Technological Progress and the Distribution of Productivities across Sectors

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Abstract

This paper studies the impact of the process of technological change on the distribution of productivities and profits across sectors. We find that if technological progress affects high-tech and traditional sectors differently, the impact of changes in the determinants of economic growth may differ depending on which is the actual change. When an economy is growing faster due to an increase in the productivity of research or to a reduction of the taxes on capital accumulation, inequality will decrease. However, if faster growth is due to the presence of tax incentives to high technology sectors or to structural changes that allow a better absorption of externalities, inequality will increase. Regarding the effect on growth of changes affecting the distribution of productivities we find that if the scope of technological spillovers is sufficiently broad, a distribution with a larger mass of high-tech sectors is associated with a larger growth rate. Nevertheless, a larger mass of research intensive sectors is not necessarily associated with faster growth when spillovers are technology specific or narrow in scope. In this case, the mass of the leading group of sectors will not affect the growth rate because the increased probability of innovation due to the larger mass of high-tech products is completely offset by the reduction in the marginal impact of an individual innovation.

1 Introduction

Does technological progress increase or reduce inequality in the profitability of productive activities? How is the distribution of productivities related to the growth process? Do growth promoting policies induce different degrees of inequality among productivities? In this paper we try to provide answers to these questions by means of an endogenous growth model in which the distribution of productivities across sectors affects and is affected by the characteristics of the process of technological change.

We take as reference the Aghion and Howitt (1998) model of endogenous technological change in which the distribution of relative productivities is time invariant and is not affected by changes in most of the parameters except for the size of innovations. By means of the introduction of a punishment to obsolescence, we develop a model in which both technological parameters and policy instruments will be able to modify the distribution of productivities. We will find that in some cases, faster growth can induce more inequality, introducing a wider gap between the technological leaders of the economy and the less innovative sectors.

R&D based models of growth were initially divided into horizontal models of product development (as in Romer 1990) and models of growth through creative destruction (Aghion and Howitt 1992). The introduction of the schumpeterian concept of creative destruction allows for the existence of obsolescence of old intermediate products but technological improvements in other sectors can also cause relative obsolescence. However, Aghion and Howitt (1992) considered only one intermediate sector producing improved varieties of the same good as technology evolved. When a multisector approach is taken, as in Caballero and Jaffé (1993) and Howitt and Aghion (1998), a wide variety of new considerations appear. In this new framework, growth promoting policies will make aggregate productivity grow faster but different policies may have distinct effects on the distribution of productivities across the economy. Empirical studies detect relevant changes in the distribution of productivities in the last decades. Cameron *et al.* (1997) find that the distribution of productivity levels across UK manufacturing sectors exhibits an increase in dispersion and becomes increasingly skewed during the period 1973-1989. They find evidence of convergence of a number of industries just below the mean while productivity levels in a few sectors persistently remain above and rise away from mean values. This divergence in productivity levels between high-tech industries and traditional sectors and the formation of technological clusters has been observed in most developed countries.[1] In addition, there exists a wide array of policies that try to affect the productive performance of different sectors. Research subsidies are predominantly devoted to high-tech sectors while most countries develop programs to support the competitiveness of traditional sectors or to increase research productivity.[2] The model we propose allows to analyze the distributional implications of these different policies in a theoretical framework.

The distribution of productivities considered in this model differs from the one used in leap-frogging neo-Schumpeterian literature in the following aspect: In the standard model, the occurrence of a sole

[1] See Bergeron *et al.* (1998) or Boschma (1999).
[2] See Ford, R. and W. Suyker (1990) and Eaton *et al.* (1998).

innovation would take the productivity of the sector to the leading edge, no matter how long ago occurred the last innovation or how obsolete was the previous technology. In our model, the introduction of a *punishment* to obsolescence creates two classes of sectors. If the relative productivity of a sector falls below a given threshold, it will not be able to reach the technological frontier with just one innovation. Instead, the productivity increase will only be a fraction of the gap existing between the previous productivity and the most advanced technology of the economy. We will refer to these sectors as the *lagging group.* Conversely, the *leading group* will be formed by those sectors with a relative productivity parameter above that threshold. These sectors are able to reach the leading edge if they innovate, but if they do not, their relative productivity will fall and will enter into the lagging class. The resulting distribution will be affected by policy variables and technological parameters. We find that a larger productivity of research or an increase in the incentives to accumulate capital will make the economy grow faster and reduce the mass of technological laggards, improving thus the distribution of productivities and profits across sectors. Conversely, a larger size of innovations or a higher influence of individual innovations on the aggregate state of knowledge will increase the size of the lagging group, and therefore, there will exist a larger mass of firms earning relatively low profits with respect to the technological leaders. Whether this increase in the size of innovations is growth enhancing or not will depend on the assumption we make about what determines the growth rate of productivity. Similarly, a research subsidy to high-tech sectors will also reduce the mass of the leading group since it will induce a higher research intensity and thus a faster rate of decay of non-innovating sectors. Again, the effect on growth of this subsidy depends on how the size of the leading group affects the evolution of aggregate productivity. We have also found that a subsidy to less research intensive sectors will reduce the size of the lagging group and may increase the rate of growth of the economy.

In summary, this model establishes a set of links between the process of technological progress and the distribution of productivities and profits across economic sectors. We find that if technological progress affects high-tech and traditional sectors differently, the impact of changes in the determinants of economic growth may be very different depending on which is the source of faster growth.

The rest of the paper is organized as follows: Section 2 presents the model, sections 3 and 4 perform the equilibrium and steady state analysis and section 5 concludes the paper.

2 The model

This paper presents a model in which the nature of the process of innovation will affect the distribution of productivities across sectors. The paper is based on the work of Aghion and Howitt (1998) but their model is modified in such a way that changes in the technological parameters will influence the distribution of profits across economic sectors.

2.1 Consumers

There exists a representative consumer who gets utility from the consumption of a final good. He therefore, will maximize the present value of utility

$$V(C(t)) = \int_0^\infty \ln(C(t))e^{-\rho t}dt, \tag{1}$$

where $C(t)$ is consumption at time t and ρ is the rate of discount.

2.2 Final good sector

The consumption good is produced in a competitive sector out of labor L, that is assumed to be exogenously given, and a continuum of mass one of intermediate goods. Let $m_i(t)$ be the supply of sector i at date t. The production function is a Cobb-Douglas with constant returns on intermediate goods and efficiency units of labor as given by

$$Y(t) = L^{1-\alpha} \int_0^1 A_i(t)\left[m_i(t)\right]^\alpha di, \tag{2}$$

where $Y(t)$ is final good production and $A_i(t)$ is the productivity coefficient of each sector. The evolution of each sector's productivity coefficient $A_i(t)$ is determined in the research sector. I assume equal factor intensities to simplify calculations.

2.3 Intermediate goods

Intermediate goods are produced in a sector formed by a continuum of monopolies each producing one good. They are monopolies because their production technology is protected by a patent. The only input in the production of intermediate goods is capital. In particular, it is assumed that $A_i(t)$ units of capital are needed to produce one unit of intermediate good i at date t. As we will see, this assumption is necessary to obtain stability. Capital is hired in a perfectly competitive market at rate $\zeta(t)$. Therefore, the cost of one unit of intermediate good i is $A_i(t)\zeta(t)$. Because the final good sector is assumed to be competitive, the equilibrium price $p(m_i(t))$ of intermediate good i will be its marginal product

$$p(m_i(t)) = \alpha L^{1-\alpha} A_i(t)\left[m_i(t)\right]^{\alpha-1}.$$

Consequently, the monopolist's profit maximization problem will be

$$\begin{aligned} \pi_i(t) &= \max_{m_i(t)}[p(m_i(t))m_i(t) - A_i(t)\zeta(t)m_i(t)] \\ \textit{subject to} \quad p(m_i(t)) &= \alpha L^{1-\alpha} A_i(t)\left[m_i(t)\right]^{\alpha-1}, \end{aligned}$$

from where we obtain the profit-maximizing supply and the flow of profits as

$$\begin{aligned} m_i(t) &= L\left(\frac{\alpha^2}{\zeta(t)}\right)^{\frac{1}{1-\alpha}} \\ \pi_i(t) &= \alpha(1-\alpha)L^{1-\alpha} A_i(t)\left[m_i(t)\right]^\alpha. \end{aligned}$$

Due to the assumption of equal factor intensities, supply of intermediate goods is equal in all sectors, $m_i(t) = m(t)$. Thus, the aggregate demand of capital is equal to $\int_0^1 A_i(t)\, m(t)\, di$. Let $A(t) = \int_0^1 A_i(t)\, di$, be the aggregate productivity coefficient. Then, equilibrium in the capital market requires demand to equal supply

$$A(t)\, m(t) = K(t),$$

or equivalently, the flow of intermediate output must be equal to $\frac{K(t)}{A(t)}$ which we will call capital intensity and denote by $k(t)$. That is,

$$m(t) = \frac{K(t)}{A(t)} \equiv k(t).$$

With this notation we can express the equilibrium rental rate in terms of capital intensity

$$\zeta(t) = \alpha^2 L^{1-\alpha} [k(t)]^{\alpha-1}. \tag{3}$$

2.4 Research sector

Innovations are produced using the same technology of the final good sector. Hence, they need capital apart from labor to be produced. Let $n_i(t) \equiv \frac{N_i(t)}{A^{\max}(t)}$ be the productivity adjusted level of research or research intensity of sector i at date t. It is defined as the total amount of output invested in research by that sector $N_i(t)$, divided by $A^{\max}(t)$, the productivity coefficient of the most advanced technology in the economy. Investment in research is adjusted by $A^{\max}(t)$ in order to take into account the effect of increasing technological complexity. Thus, as technology evolves and becomes more complex, an ever increasing amount of research will be necessary in order to obtain further technological improvements. The Poisson arrival rate of innovations in each sector is assumed to be $\lambda n_i(t)$, where λ is a positive parameter representing the productivity of research.

