Base Period, Qualifying Period and the Equilibrium Rate of Unemployment

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ABSTRACT: Employing a Mortensen-Pissarides type matching model, we show that a macroeconomic trade-off exists between the qualifying period and base period on the one hand and the amount and duration of the UI benefit on the other. If we take two otherwise identical countries with a similar rate of unemployment, it is possible for one country to offer its job seekers a *high* level of UI benefit with a *long* benefit duration, while neutralising the effect on the equilibrium rate of unemployment with a *long* qualifying period and a *short* base period.

Key-words: Matching model, unemployment insurance, base period, qualifying period, labour market policy

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1. Introduction

The base period and the qualifying period are next to the unemployment benefit (UI benefit) constituent parts of the unemployment insurance systems in most of the OECD (2002) countries. A worker must complete the qualifying period within a statutory base period in order to obtain a claim with a certain duration for UI benefits. Of the four parameters – base period, qualifying period, UI benefit and benefit duration – we know through economic theory (Mortensen Pissarides 1999, Pissarides 2000, Rogerson and Wright 2002) and empirical research (Atkinson and Micklewright 1990, Layard, Nickell and Jackmann 1999, Nickell Handbook), that the amount and duration of the claim for UI benefit correlate positively with the equilibrium rate of unemployment. There are two reasons for this. Both parameters raise the workers' reservation income and allow them to demand higher wages. As a result, the firms' profits drop, the number of offered vacancies falls and the transition probability into employment decreases, while the length of the unemployment spells rises. Second, the jobs' reservation productivity – the productivity threshold at which the continuation of the job is no longer profitable – increases with the opportunity costs of the job. With the reservation productivity, the unemployment incidence and the rate of job destruction increase. But a higher incidence and a longer duration of unemployment are each sufficient to cause the equilibrium unemployment rate to rise.

While the literature has focused on the effects of the amount and duration of the UI benefit, there are, it appears, neither analytical nor empirical model analyses for the two other parameters of the unemployment insurance system – the base period and qualifying period. The qualifying period is often described as a rule having a financing and information function, which reduces the *moral hazard* of the unemployed. It is, so the story goes, the low qualified and low paid workers who are said to improve their situation by registering a claim for UI benefit with the ill informed labour market authorities. According to this hypothesis, the longer the qualifying period, the lower the likelihood that workers register as unemployed to capture the UI benefits and the higher the accumulated contributions to finance the unemployment insurance, when they themselves once claim UI benefit.

Our paper focuses on the macroeconomic effects of the qualifying period and the base period. Employing a Mortensen-Pissarides type (MP) matching model (Mortensen and Pissarides1994, Pissarides 2000), we show that a macroeconomic trade-off exists between the qualifying period and base period on the one hand and the amount and duration of the UI benefit on the other. If we take two otherwise identical countries with a similar unemployment rate, it is possible for one country to offer its job seekers a *high* level of UI benefit with a *long* benefit duration, while neutralising the effect on the equilibrium rate of unemployment with a *long* qualifying period and/or a *short* base period.

This paper is structured as follows. Section 2 introduces a finite UI benefit duration as a parameter of the labour market policy into the MP-model. In Section 3, we integrate the base period and the qualifying period into the MP-model. Section 4 deals with numerical simulations with the instruments of passive labour market policy – benefit duration, qualifying period and base period. Section 5 concludes.[1]

2. Benefit Duration T

The time structure of the model is discrete. Job creation takes place at the beginning and job destruction at the end of a period. At the beginning of a period, a continuum of applicants looks for suitable vacancies. When a match is found, firm and applicant negotiate the employment contract and begin production. At the end of the period, the output is sold, the wage is paid and the agents decide on whether to continue the match. Idiosyncratic shocks, caused either by technological change or fluctuations in demand, affect the productivity of the match in the subsequent period. If the productivity is too low, the match is dissolved, the job becomes vacant and the worker unemployed. Job seekers who are eligible receive UI benefits, which are paid as a flat rate at the end of a period.

Workers are homogenous. The labour force is represented as a unit mass, each worker is either employed or not, hence $1 = e + u$, where e denotes the pool of employed and u the pool of unemployed. Out of the e employed, $\lambda G(R)e$ lose their job at the end of a period. $\lambda G(R)$ is the endogenous separation rate, where λ is the probability of a job specific shock x. $G(x)$, with the domain $0 \leq \alpha \leq x \leq 1$, is the distribution function of x. $R \geq \alpha$ is the endogenous reservation productivity and yx, with the exogenous marginal product $y > 0$, is the output of the job. Worker and firm prefer the same separation rule, as is shown below. If $x \geq R$, the match is continued. If $x < R$, the job is destroyed. Since R is endogenous and x is bounded from below, worker and firm can avoid job destruction by agreeing to the reservation productivity $R = \alpha \geq 0$. The u job seekers search for a job and apply as soon as they find a vacancy. Job seekers apply at most once per period and vacancies receive no more than one application.

Unemployment incidence. Job search takes place at the beginning of a period. Job seekers who do not find a job form the inflow I of the pool of unemployed: $I \equiv (1-p)\lambda G(R)e$, where p is the transition probability into employment, $0 < p < 1$. We call $(1-p)\lambda G(R)$ the *ex-post-*

[1] Appendix IV with the proofs of the statements is available from the authors upon request.

incidence. The unemployment incidence $\lambda G(R)$ comprises, in contrast to the *ex-post*-incidence, the job seekers who find a vacancy immediately after losing their previous job, as $\lambda G(R) = p\lambda G(R) + (1-p)\lambda G(R)$, where $p\lambda G(R)$ is the fraction of the job-to-job transitions.

Unemployment Insurance. Workers without a job register with the unemployment insurance $[T,b]$. The unemployment insurance $[T,b]$ has the following attributes.

(A1) [Employed Worker]. Each employed worker is entitled to claim UI benefits $b > 0$ if made redundant. The benefit duration amounts to $T \geq 0$ periods.

(A2) [Job Seekers]. u_{T-j} is the pool of job seekers with a residual benefit duration of $T - j \geq 0$ periods. *j* is the *current spell* of unemployment, $j = 0,\ldots,T$. An additional period of unemployment first raises the *current spell* from j to $j+1$ periods, second, reduces the counter of the residual claims to $T-(j+1) \geq 0$ and third, places the unemployed into pool $u_{T-(j+1)}$. Job seekers who have not found employment *T* or more periods after losing their previous job lose their right to UI benefits and form the job seeker pool u_0.

Job seekers from the pool u_T who lost their job at the end of the previous period are entitled to UI benefit for *T* periods. In the steady state, the inflow *I* is identical to the pool u_T, so that $u_T = (1-p)\lambda G(R)e$. Those job seekers from u_T, who still have no job at the succeeding period, form the pool u_{T-1}. For the pool of job seekers with a counter of residual claims equal to $T - j$, we have

(1) $$u_{T-j} = (1-p)^{j+1}\lambda G(R)e,\ j = 0,\ldots,T-1.$$

Since $p < 1$, u_{T-j} strictly decreases with an increasing spell length *j*.

Of the unemployed in the pool u_0, pu_0 find a job. Thus in the steady state we have: $pu_0 = (1-p)^{T+1}\lambda G(R)e$. From this steady condition, we can determine u_0 as

(2) $$u_0 = \frac{(1-p)^{T+1}}{p}\lambda G(R)e.$$

Finally, we get the aggregate pool of job seekers *u* from

(3) $$u = \sum_{j=0}^{T} u_{T-j}.$$

Matching function. The labour market is a search market with two-sided search, characterised by frictions – heterogeneities, mobility costs and information asymmetries – not explicitly modelled. The function $m(u,v)$ represents the matching technology of the market, m is the number of jobs filled with an input of u job seekers and v vacancies. The matching function is linear homogenous, concave and monotone in both arguments. For a given vacancy, $q(\theta) \equiv m(1/\theta,1) = m(u,v)/v$ is the probability of an application, where the ratio of vacancies to job seekers, $\theta = v/u$, is the tightness of the labour market. For a given job seeker, $p(\theta) = \theta q(\theta)$ is the transition probability into employment. For convenience, we will write $q = q(\theta)$ and $p = p(\theta)$.

Plugging (1) and (2) into (3) gives, in view of the transition probability $p(\theta)$, the natural rate of unemployment as a function of the tightness and the reservation productivity

$$u(\theta,R) = \frac{[1-p(\theta)]\lambda G(R)}{[1-p(\theta)]\lambda G(R) + p(\theta)}. \tag{4}$$

The parameters of the unemployment insurance T and b do not affect u directly, but rather through the *ex-post*-incidence, $[1-p(\theta)]\lambda G(R)$, and the duration of unemployment, $1/p(\theta)$.

Filled Jobs. Every match is formed by one vacancy and one job seeker. The match partners negotiate the employment contract and begin production. An employment contract $\lfloor w_{T-j}, w(x), R \rfloor$ has three components. w_{T-j} is the outside wage, which the worker earns the first period. The outside wage is dependent on the residual claims of the job seeker. If the negotiations fail, the worker receives UI benefit b up to another $T-j$ periods, $j = 0,\ldots,T$.

The second component of the contract is the match specific inside wage with the wage function $w:[R,1] \to \Re$. At the end of a period, the succeeding periods' productivity is revealed to the match. If $x \in [R,1]$, the match is continued and the worker earns the wage $w(x)$.[2] The third component of the contract shows the negotiated break-even productivity R, at which the job will be destroyed.

Continuation periods. Shocks hit a match with probability $\lambda \geq 0$. A job will be affected by no more than one shock per period, where shocks are iid.

Let $\Pi(x)$ be the present value of a filled job after the manifestation of $x \in [\alpha,1]$. Worker and firm, considering their reservation utility, are both interested in continuing the match as

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2 Mortensen/Pissarides (1999) and Pissarides (2000) present a discussion of objections against the plausibility of this assumption and the two-tier wage structure which results from the possibility of renegotiation.

long as $\Pi(x) \geq 0$ and agree on job destruction as soon as $\Pi(x) < 0$. Since $\Pi(x)$ is a continuously increasing function of *x*, as will be shown below, a reservation threshold *R* exists, for which

$$\Pi(R) = 0 . \tag{5}$$

Only jobs with $x \geq R$ will be continued.

We assume that the firm markets the output *yx* at the end of the period at the same time as it pays the wage $w(x)$. Then the steady state equation for the present-discounted value $\Pi(x)$ of an occupied job is

$$\Pi(x) = \rho \left\{ yx - w(x) + \lambda \int_R^1 \Pi(h) dG(h) + (1-\lambda)\Pi(x) \right\} . \tag{6}$$

Flow and stock variables are discounted at the factor ρ, where $0 < \rho = 1/(1+r) < 1$ with the real interest rate $r > 0$. With probability λ the job is hit by a shock and changes into state *h*. If $R \leq h \leq 1$ the match is continued and the continuation value becomes $\Pi(h)$. With probability $1-\lambda$ the match specific productivity does not change.

A worker employed at the match specific productivity *x* earns the wage $w(x)$, and his human capital has the present-discounted value $W(x)$. The asset pricing equation for the worker is

$$W(x) = \rho \left\{ w(x) + \lambda \left[\int_R^1 W(h) dG(h) + G(R) U_T \right] + (1-\lambda) W(x) \right\} . \tag{7}$$

With probability λ a shock arrives and the match draws the productivity *h*. If $h \geq R$, the value of the worker is $W(h)$ and the match continues. If, on the other hand, $h < R$, which happens with probability $G(R)$, the job is destroyed, the worker becomes unemployed and the value of his human capital is U_T .

Initial period. Firms choose the initial productivity $x = 1$ when they set up a match and negotiate the outside wage. If the firm meets a worker with the a *current spell* of unemployment of length *j*, then the market value Π_{T-j} of the newly filled job is

$$\Pi_{T-j} = \Pi(1) + \rho \{ w(1) - w_{T-j} \}, \; j = 0, \ldots, T , \tag{8}$$

where w_{T-j} is the outside wage.

The market value of a job seeker with a *current spell* of unemployment of length j is, in view of the asset equation (7) and the outside wage w_{T-j}:

(9) $$W_{T-j} = W(1) + \rho\{w_{T-j} - w(1)\},\ j = 0,\ldots,T .$$

Job creation. Entrance into the labour market is free for all vacancies, but open only at the beginning of a period. The flow of vacancies therefore persists until the present value of a vacancy is driven to zero. Considering this infinitely elastic supply of vacancies, the *job creation* condition is

(10) $$0 = -k + q\sum_{j=0}^{T}\mu_{T-j}\Pi_{T-j}\ ,$$

where k denotes the flow costs for advertising a vacancy, q is the probability of meeting a job seeker, μ_{T-j} the conditional probability that the applicant will have a *current spell* of unemployment of length j and Π_{T-j} the value of the newly filled job according to asset equation (8).

All job seekers search for jobs with the same intensity. Therefore, $\mu_{T-j} = u_{T-j}/u$ denotes the probability with which a vacancy will meet a job seeker with a *current spell* of unemployment of length j. Taking into account the pool equations (1), (2) and (4), the following relationship applies

(11) $$\mu_{T-j} = \begin{cases} p(1-p)^{j},\ j = 0,\ldots,T-1 \\ (1-p)^{T},\ j = T \end{cases}$$

Value of unemployment. Unemployed who are not eligible for UI benefit have the value U_0, where in the steady state

(12) $$U_0 = pW_0 + (1-p)\rho(z + U_0).$$

With the probability p, the job seeker finds a job and his human capital takes on the initial value W_0 (see equation (9)). If he is not matched, the unemployed worker gains utility from leisure equal to z.

The human capital of a job seeker with a *current spell* of unemployment of length j has the value U_{T-j}. In the steady state, the first order linear inhomogeneous difference equation for U_{T-j} is

(13) $$U_{T-j} = pW_{T-j} + (1-p)\rho[z+b+U_{T-(j+1)}],\ j=0,\ldots,T-1.$$

The human capital of the outsider who meets a vacancy has the value W_{T-j} (see equation (9)). Should the job seeker not meet a vacancy, he receives the UI benefit *b* in addition to the utility of leisure *z*, the counter of the *current spell* of unemployment increases to $(j+1)$ and his human capital takes on the value $U_{T-(j+1)}$.

Wage negotiations. Job search takes time and causes search costs. Therefore, each match generates a positive monopoly rent which is distributed between the match partners through the wage. The distribution rules are obtained according to the generalised Nash solution to a bargaining problem, with $\beta \in (0,1)$ denoting the bargaining strength of the job seeker.

Taking into account the idiosyncratic productivity shock $x \in [R,1]$, the reservation utility of the insider U_T, and the fact that in equilibrium the asset price of a vacancy is equal to zero, the sharing rule used for the negotiations with an insider is

(14) $$W(x) - U_T = \frac{\beta}{1-\beta}\Pi(x).$$

$W(x) - U_T$ denotes the worker's contribution and $\Pi(x)$ the firm's contribution to the quasi-rent of the job.

The job rent of a match with an outsider, who has a *current spell* of unemployment of length *j*, will be distributed according to the following rule

(15) $$W_{T-j} - U_{T-j} = \frac{\beta}{1-\beta}\Pi_{T-j},\ j=0,\ldots,T,$$

where the asset equations (8), (9), (12) and (13) give the initial values of the outsider, W_{T-j}, the newly filled job, Π_{T-j}, and the value of the unemployed at the time of wage negotiations, U_{T-j}.

Lemma 1. [Bargained Wages]. *In view of the reservation income rU_T of the insider and the value U_{T-j} of the job seekers with a current spell of unemployment of length j, the agents negotiate the following inside and outside wages.*

(i) The bargained inside wage at a match specific productivity $x \in [R,1]$ is

(16) $$w(x) = rU_T + \beta(yx - rU_T).$$

(ii) An outsider with a current spell of unemployment of length j, who produces in the first period with the productivity $x=1$, earns the wage

(17) $$w_{T-j} = w(1) - (1-\beta)(U_T - U_{T-j})\rho^{-1},\ j=0,\ldots,T,$$

where $w(1)$ is the inside wage (16) for $x=1$, and $\rho^{-1} = 1+r$.

As equation (16) shows, the inside wage equals the reservation income of the worker plus a share of the current match rent that depends on his bargaining strength β.

Should an outsider with a *current spell* of unemployment of length *j* find a job, then the guarantee value of his human capital increases by the amount of the differential rent $U_T - U_{T-j}$. As the wage equation (17) illustrates, the firm which places the outsider under contract takes the fraction $1-\beta$ of this rent.

An outsider who lost his job in the previous period and found a follow-up job at the beginning of the current period is entitled to *T* benefit payments in case the contract negotiations fail. His reservation utility, therefore, does not differ from that of an insider and, for $j = 0$, $w_{T-j} = w(1)$, as equation (17) shows.

