The Order of Integration for Quarterly Macroeconomic Time Series: a Simple Testing Strategy *

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Abstract

Besides introducing a simple and intuitive definition for the order of integration of quarterly time series, this paper also presents a simple testing strategy to determine that order for the case of macroeconomic data.

A simulation study shows that much more attention should be devoted to the practical issue of selecting the maximum admissible order of integration. In fact, it is shown that when that order is too high, one may get (spurious) evidence for an excessive number of unit roots, resulting in an overdifferenced series.

Keywords: unit roots; seasonality; DF tests; HEGY tests.

JEL Classification: C22, C52

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1 Introduction

The seminal paper by Nelson and Plosser (1982) marked the beginning of a new research agenda in applied macroeconomics. Since then, it has become standard practice to determine the order of integration for macroeconomic time series. As described by Stock (1994), such an analysis can be useful for several distinct purposes: a) for simple data description; b) as a guide to subsequent univariate and multivariate modelling or inference; c) to guide the construction or testing of economic theories; d) for forecasting purposes.

Since the quarterly sampling frequency is capable of producing reasonable sample sizes based on relatively short spans of time, quarterly data sets have become increasingly appealing for all those purposes. Moreover, given the pitfalls arising from using seasonally adjusted data [see, e.g., Franses (1991) and Ghysels and Perron (1993)], it seems clearly preferable to resort to unadjusted time series. A further issue then arises: besides long-run unit roots, non-stationarity may be also present at the seasonal frequencies.

Thus, the purpose of this paper is twofold. First, we discuss at some length the existing definitions of order of integration for quarterly time series. As a sequel, a simple and intuitive definition will be proposed, which will be accompanied by the corresponding notation. Second, a sequential testing strategy will be introduced, one where much attention is devoted to determine the starting point in the sequence.

The outline of this paper is the following. Section 2 contains the discussion of the concept of order of integration for quarterly time series. Section 3 motivates the introduction of the new testing strategy based on power considerations. This section starts with an empirical example and it is subsequently generalized through Monte Carlo simulation experiments. Previous literature has emphasized the importance of initiating the testing sequence with the highest order entertained and working down, until a non-rejection of the null hypothesis is obtained (or the order zero is attained). This paper offers a complementary perspective, showing the importance of correctly choosing that upper bound to avoid obtaining evidence for too many unit roots. The proposed testing strategy is presented in Section 4. Section 5 concludes the paper.

2 Order of integration for quarterly time series

This section reviews the available definitions of order of integration that take into consideration that economic time series can be seasonally observed, and hence that stochastic non-stationarity can be seasonal. The definitions proposed in Osborn *et al.* (1988) [OCSB], Engle *et al.* (1989), Ghysels *et al.* (1999) and Franses and Taylor (2000) are recalled and discussed, and a simple but informative definition, strongly based on the one of Franses and Taylor, is introduced. To accompany this definition, a new and intuitive notation is proposed. As previously mentioned, the discussion is confined to the quarterly case, the most popular in empirical research.

Adapting the familiar definition of Engle and Granger (1987) to the modelling tradition of Box and Jenkins, OCSB introduced the following definition.

Definition 1 *(OCSB, p. 362) "A non-deterministic series x_t is said to be integrated of order (d, D), denoted $x_t \sim I(d, D)$, if the series has a stationary, invertible ARMA representation after one-period differencing d times and seasonally differencing D times."*

As in Engle and Granger (1987), the focus is on the stochastic component of the series. That is, decomposing the observed series, y_t, as

$$y_t = \mu_t + x_t, \quad t = 1, ..., T, \tag{1}$$

where μ_t denotes the linearly deterministic component (usually a linear function in t), the order of integration of y_t is given by that of the stochastic component. Moreover, as is well known, a failure in the specification of μ_t may imply erroneous inferences concerning the order of integration of x_t and hence of y_t [see, e.g., Campbell and Perron (1991) and Ghysels *et al.* (1994) (GLN)]. Thus, even when not stated explicitly, this framework is implicit in the next definitions.

Hence, according to OCSB, the quarterly time series y_t is said to be $I(d, D)$ when $\Delta^d \Delta_4^D x_t = (1 - L)^d (1 - L^4)^D x_t$, where L denotes the usual lag operator, admits a stationary and invertible ARMA representation.

The definition provided in Ghysels *et al.* (1999) [GOR], which we adapt to the quarterly case, while focusing also on the filter required to induce stationary, apparently disregards the long-run properties of the data.

