DECONSTRUCTING THE CONSUMPTION FUNCTION: NEW TOOLS AND OLD PROBLEMS

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The relationship between aggregate income and consumption in the United Kingdom is analysed anew. This entails a close examination of the structure of the data, using a variety of spectral methods that depend on the concepts of Fourier analysis. It is found that fluctuations in the rate of growth of consumption tend to precede similar fluctuations in income, which contradicts a common supposition. The difficulty is emphasised of uncovering from the aggregate data a structural equation representing the behaviour of consumers.

Date: January, 2004

Key words: The Consumption Function, Trend Estimation, Seasonal Adjustment, Spectral Analysis

JEL Classification: C22

1. Introduction: The Evolution of the Consumption Function

Over many years, the aggregate consumption function has provided a context in which problems of econometric modelling have been debated and from which significant innovations in methodology have emerged. Whereas such innovations have advanced the subject of econometrics, none of them has been wholly appropriate to the aggregate consumption function itself. This may be one of the reasons why the consumption function has remained a focus of attention.

The vestiges of our misconceptions tend to linger in our minds long after we have consciously amended our beliefs. Our view of the consumption function is particularly prone to the effects of ideas that have not been properly discarded despite their inapplicability. Therefore, in setting a context for our discussion, it is helpful to recount some of the history of the consumption function.

The first difficulties that were encountered in modelling the aggregate consumption function arose from a conflict between Keynesian theory and the empirical findings of Kuznets (1946) and others. Whereas the theory of Keynes (1936) postulated average and marginal propensities to consume that declined with income, it was discovered that income and consumption had maintained a rough proportionality over many years.

At the same time, the econometricians were conscious that there is a double relationship between income and consumption, which follows from the fact that consumption expenditures are a major factor in determining the level of aggregate income. The failure to take account of the second relationship might lead to biases in the estimated coefficients of the consumption function.

Using a static analysis, Haavelmo (1947) demonstrated that the estimated marginal propensity to consume was subject to an upward basis that was directly related to the variance of the innovations in consumption and inversely related to the variance of the innovations in income. The latter were attributed to autonomous changes in the rate of investment.

However, Haavelmo also envisaged, in common with other analysts, "that the active dynamic factor in the business cycle is investment, with consumption assuming a passive lagging role." (These are the words of Alvin Hansen (1941), as quoted by Haavelmo.) This notion was used by others in reconciling the Keynesian formulation with the empirical findings. The manner in which they did so greatly stimulated the development of dynamic econometric modelling.

Models in which consumption displayed a laggardly response to income were provided by Duesenberry (1949), who propounded the relative income hypothesis, by Modigliani and Brumberg (1954), who propounded the life-cycle hypothesis—see Modigliani (1975), also—and by Friedman (1957), who propounded the permanent income hypotheses. According to these models, rapid increases in income will give rise, in the short run, to less-than-proportional increases in consumption, which is in accordance with the Keynesian view. Over longer periods, consumption will gradually regain the relationship with income that was revealed in the empirical findings.

The idea that consumption reacts in a passive and laggardly fashion to the forces impacting upon it also suggested that it might be reasonable to ignore the problem of simultaneous equation bias, to which Haavelmo had drawn attention. The biases would be small if the innovations or disturbances in consumption behaviour were relatively small and if consumers were reacting preponderantly to events of the past.

The two suppositions, upon which the interpretations of the dynamic models largely depended, which were the inertial nature of consumer's behaviour and the relative insignificance of the consumption innovations, have become established preconceptions, despite the lack of evidence to support them. In fact, the evidence that we shall uncover strongly suggests that, in the U.K., the business cycle has been driven by the fluctuations in consumers' expenditure.

For almost two decades, beginning in the mid fifties, successes in modelling the consumption function were seen as grounds for congratulating the econometricians. However, the observations of Granger and Newbold (1974) and others on the spurious nature of regression relationships between trended economic variables led many to suspect that the success might be illusory. Whereas such regressions account remarkably well for the level of consumption, they often perform poorly in the far more stringent task of predicting changes in the level of consumption from one period to another. Moreover, as Granger and Newbold (1974) emphasised, the standard inferential procedures of linear regression analysis are valid only in application to data that have finite-valued asymptotic moment matrices. The moment matrices of trended variables, such as income and consumption, are unbounded.

An apparent resolution of these difficulties came in the late 1970's with advent of the error-correction formulation of the consumption function. It was understood that a dynamic regression model in the levels of income and consumption can be expressed, via a linear reparametrisation, as a model that comprises the differences of the variables together with a stationary error term expressing the current disproportion between income and consumption. Such a model, in which all of the variables appear to be from stationary sequences, is amenable to the standard inferential procedures.

The paper of Davidson, Hendry, Srba and Yeo (1978), which adopted an error-correction formulation, succeeded in re-establishing the traditional consumption function within a viable econometric framework. For a model in which the dependent variable was a differenced sequence, it achieved a remarkably high value for the coefficient of determination. It also heralded the incipient notion of a cointegrating relationship between trended variables, which has subsequently proved to be of major importance.

Some doubts have remained concerning the error-correction formulation of the dynamic consumption function. For a start, it is questionable whether the equation is a structural equation that truly represents the behaviour of consumers in the aggregate, as it purports to do. There may be insufficient grounds for ignoring the problems of simultaneous equation bias. There have also been doubts about the statistical significance of the error-correction term, which is included in the equation. We shall raise these doubts anew.