Lemma 2. (i) [Filled Jobs]. *The continuation value of a filled job producing with the idiosyncratic productivity* $x \in [R,1]$ *is*

$$\Pi(x) = (1-\beta)y\frac{x-R}{\lambda+r}. \tag{18}$$

(ii) [Job Destruction Rule]. *The job destruction rule can be derived by evaluating the asset equation (6) at the reservation threshold* $x = R$. *Taking into account the wage equation (16) and the continuation value (18) we obtain:*

$$R = \frac{rU_T}{y} - \frac{\lambda}{(1-\beta)y}\int_R^1 \Pi(h)dG(h). \tag{19}$$

As the *destruction rule* (19) illustrates, the current reservation output of a match is lower than its permanent reservation income. Since the firm can destroy the job at no charge (free disposal) and the supply of vacancies is infinitely elastic, the reservation income of the match is identical with the reservation income of the worker. Therefore, when the job produces the reservation output *yR*, then the match partners suffer a current loss equal to $\lambda\int_R^1 \Pi(h)dG(h)$ and the worker, with the wage $w(R) < rU_T$, forgoes part of the income, which he would have earned as a registered unemployed and the utility of leisure. The reason why the match partners are willing to accept losses is the option value of the filled job. If they dissolve the job search and recruiting costs arise to find a new match. In order to avoid these transaction costs, the agents prefer to wait for a recovery of the demand and carry losses up to the limit of the reservation output.

In order to close the model, we still have to determine the reservation income of the different types of unemployed. The unemployment insurance $[T,b]$ creates a discrete distribution with $T+1$ types on the pool of job seekers. The job seeker types differ with respect to their residual entitlement to UI benefits and in turn in their reservation utility and the outside wages they are able to demand when matched to a vacancy. Given the distribution of the market values of the $T+1$ job seeker types, we finally can derive the distribution of the initial values of the filled jobs.

Lemma 3 (i) [Reservation Income]. *From the asset equations for the job seekers, the distribution rules and the equations for the initial values, we obtain the distribution of the reservation income of the* $T+1$ *job seeker types with*

$$(20) \qquad rU_{T-j} = z + \left(1 - d^{T-j}\right)b + \frac{\beta p}{(1-\beta)(1-p)}\left[\Pi(1) + (1-\beta)\left(U_T - U_{T-j}\right)\right]\rho^{-1}, \; j = 0,\ldots,T$$

where $d(\theta) \equiv \dfrac{[1-p(\theta)]\rho}{1-(1-\beta)p(\theta)} \in (0,1)$.

(ii) [Initial Values]. *The distribution of the initial values of jobs is obtained from*

$$(21) \qquad \Pi_{T-j} = \Pi(1) + (1-\beta)\left(U_T - U_{T-j}\right), \; j = 0,\ldots,T.$$

As (20) and (21) show, while, *cet. par*., the reservation income of a job seeker with a *current spell* of unemployment of length *j* decreases, the value of a job filled by an outsider with the same *current spell*, increases monotonically with *j*. As a result, unemployed without benefit entitlement from pool u_0 have the lowest market value of all the job seekers and the jobs filled by the type u_0-unemployed have the highest market value of all newly formed jobs.

The equilibrium of the search model consists of solutions $\lfloor \Pi(1), U_{T-j}, \theta, R, u \rfloor$, $j = 0,\ldots,T$, to the equations (10), (18) – (20) and the equilibrium unemployment (4).

Labour Market Policy. An increase in the benefit duration T raises the fraction of job seekers with a long residual duration of benefit entitlement. Their reservation income increases and, consequently, the outside wages they demand increase too. The initial values of the newly established firms fall and the supply of vacancies declines. In turn, the tightness of the labour market decreases and the duration of unemployment, $1/p(\theta)$, rises. In addition, the *ex-post*-incidence $[1-p(\theta)]\lambda G(R)$ increases. The rising duration of unemployment and the higher *ex-post*-incidence are each sufficient to raise the equilibrium rate of unemployment, see Appendices III and IV.

3. Qualifying Period, Base Period and Waiting time

In the unemployment insurance $[E, F, T, b]$ with qualifying period $E \geq 2$ and base period $F \geq E$, workers who lose their job before completing the qualifying period have *no* claim to UI benefit. In order to model the insurance, we introduce the following five assumptions (A1) – (A5), where (A1) – (A4) deal with the qualifying period and (A5) describes the role of the base period.

3.1 Qualifying Period E

(A1) [Completed Qualifying Period]. The qualifying period of a worker who was employed for at least E periods in the base period F is completed. If a worker with a completed qualifying period becomes unemployed, he is entitled to up to T payments of the UI benefit b.

The benefit duration T can be shorter, as long as or longer than the qualifying period E. Out of 20 member states of the OECD (2002), ten – e.g. Canada, Switzerland, Great Britain and the Scandinavian countries – have a benefit duration T which is longer than E – in Belgium, the benefit duration T is actually unlimited. In a further seven countries – e.g. Germany, Austria, Japan and Spain –, the benefit duration is shorter than the qualifying period; in three countries – France, Portugal and the USA –, the insurance system provides a qualifying period which is just as long as T.

(A2) [Transferability]. Residual claims to UI benefit from earlier employment spells are lost; qualifying periods on the other hand are intertemporally transferable. There is neither a market for claims to UI benefits nor for qualifying periods.

(A3) [Employed worker]. Each employed worker is characterised by a tupel $[E-i, D]$, in which the counter $E-i \geq 0$ shows the number of *currently accumulated* qualifying points of the worker and D the duration of his current claim to the UI benefit b. i denotes the number of uncompleted qualifying (sub-)periods, with $i = 0, \ldots, E-1$. The benefit duration $D \in \{0, T\}$ is a binary variable and either equal to T or zero – depending on whether the qualifying period is completed or not. An additional period of employment – during an uncompleted qualifying period – raises the counter of the qualifying points from $E-i$ to $E-(i-1) \leq E$.

(A4) [Job Seekers]. Each job seeker is characterised by a tupel $[E-i, T-j]$: the counter $E-i \geq 0$ shows the number of *currently accumulated* qualifying points and $T-j \geq 0$ the residual benefit duration, where $j = 0, \ldots, T$. An additional period of unemployment of a job seeker who still owns residual benefit claims raises the length of the *current spell* of unemployment from j to $j+1$ and reduces the counter for the residual benefit duration from $T-j$ to $T-(j+1) \geq 0$.

3.2 Base Period

Many countries of the OECD have established base periods in order to make qualifying periods easier to obtain. The original base period of the first German unemployment insurance in 1927, for example, amounted to 12 months for a qualifying period of 6 months. After countless alterations, the base period was extended to two years in 1956. In 1969, there was a further extension of the base period to 3 years, while the qualifying period was still 6 months. Finally, in 1982, in view of the rising mass unemployment, the qualifying period was extended to 12 months.

One consequence of integrating a base period rule into the unemployment insurance is that the tupel $[E-i,T-j]$ does not unambiguously characterise a job seeker. A worker who lost his job in the previous period and had no match in the current period, signs on and is, according to assumption (A1), eligible to UI benefits if he was employed for at least *E* periods during the current base period of length *F*. The employment records with a completed qualifying period differ, however, in regard to how the $F-E \geq 0$ periods, in which the worker was either employed or seeking a job, were distributed over the base period *F*. There are two different cases here.

If $F = E$, there is exactly one employment record which meets the qualification: only those workers who were *continuously* employed for at least *E* periods are eligible to UI benefits. If, on the other hand, $F > E$, then the number of employment records with a current counter of, for example, $E-i \geq 0$ qualifying points is possibly very large, as is indicated by the following example. Let A and B be job seekers with identical qualifying counters $E-i \geq 0$. Both have recently found a job. Whereas A, however, was employed *F* periods ago, B was unemployed. Since the oldest period in the previous base period is continuously replaced by the most recent period by moving forward through the calendar, B receives the counter status $E-(i-1)$ at the end of the current period, whereas A still only has $E-i$ qualifying points. Why? Both workers have an additional employment period at the end of the current period. B, however, because of his employment record, replaces a period of unemployment at the beginning of the previous base period with the current employment period in the present base period, so that his counter increases by one; A, on the other hand, replaces an employment period at the beginning of the previous base period with a current employment period in the present base period, so that his counter is constant. The tupel $[E-i,T-j]$, therefore, does *not* unambiguously characterise job seekers A and B in the insurance system $[E,F,T,b]$ if $F > E$.

The length of the *waiting time* of a job seeker of type $[E-i,T-j]$, which passes until the next benefit entitlement begins, depends on the distribution of the $E-i$ employment periods over the base period *F*. The longer *F* is, the greater, *cet. par.*, the number of different employment records with $E-i$ qualifying points and the greater the range of the distribution of waiting times of otherwise identical job seekers.[3]

The reservation utility of an applicant and his initial wage depend on three factors in the insurance system $[E,F,T,b]$. First, the counter of the residual benefit duration $T-j$; second, the accumulated qualifying points $E-i$; and thirdly, the distribution of the $E-i$ employment periods over the base period *F*. The higher the number of accumulated qualifying points, the longer the residual benefit duration or the sooner the job seeker will complete the qualifying period, the higher his wage demand will be during the contract negotiations.

An investor offering a vacancy knows just as little *ex ante* about the applicants' specific employment record as their accumulated qualifying points or their residual benefit claims. Yet the value of the job and, consequently, his decision to offer a vacancy depends on these variables. In order to provide a simple model of the investor's decision, we assume that the initial value of a filled job, Π_{E-iT-j}, will only be directly influenced by the characteristics $[E-i,T-j]$ of the job seeker and not by the distribution of the $E-i$ employment periods over the base period *F*. The risk-neutral investor, therefore, need "only" estimate the probability μ_{E-iT-j} of meeting a type $[E-i,T-j]$ applicant who has $E-i$ qualifying points and a residual benefit duration of $T-j$ periods.

We model the effect of the applicant's employment record on the decision of the investors – i.e. the distribution of the $E-i$ employment periods over the base period *F* and their effect on the initial value of a filled job – using a Markov process. Let u_{E-iT-j} denote the pool of job seekers with $E-i$ qualifying points and $T-j$ residual benefit periods.

(A5) [Employment Record]. The unemployed from the pool u_{E-iT-j}, who have had no match, make a transition into the pool $u_{E-iT-(j+1)}$ with the probability $\gamma\in[0,1)$ and into the pool $u_{E-(i+1)T-(j+1)}$ with the probability $1-\gamma\in(0,1]$.[4]

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[3] In total, there are $\binom{F}{E-i}$ employment records with $E-i$ qualifying points in the base period *F*, $i=0,\dots,E$. If, for example, – as in Germany (SGB III) – the base period comprises $F=36$ months and the qualifying period $E=12$ months, then there are $\binom{F}{E}=1{,}252*10^9$ possible employment records with a completed qualifying period.

[4] In the first case – as with B in the introductory example –, the job seeker was unemployed *F* periods ago; in the second – as with A –, he had a job *F* periods ago. In the first case, the job seeker replaces a period of unemployment at the beginning of the previous base period with a period of unemployment at the end of the present base period, which is why his counter $E-i$ is constant; in the second case, the job seeker swaps an employment period at the beginning of the previous base period for a period of unemployment at the end of the current base period and the counter of his qualifying counter decreases from $E-i$ to $E-(i+1)$.

If a type $[E-i,T-j]$ job seeker is unemployed for an additional period, the residual benefit duration decreases from $T-j$ to $T-(j+1)$ in accordance with (A4). Although only in the second case, described in (A5), does the counter of the qualifying period also fall from $E-i$ to $E-(i+1)$, while in the first case the counter is constant.[5]

At the micro level, there exists no correlate of the transition probability γ. Although at the macro level, the policy parameter γ has similar effects as the distribution of the employment periods $E-i$ over the base period *F*. First, if $F=E$, this case corresponds to a transition probability of $\gamma=0$, as there is only one employment record with a continuous employment spell of *E* periods which meets qualification. Second, the longer *cet. par*. the base period *F*, the higher the fraction of agents in the inflow to the aggregate pool of unemployed *u* who can claim UI benefit. An increase in the transition probability γ has the same effect on the mix of types in the inflow to *u* as an extension of the base period *F*. At the macro level, γ establishes the fraction of the job seekers from the pool $u_{E-i} \equiv \sum_{j=1}^{T} u_{E-iT-j}$, $i=0,\ldots,E$, whose qualifying counters do not change despite advancing calendar time and who therefore search for a job in the following period with $E-i$ qualifying points again. For the fraction $1-\gamma$ of the unemployed from pool u_{E-i} on the other hand, both the counter of the residual claims and the qualifying counter sink by one and their reservation income decreases correspondingly. Third – just as in the case $F\to\infty$ -, if $\gamma\to 1$, the fraction of the employed worker with a completed qualifying period approaches one irrespective of the length of the qualifying period *E*.

3.3 Qualifying Path and Unemployment Rate in the Steady State

The unemployment insurance $[E,\gamma,T,b]$ with the qualifying period *E*, base period γ and benefit duration *T* creates a discrete distribution of E types among the pool of employed worker. Employed workers differ in the qualifying counter $E-i$, $i=0,\ldots,E-1$. In the following, e_{E-i} is the pool of workers with $E-i$ qualifying points. Among the *u* unemployed, the unemployment insurance likewise creates a discrete distribution of types[6], who differ with respect to the qualifying points $E-i$, $i=0,\ldots,E-1$, and the residual benefit duration $T-j$,

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[5] Consequently, from (A4) and (A5), it follows that the counter of the length *j* of the *current spell* of unemployment is at least as large as the counter *i* of the missing qualifying points, such that for all job seekers $j\ge i$.

[6] If $E\le T$, there are $\sum_{k=1}^{E+1}(T-E+k)=(\mathrm{E}+1)(T+1)-E(E+1)/2$ types of job seeker; if $E\ge T+1$, the number of job seeker types amounts to $\sum_{k=0}^{T}(T+1-k)+(E-T)=T(T+1)/2+(E+1)$. The steady state equations for the employees e_{E-i} and job seekers u_{E-iT-j} are developed further in Appendix I.

$j = i,\ldots,T$. With u_{E-iT-j} , we denote the pool of job seekers with $E-i$ qualifying points and a residual benefit duration of $T-j$ periods.

Since the time of the model is discrete, every employed worker owns at least one qualifying point. Job seekers from the pool $u_0 = \sum_{j=E}^{T} u_{0T-j}$, who, because of their long unemployment, do not possess any accumulated qualifying points, begin their employment record in the pool e_1 and make a transition to the pool e_2 if $x \geq R_2$ at the end of the first period of the current employment spell.[7] R_2 is the negotiated reservation productivity for the transition from the pool e_1 to pool e_2 , see Fig.1. Consider a filled job with $E-i$, $i = 1,\ldots,E-1$, qualifying points. Firm and worker have to decide at the end of the period whether to continue the job. In case of a continuation, the worker makes a transition to the pool $e_{E-(i-1)}$. The match partners decide in regard to the bargained reservation productivity $R_{E-(i-1)}$. Given $x \in [\alpha,1]$, they proceed with the job, if and only if $x \geq R_{E-(i-1)}$, otherwise the match dissolves, the job becomes vacant and the worker unemployed – *without* claim to the UI benefit. Jobs from the pool e_{E-1} which are close to the completion of the qualifying period decide to continue and make a transition to pool e_E , if $x \geq R_E$. The pool e_E comprises all, and only, those jobs with a completed qualifying period. A job from e_E is continued if $x \geq R_{E+1}$. Otherwise it is destroyed, and the worker becomes unemployed – *with* claim to unemployment benefit. R_{E+1} is the bargained reservation productivity of the jobs with a completed qualifying period.

We call the path of the reservation productivities $\Psi_E = [R_2,\ldots,R_E]$, $E \geq 2$, the *qualifying path*: every worker must – possibly interrupted by unemployment spells – pass through the qualifying path Ψ_E before his qualifying period is completed and he is entitled to UI benefit.