Definition 2 *(GOR, pp. 1-2) The nonstationary quarterly stochastic process x_t is said to be seasonally integrated of order d, denoted $x_t \sim SI(d)$, if $\Delta_4^d x_t = (1 - L^4)^d x_t$ is a stationary, invertible ARMA process.*

Although useful in many circumstances, the previous definitions follow the Box and Jenkins tradition too closely and disregard the (now) familiar decomposition of the seasonal (or annual) differencing filter:

$$\begin{aligned}\Delta_4 &\equiv (1-L^4) = (1-L)S(L) \\ &= (1-L)(1+L+L^2+L^3) = (1-L)(1+L)(1+L^2) \qquad (2)\\ &= (1-L)(1+L)(1+iL)(1-iL), \qquad (3)\end{aligned}$$

where $i^2 = -1$ and $S(L)$ denotes the moving sum filter $(1+L+L^2+L^3)$ [1]. Clearly, this factorization highlights the presence of the familiar long-run (or non-seasonal or zero frequency unit) root 1 in the seasonal differencing filter. Proceeding in this direction, a definition based on Engle *et al.* (1989) [EGH] can be presented.

Definition 3 *(EGH, p. 49) The nonstationary stochastic process x_t is said to be seasonally integrated of orders d_0 and d_s, denoted $x_t \sim SI(d_0, d_s)$, if $\Delta^{d_0} S(L)^{d_s} x_t = (1-L)^{d_0} S(L)^{d_s} x_t$ is a stationary and invertible ARMA process.*

Then, for instance, if a series is considered as $I(1,1)$ according to the OCSB terminology, the EGH notation $SI(2,1)$ seems more useful as it makes clear the presence of two long-run unit roots. Moreover, if a series is considered as $SI(1)$ using the GOR terminology, the notation $SI(1,1)$ is also clearer, as it allows separating the long-run and the seasonal integradeness properties of the data. Aditionally, with non-stationarity arising exclusively through the seasonal frequencies, a series requiring only the filter $S(L)$ to achieve stationarity (i.e., $SI(0,1)$) cannot be considered as integrated neither according to OCSB nor to GOR.

For convenience, EGH assumed the same order of integration across all the frequencies (non-seasonal and seasonals) and under this assumption the previous definition is sufficient. However, as the complex roots occur in conjugate pairs and imply indistinguishable effects, it is clear from (3) that, for quarterly data, there are two distinct seasonal frequencies which may imply roots with unity modulus. Thus, a more comprehensive definition can be based also in EGH (see also Hylleberg *et al.* (1990) [HEGY]).

Definition 4 *(EGH, pp. 48-9) The stochastic process x_t is said to be integrated of order d at frequency θ if its spectrum $f(\omega)$ takes the form*

$$f(\omega) = c\,(\omega - \theta)^{-2d}$$

for ω near θ, denoted $x_t \sim I_\theta(d)$.

Thus, a quarterly time series can be integrated at frequencies 0, $\pi/2$ and π, these corresponding to the roots 1, $\pm i$ and -1 in (3): the complex roots are associated

[1] When $y_t = \log(Y_t)$ the (approximate) annual growth rate of Y_t, $\Delta_4 y_t$, equals the moving sum of the four (approximate) quarterly growth rates, Δy_t, i.e., $\Delta_4 y_t = S(L)\Delta y_t$.

with the annual cycle (i.e., one cycle per year) and the root -1 with the semi-annual cycle (i.e., two cycles per year). Obviously, this definition is the most flexible: a) to become stationary a series may require the application of only one of the filters $(1-L)$, $(1+L)$ and $(1+L^2)$; b) even when a combination of two of these filters is needed, the previous definitions are not comprehensive enough to handle the situation.

Although presented more than a decade ago and despite its flexibility, this last EGH definition has gained no popularity. This is probably due to the unfamiliarity of many economists with spectral analysis. For this reason, the definition based on Franses and Taylor (2000) [FT], which we adapt to the quarterly case, is likely to become more popular.

Definition 5 *(FT, pp. 251-2) The quarterly stochastic process x_t is said to be integrated of order d_j, $d_j \in \{0,1,2,...\}$, at frequency $\omega_j = \pi j/2$, $j = 0,1,2$, denoted $x_t \sim I_j(d_j)$, if $(1-L)^{d_0}(1+L^2)^{d_1}(1+L)^{d_2}x_t$ is a stationary and invertible ARMA process.*

Moreover, it should be noticed that this notation easily lends itself to an easy and intuitive interpretation, the periodicity (and not the frequency) of the seasonal unit roots appearing as a subscript. Then, for instance, if the series y_t requires the filter $(1-L)^2(1+L)$ to attain stationarity, it is said to be $I_0(2)$, $I_1(0)$ and $I_2(1)$.