Enough time has elapsed since the publication of the article of Davidson *et al.* for the data series to have more than doubled in length. In spite of the various economic vicissitudes that are reflected in the extended data set, their model continues to fit remarkably well, with newly estimated coefficients that are not vastly different from the original ones. One of the purposes of the present paper is to examine the basis for this apparent success. The principal purpose is to determine whether the time-honoured presuppositions about the nature of the income-consumption relationship, which were inherited by the consumption function of Davidson *et al.*, have any empirical support.

2. The Data and the Four-Period Difference Filter

In evaluating any model, we should begin by inspecting the data. The data series of income and consumption—which is the expenditure on nondurable goods—have two prominent characteristics. The first characteristic is their non-stationarity. Over the extended data, the logarithms of the data, which are plotted in Figure 1, show upward trends that are essentially linear. The second characteristic of the data series is that they both show evident patterns of seasonal variation, which play on the backs of the rising trends.

The seasonal pattern is more evident in the consumption series than it is in the income series. Therefore, we incline to the view that, rather than being transferred from the income stream, the seasonal fluctuations in consumption have their origin in an independent influence that impinges on both income and consumption. This motivates us to look at ways of deseasonalising the data which will remove the effect.

Models like that of Davidson *et al.* seek to explain an annual growth rate in consumption that is derived from quarterly data. The dependent variable

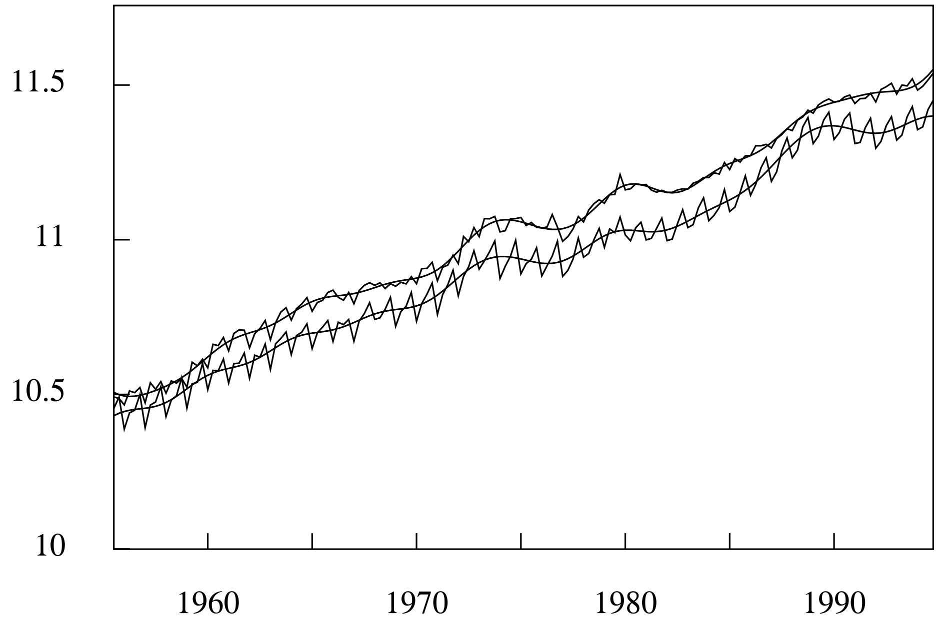

Figure 1. The quarterly series of the logarithms of income (upper) and consumption (lower) in the U.K., for the years 1955 to 1994, together with their interpolated trends.

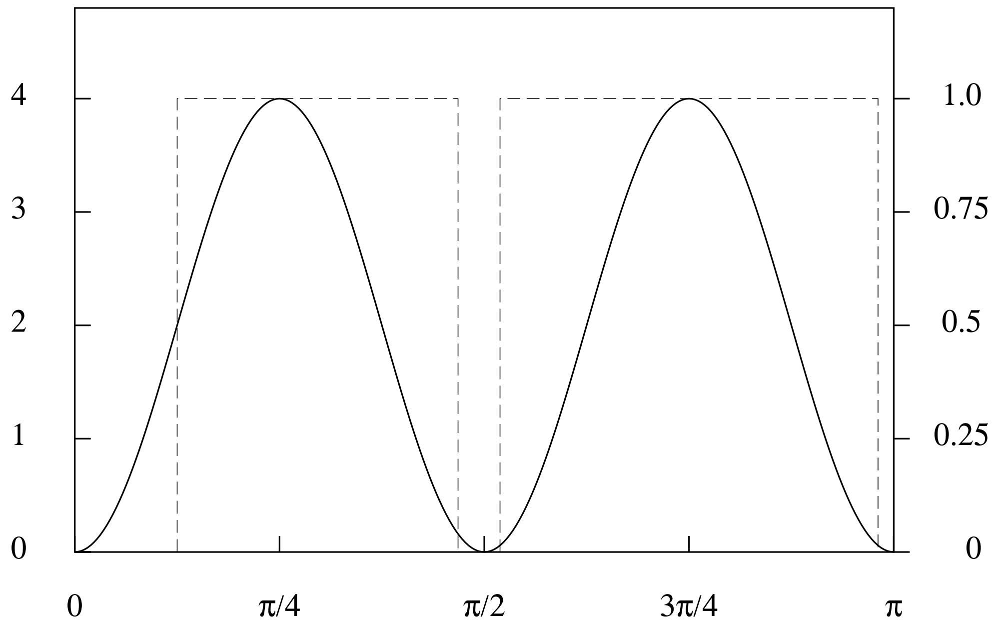

Figure 2. The squared gain of the four-period difference filter $\nabla_4 = 1 - L^4$ (continuous line and left scale) and the frequency selection of the deseasonalised detrended data (broken line and right scale).

