Financial Stress and Liquidity Traps

David Llewelyn Aikman
Department of Economics
University of Warwick, U.K.
D.L.Aikman@csv.warwick.ac.uk

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Abstract

Motivated by the bubble-collapse cycle witnessed in Japanese asset prices since the late 1980s, this paper examines how a …nancial crisis in‡uences the power of monetary policy. We construct a simple macroeconomic model based on the microfoundations of Hölmstrom and Tirole (1997) to analyse the e¤ect of three types of …nancial stress on the nature of the equilibrium: a credit crunch; an adverse collateral shock; and a monitoring cost shock. Perhaps surprisingly, we …nd that the power of monetary policy is, if anything, heightened in a credit crunch; higher monitoring costs however work in the opposite direction, suggesting a need for more aggressive stabilisation policy in the face of …nancial shocks.

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1 Introduction

Why is it, that a decade on from the complete collapse witnessed in asset prices, the Japanese economy is still experiencing a "growth recession" with actual output significantly below capacity? Why is it, that even with short term nominal interest rates slashed right down to their zero bound, the Japanese private sector cannot be convinced to spend sufficiently to fill this gap? This paper develops a simple theoretical model to study these fundamental, yet not – we argue – satisfactorily resolved issues.

The seminal work in this field is of course that of Paul Krugman (see Krugman, 1998 a,b for example), the well-known thesis maintained in these papers needing no repetition here. It can be argued, however, that this important and influential research is actually based on the wrong model, there being at least two aspects which leave students of Japan's macroeconomic performance over this period feeling uncomfortable. First, Krugman's baseline model focuses entirely on inadequate private sector consumption as the cause of this weak aggregate demand. A simple eyeballing of the data, however, strongly suggests weak private business investment as the main culprit.

Now, correlation doesn't prove causation of course, especially once we allow for accelerator effects from output onto investment. Nevertheless, a number of papers, both in academic and policy circles as well as in the financial press, support the weak business investment hypothesis. Ramaswamy and Rendu (2000) for example find that adverse shocks to business and residential investment have been the main determinants of the overall growth slowdown; private consumption shocks according to their results played a relatively minor role. Similar results are found by Bayoumi (2000), while Kiyotaki and West (1996) and Motonishi and Yoshikawa (1999) also focus their analysis on this component of demand.

A second and equally important issue concerns the plausibility of the liquidity trap-triggering shock[1] in Krugman's framework: a fall in the expected level of full capacity output. The possibility that demographic change in an advanced industrial economy may actually cause the production possibility frontier to shift inwards through time is often regarded with extreme pessimism, given historical rates of technical progress and labour productivity

[1] The lack of a clear consensus among the profession as to the cause of Japan's present difficulties is certainly consistent with the lack of attention this issue has received in most papers (see for example Krugman, 1999, Bernanke, 2000, Svensson, 2000, McCallum, 2000).

increases. Besides, why focus on a twenty year demographic shock with such an obvious alternative candidate at hand, namely the almost seventy percent fall in the Nikkei 225 Average (at the time of writing, the Nikkei has plummeted to a sixteen year low of 11,819) over the last decade or so and the equally precipitous collapse witnessed in land prices?

If we take these shortcomings seriously, the pressing question from a macroeconomic policy viewpoint is this: is it the case of wrong model, right message, or does the appropriateness of an inflation targeting policy require a radical rethink? This paper attempts to address these issues within the context of an asymmetric information model of the credit market with credit constraints and balance sheet effects. The central message can easily be conveyed in simple IS-LM terms. In an asymmetric information framework, the position and slope of the economy's IS curve can generally be influenced by financial factors[2]. Under such conditions, it is not hard to imagine how a massive correction in asset values, weakening the capital positions of both borrowers and lenders, could shift IS (drawn as a function of the safe real interest rate) leftwards, causing the low interest rate-low output conjuncture currently seen in Japan.

The paper also examines the issue of whether monetary policy - by which, in this paper I mean policy aimed at adjusting real interest rates, be it through nominal interest rate targeting combined with price stickiness, or when nominal interest rates hit their zero floor, through inflation targeting - retains its effectiveness in influencing demand in the face of such a shock. It often appears to be taken for granted in the literature that the effectiveness of this instrument is necessarily reduced by a financial shock (see Ramaswamy and Rendu (2000) for example), or in the limit, that financial stress can be a distinct cause of monetary impotence (Hutchinson, 2000).

Such a debate is fundamentally about the slope of the IS schedule: a steeper or in the limit vertical IS being consistent with the preceding statements. Contrary to our prior, we find that a credit crunch actually increases the sensitivity of investment to a change in the safe rate implying a flattening of IS, while an adverse shock to firm capital leaves the slope unchanged. Neither phenomenon it seems provides an explanation of monetary impotence per se. An increase in the severity of the informational asymmetry on the other hand tends to make IS steeper.

Based on the microfoundations of Hölmstrom and Tirole (1997), we construct a simple partial equilibrium macroeconomic model to analyse the effects of different types of financial stress on the nature of the equilibrium.

[2] A result noted, though not formally modelled, by Bernanke and Lown (1991) and Hayakawa and Maeda (2000).

With its emphasis on the role of net worth when both entrepreneurs and banks are subject to moral hazard problems vis-à-vis their respective lenders, this model provides a convenient analytical framework for studying the effects of a bubble collapse on business investment. A model related to ours, but with a very different perspective, is that of Repullo and Suarez (1999). The authors model monetary policy in a Hölmstrom and Tirole-type environment (albeit with a slightly richer moral hazard specification) similar to ours, although they do not assess how financial shocks effect the power of this policy.

The rest of the paper is organised as follows. Section 2.1 begins by outlining in some detail Hölmstrom and Tirole (1997); readers already familiar with this model may wish to skip directly to Section 2.2, which describes the monetary transmission mechanism. The key results of the paper are presented in section 2.3 while section 3 concludes. A general characterisation of the IS equation is contained in Appendix A, Appendix B detailing proofs of the main propositions.

2 The Model

2.1 An Outline of the Hölmstrom and Tirole (1997) Model of Financial Stress

The model has three types of agents: firms; financial intermediaries (which, for our purposes, can be thought of as banks); and investors, all assumed to be risk neutral with limited liability.

Firms In the real sector, there exists a continuum of firms, each endowed with a risky project of size I. Firms differ only in their capital endowments[3], A, as described by the density function $f(A)$ on the support $[0; A_{max}]$, $A_{max} < I$; $I - A$ therefore being the quantity of external finance required. The revenue generated (assumed to be common knowledge) by a successful investment is R, while that on a failure is zero. Firm's manager's face a moral hazard problem: three versions of the project are open to selection, each offering different degrees of non-verifiable private benefit to the manager (independently of whether the project succeeds or not), 0, b, B $(B > b > 0)$, with associated probabilities of success in the investment of p_H, p_L, and p_L $(\Delta p \equiv p_H - p_L)$; managers may therefore deliberately choose to reduce the probability of success in order to enjoy this private benefit, or put differently,

[3] A firm's capital endowment takes the form of cash, or any type of asset that can serve as collateral.

the project selection decision is non-contractable by assumption. Notice that this structure implies that the firm's managers will always prefer project-B to project-b when choice in not observed.

