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The Cross Sectional Dependence Puzzle

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Abstract

The analysis of unit roots and cointegration in panel data is becoming a growing research area. A number of issues have been raised in the literature (see Phillips and Moon 1999 and 2000, Banerjee 2000, Maddala and Wu 1999). The aim of the present paper is to contribute to the issue of cross sectional dependence in non-stationary panel data. We review some of the most recent econometric techniques proposed by the literature to dealing with cross sectional dependence and notice a sort of puzzle. We extend the bootstrap methodology proposed by Maddala and Wu (1999) and apply the resulting test to test for PPP. We find no evidence favouring PPP. Finally, we use Monte Carlo simulation to analyse the size distortion of the bootstrap test presented in this paper. The proposed test presents size distortion only when T = 100

1 Introduction

The analysis of unit root and cointegration in panel data is becoming a growing research area. The emphasis of the literature is the attempt to combine information from the time series dimension with that obtained from the cross sectional dimension. The advantages of using information from both dimensions rather than just one, are, now well known. First, the power of unit root and cointegration tests increases notably if we combine information from both dimensions. Finally, spurious regression can be overcome by using panel data.

Most of the relevant asymptotic theory for panel data was developed for large cross sectional dimension (N), but small time series dimensions (T). However, recently, there has been an increase of the number of observations and this raises a number of issues. First, most economic time series are known to be non-stationary. The issue here is to develop asymptotic properties of panel estimators when data are non-stationary. Second, since we have large N and large T, there is the question of how to do the asymptotic analysis of N, T rather than just N. Several ways have been suggested. Phillips and Moon (1999, 2000) are two important papers, which attempt to deal with these issues. Third, since we have large T, it is possible to estimate each group separately. This kind of analysis raises a further issue. In fact, we can think that parameters can differ over groups. If this is the case, then we have heterogeneous panels. So instead of being forced to assume homogeneous parameters, as in the small T case, we can test the heterogeneous hypothesis. Fourth, all the panel unit root-cointegration tests, assume that each cross-section is independent from the other.

The aim of the present paper is to contribute to the issue of cross sectional dependence in panel data when the time series are I(1). In fact, as stressed in Banerjee (1999), a formal study of the relaxation of the cross sectional independence assumption is necessary since the asymptotic theory, so far, relies completely on that assumption. Different procedures, for dealing with cross sectional dependence, have been proposed. Im et al.(1997) propose subtracting cross sectional means from the observed data. This procedure works provided that cross sectional dependence is of weak memory variety. In fact in this case the central limit theorem, so important to derive the asymptotic distribution, will continue to apply. However, when there are strong correlations in a cross section (for example, in the presence of global shocks) we can expect failure in the central limit theorem (Phillips and Moon, 1999). Pedroni (1997) and O`Connell (1998) use feasible GLS corrections to deal with cross sectional dependence. However as shown in Cerrato (2001) these corrections are likely to be invalid because the estimator used to estimate the covariance matrix is, in this case, inconsistent. Finally, Maddala and Wu (1999) suggest bootstrap methods to deal with cross-sectional dependence.

Though all the above mentioned papers have proposed different methodology to deal with cross sectional dependence, they did not try to characterise it. As far as we know, the only papers which attempt to characterise cross sectional dependence is Bai (2001) and Chang (2001). The former models cross sectional dependence caused by common stochastic trends that are not observable and relying on principal component analysis, it shows that the number of common stochastic trends can be consistently estimated. However, in this case, principal components capture a mixture of omitted variables and global shocks, so this methodology cannot discern between different sources of cross sectional dependence. The latter paper attempts to model a different source of cross sectional dependence, that is omitted variables and deterministic trends and it suggests non-linear IV estimation. However, even if there seems to be a sort of puzzle between modelling cross sectional dependence induced by omitted variables and modelling cross sectional dependence induced by global shocks, the two above mentioned papers are a step in the right direction.

This paper, following Maddala and Wu (1999), suggests bootstrap procedure to deal with cross sectional dependence. We account for cross sectional dependence following the procedure suggested by Maddala and Wu, but we implement that bootstrap procedure in different ways and apply the resulting unit root test to test for Purchasing Power Parity. We show that bootstrap, if correctly specified, is a reasonable way of dealing with cross sectional dependence. At least, it seems to be the methodology that gives fewer headaches. The unit root test proposed in this paper cannot reject the unit root null hypothesis in the real exchange rate. Monte Carlo simulations on the proposed test confirm very little evidence of size distortion.

2. Cross-Sectional Dependence

A very important issue in panel unit root and cointegration tests is cross sectional dependence. In fact, the properties of all panel unit root and cointegration tests are based on the assumption that the error terms are not cross-correlated. In this section we highlight the implications of cross sectional dependence on the asymptotic distribution of panel unit root and cointegration tests.