of the model is obtained by passing the logarithm of the consumption series, which we shall denote by $y(t)$, through a four-period difference filter of the form $\nabla_4 = 1 - L^4 = (1-L)(1+L+L^2+L^3)$. Here, L is the lag operator, which has the effect that $Ly(t) = y(t-1)$, where $y(t) = \{y_t; t = 0 \pm 1, \pm 2, \ldots\}$ is a series of observations taken at three-monthly intervals. The filter removes from $y(t)$ both the trend and the seasonal fluctuations; and it removes much else besides.

The gain of the filter is depicted in Figure 2. The operator nullifies the component at zero frequency and it diminishes the power of the elements of the trend whose frequencies are in the neighbourhood of zero. This is the effect of $\nabla = 1 - L$, which is a factor of ∇_4. The filter also removes the elements at the seasonal frequency of $\pi/2$ and at its harmonic frequency of π, and it attenuates the elements in the neighbourhoods of these frequencies. This is the effect of the four-point summation operator $S_4 = 1 + L + L^2 + L^3$, which is the other factor of ∇_4. It is also apparent that the filter amplifies the cyclical components of the data that have frequencies in the vicinities of $\pi/4$ and $3\pi/4$; and, as we shall discover later, this is a distortion that can have a marked effect upon some of the estimates that are derived from the filtered data.

The effect of the filter upon the logarithmic consumption series can be see by comparing the periodograms of Figure 3 and Figure 4. The periodogram is the sequence of the coefficients $\rho_j^2 = \alpha_j^2 + \beta_j^2$, scaled by $T/2$, that come from the Fourier expression

$$\begin{aligned} y_t &= \sum_{j=0}^{[T/2]} \rho_j \cos(\omega_j t - \theta_j) \\ &= \sum_{j=0}^{[T/2]} \{\alpha_j \cos(\omega_j t) + \beta_j \sin(\omega_j t)\}, \end{aligned} \tag{1}$$

where T is the sample size and $[T/2]$ is the integral part of $T/2$. Here, $\omega_j = 2\pi j/T$ is the frequency of a sinusoid that takes j periods to complete a cycle. Its amplitude is ρ_j, whilst $\rho_j^2/2$ is its power which is, in other words, its contribution to the variance of the sample $\{y_t; t = 0, 1, \ldots, T-1\}$.

In the second expression, the parameters are $\alpha_j = \rho_j \cos\theta_j$ and $\beta_j = \rho_j \sin\theta_j$, with $\beta_0 = 0$ and $\beta_{[T/2]} = 0$ if T is an even number. We shall describe $\rho_j \cos(\omega_j t - \theta_j)$ as the jth sinusoidal element in the Fourier decomposition of the sample. (For a detailed exposition, see Pollock 1999).

The most striking effect of the filtering is the diminution of the power at the frequencies in the vicinity of zero, which is where the elements of the trend component are to be found, and in the vicinities of $\pi/2$ and π, where the seasonal elements and their harmonics are to be found. The degree of the amplification of the elements in the vicinities of $\pi/4$ and $3\pi/4$, which is evident in Figure 4, can be judged in comparison with a periodogram of the detrended data, presented in Figure 5, which has been obtained by fitting a linear trend.

The methods for detrending and deseasonalising the data that we shall propose are designed to remove the minimum amount of information from the processed series. They avoid the distortions that are induced by the differencing operator.

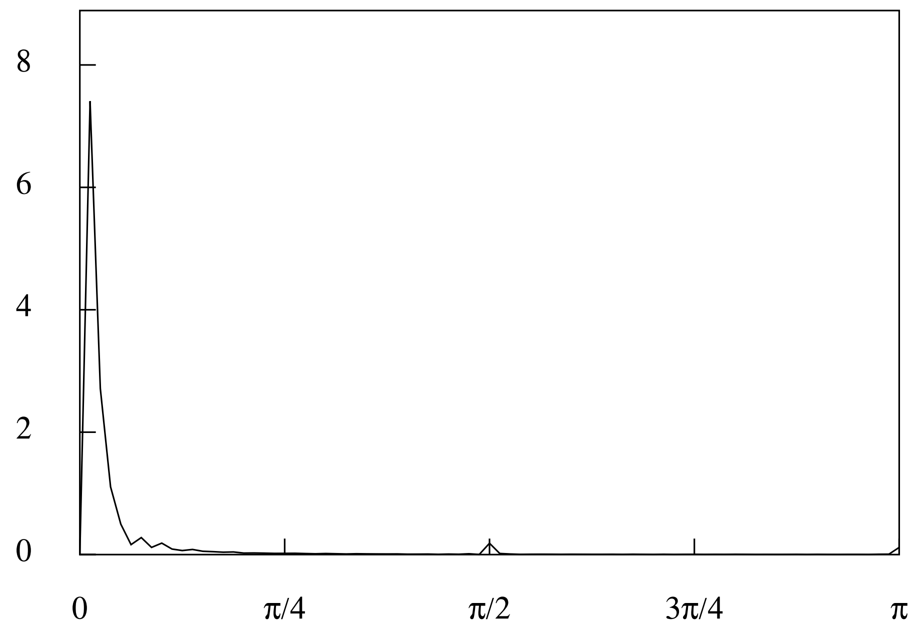

Figure 3. The periodogram of the logarithms of consumption in the U.K., for the years 1955 to 1994.