Financial Intermediaries The financial sector consists of a large number of banks, whose function it is to monitor firms and thereby alleviate the moral hazard described above. Specifically, it is assumed that monitoring by a bank eliminates the B-projects available to the firm's managers, thus reducing their opportunity cost of acting diligently. Monitoring however is privately costly for the bank: elimination of the B-project entails the non-verifiable cost c ($c > 0$) and so each bank faces a moral hazard problem of its own. This moral hazard problem on the supply-side of the credit market requires the bank to inject some of its own capital into the project, and the endogenously determined rate of return on these funds, ¯, will have to be sufficiently high in order for the bank to have the correct incentives to monitor. The aggregate quantity[4] of bank, or "informed", capital K_m, assumed to be exogenous[5], then becomes an important constraint on the level of aggregate investment.

Uninformed Investors Finally, "uninformed" investors - defined as a group by their inability to access the monitoring technology available to banks - have access to a safe alternative investment, the gross rate of return on which is denoted r_B. For our purposes, we will assume that this alternative investment possibility takes the form of a government securities market, r_B then being the one period gross real interest rate on this asset. We will also assume that r_B is fixed, determined exogenously by monetary policy.

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[4] The distribution of bank capital, on the other hand, is completely irrelevant to the equilibrium in this framework as the returns on all projects are perfectly correlated. Dropping this unrealistic assumption would permit better capitalised banks to benefit by diversifying their portfolios.

[5] Assuming a perfectly inelastic supply of informed capital may seem at odds with the perfectly elastic supply of uninformed capital at each r_B. Such a restriction, however may be entirely appropriate during a period of severe financial distress. Kanaya and Woo (2000) document the difficulties faced by many leading Japanese banks in increasing their capital base following the crisis (pp14-15):

"...between 1992 and 1997, only Sakura, Daiwa, Tokai and Mitsubishi were able to raise tier 1 capital in the market...But by 1997, following the sharp decline in bank stocks and consecutive downgrades by rating agencies of even the best banks, banks suspended any further attempts to raise capital in the market...Almost all banks issued subordinated debt, partly to compensate for the decline in tier 2 capital caused by the drop in unrealised profits of securities holdings...But even subordinated debt issues fell out of favour with investors by 1997 when the risk in the subordination became apparent."

Assumption 1: Given r_B, only the good project by assumption has a positive net present value, even after including the private benefit of the firm's manager:

$$p_H R - r_B I > 0 > p_L R - r_B I + B \tag{1}$$

Firms can choose to finance their project by either issuing bonds ("direct" finance) or by means of a bank loan ("indirect" finance)[6].

2.1.1 Direct Finance

In the case of direct finance, an optimal contract will involve the firm investing all its funds A with bondholders putting up the remainder $I - A$; the firm and bondholders being paid R_f and R_u respectively ($R_f + R_u = R$) in the case of success, and neither party being paid at all if the project fails. Given 1, a necessary condition for the existence of direct bond finance is that the firm's managers choose the good project, an outcome that can be secured by the following incentive constraint:

$$p_H R_f \geq p_L R_f + B \Leftrightarrow R_f \geq \frac{B}{\Delta p} \tag{2}$$

Intuitively, the firm's managers must take a big enough piece of the action relative to the private benefit achieved through shirking. Equation 2 thus implies an upper bound to the expected return the firm's managers can promise to bondholders, and, given the opportunty cost to these investors of investing in real assets, this in turn places an upper bound on the total amount of uninformed capital that the firm can attract

$$p_H (R - R_f) \leq p_H \left(R - \frac{B}{\Delta p} \right) \geq (I - A) r_B \tag{3}$$

The maximum expected return that can be offered to uninformed investors consistent with the firm not shirking – the "pledgeable expected income" as Hölmstrom and Tirole put it – cannot be less than the opportunity cost of these funds to ensure participation by the uninformed. Rearranging equation 3, we find that only sufficiently capitalised firms can finance their investment directly through bond issuance

$$A \geq \overline{A} \left(\overset{(+)}{r_B} \right) \equiv I - \frac{p_H}{r_B} \left(R - \frac{B}{\Delta p} \right) \tag{4}$$

A firm's capital endowment must at least cover the project cost less the discounted pledgeable expected income.

[6] To keep the corporate financing side of the model to its bare essentials, we preclude the possibility of firms issuing equity.

2.1.2 Indirect Finance

Firms whose capital endowment fails to satisfy the above condition can apply for intermediated credit, by means of a bank loan. There is an extensive theoretical literature stressing theo informational advantages banks are likely to have over ther investors, the standard rationale being that control of the debtor is less effective under bond finance because the dispersion of claim holders generates free-rider problems and wasteful multiplication of monitoring costs (Diamond, 1984). In the context of the present model, the real service performed by the banking system is the elimination through some external control mechanism of the large (B) private benefit obtainable through shirking; put differently, intermediated finance reduces the opportunity cost to the firm of acting diligently, permitting the firm to credibly promise to give away a larger piece of the action.

In the case of intermediated finance, an optimal contract will involve the firm investing all its funds A, the bank investing I_m with bondholders putting up the remainder $I - A - I_m$; bondholders, the bank and the firm being paid $R_u; R_m$ and R_f respectively ($R_u + R_m + R_f = R$) in the case of success, and none of the parties being paid at all if the project fails. The minimum return required by the firm without destroying incentives now falls to

$$R_f \geq \frac{b}{\Delta p} \tag{5}$$

Whether or not a bank has actually monitored a firm is non-verifiable ie. uninformed investors are not a party to this information. A further necessary condition required for the existence of intermediated finance therefore is that the bank chooses to monitor the firm

$$p_H R_m - c \geq p_L R_m \Leftrightarrow R_m \geq \frac{c}{\Delta p} \tag{6}$$

We can now proceed in an analogous fashion to that described above to show that a necessary and sufficient condition for the existence of bank finance is that

$$p_H (R - R_f - R_m) \leq p_H \left[R - \left(\frac{b + c}{\Delta p} \right) \right] \geq (I - A - I_m) r_B \tag{7}$$

As before, the pledgeable expected income, which in this case is the maximum expected return that can be offered to uninformed investors so that neither the firm nor the bank shirks, cannot be less than the opportunity cost of these funds to ensure participation by the uninformed. Rearranging the preceding weak inequality, we find the minimum capital requirement for a firm to have access to intermediated finance

$$A \geq \underline{A}\left(\overset{(+)}{r_B}, \overset{(+)}{\beta}, \overset{(+)}{c}\right) \equiv I - I_m - \frac{p_H}{r_B}\left[R - \left(\frac{b+c}{\Delta p}\right)\right] \tag{8}$$

What determines the size of the monitor's capital injection into each project? Firstly, note the relationship between this capital injection and its expected gross rate of return, β

$$I_m = \frac{p_H R_m}{\beta} \geq \frac{p_H c}{\beta \Delta p} \tag{9}$$

Competition amongst intermediaries will drive the expected return down to the minimum level compatible with incentives to monitor - the zero profit condition - and the above expression will hold as an equality. Notice that, perhaps surprisingly, each firm being monitored accepts an informed capital injection of equal size, independently of its own collateral level. A marginal increase in the bank's capital injection actually causes the share of the project that the firm can credibly promise to the uninformed to shrink by a greater amount, thus raising the minimum capital level a firm needs to invest[7]; put differently, monitoring capital is relatively expensive and so it will always be optimal for a firm to minimise its reliance on such a form of finance.

