. In particular, due to the additional freedom gained by reference- and sign-dependence, the model is a good alternative to the DT which has often been applied in economic analyses. Some examples are firm behavior under risk (Demers and Demers 1990), insurance demand (Doherty and Eeckhoudt 1995, Schmidt 1996), insurance pricing (Wang 1995, 1996, Wang, Young and Panjer 1997), agency theory (Schmidt 1999a), and efficient risk-sharing (Schmidt 1999b). Interestingly, van der Hoek and Sherris (2001) propose a risk measure for portfolio choice and insurance decisions based on DT and choose different weighting functions for gains and losses. Therefore, our model can serve as a theoretical basis for their risk measure.

Altogether, linear utility plays an important role in both theoretical and empirical research, especially for analyzing firm behavior and insurance economics. This conclusion is reinforced by the results of Rabin (2000) and Rabin and Thaler (2001) since they clearly show that the common assumption of concave utility has undesirable and unrealistic implications. The goal of the present paper is to provide a theoretical foundation of linear utility, which is in accordance with recent empirical findings and which is well-suited for theoretical and empirical applications.

In the next section we will present the model and introduce our central condition, termed independence of common increments. In contrast to other conditions which have been employed to derive CPT, the condition of independence of common increments is rather simple and, therefore, well suited for empirical research. More precisely, with the help of independence of common increments one can empirically test the linearity of utility even if effects of rank-, reference-, and sign-dependence are involved.

In Section 3 we apply our model to portfolio selection and insurance demand. The various applications of DT mentioned above have shown that the linearity of utility often implies a pattern of behavior which has been termed "plunging" by Yaari (1987). For example when choosing between a safe and a risky asset a decision maker in DT never diversifies but invests either all money in the safe asset or everything in the risky asset.

Analogously, DT predicts for insurance demand that individuals either buy full coverage or no coverage while partial coverage is never optimal. Our results show that behavior under CPT with linear utility is more realistic since only a weakened variant of plunging is implied: In the portfolio selection problem an individual may also diversify, however if she diversifies, she always sticks to one specific portfolio composition. Analogously, an individual may demand partial coverage in insurance problems, however, if she demands partial coverage, she will always demand the same amount of coverage.

2 The Model

We consider a set of monetary outcomes identified with $I\!R$. A *lottery* is a finite probability distribution over the set of outcomes. It is represented by $P := (p_1, x_1; \ldots; p_n, x_n)$ meaning that probability p_j is assigned to outcome x_j, for $j = 1, \ldots, n$. With this notation we implicitly assume that outcomes are ranked in decreasing order, i.e., $x_1 \geqslant \cdots \geqslant x_n$. The probabilities p_j are nonnegative and sum to one. Without loss of generality, we assume that the status quo is given by zero. Therefore, we refer to positive outcomes as *gains* and to negative outcomes as *losses*.

We assume a preference relation $\succcurlyeq$ over the set of lotteries, where $\succ$ denotes strict preference and $\sim$ denotes indifference. Our goal is to find a functional that represents preferences over lotteries. This necessarily implies that $\succcurlyeq$ must be a *weak order*, i.e. $\succcurlyeq$ is *complete* ($P \succcurlyeq Q$ or $P \preccurlyeq Q$ for all lotteries P, Q) and transitive. Moreover, we assume that $\succcurlyeq$ satisfies *simple continuity*, i.e. for any lottery $(p_1, x_1; \ldots; p_n, x_n)$, the sets $\{(y_1, \ldots, y_n) : (p_1, y_1; \ldots; p_n, y_n) \succcurlyeq (p_1, x_1; \ldots; p_n, x_n)\}$ and $\{(y_1, \ldots, y_n) : (p_1, y_1; \ldots; p_n, y_n) \preccurlyeq (p_1, x_1; \ldots; p_n, x_n)\}$ are closed subsets of $I\!R^n$.

From Debreu (1954) we know that weak ordering and simple continuity guarantee the existence of a functional V that represents preference, i.e. $P \succcurlyeq Q \Leftrightarrow V(P) \geqslant V(Q)$ for all lotteries P and Q.

A particular functional form of V is CPT. As argued above our goal is to derive a model of CPT with linear utility which we will refer to as linear cumulative prospect theory (LCPT). Consider a lottery $P = (p_1, x_1; \ldots; p_n, x_n)$ such that

$$x_1 \geqslant \cdots \geqslant x_k \geqslant 0 > x_{k+1} \geqslant \cdots \geqslant x_n,$$

for some $k \in \{0, \ldots, n\}$. The value of P under LCPT is given by

$$LCPT(P) = \sum_{i=1}^{n} \pi_i U(x_i),$$

where the utility function U has the form

$$U(x) = \begin{cases} x, & \text{for all } x \geqslant 0, \\ \lambda x, & \text{with } \lambda > 0 \text{ for all } x \leqslant 0 \end{cases}$$

and the decision weights $\pi_i, i = 1, \ldots, n$, are defined as follows. There exist two probability weighting functions $w^+, w^- : [0,1] \to [0,1]$ with $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$ which generate the decision weights. This is done as follows:

$$\pi_i = \begin{cases} w^+(p_1 + \cdots + p_i) - w^+(p_1 + \cdots + p_{i-1}), & \text{if } i \leqslant k, \\ w^-(p_i + \cdots + p_n) - w^-(p_{i+1} + \cdots + p_n), & \text{if } i > k. \end{cases}$$

for each $i \in \{1, \ldots, n\}$. Therefore, for gains ($i \leqslant k$) the decision weight π_i represents a difference in transformed decumulative probabilities, whereas for losses ($i > k$) cumulative probabilities are transformed. The fact that $w^+(p)$ may differ from $1 - w^-(1-p)$, the dual of w^-, reflects sign-dependence. The parameter λ in the utility function is the loss aversion parameter. Loss aversion is characterized by $\lambda \geqslant 1$. Note that without sign-dependence LCPT would reduce to the dual theory of Yaari (1987) if $\lambda = 1$.

In general both weighting functions are assumed to be strictly increasing. This is necessary in order to guarantee consistency with stochastic dominance. The preference relation $\succcurlyeq$ satisfies *stochastic dominance* if $(p_1, x_1; \ldots; p_n, x_n) \succ (p_1, y_1; \ldots; p_n, y_n)$ whenever $x_j \geqslant y_j$ for all j and $x_j > y_j$ for at least one j with $p_j > 0$. Since the probabilities are fixed this condition comes down to monotonicity in outcomes.