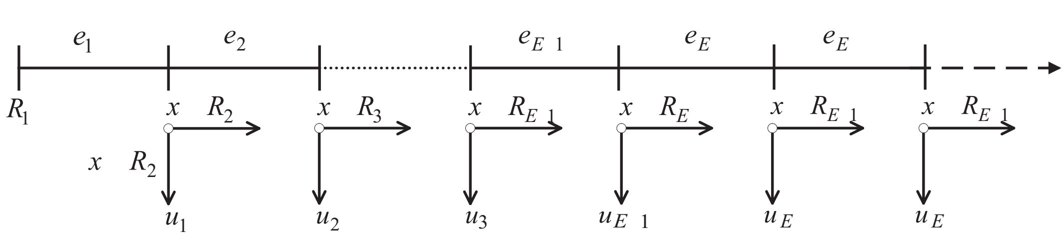

FIG. 1: Qualifying path

Out of the e_{E-i} employed workers with the qualifying counter $E-i$, $\lambda G(R_{E-(i-1)})e_{E-i}$ lose their job at the end of the period. In the ensuing matching at the beginning of the following period, $[1-p(\theta)]\lambda G(R_{E-(i-1)})e_{E-i}$ do not meet a vacancy and form the inflow to the pool

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[7] Whether the creation of vacancies is profitable depends in particular on the reservation productivity R_1. For profitability $R_1 \leq 1$ is a necessary condition because the firms choose the initial productivity at $x = 1$.

of unemployed u; $[1-p(\theta)]\lambda G(R_{E-(i-1)})$ is the *ex-post*-incidence among the workers with the qualifying counter $E-i$. In the steady state, entries to the unemployment pool u are equal to the exits, so that $[1-p(\theta)]\sum_{i=0}^{i=E-1}\lambda G(R_{E-(i-1)})e_{E-i} = p(\theta)u$. If we divide both sides of the steady state condition by e and take into account that $e=1-u$, we obtain the steady state unemployment rate

$$(22)\qquad u(\theta,\Psi_E,R_{E+1})=\frac{[1-p(\theta)]\sum_{i=0}^{E-1}\lambda G(R_{E-(i-1)})\varepsilon_{E-i}}{[1-p(\theta)]\sum_{i=0}^{E-1}\lambda G(R_{E-(i-1)})\varepsilon_{E-i}+p(\theta)},$$

where $\varepsilon_{E-i}=\varepsilon_{E-i}(\theta,\Psi_E,R_{E+1})$, with $\varepsilon_{E-i}=e_{E-i}/e$, $i=0,\ldots,E-1$, is the fraction of the employed workers with the qualifying counter $E-i$, hence $\sum_{i=0}^{E-1}\varepsilon_{E-i}=1$. As Lemma A4 in the Appendix IV shows, the shares ε_{E-i} and the unemployment rate (22) are functions of the tightness of the labour market θ, the qualifying path $\Psi_E=[R_2,\ldots,R_E]$ and the reservation productivity R_{E+1} of the jobs with a completed qualifying period.

The equilibrium unemployment rate (22) – similarly to the steady state rate (4) of the unemployment insurance $[T,b]$ – depends on, first, the weighted average of the *ex-post*-incidences, $[1-p(\theta)]\sum_{i=0}^{E-1}\lambda G(R_{E-(i-1)})\varepsilon_{E-i}$, and second, the duration of job search, $1/p(\theta)$.

3.4 Qualifying Rents and Waiting time

First, we deal with the arbitrage equations of the filled jobs and the employed workers in the continuation periods of a match, then we focus on the job creation condition, the wage negotiations, the qualifying rents and finally the waiting time.

Continuation periods. The value of a filled job with a completed qualifying period is derived from the asset equation (6) and the value of the worker from equation (7). For convenience, we repeat the equations. A filled job with a completed qualifying period has the value

$$(23)\qquad \Pi_{E+1}(x)=\rho\left\{yx-w_{E+1}(x)+\lambda\int_{R_{E+1}}^{1}\Pi_{E+1}(h)dG(h)+(1-\lambda)\Pi_{E+1}(x)\right\}$$

and the value of the worker is:

$$(24)\qquad W_{E+1}(x)=\rho\left\{w_{E+1}(x)+\lambda\left[\int_{R_{E+1}}^{1}W_{E+1}(h)dG(h)+G(R_{E+1})U_{ET}\right]+(1-\lambda)W_{E+1}(x)\right\},$$

where $w_{E+1}:[R_{E+1},1]\rightarrow\Re$ is the function of the inside wage and U_{ET} is the value of a job seeker whose qualifying period and benefit entitlement are complete. Firm and worker with a completed qualifying period share the match rent according to the rule (14).

The continuation value of a job with $E-i$ qualifying points, $i=0,\dots,E-1$, and the productivity $x\in[R_{E-i},1]$ is given by

$$\Pi_{E-i}(x)=\rho\left\{yx-w_{E-i}(x)+\lambda\int_{R_{E-(i-1)}}^{1}\Pi_{E-(i-1)}(h)dG(h)+(1-\lambda)\max\left\{0,\Pi_{E-(i-1)}(x)\right\}\right\}. \tag{25}$$

Firm and worker negotiate the reservation productivity $R_{E-(i-1)}$, on which the transition to the pool $e_{E-(i-1)}$ depends. If the match is hit by a shock and draws the productivity $h\geq R_{E-(i-1)}$, the match is continued, otherwise it is destroyed. If no shock arrives, firm and worker must still decide whether to proceed. The reason is that if the match continues, the worker makes a transition to the pool $e_{E-(i-1)}$. Since the firm is free to destroy the job at no charge (free disposal), it decides for the alternative $\max\left\{0,\Pi_{E-(i-1)}(x)\right\}$. The worker also prefers continuation only, if $\Pi_{E-(i-1)}(x)\geq 0$, as is shown below.

The value of a worker with the qualifying counter $E-i$, $i=0,\dots,E-1$ is given by

$$W_{E-i}(x)=\begin{cases}\rho\left\{w_E(x)+\lambda\left[\int_{R_{E+1}}^{1}W_{E+1}(h)dG(h)+G(R_{E+1})U_{ET}\right]+(1-\lambda)\max\left\{U_{ET},W_{E+1}(x)\right\}\right\}, & i=0\\ \rho\left\{w_{E-i}(x)+\lambda\left[\int_{R_{E-(i-1)}}^{1}W_{E-(i-1)}(h)dG(h)+G(R_{E-(i-1)})U_{E-i0}\int_{R_{E-(i-1)}}^{1}\right]+(1-\lambda)\max\left\{U_{E-i0},W_{E-(i-1)}(x)\right\}\right\}, & i=1,\dots,E-1\end{cases} \tag{26}$$

If the job is hit by a shock and draws $h<R_{E-(i-1)}$, it is destroyed and the worker with the qualifying counter $E-i$ becomes unemployed. In the case $i=0$, the worker has the value U_{ET} and is entitled to UI benefits; in the case $i=1,\dots,E-1$, the worker's qualifying period is not yet completed and his value is U_{E-i0}.

If a shock arrives, the worker chooses the alternative $\max\left\{U_{E-i0},W_{E-(i-1)}(x)\right\}$. If $W_{E-(i-1)}(x)\geq U_{E-i0}$ – or $W_{E+1}(x)\geq U_{ET}$, in the case $i=0$ – he decides to continue the match, otherwise he leaves the firm and makes a transition to unemployment (free disposal). As the insiders distribute their monopoly rent according to the rule

(27) $$W_{E-i}(x)-U_{E-(i+1)0}=\frac{\beta}{1-\beta}\Pi_{E-i}(x),\ i=0,\ldots,E-1,$$

$W_{E-(i-1)}(x)\geq U_{E-i0}$ applies if and only if $\Pi_{E-(i-1)}(x)\geq 0$. The distribution rule (27) takes into account that the worker makes a transition to e_{E-i} if the wage negotiations succeed, but if the bargaining fails, the worker passes into the job seeker status with a qualifying counter equal to $E-(i+1)$ and *no* entitlement to benefits. His value in this case is $U_{E-(i+1)0}$.

The initial value Π_{E-iT-j} of a newly filled job, the value of an outsider, who accepts a job, W_{E-iT-j}, moreover, the distribution rule, which job seekers and vacancies employ in their contract negotiations as well as the asset equations for the value of the unemployed U_{E-iT-j} are developed in Appendix IV.[8]

Job creation. Out of the *u* job seekers, there are u_{E-iT-j}, who have $E-i$ qualifying points and a *current spell* of unemployment of length *j*. Since all job seekers search for jobs with the same intensity, for a given vacancy $\mu_{E-iT-j}=u_{E-iT-j}/u$ is the conditional probability of an application from a job seeker from u_{E-iT-j}. The probabilities μ_{E-iT-j} – developed in Lemma A5, Appendix IV – are functions of the tightness θ, the base period γ, the qualifying path Ψ_E and the reservation productivity R_{E+1} for jobs with a completed qualification. The expected market value of a newly filled job is therefore $\sum\mu_{E-iT-j}\Pi_{E-iT-j}$. Access to the labour market is free, so that in the steady state, given the search costs *k* and the probability *q* of an application, the following *job creation condition* applies:

(28) $$0=-k+q\sum_{i=0}^{E}\sum_{j=i}^{T}\mu_{E-iT-j}\Pi_{E-iT-j}\ .$$

The agents negotiate the following outside and inside wages.

Lemma 4 [Bargained Wages]. ***(i)*** *The bargained inside wage of a worker with a completed qualifying period at a match specific productivity* $x\in[\alpha,1]$ *is*

(29) $$w_{E+1}(x)=rU_{ET}+\beta(yx-rU_{ET}).$$

The inside wage of a worker with the counter $E-i$ *and the job specific productivity* $x\in[\alpha,1]$ *is:*

(30) $$w_{E-i}(x)=\begin{cases} rU_{E-10}+\beta[yx-rU_{E-10}]-(1-\beta)[U_{ET}-U_{E-10}],\ i=0 \\ rU_{E-(i+1)0}+\beta[yx-rU_{E-(i+1)0}]-(1-\beta)[U_{E-i0}-U_{E-(i+1)0}],\ i=1,\ldots,E-1\end{cases}$$

(ii) *Since newly filled jobs produce with the productivity* $x=1$*, a job seeker with the counter* $E-i$ *and a residual benefit duration of* $T-j$ *periods,* $j=i,\ldots,T$*, obtains the outside wage*

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[8] The initial value of the filled jobs and the workers can be found in the equations (A32) and (A33), and the values of the unemployed human capital are represented in the equation (A34).

(31) $$w_{E-iT-j} = \begin{cases} w_{E+1}(1) - (1-\beta)\left(U_{ET} - U_{ET-j}\right)\rho^{-1}, & i = 0 \\ w_{E-(i-1)}(1) + (1-\beta)\left(U_{E-iT-j} - U_{E-i0}\right)\rho^{-1}, & i = 1, \ldots, E \end{cases}$$

where $w_{E+1}(1)$ *and* $w_{E-(i-1)}(1)$ *are the inside-wages (29) and (30) for* $x = 1$.

The inside wage $w_{E-i}(x)$ of a worker with the counter $E - i$ has – as (30) shows – three components: the guarantee income, $rU_{E-(i+1)0}$, the worker's share of the current match rent, $\beta\left[yx - rU_{E-(i+1)0}\right]$, and the wage penalty $(1-\beta)\left[U_{E-i0} - U_{E-(i+1)0}\right]$. The wage penalty has the following reason. At the end of the previous period, the worker had $E-(i+1)$ qualifying points and the guarantee value $U_{E-(i+1)0}$. If the match is continued, the counter increases by one to $E - i$ and the guarantee value of the human capital increases by the *qualifying rent* $U_{E-i0} - U_{E-(i+1)0}$. Out of the qualifying rent, the firm which employs the worker appropriates the share $1-\beta$.

In accordance with (A2), the qualifying period is an asset owned by the worker, which is not tradable. Thus, since the labour force is exogenous, a dissipation of the qualifying rent, even in the steady state, is generally not to be expected. The supply of vacancies and the reservation productivities are the only quantity variables of the model with which the market system reacts to the qualifying rents created by the unemployment insurance.

If one compares, *cet. par.*, two agents with a completed qualifying period ($i = 0$) – one is an outsider, the other an insider – then, as we would expect, the outsider is worse off, because, as opposed to the insider, he has to accept a wage penalty, as seen in the first line of (31). The wage penalty is determined by the length j of the *current spell* of unemployment and the quasi-rent $U_{ET} - U_{ET-j}$, by which the guarantee value of the outsider is lower than guarantee value of the insider.

If one now compares two agents with the counter $E - i$ who have as yet *not* completed their qualifying period – one is an outsider and has a residual entitlement to UI benefits of $T - j$ periods, the other is an insider – then the outsider is better off, since he receives a wage bonus, which is depend on the quasi-rent $U_{E-iT-j} - U_{E-i0}$, as the second line of (31) shows. The insider is worse off because his qualifying period is not yet completed and as a result, in accordance with (A1), he has *no* benefit entitlement – as opposed to the outsider.

As the following proposition shows, the market value of a filled job $\Pi_{E-(i-1)}(x)$, $i = 0, \ldots, E$, is a continuously increasing function of $x \in [\alpha, 1]$. If $\Pi_{E-(i-1)}(\alpha) \leq 0$, as we assume throughout, a reservation productivity $R_{E-(i-1)}$ exists, which fulfils the *reservation condition*

(32) $$\Pi_{E-(i-1)}\left(R_{E-(i-1)}\right)=0,\ i=0,\ldots,E.$$

The asset values of the filled jobs and the job destruction rules are discussed in the following proposition.

Proposition. (i) [Filled Jobs]. *The value of a filled job with a completed qualifying period and the idiosyncratic productivity* $x \in [R_{E+1},1]$, *is*

(33) $$\Pi_{E+1}(x)=(1-\beta)y\frac{x-R_{E+1}}{\lambda+r}.$$

Obviously, $\Pi_{E+1}(x)$ *is a continuously increasing function of x. Through backward induction, the continuity and monotonicity are transferred to* $\Pi_{E-i}(x)$, *as the equation (34) shows. The value of a job from the pool* e_{E-i}, $i=0,\ldots,E-1$, *is*

(34) $$\Pi_{E-i}(x)=\rho\left\{(1-\beta)y(x-R_{E-i})+(1-\lambda)\left[\max\left\{0,\Pi_{E-(i-1)}(x)\right\}-\max\left\{0,\Pi_{E-(i-1)}(R_{E-i})\right\}\right]\right\}.$$

(ii) [Job Destruction]. *For a job with a completed qualifying period, the job destruction rule can be derived by evaluating the asset equation (23) at the reservation threshold* $x=R_{E+1}$. *Taking into account the wage equation (29) we obtain*:

(35) $$R_{E+1}=\frac{rU_{ET}}{y}-\frac{\lambda}{(1-\beta)y}\int_{R_{E+1}}^{1}\Pi_{E+1}(h)dG(h).$$

For a job with the qualifying counter $E-i$, *the job destruction rule can be derived from the asset equation (25), the reservation condition (32) and the wage equation (30) with*

(36) $$R_{E-i}=\begin{cases}\dfrac{rU_{E-10}}{y}-\dfrac{U_{ET}-U_{E-10}}{y}-\\ \qquad\dfrac{1}{(1-\beta)y}\left[\lambda\displaystyle\int_{R_{E+1}}^{1}\Pi_{E+1}(h)dG(h)+(1-\lambda)\max\left\{0,\Pi_{E+1}(R_E)\right\}\right],\ i=0\\ \dfrac{rU_{E-(i+1)0}}{y}-\dfrac{U_{E-i0}-U_{E-(i+1)0}}{y}-\\ \dfrac{1}{(1-\beta)y}\left[\lambda\displaystyle\int_{R_{E-(i-1)}}^{1}\Pi_{E-(i-1)}(h)dG(h)+(1-\lambda)\max\left\{0,\Pi_{E-(i-1)}(R_{E-i})\right\}\right],\ i=1,\ldots,E-1\end{cases}$$

As the equations (35) and (36) show, the current break-even output of a match is lower than the match's permanent reservation income both during the waiting time of the worker, see equation (36), and also after the completion of the qualifying period, see equation (35). The reservation income of a match – given the assumption of free disposal and the infinitely elastic supply of vacancies – is identical with the reservation income of the worker.

With the job destruction rule (35), the firm and the worker who is entitled to UI benefits choose the reservation productivity R_{E+1} such that for the break-even output of the match: $yR_{E+1} < rU_{ET}$. The firms are willing to hoard workers and to supply the market even if hit by negative productivity or demand shocks. The reasons for this behaviour are the positive search costs and the resulting option value of a filled job. The option value is the expected market value of a productive job weighted with the shock probability λ. If demand or the productivity changes in favour of the job, the hoarded workers are immediately ready to start production, since on the internal labour market neither search nor recruiting costs arise. If the match partners would separate as soon as the output falls below the guarantee income of the worker, they would sacrifice this option and have to search for another match.

The waiting time is the time which passes until a worker on the qualifying path becomes eligible to UI benefits. Under the conditions of the unemployment insurance $[E,\gamma,T,b]$ the waiting time is endogenous, whereby workers face the following trade-off.

The shock parameter $x \in [\alpha,1]$ is bounded from below. Consequently, a match can force the continuation of production until the UI entitlement is reached. Thus, for example, a worker with the qualifying counter $E-i$ can reduce his waiting time to exactly *i* periods, if he and the firm fix the reservation productivity along the residual qualifying path at the level of the lower support α, such that $R_{E-m} = \alpha \geq 0$, $m = 0, \ldots, i-1$. By taking this extreme decision, however, the worker must accept a low wage, a boundary solution, which pays only if he can expect a high UI benefit *b*, a long benefit duration *T* or a low utility of leisure *z*.