The minimum capital requirement for a firm to access bank finance can thus be written as

$$\underline{A}\left(\overset{(+)}{r_B}, \overset{(+)}{\beta}, \overset{(+)}{c}\right) \equiv I - \frac{p_H c}{\beta \Delta p} - \frac{p_H}{r_B}\left[R - \left(\frac{b+c}{\Delta p}\right)\right] \tag{10}$$

Comparing expressions 4 and 10, it is straighforward to see that a neces-

[7] It is straightforward to show that a marginal increase in I_m reduces the pledgeable expected income from a project by β. Differentiating, we find that $d\underline{A}/dI_m = \beta/r_B - 1 > 0$.

sary and sufficient condition for monitoring to be socially useful[8] is

$$
\begin{aligned}
\overline{A}(r_B) &> \underline{A}(r_B;\beta;c) && (11)\\
\Rightarrow\quad b + c - B &< \frac{c r_B}{\beta} && (12)\\
\Rightarrow\quad c &< \mathbf{b} \equiv \frac{\beta}{\beta - r_B}(B - b)
\end{aligned}
$$

For monitoring to appear in equilibrium then, its cost cannot be "too" large. For our purposes, it turns out to be useful if c is also bounded below by $B - b$. Hölmstrom and Tirole (1997) also make such an assumption, justifying it on the grounds that were it not to be the case, monitoring would allow a firm to raise more uninformed capital than without monitoring (compare 4 and 10 to see this). Under such conditions, there could be an equilibrium with monitoring even if intermediaries possessed no own capital. In what follows, such a restriction is required to ensure that loan rates fall following an easing of monetary policy in our benchmark model. These restrictions are summarised in assumption 2.

Assumption 2:

$$c \in \left(B - b; \frac{\beta}{\beta - r_B}(B - b) \right), \forall \beta > \underline{\beta} \qquad (13)$$

For all c satisfying these restictions, we can classify firms into three categories according to their capital endowment: well-capitalised firms, $A \geq \overline{A}(r_B)$, can finance their investment by issuing bonds directly to uninformed investors; mid-capitalised firms, $\underline{A}(r_B;\beta;c) \leq A < \overline{A}(r_B)$, are obliged to use a mix of intermediated capital and bonds; while under-capitalised firms, $A < \underline{A}(r_B;\beta;c)$, are excluded from the capital market altogether.

With an exogenous safe real interest rate, equilibrium in the capital market is then fully described by equalising the demand for and the stock of

[8] Compare this to the necessary and sufficient condition appearing in Hölmstrom and Tirole (1997), p.674:

$$c < \mathbf{e} \equiv \frac{p_H}{\Delta p}(B - b)$$

Condition 11 must hold for all values of β and r_B such that $\beta > \underline{\beta} \equiv \frac{p_H}{p_L} r_B$, whereas the above condition holds only for the lowest feasible value of β. Now, for each r_B, $\mathbf{b}$ is decreasing in β Surely $c < \mathbf{e}$ then is purely a necessary condition? The true necessary and sufficient condition being $(B - b) < c < \mathbf{b}(\beta)\ \forall \beta \in \left(\underline{\beta}; \infty\right)$.

informed capital

$$K_m = \int_{\underline{A}(r_B;\beta;c)}^{\overline{A}(r_B)} \frac{p_H c}{\beta \Delta p} f(A)\, dA \equiv D_m\left(\overset{(?)}{r_B}; \overset{(-)}{\beta}; \overset{(?)}{c}\right) \tag{14}$$

Equation 14 implicitly determines the equilibrium solution for β, $\beta = \beta\left(\overset{(?)}{r_B}; \overset{(?)}{c}; \overset{(-)}{K_m}\right)$.

To summarise the model: the financial health of both firms and intermediaries is exogenously given, as is the safe real interest rate, which we assume is controlled indirectly by the monetary authority; banks' monitoring costs are also exogenous, but for intermediated finance to appear in equilibrium these must be bounded; the rate of return on bank capital is endogenously determined, and this plus the other variables/densities described above yield the cut-off capital endowments required to access the credit markets.

2.2 Aggregate Investment and the Monetary Transmission Mechanism

Aggregating investment across firms that are sufficiently well capitalised to access the capital market allows us to write down an expression for economy-wide investment, K

$$K\left(\overset{(-)}{r_B}; \overset{(-)}{\beta}(r_B; c; K_m); \overset{(-)}{c}\right) = \int_{\underline{A}(r_B;\beta;c)}^{\overline{A}(r_B)} I f(A)\, dA + \int_{\overline{A}(r_B)}^{A_{max}} I f(A)\, dA \tag{15}$$

$$= \int_{\underline{A}(r_B;\beta(r_B;c;K_m);c)}^{A_{max}} I f(A)\, dA$$

Equation 15 can be thought of as a micro-founded IS curve linking aggregate investment to the safe real interest rate Differentiating (see Appendix A for a detailed derivation), we can write the slope of the function, the response of aggregate investment that is to a small change in the safe rate, as

$$\frac{dK}{dr_B} = \overset{(-)}{\frac{\partial K}{\partial r_B}} + \overset{(-)}{\frac{\partial K}{\partial \beta}} \cdot \overset{(?)}{\frac{d\beta}{dr_B}} \tag{16}$$

$$= -I f(\underline{A}) \left\{ \frac{\partial \underline{A}}{\partial r_B} + \frac{\partial \underline{A}}{\partial \beta} \left[\frac{f\left(\overline{A}\right) \frac{\partial \overline{A}}{\partial r_B} - f(\underline{A}) \frac{\partial \underline{A}}{\partial r_B}}{f(\underline{A}) \frac{\partial \underline{A}}{\partial \beta} + \frac{1}{\beta} \int_{\underline{A}}^{\overline{A}} f(A)\, dA} \right] \right\} < 0 \tag{17}$$

Two distinct elements determine the response of investment to a change in safe real interest rates in the model:

² …rstly, for a given ¯, a higher opportunity cost of investing in real assets will raise the required return demanded by uninformed investors, a return that only better capitalised …rms will be in a position to o¤er without destroying incentives. This e¤ect can be interpreted as the broad credit (or balance sheet) channel of monetary transmission (see Bernanke and Gertler, 1995, for an overview), operating here through the discounted pledgeable expected income and $\underline{A}(r_B; ¯; c)$, rather than directly through …rm assets, $f(A)$.

² secondly, there will be an endogeous response from ¯ itself, as the number of …rms spanned by intermediation (and thus the demand for bank …nance) will change following the change in the safe rate. This is broadly interpretable as a bank lending channel, albeit one working through the demand rather than supply side of this market, à la Bernanke and Blinder (1988).

To be clear then, we are excluding from our analysis the possibility that monetary policy can directly a¤ect …rm assets, $f(A)$ and/or the quantity of intermediary capital, K_m. A monetary tightening induces a ‡ight-to-quality e¤ect, whereby the …rms with the weakest balance sheets in both capital markets are …nd themselves excluded. In the bond market, such …rms will switch to bank …nance, while in the market for intermediated …nance, such …rms will be unable to obtain credit from any source.