So far we have considered only standard conditions (weak order, simple continuity, and stochastic dominance) which imply only the existence of a representing functional that is consistent with stochastic dominance. In order to derive a CPT functional further conditions have to be imposed. Because such conditions need to imply the separation of utility and decision weights and above that to imply sign-dependence, most conditions that are employed in the derivation of general CPT are rather complex. For example Luce and Fishburn (1991) use a condition called compound gamble and joint receipt, whereas Tversky and Kahneman (1992), Wakker and Tversky (1993), Chateauneuf and Wakker (1999), and Schmidt (2000) use sign-dependent and comonotonic tradeoff consistency. In contrast, Wakker and Zank (2001) use a generalization of traditional constant proportional risk aversion to nonpositive outcomes to derive a CPT model where utility is a power function. The latter result shows that if one is interested in a particular form for utility, much simpler axioms may be employed in order to characterize CPT.

In this paper we are also interested in a particular form for utility. Hence, we propose an alternative condition, termed independence of common increments, which is rather simple in its formulation but strong in its implications as it leads to the derivation of a piecewise linear utility function. In order to formulate the condition as weak and simple as possible, we consider only lotteries with equally likely outcomes. Formulations of preference conditions for equally likely outcomes have already been proposed by Ramsey (1931), Debreu (1959), Blackorby, Davidson, and Donaldson (1977), Chew and Epstein (1989), and Schmidt and Zank (2001a). An analysis of the implications of equally likely outcomes for dominance and independence rules is offered in Quiggin (1989).

The advantage of considering only equally likely outcomes is that people may perceive them easier. This is because in comparisons between such lotteries probabilities can be suppressed, thereby reducing the cognitive effort for the evaluation of the lotteries. Since probabilities and outcomes are measured on different scales, a tradeoff between them is rather difficult. As a result people may overweight probabilities in choice problems, but overweight outcomes in pricing problems. This pattern of behavior has been referred to as scale compatibility in the psychological literature (Tversky, Sattath, and Slovic 1988, Tversky, Slovic, and Kahneman 1990). Scale compatibility is nowadays the most prominent explanation of the preference reversal phenomenon first observed by Slovic and Lichtenstein (1968). Considering only equally likely outcomes should reduce such inconsistencies and is, therefore, particularly suited for empirical research.

To simplify notation, we identify a lottery $(\frac{1}{n}, x_1; \ldots; \frac{1}{n}, x_n)$ with the vector $(x_1, \ldots, x_n)$ where $(x_1 > x_2 > \cdots > x_n)$. *Independence of common increments* is defined as follows. For two lotteries $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ and $\alpha \in I\!R$ we have

$$\begin{aligned} (x_1, \ldots, x_i, \ldots, x_n) &\succcurlyeq (y_1, \ldots, y_i, \ldots, y_n) \Rightarrow \\ (x_1, \ldots, x_i + \alpha, \ldots, x_n) &\succcurlyeq (y_1, \ldots, y_i + \alpha, \ldots, y_n), \end{aligned}$$

whenever $x_i, x_i + \alpha, y_i, y_i + \alpha$ are of the same sign, that is either they are all gains or they are all losses. Implicitly in the above definition the ranking of outcomes should remain the same. Therefore, an additional constraint is imposed on the magnitude of the constant α.

Independence of common increments says that a common absolute change of an outcome of the same rank does not revert the preference between two lotteries as long as this change is small enough to affect neither the rank nor the sign of outcomes. For

α small enough, repeated application of this principle yields $(x_1 + \alpha, \ldots, x_n + \alpha) \succcurlyeq (y_1 + \alpha, \ldots, y_n + \alpha)$, indicating that it implies a weakened variant of the concept of constant absolute risk aversion (CARA). The restrictions on α mentioned above are crucial for the difference to traditional CARA.

Because in the definition of our independence condition the outcomes in the considered lotteries are all different there exist constants $\alpha_1 > \cdots > \alpha_n$ such that

$$\begin{aligned} (x_1, \ldots, x_i, \ldots, x_n) &\succcurlyeq (y_1, \ldots, y_i, \ldots, y_n) \Rightarrow \\ (x_1 + \alpha_1, \ldots, x_n + \alpha_n) &\succcurlyeq (y_1 + \alpha_1, \ldots, y_n + \alpha_n) \end{aligned}$$

if $x_k, y_k \geqslant 0 > x_{k+1}, y_{k+1}$. This shows that the property comes close to additivity on rank ordered sets. Such a condition has been used by Weymark (1981) to derive the generalized Gini welfare functions. Our condition here is weaker because of its reference- and sign-dependent nature. If we would drop the sign- and the rank-dependence restrictions we would get additivity on general sets. That and stochastic dominance are equivalent to the non-existence of a Dutch book, and the latter condition has been used by de Finetti (1931) to derive subjective expected utility with linear utility. This demonstrates that the only features that we have added to additivity are rank-dependence, reference-dependence, and sign-dependence, the basic characteristics of CPT.

It is easy to show that independence of common increments is a necessary condition for CPT with linear utility. The next theorem shows that the property is also sufficient in the presence of weak ordering, simple continuity and stochastic dominance.

Theorem 1 Assume a preference relation $\succcurlyeq$ on the set of lotteries. The following conditions are equivalent:

(i) $\succcurlyeq$ satisfies weak ordering, simple continuity, stochastic dominance, and independence of common increments.

(ii) LCPT holds with strictly increasing weighting functions.

If one of the above statements holds, then the weighting functions are uniquely determined and the utility function is a ratio scale, i.e. it is unique up to multiplication by a positive constant. □

This theorem – proved in the appendix – demonstrates that due to its simple formulation independence of common increments is a suitable concept for empirical research in the examination of linear utility in the presence of rank-, reference-, and sign-dependence.

