A Theory of Self-Segregation

as a Response to Relative Deprivation

by

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Abstract

We model group formation as a response to relative deprivation. We employ a simple measure of relative deprivation. We show that the process of deprivation-induced self-selection into groups reaches a unique steady state. We study the social welfare implications of the deprivation-induced process of group formation and show that when individuals are left to pursue their betterment the resulting state tends to fall short of the best social outcome. We present several implications of the model including federalism and the demand for secession.

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1. Introduction

People who transact individually in markets also belong to groups. Both the outcome of the market exchange and the satisfaction arising from the group affiliation impinge on well-being. But how and why do groups form and dissolve? The pleasure or dismay that arise from group membership can be captured in a number of ways and relative position is an appealing measure. A plausible response to transacting in a market that confers an undesirable outcome is to transact in another market (when the latter exists and participation in it is feasible). Labor migration is an obvious example. Similarly, a reaction to a low relative position in a given group could be a change in group affiliation. What happens then when people who care about their relative position in a group have the option to react by either staying in the group or exiting from it?

We study this particular response in order to gain some insight into how groups form when individuals care about their relative position. To enable us to focus on essentials, we confine ourselves to an extremely stark environment. We hold the incomes of all the individuals fixed[1], we use a payoff function that is the negative of the sum of the income differences between one individual and others in his group who have higher incomes, we start with all individuals belonging to a single group (exit is not an option) and then allow the formation of a second group (exit is feasible), and we allow costless movement between groups. We derive stark and unexpected results. We find that the process converges to a unique steady state equilibrium of individuals across groups wherein clusters of income sub-groups exist in each group. There is no unique cut-off point above or below which individuals move. We also characterize and explore the social welfare repercussions of the process, and list several implications of our analysis.

Interestingly, the result of a non-uniform equilibrium distribution has already been derived, at least twice, in the very context that constitutes our primary example,

[1] When utility is derived both from absolute income and from relative income, and the utility function is additively separable, the difference in utilities across groups is reduced to the difference that arises from levels of relative income. Holding absolute incomes constant should not then be taken to imply that the individual does not care about his absolute income, and it enables us to study behavior that is purely due to considerations of relative income.

that is, migration. Stark (1993, chapter 12) studies migration under asymmetric information with signaling. Employers at destination do not know the skill levels of individual workers – they only know the skill distribution. Employers are assumed to pay all indistinguishable workers the same wage based on the average product of the group of workers. Employers at origin, however, know the skill levels of individual workers and pay them a wage based on their marginal product. When a signaling device that enables a worker's skill level to be completely identified exists, and when the cost of the device is moderate, the equilibrium distribution of the workers is such that the least skilled migrate without investing in the signaling device, the most skilled invest in the signaling device and migrate, and the medium skilled do not migrate. Banerjee and Newman (1998) derive a qualitatively similar result. They study a developing economy that consists of two sectors: a modern, high productivity sector in which people have poor information about each other, and a traditional, low productivity sector in which information is good. Since from time to time people need consumption loans that are subject to default, collateral is essential. The superior information available in the traditional sector enables lenders to better monitor borrowers there as opposed to the modern sector. The superior access to credit in the traditional sector conditional on the supply of collateral, and the higher productivity in the modern sector prompt migration from the traditional sector to the modern sector by the wealthiest and most productive workers, and by the poorest and least productive employees. The wealthy leave because they can finance consumption on their own and do not need loans; the most productive leave because they have much to gain; and the poorest and the least productive leave because they have nothing to lose – they cannot get a loan in either location.

A crucial assumption of both Stark's and Banerjee and Newman's models is that information is asymmetric. So far, no migration study has analytically generated an equilibrium distribution of three distinct groups under symmetric information, nor has a migration study analytically generated an equilibrium distribution of more than three groups. As the present study yields an equilibrium distribution of more than three groups, and it does so under symmetric information, our study contributes to the theory of migration. It also contributes to the large and growing literature on the theory of non-market, social interactions pioneered by Schelling (1971, 1972) and recently added to, among many others, by Stark (1999), Glaeser and Scheinkman (2000) who provide a useful synthesis, and Becker and Murphy (2001).