Proceeding one step further, a slightly more simple definition, together with a less cumbersome notation, can be introduced.

Definition 6 *The quarterly stochastic process x_t is said to be integrated of orders d_0, d_1 and d_2, $d_j \in \{0,1,2,...\}$, denoted $x_t \sim I(d_0,d_1,d_2)$, if $(1-L)^{d_0}(1+L^2)^{d_1}(1+L)^{d_2}x_t$ is a stationary and invertible ARMA process.*

Obviously, the number of cycles within a year (that is, the frequencies) to which d_0, d_1 and d_2 refer are clear, allowing us to omit the corresponding subscripts in $I(.,.,.)$. Thus, for the previous example, the lighter notation $I(2,0,1)$ may be used.

3 Motivation for the new testing strategy

To motivate the introduction of a new testing strategy, this section presents some evidence on the likely inconsistencies and lack of power of existing ones. First an empirical example is extensively discussed and subsequently the analysis is extended to a general framework through Monte Carlo experimentation.

3.1 A tale of three econometricians

Suppose that three empirical researchers (RA, RB and RC) face the problem of determining the order of integration of y_t, y_t denoting the log of quarterly domestic demand for cement in the Portuguese economy, the sample extending from 71:1 to 99:4 (i.e., $T = 116$) [2].

Following the bulk of the literature and unaware of the work of Dickey and Pantula (1987) and Franses and Taylor (2000), i.e., ignoring that the real size of her procedure may exceed the nominal 5%, RA simply runs the most popular HEGY regression,

$$\Delta_4 y_t = \sum_{j=1}^{4} \alpha_j D_{jt} + \beta\, t + \sum_{j=1}^{4} \pi_j\, y_{j,t-1} + \sum_{i=1}^{k} \gamma_i \Delta_4\, y_{t-i} + \epsilon_t, \tag{4}$$

where D_{jt} ($j = 1, 2, 3, 4$) denote the usual seasonal dummies, y_{jt} represent the HEGY transformed variables, that is $y_{1,t} \equiv S(L) y_t \equiv (1 + L + L^2 + L^3) y_t$, $y_{2,t} \equiv -(1 - L + L^2 - L^3) y_t$, $y_{3,t} \equiv -L(1 - L^2) y_t$ and $y_{4,t} \equiv -(1 - L^2) y_t$, and ϵ_t is assumed to be iid$(0, \sigma^2)$. [For details on the HEGY procedure and besides HEGY, see also, *inter alia*, Engle *et al.* (1993), GLN, Smith and Taylor (1998) and Franses and Taylor (2000).] That is, RA implicitly assumes that the maximum order of integration for the series is $I(1, 1, 1)$. Alternatively, we could consider that RA is aware of the work of Dickey and Pantula (1987) and Franses and Taylor (2000) but that after a simple visual inspection of the graph of Δy_t she decided straightforwardly to discard the $I(2, 1, 1)$ hypothesis or any other hypothesis which is higher than $I(1, 1, 1)$.

To determine k, the order of lag augmentation in (4), RA uses the general-to-specific data dependent method recommended by Ng and Perron (1995), setting $k_{max} = 12$ [i.e., 3 years of lags, as in Beaulieu and Miron (1993)] and sequentially testing (down) the significance of the last lag — based on its t-ratio and using $\alpha = 0.05$ and the standard normal table — until a rejection is found (that is, the usual t-sig procedure). Checking also for residual autocorrelation at the selected lag, RA finds $k = 0$ and no symptoms of this problem using the Breusch-Godfrey (Lagrange multiplier) tests for orders 1 and 4, the corresponding p-values, BG1(p) and BG4(p) being 0.497 and 0.377, respectively. Then, the (non-augmented) HEGY

[2] This series, which plays a very important role in the analysis of the Portuguese economy, both to evaluate the business cycle and to estimate total investment on a quarterly basis, is available from the author on request. It should be also noted that it is relatively easy to find examples with similar features in the Portuguese economy. Moreover, a careful analysis of tables 1(b) and (2) in Osborn (1990) and 2 and 3 in McDougall (1995) reveals that some of the problems pointed here are also present in some series for the U.K. and the New Zealand economies.

regression yields $t_{\pi_1} = -2.02$, $t_{\pi_2} = -5.15$ and $F_{34} = 41.24$. [3] Thus, either using the 5% critical values (cvs) in HEGY or those in Franses and Hobijn (1997), RA finds evidence for the non-seasonal unit root only and classifies y_t as $I(1,0,0)$.