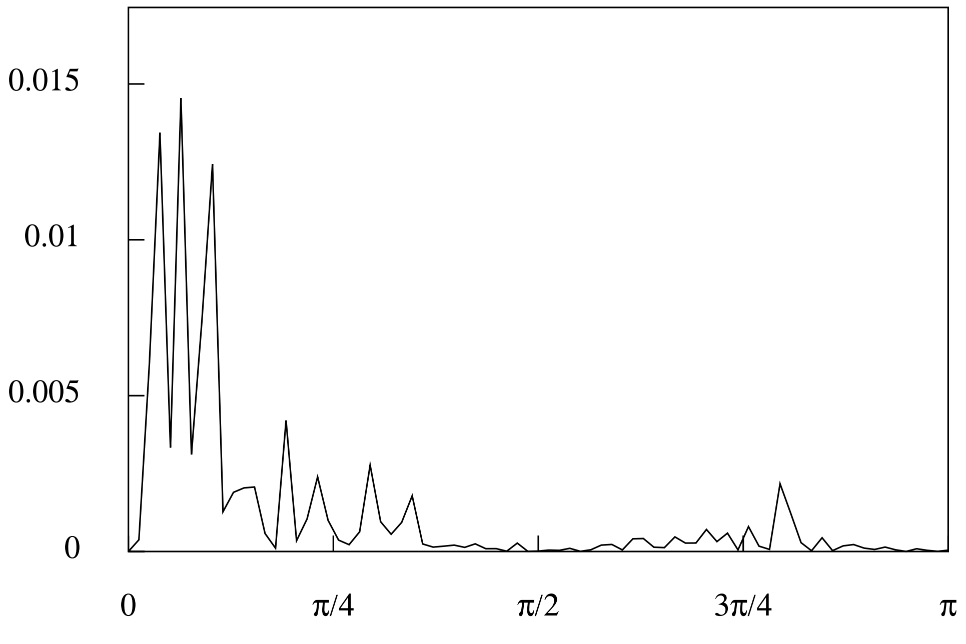

Figure 4. The periodogram of the filtered series $\nabla_4 y(t)$ representing the annual growth rate of consumption.

3. The Error-Correction Model and its Implications

The consumption function of Davidson *et al.* (1978) was calculated originally on a data set from the U.K. running from 1958 to 1970, which was a period of relative economic quiescence. When the function is estimated for an extended data period, running from 1956 to 1994, it yields the following results:

(2)

$$\begin{aligned}\nabla_4 y(t) = {} & 0.70\nabla_4 x(t) - 0.156\nabla\nabla_4 x(t) + 0.068\{x(t-4) - y(t-4)\} + e(t)\\ & \quad(0.39) \qquad\qquad (0.59) \qquad\qquad\quad (0.14)\\ & R^2 = 0.77 \qquad \text{D–W} = 0.920.\end{aligned}$$

Here $y(t)$ and $x(t)$ represent, respectively, the logarithms of the consumption sequence and the income sequence, without seasonal adjustment. The operators $\nabla = 1-L$ and $\nabla_4 = 1-L^4$ are, respectively, the one-period and the four-period difference operator. Therefore, $\nabla_4 y(t)$ and $\nabla_4 x(t)$ represent the annual growth rates of consumption and income, whilst $\nabla_1\nabla_4 x(t)$ represents the acceleration or deceleration in the growth of income.

This specification reflects an awareness of the difficulty of drawing meaningful inferences from a regression equation that incorporates nonstationary variables. The difference operators are effective in reducing the sequences $x(t)$ and $y(t)$ to stationarity. The synthetic sequence $x(t-4) - y(t-4)$ is also presumed to be stationary by virtue of the cointegration of $x(t)$ and $y(t)$; and its role within the equation is to provide an error-correction mechanism, which tends to eliminate any disproportion that might arise between consumption and income.

The specification also bears the impress of some of the earlier experiences in modelling the consumption function that we have described in the introduction. The variable $\nabla_1\nabla_4 x(t)$ with its associated negative-valued coefficient allows the growth of consumption to lag behind the growth of income when the latter is accelerating. This is the sort of response that the analysts of the late 1940's and 1950's, who were intent on reconciling the Keynesian formulations with the empirical findings, were at pains to model.

We can evaluate the roles played by the terms of the RHS of equation (2) by modifying the specification and by observing how the coefficients of the fitted regression are affected and how the goodness of fit is affected.