As we mentioned in the introduction, the asymptotic and finite sample properties of many unit root and cointegration tests (Im. et al., Levin and Lin, Pedroni) have been derived under the assumption of zero error covariance, that is assuming $E(\varepsilon_t\varepsilon_t^{|})= \Omega$ is diagonal. If this assumption is relaxed, the derived distributions of panel unit root and cointegration tests are no longer valid and depend in a very complicated way upon various nuisance parameters leading to correlations across individual units, Chang (2001). In the above context, the distribution of unit root and cointegration tests will not be asymptotically non-stochastic. Then, it is evident that cross sectional dependence is a serious matter. If error terms are correlated[1] across the units, or to be more precise are not orthogonal, the variance-covariance matrix is likely to increase with the number of sectional units in the panel.

As noted in Cerrato (2001) cross sectional dependence can be caused by different factors. For example in the case of Purchasing Power Parity, cross sectional dependence can be caused by assuming the same numeraire currency, by omitted variables and by exogenous common shocks. This means that cross sectional dependence can be caused by model mis-pecification or also by common shocks. In

[1] Here, we prefer using the term "non correlated" rather than " independent"error terms. In fact it is imperative to differentiate non-correlation from independence. Let X and Y be two random variables. If for any functions $v=\Phi(X)$ and $z=\vartheta(Y)$; $f(\Phi(X),\vartheta(Y))=f_v(\Phi(X))\bullet fz(\vartheta(Y))$, for each $(v,z)\in R$, the two random variables are said to be independent. This means that if X and Y are independent, then any functions of these random variables are also independent. On the other hand, correlation is a different issue. In fact, broadly speaking, it defines a measure of linear dependence only. Hence, the general conclusion we reach is that if error terms are independent, they are non-correlated. On the other hand, if error terms are non-correlated, this does not imply that they are also independent. However, for simplicity in this paper we focus on non correlated error terms.

general, researchers have largely neglected modelling cross-sectional dependence, because it is often very complicated since individual observations across sections display no natural ordering. Nevertheless, many researchers (see Phillips and Moon 1999, Banerjee, 1999) have called for major research effort in this direction.

One of the first papers, which attempt to model cross sectional dependence[2] is Bai (2001). Other papers are also of great interest. For example, Chang (2001) that proposes an original way of dealing with cross sectional dependence. In fact, Levin and Lin (1993) and Im et al. (1997) derived the asymptotic distribution of their tests, through sequential asymptotic under the assumption of no cross sectional dependence, but they did not derive joint asymptotics for their tests. Chang (2001) proposes a test based on non-linear IV estimation of an ADF type regression on each individual cross section. In this test cross sectional independence is not imposed as in the mentioned tests, but it is reached by establishing asymptotic orthogonalities of the non-linear instruments used to construct the test statistic. However, we shall review more in detail this and other papers that deal with cross sectional dependence in the next section.

[2] However modelling cross sectional dependence is not always an easy task For example, Peasaran and Smith (1995) modelled cross sectional dependence by including, explicitly, amongst regressors an additional variable which accounted for cross sectional dependence. However this way is not always feasible (e.g in the case of Purchasing Power Parity).

3. Some Literature on Cross Sectional Dependence

One of the first papers, which explicitly attempt to deal with cross sectional dependence, is Im. et al. (1997). In this paper the authors, explicitly say that their procedure is no longer applicable when observations are not generated independently across groups. To allow for the possibility of correlated errors they propose a de-meaning procedure. Consider the following model:

$$\Delta y_{it} = a_i + b_i y_{i,t-1} + u_{it} \qquad \text{i=1,…,N,} \qquad \text{t=1,…,T} \tag{1}$$

if we assume that the error term is composed of two components:

$$u_{it} = q_t + e_{it} \tag{2}$$

a time specific effect and a random effect, which is independent across the sections, then to remove the effect of the common component in equation (2), we subtract cross sectional means from both sides of equation (1), that is:

$$a_i^* = a_i - N^{-1}\sum_{j=1}^{N} a_j \ , \quad y_{it}^* = y_{it} - N^{-1}\sum_{j=1}^{N} y_{it} \ ,$$

However the assumed form of homogeneous cross sectional dependence (see Cerrato, 2001 for details) represented by equation (2) is of little use. In fact, generally cross sectional dependence is often heterogeneous across sections (for example in the presence of global shocks, O` Connell, 1998). In these circumstances, the equation (2) takes the following form:

$$u_{it} = r_i q_t + e_{it} \tag{3}$$

In the presence of heterogeneous cross sectional dependence, O`Connell (1998) shows that feasible GLS (FGLS) estimator can restore orthogonality across the units. In fact, the cross sectional effect is completely captured by the off-diagonal element of the covariance matrix Ω, that is ω, and FGLS is invariant with respect to ω (see O`Connell, 1998 for details). Based on this procedure O`Connell proposes a panel unit root test. Many researchers, following O`Connell, use FGLS procedure to dealing with cross sectional dependence (Higgins and Zakrajsek, 2000, Coakley and Fuertes, 2000). FGLS relies on a consistent estimator of Ω. Generally the covariance matrix is estimated using OLS residuals. We can use residuals to estimate Ω because we assume that they consistently estimate the error term. If the OLS estimator, say η*, consistently estimates η, the residuals will consistently estimate the error term. But, it is not always obvious that η* will consistently estimate η. In the case of equicorrelated error terms, η^* is no longer a consistent estimator of η, the covariance matrix is not estimated consistently and the FGLS procedure breaks down[3] Cerrato (2001).