2. Preliminaries

You board a boat in Guilin in order to take a ride through the river Lijiang going by Guilin. You can stand either on the port side (left deck) or on the starboard side (right deck) admiring the beautiful cliffs hanging high above the two banks of the river. With you on the port side stand other passengers several of whom are taller than you. They block some of your view of the scenery. You notice that the starboard is empty so you move there, only to find that other passengers who were disturbed by taller passengers have also moved to the starboard side. You find yourself grouped with others who block your view, which prompts you, as well as some other passengers, to return to the port side. And so on. Do these shifts come to a halt? Is so, what will the steady-state distribution of passengers between the two decks look like? Will the steady-state distribution confer the best possible social viewing arrangement?

Incomes in the small region R where you live are fully used for visible consumption purposes. Any income (consumption) in your region higher than yours induces discomfort – it makes you feel relatively deprived. Another region, R', identical in all respects to your region except that initially it is unpopulated, opens up and offers the possibility that you, and for that matter anyone else, can costlessly move to R'. Who moves and who stays? Will all those who move to R' stay in R'? Will some return? And will some of those who return move once more? Will a steady-state distribution of the population across the two regions emerge? If so, will the steady-state distribution be unique? And at the steady-state distribution, will the aggregate deprivation of the population be lower than the initial aggregate deprivation? Will it be minimal?

The idea that discontent can arise not only from having a low wage but also from having a wage that is lower than that of others and that an unfavorable comparison could induce a departure for work elsewhere where wages are higher, *without* changing the set of individuals with whom comparisons are made, was taken up in the literature on relative deprivation and migration. Earlier studies include Stark (1984), Stark and Yitzhaki (1988), and Stark and Taylor (1989,1991). The key idea of this body of work is that a comparison of the income of *i* (an individual, a household, a family) with the incomes of others who are richer in *i*'s reference group results in *i*'s feeling of relative deprivation. The associated negative utility impinges on migration behavior. Our empirical work suggests that relative deprivation is a significant explanatory variable of migration behavior.[2]

Yet a second response could be to sever one's ties with the offensive set; association with another set, without a change in one's income, could also dampen relative deprivation. This reaction is the subject of the present paper.[3] The main questions are: Does the process of group formation in response to relative deprivation reach a steady state (wherein all moves cease and no one is able to reduce his relative deprivation through further moves)? Does the process of group formation by individuals in response to their aversion to relative deprivation lower societal relative deprivation? Does it minimize societal relative deprivation? We study the relationship between relative deprivation and group formation by employing a simple measure of relative deprivation: the relative deprivation of an individual whose income is x_j and whose reference group consists of n individuals whose incomes are $x_1,\cdots,x_n$ is defined as $D(x_j)=\sum_{x_i>x_j}(x_i-x_j)$ and $D(x_j)=0$ if $x_j \geq x_i$ for $i=1, 2, \cdots, n$. The linearity of this measure implies that regardless of their distribution, all units of income in excess of one's own are equally distressing.

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[2] The twin ideas that people compare themselves with others in their reference group, and that the outcome of the comparison impinges on their wellbeing and behavior, have been pursued in the economics literature quite extensively. Thus, it has been argued that given the set of individuals with whom comparisons are made, an unfavorable comparison could induce harder work. This idea is captured and developed in the literature on performance incentives in career games and other contests. (Early studies include Lazear and Rosen (1981), Rosen (1986), and Stark (1990).) Loewenstein, Thompson, and Bazerman (1989) provide evidence that individuals strongly dislike being in an income distribution in which "comparison persons" earn more. Clark and Oswald (1996) present evidence that "comparison incomes" have a significant negative impact on overall job satisfaction.

[3] Holding income constant (as if the individual were born with an income) enables us to study group selection behavior that is purely due to relative deprivation. The present paper can thus be considered as the dual of earlier work: while in past work the reference group was held constant and migration with a gain in income served to reduce relative deprivation within a given reference group, the present paper holds income constant and relative deprivation is reduced through a substitution of reference groups.

We embark on the study of the relationship between deprivation and group formation by considering an example. We list several results suggested by the example. We next develop a general approach that we use to substantiate results suggested by the example and to derive additional results. We address the issues of convergence of the group-formation process to a steady state, uniqueness of the steady state, and social welfare. In conclusion, we offer several conjectures and reflect on issues that pertain to the distaste for relative deprivation.