3 Applications

3.1 Portfolio Selection

In the introduction we mentioned that DT implies behavior which can be characterized as all-or-nothing decision. The reason for this plunging behavior is the fact that the linearity of utility in DT produces corner solutions (Yaari 1987). The goal of the present section is to investigate whether plunging is also implied by LCPT. Therefore, we consider a simple problem of portfolio selection which can be analyzed graphically in a two-outcome diagram. In such a diagram it is assumed that there are only two possible states of the world, state A and state B. The consequences associated with these states are denoted by x_A and x_B, respectively. Moreover, it is assumed that the states occur with fixed probabilities, that is we can assume that $0 < p < 1$ is the probability of state A and $1 - p$ the probability of state B. In order to compare LCPT with DT we will first analyze DT as reference model. As mentioned above LCPT reduces to DT if reference- and sign-dependence can be ignored, i.e. the loss aversion factor λ is always equal to unity and the weighting functions for gains coincides with the dual one for losses (i.e.

$w^+(\cdot) = 1 - w^-(1-\cdot) = w(\cdot))$. Therefore, the utility of a two-outcome lottery P in DT is given by

$$DT(P) = \begin{cases} w(p)x_A + (1-w(p))x_B & \text{if } x_A \geqslant x_B \\ w(1-p)x_B + (1-w(1-p))x_A & \text{if } x_B > x_A. \end{cases}$$

We can now calculate the slope of indifference curves in a two-outcome diagram as:

$$\frac{dx_A}{dx_B}\Big|_{dDT(P)=0} = \begin{cases} -\frac{1-w(p)}{w(p)} & \text{if } x_A > x_B \\ -\frac{w(1-p)}{1-w(1-p)} & \text{if } x_B > x_A. \end{cases}$$

This equation shows that indifference curves are negatively sloped and piecewise linear. Note that strong risk aversion in DT implies that the weighting function is strictly convex (Chew, Karni, and Safra 1987, Yaari 1987), i.e. $w(p) + w(1-p) < 1$, which yields $1 - w(p) > w(1-p)$ and $w(p) < 1 - w(1-p)$. Hence, the indifference curves for $x_A > x_B$ are steeper than those for $x_A < x_B$, meaning that they have a kink along the 45^0-line as depicted in Figure 1. Moreover, the slope of the indifference curves is independent of income in the sense that indifference curves are parallel above the 45^0-line and parallel below the 45^0-line.

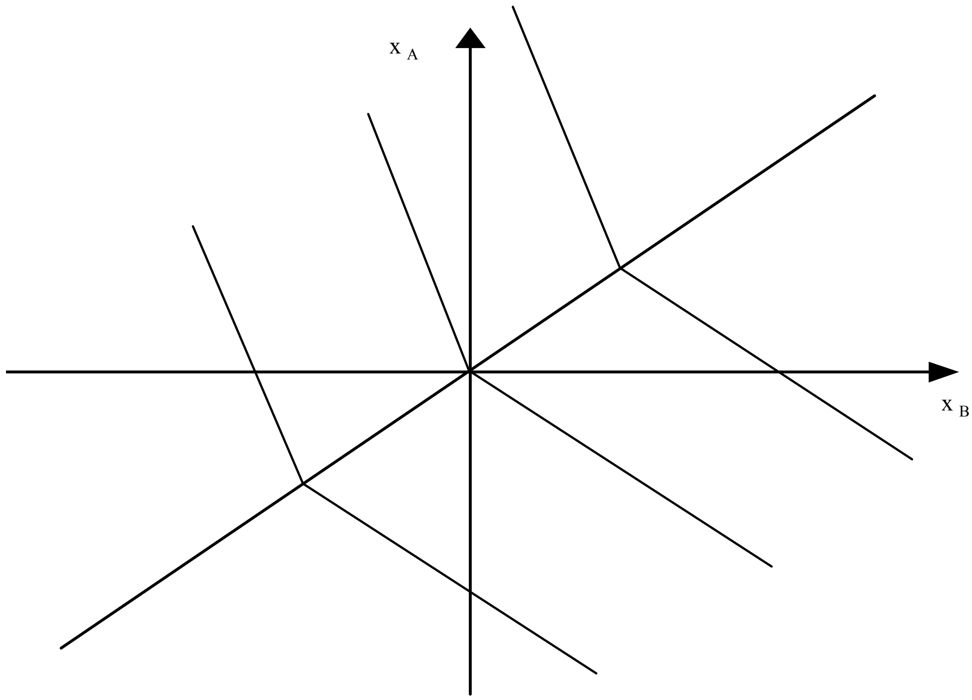

Figure 1

Let us now consider LCPT. For convenience, we exchange the function w^- for its dual $\tilde{w}^-$ defined by $\tilde{w}^-(q) := 1 - w^-(1-q)$ for $q \in [0,1]$, that is, we consider transformed decumulative probabilities not only for gains but also for losses. Then, the utility of a two-outcome lottery is given by

$$LCPT(P) = \begin{cases} w^+(p)x_A + (1 - w^+(p))x_B & \text{if } x_A \geqslant x_B \geqslant 0 \\ w^+(1-p)x_B + (1 - w^+(1-p))x_A & \text{if } x_B > x_A \geqslant 0 \\ \tilde{w}^-(p)\lambda x_A + (1 - \tilde{w}^-(p))\lambda x_B & \text{if } 0 \geqslant x_A \geqslant x_B \\ \tilde{w}^-(1-p)\lambda x_B + (1 - \tilde{w}^-(1-p))\lambda x_A & \text{if } 0 \geqslant x_B > x_A \\ w^+(p)x_A + (1 - \tilde{w}^-(p))\lambda x_B & \text{if } x_A > 0 > x_B \\ w^+(1-p)x_B + (1 - \tilde{w}^-(1-p)\lambda x_A & \text{if } x_B > 0 > x_A, \end{cases}$$

which implies the following slope of indifference curves:

$$\frac{dx_A}{dx_B}\Big|_{dLCPT(P)=0} = \begin{cases} -\frac{1-w^+(p)}{w^+(p)} & \text{if } x_A > x_B \geqslant 0 \\ -\frac{w^+(1-p)}{1-w^+(1-p)} & \text{if } x_B > x_A \geqslant 0 \\ -\frac{1-\tilde{w}^-(p)}{\tilde{w}^-(p)} & \text{if } 0 \geqslant x_A > x_B \\ -\frac{\tilde{w}^-(1-p)}{1-\tilde{w}^-(1-p)} & \text{if } 0 \geqslant x_B > x_A \\ -\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} & \text{if } x_A > 0 > x_B \\ -\frac{w^+(1-p)}{\lambda(1-\tilde{w}^-(1-p))} & \text{if } x_B > 0 > x_A. \end{cases}$$

In the following we assume that w^+ and $\tilde{w}^-$ are strictly convex and that for all $q \in [0,1]$ we have $\lambda > \frac{w^+(q)}{\tilde{w}^-(q)}$ as well as $\lambda > \frac{1-w^+(q)}{1-\tilde{w}^-(q)}$. If we consider only two-outcome lotteries, these assumptions are necessary and sufficient for the exhibition of strong risk aversion (Schmidt and Zank 2001b) and yield the pattern of indifference curves depicted in Figure 2.