The worker will weigh up the disadvantages of restraining his wage claims against the benefit from a reduction in the waiting time. His willingness to restrain his wage claims during the waiting time – as the job destruction rule (36) shows – is bounded by the path of the reservation incomes, the qualifying rents he can expect to capture and the option value of the filled job.

The option value of the filled job is measured by the integral expression in equation (36). Since the worker makes a transition independent of the prevailing market conditions from e_{E-i} to $e_{E-(i-1)}$ when the job is continued, the lower bound of the integral is the reservation productivity $R_{E-(i-1)}$ which is the threshold productivity for the transition to $e_{E-(i-1)}$.

If the firm currently produces at the break-even point with the reservation productivity R_{E-i} and is not hit by a shock – an event which has the probability $1-\lambda$, – the firm opts for the alternative $\max\{0, \Pi_{E-(i-1)}(R_{E-i})\}$, since it can destroy the job without charge at any time (free disposal).

Finally, the worker's willingness to accept a sequence of low wage incomes on the qualifying path is bounded by the qualifying rents. If the firm and worker negotiate the reservation productivity R_{E-i}, the worker's guarantee value is $U_{E-(i+1)0}$. If the match is continued, his guarantee value increases to U_{E-i0}. In order to capture the qualifying rent $U_{E-i0} - U_{E-(i+1)0}$

created by the insurance system, the worker is prepared to decrease the reservation output of the match by an amount just equal to the qualifying rent.

Solution. To solve the model, we must determine the equilibrium path of the reservation productivities $R_{E-(i-1)}$, $i=0,\ldots,E$, and the tightness θ of the labour market – in total $E+2$ endogenous variables. The reservation productivities, as the job destruction rules (35) – (36) show, depend on the reservation incomes of the workers, the qualifying rents and the market values of the filled jobs. The market values of the filled jobs are in turn functions of the reservation productivities, as equations (33) – (34) show. In order to close the model, Lemma A7 in the Appendix IV shows how both the reservation incomes of the workers and the qualifying rents depend on the market values of the filled jobs and, thus, the reservation productivities. To calculate the tightness θ, we need the *job creation condition* (28). The conditional probabilities μ_{E-iT-j} of meeting a job seeker with the qualifying counter $E-i$ and a residual benefit duration of $T-j$ periods are developed in Lemma A5 in the Appendix IV.

4. Simulation

Parameters and matching function. The base parameters for the numeric simulations, are shown in Table A1, Appendix I. The bargaining power of the workers is $\beta=0.50$, the marginal product of a job at full productivity is $y=100$. The value of leisure is $z=40$, UI benefits are $b=40$. The real interest rate r is 2 %; the probability of a productivity shock λ is 10 %; the search and recruiting costs of a vacancy amount to $k=40$.

The distribution function $G(x)$ of the productivity shocks is assumed to be uniform on $[\alpha,1]$, with the lower support $\alpha=0$. Hence, $tG(x)=G(tx)$ holds for all $t\in[0,1]$.

The matching function of the search market is of the Cobb Douglas type (Petrongolo/ Pissarides 2001). For a given vacancy the probability of a contact with a job seeker is $q(\theta)=\theta^{-(1-\phi)}$. For the elasticity of the job matches with respect to vacancies, we use $\phi=\beta=0.50$ (Hosios 1990).

Indicators. The following indicators are used to evaluate the simulations: (1) sequence of reservation productivities $R_{E-(i-1)}$, $i=0,\ldots,E$; (2) unemployment rate u in percent; (3) unemployment incidence *In-exP*, with *In-exP* $\equiv[1-p(\theta)]\sum_{i=0}^{i=E-1}\lambda G(R_{E-(i-1)})\varepsilon_{E-i}$. *In-exP*, the weighted *ex-post*-incidence, is the fraction of the employed worker who lose their job, do not find a follow-up job at the subsequent matching and, as a result, are unemployed for at least one period. Define $\overline{R}_E\equiv\sum_{i=0}^{E-1}R_{E-(i-1)}\varepsilon_{E-i}$. $\overline{R}_E$ is the mean of the reservation productivities of the qualifying path $\Psi_E = [R_2, \ldots, R_E]$ and the jobs with a completed qualifying period R_{E+1}. For the *ex-post*-incidence, by virtue of the homogeneity of the uniform distribution G on the support $[0,1]$, the following holds: *In-exP* $= [1 - p(\theta)]\lambda G(\overline{R}_E)$. (4) unemployment duration *D-exP* in periods, with *D-exP* $= 1/p(\theta)$.

The results of the simulations with the qualifying period E, the benefit duration T and the base period γ are shown in the Appendices III-IV.

Appendix II provides simulations with the benefit duration T and the qualifying period E for a given base period $\gamma = 0.10$. For the qualifying period, we assume $E = 4, 8$ and for the benefit duration, $T = 1, 2, \ldots, 20$. In addition, Appendix II compares the two unemployment insurance systems $[E, \gamma, T, b]$ and $[T, b]$ (see Section 2). With the unemployment insurance $[T, b]$, every worker is entitled to up to T payments of the UI benefit b. The model of the unemployment insurance $[T, b]$, therefore, *implicitly* assumes that for the qualifying period $E = 1$ and the base period $\gamma = 1$.

Appendix III deals with comparative static simulations with the base period γ for a benefit duration of $T = 10$ periods and the qualifying periods $E = 4, 8$.

Result 1. 1. As figures (a) and (b) in Appendix II demonstrate, the qualifying path Ψ_E follows the same pattern in all simulations: first, the reservation productivities strictly decrease until they reach their minimum in the last period before the completion of the qualifying period. As soon as firm and worker have captured the qualifying rents, the reservation productivity, the quit rate and the wage income of the employed worker jump to the levels of the jobs with a completed qualifying period.

Figure (a) shows the qualifying path Ψ_E for $E = 4$ and $E = 8$ and a benefit duration of $T = 10$ periods. The counter of the qualifying period, $i = 1, \ldots, E + 1$, is depicted on the horizontal axis and the corresponding reservation productivities are graphed on the vertical axis.

Figure (b) pictures, for the case $E = 4$, the four reservation productivities of the qualifying path Ψ_4 and the reservation productivity R_{E+1} of the jobs with a completed qualifying period, against the benefit duration T on the horizontal axis. If we draw a vertical line through figure (b) at $T = 10$, we obtain the qualifying path Ψ_4 for $E = 4$, which is shown in diagram (a).

2. For a given qualifying period E, the unemployment rate u strictly increases with the benefit duration T. T affects u via two channels: first, through the weighted *ex-post*-incidence, *In-exP* $\equiv [1 - p(\theta)] \sum_{i=0}^{i=E-1} \lambda G(R_{E-(i-1)}) \varepsilon_{E-i}$, and second, through the expected unemployment duration, $1/p(\theta)$. Consider, for example, the insurance system with the qualifying period

$E = 4$, Appendix II. If the policymakers increase the benefit duration from $T = 1$ to $T = 20$, the expected duration of unemployment increases from 1.59 to 2.08 periods, Fig. (c), the *ex-post*-incidence increases from 3.50 % to 4.89 %, Fig. (d), and the unemployment rate, as a result, rises from 5.26 % to 9.23 %, see Fig. (e).

Result 2. The comparison with the insurance system $[T, b]$ shows that the rule of the qualifying period lowers the aggregate unemployment. Under the unemployment insurance $[T, b]$, $u = 5.33\%$ if $T = 1$, and $u = 10.18\%$ if $T = 20$. The plots in Appendix II show the reasons for the strictly increasing difference between the rates of unemployment of the two insurance systems, see Fig. (e). Under the conditions of the insurance system $[E, \gamma, T, b]$, not only is the average duration of unemployment shorter than in the system $[T, b]$, as Fig. (c) shows, but the *ex-post*-incidence is also lower, see Fig. (d).

Result 3. 1. An extension of the qualifying period *E* for a given base period ($\gamma = 0.10$) lowers the unemployment rate, as Fig. (e) and (f) show. If the policymakers increase the qualifying period to $E = 8$, for example, the unemployment rate for a benefit duration of $T = 1$ is equal to $u = 5.25\%$, and up to a benefit period of $T = 20$ periods, rises to $u = 8.85\%$, Fig. (e). Figure (f) graphs the unemployment rate *u* against the qualifying period *E* for a given benefit duration of $T = 10$ periods. For $E = 1$, $u = 9.62\%$, while $u = 8.60\%$, if $E = 8$.

The unemployment rate strictly decreases with an increasing *E*, since, *cet. par.*, both the unemployment duration and the weighted *ex-post*-incidence decrease with the rising *E*, see Fig. (c) and Fig. (d). For $T = 10$, the unemployment duration falls from 2.11 periods if $E = 1$ to 2.03 periods if $E = 4$ down to 2.00 periods if $E = 8$. The weighted *ex-post*-incidence is equal to 5.0% if $E = 1$ and falls to 4.8% for $E = 4$ and finally to 4.71% if $E = 8$.

2. As the simulations confirm, firms hoard above all those workers whose qualifying period is not yet completed and choose a qualifying path with reservation productivities which are strictly lower than the reservation productivity of the jobs being entitled to the UI benefit, so that $R_{E+1} > R_2 > \ldots > R_E \geq \alpha$, see Fig. (a) and (b).

3. The rule of the qualifying period, moreover, induces the match partners to choose a threshold value R_{E+1} for the jobs which are entitled to UI benefits *cet. par.* below the threshold *R* of the insurance system $[T, b]$, so that $R > R_{E+1}$, see Fig. (g). What are the reasons for this ordering?

The risk-neutral match partners have rational expectations and anticipate the consequences of a job destruction. In the insurance system $[E, \gamma, T, b]$, the destruction of a job which qualifies for UI benefits occurs with the endogenous probability $\lambda G(R_{E+1})$ while in the system

$[T,b]$, the probability is $\lambda G(R)$, where, as the simulations confirm, $\lambda G(R) > \lambda G(R_{E+1})$. In fact, the workers in both insurance systems are entitled to the UI benefit b and an equally long benefit duration of T periods. Furthermore, in both insurance systems, they have a positive probability of losing their benefit entitlement and to become long term unemployed. Yet with the unemployment insurance $[T,b]$, they can be sure of having the benefit entitlement returned with their next job. The waiting time, which elapses until a worker who loses his current job receives the next benefit entitlement, is identical with the duration of job search. In the insurance system $[E,\gamma,T,b]$, on the other hand, a positive probability exists that the worker with an increasing duration of unemployment will not only lose his benefit entitlement, but also his qualifying points, so that, on average, *cet. par*. more time will pass until the completion of the next qualifying period than just the time of the job search.

While the waiting time which elapses between two benefit entitlements in the system $[T,b]$ is exogenous for the individual match partner and identical with the expected duration of job search, $1/p(\theta)$, from the perspective of the job seeker, the waiting time in the insurance system $[E,\gamma,T,b]$ is endogenous and bounded from below by the expected duration of an unemployment spell, $1/p(\theta)$. The agents choose the optimal waiting time subject to the trade-off between the waiting time on the one hand and the bargained wage on the other, as discussed above. As a consequence, in the insurance system $[E,\gamma,T,b]$ the reservation income of a worker entitled to UI benefits is, *cet. par*., lower than in the unemployment insurance $[T,b]$, his wage income is also lower and his willingness to continue the match despite negative productivity shocks is higher.

4. Although the workers entitled to UI benefits in the insurance system $[E,\gamma,T,b]$, compared to the insurance $[T,b]$, trade a lower wage income for a shorter waiting time and a lower incidence, so that, in comparison of the two systems $R > R_{E+1}$, within the class of unemployment insurances $[E,\gamma,T,b]$ with qualifying period $E \geq 2$ this ordering is not valid, as Fig. (g), Appendix II, illustrates. Since an extension of the qualifying period E lowers both the duration of the job search and the weighted *ex-post*-incidence, workers entitled to UI benefits, for example in the unemployment insurance with $E = 8$, prefer a higher reservation productivity R_{E+1} and consequently, a greater incidence $\lambda G(R_{E+1})$, compared with the case $E = 4$. Note, however, that the weighted *ex-post*-incidence in the unemployment insurance with qualifying period $E = 8$, is, nevertheless, lower than in the unemployment insurance with $E = 4$, see Fig. (d).

Result 4. As Appendix III shows for a given benefit duration of $T = 10$, the equilibrium rate of unemployment increases with the base period γ, see Fig. (c). For comparison, the four figures, Appendix III, show the corresponding sequence of the unemployment rate for the unemployment insurance $[T, b]$, which *implicitly* assumes the parameter values $E = \gamma = 1$.

An increase in γ does not only lower the waiting time, but also the qualifying rents and therefore the option value of a filled job, moreover, the expected wage income increases, the supply of vacancies falls and as a result, both the duration of unemployment, see Fig. (b), and the weighted *ex-post*-incidence, see Fig. (a), increase. In addition, the figures illustrate that the equilibrium values of the endogenous variables under the insurance systems $[E, \gamma, T, b]$, $E \geq 2$, converge with rising γ to the corresponding values of the insurance system $[T, b]$, see Fig. (a) – (d).

5. Conclusion

Base period and qualifying period are instruments of the passive labour market policy, which have so far received little attention in labour market theory, macroeconomic theory and empirical research. We develop a Mortensen-Pissarides type search model, in which we integrate the following instruments of labour market policy: the base period, the qualifying period, the benefit duration and the wage-replacement benefit. A worker is entitled to UI benefit if during the base period he has completed the qualifying period.

The qualifying period lowers both the incidence and the duration of unemployment and therefore reduces the aggregate unemployment rate. An increasing base period on the other hand weakens the effect of the qualifying period by providing workers with a time margin to meet the criterion of the qualifying rule. The longer the base period, the higher therefore the equilibrium rate of unemployment.

In an unemployment insurance without qualifying rule – as for example in the standard Mortensen-Pissarides model – the time that passes until the benefit entitlement occurs is exogenous. Every worker who makes a transition to unemployment is entitled to UI benefits and every job seeker must wait until he finds a new job and in turn the next benefit entitlement. The rule of the qualifying period endogenizes the waiting time and confronts the workers with the following trade-off. The lower the separation rates negotiated by the match partners, the longer the durability of the job, the shorter the waiting time, but also the lower the worker's wage income. The decision to reduce the waiting time is more attractive the higher the UI benefits are, the longer the benefit duration and the lower the utility of leisure. The price for a prolongation of the durability of the job and a shorter waiting time is the wage penalty, which the worker must accept if the match is hit by negative productivity or demand shocks.

For a match on the qualifying path, the optimal separation rate falls from period to period, until it reaches a minimum in the last period before the completion of the qualifying period. At this moment, the qualifying rents generated by the unemployment insurance are skimmed off and the reservation productivity, and with it, the separation rate and the wage income of the workers, who are now entitled to UI benefits, increase sharply. Nevertheless, all employed workers face separation rates which are lower than under the conditions of an unemployment insurance with an exogenous waiting time. On the one hand, there are the workers with a completed qualifying period who bargain for a reduced separation rate because they want to delay unemployment and the inconveniences the qualifying period brings, on the other hand, there are those workers – whose qualifying period is not yet completed -, who want to achieve the benefit entitlement more quickly.

References

Atkinson A.B. and Micklewright, J., 1991, Unemployment compensation and labor market transitions: a critical review, Journal of Economic Literature 29, 1679-727.

Hosios, A., 1990, On the efficiency of matching and related models of search and unemployment, Review of Economic Studies 57, 279-298.

Mortensen, D.T. and Pissarides, C.A., 1994, Job Creation and Job Destruction in the Theory of Unemployment, Review of Economic Studies 66, 397-415.

Mortensen, D.T. and Pissarides, C.A., 1999, New Developments in Models of Search in the Labour Market, in: O. Ashenfelter, D. Card (eds.), Handbook of Labour Economics, Vol. 3B, Amsterdam, 2567-2627.

OECD 2002, Benefits and Wages: OECD Indicators, Paris.

Petrongolo, B. and Pissarides, C.A., 2001, Looking into the Black Box: A Survey of the Matching Function, Journal of Economic Literature 34, 390-431.

Pissarides, C.A., 2000, Equilibrium Unemployment Theory, 2. Edition, Oxford University Press, Oxford.

Rogerson, R. and Wright, R., 2002, Search-Theoretic Models of the Labor Market: A Survey, Penn Institute for Economic Research, Working Paper 02-041, Pennsylvavia.

Layard, R., Nickell, S., and Jackman, R., 1991, Unemployment: Macroeconomic performance and the labour market, Oxford University Press, Oxford.