3. Self-Segregation

3.1 An Example

Define the deprivation of an individual whose income is x_j as $D(x_j)$. Suppose there are two groups, A and B, and that an individual's deprivation arises only from comparisons with other individuals in his group; nothing else matters. We abstract from the intrinsic value of x. However, this is of no consequence whatsoever since x is retained (the individual's income is held constant) across groups. We are thus able to study group-formation behavior that is purely due to deprivation. The individual prefers to be affiliated with the group in which his deprivation is lower. When equally deprived (a tie), the individual does not change groups. The individual cannot take into account the fact that other individuals behave in a similar fashion. However, the individual's payoff, or utility, depends on the actions of all other individuals whose incomes are higher than his. A key feature of this situation is that tomorrow's group-selection behavior of every individual is his best reply to today's selection actions of other individuals. What will the group-selection path emanating from the difference equation above, and the associated behavior, look like? What will be the steady-state allocation of individuals across the two groups?

In this section, we examine a simple case in which there are ten individuals and individual i receives an income of i, $i = 1, \cdots, 10$. Suppose that initially all individuals $1, \cdots, 10$ are in group A. Group B just comes into existence. (For example, A can be a village, B - a city; A can be a region or a country, B - another region or country; and so on. In cases such as these we assume that the individual does not care at all about the regions themselves and that moving from one region to another is costless.) Measuring time discretely, we will observe the following series of migratory moves. In period 1, all individuals except 10 move from *A* to *B* because the deprivation of individual 10 is zero, while the deprivation of all other individuals is strictly positive. In period 2, individuals 1 through 6 return from *B* to *A* because every individual in region *B*, except 9, 8, and 7, is more deprived in *B* than in *A*. When an individual cannot factor in the contemporaneous response of other individuals, his decision is made under the assumption of no group substitution by these individuals. In period 3, individual 1 prefers to move from *A* to *B* rather than be in *A*, and the process comes to a halt. What seems to be particularly interesting about this example is that after three periods, a steady state is reached such that the 10th and 6th through 2nd individuals are in region *A*, while the 9th through 7th and 1st individuals are in group *B*. Figure 1 below diagrammatically illustrates this example.

Period 0		Period 1		Period 2		Period 3	
Region *A*	Region *B*	Region *A*	Region *B*	Region *A*	Region *B*	Region *A*	Region *B*
10		10		10		10	
9			9		9		9
8			8		8		8
7			7		7		7
6			6	6		6	
5			5	5		5	
4			4	4		4	
3			3	3		3	
2			2	2		2	
1			1	1			1

Figure 1. The Group-Formation Process and the Steady-State Distribution

What can be learned from this simple example? First, a well-defined rule is in place, which enables us to predict group affiliation and steady-state distribution across groups. Second, until a steady state is reached, a change in group affiliation by any individual n is associated with a change in group affiliation by all individuals $i = 1,2,\cdots,n-1$. Third, the number of individuals changing affiliation in a period is declining in the (rank) order of the period. Fourth, the number of inter-group moves by individuals never rises in their income; individuals with low incomes change affiliations at least as many times as individuals with higher incomes. Fifth, the deprivation motive leads to a stratification steady-state distribution where clusters of income groups exist in each region rather than having a unique cut-off point above or below which individuals move.

In the following section, we present an analytical framework that facilitates a rigorous examination of several issues pertaining to deprivation and group formation.

3.2 Deprivation and Self-Segregation: An Analytical Framework

For the sake of concreteness and ease of reference we refer in this section to regions and to migration. The general setting, however, is of groups and group selection.

Consider two regions A and B. There are n individuals whose incomes are $x_1,\cdots,x_n$, where $x_1 \leq \cdots \leq x_n$ with at least one inequality holding strictly. In any period $t = 0,1,2,\cdots$, each individual chooses between two actions: to migrate or to stay. We introduce the following notations: $x' = (x_n,\cdots,x_1)_{1xn}$, $e_i' = (\underbrace{1,\cdots,1}_{i},0,\cdots,0)_{1xn}$, $e_0' = (0,\cdots,0)_{1xn}$. In addition, we denote the distribution of individuals at period t in region l, $l = A,B$, by $R_l(t)' = (r_n(t),\cdots,r_1(t))_{1xn}$ where $r_i(t) = 1$ if the ith individual is in region l at period t, and where $r_i(t) = 0$ otherwise. Assuming that initially all n individuals are in region A, we have that $R_A(0) = e_n$ and $R_B(0) = e_0$.