The first modification is to replace $x(t-4) - y(t-4)$ by a constant dummy variable. The result is a slight change in the estimates of the remaining parameters of the model and a negligible loss in the goodness of fit. This suggests that we can dispense with the error-correction term at little cost:

$$\begin{aligned}\nabla_4 y(t) = {} & 0.006 + 0.682\nabla_4 x(t) - 0.160\nabla\nabla_4 x(t) + e(t)\\ & (0.001) \quad (0.52) \qquad\qquad (0.65)\\ & R^2 = 0.76 \qquad \text{D–W} = 0.93.\end{aligned} \tag{3}$$

In this connection, we should note that several analysts, including Hylleberg *et al.* (1990), have found that the logarithmic series of consumption and income in the U.K. fail a test for cointegration. This seems to fly is the face of the evident relatedness of the two quantities. However, the finding may be taken as an indication that the relationship is not readily amenable to the linear dynamics of a simple error-correction mechanism.

We should also mention that, in a recent paper, Fenandez-Corugedo, Price and Blake (20003) have found evidence for an error-correction mechanism within a vector autoregressive system of four equations. Their system has non-durable consumption, labour or non-assets income, the stock of assets and the relative price of durables to non-durables as its variables. However, the factor loadings on the single cointegrating vector indicate that the correction mechanism is present only in the equation of the assets. It is not present in the consumption equation.

The second modification is to eliminate both the error-correction term and the acceleration term $\nabla_1\nabla_4 x(t)$ and to observe how well the annual growth in consumption is explained by the annual growth of income. In this case, we observe that the coefficient of determination of the fitted regression is 0.72, compared with 0.77 for the fully specified model, while the error sum of squares increases to 0.053 from 0.044. We conclude from this that the acceleration term does have some effect:

$$\begin{gathered}\nabla_4 y(t) = \underset{(0.27)}{0.769}\nabla_4 x(t) + e(t) \\ R^2 = 0.72 \qquad \text{D–W} = 1.15.\end{gathered} \tag{4}$$

The fact that the acceleration term enters the consumption function with a negative coefficient seem to suggest that the response of consumption to rapid changes in income is laggardly more often that not. This would fit well with the various hypotheses regarding consumer behaviour that have been mentioned in the introduction. However, the significance of the estimated coefficient is not very great and it is considerably reduced when the coefficient is estimated using only the first third of the data. We shall reconsider the acceleration term at the end of the paper, where we shall discover that its effect is reversed when we analyse the relationship between the trends depicted in Figure 1.

4. A Fourier Method for Detrending the Data

We have seen how the difference operator $1 - L$ and the four-point summation operator $S_4 = 1 + L + L^2 + L^3$ are liable to remove a substantial part of the information that is contained in the data of the consumption series. In this section and the next, we shall propose alternative devices for detrending and for deseasonalising the data that leave much of the information intact. Our basic objective is to remove from the data only those Fourier elements that contribute to the trend or to the seasonality, and to leave the other components of the data unaffected.

A normal requirement for the use of the standard methods of statistical Fourier analysis is that the data in question should be generated by stationary processes, and this requirement is a hardly ever satisfied in econometric analysis. To understand the problems that can arise in applying Fourier methods to trended data, one must recognise that, in analysing a finite data sequence, one is making the implicit assumption that it represents a single cycle of a periodic function that is defined over the entire set of positive and negative integers. This function may be described as the periodic extension of the data sequence.

In the case of a trended sequence, there are bound to be radical disjunctions in the periodic function where one replication of the data sequence ends and another begins. Thus, for example, if the data follow a linear trend, then the function that is the subject of the Fourier analysis will have the appearance of the serrated edge of a saw blade. The saw tooth function has a spectrum that extends across the entire range of frequencies, with ordinates whose absolute values are inversely proportional to corresponding frequencies—see for example, Hamming (1989). These effects of the trend are liable to be confounded with the spectral effects of the other motions that are present in the data.

The problem is resolved by using an approach that is familiar from the forecasting of ARIMA processes. We begin by differencing the data sequence as many times as may be necessary to reduce it to a state of stationarity. We proceed to eliminate the low-frequency sinusoidal elements from the differenced data. Then, by cumulating or 'integrating' the resulting sequence, we will obtain the detrended version of the data. The trend of the data can be obtained, likewise, by cumulating the sum of the low-frequency elements that have been extracted from the differenced data.

The process by which the trend component is cumulated after it has been extracted from the differenced data sequence calls for some initial conditions or starting values. To provide expressions for these values, we need to describe the matrix versions of the difference operator and of the summation or cumulation operator, which is its inverse.

Let the identity matrix of order T be denoted by

$$I_T = [e_0, e_1, \ldots, e_{T-1}], \tag{5}$$

where e_j represents a column vector that contains a single unit preceded by j zeros and followed by $T - j - 1$ zeros. Then, the finite-sample lag operator is the matrix

$$L_T = [e_1, \ldots, e_{T-1}, 0], \tag{6}$$

which has units on the first subdiagonal and zeros elsewhere. The matrix that takes the d-th difference of a vector of order T is given by $\Delta = (I - L_T)^d$.

Taking differences within a vector entails a loss of information. Therefore, if $\Delta = [Q_*, Q]'$, where Q'_* has d rows, then the d-th differences of a vector $y = [y_0, \ldots, y_{T-1}]'$ are the elements of the vector $g = [g_d, \ldots, g_{T-1}]'$ that is found in the equation

$$\begin{bmatrix} g_* \\ g \end{bmatrix} = \begin{bmatrix} Q'_* \\ Q' \end{bmatrix} y. \tag{7}$$

The vector $g_* = Q'_* y$ in this equation, which is a transform of the vector $[y_0, \ldots, y_{d-1}]$ of the leading elements of y, is liable to be discarded.