It is important to realise that in the full information case, where p_H is fully contractable, aggregate investment in this economy would be independent of changes in the safe real interest rate. This result follows directly from the …xed investment scale adopted, coupled with assumption 1: all good investment projects have positive NPV and hence all will get …nanced, regardless of the internal capital positions of …rms. The marginal e¢ciency of invesment schedule under such a scenario is simply $K = \int_0^{A_{max}} I f(A)\, dA$ for $r_B < \frac{p_H R}{I}$ and 0 otherwise.

Inspecting equation 16, it is clear that the (up-until-now unspeci…ed) density function of …rm assets, f, plays a crucial role in determining the slope of IS. For one thing, it is impossible to determine *a priori* the direction of this economy's bank lending channel, as this result depends crucially upon the mass of …rms requiring intermediated …nance after the fall in safe rates. Furthermore, f plays a more direct role in the slope expression in that it translates the impact of changes in the safe rate on $\underline{A}(r_B;\beta;c)$ into the response of aggregate investment: if the configuration of $\underline{A}(r_B;\beta;c)$ and f are such that few firms are brought into the capital market by an interest rate reduction, then the consequent impact on aggregate investment spending will be less.

Despite this ambiguity, it is possible to show that the relationship between aggregate investment and the safe real interest rate in the model is always negative as one would expect (see Appendix A for a proof). Without specifying f, however, it is not possible to go further and fully characterise the shape of IS. For this reason, if we are to make any progress towards the goal of the paper, it will be necessary to remove this source of ambiguity by specifying f: an obvious benchmark candidate is to assume that firm assets are uniformly distributed[9], implying $f(A_i) = f(A_j)\ \forall A_i;A_j \in [0;A_{max}]$. Though clearly unrealistic, such a choice allows us to focus on more fundamental aspects of the model.

Plugging in the evaluated derivatives of 16 , one can write this slope as

$$\frac{dK}{dr_B} = -If(\underline{A})\left\{\frac{p_H}{r_B^2}\left[R-\left(\frac{b+c}{\Delta p}\right)\right]+\frac{p_H c}{\beta^2\Delta p}\left[\frac{(b+c-B)}{2c\left(\frac{r_B}{\beta}\right)^2-\frac{r_B}{\beta}(b+c-B)}\right]\right\}<0 \tag{18}$$

2.3 Comparative Statics

The proceeding analysis will use this model to examine the macroeconomic implications of three types of financial stress: a credit crunch, defined as a fall in the exogenous stock of aggregate bank capital[10]; an adverse collateral

[9] Interesting implications (and avenues for future research) of relaxing the uniform-asset distribution assumption include discontinuties and non-monotonicity in the second derivative of IS: the economy will then display asymmetric responses to interest rate changes and small changes in interest rates can have larger-than-anticipated effects. Taken from Walsh (1998), p.268: "A rise in interest rates may have a much more contractionary impact on the economy if balance sheets are already weak, introducing the possibility that nonlinearities in the impact of monetary policy may be important."

[10] This definition is identical to that found in Bernanke and Lown (1991), who define the term as "...a significant leftward shift in the supply curve for bank loans, holding constant both the safe real interest rate and the quality of potential borrowers". As in their paper, equilibrium credit rationing should by no means be considered a necessary condition for a credit crunch here. The mapping between this definition and the meaning of the term as most observers conventionally understand it, however, was diputed by Friedman (1991) in his comments on the above paper: "I doubt, however, that a simple leftward shift of loan supply...would qualify as a credit crunch in the mind of the typical market participant or monetary policymaker. It is also no conincidence that the widespread anecdotal evidence to

shock, reducing the cash assets of those firms capable of tapping capital markets for funds; and finally, an increase in the severity of informational asymmetry, proxied by an increase in bank's monitoring costs. For each type of shock, particular attention will be paid to a) displacements in the position of IS and b) the effect on the slope of IS.

Motivating these comparative statics exercises is the bubble-collapse cycle in asset prices witnessed in Japan since the late 1980s. The real effects of this cycle have been well documented elsewhere in the literature (see Fleming, 1999, for example); for an interesting discussion of the link between asset prices and bank capital see Ito and Sasaki (1998). Higher monitoring costs can be justified on the grounds that cash flow becomes a poorer signal of type during an economic downturn. The main results of this paper are summarised in the following three propositions.

2.3.1 Credit Crunch

Proposition 2 An exogenous reduction in the aggregate stock of monitoring capital, K_m will: (i) result in a leftward shift of the IS curve, reducing the level of aggregate investment at each safe real interest rate; and (ii) when cash assets are uniformly distributed across the set of firms, raise the sensitivity of investment spending with respect to the real interest rate.

The proof of part (i) is straightforward and follows directly from the partial derivatives of β and K. Differentiating 15, we have

$$\frac{dK}{dK_m} = -I f(\underline{A}) \frac{\partial \underline{A}}{\partial \beta} \cdot \frac{\partial \beta}{\partial K_m} > 0 \tag{19}$$

Plugging in the evaluated derivatives, this simplifies to

$$\frac{dK}{dK_m} = \frac{I f(\underline{A})}{\frac{p_H c}{\beta \Delta p} f(\underline{A}) + \int_{\underline{A}}^{\overline{A}} f(A)\, dA} \tag{20}$$

for the general case and

$$\frac{dK}{dK_m} = \frac{I r_B \Delta p}{p_H \left(\frac{2 c r_B}{\beta} - (b + c - B) \right)} \tag{21}$$

when $f(\cdot)$ is uniform. Notice also that the relationship between aggregate investment and monitoring capital is concave, implying that a credit crunch

which Bernanke and Lown refer includes many examples of borrowers who have been asked to wind up their loans despite having kept their accounts fully current, or new projects that U.S. lenders have simply declined to finance at any interest rate".

will have a bigger impact on demand when occuring in an environment where bank capital is already weak. Intuitively, this is caused by the convexity in the demand function for monitoring capital.

Appendix B contains a proof of the interesting and perhaps unexpected result of part (ii): the IS curve actually gets flatter in a credit crunch:

$$\frac{d}{dK_m}\left(\frac{dK}{dr_B}\right) = \frac{I r_B \beta (\Delta p)^2 (b + c - B)^2}{\left[p_H \frac{2cr_B}{\beta} - (b + c - B)\right]^3} > 0$$

The intuition is once again given by the convexity of $D_m(r_B;\beta;c)$: when supply falls, equilibrium in the market for monitoring capital necessarily occurs at a steeper point on the demand schedule, and as such, the equilibrium response of β to a change in r_B is consequently bigger. A given cut in safe rates will thus give more firms access to the capital market and hence the effect on overall investment will be larger. A credit crunch, it seems, can always be offset by looser monetary policy, unless, of course, a liquidity trap has occurred; put differently, such a phenomenon cannot by itself explain monetary impotence.

2.3.2 Adverse Collateral Shock

Proposition 3 For concreteness, imagine a negative shock to the distribution of firms' assets, producing a post-shock distribution $g(A) \equiv f(A + \delta)$ for some positive constant, δ. Such a shock will: (i) shift the IS curve leftward, resulting in a lower level of investment for each safe real interest rate; and (ii) when cash assets are uniformly distributed across the set of firms, and $\delta < A_{max} - \overline{A}(r_B)$ (ensuring the set of firms spanned by intermediation post-shock is non-empty), leave the interest sensitivity of investment spending unchanged.