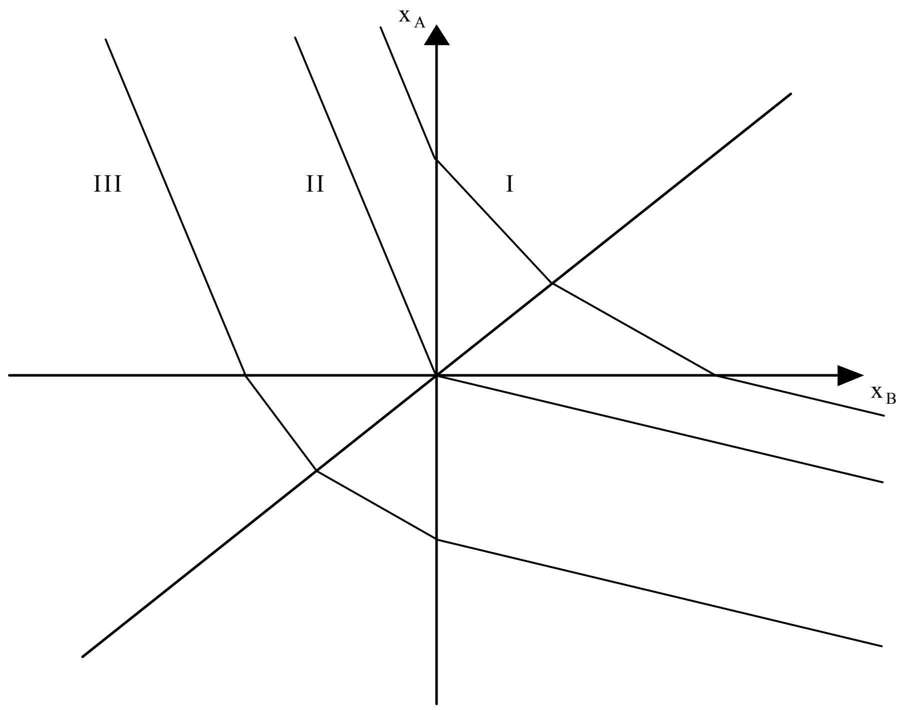

Figure 2: Indifference curves in LCPT

In this figure, three types of indifference curves are considered, curve a representing a strictly positive utility level, curve b representing a utility level of zero, and curve c

representing a strictly negative utility level. The shape of curve a becomes clear by observing that our assumptions on λ, w^+, and $\tilde{w}^-$ imply $\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} > \frac{1-w^+(p)}{w^+(p)} > \frac{w^+(1-p)}{1-w^+(1-p)} > \frac{w^+(1-p)}{\lambda(1-\tilde{w}^-(1-p))}$. Moreover, the shape of curve b is due to $\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} > \frac{w^+(1-p)}{\lambda(1-\tilde{w}^-(1-p))}$ while the shape of curve c is caused by $\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} > \frac{1-\tilde{w}^-(p)}{\tilde{w}^-(p)} > \frac{\tilde{w}^-(1-p)}{1-\tilde{w}^-(1-p)} > \frac{w^+(1-p)}{\lambda(1-\tilde{w}^-(1-p))}$. Compared to DT, indifference curves have additional kinks at both axes. Moreover, due to sign-dependence, indifference curves representing a positive utility level may have a different slope than those representing a negative utility level.

We will now investigate a simple portfolio selection problem already studied, among others, by Tobin (1958), Arrow (1965), and Cass and Stiglitz (1972) which has been applied to DT by Yaari (1987). Consider an individual who may invest a fixed positive amount $\bar{y}$ into two assets, one riskless asset with a fixed positive interest rate $r > 0$ and one risky asset. Suppose that the risky asset has a high return $z_A > r$ in state A and a negative return $z_B < 0$ in state B. It is well known that a risk averse expected utility maximizer would always invest some amount y_{EU} with $0 < y_{EU} < \bar{y}$ into the risky asset provided that the expected return, i.e. $pz_A + (1-p)z_B$, is strictly greater than r. On the other hand, DT implies an investment of y_{DT} in the risky asset given by

$$y_{DT} = \begin{cases} 0 & \text{if } \frac{z_A - r}{z_B - r} > -\frac{1-w(p)}{w(p)} \\ \text{any value in } [0, \bar{y}] & \text{if } \frac{z_A - r}{z_B - r} = -\frac{1-w(p)}{w(p)} \\ \bar{y} & \text{if } \frac{z_A - r}{z_B - r} < -\frac{1-w(p)}{w(p)}. \end{cases}$$

This behavior has been termed as plunging: The investor either invests everything safe or everything risky depending on the relative rate of return of the risky asset.