RB is aware of the work of Dickey and Pantula (1987) [but not of that of Franses and Taylor (2000)] and follows the strategy introduced in Osborn (1990) [and used also by McDougall (1995) and by Han and Thury (1997)], allowing the maximum order of integration to be $I(2,1,1)$. Hence, the analysis begins with the HEGY regression on Δy_t: [4]

$$\Delta_4 \Delta y_t = \sum_{j=1}^{4} \delta_j D_{jt} + \sum_{j=1}^{4} \pi_j \, \Delta y_{j,t-1} + \sum_{i=1}^{l} \phi_i \Delta_4 \Delta y_{t-i} + \epsilon_t. \tag{5}$$

Again setting $l_{max} = 12$ and using the sequential t-sig procedure, RB chooses $l = 9$ (and finds no signs of autocorrelation as BG1(p) = 0.372 and BG4(p)=0.458). Then, the HEGY test statistics assume the following values: $t_{\pi_1} = -2.85$, $t_{\pi_2} = -2.16$ and $F_{34} = 9.80$. Using the cvs reported in Franses and Hobijn (1997), RB considers t_{π_1} and F_{34} significant but t_{π_2} insignificant (at the 5% level). As in Osborn (1990) she then proceeds using equation (4) and finding the same values for the test statistics as RA.

If one follows Osborn (1990), according to the OCSB notation y_t is classified as $I(1,0)$ because the filter Δ_4 seems excessive, i. e., its imposition inducing non-invertibility. However, if seasonal non-stationarity is really an issue, an inconsistency clearly arises concerning the semi-annual unit root (tested through t_{π_2}): while equation (5) (with $l = 9$) provides evidence for its presence, equation (4) (with $k = 0$) does not.

In a sense, the problem is further complicated if, instead, RB uses the cvs in HEGY: as $t_{\pi_1} = -2.85$ is insignificant (at the 5% level) according to the HEGY tables, RB may consider equation (4) as inadequate to test for the seasonal unit roots and classifies y_t as $I(2,0,1)$.

Finally, RC is acquainted with the recent sequential testing strategy proposed by Franses and Taylor (2000), which extends the Dickey-Pantula procedure to testing

[3] As is well known, t_{π_1}, t_{π_2} and F_{34} are the HEGY test statistics designed for testing the roots 1, -1 and $\pm i$, respectively.

[4] Osborn (1990), McDougall (1995) and Han and Thury (1997) begin the analysis using the OCSB test for the $I(1,1)$ hypothesis (according to the OCSB notation). However, we assume (for example) that RB is aware of the restrictive parametrization of the OCSB test — see, e.g., Franses and Taylor (2000) and Lopes (2001) — and relies preferably on this regression to accomplish her purposes. Otherwise, additional inconsistencies could arise.

for non-seasonal and seasonal unit roots using only the HEGY statistics. Similarly to RB, RC does not resort to any graphical means and adopts a very cautious position [following the example of subsection 4.1 in Franses and Taylor (2000)], setting the upper bound for the order of integration at $I(2,2,2)$.

Using what seems to be a non-central step in the procedure of Franses and Taylor (2000), RC then proceeds determining the order of the autoregressive model for y_t, independently of the test regression. That is, denoting with $\phi_p(L)$ the pth-order autoregressive (approximating) polynomial for y_t,

$$\phi_p(L)y_t = \sum_{j=1}^{4} \theta_j D_{jt} + \psi\, t + \varepsilon_t,$$

RC then estimates p, $p_{min} \leq p \leq p_{max}$, using the t-sig general-to-specific method, where it should be noted that p_{min} is given by the minimum order implied by the initially assumed upper bound for $I(.,.,.)$. Hence, in this case $p_{min} = 8$. Setting $p_{max} = 16$, RC finds $\hat{p} = 8\,(= p_{min})$, therefore allowing him (her) to specify the first step auxiliary regression as

$$\Delta_4^2 y_t = \sum_{j=1}^{4} \kappa_j D_{jt} + \sum_{j=1}^{4} \pi_{j,2} \Delta_4 y_{j,t-1} + \varepsilon_{t,1}, \tag{6}$$