Let us define $a_i(t)$ as the relative productivity parameter of sector i at date t. This relative productivity is given by the productivity coefficient $A_i(t)$ of that sector, divided by the productivity coefficient $A^{\max}(t)$ of the leading edge technology, and this ratio measures the technological level of the sector with respect to the most advanced technology of the economy. We will assume that $A^{\max}(t)$ will grow due to the flow of innovations in the economy. Therefore, if $A_i(t)$ does not change, the relative productivity parameter will gradually fall as the sector's technology becomes obsolete. This process of obsolescence can be avoided if an innovation occurs in the sector since then, its productivity coefficient will increase. In order to take into account the effect of intertemporal and intersectoral spillovers, we assume that $A_i(t)$ will jump to $A^{\max}(t)$. That is, the final increase in productivity depends upon the evolution of innovations in the rest of the economy and the technological gain will arise from the adoption of new technologies created in other sectors and the absorption of spillovers. However, consider a sector with a very low relative productivity parameter. A low value of $a_i(t)$ implies that the sector's technology has fallen far behind the leading edge and that no recent innovations have taken place. Let us call this type of sectors *lagging sectors.* In

Aghion and Howitt's model, a sole innovation would take the productivity coefficient of this sector to the leading edge. In the present model, we will introduce a punishment for having lagged behind, in the sense that if the relative productivity parameter has fallen below a given threshold, innovating once will not allow the sector to reach the top of the distribution. We will thus assume that if an innovation occurs in a lagging sector, the productivity coefficient attained will only be a fraction of $A^{\max}(t)$. Specifically, we assume that if $a_i(t)$ falls below β, the relative productivity parameter attainable by an innovation will be γ instead of 1, where $0 \leq \beta < \gamma < 1$. In order to analyze the implications of this assumption, we will consider first the determination of the equilibrium level of research investment.

There exists a number of research firms in each sector competing in a patent race to get the next innovation for a specific production technology. The first innovating firm gains the patent and it either becomes the monopolist producer of the new variety or sells the patent to an established firm. In any case, the reward to the innovation will be the present value of the flow of profits arising from the monopolistic exploitation of the patent. Let us denote the value of the innovation by $V(t)$. On the other hand, the cost of one unit of research is one unit of output. If a firm invests one unit of research it will have a probability of obtaining the innovation equal to $\frac{\lambda}{A^{\max}(t)}$. The research arbitrage equation establishes that the cost of one unit of research must be equal to the expected revenue from this research. Therefore,

$$1 - s_i = \frac{\lambda V(t)}{A^{\max}(t)}, \tag{4}$$

where s_i is the subsidy rate to research in sector i. Consider now the determination of the value of the innovation $V(t)$. The flow of profits will depend on whether the innovating sector was a leading or a lagging sector. If the innovation has occurred in a leading sector, then the productivity coefficient achieved is $A^{\max}(t)$ and the flow of profits will be given by $\alpha(1-\alpha)L^{1-\alpha}A^{\max}(t)[k(t)]^{\alpha}$ and equation (4) may be written as

$$1 - s_i = \frac{\lambda\alpha(1-\alpha)L^{1-\alpha}[k(t)]^{\alpha}}{r(t) + \lambda n_i(t)},$$

where $r(t)$ is the interest rate. Notice that in order to compute the present value of the flow of profits, the rate of discount includes $\lambda n_i(t)$ in addition to the rate of interest. The term $\lambda n_i(t)$ represents the probability that the incumbent monopolist is replaced by the owner of a new patent and it is also known as the rate of creative destruction.

If the innovating sector was a lagging sector, then the flow of profits arising from the innovation will be $\alpha(1-\alpha)L^{1-\alpha}\gamma A^{\max}(t)[k(t)]^{\alpha}$. Consequently, equation (4) will now be given by

$$1 - s_i = \gamma\left(\frac{\lambda\alpha(1-\alpha)L^{1-\alpha}[k(t)]^{\alpha}}{r(t) + \lambda n_i(t)}\right).$$

It is thus obvious that research intensity in lagging and leading sectors will be generally different. In particular, we can establish that the relationship between research intensities will be

$$\lambda n_l(t) = \gamma\tau\lambda n_h(t) - (1-\gamma\tau)r(t),$$

where $n_l(t)$ and $n_h(t)$ are research intensity in lagging and leading sectors, respectively, and $\tau = \frac{1-s_h}{1-s_l}$, where s_l and s_h are the corresponding subsidies to lagging and leading sectors. Notice that in equilibrium, the research intensity performed in all the sectors belonging to the same group will be equal given that they will obtain the same reward. Notice also that if $\tau \leq \frac{1}{\gamma}$, research intensity in the lagging group will not be larger than research intensity in the leading group. In what follows we will restrict the analysis to subsidy values satisfying this condition, namely, that the subsidy to lagged sectors may increase research intensity up to but not above the level of leading sectors. Thus, we will not consider subsidies that would make lagged sectors more research intensive than the technological leaders.

For the sake of simplicity, we will assume that aggregate knowledge and, hence, $A^{\max}(t)$ will only grow thanks to innovations in the leading group. Intuitively, this implies that lagged sectors only adapt technological improvements from other sectors, but do not add to the growth of the technological frontier. Indeed, data on the contribution of traditional sectors to knowledge creation suggest that this assumption is not too far from reality.[3] We will consider two alternative assumptions for the growth behavior of $A^{\max}(t)$. The first assumption simply states that the rate of growth of the knowledge frontier is proportional to the aggregate probability of innovation in leading sectors, that is

$$\frac{\dot{A}^{\max}(t)}{A^{\max}(t)} = \sigma(1-\phi)\lambda n_h(t), \tag{5}$$

where $\sigma > 0$ is a parameter that measures the effect of individual innovations on the leading edge productivity coefficient. This parameter is traditionally interpreted as measuring the size of innovations, but it can also represent the degree of interrelation between sectors or the capacity to absorb spillovers from other sectors. The parameter ϕ measures the size of the lagging group. We will refer to this assumption as the *aggregate assumption.*

In models where technological progress is due to both vertical and horizontal innovations, it is generally assumed that an increase in the mass of available technologies reduces the effect of an innovation on the aggregate economy. In particular, it is assumed that the increasing probability of innovation due to the larger mass of products is completely offset by the reduction in the marginal impact of an individual innovation.[4] In this case the rate of growth of aggregate knowledge would be proportional to the average probability of innovation in leading sectors. Consequently, we will refer to this assumption as the *average assumption* and the rate of growth of $A^{\max}(t)$ would be given by

$$\frac{\dot{A}^{\max}(t)}{A^{\max}(t)} = \theta\lambda n_h(t), \tag{6}$$

where $\theta > 0$ is a parameter measuring the effect on the rate of growth of aggregate knowledge of a change in the average probability of innovation in leading sectors.

[3] Cameron *et al.* (1997) report that only seven industries out of nineteen accounted for 95% of TFP growth in the UK economy in the last decades. Among these industries, Computing, Pharmaceuticals and Aerospace, the highest productivity attainers, accounted for a 42% of the total growth in productivity.

[4] See Aghion and Howitt (1998), chapter 12 or Howitt (1999).

The key difference between these two assumptions lies on whether we consider that the technological frontier is formed by all the production technologies in the economy or only by those sectors innovatively active enough to reach the frontier with just one innovation. In the first case, an increase in the mass of the leading group should make the economy grow faster because the sectors in this group are more research intensive. In the second case, even though there will be more research, there will also exist more technologies to improve and thus, research efforts will have to be distributed among more different fields.

The lagging group is formed by sectors with obsolete technologies in which no innovation has occurred for a considerably long period of time. Productivity increases in these sectors are generally due to the adoption of technologies from other sectors. Therefore, ignoring them as part of the technological frontier should not represent a problem, at least when there exists a large distance between traditional and high-tech sectors. In very developed economies we may expect a wide gap between the leading-edge production technologies and the most traditional sectors of the economy. In these cases, spillovers from the high-tech sectors will probably be technology specific and narrower in scope.[5] This picture of the technological system is better fit by the average assumption. On the other hand, consider an economy in the early phases of development or with a nearly non-existing high-tech sector. Then, the difference between the leading-edge and the more obsolete sectors will not be so large and technological improvements in the leading group will not be so specific that the whole mass of technologies cannot benefit from it. In this case, the most appropriate assumption would be the aggregate assumption.

Trying to connect these theoretical discussion with empirical findings, let us mention the paper by Caballero and Jaffé (1993) in which the authors observe a decline over the twentieth century in a parameter representing the "potency of spillovers emanating from each cohort of ideas or the intensity of use of old ideas by new ideas". This decline could be interpreted, in the authors' words, as a process by which "research is steadily becoming narrower and hence generates fewer spillovers because each new idea is relevant to a smaller and smaller set of technological concerns". The authors estimate that the average idea at the beginning of the century generated about 5 times the level of spillovers as the average recent idea. This narrower scope for spillovers could be supporting the average assumption, by which the relevant set of technologies that conform the technological frontier is the leading group and an increase in the size of this group would induce a smaller effect of innovations on the enlarged set of technologies.

We will develop the model first under the average assumption because this assumption allows us to identify the effect of growth determinants on the distribution of relative productivities. In fact, under the average assumption we could abstract from the complications arising from the interaction between the productivities distribution and the growth rate. When considering the aggregate assumption, we will have to take into account the relationship between changes in the mass of the lagging group and changes in the growth rate.

In addition to the effect on the research investment of firms, the introduction of the assumption that

[5] Indeed, Cameron et al. (1997) find informal evidence suggesting that for at least a small subsector of industries, the development of technology is quite specific to the individual sector and does not spill over rapidly into many other manufacturing sectors.

lagged sectors will not be able to reach the leading edge with a sole innovation has another important implication. Without this assumption, the long run distribution of relative productivity parameters is time invariant and does not depend on the growth behavior of the economy. Specifically, the long run distribution of relative productivities is described by the following distribution function[6]

$$H(a) = a^{\frac{1}{\sigma}}.$$

In the present model however, the distribution of relative productivities will depend upon the growth rate of the economy and will be affected by changes in the determinants of equilibrium.

2.5 Capital market

Capital is used as a factor of production in the intermediate goods sector. We have seen that equilibrium in the capital market requires the rental rate to satisfy equation (3). The owner of a unit of capital will obtain $\zeta(t)$ for it. This amount must be enough to cover the cost of capital. This includes the interest rate $r(t)$, the depreciation rate δ, and the tax rate on capital accumulation τ_k which is introduced in order to parametrize the incentives to accumulate capital. Hence, the capital market arbitrage equation is

$$r(t) + \delta + \tau_k = \alpha^2 L^{1-\alpha} [k(t)]^{\alpha-1},$$

which establishes a decreasing relationship between the interest rate and capital intensity.