Let us now consider LCPT. This requires a choice of the status quo. Already Kahneman and Tversky (1979) argued that the status quo is in most cases given by the initial position. Since also most applications choose initial wealth as status quo we think it is most convincing to fix it at $\bar{y}$. Then the optimal investment y_{LCPT} in the risky asset is given by:

$$y_{LCPT} = \begin{cases} 0 & \text{if } \frac{z_A - r}{z_B - r} > -\frac{1-w^+(p)}{w^+(p)} \\ \text{any value in } [0, \frac{r\bar{y}}{r - z_B}] & \text{if } \frac{z_A - r}{z_B - r} = -\frac{1-w^+(p)}{w^+(p)} \\ \frac{r\bar{y}}{r - z_B} & \text{if } -\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} < \frac{z_A - r}{z_B - r} < -\frac{1-w^+(p)}{w^+(p)} \\ \text{any value in } [\frac{r\bar{y}}{r - z_B}, y] & \text{if } \frac{z_A - r}{z_B - r} = -\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} \\ \bar{y} & \text{if } \frac{z_A - r}{z_B - r} < -\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)}. \end{cases}$$

The validity of this equation can be easily checked by considering Figure 3. In case I the budget line is flatter than the indifference curve for $x_A > x_B > 0$, i.e. $\frac{z_A - r}{z_B - r} > -\frac{1-w^+(p)}{w^+(p)}$, which yields $y_{LCPT} = 0$ as optimum. In case II the budget line is steeper than the indifference curve for $x_A > x_B > 0$ but flatter than the indifference curve for $x_A > 0 > x_B$ which implies that $x_B = 0$ and, therefore, $y_{LCPT} = \frac{r\bar{y}}{r - z_B}$ is optimal. Finally, if the budget line is steeper than the indifference curve for $x_A > 0 > x_B$ we get $y_{LCPT} = \bar{y}$ as optimum (case III).

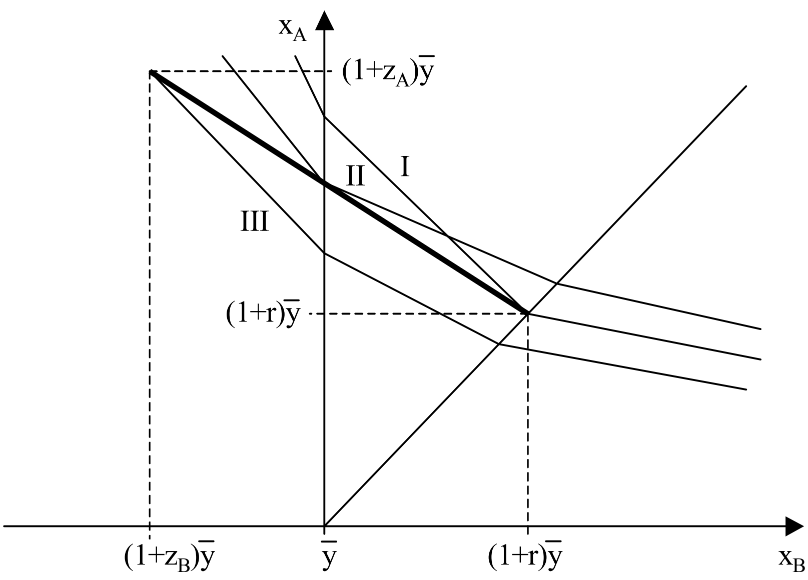

Figure 3: Portfolio selection in LCPT

Altogether the result shows that individuals in LCPT exhibit only a weak variant of plunging. In contrast to DT there is not only an all-or-nothing decision but there exist also relative returns of the risky asset for which diversification is optimal. However, if the individual diversifies, she or he always invests that amount in the risky asset for which the final wealth in the unfavorable state equals the status quo.

Note that the possibility of diversification in LCPT is an immediate consequence of reference dependence which implies the additional kink of indifference curves along both axes. Sign-dependence, on the other hand, allows for different investment behavior in the gain and in the loss domain. Note that in DT the optimal investment does only depend on $\frac{z_A - r}{z_B - r}$. In contrast, the optimal investment in LCPT for positive r can differ from the optimal investment for negative r (which ´may for instance be a result of holding money in the presence of inflation) even if $\frac{z_A - r}{z_B - r}$ remains constant.

3.2 Insurance Demand

In this section we will analyze demand for coinsurance with LCPT. Again we will only consider two possible states of the world in order to allow for a graphical analysis in the two-outcome diagram. Suppose the initial wealth of an individual is given by $\bar{x}_A$ in state A and by $\bar{x}_B = \bar{x}_A - L < \bar{x}_A$ in state be where L is some monetary loss which can be insured. More precisely, if the individual enters into an insurance contract, he or she has to pay a premium R with

$$R = \gamma C$$

and, therefore, receives a compensation C, $0 \leqslant C \leqslant L$, if state B occurs. An insurance premium is said to be fair if $\gamma = p$. With the possibility of insurance the final wealth distribution is given by

$$x_A = \bar{x}_A - \gamma C$$
$$x_B = \bar{x}_B - \gamma C + C = \bar{x}_B + (1-\gamma)C,$$

which implies that the slope of the budget line is given by $-\frac{\gamma}{1-\gamma}$.

The problem of the optimal choice of C has been studied in the DT framework by Doherty and Eeckhoudt (1995) and Schmidt (1996). They have shown that the demand for coinsurance C_{DT} is in DT given by

$$C_{DT} = \begin{cases} L & \text{if } -\frac{\gamma}{1-\gamma} > -\frac{1-w(p)}{w(p)} \\ \text{any value in } [0, L] & \text{if } -\frac{\gamma}{1-\gamma} = -\frac{1-w(p)}{w(p)} \\ 0 & \text{if } -\frac{\gamma}{1-\gamma} < -\frac{1-w(p)}{w(p)}. \end{cases}$$

This equation shows that individuals in DT also exhibit plunging for insurance demand since they either demand no coverage of full coverage while partial coverage is only optimal in the knife-edge case of $-\frac{\gamma}{1-\gamma} = -\frac{1-w(p)}{w(p)}$.

If we want analyze insurance demand with LCPT, again the reference point has to be fixed. There is not much literature on the choice of the reference point in the case of a random initial wealth. We think that it is most convincing to assume that the status quo q is somewhere in-between the two possible initial wealth levels, i.e. $x_A > q > x_B$. We further assume that $q < \bar{x}_A - \gamma L$, i.e. both outcomes are gains in the case of full insurance. Note that for $q = \bar{x}_A - \gamma L$ optimal insurance demand for DT and LCPT would coincide. However, for $q < \bar{x}_A - \gamma L$ insurance demand C_{LCPT} in the case of LCPT is characterized by

$$C_{LCPT} = \begin{cases} L & \text{if } -\frac{\gamma}{1-\gamma} > -\frac{1-w^+(p)}{w^+(p)} \\ \text{any value in } [\frac{q-\bar{x}_B}{1-\gamma}, L] & \text{if } -\frac{\gamma}{1-\gamma} = -\frac{1-w^+(p)}{w^+(p)} \\ \frac{q-\bar{x}_B}{1-\gamma} & \text{if } -\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} < -\frac{\gamma}{1-\gamma} < -\frac{1-w^+(p)}{w^+(p)} \\ \text{any value in } [0, \frac{q-\bar{x}_B}{1-\gamma}] & \text{if } -\frac{\gamma}{1-\gamma} = -\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)} \\ 0 & \text{if } -\frac{\gamma}{1-\gamma} < -\frac{\lambda(1-\tilde{w}^-(p))}{w^+(p)}. \end{cases}$$

The validity of this equation can be easily inferred from Figure 4. In order to find the optimal solution the slope of the budget line has to be compared with the slope of indifference curves. Apart from the slope of the budget line, $-\frac{\gamma}{1-\gamma}$, the problem is identical to the portfolio selection problem presented in the preceding section. Therefore, the argument will not be repeated here.