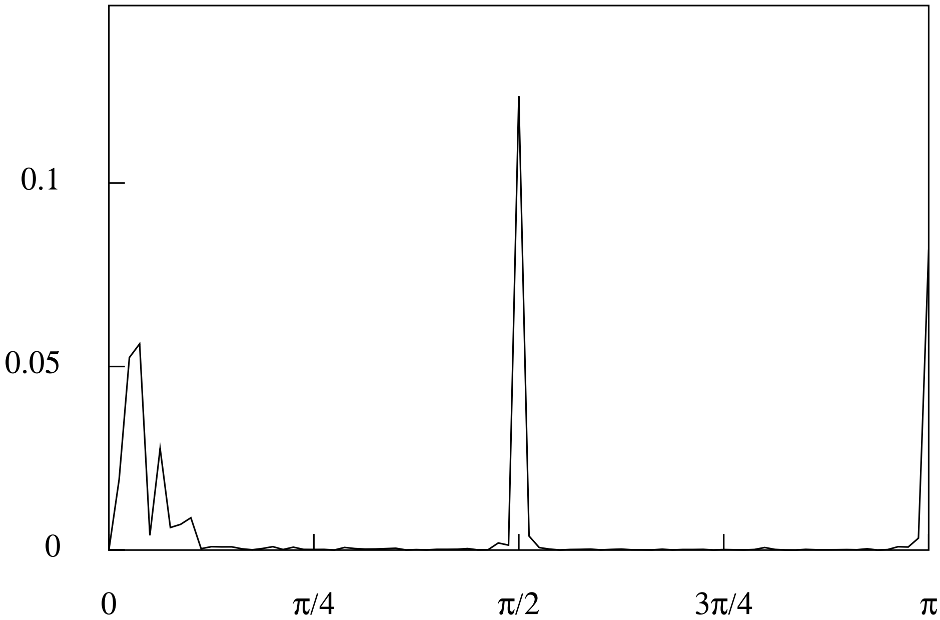

Figure 5. The periodogram of the residuals obtained by fitting a linear trend through the logarithmic consumption data of Figure 1.

The inverse of the difference matrix is the matrix $\Delta^{-1} = \Sigma = [S_*, S]$. This has the effect that

$$S_* g_* + Sg = y. \tag{8}$$

The vector y can be recovered from the differenced vector g only if the vector g_* of initial conditions is provided.

Now let z represent the differenced version of the trend component that requires to be cumulated to form $x = S_* z_* + Sz$. Then the initial conditions in z_* should be chosen so as to ensure that the trend is aligned with the data as closely as possible. The criterion is

$$\text{Minimise} \quad (y - S_* z_* - Sz)'(y - S_* z_* - Sz) \quad \text{with respect to} \quad z_*. \tag{9}$$

The solution for the starting values is

$$z_* = (S_*' S_*)^{-1} S_*'(y - Sz). \tag{10}$$

The facility that we have constructed for removing the trend from the data allows us to select a cut-off point that marks the highest frequency amongst the Fourier elements that constitute the trend. The decision of where to place the cut-off point should be guided by an appraisal of the spectral structure of the data. Figure 5 shows the periodogram of the residual sequence obtained by fitting a linear trend through the logarithms of the consumption series. Fitting a linear trend overcomes the problems of non-stationarity without destroying the information relating to the trend component.

We choose to place the cut-off point at $\pi/8$ radians, which is in a dead space of the periodogram where there are no ordinates of any significant size. Given that the observations are at quarterly intervals, this implies that the trend includes all cycles of four years duration of more. The detrended consumption series is show in Figure 6. A similar analysis of the income data suggests that the same cut-off point is appropriate. The trends in the consumption and income series that have been calculated on this basis are depicted in Figure 1.

5. A Fourier Method for Deseasonalising the Data

As well as removing the trend from the data, we also wish to remove the seasonal fluctuations. This can be done in much the same way. At its simplest, we can define the differenced seasonal component to consist only of those sinusoidal elements, extracted from the differenced data $\{g_d, \ldots, g_{T-1}\}$, that are at the seasonal frequency and at the harmonically related frequencies. Let $N = T - d$, where d is the degree of differencing. Then, in the case of quarterly data, and on the supposition that N is an even number, the component would be described by the equation

$$u_t = \alpha_{N/4} \cos\left(\frac{\pi t}{2}\right) + \beta_{N/4} \sin\left(\frac{\pi t}{2}\right) + \alpha_{N/2}(-1)^t, \tag{11}$$

wherein

$$\begin{aligned} \alpha_{N/4} &= \frac{2}{N} \sum_t g_t \cos\left(\frac{\pi t}{2}\right), \\ \beta_{N/4} &= \frac{2}{N} \sum_t g_t \sin\left(\frac{\pi t}{2}\right), \\ \alpha_{N/2} &= \frac{1}{N} \sum_t g_t (-1)^t. \end{aligned} \tag{12}$$

In fact, this scheme is equivalent to one that uses seasonal dummy variables with the constraint that their associated coefficients must sum to zero. It will generate a pattern of seasonal variation that is the same for every year.