2.6 Public sector

The role of the government in this model will be confined to the concession of subsidies to leading and lagging sectors s_h and s_l, respectively and the imposition of the tax on capital accumulation τ_k. The public budget will be balanced through a lump-sum tax or transfer T which will help us to isolate the effects of the different policy instruments. Therefore, the government budget is given by the following equation:

$$T(t) = s_h N_h(t) + s_l N_l(t) - \tau_k K(t).$$

2.7 Distribution of relative productivity coefficients

The existence of a lagging group that behaves differently after an innovation determines a distribution of relative productivities that will be affected by changes in the technological and policy parameters. The next proposition provides the distribution function of a under the average assumption:

[6] See Aghion and Howitt (1998).

Proposition 1 *The long run distribution of relative productivity coefficients under the average assumption is time invariant and is described by the following cumulative distribution function:*

$$H(a) = \begin{cases} \phi + (1-\phi)\, a^{\frac{1}{\theta}} & if \quad \gamma \leq a \leq 1 \\ \phi + \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}} \left((1-\phi)\,\gamma^{\frac{1}{\theta}} + \phi \int_a^{\gamma} \lambda n_l\left(\tilde{t}(a)\right) \left(\frac{a}{\gamma}\right)^{-\frac{1}{\theta}} \tilde{t}'(a)\, da \right) & if \quad \beta \leq a \leq \gamma \\ \phi \exp\left(\int_a^{\beta} \lambda n_l\left(\tilde{t}(a)\right) \tilde{t}'(a)\, da \right) & if \quad a \leq \beta \end{cases}, \tag{7}$$

where $\tilde{t}(a)$ is a differentiable and decreasing function relating date t and the relative productivity a of a given sector which is implicitly defined by equation (23) in Appendix A.

Proof. See Appendix A. ■

Similarly, Proposition 2 gives the distribution function of a under the aggregate assumption.

Proposition 2 *The distribution of relative productivity coefficients under the aggregate assumption is time invariant and may be characterized by the following distribution function:*

$$H(a) = \begin{cases} \phi + (1-\phi)\, a^{\frac{1}{\sigma(1-\phi)}} & if \quad \gamma \leq a \leq 1 \\ \quad \phi + (1-\phi)\, a^{\frac{1}{\sigma(1-\phi)}} + \\ \quad + \left(\frac{a}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}} \phi \int_a^{\gamma} \lambda n_l\left(\tilde{t}(a)\right) \left(\frac{a}{\gamma}\right)^{-\frac{1}{\sigma(1-\phi)}} \tilde{t}'(a)\, da & if \quad \beta \leq a \leq \gamma \\ \phi \exp\left(\int_a^{\beta} \lambda n_l\left(\tilde{t}(a)\right) \tilde{t}'(a)\, da \right) & if \quad a \leq \beta \end{cases}. \tag{8}$$

Proof. See Appendix A. ■

The distribution of relative productivity coefficients will thus be affected by policy changes and the characteristics of the process of technological change. In order to analyze the implications of changes in these parameters we solve the model in the following section.

3 Equilibrium

3.1 Equilibrium under the average assumption.

General equilibrium is defined by the following equations:

$$1 - s_h = \frac{\lambda\alpha(1-\alpha)\, L^{1-\alpha}\, [k(t)]^{\alpha}}{r(t) + \lambda n_h(t)}, \tag{9}$$

$$\lambda n_l(t) = \gamma\tau\lambda n_h(t) - (1-\gamma\tau)\, r(t), \tag{10}$$

$$r(t) + \delta + \tau_k = \alpha^2 L^{1-\alpha}\, [k(t)]^{\alpha-1}, \tag{11}$$

where (9) is the arbitrage equation for research in a leading sector, (10) gives the relationship between lagged and leading sectors research intensity and (11) is the capital market arbitrage equation. The last expression implies that the interest rate is a function of the equilibrium value of capital intensity. Thus, from (9) we can view $n_h(t)$ as a function of $k(t)$, while (10) gives us the research intensity $n_l(t)$ in lagged sectors as a function of capital intensity $k(t)$. Consequently, using these equations we may denote $n_h(t) = n_h(k(t))$, $n_l(t) = n_l(k(t))$ and $r(t) = r(k(t))$. Therefore, we can express the dynamics of the model in terms of capital and consumption. The laws of motion for these two variables are

$$\dot{K}(t) = Y(t) - C(t) - (N_h(t) + N_l(t)) - \delta K(t),$$

and

$$\dot{C}(t) = (r(t) - \rho)C(t), \tag{12}$$

where (12) is derived from the consumer's optimization problem. These expressions can be written in efficiency units as follows:

$$\dot{k}(t) = L^{1-\alpha}k(t)^{\alpha} - c(t) - \frac{1}{E(a)}(n_h(k(t)) + n_l(k(t))) - (\delta + g(t))k(t) \tag{13}$$

$$\dot{c}(t) = (r(t) - \rho - g(t))c(t), \tag{14}$$

where $g(t)$ is the rate of growth of aggregate knowledge and $E(a)$ is the mean of the distribution of relative productivity parameters.[7] The rate of growth of aggregate knowledge and $E(a)$ can also be viewed as functions of $k(t)$ since $E(a)$ will depend upon $n_h(t)$ and $r(t)$, while $g(t)$ is given by the following expression:

$$g(t) = \frac{\dot{A}(t)}{A(t)} = \frac{\dot{A}^{\max}(t)}{A^{\max}(t)} + \frac{\dot{E}(a)}{E(a)} = \theta\lambda n_h(t) + \frac{\dot{E}(a)}{E(a)}.$$

Since the distribution of a is time invariant in the long run, so is $E(a)$. Therefore, $g(t) = \theta\lambda n_h(t)$.

Due to the non-linearity of the system, we linearize it around the steady state in order to analyze the local dynamics of the model. We find local saddle path stability around the steady state.[8] Therefore, we can perform comparative statics analysis at the long run equilibrium.

3.2 Equilibrium under the aggregate assumption.

The equations determining equilibrium under this assumption are the same as for the average assumption except that the rate of growth of aggregate technology is now given by

$$g(t) = \sigma(1 - \phi)\lambda n_h(t), \tag{15}$$

where ϕ is implicitly defined as a function of k by equation (29) in Appendix A. Therefore, the dynamic system defined by equations (13) and (14) with $g(t)$ defined by (15) presents also local saddle path stability. See Appendix B for a proof.

[7] We are using the relationship between aggregate and leading edge productivity since $A_t = \int_0^1 A_{it}di = A_t^{\max}\int_0^1 \frac{A_{it}}{A_t^{\max}}di = A_t^{\max}\int_0^1 ah(a)\,da = A_t^{\max}E(a)$, where $h(a)$ is the density function of a.

[8] See Appendix B for a proof.

4 Steady state analysis

4.1 Steady state analysis under the average assumption

In equilibrium, the production function is simplified due to the fact that the equilibrium value of intermediate input is the same for every sector. Consequently, we may write equation (2) as

$$Y(t) = A(t) L^{1-\alpha} [k(t)]^{\alpha},$$

which implies that in a steady state, the rate of growth of output will be the rate of growth of aggregate productivity. That is

$$g = \theta \lambda n_h.$$

Using this result, and the fact that in a steady state k and n_h are constant we may write equations (9), (10) and (11) as follows:

$$1 - s_h = \frac{\lambda\alpha(1-\alpha) L^{1-\alpha} k^{\alpha}}{\rho + (1+\theta)\lambda n_h}, \tag{16}$$

$$\lambda n_l = \gamma\tau\lambda n_h - (1-\gamma\tau)(\rho + \theta\lambda n_h), \tag{17}$$

$$\rho + \theta\lambda n_h + \delta + \tau_k = \alpha^2 L^{1-\alpha} k^{\alpha-1}, \tag{18}$$

where we are using the steady state relationship between the interest rate and the growth rate, i.e. $r = \rho + \theta\lambda n_h$. Equations (16) and (18) determine the steady state values for k and n and allow us to perform comparative statics on the different parameters of the model. The following proposition establishes the steady state relationships between some of the parameters and the growth rate:

Proposition 3 *The steady state growth rate is increasing in θ, λ and s_h and decreasing in τ_k.*

Proof. See Appendix A ■

These results were already obtained in the standard model. They are relevant however, because we want to look at the relationship between growth and the distribution of profits across sectors. The next lemma establishes the relationship between the mass of the lagging group and the previous parameters:

Lemma 4 *The mass of the lagging group ϕ is increasing in θ, τ_k and s_h and decreasing in λ.*

Proof. See Appendix A ■

The result established in Lemma 4 allows us to rank distribution functions. A change in these parameters will have the following effects on the distribution of relative productivities:

Proposition 5 *a) Let $\theta_1 < \theta_2$ and let $H_{\theta_i}(a)$ be the distribution function of relative productivities associated to θ_i for $i = 1, 2$. Then, $H_{\theta_1}(a) < H_{\theta_2}(a)$ for $a \in (0, 1)$.*

b) Let $\lambda_1 < \lambda_2$ and let $H_{\lambda_i}(a)$ be the distribution function of relative productivities associated to λ_i for $i = 1, 2$. Then, $H_{\lambda_1}(a) > H_{\lambda_2}(a)$ for $a \in (0, 1)$.

c) Let $s_{h1} < s_{h2}$ and let $H_{s_{hi}}(a)$ be the distribution function of relative productivities associated to s_{hi} for $i = 1, 2$. Then, $H_{s_{h1}}(a) < H_{s_{h2}}(a)$ for $a \in (0, 1)$.

d) Let $\tau_{k1} < \tau_{k2}$ and let $H_{\tau_{ki}}(a)$ be the distribution function of relative productivities associated to τ_{ki} for $i = 1, 2$. Then, $H_{\tau_{k1}}(a) < H_{\tau_{k2}}(a)$ for $a \in (0, 1)$.