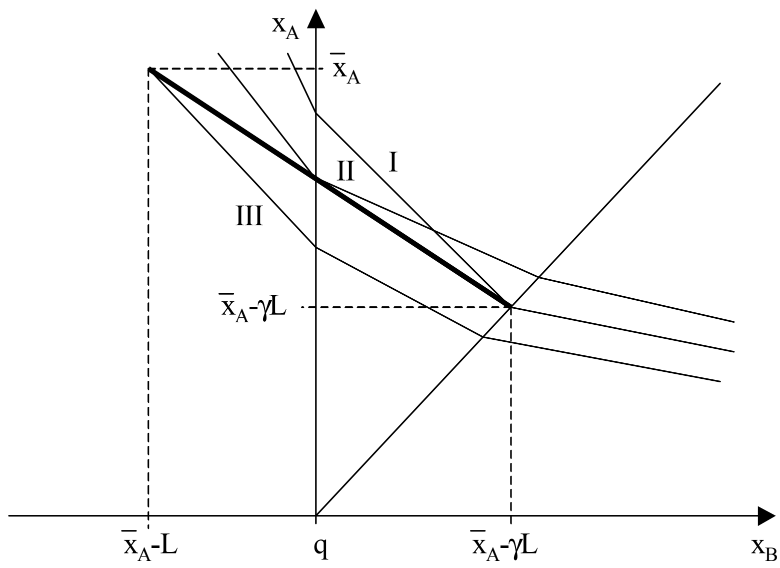

Figure 4: Insurance demand in LCPT

Also for insurance demand under LCPT individuals exhibit only a weak variant of plunging. In contrast to DT they choose not only between no and full coverage but there exist also values of γ for which partial coverage is optimal. However, if an individual demands partial coverage she or he always chooses that value of C for which final wealth in the worse state equals the status quo.

4 Appendix

Proof of Theorem 1: First we assume statement (ii) and derive statement (i). Let

LCPT hold for a preference relation $\succcurlyeq$ on the set of lotteries. Then, weak ordering follows from the fact that LCPT represents the preference on the set of lotteries. Because utility is continuous under LCPT, simple continuity is satisfied. Moreover, utility is strictly increasing as well as the weighting functions, which implies stochastic dominance. It remains to show that independence of common increments is satisfied. Consider two lotteries $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ such that $(x_1, \ldots, x_n) \succcurlyeq (y_1, \ldots, y_n)$. Let $i \in \{1, \ldots, n\}$ be arbitrary. Then

$$\begin{array}{rcl} (x_1, \ldots, x_i, \ldots, x_n) & \succcurlyeq & (y_1, \ldots, y_i, \ldots, y_n) \\ & \Leftrightarrow & \\ LCPT(x_1, \ldots, x_i, \ldots, x_n) & \geqslant & LCPT(y_1, \ldots, y_i, \ldots, y_n) \end{array}$$

We consider the case that x_i and y_i are gains. After substitution of LCPT we get

$$\sum_{j=1}^{i-1} \pi_j x_j + \pi_i x_i + \sum_{j=i+1}^{k} \pi_j x_j + \sum_{j=k+1}^{n} \pi_j \lambda x_j \geqslant \sum_{j=1}^{i-1} \pi_j y_j + \pi_i y_i + \sum_{j=i+1}^{k} \pi_j y_j + \sum_{j=k+1}^{n} \pi_j \lambda y_j.$$

Take any $\alpha \in I\!R$ such that $x_i, x_i + \alpha, y_i, y_i + \alpha$ are positive and such that both $x_{i-1} > x_i + \alpha > x_{i+1}$ and $y_{i-1} > y_i + \alpha > y_{i+1}$ hold. Then, the previous inequality implies

$$\sum_{j=1}^{i-1} \pi_j x_j + \pi_i (x_i + \alpha) + \sum_{j=i+1}^{k} \pi_j x_j + \sum_{j=k+1}^{n} \pi_j \lambda x_j \geqslant \sum_{j=1}^{i-1} \pi_j y_j + \pi_i (y_i + \alpha) + \sum_{j=i+1}^{k} \pi_j y_j + \sum_{j=k+1}^{n} \pi_j \lambda y_j.$$

Note that the choice of α is crucial. That the sign of the affected outcomes is the same ensures that the decision weights are generated by the same weighting function. Moreover, because the ranking of outcomes has not been altered the decision weights for the involved outcomes is the same. That is, the above equation can be written as

$$LCPT(x_1, \ldots, x_{i-1}, x_i + \alpha, x_{i+1}, \ldots, x_n) \geqslant LCPT(y_1, \ldots, y_{i-1}, y_i + \alpha, y_{i+1}, \ldots, y_n)$$

or equivalently

$$(x_1, \ldots, x_{i-1}, x_i + \alpha, x_{i+1}, \ldots, x_n) \succcurlyeq (y_1, \ldots, y_{i-1}, y_i + \alpha, y_{i+1}, \ldots, y_n).$$

The case where both x_i, and y_i are losses is similar. Hence, we have shown that independence of common increments is satisfied, which concludes the derivation of statement (i) from statement (ii).

Now we assume statement (i) and derive statement (ii). Let us briefly outline the main steps of the proof. First, we show that independence of common increments is satisfied on the set of lotteries with equally likely outcomes which may be equal. Then, for a fixed natural number n, we show that LCPT holds on the set of lotteries with n equally likely (and possible equal) outcomes. In a subsequent step we show that LCPT holds on the set of lotteries with rational probabilities. Then, LCPT can uniquely be extended to hold on the entire set of lotteries, which concludes the derivation of statement (ii). Finally, the uniqueness results in the theorem are derived.