where no lag augmentation is present because the polynomial imposed on y_t is already of order 8. [5] Equation (6) produces $t_{\pi_{1,2}} = -4.65$, $t_{\pi_{2,2}} = -6.01$ and $F_{34,2} = 64.58$, but it is apparently plagued with residual autocorrelation problems as BG1(p) $= 0.000$ and BG4(p) $= 0.003$. If RC neglects this problem then she is able to reject double unit roots at all the frequencies and proceeds by specifying the second stage auxiliary regression as [see Franses and Taylor (2000) for details]:

$$\Delta_4^2 y_t = \sum_{j=1}^{4} \kappa_j' D_{jt} + \lambda t + \sum_{j=1}^{4} \pi_{j,2} \Delta_4 y_{j,t-1} + \sum_{j=1}^{4} \pi_{j,1} y_{j,t-1} + \varepsilon_{t,2}, \tag{7}$$

which yields $t_{\pi_{1,1}} = -2.54$, $t_{\pi_{2,1}} = -4.10$ and $F_{34,1} = 16.81$. As only $t_{\pi_{1,1}}$ is insignificant, RC might agree with RA in classifying y_t as $I(1,0,0)$.

However, it is doubtful that valid inferences can be drawn from (6) when $\varepsilon_{t,1}$ is autocorrelated, as the Breusch-Godfrey statistics so strongly suggest. In such circumstances, it seems more appropriate to use an augmented version of equation (6),

[5] Using the notation of Franses and Taylor (2000), $\phi_p(L)$ is approximated by $\alpha(L)\beta_{m_T}(L)$, where $\alpha(L)$ is the non-stationary polynomial and $\beta_{m_T}(L)$ is the stationary (augmenting) polynomial, whose order, m_T, is given by $\hat{p} - p_{min}$.

neglecting the lag truncation parameter initially estimated, the lag augmentation order again estimated with the $t - sig$ method. Following this route, RC now finds $t_{\pi_{1,2}} = -2.70$, which is insignificant (at the 5% level), though $t_{\pi_{2,2}} = -5.77$ and $F_{34,2} = 21.38$ are still highly significant.

Then, as the presence of the double non-seasonal unit root has not been rejected, the second stage auxiliary regression would become

$$\Delta_4^2 y_t = \sum_{j=1}^{4} \kappa_j^{"} D_{jt} + \sum_{j=2}^{4} \pi_{j,2} \Delta_4 y_{j,t-1} + \sum_{j=2}^{4} \pi_{j,1} \Delta y_{j,t-1} + \sum_{i=1}^{m} \psi_i \Delta_4^2 y_{t-i} \varepsilon'_{t,2}, \qquad (8)$$

instead of (7), and would allow RC to reject all the seasonal unit roots, as $t_{\pi_{2,1}} = -4.40$ and $F_{34,1} = 20.30$. However, a quite different picture now emerges, y_t being considered as $I(2,0,0)$.

Though the data generation process (DGP) for y_t is obviously unknown, the graphical representations (not presented here) clearly suggest that the upper bound for its order of integration should be set at $I(1,1,1)$. Then, it seems likely that the less "sophisticated" researcher (RA) is getting the most adequate results, RB and RC obtaining evidence for too many unit roots. What might be happening in these last two cases is that a too high upper bound is being considered, implying that the y_t series is heavily overdifferenced to produce the dependent variable of the auxiliary regressions. This overdifferencing effect, together with the t-sig procedure, then tends to lead to very long autoregressions. This in turn implies inefficient estimates and powerless test statistics, as is well documented in the literature [see, e.g., Hylleberg (1995)]. More simply, though protecting against potentially high significance levels, the test strategies pursued by RB and RC seem to be too much cautious, leading to serious power losses and hence to spurious evidence for (non-seasonal and seasonal) unit roots.

3.2 Monte Carlo analysis

To confirm the previous conjecture and to show that the results of the previous example did not occur merely by chance, some Monte Carlo experiments were performed using TSP 4.5, the purpose of these being to highlight the effect of the initially assumed upper bound on the power of the HEGY tests.