Maddala and Wu (1999) suggest an alternative way of dealing with cross sectional dependence. If error terms are correlated across the units, the derived distributions of many unit root and cointegration tests are no longer valid, or to be more precise, they are unknown. If this is the case, Maddala and Wu propose using the bootstrap distribution to make inference.

The bootstrap method is a resampling method. It works as follows: Let $(x_1, x_2, \ldots, x_n)$ be the original sample. Draw a sample of size n[4] from this sample with replacement, say $B_j=(x^*_1, x^*_2, \ldots, x^*_n)$. This is the bootstrap sample. Each x^*_i is randomly drawn from the given sample. If we do this many times and compute the estimator θ_j^* from each of the bootstrap sample B_j, we have a realisation of θ^*, and we use it to make inference. Of course we may alternatively decide to bootstrap residuals or bootstrap data. Assume we decide a bootstrap procedure based on bootstrapping residuals. Assume that our data generating process is the following:

[3] Note that the same criticism applies to all tests based on seemingly unrelated regression estimation (SURE). In fact, the efficiency of SURE estimation relative to OLS increases with the average absolute size of the error correlations and decreases with the average absolute magnitude of the correlations among the regressors across equations. However, Breuer et al. (2000) show that in a ADF context, when correlation among error terms leads to correlation among regressors, the efficiency gains from SURE are weakened.

[4] We may also decide to draw a sample of size m<n. The validity of an m out of n bootstrap sample is well documented.

$$\Delta y_{it} = n_i \Delta y_{i,t-1} + e*_{it}$$

since there are cross correlations among $e*_{it}$, instead of resampling $e*_{it}$, we resample $e*_t=[e*_{1,t}, e^*_{2,t},\ldots,e*_{n,t}]^{|}$. This procedure consists in resampling $e*_{it}$ keeping the cross sectional dimension fixed or in other words resampling a full column of the $[e*_{i,t}]$ matrix at a time.

However, it should be kept in mind that although the bootstrap often provides a better finite sample critical values for test statistics than does first-order asymptotic theory, bootstrap values are still approximations and are not exact. That is why we need Monte Carlo evidence on the numerical performance of the bootstrap as a means of reducing differences between the true and the nominal levels of tests[5]. Finally, as noted by Li and Maddala, (1996) "it is easy to jump on the computer and mechanically apply a certain bootstrap procedure when in fact the structure of the model suggests some other procedure for bootstrap data generation. It is also important to think about what statistic to bootstrap which depends on the particular problem and procedure for studentization. For this reason it is important to avoid some ready available canned programs".

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[5] Monte Carlo analysis of bootstrap tests is highly time consuming. However Davidson and MacKinnon (1999) suggest a Monte Carlo approach that is relatively cheap, under the conditions of asymptotic independence of the bootstrapped statistic and the bootstrap data generating process.

Chang (2001), proposes a unit root test for panels with cross sectional dependence. Consider the following regression:

$$y_{it} = a_i y_{it-1} + u_{it} \qquad \text{i=1,…,N; t=1,…,Ti} \qquad (4)$$

where i is the cross sectional unit and t the time period. Since T can differ across i, unbalanced panels are allowed. The hypotheses under consideration are that $a_i=1$ for all y_{it}`s in the above equation, against $a_i<1$ for some y_{it}. The error in the above equation is modelled assuming an AR(p_i) process as follows:

$$a^i(L)u_{it} = e_{it} \qquad (5)$$

where L is the lag operator. If the linear filter of the above process is represented by:

$$a^i(z) = 1 - \sum_{k=1}^{p_i} a_{i,k} z^k \qquad (6)$$

then, model 1 can be re-written as follows:

$$y_{it} = a_i y_{i,t-1} + \sum_{k=1}^{p_i} a_{i,k} u_{i,t-k} + e_{it} \tag{7}$$

since under the unit root null hypothesis $\Delta y_{it} = u_{it}$ equation (7) can be also written as follows:

$$\Delta y_{it} = a_i y_{i,t-1} + \sum_{k=1}^{p_i} a_{i,k} \Delta y_{i,t-k} + e_{it} \tag{8}$$

Using the equation (8) Chang (2001) constructs a unit root test based on IV estimation procedure. Strictly speaking the test is based on the ADF regression for each individual cross section, using as instruments non-linear transformations of the lagged levels. He shows that such a test is simply the standardised sum of the individual IV t-ratios. To deal with cross sectional dependence, he uses instruments generated by non-linear instrument generating function defined as $F(y_{i,t-1})$. The main result is that the limit distributions of the IV t-ratio statistics, that is proved to be normal, are crosssectionally independent, since the non linear instruments $F(y_{i,t-1})$ and $F(y_{j,t-1})$ are asymptotically uncorrelated[6].