Proof. See Appendix A ■

Proposition 5 implies first degree stochastic dominance of $H_{\theta_1}(a)$ over $H_{\theta_2}(a)$, of $H_{\lambda_2}(a)$ over $H_{\lambda_1}(a)$, of $H_{s_{h1}}(a)$ over $H_{s_{h2}}(a)$ and of $H_{\tau_{k1}}(a)$ over $H_{\tau_{k2}}(a)$. Consequently, the Generalized Lorenz curves for the distribution of relative productivities associated to θ_1, s_{h1}, τ_{k1} and λ_2 dominate the Generalized Lorenz curves associated to θ_2, s_{h2}, τ_{k2} and λ_1 respectively.[9] Accordingly, an increase in λ or a reduction in θ, s_h or τ_k reduces the inequality induced by the distribution of relative productivities across sectors. In other words, an increase in the growth rate due to a larger value of θ or s_h will shift $H(a)$ upwards and therefore, make the generalized Lorenz curve shift downwards. Figure 1 illustrates the effect of an increase in any of these two parameters. Observe that the shift in the distribution function implies that after the change, there exists a larger mass of sectors with smaller relative productivity coefficients and that the mass of the leading group[10] is reduced. Conversely, a higher growth rate due to a larger value of λ or to a reduction in τ_k will shift $H(a)$ downwards and make the generalized Lorenz curve shift upwards. This implies that the relationship between growth and the distribution of productivities can be positive or negative depending on the cause of faster growth. The effect of an increase in θ due for instance to a higher ability of firms to absorb externalities is a larger growth rate. However, it will also induce an increase in the mass of firms that lag behind and that consequently, have smaller relative profits while the leading group, the one with higher relative profits, is reduced. Similarly, a higher subsidy to research in leading sectors, will make the economy grow faster due to the higher research intensity of these sectors, but the gap between the leading and the lagging group will be wider. However, when faster growth is due to a larger productivity of research or to a tax reduction that stimulates capital accumulation, the result is the opposite. That is, the mass of lagging sectors is reduced while the number of sectors in the high-technology group increases, which reduces the inequality among relative productivities. Consequently, faster growth due to an increase in θ or s_h will induce a more unequal distribution of productivities and profits. On the other hand, if the cause of faster growth is an improvement in the productivity of research that affects all sectors or a policy change that stimulates capital accumulation, productive inequality will decrease. Observe that we are considering a set of parameters that includes

[9] See Shorrocks (1983) for a proof of these results and a definition of the Generalized Lorenz Curve.

[10] In the figure, the leading group is formed by those sectors with $a > \beta$, where β is set to 0.6 just for illustrative purposes.

proper policy instruments like subsidies to R&D and taxes on capital accumulation on one hand and exogenous technological parameters like the scope of spillovers θ and research productivity λ on the other. Strictly speaking, θ and λ are not policy instruments that can be changed at the discretion of the public sector. However, one can think of policies oriented at influencing their values. Empirical studies have found evidence that investment in infrastructure and education or the performance of public research can improve private research productivity and the absorptive capacity of private firms (see Eaton *et al.* 1998).

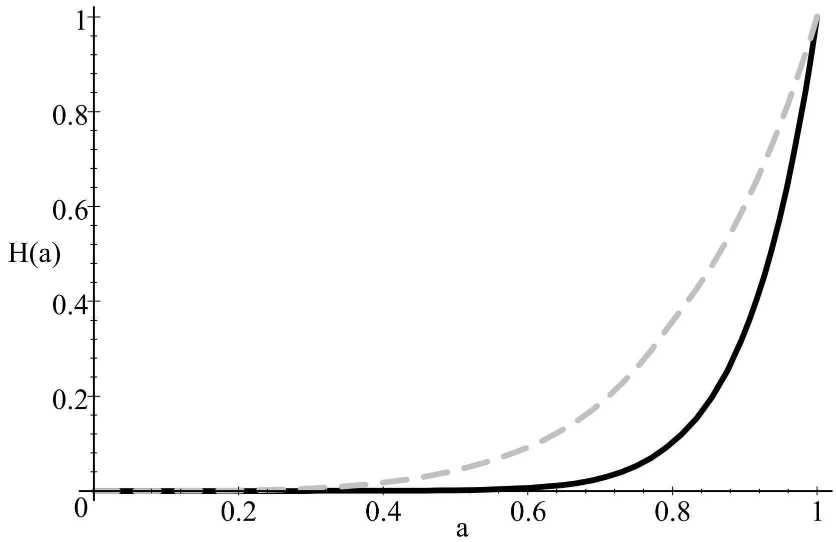

Figure 1: Shift in $H(a)$ caused by an increase in either θ or s_h.

4.2 Steady state analysis under the aggregate assumption

Under the assumption that the rate of growth of the leading edge technology is determined by the aggregate probability of innovation in the leading group, the rate of growth of the economy will be given by

$$g = \sigma\lambda (1-\phi)\, n_h.$$

Therefore, the equations determining the steady state values of k, n_h, n_l and ϕ are

$$1 - s_h = \frac{\lambda\alpha (1-\alpha) L^{1-\alpha} k^{\alpha}}{\rho + (1 + \sigma (1-\phi))\lambda n_h}, \tag{19}$$

$$\lambda n_l = \gamma\tau\lambda n_h - (1-\gamma\tau)(\rho + \sigma (1-\phi)\lambda n_h), \tag{20}$$

$$\rho+\sigma\left(1-\phi\right)\lambda n_h+\delta+\tau_k=\alpha^2 L^{1-\alpha}k^{\alpha-1}, \tag{21}$$

$$\left(1-\phi\right)\beta^{\frac{1}{\sigma(1-\phi)}}-\phi\frac{n_l}{n_h}\left(1-\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right)=0, \tag{22}$$

where (19), (20), and (21) are the research arbitrage equation in leading sectors, the relationship between lagged and leading sectors research intensity and the capital market equilibrium condition, respectively, all expressed for the steady state. Equation (22) is derived from the steady state distribution function of a. It establishes that ϕ must be such that the distribution function is continuous at $a=\beta$. The following proposition establishes the steady state relationships between some of the parameters and the growth rate:

Proposition 6 *The steady state growth rate is increasing in both λ and s_l, decreasing in σ and τ_k and the effect of s_h on growth is ambiguous.*

Proof. See Appendix A ■

Observe that the effects on growth of research productivity and the tax on capital accumulation are not altered by the assumption that the growth rate depends upon the size of the leading group. However, this is not the case for the other three parameters. The next lemma presents the effect of these parameters on the size of the lagging group, which will help us understand the cause of the new results:

Lemma 7 *The mass of the lagging group ϕ is increasing in σ, τ_k and s_h and decreasing in both λ and s_l.*

Proof. See Appendix A ■

Lemma 7 implies that a larger λ will increase the productivity of research on one hand, and on the other, it will reduce the mass of lagging sectors. Therefore, a larger productivity of research is both growth enhancing and promotes less inequality among productivities across sectors. A similar effect is induced by a reduction of τ_k, that is, by an increase in the incentives to accumulate capital. With respect to s_l, notice that under the average assumption it had no effect on the rate of growth. However, under the aggregate assumption we observe that it reduces the mass of lagging sectors. This is a positive effect on growth that is able to compensate the reduction induced on the research intensity of leading sectors. The cases of the other two parameters are more complex to understand. Consider the effect of having a larger σ. Recall that this parameter measures the size of innovations or the influence of individual innovations on the leading edge productivity. When σ increases, research intensity falls due to the rise in the interest rate that makes the inputs to research more expensive. However, σ has a positive direct effect on the growth rate, which made the total growth effect positive under the average assumption. Under the aggregate assumption, we observe that the size of innovation has an additional effect on ϕ which will make the final impact on growth negative. The larger size of innovation makes the relative productivity parameter of the non-innovating sectors fall faster and therefore, there will exist a larger probability of entering the lagging group. Something similar happens when we increase the subsidy to research in high-tech sectors. The subsidy provides incentives to perform a higher research intensity in the leading sectors which will induce large productivity increases for innovators. However, those sectors that were not successful, will lag behind more rapidly and enlarge the lagging group. Consequently, the net effect on growth is ambiguous. Thus, under the aggregate assumption the influence of policy parameters on the mass of the lagging group affects the growth rate finally achieved and introduces important changes in the effectiveness of intendedly growth promoting policies. Only those policies that influence positively both R&D investments and the mass of the leading group will unambiguously promote growth. On the contrary, those policies that induce a larger lagging group will see their growth effectiveness undercut due to their distributional effects.

The complexity of the system under the aggregate assumption prevents us from establishing a ranking of distribution functions similar to the one presented in Proposition 5. Nevertheless, the results for the value of ϕ provide a partial characterization of the effects on the distribution function.

5 Conclusions

This paper has analyzed the effects of technological progress on the distribution of relative productivities across sectors. In particular, we have observed how changes in the characteristics of the process of technological change induce modifications on the distribution of productivities and profits across economic activities and how they may influence the growth performance of the economy. We have found that increases in research productivity, in the incentives to accumulate capital and larger subsidies to technological laggards will increase the mass of research intensive sectors and improve the growth rate of the economy. However, higher subsidies to technological leaders and a larger size of innovations or a higher degree of spillovers will increase the mass of the lagging class, which may in some cases reduce the growth rate of the economy.

References

[1] Aghion, P., Harris, C. and Vickers, J. (1997) "Competition and Growth with Step-by-step Innovation: An Example" European Economic Review 41, 771-782.

[2] Aghion, P. and P. Howitt (1998) "Endogenous growth theory" MIT Press.

[3] Aghion, P. and P. Howitt (1992) "A model of growth through creative destruction" Econometrica LX, 323-351.

[4] Bergeron, S., Lallich, S. and Le Bas, C. (1998) "Location of Innovating Activities, Industrial Structure and Techno-industrial Clusters in the French Economy, 1985-1990. Evidence from US Patenting." Research Policy 26, 733-751.

[5] Boschma, R. (1999) "The Rise of Clusters of Innovative Industries in Belgium during the Industrial Epoch" Research Policy 28, 853-871.

[6] Caballero, R. J. and Jaffé, A. B. (1993) "How High are the Giants' Shoulders: An empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth" NBER Working Paper Series, WP 4370.

[7] Cameron, G., Proudman, J. and Redding, S. (1997) "Deconstructing Growth in UK Manufacturing." Centre for Economic Performance & Institute of Economics Working Paper.