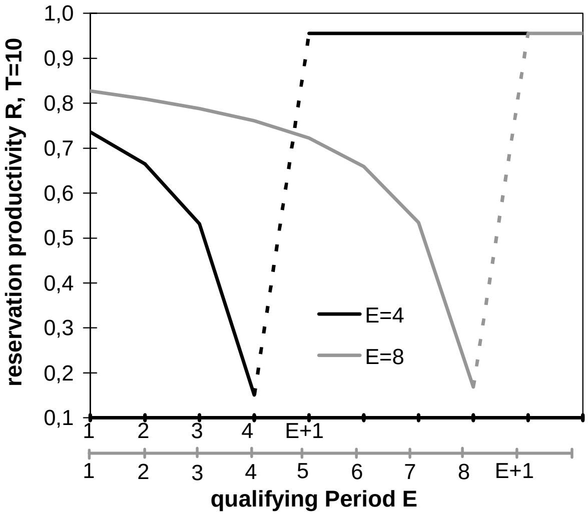
Nickell, S., and Layard, R., 1999, Labour market institutions and economic performance, In: Ashenfelter, O. and D. Card (eds.), Handbook of Labour Economics, Vol. 3C, North-Holland, Amsterdam, 3029-3084.

Appendix I

Tab. A1: The baseline parameter of the model

β	r	λ	y	z	b	k	α	ϕ
0.50	0.02	0.10	100	40	40	40	0	0.50

Appendix II



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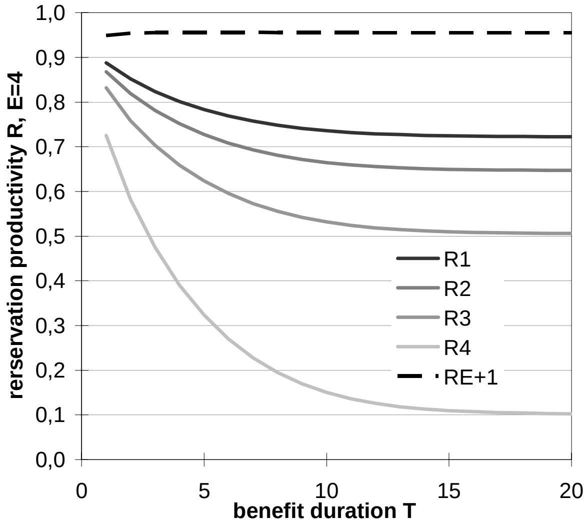

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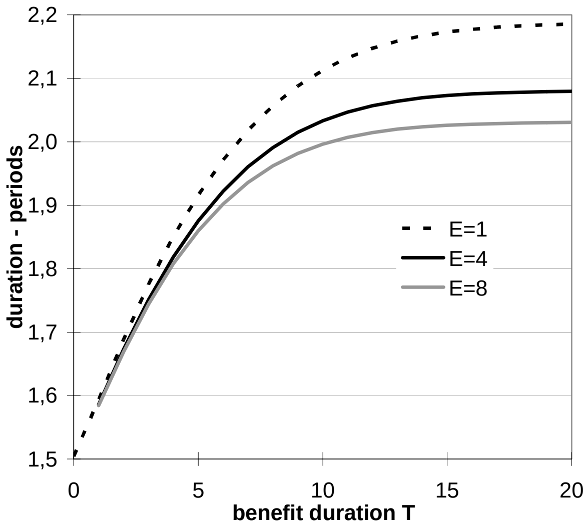

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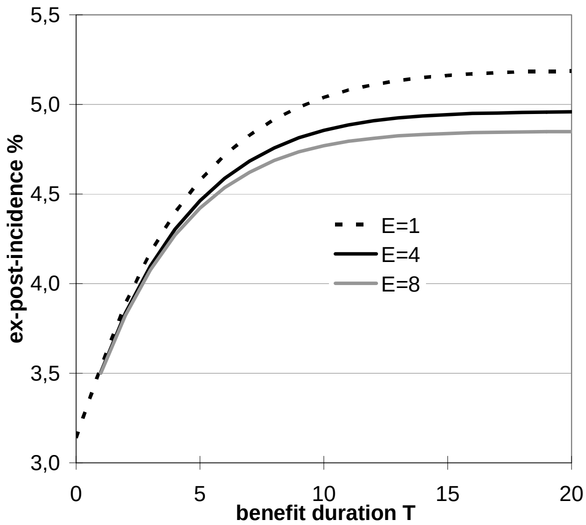

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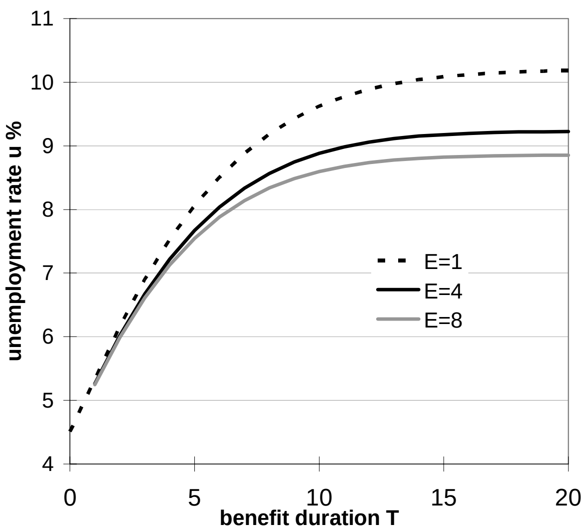

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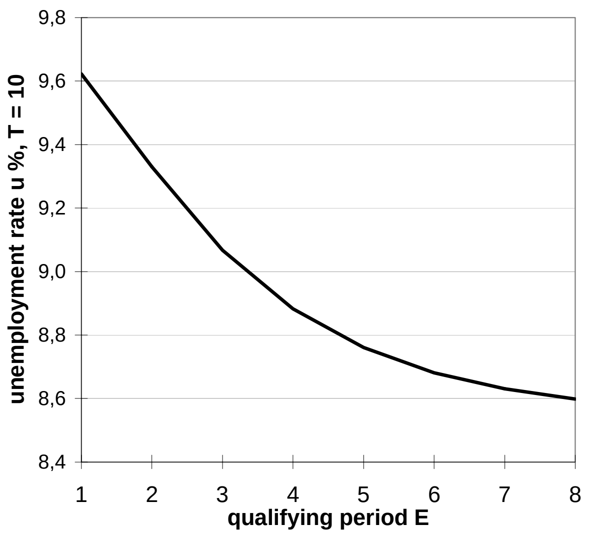

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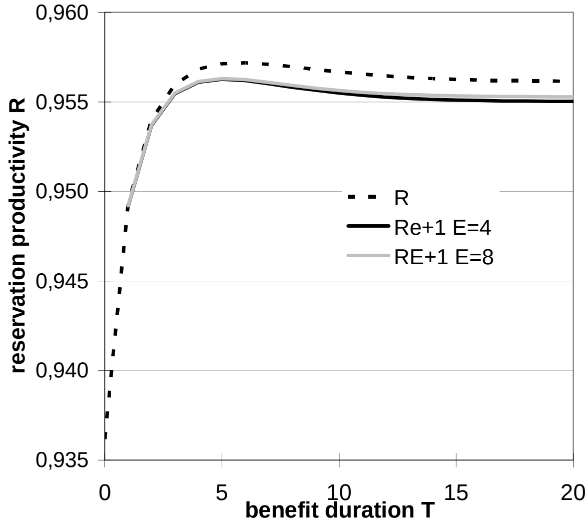
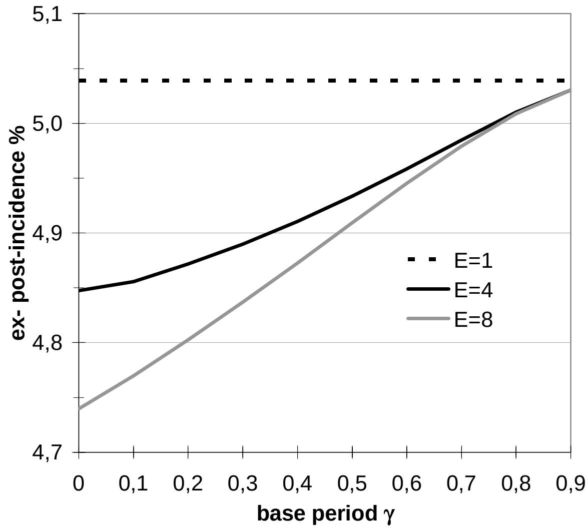
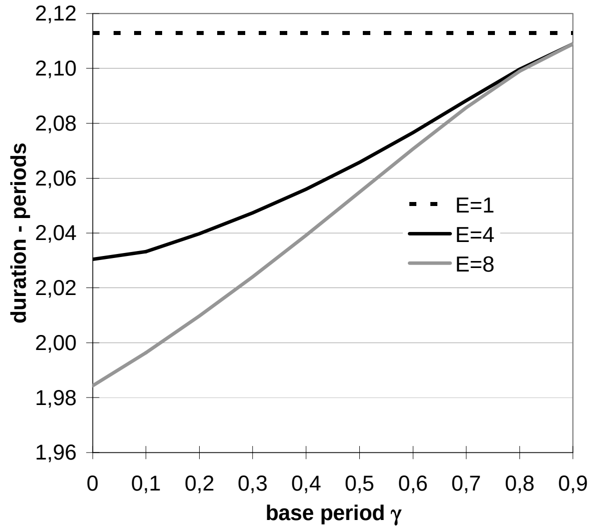


(g)

Appendix III



(a)



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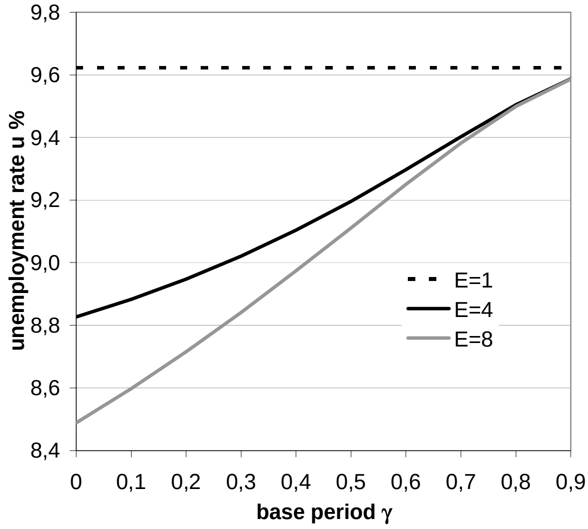

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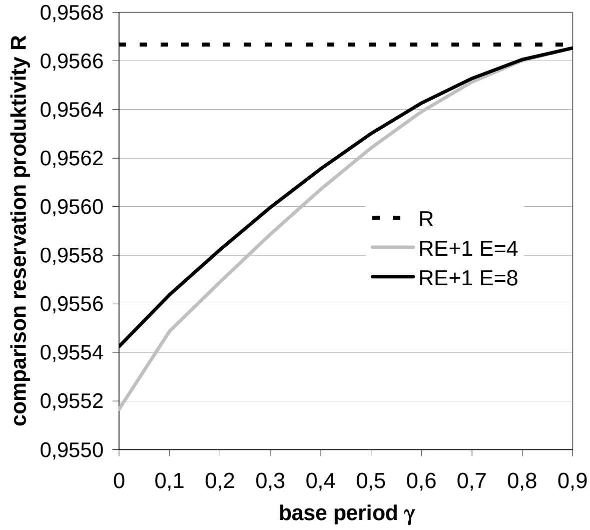

(d)

Appendix IV

Ad: 2. Benefit Duration *T*

Proof of Lemma 1. (i): From (14), it follows that $(1-\beta)U_T = (1-\beta)W(x) - \beta\Pi(x)$. Insert the asset equations (6) and (7) into the above expression and rearrange terms to get the inside wage (16).

(ii) From (15), it follows that $(1-\beta)U_{T-j} = (1-\beta)W_{T-j} - \beta\Pi_{T-j}$. Plugging (8) and (9) into the last equation gives $(1-\beta)U_{T-j} = [(1-\beta)W(1) - \beta\Pi(1)] + \rho[w_{T-j} - w(1)]$, from which in view of (14) the outside wage (17) follows.

Proof of Lemma 2. (i) and (ii): From (5) and (6) we have $0 = yR - w(R) + \lambda\int_R^1 \Pi(h)dG(h)$. From this equation, taking (16) and (6) into account we obtain the equations (18) and (19).

Proof of Lemma 3. (i) From (13), (15), (8) and the wage equation (17), it follows that

(A1) $$U_{T-j} = D + d\left[z + b + U_{T-(j+1)}\right].$$

where $D \equiv \frac{\beta p}{(1-\beta)[1-(1-\beta)p]}\left[\Pi(1) + (1-\beta)U_T\right]$ and $d \equiv \frac{(1-p)\rho}{1-(1-\beta)p}$. Solving the difference equation (A1) gives:

(A2) $$U_{T-j} = \frac{1-d^{T-j}}{1-d}\left[D + d(z+b)\right] + d^{T-j}U_0 .$$

In the same way, it follows from (12), (15), (8) and (17) for U_0:

(A3) $$U_0 = \frac{(1-p)\rho}{(1-\beta)[(1-p)r\rho + \beta p]}z + \frac{1-(1-\beta)p}{(1-p)r\rho + \beta p}D .$$

Using (A3) in (A2) gives:

(A4) $$U_{T-j} = \frac{(1-p)\rho}{(1-p)r\rho + \beta p}\left[z + b\left(1 - d^{T-j}\right)\right] + \frac{\beta p}{(1-\beta)[(1-p)r\rho + \beta p]}\left[\Pi(1) + (1-\beta)U_T\right].$$

From (A4) we get the asset equation (20).

Ad: 3.3 Qualifying Path and Rate of Unemployment in the Steady State

The effects of the parameters of the labour market policy $[E,\gamma,T,b]$ on the equilibrium unemployment rate *u* do not depend on whether the qualifying period is shorter or longer than the benefit duration. For the sake of brevity, we represent the pool equations and the proofs for the case $E \le T$, which most of the OECD (2002) countries follow. The simulations and results in section 4, however, also take into account the case $E \ge T+1$.

First we deal with the steady equations for the number of employed workers, e_{E-i}, $i = 0,\ldots,E-1$, then we develop the steady state conditions for the job seeker, u_{E-iT-j}, $i = 0,\ldots,E$, $j = i,\ldots,T$, and finally we present four lemmas A1 – A4, to be used to develop the functions of the fractions $\varepsilon_{E-i}(\theta,\Psi_E,R_{E+1})$.

1. *Employed Workers*

In the steady state, the following relation hold for the number of the employed workers with the qualifying counter $E-i$:

$$
\text{(A5)} \qquad e_{E-i} = \begin{cases} [1-\lambda G(R_{E+1})]e_E + p\lambda G(R_{E+1})e_E + [1-\lambda G(R_E)]e_{E-1} + \\ \qquad p\lambda G(R_E)e_{E-1} + p\sum_{m=0}^{1}\sum_{j=m}^{T} u_{E-mT-j}, \ i=0 \\ [1-\lambda G(R_{E-i})]e_{E-(i+1)} + p\lambda G(R_{E-i})e_{E-(i+1)} + \\ \qquad p\sum_{j=i+1}^{T} u_{E-(i+1)T-j}, \ i=1,\dots,E-2 \\ p\sum_{j=0}^{T-E} u_{0T-(E+j)}, \ i=E-1 \end{cases}
$$

Ad $i=0$**:** e_E is the measure of the employed worker with a completed qualifying period. The inflow of e_E consists first of workers with a productive job who are entitled to UI benefits, $[1-\lambda G(R_{E+1})]e_E$; second, workers entitled to UI benefits who made a job-to-job transition, $p\lambda G(R_{E+1})e_E$; third in the inflow are the workers of the pool e_{E-1} who make a transition to e_E, $[1-\lambda G(R_E)]e_{E-1}$, or who made a job-to-job transition, $p\lambda G(R_E)e_{E-1}$; and fourth, the successful job seekers $p\sum_{m=0}^{1} u_{E-m}$, where $u_{E-m} \equiv \sum_{j=m}^{T} u_{E-mT-j}$, with a qualifying counter equal to E or $E-1$ belong to the inflow of e_E.

Ad $i=E-1$**:** The inflow of the pool e_1 consists of successful job seekers whose qualifying counter is equal to zero because of the long unemployment, pu_0, where $u_0 \equiv \sum_{j=0}^{T-E} u_{0T-(E+j)}$.

2. *Job Seekers*

2.1 For the measure of job seekers with a completed qualifying period and a *current spell* of unemployment of length *j*, u_{ET-j}, the following is true in the steady state

$$
\text{(A6)} \qquad u_{ET-j} = \begin{cases} (1-p)\lambda G(R_{E+1})e_E, \ j=0 \\ \gamma(1-p)u_{ET-(j-1)}, \ j=1,\dots,T-1. \\ \gamma(1-p)(u_{E0}+u_{E1}), \ j=T \end{cases}
$$

Ad $j=0$**:** u_{ET} is the pool of the unemployed with a completed qualifying period and full entitlement to UI benefits. As the first line of (A6) shows, the inflow to u_{ET} consists of workers with a completed qualifying period who lost their job in the previous period and did not meet a vacancy during the last matching.