The main difference between the panel unit root test proposed by Chang (2001) and the others proposed in the literature is that the former achieves asymptotic normality without imposing independence across sectional units, but relying on the asymptotic orthogonalities of the non-linear instruments, the latter obtain asymptotic normality under the assumption of no cross sectional dependence.

Two considerations are to be made on the described procedure. First, instruments for the lagged difference (i.e. $(\Delta y_{i.t-1},...,\Delta y_{i,t-1-p_i})$), are generated using the variable themselves. For the entire regressors the instruments are $(F(y_{i,t-1}),\Delta y_{i.t-1},...,\Delta y_{i,t-p_i})'$. This procedure is valid provided that $\text{cov}(\Delta y_{i,t-1},...,\Delta y_{i,t-p_i},e_{it})=0,$ that is there must be no correlation between the instruments and the error term. In practice, this assumption is likely to be violated. This could be a further explanation of why the test produces ambiguous results when the sample size is small.[7]

Second, and more important, the econometric methodology suggested by Chang (2001) is valid provided that cross sectional dependence is generated by omitted variables. But cross sectional dependence is a more complex issue. As we stressed, it

[6] However this test is found to be very sensitive to the specification of the cross sectional and time series dimensions. Furthermore, it produces ambiguous results if the autoregressive parameter is restricted to be homogeneous across individual units.

[7] Chang (2001) applies his non-linear IV method to test for PPP. When he applies his test to IFS and PWT data, he gets contradictory results. In fact, the test appears to support PPP only when the sample size is large. However, he does not report any test statistic on the orthogonality of the instrumental variables. As a consequence, it may well be that the independence condition is violated.

can be caused by different factors. In the PPP case, it may also be due to global shocks. Chang`s methodology does not account for this possibility.

Bai (2001) uses a different approach. Based on Hall et al. (1999a,b), he models cross sectional dependence through common stochastic trends. That is, he assumes cross sectional dependence to be caused by common factors (global shocks) and he shows that if this is the case, it is possible to estimate the common stochastic trends as well as the shocks themselves.

Consider the following model:

$$X_{it} = l_i^{'} F_t + e_t \qquad (10)$$

where F_t are the common stochastic trends, λ_i is a vector of cointegrating coefficients, and $l_i^{'} F_t$ the common components of X_{it}. Only X_{it} is observable and X_{it}, F_t are cointegrated. Call r the number of the true common trends, and assume that it is given. Then, for a single time series, equation (10) can be re-written as follows:

$$\begin{array}{cccccc} \underline{X_i} = & F^0 & \lambda_i^0 & + & \underline{e}_i \\ (T\times 1) & (T\times r) & (r\times 1) & & (T\times 1) \end{array}$$

for the panel data:

$$\underset{(T\times N)}{X} = \underset{(T\times r)}{F^0} \underset{(r\times N)}{\Lambda^{0'}} + \underset{(T\times N)}{e}$$

where $X=(\underline{X}_1,\ldots,\underline{X}_N)$. The goal, here, is to estimate r, F^0 and Λ^0. [8] To achieve this goal Bai`s methodology relies on 4 assumptions, that is (A) common stochastic trends, (B) heterogeneous cointegrating coefficients[9], (C) time series and cross section dependence and heteroschedasticity and (D) weak dependence between common trends and idiosyncratic errors. We do not intend to go into details here, because this would be beyond the aim of this paper. The reader interested in more details is therefore referred to Bai`s paper.

Estimates of F_t^k and Λ^k (here r is assumed given and equal to k) are obtained as follows:

[8] Note that in this paper we only describe the procedure used to estimate common trends and the true cointegrating coefficients. For the estimation of the number of trends (r), see Bai (2001) and Bai and Ng (2000).

[9] An important feature should be noted. Once cross sectional dependence is introduced explicitly in the model, it could make sense restricting the cointegrating vector to be homogeneous across sectional units. In fact as noted in Banerjee (1999) it is the common stochastic trend that impart homogeneity across the units of the panel. This result is also confirmed in Nelson and Sul (2001). In fact they use a Wald test of the homogeneity restrictions on the cointegrating vector first omitting heterogeneous trend and finally including heterogeneous trends. They show that there is evidence against homogeneity only when trends are omitted. The homogeneous/heterogeneous issue in the presence of cross-correlated

let the covariance matrix of X be Σ[10]. Then, the variance of a linear combination, say $\Psi^{'}X$ is $\Psi^{'}\Sigma\Psi$. Maximising this with respect to Ψ subject to a normalisation rule $\Psi^{'}\Psi/=I_r$, gives Ψ as the eigenvector of $\lvert X-\Psi I_r \rvert = 0$. If $v_1, v_2, \ldots v_k$ are the eigenvalues of Σ and $\Psi_1, \Psi_2,\ldots,\Psi_k$ are the corresponding eigenvectors then Ψ_k are mutually orthogonal and $var(\Psi^{'}_k X) = v_k$. If we order v_k in descending order, $v_1 > v_2 > \ldots v_k$, then we get the principal components as $\Psi_1^{'}X, \Psi_2^{'}X,\ldots,\Psi^{'}_k X$. Thus the principal components corresponding to the lowest v_k give the cointegrating vectors and those corresponding to the largest v_k give the common stochastic trends. Clearly, the above proposed methodology is the method of principal components. However, there are a number of drawbacks with this procedure as well. For example, the principal components often do not have economic meaning, so, the first problem would be how to interpret them. Since in the mentioned case we use principal components to account for cross sectional dependence, we may say that we are not interested in their economic interpretation. Furthermore, once we have estimated the common factors we have to determine which of these factors are important. To do this, it is necessary to establish the consistency property of the estimated common factors when both N and T are large. Although Bai derives the limiting distribution for the estimated common-stochastic trends, cointegrating coefficients, and common components, more work is needed on this issue. λ_i in equation 10 is assumed to be not random. If λ_i is random and is correlated with the common factors, Bai`s result will no longer hold. On the other hand, there is a practical issue also: which kind of cross sectional