At period t, the ith individual's deprivation is

(1) $$D_i^l(t) = (x - x_i \cdot e_n)' \cdot diag(R_l(t)) \cdot e_{n-i}$$

if the individual is in region l, $l = A,B$, where $diag(R_l(t))$ is the nxn diagonal matrix with the vector $R_l(t)$ as its main diagonal. Since $R_A(t) + R_B(t) = e_n$ at any period t, we have that

$$D_i = (x - x_i \cdot e_n)' \cdot diag(e_n) \cdot e_{n-i} = (x - x_i \cdot e_n)' \cdot (diag(R_A(t)) + diag(R_B(t))) \cdot e_{n-i}.$$

Note that

(2) $$D_i = (x - x_i \cdot e_n)' \cdot diag(e_n) \cdot e_{n-i}$$

is the maximal deprivation that individual i could have. It follows from (1) that

(3) $D_i^A(t) + D_i^B(t) = D_i$.

Let m_i be the number of migratory moves by individual i. We then have the following:

Proposition 1: Given the above setup, $m_i \le n - i$ for $i = 1, \cdots, n$.

Proof: By (2), $D_n = 0$ and $D_{n-1} = x_n - x_{n-1} \ge 0$, and hence $m_n = 0$ and $m_{n-1} \le 1$. Assume that $m_{i+1} \le n - i - 1$. Now consider individual i's action. By (3), $D_i^A(m_{i+1}) + D_i^B(m_{i+1}) = D_i$. Note that all individuals with incomes higher than i have "settled down." Thus, in the cases where $D_i^A(m_{i+1}) < D_i^B(m_{i+1})$ and individual i is in region B or $D_i^A(m_{i+1}) > D_i^B(m_{i+1})$ and individual i is in region A, the individual migrates one more time and then "settles down" for ever; in all other cases, individual i will not migrate any more. This yields $m_i \le m_{i+1} + 1$. Since $m_{i+1} \le n - i - 1$ by the induction assumption, we obtain the claim of the proposition. Q.E.D.

Proposition 1 asserts that the process of group formation in response to deprivation comes to a halt. Is the resulting steady state unique?

3.3 Uniqueness

Proposition 2: Under the assumption that all individuals are in region A initially and the individual does not migrate when equally deprived (a tie), the process of migration in response to deprivation reaches a unique steady state.

Proof: Proposition 1 establishes the existence of a steady state for the process of migration in response to deprivation. We now prove the uniqueness of the steady state by way of contradiction. Suppose that there are two steady-state distributions $(\overline{R}_A(\bar{t}), \overline{R}_B(\bar{t}))$ and $(\widetilde{R}_A(\tilde{t}), \widetilde{R}_B(\tilde{t}))$. Note that $\overline{R}_A(\bar{t}) + \overline{R}_B(\bar{t}) = e_n = \widetilde{R}_A(\tilde{t}) + \widetilde{R}_B(\tilde{t})$. Without loss of generality, we assume that $\overline{R}_A(\bar{t}) \neq \widetilde{R}_A(\widetilde{t})$. Using our notation, we have that $\overline{R}_A(\bar{t})' = (\bar{r}_n(\bar{t}),\cdots,\bar{r}_1(\bar{t}))_{1xn}$ and $\widetilde{R}_A(\widetilde{t})' = (\widetilde{r}_n(\widetilde{t}),\cdots,\widetilde{r}_1(\widetilde{t}))_{1xn}$. Since $\overline{R}_A(\bar{t}) \neq \widetilde{R}_A(\widetilde{t})$, we let $j = Max\{k | \bar{r}_k(\bar{t}) \neq \widetilde{r}_k(\widetilde{t}), 1 \leq k \leq n\}$. Furthermore, without loss of generality, we assume that $\bar{r}_j(\bar{t}) = 1$. Thus, in the steady-state distribution $(\overline{R}_A(\bar{t}), \overline{R}_B(\bar{t}))$, individual j is in region A and hence $\overline{D}_j^A(\bar{t}) \leq \overline{D}_j^B(\bar{t})$ where, by (1), $\overline{D}_j^l(\bar{t}) = (x - x_j \cdot e_n)' \cdot diag(\overline{R}_l(\bar{t})) \cdot e_{n-j}$, $l = A, B$. By the definition of j, we have that $diag(\overline{R}_A(\bar{t})) \cdot e_{n-j} = diag(\widetilde{R}_A(\widetilde{t})) \cdot e_{n-j}$ and hence $\overline{D}_j^A(\bar{t}) = \widetilde{D}_j^A(\widetilde{t})$ where, by (1), $\widetilde{D}_j^A(\widetilde{t}) = (x - x_j \cdot e_n)' \cdot diag(\widetilde{R}_A(\widetilde{t})) \cdot e_{n-j}$. This implies that $\overline{D}_j^B(\bar{t}) = \widetilde{D}_j^B(\widetilde{t})$ where $\widetilde{D}_j^B(\widetilde{t}) = (x - x_j \cdot e_n)' \cdot diag(\widetilde{R}_B(\widetilde{t})) \cdot e_{n-j}$ by (1). Therefore, $\widetilde{r}_j(\widetilde{t}) = \bar{r}_j(\bar{t}) = 1$, which contradicts the definition of j. Q.E.D.