Ad $j=T$**:** The third line of (A6) shows the inflow to the pool of job seekers with a completed qualifying period, but no residual claims to unemployment insurance, u_{E0}. The inflow consists of job seekers from the pool $u_{E0}+u_{E1}$ who, although without a match, retain their qualifying points: a composite event, which has the probability $\gamma(1-p)$.

2.2 For the pool of job seekers with a *current spell* of unemployment of length *j* and a qualifying counter equal to $E-i$, u_{E-iT-j}, the steady state condition holds

$$
\text{(A7)} \quad u_{E-iT-j} = \begin{cases} (1-\gamma)(1-p)u_{E-(i-1)T-(j-1)}, \;\; i=1,\dots,E\,, \; j=i \\ \\ (1-p)\left[\gamma u_{E-iT-(j-1)} + (1-\gamma)u_{E-(i-1)T-(j-1)}\right], \;\; i=1,\dots,E, \;\; j=i+1,\dots,T-1 \\ \\ (1-p)\left[\lambda G\left(R_{E-(i-1)}\right)e_{E-i} + \gamma\left(u_{E-i0}+u_{E-i1}\right) + \right. \\ \qquad\qquad \left. (1-\gamma)\left(u_{E-(i-1)0}+u_{E-(i-1)1}\right)\right], \;\; i=1,\dots,E-1, \;\; j=T \end{cases}
$$

Ad $i=1,\dots,E\,,\; j=i$ **:** Since $j \ge i$, u_{E-iT-i} is the pool of job seekers which has the shortest *current spell* of unemployment of $j=i$ periods given the qualifying counter $E-i$. As the first line of (A7) illustrates, the inflow to u_{E-iT-i} consists of unsuccessful job seekers who still belonged to the pool $u_{E-(i-1)T-(j-1)}$ in the previous period.[9]

Ad $i=1,\dots,E-1,\;\; j=T$ **:** The inflow to the pool u_{E-i0} is first composed of workers who lost their job because of a negative shock and did not meet a vacancy during the subsequent matching, $(1-p)\lambda G(R_{E-(i-1)})e_{E-i}$. Secondly, the fraction of the unsuccessful job seekers from pool $u_{E-i0}+u_{E-i1}$ makes a transition to u_{E-i0}, who retain their qualifying points.[10] Finally the fraction of unsuccessful job seekers from the pool $u_{E-(i-1)0}+u_{E-(i-1)1}$, who lose a qualifying point belong also to the inflow to u_{E-i0}.[11]

2.3 For job seekers with a *current spell* of unemployment of length $j \ge E$, whose qualifying counter is equal to zero, u_{0T-j}, the following steady state condition hold

$$
\text{(A8)} \qquad u_{0T-j} = \begin{cases} (1-\gamma)(1-p)u_{1T-(E-1)}, \;\; j=E \\ (1-p)\left[u_{0T-(j-1)} + (1-\gamma)u_{1T-(j-1)}\right], \;\; j=E+1,\dots,T-1 \\ (1-p)\left[u_{00}+u_{01}+(1-\gamma)(u_{10}+u_{11})\right], \;\; j=T \end{cases}
$$

Ad $j=T$ **:** The pool u_{00} consists of job seekers who have neither qualifying points or residual claims for unemployment insurance. The inflow to u_{00} is composed of unsuccessful job seekers first from pool $u_{00}+u_{01}$ and second from pool $u_{10}+u_{11}$ who lose the last qualifying point at the transition.

3. *Lemmas*

Lemma A1. (i) *Let* $T \ge i+1 \ge 1$, *then the following equation holds:*

$$
\textit{(A9)} \quad 1 = \sum_{j=0}^{i} \binom{T}{j} \left[1-\gamma(1-p)\right]^{j} (1-p)^{T-j} \gamma^{T-j} + \left[1-\gamma(1-p)\right]^{i+1} \sum_{j=0}^{T-(i+1)} \binom{i+j}{j} (1-p)^{j} \gamma^{j}\,.
$$

(ii) *Let* $T \ge E+1$, *then we can prove:*

$$
\textit{(A10)} \quad \sum_{j=0}^{T-(E+1)} (1-p)^{j} \sum_{k=0}^{j} \binom{E-1+k}{E-1} \gamma^{k} = \sum_{j=0}^{T-(E+1)} \binom{E-1+j}{E-1} (1-p)^{j}\, \gamma^{j}\, \frac{\left[1-(1-p)^{T-(E+j)}\right]}{p}.
$$

---

[9] In view of base period F, this transition corresponds to the transition of a job seeker with the qualifying counter $E-(i-1)$ who did not meet a vacancy and was employed F periods ago.

[10] These workers were unemployed F periods ago at the beginning of the previous base period.

[11] These workers were employed F periods ago at the beginning of the previous base period.

The following lemma presents solutions of the difference equations (A6) – (A8) for the different types of job seekers. To solve the equations, we use the conditional probability $a(\theta) \equiv \frac{(1-\gamma)[1-p(\theta)]}{p(\theta)+(1-\gamma)[1-p(\theta)]}$, which depends on the tightness θ. A job seeker makes a transition from his type-specific pool $u_{E-i} \equiv \sum_{j=i}^{T} u_{E-iT-j}$ either because his search was successful or because he did not meet a vacancy and loses a qualifying point. The first event occurs with the probability p, the second with the probability $(1-\gamma)(1-p)$. a is the probability that a job seeker who makes a transition will not find a job and loses a qualifying point. $1-a$ is the probability that a job seeker who makes a transition will find a new job.

Lemma A2. (i) [JOB SEEKERS] *1. For the job seeker pool* u_{E-iT-j}*, with* $i = 0,\dots,E-1$ *and* $j = i,\dots,T-1$*, the following is true:*

(A11) $$u_{E-iT-j} = \binom{j}{i}(1-\gamma)^i (1-p)^{j+1} \gamma^{j-i} \lambda G(R_{E+1}) e_E .$$

2. For the job seeker pool u_{E-i0}*, with* $i = 1,\dots,E-1$*, we have:*

(A12) $$u_{E-i0} = \frac{1-p}{1-\gamma(1-p)}\left[(1-p)^T \lambda G(R_{E+1}) e_E \sum_{k=0}^{i} \binom{T}{k} a^{i-k} (1-\gamma)^k \gamma^{T-k} + \sum_{k=1}^{i} a^{i-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k}\right].$$

3. For the pool u_{E0} *we can prove:*

(A13) $$u_{E0} = \frac{1-p}{1-\gamma(1-p)}(1-p)^T \gamma^T \lambda G(R_{E+1}) e_E .$$

4. For the pool $u_{0T-(E+j)}$*, with* $j = 0,\dots T-(E+1)$*, the following is the case:*

(A14) $$u_{0T-(E+j)} = (1-p)^{E+j+1} (1-\gamma)^E \lambda G(R_{E+1}) e_E \sum_{k=0}^{j} \binom{E-1+k}{E-1} \gamma^k$$

5. For the pool u_{00} *the following is true:*

(A15) $$u_{00} = \frac{1-p}{p} \sum_{k=0}^{E-1} a^{E-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k} - \frac{(1-p)^{E+1}}{p} (1-\gamma)^E \lambda G(R_{E+1}) e_E \sum_{j=0}^{T-E} \binom{E-1+j}{E-1} (1-p)^j \gamma^j \left[1-(1-p)^{T-(E+j)}\right]$$

(ii) [AGGREGATED POOLS] *1. In the steady state, the aggregated pool* $u_{E-i} \equiv \sum_{j=i}^{T} u_{E-iT-j}$*,* $i = 1,\dots,E-1$*, has the mass*

(A16) $$u_{E-i} = \frac{1-p}{1-\gamma(1-p)} \sum_{k=0}^{i} a^{i-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k} .$$

2. For the aggregated pool $u_E \equiv \sum_{j=0}^{T} u_{ET-j}$ *the following is true*

(A17) $$u_E = \frac{1-p}{1-\gamma(1-p)} \lambda G(R_{E+1}) e_E .$$

3. Finally for $u_0 \equiv \sum_{j=0}^{T-E} u_{0T-(E+j)}$ *the following steady state equation holds:*

(A18) $$u_0 = \frac{1-p}{p} \sum_{k=0}^{E-1} a^{E-k} \lambda G(R_{E-(k-1)}) e_{E-k} .$$

The next lemma develops the solutions of the difference equations (A5).

Lemma A3. [Employed worker] (i) *For the measure of workers with the qualifying counter* $E-i$*, the following is true:*

(A19) $$e_{E-i} = \begin{cases} \dfrac{a^2}{1-a(1-p)\lambda G(R_E)} (1-p)\lambda G(R_{E+1}) e_E, \; i=1 \\ \dfrac{a}{1-a(1-p)\lambda G(R_{E-(i-1)})} e_{E-(i-1)}, \; i = 2, \ldots, E-2 \\ \dfrac{\sum_{k=0}^{E-2} a^{E-k} (1-p)\lambda G(R_{E-(k-1)}) e_{E-k}}{1-a(1-p)\lambda G(R_2)}, \; i = E-1 \end{cases}$$

(ii) *By using the difference equations (A19) we obtain:*

(A20) $$e_{E-i} = f_{E-i}(\theta, R_{E-i}, \ldots, R_E, R_{E+1}) e_E , \; i = 1, \ldots E-1$$

where for the frequencies f_{E-i}, $i = 1, \ldots, E-2$*, the following holds:*

(A21) $$f_{E-i}(\theta, R_{E-(i-1)}, \ldots, R_E, R_{E+1}) \equiv \frac{a(\theta)^{i+1}(1-p(\theta))\lambda G(R_{E+1})}{\prod_{k=0}^{i-1} [1-a(\theta)(1-p(\theta))\lambda G(R_{E-k})]}$$

and for f_1*:*

(A22) $$f_1(\theta, \Psi_E R_{E+1}) \equiv \frac{\sum_{k=0}^{E-2} a(\theta)^{E-k} (1-p(\theta))\lambda G(R_{E-(k-1)}) f_{E-k}(\theta, R_{E-(k-1)}, \ldots, R_E, R_{E+1})}{1-a(\theta)(1-p(\theta))\lambda G(R_2)} ,$$

where $\Psi_E = (R_2, \ldots, R_E)$*. Also, let* $f_E \equiv 1$*.*

Lemma A4. [Fractions ε_{E-i}**]** *With Lemma A3 we obtain the fraction of employed workers with the qualifying counter* $E-i$ *to:*

(A23) $$\varepsilon_{E-i}(\theta,\Psi_E,R_{E+1})=\frac{f_{E-i}\left(\theta,R_{E-(i-1)},\ldots,R_E,R_{E+1}\right)}{1+\sum_{k=1}^{E-1}f_{E-k}\left(\theta,R_{E-(k-1)},\ldots,R_E,R_{E+1}\right)},\ i=0,\ldots,E-1.$$

4. Proofs of the Lemmas A1 – A4

Proof of Lemma A1. (i) 1. Let $i=0$, then clearly $(1-p)^T\gamma^T+[1-\gamma(1-p)]\sum_{j=0}^{T-1}(1-p)^j\gamma^j=1$ holds.

2. Assume the statement is true for *i*, then for $i+1$ and $T\geq i+2$ with

$$RHS(T)\equiv\sum_{j=0}^{i+1}\binom{T}{j}[1-\gamma(1-p)]^j(1-p)^{T-j}\gamma^{T-j}+[1-\gamma(1-p)]^{i+2}\sum_{j=0}^{T-(i+2)}\binom{i+1+j}{j}(1-p)^j\gamma^j$$

it follows that

$$RHS(T)=1+[1-\gamma(1-p)]^{i+1}\Bigg[\binom{T}{i+1}(1-p)^{T-(i+1)}\gamma^{T-(i+1)}+$$
$$[1-\gamma(1-p)]\sum_{j=0}^{T-(i+2)}\binom{i+1+j}{i+1}(1-p)^j\gamma^j-\sum_{j=0}^{T-(i+1)}\binom{i+j}{i}(1-p)^j\gamma^j\Bigg]$$
$$=1+[1-\gamma(1-p)]^{i+1}\Bigg[(1-p)^{T-(i+1)}\gamma^{T-(i+1)}\left[\binom{T}{i+1}-\binom{T-1}{i}\right]+$$
$$\sum_{j=0}^{T-(i+2)}(1-p)^j\gamma^j\left[[1-\gamma(1-p)]\binom{i+1+j}{i+1}-\binom{i+j}{i}\right]\Bigg].$$

The second summand in the above equation is equal to zero! We prove this statement by induction over the benefit duration $T\geq i+2$. Clearly, for $T=i+2$, $RHS(i+2)=1$ holds. For the conclusion from *T* to $T+1$, in view of the induction hypothesis, it then holds that:

$$RHS(T+1)=1+[1-\gamma(1-p)]^{i+1}\Bigg[(1-p)^{T-i}\gamma^{T-i}\left[\binom{T}{i+1}\frac{T+1}{T-i}-\binom{T-1}{i}\frac{T}{T-i}\right]+$$
$$\sum_{j=0}^{T-(i+2)}(1-p)^j\gamma^j\left[[1-\gamma(1-p)]\binom{i+1+j}{i+1}-\binom{i+j}{i}\right]+(1-p)^{T-(i+1)}\gamma^{T-(i+1)}\left[[1-\gamma(1-p)]\binom{T}{i+1}-\binom{T-1}{i}\right]\Bigg]$$
$$=1+[1-\gamma(1-p)]^{i+1}\Bigg[(1-p)^{T-i}\gamma^{T-i}\left[\binom{T}{i+1}\frac{T+1}{T-i}-\binom{T-1}{i}\frac{T}{T-i}\right]-(1-p)^{T-i}\gamma^{T-i}\binom{T}{i+1}\Bigg]$$
$$=1.$$

(ii) 1. If $T=E+1$, then $RHS(T)=LHS(T)=1$ is true. 2. For the conclusion from *T* to $T+1$ we develop the RHS of the equation (A10):

$$RHS(T+1)\equiv\sum_{j=0}^{T-E}\binom{E-1+j}{E-1}(1-p)^j\ \gamma^j\ \frac{\left[1-(1-p)(1-p)^{T-(E+j)}\right]}{p}$$
$$=\sum_{j=0}^{T-(E+1)}\binom{E-1+j}{E-1}(1-p)^j\ \gamma^j\ \frac{\left[1-(1-p)^{T-(E+j)}\right]}{p}+\sum_{j=0}^{T-(E+1)}\binom{E-1+j}{E-1}(1-p)^j\ \gamma^j(1-p)^{T-(E+j)}+$$
$$\binom{T-1}{E-1}(1-p)^{T-E}\gamma^{T-E}$$

$$= LHS(T) + (1-p)^{T-E}\left[\sum_{j=0}^{T-(E+1)}\binom{E-1+j}{E-1}\gamma^{j} + \binom{T-1}{E-1}\gamma^{T-E}\right]$$
$$= LHS(T+1).$$

Proof of Lemma A2. **(i) [JOB SEEKERS]** 1. When $j=i$, in view of (A7) the statement follows directly from the equation $u_{E-iT-i} = (1-p)(1-\gamma)u_{E-(i-1)T-(i-1)}$. Now, let $j>i$, then by virtue of (A7), the following results by induction over *j*:

$$u_{E-iT-j} = (1-p)\left[\gamma u_{E-iT-(j-1)} + (1-\gamma)u_{E-(i-1)T-(j-1)}\right]$$
$$= (1-p)\left[\gamma\binom{j-1}{i}(1-\gamma)^{i}(1-p)^{(j-1)+1}\gamma^{(j-1)-i}\lambda G(R_{E+1})e_E + \right.$$
$$\left.(1-\gamma)\binom{j-1}{i-1}(1-\gamma)^{i-1}(1-p)^{(j-1)+1}\gamma^{(j-1)-(i-1)}\lambda G(R_{E+1})e_E\right]$$
$$= \left[\binom{j-1}{i} + \binom{j-1}{i-1}\right](1-\gamma)^{i}(1-p)^{j+1}\gamma^{j-i}\lambda G(R_{E+1})e_E$$
$$= \binom{j}{i}(1-\gamma)^{i}(1-p)^{j+1}\gamma^{j-i}\lambda G(R_{E+1})e_E\,.$$

2. With (A7),

$$u_{E-i0} = (1-p)\left[\lambda G(R_{E-(i-1)})e_{E-i} + \gamma(u_{E-i0} + u_{E-i1}) + (1-\gamma)(u_{E-(i-1)0} + u_{E-(i-1)1})\right]$$
$$= \frac{(1-p)}{1-\gamma(1-p)}\left[\lambda G(R_{E-(i-1)})e_{E-i} + \gamma u_{E-i1} + (1-\gamma)(u_{E-(i-1)0} + u_{E-(i-1)1})\right].$$

We eliminate the pools u_{E-i1} and $u_{E-(i-1)1}$ using (A11), and replace pool $u_{E-(i-1)0}$ by induction over *i* taking into account that $(1-\gamma)\dfrac{(1-p)}{1-\gamma(1-p)} = a$, to arrive at:

$$u_{E-i0} = \frac{(1-p)}{1-\gamma(1-p)}\left[\lambda G(R_{E-(i-1)})e_{E-i} + \binom{T-1}{i}(1-\gamma)^{i}(1-p)^{T}\gamma^{T-i}\lambda G(R_{E+1})e_E + \right.$$
$$(1-p)^{T}\lambda G(R_{E+1})e_E\sum_{k=0}^{i-1}\binom{T}{k}a^{i-k}(1-\gamma)^{k}\gamma^{T-k} + \sum_{k=1}^{i-1}a^{i-k}\lambda G(R_{E-(k-1)})e_{E-k} +$$
$$\left.\binom{T-1}{i-1}(1-\gamma)^{i}(1-p)^{T}\gamma^{T-i}\lambda G(R_{E+1})e_E\right].$$

Collecting terms it follows:

$$u_{E-i0} = \frac{(1-p)}{1-\gamma(1-p)}\left[\sum_{k=1}^{i}a^{i-k}\lambda G(R_{E-(k-1)})e_{E-k} + \right.$$
$$\left.(1-p)^{T}\lambda G(R_{E+1})e_E\left[(1-\gamma)^{i}\gamma^{T-i}\left[\binom{T-1}{i-1} + \binom{T-1}{i}\right] + \sum_{k=0}^{i-1}\binom{T}{k}a^{i-k}(1-\gamma)^{k}\gamma^{T-k}\right]\right]$$
$$= \frac{(1-p)}{1-\gamma(1-p)}\left[\sum_{k=1}^{i}a^{i-k}\lambda G(R_{E-(k-1)})e_{E-k} + (1-p)^{T}\lambda G(R_{E+1})e_E\sum_{k=0}^{i}\binom{T}{k}a^{i-k}(1-\gamma)^{k}\gamma^{T-k}\right].$$

3. With (A6) $u_{E0} = \gamma(1-p)(u_{E0} + u_{E1})$ results. If we eliminate u_{E1} with (A11) and solve for u_{E0}, the statement follows.