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errors is on the agenda for future research. However if confirmed it would give an enormous contribution to the literature in this area.

[10] Note that to make the subject more simple, we assume $\Sigma=(\Sigma_\Lambda \ \Sigma_F)$, this would imply that the random matrix $(\Sigma_\Lambda \ \Sigma_F)$ has the same eigenvalues. Of course this is not the assumption made by Bai. In fact he assumes that the eigenvalues of the described $r\times r$ matrix are distinct with probability 1.

dependence are we dealing with using this methodology? To be more specific, in the case of Bai`s paper principal components may capture a mixture of omitted variables and global shocks, but they say nothing about which of them predominate.

There seems to be a sort of puzzle in the cross sectional issue. That is, we can deal with cross sectional dependence induced by omitted/global variables or by stochastic trends, but we cannot discern between them. The effort made by Chang (2001) and Bai (2001) to explicitly model one or another form of cross sectional dependence is surprising, it is a step in the right direction, but as we stressed, their methodologies are subject to different problems. We believe that, so far, the methodology that gives fewer headaches is still bootstrap. In the next section we present a bootstrap methodology that is robust to cross-sectional dependence.

4 Botstrapping or not Bootstrapping?

Consider the following model:

$$y_{i,t} = b_i y_{i,t-1} + e_{i,t} \tag{11a}$$

The above model is clearly a panel model. For simplicity assume i=1. The model is clearly an AR(1) model. Three hypotheses are possible, (a) $|\beta|<1$ (b) $|\beta| = 1$, and (c) $|\beta|>1$. In the first case we say that the AR process is stationary, the second case it is known as unit root, and the last one is known as an explosive process. Here we are interested in the second case. That is we assume that the process contains a unit root.

The estimated counterpart is:

$$y^*_t = b_0^* \; y_{t-1} + e^*_t \tag{11b}$$

Define the vector of bootstrap residuals generated by the OLS regression errors as $\varepsilon^{\bullet}$[11]. The hypotheses under consideration in this case are H_o: $\beta_0^* = 1$ and H_1: $\beta_0^* < 1$. Under the null hypothesis $\beta_0^* = 1$ equation (11b) can be re-written as follows:

$$y^{\bullet}_t = y^{\bullet}_{t-1} + e^{\bullet}_t \tag{12}$$

Equation (12) is the sampling scheme S_2 suggested by Li and Maddala (1996). In this paper we use S_2 to generate our bootstrap sample.

Bootstrapping an AR (1) process as the one represented by equation (11) is quite straightforward, in the sense that all we have to do is to generate pseudo data using the bootstrap residuals and the scheme in equation (12) by a recursive procedure assuming as an initial value $y_0 = 0$. However, the described procedure is appropriate if the process{ y_t } can be represented as a first order autoregressive process. If not, an alternative procedure must be used. Maddala and Wu (1999) suggest the following procedure: generate pseudo-data: $u_t^{\bullet} = b_0^* u_{i,t-1}^* + e_{i,t}^{\bullet}$ conditional on $u_0 = \sum_{j=0}^{m} b_0^j e_{-j}^{\bullet}$ [12]. Finally, generate bootstrap sample using the sample scheme S_2 , that is

$$y^{\bullet}{}_t = y^{\bullet}{}_{t-1} + u^{\bullet}{}_t$$

Strictly speaking the procedure suggested by Maddala and Wu to build up u_0 is to pick it up from the estimated moving average (MA) representation. However the suggested procedure encounters two practical difficulties. First it is well known that estimation of MA time series models is not as straightforward as the estimation of the AR models. Second it requires the truncation of an infinite sum, Berkowitz and Kilian (2000). In this paper we follow an alternative way that has been suggested by Berkowitz and Kilian (2000). We pick up arbitrary values for u_0 in the recursion:

$$y_t = b *_0 y_{t-1} + e^{\bullet}{}_t$$

[11] Refer to appendix 2 to see how we account for cross sectional dependence using bootstrap.
[12] In this case we cannot assume $u_0 = 0$ as initial value of the process $\{u_t\}$. In fact this strategy is feasible only in the case we analyse processes that contain a unit root.

after discarding the start-up transients for { $y_{t\,t}$ }[13]

5 Empirical Results

In this section we use two different methodologies to account for cross-sectional dependence. To have a benchmark against which we can evaluate our test statistic, we use the widely used t-test proposed by Im. et al. (1997), with and without the adjustment described in section 3. Finally, we use a bootstrap methodology, that is an extension of the one proposed by Maddala and Wu (1999), to obtain p-values for our test. We apply these tests to two different panels to test for Purchasing Power Parity (PPP).[14]

We include only an intercept in the PPP specification. In fact, following Papell (2000) we do not include a time trend, because such an inclusion would be inconsistent with long-run PPP. Also, we use the recursive t-statistic procedure as described by Campbell and Perron (1991), to select the lag length in the ADF specification.