The proof of part (i) of this proposition is also straightforward: post-shock, clearly the integral on the right hand side of 15 will span strictly fewer firms. Notice that with uniformly distributed assets, the drop in aggregate investment in linear in the magnitude of the shock, δ[11]. Part (ii), is also straightforward: provided the set of firms spanned by intermediation is non-empty, the slope expression 18 is independent of δ.

[11] For uniformly distributed assets, post-shock investment, K', will be

$$K' = \int_{\underline{A}(r_B;\beta;c)}^{A_{max} - \delta} I g(A)\, dA = I g(\underline{A})(A_{max} - \delta - \underline{A})$$

2.3.3 Monitoring Cost Shock

Proposition 4 An increase in the cost, c, banks face when eliminating the B-project will: (i) shift the IS curve leftward; but (ii) when cash assets are uniformly distributed across the set of firms, the effect on the interest sensitivity of investment spending is ambiguous.

Proofs of both parts to this proposition are contained in Appendix B. The intuition for part (i) is as follows. Following the shock, each bank will require a greater proportion of the total surplus generated to cover the higher costs of monitoring, implying a corresponding drop in the share that each firm can credibly offer to outside investors and consequently the amount of external finance that can raised; $\underline{A}$ thus goes up. It turns out also that the relationship between monitoring costs and aggregate investment

$$\frac{dK}{dc} = -I f\left(\underline{A}\right) \frac{p_H}{\Delta p r_B} \left(\frac{\frac{c}{\beta} - \frac{(b+c-B)}{r_B}}{\frac{2c}{\beta} - \frac{(b+c-B)}{r_B}} \right) < 0 \tag{22}$$

is convex (a proof of this is also contained in Appendix B). The nonlinearity here occurs because $\frac{d\beta}{dc}$ is a non-monotonic function of c. Higher monitoring costs have two conflicting effects on the aggregate demand for bank capital, D_m: on the one hand, for a given r_B, β and K_m, $\underline{A}$ unambiguously rises following an increase in c, implying that the integral on the right hand side of equation 14 spans strictly fewer firms; on the other, however, each firm being monitored post-shock will require a greater informed capital injection, and so the overall effect $\frac{\partial D_m}{\partial c}$ is ambiguous. As shown in Appendix B, $\frac{d\beta}{dc} > 0$ for $c < c^*$ and $\frac{d\beta}{dc} \leq 0$ for $c \geq c^*$ where $c^* \equiv \frac{1}{2\left(1 - \frac{r_B}{\beta}\right)}(B - b)$ ie. either $c^* \in \left(B - b, \frac{1}{1 - \frac{r_B}{\beta}}(B - b) \right)$, or $c^* < B - b$, implying that most or all of the set of permissible c values will yield $\frac{d\beta}{dc} < 0$. Note also that $\frac{d}{dc}\left(\frac{d\beta}{dc}\right) < 0$. This explains the convexity of our earlier relationship: as c increases, the effect on K is diminishing as the increase in β gets smaller, indeed β will eventually decrease.

Although formally speaking, the effect of a monitoring cost shock on $\frac{dK}{dr_B}$ is ambiguous, it turns out that for all intents and purposes, investment becomes more sensitive to interest rates and the IS curve becomes steeper following a shock. A complete characterisation of this relationship under uniform f is contained in Appendix B, where it is shown that

$$\frac{d}{dc}\left(\frac{dK}{dr_B}\right) = \frac{-I f(\underline{A})\, p_H \Psi(b+c-B)}{r_B^3 \cdot p^{\beta^2}\left[\frac{2cr_B}{\beta} - (b+c-B)\right]^3}$$

where $\Psi(b+c-B) \equiv \frac{\beta^2}{r_B}(b+c-B)^3 - 7c\beta(b+c-B)^2 + 10c^2 r_B (b+c-B) - \frac{4c^3 r_B^2}{\beta}$

Here, we simply note two properties to illustrate the point:

- $\frac{d}{dc}\left(\frac{dK}{dr_B}\right) > 0$ (steeper IS) for over three-quarters of the range of permissible c values;
- and just to give an order of magnitude to the analysis, the value of the cubic Ψ at the local maximum is $0.061\frac{c^3 r_B^2}{\beta}$ compared to $-4\frac{c^3 r_B^2}{\beta}$ at the local minimum: the maximum amount of steepeing therefore dwarfs maximum amount of flattening by a ratio of 65:1.

The ambiguity over $\frac{d}{dc}\left(\frac{dK}{dr_B}\right)$ is a result of conflicting effects from the cost of capital and bank lending channels of monetary policy. The power of the former, which operates through the discounted pledgeable expected income, is always diminished by an increase in monitoring costs. This effect, it can be shown (see Appendix B), is partially and at the extreme, completely offset by an increase in the strength of the latter.

2.3.4 Relative Magnitude of Slope Effects

It is instructive to examine what happens to these slope effects highlighted above for different parameterisations of the model. Recall that c is (strictly) bounded below by $B - b$ and above by $\frac{cr_B}{\beta}$. To begin with, consider what happens to both slope effects as c gets small:

$$\lim_{(b+c-B)\to 0} \frac{d}{dK_m}\left(\frac{dK}{dr_B}\right) = 0 \tag{23}$$

$$\lim_{(b+c-B)\to 0} \frac{d}{dc}\left(\frac{dK}{dr_B}\right) = \frac{I f(\underline{A})\, p_H}{2 \cdot p r_B^4} \tag{24}$$

The flattening of the IS curve in a credit crunch disappears, while the steepening of IS caused by a monitoring cost shock is maximised.

Next, consider what happens as $(b + c - B)$ approaches the first root of $\Phi(\Delta)$:

$$\lim_{(b+c-B)\to 0.76\frac{cr_B}{\beta}} \frac{d}{dK_m}\left(\frac{dK}{dr_B}\right) = \frac{I\beta^2(\Delta p)^2 0.76^2}{p_H c 1.24^3} \tag{25}$$

$$\lim_{(b+c-B)\to 0.76\frac{cr_B}{\beta}} \frac{d}{dc}\left(\frac{dK}{dr_B}\right) = 0 \tag{26}$$

and finally, as $(b + c - B)$ approaches the second root of $\Phi(\Delta)$, and its upper bound:

$$\lim_{(b+c-B)\to \frac{cr_B}{\beta}} \frac{d}{dK_m}\left(\frac{dK}{dr_B}\right) = \frac{I(\Delta p)^2\beta^2}{p_H c} \tag{27}$$

$$\lim_{(b+c-B)\to \frac{cr_B}{\beta}} \frac{d}{dc}\left(\frac{dK}{dr_B}\right) = 0 \tag{28}$$

In this latter case, it is now the steepening of IS caused by increased c that vanishes; conversely, the flattening of IS in a credit crunch is maximised as c approaches its upper bound.