[8] Eaton, J., Gutierrez, E. and Kortum, S. (1998) "European Technology Policy." Economic Policy 27, 405-438.

[9] Ekholm, K. and Torstensson, J. (1997) "High-technology Subsidies in General Equilibrium: a Sector-specific Approach." Canadian Journal of Economics 30(4b), 1184-1203.

[10] Ford, R. and Suyker, W. (1990) "Industrial Subsidies in the OECD Economies" OECD Economic Studies 15, 37-81.

[11] Howitt, P. (1999) "Steady Endogenous Growth with Population and R&D Inputs Growing" Journal of Political Economy 107 (4), 715-730.

[12] Howitt, P. and P. Aghion (1998) "Capital Accumulation and Innovation as Complementary Factors in Long-Run Growth" Journal of Economic Growth 3, 111-130.

[13] Huang, C. and Litzenberger, R. H. (1988) "Foundations for Financial Economics" North-Holland.

[14] Romer, P. (1990) "Endogenous technological change" Journal of Political Economy 98(5), s71-s102.

[15] Schumpeter, J. A. (1911) "The Theory of Economic Development" Cambridge Ma. Harvard U. Press.

[16] Shorrocks, A. F. (1983) "Ranking Income Distributions" Economica 50, 3-17.

A Proofs

Proof of Proposition 1. In order to derive the distribution of relative productivities define $A^{\max}(t_0)$ to be the absolute productivity coefficient of a sector that innovated on date t_0 and achieved the leading edge productivity. Then, from equation (6) we may write

$$\frac{A^{\max}(t_0)}{A^{\max}(t)} = \exp\left(-\int_{t_0}^{t} \theta\lambda n_h(s)\, ds\right), \tag{23}$$

which establishes that as $A^{\max}(t)$ grows, the relative productivity parameter of this sector will fall at a rate $\theta\lambda n_h(t)$. Define $F(\cdot,t)$ as the cumulative distribution of the absolute productivity coefficients A across sectors at any arbitrarily given date t. Define $\Phi(t) = F(A^{\max}(t_0), t)$. Then,

$$\Phi(t_0) = 1$$

and

$$\frac{d\Phi(t)}{dt} = \begin{cases} -(\Phi(t) - \Phi(t_2))\,\lambda n_h(t) & if \quad t_0 \leq t < t_1 \\ -(\Phi(t) - \Phi(t_2))\,\lambda n_h(t) - \Phi(t_2)\,\lambda n_l(t) & if \quad t_1 \leq t < t_2 \\ -\Phi(t)\,\lambda n_l(t) & if \quad t_2 \leq t \end{cases}, \tag{24}$$

where t_1 and t_2 are, respectively, the dates at which $a_0 = \frac{A^{\max}(t_0)}{A^{\max}(t)}$ equals γ and β. Thus t_1 and t_2 are implicitly defined by the following equations which in turn, are derived from (23):

$$\exp\left(-\theta\int_{t_0}^{t_1}\lambda n_h(s)\,ds\right) = \gamma \tag{25}$$

$$\exp\left(-\theta\int_{t_0}^{t_2}\lambda n_h(s)\,ds\right) = \beta. \tag{26}$$

The time derivative of $\Phi(t)$ gives us the rate at which the sector that innovated at date t_0 is left behind by other innovating sectors. Notice that while $t > t_1$, $a_0 > \gamma$ and the sector will only be overtaken by those sectors belonging to the leading group and having an absolute productivity parameter below $A^{\max}(t_0)$. Those sectors have a flow probability of innovation $\lambda n_h(t)$ and a mass of $\Phi(t) - \Phi(t_2)$. However, when $t_1 \leq t < t_2$, the relative productivity coefficient a_0 has fallen below γ and consequently, it may be overtaken by all innovating sectors having an absolute productivity coefficient below $A^{\max}(t_0)$. Therefore, we have a number of sectors which belong to the leading group, $\Phi(t) - \Phi(t_2)$ with a flow probability of innovation equal to $\lambda n_h(t)$ and all the sectors in the lagging group $\Phi(t_2)$, with a flow probability of $\lambda n_l(t)$. When $t \geq t_2$, all the sectors with an absolute productivity coefficient below $A^{\max}(t_0)$, that is $\Phi(t)$, belong to the lagging group and therefore, have a flow probability of innovation of $\lambda n_l(t)$. Equation (24) defines a differential equation whose solution is given by the following expression:

$$\Phi(t) = \begin{cases} \Phi(t_2) + (1 - \Phi(t_2))\exp\left(-\int_{t_0}^{t}\lambda n_h(s)\,ds\right) & if \quad t_0 \leq t < t_1 \\ \Phi(t_2) + (1 - \Phi(t_2))\exp\left(-\int_{t_1}^{t}\lambda n_h(s)\,ds\right)\gamma^{\frac{1}{\theta}} - \\ -\exp\left(-\int_{t_1}^{t}\lambda n_h(s)\,ds\right)\Phi(t_2)\int_{t_1}^{t}\lambda n_l(v)\exp\left(\int_{t_1}^{v}\lambda n_h(s)\,ds\right)dv & if \quad t_1 \leq t < t_2 \\ \Phi(t_2)\exp\left(-\int_{t_2}^{t}\lambda n_l(s)\,ds\right) & if \quad t_2 \leq t \end{cases}, \tag{27}$$

where

$$\Phi(t_2) = \frac{\beta^{\frac{1}{\theta}}}{\beta^{\frac{1}{\theta}} + \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\theta}}\int_{t_1}^{t_2}\lambda n_l(t)\exp\left(\int_{t_1}^{t}\lambda n_h(s)\,ds\right)dt}.$$

Equation (23) implicitly defines t as a function of a_0. Let $\tilde{t}(a_0)$ be this function and use it to perform a change of variable in (27). The function that we obtain is

$$\Phi\left(\tilde{t}(a_0)\right) = \begin{cases} \Phi(t_2) + (1-\Phi(t_2))(a_0)^{\frac{1}{\theta}} & if \quad \gamma \le a_0 \le 1 \\ \Phi(t_2) + (1-\Phi(t_2))(a_0)^{\frac{1}{\theta}} + \\ + \left(\frac{a_0}{\gamma}\right)^{\frac{1}{\theta}} \Phi(t_2) \int_{a_0}^{\gamma} \lambda n_l\left(\tilde{t}(a_0)\right)\left(\frac{a_0}{\gamma}\right)^{-\frac{1}{\theta}} \tilde{t}'(a_0)\, da_0 & if \quad \beta \le a_0 \le \gamma \\ \Phi(t_2)\exp\left(\int_{a_0}^{\beta} \lambda n_l\left(\tilde{t}(a_0)\right)\tilde{t}'(a_0)\, da_0\right) & if \quad a_0 \le \beta \end{cases}.$$

From the definition of $\Phi(t)$ we know that this function gives the mass of sectors with an absolute productivity parameter below $A^{\max}(t_0)$ at date t. In terms of relative productivity coefficients, $\Phi\left(\tilde{t}(a_0)\right)$ gives us the mass of sectors with a relative productivity coefficient below a_0 and therefore, it is giving us the value of the distribution function of relative productivity parameters for a sector that innovated on date t_0. In the long run, almost all sectors will have innovated at least once and therefore $\Phi\left(\tilde{t}(a_0)\right)$, which can now be renamed $H(a_0)$, represents the cumulative distribution function of any sector with a relative productivity parameter between 0 and 1. The expression for $H(a)$ in (7) can be obtained replacing the size of the lagging group $\Phi(t_2)$ by a parameter ϕ, whose definition in terms of a is given by

$$\phi = \frac{\beta^{\frac{1}{\theta}}}{\beta^{\frac{1}{\theta}} - \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\theta}} \int_{\beta}^{\gamma} \lambda n_l(\tilde{t}(a)) \exp\left(-\int_a^{\gamma} \lambda n_h\left(\tilde{t}(a)\right)\tilde{t}'(a)\, da\right)\tilde{t}'(a)\, da}.$$

Observe that $H(a)$ does not depend on t and therefore it is time invariant. ■

Proof of Proposition 2. The distribution function in (8) is obtained following the same steps as in the previous proof except that in this case, the relationship between a_0 and t is given by

$$a_0 = \exp\left(-\int_{t_0}^{t} \sigma(1-\phi)\lambda n_h(s)\, ds\right), \tag{28}$$

and ϕ is implicitly defined by the following equation:

$$(1-\phi)\beta^{\frac{1}{\sigma(1-\phi)}} + \phi\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}} \int_{\beta}^{\gamma} \lambda n_l\left(\tilde{t}(a)\right)\exp\left(-\int_a^{\gamma} \lambda n_h\left(\tilde{t}(a)\right)\tilde{t}'(a)\, da\right)\tilde{t}'(a)\, da = 0. \tag{29}$$

■

Proof of Proposition 3. In order to find the derivatives of the growth rate with respect to θ, λ and s_h let us express equations (16) and (18) as follows:

$$(1-s_h)(\rho + (1+\theta)\lambda n_h) - \lambda\alpha(1-\alpha)L^{1-\alpha}k^{\alpha} = 0$$
$$\rho + \theta\lambda n_h + \delta + \tau_k - \alpha^2 L^{1-\alpha}k^{\alpha-1} = 0,$$

and denote then by $f_1(k, n_h; \theta, \lambda, s_h)$ and $f_2(k, n_h; \theta, \lambda, s_h)$ respectively. These functions may be considered as the components of a function $F : (0,\infty)\times(0,\infty) \to R^2$ and use the implicit function theorem to find the derivatives needed. The Jacobian of F with respect to k and n_h will be given by

$$J_F(k, n_h) = \begin{pmatrix} -\lambda\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-1} & (1-s_h)(1+\theta)\lambda \\ \alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} & \theta\lambda \end{pmatrix},$$

and its inverse is equal to the following expression:

$$[J_F(k, n_h)]^{-1} = \frac{1}{\det [J_F(k, n_h)]} \begin{pmatrix} \theta\lambda & -(1-s_h)(1+\theta)\lambda \\ -\alpha^2 (1-\alpha) L^{1-\alpha} k^{\alpha-2} & -\lambda\alpha^2 (1-\alpha) L^{1-\alpha} k^{\alpha-1} \end{pmatrix},$$

where $\det [J_F(k, n_h)] = -\lambda (1-\alpha) \zeta \left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)$. The Jacobian of F with respect to the parameters is given by

$$\begin{aligned} J_F(\theta, \lambda) &= \begin{pmatrix} (1-s_h)\lambda n_h & (1-s_h)(1+\theta) n_h - \alpha (1-\alpha) L^{1-\alpha} k^{\alpha} \\ \lambda n_h & \theta n_h \end{pmatrix}, \\ J_F(s_h, \tau_k) &= \begin{pmatrix} -\rho - (1+\theta)\lambda n_h & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