The t-bar statistic, for both the CPI and WPI series is respectively –1.90 and –0.52 (5% critical values are –1.87 and –1.97). If we use the adjustment described in section 3, the t-bar statistic for both CPI and WPI series is now –4.41 and –2.17 (1% and 5% critical values are –1.97 and –1.84 for CPI data and –2.15 and –1.97 for WPI data). These results, strongly reject the unit root in both the panels and are in line with the

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[13] The proposed algorithm is programmed in Matlab 5.0 and run on a 333 Mhz Pentium II. B is set equal to 2000.
[14] see appendix 1 for more details on data used.

findings on PPP by Wu (1996), Oh (1996). Taken as a whole the above results seem to suggest that the real exchange rate is mean reverting in the long-run. The issue we raise at this stage is a methodological one. Is mean reversion due to effective stationarity of the real exchange rate or is it due to neglecting cross-sectional dependence? To answer this question we use a panel unit root test (tables 1 and 2), where the p-values have been calculated using a procedure that is robust to cross sectional dependence.

[Tables 1 and 2 around here]

We use the previously cited recursive procedure to select the lag length in the ADF specification. We note that using the ADF test we cannot reject the null hypothesis for all CPI exchange rates except Mexico (at 5% significance level). The test statistic is calculated at the bottom of the final columns in the tables 1 and 2 (50.44 for CPI real exchange rate and 16.07 for WPI real exchange rate). Since, as Maddala and Wu (1999) show, this test is distributed as $\chi^2(2N)$, for our panels we have $\chi^2(40)$ and $\chi^2(20)$ respectively, which give a 5% critical value of 55.76 and 31.41. For the panels as a whole, we cannot reject the unit root null[15]

The above result is not a surprise. In fact, O`Connell (1998) talked about “overvaluation of PPP”. He argued that evidence favouring PPP is mainly due to the fact of neglecting cross sectional dependence. He concluded that once we account for cross sectional dependence, evidence favouring PPP disappears. This paper confirms that result.

[15]The same qualitative results are obtained with a homogeneous lag length.

6 Size Analysis

In section 3 we presented different econometric-statistic procedures to dealing with cross sectional dependence and we highlighted for each of them some pitfalls. With regard bootstrap we stressed the necessity of running Monte Carlo simulations in order to analyse the size distortion of a bootstrap test. However such an experiment is very time-consuming, because each replication requires the calculation of B+1 test statistics if B bootstrap samples are used. Davidson and MacKinnon (1998) show that it is possible to estimate the size distortion of a bootstrap test by running a cheap Monte Carlo simulation, provided that the condition of asymptotic independence of the bootstrapped statistic and the bootstrap data generating process (DGP) holds. In this section, after a brief presentation of the Monte Carlo analysis suggested by Davidson and MacKinnon (DM)[16], we use the suggested statistical methodology to analyse the size distortion of the bootstrap test presented in this paper.

The fundamental idea of Davidson and MacKinnon is based on the fact that we can estimate the size distortion of a bootstrap test using Monte Carlo experiments, relying on two simple concepts, that is “error in rejection probability” (ERP) and “rejection probability” (RP) of the bootstrap test. The former represents the size distortion of a bootstrap test, the latter gives the rejection probability of the asymptotic test.

Consider a data-generating process (DGP), a set of DGPs form what we call a model M. A generic element, or DGP, of a model M will be denoted as μ. A test statistic λ is

[16] For more details on this statistical technique see Davidson and MacKinnon (1998,2000).

said to be asymptotically pivotal if its distribution is the same for each DGP $\mu \in M$. We denote by l^* the realisation of l calculated from data generated by some unknown DGP μ_0. The DM procedure works as follows. For each of M replications indexed by m, draw a sample from μ_0 and use this sample to draw a realisation of the statistic l and the bootstrap DGP μ_m^*. Now, draw another sample from μ_m^*, and use it to compute a realisation of l_m^*. The quantile $Q(a, m_0)$ is estimated by $Q_0^*(a)$, the α quantile of the drawings of l. If we perform m replications the simulated estimate of RP and the corresponding ERP are given by:

$$RP_2^* = 2a - \frac{1}{M}\sum_{m=1}^{M} I(l_m^* < Q_0^*(a)) \quad \text{and} \quad ERP_2^* = a - \frac{1}{M}\sum_{m=1}^{M} I(l_m^* < Q_0^*(a)) \quad (13)$$