A possible interpretation of these results is that the IS curve derived from the Hölmstrom Tirole model displays two general types of equilibrium properties. For low values of $(b + c - B)$ relative to $\frac{cr_B}{\beta}$ (implying a low c), credit crunches have no significant slope effect whereas monitoring cost shocks lead to significant steepening. When $(b + c - B)$ is high relative to $\frac{cr_B}{\beta}$ on the other hand, the results are reversed and credit crunches lead to a significant flattening of IS whereas monitoring cost shocks have little slope effect. Which effect is predominant in Japan at present can only be resolved empirically.

3 Conclusions

In this paper, we have argued that an asset price correction of the magnitude witnessed in Japan, which weakens the capital positions of both borrowers and lenders, represents a more plausible shock as the trigger of the current low interest rate-low output conjuncture in that country. Perhaps surprisingly, we find that the effectiveness of monetary policy is not necessarily diminished by such a shock; indeed, in a credit crunch, it is actually heightened, while firm collateral shocks leave it unaltered. Both shocks, it seems, can always be offset by looser monetary policy, unless, of course, a liquidity trap has occurred; put differently, neither phenomenon provides an explanation of monetary impotence per se. Monitoring cost shocks, on the other hand, tend to make IS steeper bringing in to question the suitability of Krugman's inflation targeting remedy. We can broadly characterise two "regimes": when costs are low, factors steepening the IS curve predominate, whereas when costs are high, factors which flatten IS win-out. The clear policy implication is that debt write-offs and bank recapitalisation are to be encouraged, and should help boost demand in the short run.

4 Appendix A

4.1 Some Useful Expressions Evaluated

This section evaluates some of the key expressions in the model for use in the proceeding proofs.

- $\overline{A}(r_B) \equiv I - \frac{p_H}{r_B}\left(R - \frac{B}{\Delta p}\right)$

$$\frac{\partial \overline{A}}{\partial r_B} = \frac{p_H}{r_B^2}\left(R - \frac{B}{\Delta p}\right) > 0$$

- $\underline{A}(r_B;\beta;c) \equiv I - \frac{p_H c}{\beta \Delta p} - \frac{p_H}{r_B}\left[R - \left(\frac{b+c}{\Delta p}\right)\right]$

$$\frac{\partial \underline{A}}{\partial r_B} = \frac{p_H}{r_B^2}\left[R - \left(\frac{b+c}{\Delta p}\right)\right] > 0$$

$$\frac{\partial \underline{A}}{\partial \beta} = \frac{p_H c}{\beta^2 \Delta p} > 0$$

$$\frac{\partial \underline{A}}{\partial c} = \frac{p_H}{\Delta p}\left(\frac{1}{r_B} - \frac{1}{\beta}\right) > 0$$

and

$$\overline{A} - \underline{A} = \frac{p_H}{\Delta p}\left[\frac{c}{\beta} - \frac{(b + c - B)}{r_B}\right]$$

• $D_m(r_B;\beta;c) \equiv \int_{\underline{A}(r_B;\beta;c)}^{\bar{A}(r_B)} \frac{p_H c}{\theta \Delta p} f(A)\,dA$

$$\frac{\partial D_m}{\partial r_B} = \frac{p_H c}{\theta \Delta p}\left[f\left(\bar{A}\right)\frac{\partial \bar{A}}{\partial r_B} - f(\underline{A})\frac{\partial \underline{A}}{\partial r_B}\right]$$

$$\frac{\partial D_m}{\partial \beta} = -\frac{p_H c}{\theta \Delta p} f(\underline{A})\frac{\partial \underline{A}}{\partial \beta} - \int_{\underline{A}}^{\bar{A}} \frac{p_H c}{\theta^2 \Delta p} f(A)\,dA$$

$$\frac{\partial D_m}{\partial c} = \int_{\underline{A}}^{\bar{A}} \frac{p_H}{\theta \Delta p} f(A)\,dA - \frac{p_H c}{\theta \Delta p} f(\underline{A})\frac{\partial \underline{A}}{\partial c}$$

4.2 IS Expression

Differentiating expression 15 in the text, it is straightforward to show that

$$\frac{dK}{dr_B} = -I f(\underline{A})\left[\frac{\partial \underline{A}}{\partial r_B} + \frac{\partial \underline{A}}{\partial \beta}\frac{d\beta}{dr_B}\right]$$

How does β change with r_B? β is implicitly determined by the market clearing condition 14. Totally differentiating this expression

$$dK_m = \frac{\partial D_m}{\partial r_B} dr_B + \frac{\partial D_m}{\partial \beta} d\beta$$

it is straightforward to show that for a given informed capital stock

$$\frac{d\beta}{dr_B} = \frac{f\left(\bar{A}\right)\frac{\partial \bar{A}}{\partial r_B} - f(\underline{A})\frac{\partial \underline{A}}{\partial r_B}}{f(\underline{A})\frac{\partial \underline{A}}{\partial \beta} + \frac{1}{\theta}\int_{\underline{A}}^{\bar{A}} f(A)\,dA}$$

The denominator of this expression is unambiguously positive, so $\mathrm{sgn}\left(\frac{d\beta}{dr_B}\right) = \mathrm{sgn}\left[f\left(\bar{A}\right)\frac{\partial \bar{A}}{\partial r_B} - f(\underline{A})\frac{\partial \underline{A}}{\partial r_B}\right]$, an expression which in general will vary with the shape of f. Notice that for the uniform f adopted in the text, this slope simplifies to

$$\frac{d\beta}{dr_B} = \frac{p_H(b+c-B)}{p_H c\left(\frac{r_B}{\theta}\right)^2 + \frac{r_B^2 \Delta p}{f(\underline{A})\beta}\int_{\underline{A}}^{\bar{A}} f(A)\,dA}$$

$$= \frac{b+c-B}{2c\left(\frac{r_B}{\theta}\right)^2 - \frac{r_B}{\theta}(b+c-B)} > 0$$

using the property that the area under a uniform density function between the limits $\underline{A}$ and $\overline{A}$ is simply $\int_{\underline{A}}^{\overline{A}} f(A)\,dA = f(\underline{A})\left(\overline{A} - \underline{A}\right)$.

For any f, plugging in the evaluated derivates, is it straightforward to show that the IS expression simplifies to

$$\frac{dK}{dr_B} = -\frac{I f(\underline{A})}{f(\underline{A})\frac{\partial \underline{A}}{\partial \beta} + \frac{1}{\beta}\int_{\underline{A}}^{\overline{A}} f(A)\,dA}\left[f\left(\overline{A}\right)\frac{\partial\overline{A}}{\partial r_B}\frac{\partial \underline{A}}{\partial\beta} + \frac{1}{\beta}\frac{\partial\underline{A}}{\partial r_B}\int_{\underline{A}}^{\overline{A}} f(A)\,dA\right] < 0$$

5 Appendix B

5.1 Credit Crunch: Proof of $\frac{d}{dK_m}\left(\frac{dK}{dr_B}\right) > 0$

The slope of the IS function is given by:

$$\frac{dK}{dr_B} = -If(\underline{A})\left(\frac{\partial\underline{A}}{\partial r_B} + \frac{\partial \underline{A}}{\partial\beta}\frac{d\beta}{dr_B}\right) \qquad (29)$$