A more complex pattern of seasonality, which will vary gradually over the years, could be obtained by adopting a linear stochastic model with unit roots at the seasonal frequencies or by combining such a model with a "deterministic" trigonometrical or dummy-variable model in the manner suggested by Osborn *et al.* (1988). However, the desired effect can also be achieved by comprising within the Fourier sum a set of sinusoidal elements whose frequencies are adjacent to the seasonal frequency and to its harmonics.

The combined effect of two elements at adjacent frequencies depends upon whether their sinusoids are in phase, in which case they reinforce each other, or out of phase, in which case they tend to interfere with each other destructively. Two sinusoids whose frequencies are separated by θ radians will take a take a total of $\tau = 2\pi/\theta$ periods to move from constructive interference to destructive

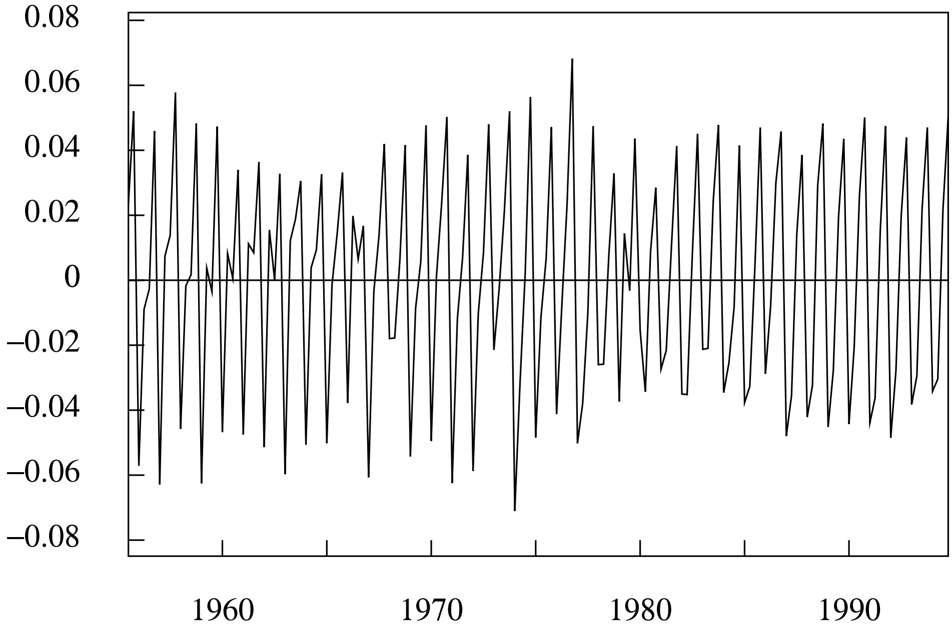

Figure 6. The detrended consumption series.

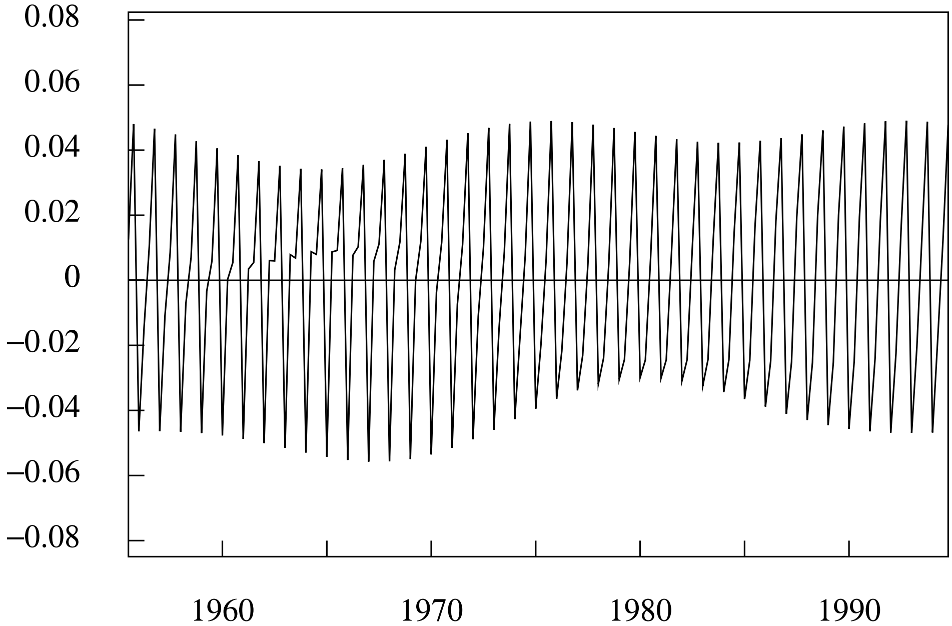

Figure 7. The estimated seasonal component of the consumption series.