However, since the above estimator of the rejection probability is not guaranteed to be positive, DM suggest to use a more accurate estimate in which λ and λ* are interchanged. The procedure is the same as the one described above but the (ERP) is estimated as the proportion of drawings of l less than $Q^*(a)$, the α quantile of l^*, minus α:

$$RP_1^* = \frac{1}{M}\sum_{m=1}^{M} I(l_m < Q^*(a)) \quad \text{and} \quad ERP_1^* = \frac{1}{M}\sum_{m=1}^{M} I(l_m < Q^*(a)) - a \qquad (14)$$

As suggested in DM (2000) since very little effort is needed to compute (12) and (14), it makes sense to compute both, since substantial difference between the two estimated ERPs may indicate that neither of them is accurate.

The DGP used, in this experiment, is the following linear AR equation:

$$\Delta y_{i,t} = a + p_i y_{i,t-1} + e_{i,t}$$

We consider the above model, assuming that p=0.98[17], 0.75, 050 and e_t ~N (0,1). We use different values of T, that is we consider our test when T= 325 and T=100. For each combination of (p,T) the number of replication is set to 2000. The nominal significance level (α) is set to 0.05. The size estimates from the DM approach are reported in tables 3 (A, B, C) and 4 (A, B, C).[18] The empirical size of the test should not greatly exceed the nominal significance level. To allow for some random

[Table 3 A-C around here]

variation we form a confidence interval of the simulated size having length α $\pm 1.96 \cdot d_{\alpha}$, with $d_a = \sqrt{a(1-a)/M}$. Since M=2000 and α = 0.05 the confidence interval is {0.059; -0.04}. From tables 3 and 4 we can see that most of those values fall within these limits. With T=325 the empirical size of the test seems to be

[17] Since 0,98 is statistically indistinguishable from 1, in this case we assume that the process under consideration contains a unit root. However, we also consider our test statistic when the process contains roots that lie outside the unit circle.
[18] the proposed algorithm is programmed in Matlab 5.0 and run on a 333 Mhz Pentium II. B is set equal to 2000

reasonable regardless of the value of p. With T=100, the test seems to suffer of very small size distortion when p = 0.98, but the size distortion increases the smaller p.

[Table 4 A-C around here]

Summarising, this experiment suggests that the empirical size of our test matches the nominal size pretty well. We believe that the results provided by our test can be reasonable trusted.

Concluding Remarks

Panel data econometrics is a growing research area. The asymptotic theory for unit root and cointegration tests is derived under the assumption of cross sectional independence across individual units. This paper provides an outline of recent developments in the field of cross sectional dependence in panel unit root and cointegration tests.

We analyse the most recent econometric techniques proposed by the literature to dealing with cross sectional dependence and notice a sort of puzzle. That is, they deal either with cross sectional dependence caused by common stochastic trends or with cross sectional dependence caused by omitted variables, but they do not account for both. At least, they cannot discern between different sources of cross sectional dependence. In this paper we use bootstrap. We think that bootstrap is still a feasible way of dealing with cross sectional dependence.

Our bootstrap methodology is an extension of the Maddala and Wu`s (1999). We use this methodology to account for cross sectional dependence in real exchange rates and then we apply our test to test for long-run PPP. We find no evidence favouring long-run PPP.

We believe that unit root and cointegration tests reject long-run PPP essentially because they do not fully account for cross sectional dependence. In fact, cross sectional dependence by imparting a common signal across sectional units is likely to increase the probability of a type 1 error.

Appendix 1.

The Data

Our data consists of monthly bilateral exchange rates using the US$ as numeraire and the wholesale (WPI) and consumer price (CPI) indices for two different panels. For the CPI data set, we use G20 countries, for WPI, due to the availability of data, we use a smaller panel. Only 10 countries. Furthermore, we note that the two panels considered in this study, also span through two different periods. While the CPI series span the period January 1973 to January 2000, the WPI series span the period January 1981 to October 1999.

Nominal exchange rates are end-of-period from Datastream.

Appendix 2.

Bootstrap Methodology

We want to bootstrap the following ADF test in a panel context:

$$\Delta y_{i,t} = a_i + b_i y_{i,t-1} + \sum_{j=1}^{k_i} r_k \Delta y_{i,t-j} + e_{i,t}$$

To achieve this goal, we generate our bootstrap distribution assuming the following data generating process (DGP):

$$\Delta y_{i,t} = +h_i \Delta y_{i,t-1} + e_{i,t} \qquad (1)$$

1) Each $y_{i,t}$ in (1) is modelled as a unit root process. The individual equations of the DGP in (1) are fitted by least squares and residuals ($e^{\times}_{i,t}$) computed.