In the first set of experiments the purpose is to compare the power of the $I(1,1,1)$ and $I(2,1,1)$ HEGY regressions for testing seasonal unit roots when the DGP is $I(0,0,0)$. Table 1 reports the rejection frequencies for these experiments and it should be clear that the results of columns 2 and 6 cannot be compared. However,

Table 1. Power estimates when $y_t \sim I(0,0,0)$ and $T = 120$. The DGP is $y_t = \phi_4 y_{t-4} - D_{1t} + D_{2t} - D_{3t} + D_{4t} + \epsilon_t$, $|\phi_4| < 1$, $\epsilon_t \sim \text{nid}(0,1)$.

	assumed upper bound							
	$I(1,1,1)$				$I(2,1,1)$			
	roots			mean lag	roots			mean lag
ϕ_4	1	-1	$\pm i$	length	1	-1	$\pm i$	length
0.9	0.100	0.128	0.124	3.200	0.932	0.128	0.119	3.183
0.8	0.186	0.193	0.263	3.160	0.957	0.190	0.241	3.346
0.7	0.324	0.334	0.490	3.159	0.972	0.320	0.428	3.583
0.6	0.501	0.514	0.718	3.076	0.982	0.466	0.618	3.912
0.5	0.663	0.678	0.840	3.055	0.989	0.576	0.740	4.263
0.4	0.772	0.780	0.885	3.086	0.992	0.644	0.791	4.664
0.3	0.824	0.832	0.910	3.053	0.995	0.675	0.820	5.047
0.2	0.852	0.855	0.930	3.033	0.997	0.698	0.838	5.380
0.1	0.870	0.875	0.944	3.038	0.998	0.720	0.856	5.707
0.0	0.883	0.887	0.953	3.036	0.998	0.733	0.867	6.057

Notes: 1) when the assumed upper bound is $I(1,1,1)$ [$I(2,1,1)$] the test statistics are obtained through the auxiliary regression (4) [(5)] with the trend term omitted; 2) in both cases the order of lag augmentation is determined using the t-sig procedure, beginning with a lag order of 12 and with 5% level tests; 3) the number of replications is 10 000.

to the extent that none of the cells in column 6 is equal to 1, it is also clear that commencing with the dispensable higher order will lead to more frequent erroneous inferences on the presence of non-seasonal unit root(s).

The superior power performance of the HEGY $I(1,1,1)$ regression, generally associated with shorter autoregressions, is clear for almost all the parameter space. That is, except for the near-unit roots case where $\phi_4 = 0.9$ (implying roots with a modulus of 0.974), assuming the lower bound leads to significant power gains, in relative terms these ranging from 1.6% to 23.3% for the root -1 and from 4.2% to 16.2% for the complex roots. Furthermore, in absolute terms the power gap increases as ϕ_4 approaches zero, i.e., as the seasonal fluctuactions become more regular, this effect paralleling the widening gap for the mean selected lag length. In other words, assuming an overly high order of integration tends to lead to overparametrized autoregressions and, consequently, to a power deterioration of the HEGY statistics.

Table 2 tells a similar story for the power performance of the tests for two non-seasonal unit roots: now the DGP is $I(1,0,0)$ and the first step regressions for the cases $I(2,1,1)$ and $I(2,2,2)$ are used to produce the power estimates. Obviously, in

Table 2. Power estimates when $y_t \sim I(1,0,0))$ and $T = 120$. The DGP is $\Delta y_t = \phi_4 \Delta y_{t-4} - D_{1t} + D_{2t} - D_{3t} + D_{4t} + \epsilon_t$, $|\phi_4| < 1$, $\epsilon_t \sim \text{nid}(0,1)$.

	assumed upper bound for $I(.,.,.)$							
	$I(2,1,1)$				$I(2,2,2)$			
	roots			mean lag	roots			mean lag
ϕ_4	1	-1	$\pm i$	length	1	-1	$\pm i$	length
0.9	0.103	0.121	0.124	3.169	0.105	0.883	0.965	3.139
0.8	0.183	0.190	0.259	3.149	0.175	0.918	0.980	3.678
0.7	0.313	0.332	0.482	3.132	0.272	0.934	0.990	4.248
0.6	0.488	0.504	0.706	3.105	0.347	0.942	0.997	4.986
0.5	0.646	0.664	0.824	3.130	0.387	0.952	0.998	5.615
0.4	0.765	0.774	0.879	3.103	0.404	0.959	0.999	6.179
0.3	0.823	0.827	0.905	3.076	0.418	0.967	≈ 1.0	6.676
0.2	0.847	0.850	0.923	3.076	0.434	0.972	≈ 1.0	7.113
0.1	0.867	0.869	0.935	3.075	0.443	0.977	≈ 1.0	7.471
0.0	0.883	0.883	0.944	3.078	0.454	0.982	≈ 1.0	7.754