Several comments are in order. First, we note that the example presented in Section 2.1 constitutes a special case of our general framework wherein $(x_1,\cdots,x_n) = (1,\cdots,n)$, $x' = (n,\cdots,1)_{1xn}$, and x_i in (1) and in D_i is replaced by i. In this special case the maximum deprivation that an individual can have (as follows from (2)) is $D_i = \frac{1}{2}(n-i)(n-i+1)$.

Second, suppose we retain all the assumptions of our general approach except that when equally deprived in *A* and *B*, the individual prefers *A* to *B* (an infinitesimal home preference). Propositions 1 and 2 carry through, even though the specific steady state reached in this case may well differ from the specific steady state reached under the original assumption that when equally deprived (a tie) the individual does not migrate. Looking again at the example of Section 3.1 we will have the sequence shown in Figure 2. Interestingly, in the case of $(x_1,\cdots,x_n) = (1,\cdots,n)$ and an infinitesimal home preference, the number of periods it takes to reach the steady state is equal to the number of complete pairs in *n*, and the number of individuals who end up locating in A is $\frac{n}{2}$ when $n = 2m$, $\frac{n-1}{2}$ when $n = 4m - 1$ or $\frac{n+1}{2}$ when $n = 4m - 3$, where m is a positive integer.

Period 0

Region A	Region B
10	
9	
8	
7	
6	
5	
4	
3	
2	
1	

Period 1

Region A	Region B
10	
	9
	8
	7
	6
	5
	4
	3
	2
	1

Period 2

Region A	Region B
10	
	9
	8
7	
6	
5	
4	
3	
2	
1	

Period 3

Region A	Region B
10	
	9
	8
7	
6	
	5
	4
	3
	2
	1

Period 4

Region A	Region B
10	
	9
	8
7	
6	
	5
	4
3	
2	
1	

Period 5

Region A	Region B
10	
	9
	8
7	
6	
	5
	4
3	
2	
	1

Figure 2. The Migration Process and the Steady-State Distribution with an Infinitesimal Home Preference

Third, changing the incomes of all individuals by the same factor will have no effect on the pattern of migration. This homogeneity of degree zero property can be expected; when the payoff functions are linear in income differences, populations with income distributions that are linear transformations of each other should display the same migration behavior. Using our analytical framework this property can be readily stated and proved.

Corollary: Given the above setup and a real number $\alpha > 0$, the *i*th individuals in the two populations $P = \{x_1, \cdots, x_n\}$ and $P_\alpha = \{\alpha \cdot x_1, \cdots, \alpha \cdot x_n\}$ have the same number of migratory moves, where $x_1 \leq \cdots \leq x_n$.