Implicit differentiation implies the following expressions for the derivatives of n_h and k with respect to the parameters:

$$\frac{dn_h}{d\theta} = -\frac{\left(\lambda + \frac{(1-s_h)}{k}\right) n_h}{\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}} \tag{30}$$

$$\frac{dn_h}{d\lambda} = \frac{\left(\frac{1-s_h}{k}\left(\frac{\rho}{\lambda}\right) - \theta\lambda n_h\right)}{\lambda\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)} \tag{31}$$

$$\frac{dn_h}{ds_h} = \frac{\rho + (1+\theta)\lambda n_h}{\lambda k\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)} \tag{32}$$

$$\frac{dn_h}{d\tau_k} = \frac{-1}{\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}}$$

$$\begin{aligned} \frac{dk}{d\theta} &= \frac{(1-s_h)\lambda^2 n_h}{\det(J_F)} \\ \frac{dk}{d\lambda} &= \frac{\theta\lambda\pi}{\det(J_F)} \\ \frac{dk}{ds_h} &= \frac{\theta\lambda(\rho + (1+\theta)\lambda n_h)}{\det(J_F)} \\ \frac{dk}{d\tau_k} &= \frac{-(1-s_h)(1+\theta)}{(1-\alpha)\zeta\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)}. \end{aligned}$$

Recall that the rate of growth is given by $g = \theta\lambda n_h$, but also, from $f_2(k, n_h) = 0$, we know that

$$g = \alpha^2 L^{1-\alpha} k^{\alpha-1} - \delta - \rho - \tau_k.$$

Therefore

$$\frac{dg}{d\theta} = -(1-\alpha)\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{dk}{d\theta} = \frac{\lambda n_h \frac{(1-s_h)}{\theta k}}{\lambda + \frac{(1-s_h)(1+\theta)}{\theta k}}$$
$$\frac{dg}{d\lambda} = -(1-\alpha)\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{dk}{d\lambda} = \frac{\theta\tilde{\pi}}{k\left(\lambda + \frac{(1-s_h)(1+\theta)}{\theta k}\right)}$$
$$\frac{dg}{ds_h} = \theta\lambda \frac{dn_h}{ds_h} = \frac{\rho + (1+\theta)\lambda n_h}{k\left(\lambda + \frac{(1-s_h)(1+\theta)}{\theta k}\right)}$$
$$\frac{dg}{d\tau_k} = \theta\lambda \frac{dn_h}{d\tau_k} = \frac{-\theta\lambda}{\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}},$$

where $\tilde{\pi} = \frac{\pi}{A^{\max}(t)}$. The first three derivatives are positive and the last one is negative. Thus, steady state growth is increasing in θ, λ and s_h and decreasing in τ_k. ■

Proof of Lemma 4. In a steady state, the distribution of relative productivity coefficients will be given by

$$H(a) = \begin{cases} \phi + (1-\phi) a^{\frac{1}{\theta}} & for \quad \gamma \leq a \leq 1 \\ \phi + (1-\phi) a^{\frac{1}{\theta}} - \phi \frac{n_l}{n_h}\left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right) & for \quad \beta \leq a \leq \gamma \\ \phi \left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}} & for \quad 0 \leq a \leq \beta \end{cases} . \tag{33}$$

where ϕ in a steady state is given by

$$\phi = \frac{\beta^{\frac{1}{\theta}}}{\beta^{\frac{1}{\theta}} + \left(1 - \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}}. \tag{34}$$

The derivative of ϕ with respect to λ will be determined by $\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right)$. Accordingly, let us perform this derivative first. From equation (17) $\frac{n_l}{n_h} = \gamma\tau - (1-\gamma\tau)\left(\frac{\rho}{\lambda n_h} + \theta\right)$. Therefore, $\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right) = \frac{(1-\gamma\tau)\rho}{(\lambda n_h)^2}\left(\lambda \frac{dn_h}{d\lambda} + n_h\right)$. Equation (31) allows us to write $\lambda \frac{dn_h}{d\lambda} + n_h = \frac{\frac{(1-s_h)}{k}(\rho + \lambda(1+\theta)n_h)}{\lambda\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)}$ which is positive. If $\frac{n_l}{n_h}$ increases with λ, then ϕ necessarily decreases.

In order to look for the derivative of ϕ with respect to θ, let us write ϕ as follows:

$$\phi = \frac{1}{1 + \left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right)\frac{n_l}{n_h}}.$$

Then,

$$\frac{d\phi}{d\theta} = -\phi^2\left[\frac{d}{d\theta}\left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right)\frac{n_l}{n_h} + \left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right)\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right)\right],$$

where

$$\frac{d}{d\theta}\left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right) = \frac{1}{\theta^2}\beta^{-\frac{1}{\theta}}\ln(\beta) - \frac{1}{\theta^2}\gamma^{-\frac{1}{\theta}}\ln(\gamma) \tag{35}$$
$$\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right) = -(1-\gamma\tau)\left(1 - \frac{\rho\frac{dn_h}{d\theta}}{\lambda(n_h)^2}\right). \tag{36}$$

Since $\beta < \gamma$ and $\frac{dn_h}{d\theta} < 0$, both (35) and (36) are negative, which implies that $\frac{d\phi}{d\theta}$ is positive.

The sign of $\frac{d\phi}{ds_h}$ will be determined by the sign of $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$. Hence, if the level of research intensity in lagged sectors relative to research intensity in leading sectors falls, then $\frac{d\phi}{ds_h}$ will be positive. From (32) n_h increases with s_h. Therefore, in order to prove that $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$ is negative, it is enough to show that $\frac{dn_l}{ds_h}$ is negative. Consider thus this derivative

$$\frac{dn_l}{ds_h} = \frac{-\left(\rho + (1+\theta)\lambda n_h\right)\left((1-s_h) + \gamma\tau\theta\lambda k\right)}{\lambda(1-s_h)\left(\theta\lambda k + (1-s_h)(1+\theta)\right)},$$

which is negative. Hence, since $\frac{dn_l}{ds_h}$ and $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$ are negative, $\frac{d\phi}{ds_h}$ is positive.

Similarly, the sign of $\frac{d\phi}{d\tau_k}$ will the determined by $\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}$ which is given by

$$\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k} = (1-\gamma\tau)\left(\frac{\rho}{\lambda n_h^2}\frac{dn_h}{d\tau_k}\right),$$

a negative expression. Therefore, $\frac{d\phi}{d\tau_k}$ is positive. ■

Proof of Proposition 5. Consider the steady state distribution of relative productivities given by equation (33). The effect of θ on $H(a)$ may be computed as

$$\frac{dH(a)}{d\theta} = \begin{cases} \frac{d\phi}{d\theta}\left(1 - a^{\frac{1}{\theta}}\right) + (1-\phi)\left(\frac{-\ln a}{\theta^2}\right)a^{\frac{1}{\theta}} & if \quad \gamma < a \leq 1 \\ \begin{array}{l} \frac{d\phi}{d\theta}\left(1 - a^{\frac{1}{\theta}} - \left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right) + (1-\phi)a^{\frac{1}{\theta}}\left(\frac{-\ln a}{\theta^2}\right) + \\ +\phi\frac{n_l}{n_h}\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\left(\frac{-\ln\left(\frac{a}{\gamma}\right)}{\theta^2}\right) - \left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\phi\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right) \end{array} & if \quad \beta < a \leq \gamma \\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\left[\frac{\frac{d\phi}{d\theta}}{\phi} + \ln\left(\frac{a}{\beta}\right)\frac{d}{d\theta}\left(\frac{n_l}{\theta n_h}\right)\right] & if \quad 0 \leq a \leq \beta \end{cases}.$$

The three pieces of this function are positive since $\left(1 - a^{\frac{1}{\theta}} - \left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right)$ is positive and both $\frac{d}{d\theta}\left(\frac{n_l}{\theta n_h}\right)$ and $\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right)$ are negative.[11] This implies that if we increase σ, the resulting distribution will attach a higher value to any $a \in (0,1)$. Therefore, if $\theta_1 < \theta_2$ then, $H_{\theta_1}(a) < H_{\theta_2}(a)$ for $a \in (0,1)$.

Similarly, the effect of λ on $H(a)$ will be given by

$$\frac{dH(a)}{d\lambda} = \begin{cases} \frac{d\phi}{d\lambda}\left(1 - a^{\frac{1}{\theta}}\right) & if \quad \gamma < a \leq 1 \\ \frac{d\phi}{d\lambda}\left(1 - a^{\frac{1}{\theta}} - \left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right) - \phi\left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right) & if \quad \beta < a \leq \gamma \\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\left[\frac{\frac{d\phi}{d\lambda}}{\phi} + \frac{\ln\left(\frac{a}{\beta}\right)}{\theta}\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right)\right] & if \quad 0 \leq a \leq \beta \end{cases}.$$

The three pieces are negative since $\frac{d\phi}{d\lambda}$ is negative, $\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right)$ is positive and $\ln\left(\frac{a}{\beta}\right)$ for $a < \beta$ is negative. Consequently, $\frac{dH(a)}{d\lambda}$ is negative for all values of a between 0 and 1. Therefore, if $\lambda_1 < \lambda_2$, the distribution

[11] The expression $\left(1 - a^{\frac{1}{\theta}} - \left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right)$ is positive if $\frac{n_l}{n_h} \leq 1$. A sufficient condition for $\frac{n_l}{n_h} \leq 1$ is $\gamma\tau \leq 1$, which is an assumption we have already made.

function associated to λ_2 will give smaller values to any $a \in (0,1)$ than the distribution function associated to λ_1. Therefore, $H_{\lambda_1}(a) > H_{\lambda_2}(a)$ for $a \in (0,1)$.