4. From (A8) $u_{0T-(E+j)} = (1-p)\left[u_{0T-(E+j-1)} + (1-\gamma)u_{1T-(E+j-1)}\right]$. If we eliminate $u_{1T-(E+j-1)}$ with (A11) and $u_{0T-(E+j-1)}$ by induction over *j*, the statement follows:

$$u_{0T-(E+j)} = \left[\sum_{k=0}^{j-1}\binom{E-1+k}{E-1}\gamma^k + \binom{E+j-1}{E-1}\gamma^j\right](1-p)^{E+j+1}(1-\gamma)^E \lambda G(R_{E+1})e_E$$

$$= (1-p)^{E+j+1}(1-\gamma)^E \lambda G(R_{E+1})e_E \sum_{k=0}^{j}\binom{E-1+k}{E-1}\gamma^k .$$

5. From (A8): $u_{00} = \frac{(1-p)}{p}\left[u_{01} + (1-\gamma)(u_{10} + u_{11})\right]$. Replace u_{01} with (A14), u_{10} with (A12) and u_{11} with (A11), to get:

$$u_{00} = \frac{(1-p)}{p}\left[\sum_{k=1}^{E-1} a^{E-k}\lambda G(R_{E-(k-1)})e_{E-k} + \right.$$

$$\left.(1-\gamma)^E(1-p)^T \lambda G(R_{E+1})e_E\left[\sum_{j=0}^{T-E}\binom{E-1+j}{E-1}\gamma^j + \sum_{j=0}^{E-1}\binom{T}{j}a^{E-j}(1-\gamma)^{j-E}\gamma^{T-j}\right]\right]$$

$$= \frac{(1-p)}{p}\left[\sum_{k=1}^{E-1} a^{E-k}\lambda G(R_{E-(k-1)})e_{E-k} + \lambda G(R_{E+1})e_E\left[(1-\gamma)^E(1-p)^T\sum_{j=0}^{T-E}\binom{E-1+j}{E-1}\gamma^j + \right.\right.$$

$$\left.\left. a^E\sum_{j=0}^{E-1}\binom{T}{j}\left[1-\gamma(1-p)\right]^j (1-p)^{T-j}\gamma^{T-j}\right]\right]$$

In view of Lemma A1 (i) and $\binom{E-1+j}{j} = \binom{E-1+j}{E-1}$, we can write:

$$u_{00} = \frac{(1-p)}{p}\left[\sum_{k=1}^{E-1} a^{E-k}\lambda G(R_{E-(k-1)})e_{E-k} + \lambda G(R_{E+1})e_E\left[(1-\gamma)^E(1-p)^T\sum_{j=0}^{T-E}\binom{E-1+j}{E-1}\gamma^j + \right.\right.$$

$$\left.\left. a^E\left[1-\left[1-\gamma(1-p)\right]^E\sum_{j=0}^{T-E}\binom{E-1+j}{E-1}(1-p)^j\gamma^j\right]\right]\right]$$

$$= \frac{(1-p)}{p}\left[\sum_{k=0}^{E-1} a^{E-k}\lambda G(R_{E-(k-1)})e_{E-k} - \right.$$

$$\left.(1-p)^E(1-\gamma)^E \lambda G(R_{E+1})e_E\sum_{j=0}^{T-E}\binom{E-1+j}{E-1}(1-p)^j\gamma^j\left[1-(1-p)^{T-(E+j)}\right]\right].$$

(ii) [AGGREGATED POOLS] The equations for the aggregated pools (A16) – (A18) can be derived from the *macroeconomic* steady state conditions or, as below, from the microeconomic pool equations (A11) – (A15).

1. For the pool $u_{E-i} \equiv \sum_{j=i}^{T} u_{E-iT-j}$, in view of $u_{E-i} = u_{E-i0} + \sum_{j=i}^{T-1} u_{E-iT-j}$, the following results from (A11) and (A12):

$$u_{E-i} = \frac{1-p}{1-\gamma(1-p)}\sum_{k=1}^{i} a^{i-k}\lambda G(R_{E-(k-1)})e_{E-k} +$$

$$(1-p)\lambda G(R_{E+1})e_E\left[(1-\gamma)^i\sum_{j=i}^{T-1}\binom{j}{i}(1-p)^j\gamma^{j-i} + \frac{(1-p)^T}{1-\gamma(1-p)}\sum_{k=0}^{i}\binom{T}{k}a^{i-k}(1-\gamma)^k\gamma^{T-k}\right]$$

$$= \frac{1-p}{1-\gamma(1-p)} \sum_{k=1}^{i} a^{i-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k} +$$

$$\frac{(1-p)}{1-\gamma(1-p)} a^i \lambda G(R_{E+1}) e_E \left[[1-\gamma(1-p)]^{i+1} \sum_{j=i}^{T-1} \binom{j}{i} (1-p)^{j-i} \gamma^{j-i} + \sum_{k=0}^{i} \binom{T}{k} [1-\gamma(1-p)]^k (1-p)^{T-k} \gamma^{T-k} \right]$$

so that, in view of Lemma A1 (i), the statement follows.

2. For the pool $u_E \equiv \sum_{j=0}^{T} u_{ET-j}$, we can write $u_E = u_{E0} + \sum_{j=0}^{T-1} u_{ET-j}$, so that the statement from (A11), $i = 0$ and (A13) follows.

3. For the pool $u_0 \equiv \sum_{j=0}^{T-E} u_{0T-(E+j)}$, we can write $u_0 = u_{00} + \sum_{j=0}^{T-(E+1)} u_{0T-(E+j)}$, so that with (A14) and Lemma A1 (ii), we obtain the following equation:

$$u_0 = u_{00} + (1-p)^{E+1} (1-\gamma)^E \lambda G(R_{E+1}) e_E \sum_{j=0}^{T-(E+1)} (1-p)^j \sum_{k=0}^{j} \binom{E-1+k}{E-1} \gamma^k$$

$$= u_{00} + (1-p)^{E+1} (1-\gamma)^E \lambda G(R_{E+1}) e_E \sum_{j=0}^{T-(E+1)} \binom{E-1+j}{E-1} (1-p)^j \gamma^j \frac{\left[1-(1-p)^{T-(E+j)}\right]}{p}.$$

If we replace u_{00} using (A15), the proposition follows.

Proof of Lemma A3. (i) 1. For e_E, we get with $i = 0$ from (A5):

$$e_E = [1-\lambda G(R_{E+1})] e_E + p\lambda G(R_{E+1}) e_E + [1-\lambda G(R_E)] e_{E-1} + p\lambda G(R_E) e_{E-1} + p[u_E + u_{E-1}].$$

If we replace $u_E + u_{E-1}$ using (A16) and (A17) and solve for e_{E-1}, we obtain the first line of (A19).

2. For $i = 1,\ldots, E-2$, we obtain the following from (A5), in view of (A16):

$$e_{E-i} = [1-\lambda G(R_{E-i})] e_{E-(i+1)} + p\lambda G(R_{E-i}) e_{E-(i+1)} + p u_{E-(i+1)}$$

$$= [1-(1-p)\lambda G(R_{E-i})] e_{E-(i+1)} + (1-a)(1-p) \sum_{k=0}^{i+1} a^{i+1-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k}$$

$$= [1-a(1-p)\lambda G(R_{E-i})] e_{E-(i+1)} + a(1-a)(1-p) \sum_{k=0}^{i} a^{i-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k}$$

$$= [1-a(1-p)\lambda G(R_{E-i})] e_{E-(i+1)} + apu_{E-i}$$

$$= [1-a(1-p)\lambda G(R_{E-i})] e_{E-(i+1)} + a\left[e_{E-(i-1)} - \left[1-(1-p)\lambda G\left(R_{E-(i-1)}\right)\right] e_{E-i}\right].$$

To derive the last equation we make use of (A5). Rearranging terms gives:

$$a\left(e_{E-i} - e_{E-(i-1)}\right) + \left[1-a(1-p)\lambda G\left(R_{E-(i-1)}\right)\right] e_{E-i} = [1-a(1-p)\lambda G(R_{E-i})] e_{E-(i+1)}.$$

By induction over *i*, we get: $\left[1-a(1-p)\lambda G\left(R_{E-(i-1)}\right)\right] e_{E-i} = ae_{E-(i-1)}$. Replacing the LHS and solving for $e_{E-(i+1)}$ gives the second line of (A19).

3. For e_1 and $i = E-1$, $e_1 = pu_0$ results from (A5), with (A18) we get: $e_1 = (1-p) \sum_{k=0}^{E-1} a^{E-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k}$. From the last equation, it follows that $e_1 = a(1-p)\lambda G(R_2) e_1 + (1-p) \sum_{k=0}^{E-2} a^{E-k} \lambda G\left(R_{E-(k-1)}\right) e_{E-k}$. If we solve for e_1, we get the last line of (A19).

(ii) The expression (A20) is derived from (A19) by virtue view of (A21) and (A22).

Proof of Lemma A4. In view of (A20) we can write $\varepsilon_{E-i} = f_{E-i}\varepsilon_E$. From this, we can conclude that $\sum_{i=1}^{E-1}\varepsilon_{E-i} = 1-\varepsilon_E = \varepsilon_E \sum_{i=1}^{E-1} f_{E-i}$, so that

$$\varepsilon_E(\theta,\Psi_E,R_{E+1}) = \frac{1}{1+\sum_{i=1}^{E-1} f_{E-i}\left(\theta,R_{E-(i-1)},\ldots,R_E,R_{E+1}\right)}.$$

Inserting this expression into $\varepsilon_{E-i} = f_{E-i}\varepsilon_E$ gives the statement (A23).

The conditional probabilities μ_{E-iT-j} - that an applicant has $E-i$ qualifying points and a residual claim to the UI benefit b of $T-j$ periods – directly follow from Lemma A2 (i) and Lemma A4.

Lemma A5. *For the conditional probabilities* $\mu_{E-iT-j} = u_{E-iT-j}/u$ *, we obtain, with*

$$F(\theta,\Psi_E,R_{E+1}) \equiv (1-p(\theta))\sum_{i=0}^{E-1} f_{E-i}\left(\theta,R_{E-(i-1)},\ldots,R_E,R_{E+1}\right)\lambda G\left(R_{E-(i-1)}\right),$$

the following:

(A24) $\mu_{E-iT-j} = \binom{j}{i}(1-\gamma)^i(1-p)^{j+1}\gamma^{j-i}\lambda G(R_{E+1})pF(\theta,\Psi_E,R_{E+1}),\ i=0,\ldots,E-1,$
$j=i,\ldots,T-1.$

(A25) $\mu_{E0} = (1-a)(1-p)^{T+1}\gamma^T\lambda G(R_{E+1})F(\theta,\Psi_E,R_{E+1}).$

(A26) $$\mu_{E-i0} = (1-a)F(\theta,\Psi_E,R_{E+1})\left[(1-p)^{T+1}\lambda G(R_{E+1})\sum_{k=0}^{i}\binom{T}{k}a^{i-k}(1-\gamma)^k\gamma^{T-k} + (1-p)\sum_{k=1}^{i}a^{i-k}\lambda G\left(R_{E-(k-1)}\right)f_{E-k}\right],\ i=1,\ldots,E-1.$$

(A27) $\mu_{0T-(E+j)} = (1-p)^{E+j+1}(1-\gamma)^E\lambda G(R_{E+1})pF(\theta,\Psi_E,R_{E+1})\sum_{k=0}^{j}\binom{E-1+k}{E-1}\gamma^k,$
$j=0,\ldots,T-(E+1).$

(A28) $$\mu_{00} = F(\theta,\Psi_E,R_{E+1})\left[(1-p)\sum_{k=1}^{i}a^{i-k}\lambda G\left(R_{E-(k-1)}\right)f_{E-k} - (1-p)^{E+1}(1-\gamma)^E\lambda G(R_{E+1})\sum_{j=0}^{T-E}\binom{E-1+j}{E-1}(1-p)^j\gamma^j\left[1-(1-p)^{T-(E+j)}\right]\right],$$

where $F \equiv F(\theta,\Psi_E,R_{E+1})$.

Ad: 3.4 Qualifying Rents and Waiting Period

The distribution rule, which is used for wage negotiations between a vacancy and a job seeker, is as follows:

(A29) $$W_{E-iT-j} - U_{E-iT-j} = \frac{\beta}{1-\beta}\Pi_{E-iT-j},\ i=0,\ldots,E,\ j=i,\ldots,T,$$

where W_{E-iT-j} is the value of an employed outsider with $E-i$ qualifying points and a benefit duration of $T-j$ periods, U_{E-iT-j} is the value of the unemployed outsider, and Π_{E-iT-j} is the initial value of the filled job.

Π_{E-iT-j} depends on the job seeker's residual claims and the current status of the qualifying counter, where the following is true, in view of the initial productivity $x=1$, the outside wage w_{E-iT-j} and the asset equations (23) and (25):

$$\text{(A30)}\quad \Pi_{E-iT-j} = \begin{cases} \Pi_{E+1}(1),\ i=j=0 \\ \\ \Pi_{E-(i-1)}(1)+\rho\left[w_{E-(i-1)}(1)-w_{E-iT-j}\right],\ i=0,\ldots,E,\ j=i,\ldots,T-1 \\ \\ \Pi_{E-(i-1)}(1),\ i=1,\ldots,E,\ j=T \end{cases}$$

For the distribution of the initial values of the job seeker, W_{E-iT-j}, analogously we have:

$$\text{(A31)}\quad W_{E-iT-j} = \begin{cases} W_{E+1}(1),\ i=j=0 \\ \\ W_{E-(i-1)}(1)+\rho\left[w_{E-iT-j}-w_{E-(i-1)}(1)\right],\ i=0,\ldots,E,\ j=i,\ldots,T-1 \\ \\ W_{E-(i-1)}(1),\ i=1,\ldots,E,\ j=T \end{cases}$$

The steady state values of the job seeker are:

$$\text{(A32)}\quad U_{E-iT-j} = \begin{cases} pW_{00}+(1-p)\rho\left[z+U_{00}\right],\ i=E,\ j=T \\ \\ pW_{E-i0}+(1-p)\rho\left[z+\gamma U_{E-i0}+(1-\gamma)U_{E-(i+1)0}\right],\ i=0,\ldots,E-1,\ j=T \\ \\ pW_{E-iT-j}+(1-p)\rho\left[z+b+\gamma U_{E-iT-(j+1)}+\right. \\ \qquad\qquad \left.(1-\gamma)U_{E-(i+1)T-(j+1)}\right],\ i=0,\ldots,E-1,\ j=i,\ldots,T-1 \\ \\ pW_{0T-j}+(1-p)\rho\left[z+b+U_{0T-(j+1)}\right], i=E,\ j=E,\ldots,T-1 \end{cases}$$

In (A32) z is the utility of leisure, b the UI benefit and $\gamma\in[0,1)$ the base period. If the job seeker does not meet a vacancy, his *current spell* of unemployment increases from length j to $j+1$, while the counter of the qualifying period is constant with probability $\gamma<1$ and decreases from $E-i$ to $E-(i+1)$ by one point with probability $1-\gamma>0$.[12]

In view of the asset pricing equations (23) – (26) and the sharing rules (14) and (27), we obtain

[12] The job seeker – like B in the introductory example – was unemployed F periods ago and in the second case – like A - he was employed at the beginning of the base period.