For any natural number n let $L(n)$ denote the set of lotteries where outcomes are distinct and their probabilities are $1/n$. Hence,

$$L(n) := \{(\frac{1}{n}, x_1; \ldots ; \frac{1}{n}, x_n) | x_1 > \cdots > x_n\}.$$

We can define

$$\bar{L}(n) := \{(\frac{1}{n}, x_1; \ldots ; \frac{1}{n}, x_n) | x_1 \geqslant \cdots \geqslant x_n\}$$

as the set of lotteries where outcomes having probabilities $1/n$ are not necessarily distinct.

Independence of common increments is defined to hold only for lotteries from $L(n)$. However, in the presence of the remaining conditions, independence of common increments holds on $\bar{L}(n)$ as well, as the next lemma shows.

Lemma 2 Suppose that $\succcurlyeq$ satisfies weak ordering, simple continuity, stochastic dominance, and independence of common increments. Then $\succcurlyeq$ satisfies independence of common increments on $\bar{L}(n)$ for any natural number n.

Proof: Suppose that independence of common increments is not satisfied on $\bar{L}(n)$ for some n. Obviously, $n > 1$ by stochastic dominance and completeness. Then, there exists a pair of lotteries $(x_1, \ldots, x_n), (y_1, \ldots, y_n) \in \bar{L}(n)$ such that for some $i \in \{1, \ldots, n\}$ and some $\alpha \in I\!R$ we have a preference reversal

$$(x_1, \ldots, x_i, \ldots, x_n) \quad \succcurlyeq \quad (y_1, \ldots, y_i, \ldots, y_n)$$

$$\Rightarrow$$

$$(x_1, \ldots, x_i + \alpha, \ldots, x_n) \quad \prec \quad (y_1, \ldots, y_i + \alpha, \ldots, y_n)$$

although $x_i, x_i + \alpha, y_i, y_i + \alpha$ are of the same sign and $x_{i-1} > x_i + \alpha > x_{i+1}$ and $y_{i-1} > y_i + \alpha > y_{i+1}$ holds.

We write $x + 1_i \cdot \alpha$ instead of $(x_1, \ldots, x_i + \alpha, \ldots, x_n)$. By continuity there exists $\delta < 0$ such that

$$x + 1_i \cdot \alpha \prec y + 1_i \cdot \alpha + 1_n \cdot \delta.$$

By stochastic dominance we have

$$x \succ y + 1_n \cdot \delta.$$

We show that the last two preferences cannot occur jointly. Let j be the smallest index such that $x_j = x_{j+1}$. Then for $\varepsilon_j > 0$ small enough continuity implies

$$x + 1_i \cdot \alpha + 1_j \cdot \varepsilon_j \prec y + 1_i \cdot \alpha + 1_n \cdot \delta.$$

Also, stochastic dominance implies

$$x + 1_j \cdot \varepsilon_j \succ y + 1_n \cdot \delta.$$

By repeating this process for all such indices j for which $x_j = x_{j+1}$ we can distort the lottery x into a lottery $\tilde{x}$ with strictly rank-ordered outcomes such that

$$\tilde{x} + 1_i \cdot \alpha \prec y + 1_i \cdot \alpha + 1_n \cdot \delta$$

and

$$\tilde{x} \succ y + 1_n \cdot \delta.$$

Note that in $y + 1_n \cdot \delta$ outcomes may not be strictly rank-ordered. However, applying the same procedure as above to $y + 1_n \cdot \delta$ we can distort this lottery into one called $\tilde{y}$ where outcomes are strictly rank-ordered. We get

$$\tilde{x} + 1_i \cdot \alpha \prec \tilde{y} + 1_i \cdot \alpha$$

and

$$\tilde{x} \succ \tilde{y}.$$

This clearly contradicts independence of common increments. This concludes the proof of the lemma. $\triangle$

Let n be an arbitrary natural number. We derive LCPT on $\bar{L}(n,k) := \{x \in \bar{L}(n) | x_k \geqslant 0 \geqslant x_{k+1}\}$. First, note that weak ordering, stochastic dominance, continuity and independence of common increments are satisfied on this set of lotteries. Next, we show that on $\bar{L}(n,k)$ the preference relation satisfies comonotonic independence. *Comonotonic independence* says that replacing common outcomes by possibly different common outcomes does not affect the preference between two lotteries, i.e.,

$$x + 1_i \cdot (\alpha - x_i) \succcurlyeq y + 1_i \cdot (\alpha - y_i)$$

implies

$$x + 1_i \cdot (\beta - x_i) \succcurlyeq y + 1_i \cdot (\beta - y_i).$$

Comonotonic independence follows immediate from independence of common increments and the previous lemma.

Hence, $\succcurlyeq$ on $\bar{L}(n,k)$ satisfies weak ordering, stochastic dominance, simple continuity, and comonotonic independence. Comonotonic independence is stronger than the separability conditions proposed by Gorman (1968), which imply additive representability. This result has been used in Theorem C.6 of Chateauneuf and Wakker (1993). Accordingly, we conclude that there exists an additive representation $V^{n,k}(x) = \sum_{j=1}^{n} V_j^k$ for $\succcurlyeq$ on $\bar{L}(n,k)$. Moreover, on $\bar{L}(n,k)$ independence of common increments is satisfied, so that for $\alpha \in I\!R$

$$x \sim y \Rightarrow x + 1_i \cdot \alpha \sim y + 1_i \cdot \alpha,$$

whenever $x_i, x_i + \alpha, y_i, y_i + \alpha$ are of the same sign. Substitution of $V^{n,k}(x)$ gives

$$V_i^k(x_i) - V_i^k(y_i) = V_i^k(x_i + \alpha) - V_i^k(y_i + \alpha),$$

which implies linearity of V_i^k on $I\!R_+$ if $i \leqslant k$ and on $I\!R_-$ if $i \geqslant k+1$.