The proof for s_h is similar. Consider the derivative of $H(a)$ with respect to s_h

$$\frac{dH(a)}{ds_h} = \begin{cases} \frac{d\phi}{ds_h}\left(1-a^{\frac{1}{\theta}}\right) & if \quad \gamma < a \leq 1 \\ \frac{d\phi}{ds_h}\left(1-a^{\frac{1}{\theta}}-\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right)-\phi\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right) & if \quad \beta < a \leq \gamma \\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\left[\frac{\frac{d\phi}{ds_h}}{\phi}+\frac{\ln\left(\frac{a}{\beta}\right)}{\theta}\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)\right] & if \quad 0 \leq a \leq \beta \end{cases}.$$

Again, the three pieces are positive, since $\frac{d\phi}{ds_h}$ is positive, $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$ is negative and $\ln\left(\frac{a}{\beta}\right)$ is negative for $a < \beta$. Consequently, if $s_{h1} < s_{h2}$ then, $H_{s_{h1}}(a) < H_{s_{h2}}(a)$ for $a \in (0,1)$.

Similarly,

$$\frac{dH(a)}{d\tau_k} = \begin{cases} \frac{d\phi}{d\tau_k}\left(1-a^{\frac{1}{\theta}}\right) & for \quad \gamma \leq a \leq 1 \\ \frac{d\phi}{d\tau_k}\left(1-a^{\frac{1}{\theta}}\right)-\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\left(\frac{d\phi}{d\tau_k}\frac{n_l}{n_h}+\phi\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}\right) & for \quad \beta \leq a \leq \gamma \\ \frac{d\phi}{d\tau_k}\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}+\phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\frac{1}{\theta}\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}\ln\left(\frac{a}{\beta}\right) & for \quad 0 \leq a \leq \beta \end{cases}$$

is also positive because $\frac{d\phi}{d\tau_k}\frac{n_l}{n_h}+\phi\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k} = \phi^2\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}$ which is negative. ■

Proof of Proposition 6 and Lemma 7. The distribution function of relative productivity parameters in a steady state under the aggregate assumption is given by

$$H(a) = \begin{cases} \phi+(1-\phi)\,a^{\frac{1}{\sigma(1-\phi)}} & for \quad \gamma \leq a \leq 1 \\ \phi+(1-\phi)\,a^{\frac{1}{\sigma(1-\Phi)}}-\phi\frac{n_l}{n_h}\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right) & for \quad \beta \leq a \leq \gamma \\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\sigma(1-\phi)n_h}} & for \quad 0 \leq a \leq \beta \end{cases}.$$

where ϕ in this case is implicitly defined by the following expression:

$$(1-\phi)\,\beta^{\frac{1}{\sigma(1-\phi)}}-\phi\frac{n_l}{n_h}\left(1-\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right)=0.$$

Under the aggregate assumption, the system determining the steady state values of k, n_h and ϕ may be expressed as follows:

$$F(k,n_h,\phi) = \begin{pmatrix} f_1(k,n_h,\phi) \\ f_2(k,n_h,\phi) \\ f_3(k,n_h,\phi) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

where

$$f_1(k,n_h,\phi) = (1-s_h)\left(\rho+(1+\sigma(1-\phi))\,\lambda n_h\right)-\lambda\alpha(1-\alpha)\,L^{1-\alpha}k^{\alpha}$$

$$f_2(k,n_h,\phi) = \rho+\sigma(1-\phi)\,\lambda n_h+\delta+\tau_k-\alpha^2L^{1-\alpha}k^{\alpha-1}$$

$$f_3(k,n_h,\phi) = (1-\phi)\,\beta^{\frac{1}{\sigma(1-\phi)}}-\phi\frac{n_l}{n_h}\left(1-\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right).$$

The Jacobian of this function is given by

$$J_F(k,n_h,\phi)=\begin{pmatrix} -\lambda(1-\alpha)\zeta & (1-s_h)(1+\sigma(1-\phi))\lambda & -(1-s_h)\sigma\lambda n_h \\ \frac{(1-\alpha)\zeta}{k} & \sigma(1-\phi)\lambda & -\sigma\lambda n_h \\ 0 & -\frac{\phi(1-\gamma\tau)\rho\omega}{\lambda n_h^2} & \Psi \end{pmatrix},$$

where $\omega=\left(1-\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right)$ and

$$\Psi=-\beta^{\frac{1}{\sigma(1-\phi)}}\left(1-\frac{\ln\beta}{\sigma(1-\phi)}\right)-\frac{n_l}{n_h}\omega+\phi\frac{n_l}{n_h}\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\left(\frac{\beta}{\gamma}\right)}{\sigma(1-\phi)^2}-\phi\omega(1-\gamma\tau)\sigma$$

Notice that Ψ is negative. Consequently, the determinant of the Jacobian, given by

$$\det(J_F)=-(1-\alpha)\zeta\left(\lambda\Psi\left(\sigma(1-\phi)\left(\frac{1-s_h}{k}+\lambda\right)+\frac{1-s_h}{k}\right)-\left(\frac{1-s_h}{k}+\lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{n_h}\right),$$

is positive. In order to compute the derivatives for comparative statics we need the inverse of the Jacobian, that is

$$[J_F]^{-1}=\frac{-1}{\det(J_F)}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

where

$$\begin{aligned}
a_{11}&=-\sigma(1-\phi)\lambda\Psi+\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{n_h}\\
a_{12}&=(1-s_h)\left[(1+\sigma(1-\phi))\lambda\Psi+\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{n_h}\right]\\
a_{13}&=(1-s_h)\lambda^2\sigma n_h\\
a_{21}&=\frac{(1-\alpha)\zeta\Psi}{k}\\
a_{22}&=\lambda(1-\alpha)\zeta\Psi\\
a_{23}&=\lambda\sigma n_h(1-\alpha)\zeta\left(\lambda+\frac{1-s_h}{k}\right)\\
a_{31}&=\left(\frac{(1-\alpha)\zeta}{k}\right)\left(\frac{\phi(1-\gamma\tau)\rho\omega}{\lambda n_h^2}\right)\\
a_{32}&=\frac{(1-\alpha)\zeta\phi(1-\gamma\tau)\rho\omega}{n_h^2}\\
a_{33}&=(1-\alpha)\zeta\lambda\left(\sigma(1-\phi)\left(\lambda+\frac{(1-s_h)}{k}\right)+\frac{(1-s_h)}{k}\right).
\end{aligned}$$

Consider now the derivatives of the component functions with respect to the relevant parameters.

$$J_F(\lambda,\sigma,\tau_k)=\begin{pmatrix} \frac{-(1-s_h)\rho}{\lambda} & (1-s_h)(1-\phi)\lambda n_h & 0 \\ \sigma(1-\phi)n_h & (1-\phi)\lambda n_h & 1 \\ -\frac{\phi(1-\gamma\tau)\rho\omega}{\lambda^2 n_h} & X & 0 \end{pmatrix},$$

where

$$X=\beta^{\frac{1}{\sigma(1-\phi)}}\left(\frac{-\ln\beta}{\sigma^{2}}\right)+\phi\left(1-\phi\right)\left(1-\gamma\tau\right)\omega+\phi\frac{n_{l}}{n_{h}}\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\left(\frac{-\ln\left(\frac{\beta}{\gamma}\right)}{\sigma^{2}\left(1-\phi\right)}\right),$$

is positive. Applying the rules of implicit differentiation, we obtain the derivatives needed to establish the results of the proposition. With respect to the productivity of research the relevant derivatives are

$$\begin{aligned}\frac{dk}{d\lambda} &= \frac{\left(1-s_{h}\right)\sigma\phi\left(1-\gamma\tau\right)\rho\omega\left(\sigma\left(1-\phi\right)-\frac{\rho}{\lambda n_{h}}-1\right)+\tilde{\pi}\sigma\left(1-\phi\right)\lambda\Psi}{\det\left(J_{F}\right)}\\ \frac{d\phi}{d\lambda} &= -\frac{\left(1-\alpha\right)\zeta}{\det\left(J_{F}\right)}\left(\frac{\left(1-s\right)\phi\left(1-\gamma\tau\right)\rho\omega\left(\rho+\lambda n_{h}\left(1+\sigma\left(1-\phi\right)\right)\right)}{\lambda^{2}n_{h}^{2}k}\right),\end{aligned}$$

where $\tilde{\pi}=\frac{\pi(t)}{A^{\max}(t)}$. The derivative of capital intensity with respect to λ gives us the effect on growth because from $f_{2}\left(k,n_{h},\phi\right)=0$, we know that $g=\alpha^{2}L^{1-\alpha}k^{\alpha-1}-\delta-\tau_{k}-\rho$ and therefore,

$$\frac{dg}{d\lambda}=-\left(1-\alpha\right)\alpha^{2}L^{1-\alpha}k^{\alpha-2}\frac{dk}{d\lambda}.$$

The sign of $\frac{d\phi}{d\lambda}$ is immediate. With respect to the sign of $\frac{dk}{d\lambda}$, it will be negative if $\sigma\left(1-\phi\right)-\frac{\rho}{\lambda n_{h}}-1\leq 0$. Recall that we are assuming that the subsidy structure must be such that the research intensity of lagging sectors will never be larger than the research intensity of high-tech sectors. This implied an upper bound for $\gamma\tau$ of 1. Thus, if $\gamma\tau\leq 1$ then $\frac{n_{l}}{n_{h}}\leq 1$ which implies $\sigma\left(1-\phi\right)\leq 1$ and consequently $\frac{dk}{d\lambda}$ is negative and $\frac{dg}{d\lambda}$ is positive.