Proof: Let m_i and m_i^α be the numbers of migratory moves by the *i*th individuals in the two populations $P = \{x_1, \cdots, x_n\}$ and $P_\alpha = \{\alpha \cdot x_1, \cdots, \alpha \cdot x_n\}$, respectively. One can easily see that $m_n = m_n^\alpha = 0$ and $m_{n-1} = m_{n-1}^\alpha \leq 1$. Assume that $m_{n-j} = m_{n-j}^\alpha$, for $j = 0, 1, \cdots, n-i-1$. This implies that $diag(R_l(m_{i+1}^\alpha)) \cdot e_{n-i} = diag(R_l(m_{i+1})) \cdot e_{n-i}$, where $l = A, B$. Then, by (1), we have that

$$
\begin{aligned}
D_i^l(m_{i+1}^\alpha) &= (\alpha \cdot x - \alpha \cdot x_i \cdot e_n)' \cdot diag(R_l(m_{i+1}^\alpha)) \cdot e_{n-i} \\
&= \alpha \cdot (x - x_i \cdot e_n)' \cdot diag(R_l(m_{i+1})) \cdot e_{n-i} \\
&= \alpha \cdot D_i^l(m_{i+1}),
\end{aligned} \tag{4}
$$

where $l = A, B$. Recall that α is a positive real number. The implication of (4) is that, in choosing to migrate or to stay, the *i*th individuals in the two populations $P = \{x_1, \cdots, x_n\}$ and $P_\alpha = \{\alpha \cdot x_1, \cdots, \alpha \cdot x_n\}$ take the same action at each and every period. This yields the claim of the proposition. Q.E.D.

Thus, the propensity prompted by aversion to deprivation to engage in migration by a rich population is equal to the propensity to engage in migration by a uniformly poorer population. Migration is independent of the general level of wealth of a population.

3.4 Societal Deprivation

Suppose we measure social welfare by the inverse of the population's total deprivation, where total deprivation is the sum of the deprivations of all the individuals constituting the population. It follows that social welfare is maximized when total deprivation is minimized. While the social welfare associated with the steady-state distribution

Region A	Region B
n	
	n-1
	n-2
n-3	
n-4	
	n-5
	n-6
n-7	
n-8	
⋮	⋮

is higher than the social welfare associated with the initial period 0 allocation, individualistic group-formation behavior fails to produce maximal social welfare. The minimal total deprivation (TD) obtains when $(n, n-1, \cdots, i)$ are in A and $(i-1, i-2, \cdots, 1)$ are in B where $i = \frac{n}{2}+1$ if n is an even number and, as can be ascertained by direct calculation, where $i = \frac{n+1}{2}$ or $i = \frac{n+3}{2}$ when n is an odd number.[4]

In the general case, where $x_1 \le x_2 \le \cdots \le x_n$ are not $1, 2, \cdots, n$, no unequivocal conclusion can be drawn. Sometimes the steady-state distribution gives rise to the minimal TD, sometimes it does not. For example, when the individuals' incomes are $(10, 2, 1)$, the steady-state distribution

Region A	Region B
10	
	2
	1

is socially optimal. When the individuals' incomes are $(10, 9, 1)$,

[4] The proof is in the Appendix.

Region A	Region B
10	
	9
	1

is the steady-state distribution; while

Region A	Region B
10	
9	
	1

is the optimal social allocation.

As long as the number of different incomes is larger than the number of (reference) groups, total relative deprivation will not be minimized at zero. If there are as many groups as there are different incomes, total relative deprivation will be zero.

4. Conclusions and Complementary Reflections

Individuals belong to groups, clubs, neighborhoods, and associations. When given a choice, individuals may want to revise their affiliation – form a new group, change their neighborhood, join another club, associate with others. Several considerations, both absolute and relative, impinge on these choices. In this paper we have singled out for close scrutiny one such consideration – the distaste of relative deprivation. We have studied several repercussions when this measure is used as the exclusive determinant of affiliation.

We have assumed a given and uniform dislike of relative deprivation. Relative deprivation is a sensitive measure that encompasses rank-related information beyond mere rank. (It tells us that 1 compared to 3 is worse than 1 compared to 2, even though in both instances 1's rank is second.) An important question that is not addressed in this paper is where the aversion to relative deprivation or, for that matter, the distaste for low rank, originates. Postlewaite (1998) argues that since over the millennia high rank conferred an evolutionary advantage in the competition for food and mating opportunities, the concern for rank is likely to be hardwired (part of the genetic structure). More generally though, any setting in which rank impinges positively - directly or indirectly - on consumption ought to imply a concern for rank.[5] The study of why an aversion to relative deprivation exists and why individuals exhibit distaste for low rank invites more attention.

It is plausible to stipulate that the distaste for low rank will not be uniform across societies. Consequently, the extent of self-segregation across societies will vary. Since segregation is visible, whereas preferences are not, an inference may be drawn from the observed segregation to the motivating distaste, with more segregation suggesting stronger distaste.