Let now $k \leqslant n-1$ be arbitrary fixed. Then, on the set $\bar{L}(n,k) \cap \bar{L}(n,k+1) = \{x \in \bar{L}(n,k) | x_{k+1} = 0\}$ both $V^{n,k}$ and $V^{n,k+1}$ represent the preference $\succcurlyeq$. Because the V_i^k's and

V_i^{k+1}'s are cardinal, we can choose $V_i^k = V_i^{k+1}$ for $i \neq k+1$ on the common domain. In doing this we have observed that

$$\begin{aligned} V_i^k &= V_i^{k+1} = V_i \text{ on } I\!R_+ \text{ if } i \leqslant k, \\ V_i^k &= V_i^{k+1} = V_i \text{ on } I\!R_- \text{ if } i > k+1, \end{aligned}$$

hence, these functions are independent of k. Moreover, for $i = k+1$ the domain of V_{k+1}^k is $I\!R_-$ and the domain of V_{k+1}^{k+1} is $I\!R_+$. So by defining

$$V_{k+1} = \begin{cases} V_{k+1}^{k+1}, & \text{on } I\!R_+, \\ V_{k+1}^k, & \text{on } I\!R_-, \end{cases}$$

the functions V_i for $i = 1, \ldots, n$ are extended to all of $I\!R$.

Let us summarize. We have derived an additive representation $V^n(x) = \sum_{j=1}^n V_j$ for $\succcurlyeq$ on $\bar{L}(n)$, with cardinal functions V_i which are linear on $I\!R_+$ and linear on $I\!R_-$. We can, therefore, fix the functions $V_i(0) = 0$ for all i and set $V(1, \ldots, 1) = 1$. Because of linearity of these functions on $I\!R_+$ we know that they are proportional to each other and in particular to their sum. Therefore we can derive decision weights π_i^+ $(= V_i(1))$ that are positive and sum to one such that on $I\!R_+$

$$V_i(x_i) = \pi_i^+ x_i.$$

The decision weights π_i^+ are differences in transformed decumulative probabilities: We can define the weighting function w_n^+ as

$$w_n^+(j/n) = \begin{cases} 0, & \text{if } j = 0, \\ \pi_1^+ + \cdots + \pi_j^+, & \text{if } j = 1, \ldots, n. \end{cases}$$

This way w_n^+ is uniquely defined, it is monotonic on its domain and satisfies $w_n^+(0) = 0$ and $w_n^+(1) = 1$. With the above definition we have $\pi_i^+ = w_n^+(i/n) - w_n^+((i-1)/n)$.

Similarly, there exist decision weights π_i^- $(= V_i(-1)/V(-1, \ldots, -1))$ that are positive and sum to one such that on $I\!R_-$

$$V_i(x_i) = \pi_i^- \lambda x_i.$$

The constant λ is positive and is defined as $-V(-1, \ldots, -1)$. Moreover, λ is independent of i due to the cardinality of the functions V_i. The decision weights π_i^- are generated by a weighting function

$$w_n^-(j/n) = \begin{cases} 0, & \text{if } j = 0, \\ \pi_1^- + \cdots + \pi_j^-, & \text{if } j = 1, \ldots, n. \end{cases}$$

Also, w_n^- is uniquely defined, it is monotonic on its domain and satisfies $w_n^-(0) = 0$ and $w_n^-(1) = 1$.

Therefore, we have derived LCPT on $\bar{L}(n)$ for a fixed natural number n.

Next, take n and m to be two distinct natural numbers. Then, on $\bar{L}(n)$ LCPT holds as well as on $\bar{L}(m)$, and on $\bar{L}(nm)$. Because $\bar{L}(n)$, and $\bar{L}(m)$ are subsets of $\bar{L}(nm)$ it follows that λ is independent of n (m, nm), and that the involved weighting functions agree on common domain. Thus, they are also independent of n (m, nm), and moreover, they are defined on the rational probabilities inheriting all properties. We have therefore derived LCPT on the entire set of lotteries with rational probabilities.

Let now $p := (p_1, \ldots, p_n)$ be any probability vector. We consider the set L_p of lotteries with fixed probability vector p. On L_p the preference relation $\succcurlyeq$ satisfies weak ordering stochastic dominance and simple continuity, and can be represented by a function $W_p : L_p \to I\!R$ that is unique up to strictly increasing continuous transformations.

Also, each lottery $(p_1, x_1; \ldots; p_n, x_n) \in L_p$ has a uniquely determined *certainty equivalent* $(1, CE(p_1, x_1; \ldots; p_n, x_n))$, i.e. the lottery that is indifferent to $(p_1, x_1; \ldots; p_n, x_n)$ and has sure outcome $CE(p_1, x_1; \ldots; p_n, x_n)$. The existence follows by continuity and uniqueness by stochastic dominance.

Therefore, we can choose W_p to agree with the LCPT functional above on the set of certain lotteries, implying $W_p(1, \alpha) = LCPT(1, \alpha)$ for all $\alpha \in I\!R$. Because for each lottery $(p_1, x_1; \ldots; p_n, x_n) \in L_p$ we have $W_p(p_1, x_1; \ldots; p_n, x_n) = W_p(1, CE(p_1, x_1; \ldots; p_n, x_n)) =$

$LCPT(1, CE(p_1, x_1; \ldots; p_n, x_n))$, we have found a functional, say V, that represents preference on the union of both sets of lotteries, namely on L_p and on the set of lotteries with rational probabilities. This functional V agrees with W_p on L_p and with the above LCPT-functional on the set of lotteries with rational probabilities.

Because the probability tuple p was arbitrary chosen, we conclude the existence of a general functional representing preference on the entire set of lotteries and which agrees with LCPT on the set of lotteries with rational probabilities. Again we call this functional V.

Note that the set of lotteries with all probabilities rational is a dense subset of the entire set of lotteries, and the latter can be viewed as a linear space. Also, V is a continuous functional that is linear on the dense subspace of lotteries with all probabilities rational. Therefore, V has a unique linear extension to the entire set of lotteries (Dunford and Schwartz, 1958). Hence, we conclude that LCPT holds on the entire set of lotteries.

Finally, let us recall that under general cumulative prospect theory the weighting functions are unique and utility is a ratio scale. Here we have derived CPT with a specific utility function. Utility is linear for gains and it is linear for losses. Hence, the uniqueness results apply here. This completes the proof of Theorem 1. □

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