Assuming uniform f implies that we can write the derivative of this w.r.t. the quantity of bank capital as

$$\frac{d}{dK_m}\left(\frac{dK}{dr_B}\right) = -If(\underline{A})\left[\frac{d}{dK_m}\left(\frac{\partial\underline{A}}{\partial r_B}\right) + \frac{d}{dK_m}\left(\frac{\partial\underline{A}}{\partial\beta}\frac{d\beta}{dr_B}\right)\right] \qquad (30)$$

Evaluating the terms in square brackets seperately, we have:

$$\frac{d}{dK_m}\left(\frac{\partial\underline{A}}{\partial r_B}\right) \equiv \frac{d}{dK_m}\left[\frac{p_H}{r_B^2}\left(R - \left(\frac{b+c}{\Delta p}\right)\right)\right] = 0$$

and

$$\begin{aligned}
\frac{d}{dK_m}\left(\frac{\partial\underline{A}}{\partial\beta}\frac{d\beta}{dr_B}\right) &\equiv \frac{d}{dK_m}\left[\frac{p_H c}{\beta^2\Delta p}\cdot\frac{b+c-B}{2c\left(\frac{r_B}{\beta}\right)^2 - \frac{r_B}{\beta}(b+c-B)}\right] \\
&= \frac{d}{dK_m}\left[\frac{p_H c(b+c-B)}{2cr_B^2\Delta p - r_B\beta\Delta p(b+c-B)}\right] \\
&= -p_H c(b+c-B)\left[2cr_B^2\Delta p - r_B\beta\Delta p(b+c-B)\right]^{-2} \\
&\quad \times -r_B\Delta p(b+c-B)\frac{d\beta}{dK_m} \\
&= \frac{r_B c p_H \Delta p(b+c-B)^2 \overset{(-)}{\left(\frac{d\beta}{dK_m}\right)}}{\left[2cr_B^2\Delta p - r_B\beta\Delta p(b+c-B)\right]^2} < 0
\end{aligned}$$

The effect on the slope of IS of a small change in the quantity of bank capital is thus

$$\frac{d}{dK_m}\left(\frac{dK}{dr_B}\right) = -lf(\underline{A})\left[\frac{r_B c p_H \Delta p(b+c-B)^2 \overbrace{\left(\frac{d\beta}{dK_m}\right)}^{(-)}}{[2cr_B^2\Delta p - r_B\beta\Delta p(b+c-B)]^2}\right] > 0$$

5.2 Monitoring Cost Shocks

5.2.1 Proof of $\frac{dK}{dc} < 0$

Differentiating equation 15, for a uniform f we can write

$$\begin{aligned}\frac{dK}{dc} &= -lf(\underline{A})\frac{d\underline{A}}{dc} \\ &= -lf(\underline{A})\left(\frac{\partial\underline{A}}{\partial c} + \frac{\partial\underline{A}}{\partial\beta}\frac{d\beta}{dc}\right)\end{aligned}$$

From the market clearing condition for informed capital:

$$\begin{aligned}\frac{d\beta}{dc} &= \frac{-\partial D_m/\partial c}{\partial D_m/\partial\beta} \\ &= \frac{\beta}{c}\left[\frac{\frac{2c}{\beta} - \left(\frac{b+c-B}{r_B}\right) - \frac{c}{r_B}}{\frac{2c}{\beta} - \frac{b+c-B}{r_B}}\right]\end{aligned}$$

whose sign appears indeterminate. Plugging these evaluated derivatives, one can easily show that

$$\frac{dK}{dc} = -lf(\underline{A})\frac{p_H}{\Delta p r_B}\left\{\frac{\frac{c}{\beta} - \frac{(b+c-B)}{r_B}}{\frac{2c}{\beta} - \frac{(b+c-B)}{r_B}}\right\} < 0$$

5.2.2 Proof of $\frac{d^2K}{dc^2} > 0$

$$\frac{d^2\underline{A}}{dc^2} = \frac{p_H}{\Delta p r_B}\left\{\begin{array}{l}\overbrace{\left[\frac{2c}{\beta} - \frac{(b+c-B)}{r_B}\right]}^{(+)}\left[\frac{1}{\beta}\left(1 - \frac{c}{\beta}\frac{d\beta}{dc}\right) - \frac{1}{r_B}\right] \\ -\overbrace{\left[\frac{c}{\beta} - \frac{(b+c-B)}{r_B}\right]}^{(+)}\left[\frac{2}{\beta}\left(1 - \frac{c}{\beta}\frac{d\beta}{dc}\right) - \frac{1}{r_B}\right]\end{array}\right\}\frac{1}{\left[\frac{2c}{\beta} - \frac{(b+c-B)}{r_B}\right]^2}$$

Note that it is possible to make further progress here by using the evaluated derivate $d\beta/dc$:

$$1 - \frac{c}{\sigma}\frac{d\beta}{dc} = \frac{c}{\frac{2cr_B}{\sigma} - (b + c - B)}$$

$$\Rightarrow \quad \frac{1}{\sigma}\left(1 - \frac{c}{\sigma}\frac{d\beta}{dc}\right) - \frac{1}{r_B} = -\frac{\left[\frac{cr_B}{\sigma} - (b + c - B)\right]}{r_B\left[\frac{2cr_B}{\sigma} - (b + c - B)\right]} < 0$$

and

$$\Rightarrow \quad \frac{2}{\sigma}\left(1 - \frac{c}{\sigma}\frac{d\beta}{dc}\right) - \frac{1}{r_B} = \frac{b + c - B}{r_B\left[\frac{2cr_B}{\sigma} - (b + c - B)\right]} > 0$$

It follows immediately that $\frac{d^2\underline{A}}{dc^2} < 0$ implying $\frac{d^2K}{dc^2} > 0$.

5.2.3 Proof of $\frac{d}{dc}\left(\frac{dK}{dr_B}\right)$

For uniform f, we can write the derivative of the slope of IS w.r.t. a small change in c as

$$\frac{d}{dc}\left(\frac{dK}{dr_B}\right) = -l f(\underline{A})\left[\frac{d}{dc}\left(\frac{\partial \underline{A}}{\partial r_B}\right) + \frac{d}{dc}\left(\frac{\partial \underline{A}}{\partial \beta}\frac{d\beta}{dr_B}\right)\right]$$

The first term in the square brackets can be evaluated as

$$\frac{d}{dc}\left(\frac{\partial \underline{A}}{\partial r_B}\right) \equiv \frac{d}{dc}\left[\frac{p_H}{r_B^2}\left(R - \left(\frac{b + c}{\Delta p}\right)\right)\right]$$
$$= -\frac{p_H}{r_B^2 \Delta p} < 0$$