Notes: 1) when the assumed upper bound is $I(2,1,1)$ [$I(2,2,2)$] the test statistics are obtained through the auxiliary regression (5) [(6) augmented with lags of $\Delta_4^2 y_t$]; 2) in both cases the order of lag augmentation is determined using the t-sig procedure, beginning with a lag order of 12 and with 5% level tests; 3) the number of replications is 10 000; 4) "≈ 1.0" denotes an estimate lying in the interval [0.9995 , 0.9999].

this case the results in columns 3 and 4 are not comparable with those in columns 7 and 8, respectively. The superiority of the test regression corresponding to the lower order hypothesis is now even more clear, ranging from 4.6% to 96.9% in relative terms, the only exception concerning again the near-unit roots case. Moreover, the widening gap in power performance as $\phi_4 \to 0$ is now also more clearly related to the increasing overparametrization implied by the higher order hypothesis. Finally, it should be also mentioned that results similar to those presented arise when the t-sig procedure is initialized with a maximum lag length of 8.

In short, while assuming the possibility of too many long-run unit roots implies diminished power for testing seasonal unit roots, allowing too many of these in the testing strategy is liable to produce spurious evidence for the number of the former. Both effects occur as a consequence of the overparametrization implied by the corresponding HEGY auxiliary regressions when the t-sig method is used to estimate the lag truncation parameter.

4 The proposed testing strategy

This section presents our proposal of a simple testing strategy to determine the order of integration for quarterly (seasonally unadjusted) macroeconomic time series. Obviously, as demonstrated by Dickey and Pantula (1987) and Franses and Taylor (2000), it is crucial for controlling the size of the tests to begin at the highest order entertained and test down, until a non-rejection of the null hypothesis arises (or the order zero is attained). However, the previous evidence showed also the importance of correctly choosing this order to maximize the power of the HEGY statistics. Power considerations suggest also complementing these with (A)DF tests.

In what follows, an important assumption concerns the preliminary transformation of the data. In particular, except for those variables measured as rates, it is assumed that the popular practice of applying the (natural) logarithmic transformation is followed. Besides transforming exponential into linear growth, this transformation also allows removing the increasing seasonal variation which is sometimes observed in macroeconomic time series.

This assumption allows us to agree with Osborn (1990), setting the upper bound for the order of integration at most at $I(2,1,1)$. Several arguments may be invoked to support this claim. First, the log transformation has proved to be very effective in removing increasing seasonal variation, i.e., divergent seasonals, thereby allowing one to dismiss the $I(2,2,2)$ hypothesis as irrelevant [6]. Second, there is the well known issue of economic plausibility of seasonal unit roots [see, e.g., Osborn (1993), Hylleberg (1995) and Lopes (1999)]. That is, while the $I(1,1,1)$ model requires the presence of strong deterministic seasonality to be economically meaningful and to mimic really observed time series — the seasonal unit roots implying a slowly changing seasonal pattern —, the $I(2,2,2)$ model, containing also a set of seasonal dummies, implies a seasonal cycle changing too rapidly, both to be useful for describing a logged macro time series and to be economically interpretable. Finally, while there is empirical evidence that some time series require two ordinary differences to stationarize their long-run component, no such evidence exists to support the claim for multiple seasonal unit roots in logged macroeconomic time series

However, for most time series — and particularly for those corresponding to real variables —, the $I(2,1,1)$ hypothesis is too high to provide a sound starting point for a powerful test procedure. Thus, we recommend that a preliminary graphical

[6] It should be noted that as Franses and Taylor (2000) emphasize, their testing procedure, allowing for multiple unit roots at the seasonal frequencies, is recommended only when the log transformation is not used, so that the amplitude of the seasonal fluctuations grows with the level of the series. See also Franses and Koheler (1998).

analysis is performed to decide whether the $I(2,1,1)$ case is really worth considering. In many cases the plots of y_t and Δy_t provide useful insights on the admissible maximum order. However, if the seasonal fluctuations are so strong and irregular as to obscure the analysis, the plot of $S(L)y_t$ may be needed, the rather smooth behaviour of those series containing two non-seasonal unit roots then clearly emerging.