We have shown that when individuals who initially belong to one group (costlessly) act upon their distaste of relative deprivation and self-select into any one of two groups, they do not end up splitting into a uniformly low income group and a uniformly high income group. Again, since the result of self-segregation is visible while the underlining motives are not, the presence of clusters in each group will suggest that relative deprivation plays a key role in determining affiliation, while a uniform division will suggest that considerations of relative deprivation do not bear overwhelmingly on affiliation choices.

We have described an endogenous process of voluntary segmentation into distinct groups; the division of the population into groups is not the outcome of an exogenous imposition of segregation. Assuming no comparisons between members of one group and another, we have shown that, as a consequence, aggregate relative deprivation is lowered. In broader contexts, the group partitioning could also be associated with improved social welfare as a result of reduced social tensions, fewer conflicts, less crime, and a mediated quest for status (as the inequality between those who compete with each other for status is reduced).

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[5] In poor societies with meager assets, rank can serve as a proxy for collateral, thereby facilitating the attainment of credit.

The opening of another region, *B*, facilitates shedding one's relative deprivation. Consider a reverse process, wherein regions *A* and *B* merge into a single composite region that constitutes everyone's reference group. In all cases (except the degenerate case in which all individuals have exactly the same income) the population's relative deprivation is bound to rise. Groups who are less well off in terms of absolute income will be better off in terms of wellbeing if they are allowed to secede, without any change in absolute income. Conversely, a group that is less well off in terms of absolute income that is forced to merge with a group that is better off in terms of absolute income becomes worse off. The pressure to form a separate state, for example, can be partially attributed to this aversion to relative deprivation; when such an aversion exists, the sole individual with less than 1 in *B* may prefer that option to having 1 in *A*, where 2 is present.

These considerations relate to federalism. The process of adding new members to a federation of nations usually draws on the expectation that in the wake of the integration, the incomes of the citizens of the new member nations will rise. The European Union, however, has taken great pains to ensure that the incomes of the citizens of the would-be member nations rise substantially *prior* to integration. Our approach suggests a rationale. To the extent that integration entails the formation of a new reference group, relative deprivation when 1 joins 2 would be reduced if 1½ were to join 2, and would be eliminated altogether if 2 were to join 2.

Appendix

To find the division of a population of n individuals across groups A and B that confers the minimal total deprivation (TD) we proceed in two steps. First given the size of the two groups, we show that the minimal TD is reached when high income individuals are in one of the groups and low income individuals are in the other group. (That is, the income of *any* individual who is in one group is higher than the income of *any* individual who is in the other group.) Second, given this distribution, we show that the minimal TD is reached when *half* of the individuals are in one group and the other half are in the other group.

Lemma: Let n be a fixed positive integer. Consider $\{a_1, a_2, ..., a_n\}$ where $a_1 < a_2 < ... < a_n$ and a_i's are positive integers. Let $S(a_1, a_2, ..., a_n) = \sum_{1 \le i, j \le n} |a_i - a_j|$. Then $S(a_1, a_2, ..., a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1,2,...,n-1$.

Proof: For any $i < j$, we have $|a_i - a_j| = |a_j - a_{j-1}| + |a_{j-1} - a_{j-2}| + ... + |a_{i+1} - a_i|$. Therefore, $|a_i - a_j| \ge j - i$ and $\left(\underset{\text{for all i,j}}{|a_i - a_j| = j - i} \right)$ if and only if $\left(\underset{\text{for } i=1,2,...,n-1}{a_{i+1} = a_i + 1} \right)$. It follows that $S(a_1, a_2, ..., a_n)$ reaches its minimum if and only if $a_{i+1} = a_i + 1$ for $i = 1,2,...,n-1$. (This minimum is $\frac{n(n^2-1)}{3}$.) Q.E.D.