The second term, however, is a somewhat tougher nut to crack. Plugging in the evaluated partial derivatives, we can write

$$
\begin{aligned}
\frac{d}{dc}\left(\frac{\partial A}{\partial \beta}\cdot\frac{\partial \beta}{\partial r_B}\right) &\equiv \frac{d}{dc}\left[\frac{p_H c}{\beta^2 \cdot p}\cdot\frac{b+c-B}{2c\left(\frac{r_B}{\beta}\right)^2-\frac{r_B}{\beta}(b+c-B)}\right]\\
&= \frac{d}{dc}\left\{\frac{p_H c(b+c-B)}{2cr_B^2\cdot p - r_B\beta\cdot p(b+c-B)}\right\}\\
&= \frac{1}{[2cr_B^2\cdot p - r_B\beta\cdot p(b+c-B)]^2}\\
&\quad\times\left\{\begin{array}{c}[(b+c-B)p_H + p_H c][2cr_B^2\cdot p - r_B\beta\cdot p(b+c-B)]\\ -p_H c(b+c-B)\left[2\cdot p r_B^2 - r_B\cdot p(b+c-B)\frac{d\beta}{dc} - r_B\cdot p\beta\right]\end{array}\right\}\\
&= \frac{2c^2\cdot p r_B^2 p_H - \beta\cdot p r_B p_H(b+c-B)^2 + \cdot p r_B p_H c(b+c-B)^2\frac{d\beta}{dc}}{[2cr_B^2\cdot p - r_B\beta\cdot p(b+c-B)]^2}
\end{aligned}
$$

whose sign appears to depend in part on the sign of $\frac{d\beta}{dc}$. Plugging in this evaluated expression, we get

$$
\begin{aligned}
\frac{d}{dc}\left(\frac{\partial A}{\partial \beta}\cdot\frac{\partial \beta}{\partial r_B}\right) &= \frac{\cdot p r_B p_H}{[2cr_B^2\cdot p - r_B\beta\cdot p(b+c-B)]^2}\\
&\quad\times\left\{\begin{array}{c}2c^2 r_B - \beta(b+c-B)^2 +\\ c(b+c-B)^2\frac{\beta}{c}\left(\frac{\left(\frac{2c}{\beta}-\frac{b+c-B}{r_B}\right)-\frac{c}{r_B}}{\frac{2c}{\beta}-\frac{b+c-B}{r_B}}\right)\end{array}\right\}\\
&= \frac{\cdot p r_B p_H}{[2cr_B^2\cdot p - r_B\beta\cdot p(b+c-B)]^2\left[\left(\frac{2c}{\beta}-\frac{b+c-B}{r_B}\right)\right]}\\
&\quad\times\left\{\begin{array}{c}2c^2 r_B\left[\left(\frac{2c}{\beta}-\frac{b+c-B}{r_B}\right)\right] - \beta(b+c-B)^2\left[\left(\frac{2c}{\beta}-\frac{b+c-B}{r_B}\right)\right]\\ +\beta(b+c-B)^2\left[\left(\frac{2c}{\beta}-\frac{b+c-B}{r_B}\right)-\frac{c}{r_B}\right]\end{array}\right\}\\
&= \frac{\cdot p r_B p_H}{[2cr_B^2\cdot p - r_B\beta\cdot p(b+c-B)]^2\left[\left(\frac{2c}{\beta}-\frac{b+c-B}{r_B}\right)\right]}\\
&\quad\times\left\{\frac{4c^3 r_B}{\beta} - 2c^2(b+c-B) - \frac{\beta c}{r_B}(b+c-B)^2\right\}
\end{aligned}
$$

ie. $\operatorname{sgn}\left[\frac{d}{dc}\left(\frac{\partial A}{\partial \beta}\cdot\frac{\partial \beta}{\partial r_B}\right)\right] = \operatorname{sgn}\left[\frac{4c^3 r_B}{\beta} - 2c^2(b+c-B) - \frac{\beta c}{r_B}(b+c-B)^2\right]$.

We are now in a position to evaluate the effect of an increase in monitoring costs on the slope of IS.

$$\frac{d}{dc}\left(\frac{dA}{dr_B}\right) = -\frac{p_H}{r_B^2 \cdot p} + \frac{\beta p r_B p_H}{\left[2cr_B^2 \cdot p - r_B \beta \cdot p\,(b+c-B)\right]^2 \left(\frac{2c}{\beta} - \frac{b+c-B}{r_B}\right)^3} \times \left\{\frac{4c^3 r_B}{\beta} - 2c^2(b+c-B) - \frac{\beta c}{r_B}(b+c-B)^2\right\}$$

$$= \frac{1}{\left[2cr_B^2 \cdot p - r_B \beta \cdot p\,(b+c-B)\right]^2 \left(\frac{2c}{\beta} - \frac{b+c-B}{r_B}\right)^3} \times \left\{\begin{array}{l} \beta p r_B p_H \left[\frac{4c^3 r_B}{\beta} - 2c^2(b+c-B) - \frac{\beta c}{r_B}(b+c-B)^2\right] \\ - \frac{p_H}{r_B^2 \cdot p}\left[2cr_B^2 \cdot p - r_B \beta \cdot p\,(b+c-B)\right]^2 \left(\frac{2c}{\beta} - \frac{b+c-B}{r_B}\right)^3 \end{array}\right\}$$

As the denominator of this expression is unambiguously positive, $\frac{d}{dc}\left(\frac{dA}{dr_B}\right)$ takes the sign of the numerator, which can be simplified as

$$p_H \cdot p \left\{\frac{\beta^2}{r_B}(b+c-B)^3 - 7c\beta(b+c-B)^2 + 10c^2 r_B (b+c-B) - \frac{4c^3 r_B^2}{\beta}\right\}$$
$$= p_H \cdot p\,\Psi(b+c-B)$$

where $\Psi(b+c-B) \equiv \frac{\beta^2}{r_B}(b+c-B)^3 - 7c\beta(b+c-B)^2 + 10c^2 r_B (b+c-B) - \frac{4c^3 r_B^2}{\beta}$

$\Rightarrow$ The effect of an increase in monitoring costs on the slope of IS is

$$\frac{d}{dc}\left(\frac{dK}{dr_B}\right) = -\frac{I f(\underline{A})\, p_H \cdot p\,\Psi(b+c-B)}{\left[2cr_B^2 \cdot p - r_B \beta \cdot p\,(b+c-B)\right]^2 \left(\frac{2c}{\beta} - \frac{b+c-B}{r_B}\right)^3}$$
$$= \frac{-I f(\underline{A})\, p_H \Psi(b+c-B)}{r_B^3 \cdot p\,\beta^2 \left(\frac{2cr_B}{\beta} - (b+c-B)\right)^3}$$
$$\Rightarrow \quad \operatorname{sgn}\left\{\frac{d}{dc}\left(\frac{dK}{dr_B}\right)\right\} = -\operatorname{sgn}\{\Psi(b+c-B)\}$$

The roots of this cubic polynomial are:

$$(b+c-B)_1 = \frac{cr_B}{\beta}$$
$$(b+c-B)_2 = \frac{cr_B}{\beta}\left(3 + \sqrt{5}\right)$$
$$(b+c-B)_3 = \frac{cr_B}{\beta}\left(3 - \sqrt{5}\right)$$

and the turning points are:

$$(b + c - B)^*_4 = \frac{c r_B}{\ } \left(\frac{7 + \sqrt{19}}{3} \right)$$

$$(b + c - B)^*_5 = \frac{c r_B}{\ } \left(\frac{7 - \sqrt{19}}{3} \right)$$

the former being the local minimum and the latter being the local maximum.

Just to give an order of magnitude to this analysis, the value this cubic at the local maximum is

$$\Phi \left[\frac{c r_B}{\ } \left(\frac{7 - \sqrt{19}}{3} \right) \right] = 0.061 \frac{c^3 r_B^2}{\ }$$

whereas the value at the local minimum is:

$$\Phi(0) = -4 \frac{c^3 r_B^2}{\ }$$

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