Lemma A6 [Bargained Inside Wage]. *Considering the reservation income* rU_{ET} *of an insider with a completed qualifying period and the qualifying rents* $U_{E-i0}-U_{E-(i+1)0}$, $i=2,\ldots,E-1$, *the agents negotiate the following inside wages.*

The bargained inside wage of a worker with a completed qualifying period at a match specific productivity x is

(A33) $$w_{E+1}(x)=rU_{ET}+\beta(yx-rU_{ET}).$$

Insiders who make a transition from $e_{E-(i+1)}$ *to* e_{E-i} *earn the bargained inside wage*

(A34) $$w_{E-i}(x)=\begin{cases} w_{E+1}(x)-(1-\beta)(U_{ET}-U_{E-10})\rho^{-1}, \; i=0 \\ w_E(x)-(1-\beta)(U_{E-10}-U_{E-20})\rho^{-1}+(1-\beta)(U_{ET}-U_{E-10}), \; i=1 \\ w_{E-(i-1)}(x)-(1-\beta)(U_{E-i0}-U_{E-(i+1)0})\rho^{-1}+ \\ \qquad (1-\beta)(U_{E-(i-1)0}-U_{E-i0}), i=2,\ldots,E-1 \end{cases}$$

Proof of Lemma A6. 1. From the distribution rule (14), it follows that: $(1-\beta)W_{E+1}(x)-\beta\Pi_{E+1}(x)=(1-\beta)U_{ET}$. Using the asset equations (23) – (24) and rearranging terms provides the wage equation (A33).

2. From the distribution rule (27), it follows that: $(1-\beta)W_{E-i}(x)-\beta\Pi_{E-i}(x)=(1-\beta)U_{E-(i+1)0}$, $i=0,\ldots,E-1$. If we use - for $i=0,\ldots,E-1$ - the asset equation (25) and assume $i=0$, then by virtue of the first line of (26) and the wage equation (A33), we obtain the first line of the wage equation (A34). The other wage equations of (A34) result analogously.

Proof of Lemma 4. (i) Wage equation (29) corresponds to the wage equation (A33) of Lemma A6. We obtain the wage equation (30) in the following way. For $i=0$, we get from (A34):

$$w_E(x)=w_{E+1}(x)-(1-\beta)(U_{ET}-U_{E-10})\rho^{-1}.$$

If we replace $w_{E+1}(x)$ using (A33) and rearrange, we get the first line of (30). Now assume that the statement is true for $w_{E-(i-1)}(x)$. For $w_{E-i}(x)$, we obtain with (A34):

$$w_{E-i}(x)=w_{E-(i-1)}(x)-(1-\beta)(U_{E-i0}-U_{E-(i+1)0})\rho^{-1}+(1-\beta)(U_{E-(i-1)0}-U_{E-i}).$$

If we replace $w_{E-(i-1)}(x)$ using (30) and rearrange, we obtain the proposition.

(ii) From the distribution rule (A29), we can write: $(1-\beta)W_{E-iT-j}-\beta\Pi_{E-iT-j}=(1-\beta)U_{E-iT-j}$. Inserting the asset equations (A30) and (A31), we obtain the wage equations (31).

Proof of the Proposition. (i) If we solve the asset equation (23) for $\Pi_{E+1}(x)$ and take the wage equation (A33) into account, we obtain:

(A35) $$\Pi_{E+1}(x)=\frac{1}{\lambda+r}\left\{(1-\beta)yx-(1-\beta)rU_{ET}+\lambda\int_{R_{E+1}}^{1}\Pi_{E+1}(h)dG(h)\right\}.$$

Let $x=R_{E+1}$ in (A35) the by virtue of $\Pi_{E+1}(R_{E+1})=0$, we obtain the asset equation (33).

If we use the wage equation (30) in (25), we obtain, for $i = 2,\ldots,E-1$:

(A36) $$\Pi_{E-i}(x) = \rho\{(1-\beta)yx - (1-\beta)rU_{E-(i+1)0} + (1-\beta)[U_{E-i0} - U_{E-(i+1)0}] + \lambda\int_{R_{E-(i-1)}}^{1} \Pi_{E-(i-1)}(h)dG(h) + (1-\lambda)\max\{0, \Pi_{E-(i-1)}(x)\}\}.$$

If we use $x = R_{E-i}$ in (A36) and consider the reservation condition (32), we obtain the continuation value (34).

(ii) If we use $x = R_{E+1}$ in (A35) and solve the equation for R_{E+1}, considering (32), we get the job-destruction rule (35). Correspondingly, if we use $x = R_{E-i}$ in (A36) and solve for the reservation productivity R_{E-i}, we get the job-destruction-rule (36).

LEMMA A7. (i) [RESERVATION INCOME] *1. The reservation income of a job seeker who neither owns qualifying points nor claims for unemployment benefits is:*

(A37) $$rU_{00} = z + \frac{\beta p}{(1-\beta)(1-p)}\Pi_1(1)\rho^{-1}.$$

2. The value of a job seeker who does not have qualifying points, but still has claims to UI benefit after $j = E,\ldots,T-1$ periods of unemployment is:

(A38) $$U_{0T-j} = U_{00} + b\sum_{k=1}^{T-j} d^k \,,$$

where $d(\theta) \equiv \frac{\rho[1-p(\theta)]}{1-p(\theta)(1-\beta)} < 1$.

3. For the reservation income of an insider with a qualifying counter equal to $E-i$, $i = 1,\ldots,E-1$, the following is true

(A39) $$rU_{E-i0} = z + \frac{\beta p}{(1-\beta)(1-p)}\tau^{E-i}\Pi_1(1)\rho^{-1} + \frac{\beta p}{(1-\beta)(1-p)(1-\rho\gamma)}\sum_{k=0}^{E-i-1}\tau^k r\Pi_{E-(i+k-1)}(1),$$

where $\tau \equiv \frac{\rho(1-\gamma)}{1-\rho\gamma} < 1$.

4. The value of a job seeker with a current spell *of unemployment of length $j = i,\ldots,T-1$ and $E-i$ qualifying points, $i = 1,\ldots,E-1$, is:*

(A40) $$U_{E-iT-j} = U_{E-i0} + b\sum_{k=1}^{T-j} d^k \,.$$

5. For the reservation income of an insider with a completed qualifying period we have

(A41) $$rU_{ET} = \frac{(1-\rho)(1-d\gamma)}{1-\rho\gamma}\frac{1-d^T}{1-d}b + z + \frac{\beta p}{(1-\beta)(1-p)}\tau^E\Pi_1(1)\rho^{-1} + \frac{\beta p}{(1-\beta)(1-p)(1-\rho\gamma)}\sum_{k=0}^{E-1}\tau^k r\Pi_{E-(k-1)}(1).$$

6. A job seeker with a completed qualifying period and residual claims to UI benefit over $T-j$ periods, $j = 1,\ldots,T$, has the value:

(A42)
$$U_{ET-j} = U_{ET} - b \sum_{k=T-(j-1)}^{T} d^k .$$

(ii) [RENTS] *1. From (A39) we get the qualifying rent for a match that makes a transition from $e_{E-(i+1)}$ to e_{E-i} with:*

(A43)
$$U_{E-i0} - U_{E-(i+1)0} = \frac{\beta p}{(1-\beta)(1-p)(1-\rho\gamma)} \sum_{k=0}^{E-(i+1)} \tau^k \left[\Pi_{E-(i+k-1)}(1) - \Pi_{E-(i+k)}(1)\right].$$

2. Lemma 4, equation (31), shows that for two workers with a completed qualifying period – one is an outsider, the other an insider -, the outsider has the worse bargaining position. The wage penalty he must accept is given by (A42) and the capital gain of an additional benefit duration of j periods, $j = 1,\ldots,T$, with

(A44)
$$U_{ET} - U_{ET-j} = b \sum_{k=T-(j-1)}^{T} d^k .$$

3. If we compare two workers with $E-i$ qualifying points – one is an outsider with a residual benefit duration of $T-j$ periods, the other is an insider -, then the outsider is better off, (s. Lemma 4, equation (31)), because he receives a wage bonus for which, with (A40):

(A45)
$$U_{E-iT-j} - U_{E-i0} = b \sum_{k=1}^{T-j} d^k .$$

Proof of Lemma A7. (i) 1. The statement follows with $i = E$, $j = T$ from the asset equations (A30), (A32) and the distribution rule (A29).

2. Assume $i = E$ and $j = E,\ldots,T-1$ then from the asset equations (A30), (A32) and the distribution rule (A29) we get:

$$U_{0T-j} = \frac{\beta p}{(1-\beta)[1-(1-\beta)p]} \Pi_1(1) + \frac{\beta p}{1-(1-\beta)p} U_{00} + d\left[z + b + U_{0T-(j+1)}\right].$$

Replace $\Pi_1(1)$ using (A37), and solve the difference equation to derive the statement.

3. From the asset equation (A32), the distribution rule (27) and the equation (A30) for the initial value of a filled job, we get

$$U_{E-i0} = \frac{\beta p}{(1-\beta)(1-p)(1-\rho\gamma)} \Pi_{E-(i-1)}(1) + \frac{\rho}{1-\rho\gamma}\left[z + (1-\gamma)U_{E-(i+1)0}\right].$$

Replace U_{00} with equation (A37), solve the difference equation and the statement follows.

4. With the asset equation (A32), the distribution rule (A29) and the initial value of a filled job (A30) we obtain the following difference equation in the benefit duration $T-j$:

(A46)
$$U_{E-iT-j} = \frac{\beta p}{(1-\beta)[1-p(1-\beta)]}\left[\Pi_{E-(i-1)}(1) + (1-\beta)U_{E-i0}\right] + d\left[z + b + \gamma U_{E-iT-(j+1)} + (1-\gamma)U_{E-(i+1)T-(j+1)}\right]$$

First, we show that the proposition holds for $T-j=1$. For $T-j=1$, we can derive from (A46) that

$$U_{E-i1} = \frac{\beta p}{(1-\beta)[1-p(1-\beta)]}\left[\Pi_{E-(i-1)}(1)+(1-\beta)U_{E-i0}\right]+$$
$$d\left[z+b+\gamma U_{E-i0}+(1-\gamma)U_{E-(i+1)0}\right]$$

If we replace $U_{E-(i+1)0}$ with (A39), we get:

$$U_{E-i1} = d\left[b+\left(1+\frac{\rho(1-\gamma)}{1-\rho}\right)z+\left(\gamma+\frac{\beta p}{\rho(1-p)}\right)U_{E-i0}\right]+$$
$$\frac{d(1-\rho\gamma)}{1-\rho}\left[\frac{\beta p}{(1-\beta)(1-p)}\tau^{E-i}\Pi_1(1)\rho^{-1}+\frac{\beta p}{(1-\beta)(1-p)(1-\rho\gamma)}\sum_{k=0}^{E-i-1}\tau^k r\Pi_{E-(i+k-1)}(1)\right]$$

If we substitute the expression in the last brackets with (A39) by $rU_{E-i0}-z$ and rearrange, we obtain the statement: $U_{E-i1}=U_{E-i0}+db$. For the conclusion from $T-j$ to $T-(j-1)$ we eliminate U_{E-iT-j} and $U_{E-(i+1)T-j}$ in (A46) with (A40) and obtain $U_{E-iT-(j-1)}=U_{E-i0}+b\sum_{k=1}^{T-(j-1)}d^k$.

5. With (A32), (A30) and the distribution rule (14), we obtain the following equation for the guarantee value of an insider with a completed qualifying period, U_{ET} :

(A47) $$U_{ET} = \frac{\beta p}{(1-\beta)(1-p)}\Pi_{E+1}(1)+\rho\left[z+b+\gamma U_{ET-1}+(1-\gamma)U_{E-1T-1}\right].$$

To solve the difference equation, we need to know the guarantee value of a job seeker with a completed qualifying period and an unemployment spell of one period, U_{ET-1}. The value U_{E-1T-1} results from (A40).

With (A32), (A30), the distribution rule (A29) and the wage equation (31) we get:

(A48) $$U_{ET-j} = \frac{\beta p}{(1-\beta)[1-p(1-\beta)]}\left[\Pi_{E+1}(1)+(1-\beta)U_{ET}\right]+$$
$$d\left[z+b+\gamma U_{ET-(j+1)}+(1-\gamma)U_{E-1T-(j+1)}\right].$$

Solve the difference equation (A48) to obtain:

(A49) $$U_{ET-j} = \frac{\beta p}{(1-\beta)[1-p(1-\beta)]}\frac{1-(d\gamma)^{T-j}}{1-d\gamma}\left[\Pi_{E+1}(1)+(1-\beta)U_{ET}\right]+\frac{1-(d\gamma)^{T-j}}{1-d\gamma}d(z+b)+$$
$$(d\gamma)^{T-j}U_{E0}+\frac{1-\gamma}{\gamma}\sum_{k=1}^{T-j}(d\gamma)^k U_{E-1T-(j+k)}\,.$$

For U_{E0}, we get from (A32), (A30), the distribution rule (A29) and the wage equation (31):

(A50) $$U_{E0} = \frac{\beta p}{(1-\beta)[(1-p)(1-\rho\gamma)+\beta p]}\left[\Pi_{E+1}(1)+(1-\beta)U_{ET}\right]+$$
$$\frac{\rho(1-p)}{(1-p)(1-\rho\gamma)+\beta p}\left[z+(1-\gamma)U_{E-10}\right].$$

Insert (A50) and (A47) in (A49), to obtain the following equation for $j=1$:

$$\text{(A51)}\quad \gamma U_{ET-1} + (1-\gamma)U_{E-1T-1} = \frac{\gamma\beta p}{(1-\beta)(1-p)(1-\rho\gamma)}\Pi_{E+1}(1) + \frac{\gamma p}{1-\rho\gamma}z + \frac{\rho\gamma-(d\gamma)^T}{1-\rho\gamma}b +$$
$$\frac{1-\gamma}{1-\rho\gamma}\left[(d\gamma)^T U_{E-10} + + \frac{(1-p)(1-\rho\gamma)+\beta p}{1-p(1-\beta)}\sum_{k=0}^{T-1}(d\gamma)^k U_{E-1T-(k+1)}\right].$$

Inserting (A51) into (A47) the statement follows by virtue of (A39) and (A40).

6. From (A47) and (A48) we can deduce that

$$\text{(A52)}\qquad U_{ET} - U_{ET-j} = d\gamma\left[U_{ET-1} - U_{ET-(j+1)}\right] + d(1-\gamma)\left[U_{E-1T-1} - U_{E-1T-(j+1)}\right].$$

Solving this difference equation we arrive at:

$$\text{(A53)}\quad U_{ET} - U_{ET-j} = (d\gamma)^m\left[U_{ET-m} - U_{E0}\right] + d(1-\gamma)\sum_{k=1}^{m}(d\gamma)^{k-1}\left[U_{E-1T-k} - U_{E-1T-(j+k)}\right],$$

where $m = T - j$.

For $U_{ET-m} - U_{E0}$ we obtain from (A48), (A50) and (A45):

$$\text{(A54)}\quad U_{ET-m} - U_{E0} = (d\gamma)^{T-(m+1)}\left[U_{E1} - U_{E0}\right] + db\sum_{g=1}^{T-(m+1)}(d\gamma)^{g-1}\left[1 + (1-\gamma)\sum_{k=1}^{T-(m+g)}d^k\right],$$

as we will prove by induction over *m*. For $m = 1$, the following results from (A48), (A50) and (A45):

$$U_{ET-1} - U_{E0} = d\gamma\left[U_{ET-2} - U_{E0}\right] + db\left[1 + (1-\gamma)\sum_{k=1}^{T-2}d^k\right].$$

From this equation, we obtain:

$$U_{ET-m} - U_{E0} = d\gamma\left[U_{ET-(m+1)} - U_{E0}\right] + db\left[1 + (1-\gamma)\sum_{k=1}^{T-(m+1)}d^k\right].$$

The solution of this difference equation gives (A54). From (A42), (A47) and (A50) we get: $U_{E1} - U_{E0} = db$. Inserting this expression into (A54), considering $m = T - j$, we get: $U_{ET-m} - U_{E0} = bd\left(1-d^j\right)/(1-d)$. Using this equation in (A53) gives the statement by virtue of (A42), from which: $U_{E-1T-k} - U_{E-1T-(j+k)} = bd^{T-(j+k-1)}\sum_{n=0}^{j-1}d^n$