Deciding between the $I(2,1,1)$ and the $I(1,1,1)$ hypotheses may be carried out through a formal test too, namely in those cases where the graphical analysis is less clear-cut. Nevertheless, the recommendation is to use a DF and not an HEGY test, as the power of the later for testing for unit roots at the zero frequency may be low [see Franses (1996), p. 73]. That is, instead of using (5), the suggestion is to base inferences concerning the possibility of the double non-seasonal unit root on the t-ratio of α (t_α) in the regression

$$\Delta^2 y_t = \sum_{j=1}^{4} \phi_j D_{jt} + \alpha \Delta y_{t-1} + \sum_{i=1}^{q} \mu_i \Delta^2 y_{t-i} + \varepsilon_t. \tag{9}$$

At this stage, the lag length selection problem requires special care because non-stationary components at the seasonal frequencies may be present, in which case the convergence of t_α to the DF distribution (under the null hypothesis) is invalidated. This problem was first addressed by GLN, who showed that in such circumstances DF tests become seriously biased against the null when $q < 3$. However, GLN [see also Rodrigues (2000)] showed also that the usual DF procedure remains valid, even in the presence of unit roots at the seasonal frequencies, provided that $q \geq 3$.

Additional Monte Carlo experiments were carried out to investigate the performance of the t-sig procedure to handle this problem. The DGP considered in the experiments was provided by $\Delta\Delta_4 y_t = \sum_{j=1}^{4} \theta_j D_{jt} + \epsilon_t$, $\epsilon_t \sim \mathrm{nid}(0,1)$. Samples sized with $T = 80, 120$ and 160 observations were considered and each experiment consisted of 10 000 replications. Besides confirming the evidence in GLN when $q_{max} < 3$, the main conclusion one can draw is that the t-sig procedure based on 5% level tests performs very well in preserving the size of DF tests, even when seasonal unit roots are allowed in the DGP, provided that $q_{max} \geq 3$. [7] Moreover, contrarily to our prior expectations, it is not even necessary to increase the level of the t-tests on the μ_i parameters when the procedure attains the third lag, the 5% level being sufficient to ensure that the selected lag length is always at least equal to 3. Additionally, it should me mentioned that for the sample sizes we have considered — and abstract-

[7] For example, when $T = 120$ and $q_{max} = 8, 10$ and 12, the estimated real size (corresponding to a nominal 5%) was 4.93%, 5.17% and 5.23%. Additional results may be made available by the author.

ing from the possible presence of negative MA components —, the best results were obtained for $8 \leq q_{max} \leq 12$.

The final step of the procedure consists of running either regression (5) or (4), according to the results of the previous analysis. That is, only one of those auxiliary regressions is performed, thereby preventing the possibility of obtaining contradictory inferences concerning the presence of seasonal unit roots. Obviously, equation (5) is used only when t_α from (9) does not allow rejecting the double root 1, in which case its only purpose is for testing for seasonal unit roots. Otherwise, or when the graphical analysis is sufficient to exclude the $I(2,1,1)$ possibility, only equation (4) needs to be used [8]. In this last case, Franses [(1996), p. 73] recommends considering *"an additional step where there are no seasonal unit roots, i.e. a standard ADF test in a regression that includes seasonal dummies"*, the reason for this being previously provided. Two qualifications must be made, however: a) the (A)DF test for the single unity root may be used even when there is evidence on the presence of seasonal unit roots, the t-sig method (with, say, $k_{max} = 10$ when $T = 80, 120$ and 160) ensuring that the nominal size of the test is preserved; b) nevertheless, using both the HEGY and the DF regressions necessarily implies an overall size exceeding that of the individual tests.

5 Concluding remarks

This paper introduced a simple testing strategy to determine the order of integration for quarterly macroeconomic time series. Three features figure prominently in the proposed procedure: a) a careful analysis of the practical upper bound for the order of integration; b) resorting to graphical means of analysis to provide helpful assistance in that task; c) complementing the HEGY statistics with DF tests in spite of the seasonal framework.

Concerning a), the main lesson to draw from this paper is, indeed, that much more attention must be devoted to such a practical issue. While it is indisputable that one should commence the testing sequence from the highest admissible order, overly cautious testing sequences are liable to produce evidence for an excessive number of unit roots. Simple graphical analysis can be insightful on the starting point of the testing sequence, thereby allowing one to avoid such outcomes. Finally, adding to the well known recommendation of using (A)DF tests to get improved

[8] Obviously, to maximize the power for the nonseasonal unit root, the trend term should be omitted in those cases where its presence is not economically justifiable (as e.g., for interest rates, inflation and unemployment rates).

power performance when testing for long-run unit roots, it was also found that they can be validly used even when the true generation process is non-stationary at the seasonal frequencies. The only condition for this to hold is that the general-to-specific, data dependent, t-sig method, associated with the usual significance level and initial lag length, is used for selecting the lag truncation parameter.

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