2) The bootstrap innovations $e_{i,t}^{*}$ are obtained by resampling with replacement from the empirical residuals. In this case, since we want to account for cross correlations among innovations, Maddala and Wu (1999) suggest resampling with the cross-section index fixed. We get the N-dimensional vector of bootstrap innovations $e_{,t.}^{*} = (e_{1,t}^{*}, \ldots e_{Nt}^{*})^{|}$. In the case where $e_{i,t}$ are AR processes (the ADF test) we first generate pseudo-data as follows:

3) Generate pseudo-data according to the following scheme

$$u^*_{i,t} = n^*_k u^*_{i,t-1} + e^*_{i,t}$$

where η_k^* is computed from estimation of equation (1) and $e_{i,t}^*$ is generated as in (2)

Note that it is not appropriate in this case to condition on $u_{i,0}$ in order to generate $u^*_{i,t}$. One way out of this problem is conditioning $u^*_{i,t}$ on a set of initial conditions Berkowitz and Kilian (2000). The problem in this case can be overcome selecting, arbitrary, values for $u^*_{i,0}$ in the recursion $y^*_{i,t} = h^*_k y^*_{i,t-1} + e^*_t$. The bootstrap sample is generated as follows:

4) $$y^*_{i,t} = y^*_{i,t-1} + u^*_{i,t} \quad \text{with } y^*_{i,t} = 0$$

In this case it makes sense to set the initial value of $y^*_{i,t} = y_{i,0}$. In fact as Dickey and Fuller show, if the DGP contains a unit root the test statistic depends on $y_{i,0}$, and α (if intercept is included) (see Dickey and Fuller, 1981 for more details).

The proposed resample scheme has been suggested by Maddala and Kim (1998) and Li and Maddala (1996). They suggested the resample scheme S_2. Briefly, if the null hypothesis is H_0:β= :β_0 versus H_1: $\beta \neq \beta_0$, they suggest using the following scheme $y^* = \beta_0 x + \varepsilon^*$.

5) Run the ADF test using the bootstrap sample. This yields a realisation of t*, where t*= (β*-1)/SE(β*).

6) Repeat 2-6 B (number of bootstrap replicates) times and the collection of realised t* statistics form the bootstrap distribution of these statistics under the null hypothesis.

Table 1

Panel Unit Root Test-CPI-RER

Series	Lags	t-statistic	P-values	Ln (P)
Austria	1	-2.2529	0.176	-1.7344
Denmark	2	-2.0037	0.282	-1.2658
Belgium	2	-1.6792	0.427	-0.8509
France	0	-1.9015	0.315	-1.1536
Germany	1	0.4896	0.989	-0.0111
Italy	0		0.334	-1.0951
Netherl.	2	-1.9587	0.314	-1.1584
Norway	2	-2.1575	0.228	-1.4784
Portugal	6	-1.4244	0.548	-0.6015
Spain	2	-1.0304	0.463	-0.77
Canada	6	-1.8574	0.329	-1.1117
Sweden	6	-1.3339	0.629	-0.4628
Switzerl.	6	-1.9834	0.252	-1.3783
UK	1	-2.4869	0.13	-2.0402
New Zeal.	6	-2.1704	0.215	-1.5348
Japan	0	-2.0106	0.19	-1.6607
Greece	4	-1.71	0.425	-0.8557
Finland	0	-1.7968	0.393	-0.9339
Ireland	4	-2.3964	0.148	-1.9045
Mexico	6	-3.0131	0.04	-3.2189
SUM Ln(P)				-25.2207
Panel test				50.4414
Chi-square (40)-5%				55.76

Table 2

Panel Unit Root Test-WPI-RER

Series	Lags	t-statistic	P-value	Ln(P)
Austria	1	-1.68	0.4385	-0.8244
Belgium	0	-2.15	0.2055	-1.58231
Denmark	0	-1.38	0.5495	-0.59875
Germany	0	-1.35	0.598	-0.51416
Italy	0	-1.49	0.518	-0.65778
Netherl.	0	-1.05	0.6845	-0.37907
Norway	2	-2.17	0.207	-1.57504
Spain	0	-1.4	0.5395	-0.61711
Switzerl.	0	-1.51	0.517	-0.65971
Ireland	2	-1.44	0.5345	-0.62642
SUM LN(P)				-8.03475
Panel test				16.06949
Chi-sq.	(20)-5%			31.41

Table 3

(A)

McKinnon-size test.		
DM-simualtion	T=325	p=0.98
RP2	0.057	
ERP2	0.007	
RP1	0.059	
ERP1	0.009	

(B)

McKinnon-size test		
DM-simulation	T=325	p=0.75
RP2	0.06	
ERP2	0.001	
RP1	0.06	
ERP1	0.001	

(C)

McKinnon-size test		
DM-simulation	T=325	p=0.50
RP2	0.04	
ERP2	-0.008	
RP1	0.04	
ERP1	-0.008	

Table 4

(A)

MacKinnon-size test			
DM-simulation		T=100	p=0.98
RP2	0.06		
ERP2	0.011		
RP1	0.06		
ERP1	0.009		

(B)

MacKinnon-size test			
DM-simulation		T=100	p=0.75
RP2	0.06		
ERP2	0.014		
RP1	0.06		
ERP1	0.013		

(C)

MacKinnon-size test			
DM-simulation		T=100	p=0.5
RP2	0.07		
ERP2	0.017		
RP1	0.07		
ERP1	0.016		

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