Corollary: Consider the configuration of incomes $(1,...,n-1,n)$. Let there be two groups, A and B, with $(i_1, i_2, ..., i_{n_A})$ in A, and $(j_1, j_2, ..., j_{n_B})$ in B, $n = n_A + n_B$. Let $TD = TD_A + TD_B$. Then, if n, n_A, n_B are fixed, TD reaches its minimum if and only if $(j_1, j_2, ..., j_{n_B}) = (1,2,...,n_B)$ or $(i_1, i_2, ..., i_{n_A}) = (1,2,...,n_A)$; that is, either

Region A	Region B
n	
$\vdots$	
$n_B + 1$	
	n_B
	$\vdots$
	1

or

Region A	Region B
	n
	$\vdots$
	$n_A + 1$
n_A	
$\vdots$	
1	

Proof: Note that $TD_A = \frac{1}{2} S(i_1, i_2, ..., i_{n_A})$, $TD_B = \frac{1}{2} S(j_1, j_2, ..., j_{n_B})$. Thus, for fixed n_A, n_B, $\min TD_A \Leftrightarrow \min S(i_1, i_2, ..., i_{n_A})$, $\min TD_B \Leftrightarrow \min S(j_1, j_2, ..., j_{n_B})$. Assume that TD reaches its minimum at $(i_1^*, i_2^*, ..., i_{n_A}^*)$, $(j_1^*, j_2^*, ..., j_{n_B}^*)$. Without loss of generality, assume that $n \in (i_1^*, i_2^*, ..., i_{n_A}^*)$. Then, if $(i_1^*, i_2^*, ..., i_{n_A}^*) \neq (n_B + 1, ..., n)$, then $(j_1^*, j_2^*, ..., j_{n_B}^*) \neq (1, ..., n_B)$. By the Lemma, we have that $TD_A(i_1^*, i_2^*, ..., i_{n_A}^*) > TD_A(n_B + 1, ..., n)$, and $TD_B(j_1^*, j_2^*, ..., j_{n_B}^*) > TD_B(1, ..., n_B)$. Thus, $TD((i_1^*, i_2^*, ..., i_{n_A}^*), (j_1^*, j_2^*, ..., j_{n_B}^*)) > TD((n_B + 1, ..., n), (1, ..., n_B))$, which contradicts the assumption that TD reaches its minimum at $(i_1^*, i_2^*, ..., i_{n_A}^*)$, $(j_1^*, j_2^*, ..., j_{n_B}^*)$.

Hence, $\left(i_1^*, i_2^*, ..., i_{n_A}^*\right) = \left(n_B + 1, ..., n\right)$, and $\left(j_1^*, j_2^*, ..., j_{n_B}^*\right) = \left(1, ..., n_B\right)$. Conversely, by the Lemma, we have that $TD_A\left(i_1, i_2, ..., i_{n_A}\right) \geq TD_A\left(n_B + 1, ..., n\right)\left(or\left(1, ..., n_A\right)\right)$, and $TD_B\left(j_1, j_2, ..., j_{n_B}\right) \geq TD_B\left(1, 2, ..., n_B\right)\left(or\left(n_A + 1, ..., n\right)\right)$. Therefore, TD reaches its minimum at either of the two configurations. We thus proved the Corollary. Q.E.D.

From the Lemma we know that the minimum of $S(a_1, a_2, \cdots, a_n)$ is $\frac{n(n^2-1)}{3}$. The total deprivation TD of $(n, n-1, \cdots, 1)$ is $\frac{1}{2}$ of this minimum, that is $TD = \frac{n(n^2-1)}{6}$. Let $n = n_A + n_B$, $n \geq 2$, $n_A \geq 1$. Then, by the Corollary, $TD_A = \frac{n_A(n_A^2-1)}{6}$, $TD_B = \frac{n_B(n_B^2-1)}{6}$. Therefore, $TD = \frac{n_A(n_A^2-1)}{6} + \frac{(n-n_A)[(n-n_A)^2-1]}{6} = \frac{n^3 - 3n^2 n_A + 3n n_A^2 - n}{6}$.

We seek to solve $\min_{1 \leq n_A \leq n} TD$. Since $\frac{dTD}{dn_A} = \frac{1}{6}(-3n^2 + 6n n_A)$ and $\frac{d^2 TD}{(dn_A)^2} = n > 0$, we have that the minimal TD obtains when $\frac{dTD}{dn_A} = 0$, that is, $n_A = \frac{n}{2}$. Therefore, if n is an even number, half of the n individuals will be in each of the two groups. With $TD_A = TD_B = \frac{n(n^2-4)}{48}$, $TD = TD_A + TD_B = \frac{n(n^2-4)}{24}$.

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