The derivatives with respect to σ are as follows:

$$\frac{dn_{h}}{d\sigma}=\frac{n_{h}\left(1-\phi\right)\left(\frac{1-s_{h}}{k}+\lambda\right)\left(\beta^{\frac{1}{\sigma(1-\phi)}}+\frac{n_{l}}{n_{h}}\omega\right)}{\Psi\left(\lambda\sigma\left(1-\phi\right)+\frac{(1-s_{h})}{k}\left(1+\sigma\left(1-\phi\right)\right)\right)-\left(\frac{(1-s_{h})}{k}+\lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{\lambda n_{h}}}$$

$$\frac{d\phi}{d\sigma}=\frac{\left(1-\alpha\right)\zeta\left(\frac{(1-\phi)\phi(1-\gamma\tau)\rho\omega}{n_{h}}\left(\frac{(1-s_{h})}{k}+\lambda\right)+X\lambda\left(\sigma\left(1-\phi\right)\left(\lambda+\frac{(1-s_{h})}{k}\right)+\frac{(1-s_{h})}{k}\right)\right)}{\det\left(J_{F}\right)}$$

$$\frac{dg}{d\sigma}=\lambda n_{h}\left(1-\phi\right)+\sigma\lambda\left(1-\phi\right)\frac{dn_{h}}{d\sigma}-\sigma\lambda n_{h}\frac{d\phi}{d\sigma}.$$

The derivative of research intensity with respect to σ is negative and $\frac{d\phi}{d\sigma}$ is positive. Thus, the sign of $\frac{dg}{d\sigma}$ is not immediate. Nevertheless, notice that

$$n_{h}+\sigma\frac{dn_{h}}{d\sigma}=\frac{\sigma\left(1-\phi\right)\left(\lambda+\frac{1-s_{h}}{k}\right)\left(\Psi+\beta^{\frac{1}{\sigma(1-\phi)}}+\frac{n_{l}}{n_{h}}\omega\right)+\frac{\Psi(1-s_{h})}{k}-\left(\frac{1-s_{h}}{k}+\lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{\lambda n_{h}}}{\Psi\left(\lambda\sigma\left(1-\phi\right)+\frac{(1-s_{h})}{k}\left(1+\sigma\left(1-\phi\right)\right)\right)-\left(\frac{(1-s_{h})}{k}+\lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{\lambda n_{h}}},$$

is negative because

$$\Psi+\beta^{\frac{1}{\sigma(1-\phi)}}+\frac{n_{l}}{n_{h}}\omega=\beta^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\beta}{\sigma\left(1-\phi\right)}+\phi\frac{n_{l}}{n_{h}}\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\left(\frac{\beta}{\gamma}\right)}{\sigma\left(1-\phi\right)^{2}}-\phi\omega\left(1-\gamma\tau\right)\sigma$$

is negative. Hence, $\frac{dg}{d\sigma}$ is also negative.

The derivative of n_h with respect to τ_k is negative while $\frac{d\phi}{d\tau_k}$ is positive. Therefore, $\frac{dg}{d\tau_k}$ is negative.

Let us consider now the effect of the two subsidies. The derivatives of the component functions with respect to s_h and s_l are given by

$$J_F(s_h, s_l) = \begin{pmatrix} -(\rho + (1+\sigma(1-\phi))\lambda n_h) & 0 \\ 0 & 0 \\ -\frac{(\rho+(1+\sigma(1-\phi))\lambda n_h)\phi\omega\gamma\frac{\partial\tau}{\partial s_h}}{\lambda n_h} & -\frac{(\rho+(1+\sigma(1-\phi))\lambda n_h)\phi\omega\gamma\frac{\partial\tau}{\partial s_l}}{\lambda n_h} \end{pmatrix}.$$

Notice that $\frac{\partial\tau}{\partial s_h} = -\frac{1}{1-s_l}$, therefore, $\frac{dn_h}{ds_h} > 0$. The derivative of ϕ with respect to s_h is not so immediate but it can be shown that it is equal to

$$\frac{d\phi}{ds_h} = \left(\frac{(\rho + (1+\sigma(1-\phi))\lambda n_h)}{\det(J_F)}\right)\left(\frac{(1-\alpha)\zeta\phi\omega\left[\frac{1}{k}\left(\frac{n_l}{n_h} + \sigma(1-\phi)\right) + \frac{\sigma(1-\phi)\lambda\gamma\tau}{1-s_h}\right]}{n_h}\right).$$

The derivative of the growth rate with respect to this subsidy is given by

$$\frac{dg}{ds_h} = -\left(\frac{\sigma\lambda(\rho + (1+\sigma(1-\phi))\lambda n_h)(1-\alpha)\zeta}{\det(J_F)}\right)\chi,$$

where

$$\chi = \frac{(1-\phi)\beta^{\frac{1}{\sigma(1-\phi)}}\left(\ln\beta^{\frac{1}{\sigma(1-\phi)}} - \frac{1}{\phi}\right)}{k} - \frac{\phi(1-\phi)(1-\sigma)\gamma\tau\omega\left(\lambda + \frac{1-s_h}{k}\right)}{1-s_h} + $$
$$+\left(\frac{\phi}{k}\right)\frac{n_l}{n_h}\left(\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\left(\ln\left(\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right) - 1\right) + 1\right).$$

The first two terms are negative but the last term is positive, which implies that the sign of this derivative will generally be ambiguous. However, the last term goes to zero as β approaches γ while the first term is increasing (in absolute value) in β. Therefore, if β is "sufficiently" close to γ, the whole derivative will be negative.

Regarding the steady state effects of an increase in s_l, we observe that

$$\frac{dn_h}{ds_l} < 0, \frac{d\phi}{ds_l} < 0 \; and \; \frac{dg}{ds_l} > 0.$$

The sign of the first two derivatives is immediate and the sign of the derivative of the growth rate with respect to this subsidy is obtained from

$$\frac{dg}{ds_l} = \sigma\lambda\left((1-\phi)\frac{dn_h}{ds_l} - n_h\frac{d\phi}{ds_l}\right) = \frac{\sigma\lambda\phi\omega\gamma\tau^2(\rho + (1+\sigma(1-\phi))\lambda n_h)(1-\alpha)\zeta}{k\det(J_F)},$$

therefore, a subsidy to lagged sectors will make the economy grow faster and reduce the mass of the lagging group. ■

B Dynamics

Proposition 8 *The dynamic system under the average assumption, defined by equations (13) and (14), presents local saddle path stability.*

Proof. In order to analyze the dynamics of the system let us express equations (13) and (14) as follows:

$$\begin{aligned} \dot{k}(t) &= \varphi(k(t), c(t)) \\ \dot{c}(t) &= \psi(k(t), c(t)). \end{aligned}$$

With this notation, we can compute the Jacobian of the system and evaluate it at the steady state. The derivatives needed are the following:

$$\begin{aligned} &\varphi_k(k,c) = \alpha L^{1-\alpha} k^{\alpha-1} - \frac{1}{E(a)}\left(\frac{dn_h(k)}{dk} + \frac{dn_l(k)}{dk}\right) + \frac{[n_h(k)+n_l(k)]\frac{dE(a)}{dk}}{[E(a)]^2} - \\ &\qquad - (\delta+g) - k(\frac{dg(k)}{dk}) \\ &\varphi_c(k,c) = -1 \\ &\psi_k(k,c) = c(-\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} - \frac{dg(k)}{dk}) \\ &\psi_c(k,c) = 0. \end{aligned}$$

The determinant of the Jacobian is equal to $\psi_k(k,c)$ which is negative since $\frac{dg(k)}{dk} = \theta\lambda\frac{dn_h(k)}{dk}$ and $\frac{dn_h(k)}{dk}$ is positive. Recall that $n_h(k(t))$ was defined by equations (9) and (11) as

$$n_h(k(t)) = \frac{(1-\alpha)\alpha L^{1-\alpha}[k(t)]^{\alpha}}{1-s_h} - \frac{\alpha^2 L^{1-\alpha}[k(t)]^{\alpha-1}}{\lambda}.$$

Therefore,

$$\frac{dn_h(k(t))}{dk(t)} = \frac{(1-\alpha)\alpha^2 L^{1-\alpha}[k(t)]^{\alpha-1}}{1-s_h} + \frac{\alpha^2(1-\alpha)L^{1-\alpha}[k(t)]^{\alpha-2}}{\lambda},$$

is positive for every positive value of k.

Given that the determinant of the Jacobian is negative, the system presents local saddle path stability.

■

Proposition 9 *The dynamic system formed by equations (13) and (14) under the aggregate assumption presents local saddle path stability.*

Proof. Since the equations of the system are the same as in Proposition 8, we know that the system will be local saddle path stable if the determinant of the Jacobian is negative. The determinant is given by

$$\psi_k(k,c) = c(-\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} - \frac{dg(k)}{dk}),$$

where

$$\frac{dg(k(t))}{dk(t)} = \sigma\lambda(1-\phi(k))\frac{dn_h(k(t))}{dk(t)} - \sigma\lambda n_h(k)\frac{d\phi(k(t))}{dk(t)}.$$

Thus, if $\frac{d\phi(k(t))}{dk(t)}$ is negative, $\frac{dg(k(t))}{dk(t)}$ will be positive, and $\psi_k(k,c)$ will be negative as we want to prove. The implicit function that defines ϕ as a function of k is given by (29), so let

$$F(k,\phi) = (1-\phi)\beta^{\frac{1}{\sigma(1-\phi)}} - \phi\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}} \int_{t_1}^{t_2} \lambda n_l(t)\exp\left(\int_{t_1}^{t} \lambda n_h(s)\,ds\right)dt = 0.$$

Then

$$\frac{dF}{d\phi} = \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}} \left(-\gamma^{\frac{1}{\sigma(1-\phi)}} + \gamma^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\gamma}{\sigma(1-\phi)} - \int_{t_1}^{t_2} \lambda n_l(t)\exp\left(\int_{t_1}^{t} \lambda n_h(s)\,ds\right)dt\right)$$

and

$$\frac{dF}{dk} = -\phi\lambda\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}} \int_{t_1}^{t_2}\left(\frac{dn_l(t)}{dk}\exp\left(\int_{t_1}^{t}\lambda n_h(s)\,ds\right) + n_l(t)\exp\left(\int_{t_1}^{t}\lambda n_h(s)\,ds\right)\int_{t_1}^{t}\lambda\frac{dn_h(s)}{dk}ds\right)dt.$$

Since $\frac{dn_l(k(t))}{dk(t)}$ and $\frac{dn_h(k(t))}{dk(t)}$ are both positive, $\frac{dF}{dk}$ is negative, and so is $\frac{dF}{d\phi}$ which implies that $\frac{d\phi(k(t))}{dk(t)}$ as given by

$$\frac{d\phi(k(t))}{dk(t)} = -\frac{\frac{dF}{dk}}{\frac{dF}{d\phi}},$$

is negative. Consequently, $\psi_k(k,c)$ is negative. ■