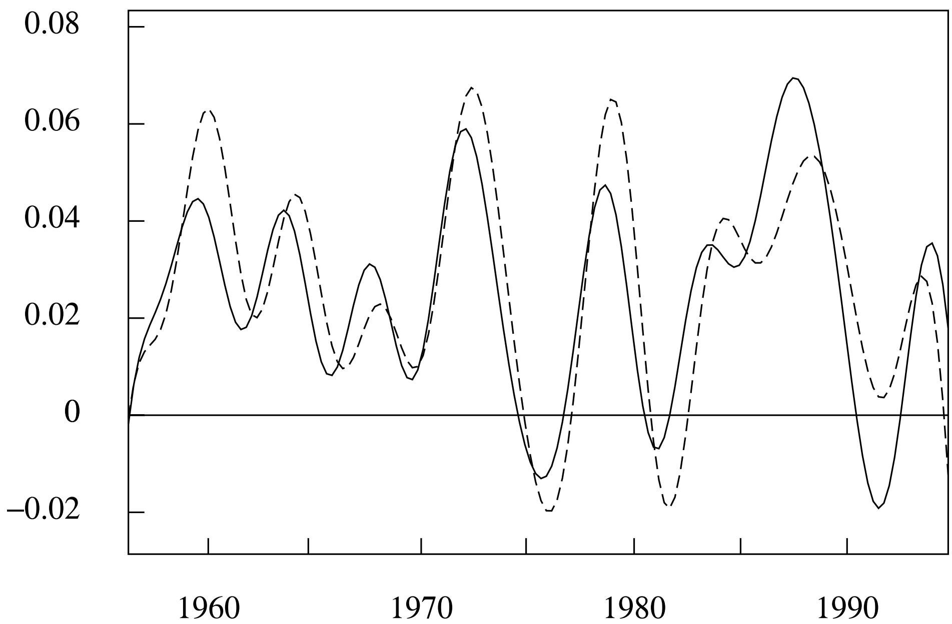

Figure 8. The annual differences of the trend of the logarithmic consumption series (solid line) and of the trend of the logarithmic income series (broken line).

interference and back again. By this device, a pattern can be generated that evolves over the length of the sample.

It remains to describe how the seasonal elements that have been extracted from the differenced data are to be cumulated to provide an estimate of the seasonal component. It seems reasonable to chose the starting values so as to minimise the sum of squares of the seasonal fluctuations. Let $w = S_*u_* + Su$ be the cumulated seasonal component, where u_* is a vector of d starting values and u is the vector of the seasonal component that has been extracted from the differenced data. Then the criterion is

$$\text{(13)} \qquad \text{Minimise} \quad (S_*u_* + Su)'(S_*u_* + Su) \quad \text{with respect to} \quad u_*.$$

The solution for the starting values is

$$\text{(14)} \qquad u_* = -(S_*'S_*)^{-1}S_*'Su.$$

Figure 7 shows the estimated seasonal component of the consumption series. The seasonal series is synthesised from the trigonometric functions at the seasonal frequency of $\pi/2$ and at its harmonic frequency of π, together with a handful of elements at the adjacent non-seasonal frequencies. It comprises two elements below $\pi/2$ and one above, and it also comprises one element below π. These choices have resulted from an analysis of the periodogram of Figure 5. Figure 2 indicates, via the dotted lines, the frequencies that are present in the detrended and deseasonalised data.

The seasonal component of consumption accounts for the 93 percent of the variation of the detrended consumption series. When the seasonal component is estimated for the income series using the same set of frequencies, it accounts for only 46 percent of the variance of the corresponding detrended series.

6. A Re-appraisal of the Income–Consumption Relationship

In the previous section, we have described some new techniques for detrending the data and for extracting the seasonal component. We have discovered that the seasonal fluctuations in consumption are of a greater amplitude than those of the income series. They also appear to be more regular. It is also the case that Hylleberg *et al.* (1990) failed to find cointegration between the two logarithmic series at the seasonal frequencies. These circumstances persuade us to reject the notion that the fluctuations have been transferred from income to consumption. It seems more reasonable to treat the seasonal fluctuations in both series as if they derive from external influences. Therefore, in seeking to establish a relationship between the detrended series, it is best to work with the deseasonalised versions.

When we turn to the deseasonalised and detrended consumption series, we find that its variance amounts to only 7 percent of the variance of the detrended series. It is hardly worthwhile to attempt to model this series. Indeed, the periodogram of Figure 5 also makes it clear that there is very little information in the data of the consumption sequence that is not attributable either to the trend or to the seasonal component.

If it is accepted that the seasonal component needs no further explanation, then attention may be confined to the trend. The use of ordinary linear statistical methods dictates that any explanation of the consumption trend is bound to be in terms of data elements whose frequencies are bounded by zero and by the cut-off point of $\pi/8$ radians. That is to say, the trend in consumption can only be explained by similar trends in other variables.

Therefore, we turn to the essential parts of the income and the consumption series, which are their trends. We take the annual differences of the logarithmic trends by applying the operator $\nabla_4 = I - L^4$; and the results are a pair of smooth series that represent the annual growth rates of income and consumption. By combining the two series in one graph, which is Figure 8, we are able to see that, in the main, the fluctuations in the growth in consumption *precede* similar fluctuations in the growth of income.

It may be recalled the income-acceleration term $\nabla\nabla_4 x(t)$ enters the consumption functions of equations (1) and (2) with a negative coefficient. This is in spite of the clear indication of Figure 8 that the consumption-growth series leads the income-growth series. However, when the smoothed growth series $\nabla_4\hat{y}(t)$ and $\nabla_4\hat{x}(t)$ of Figure 8 are used in these equations in place of $\nabla_4 x(t)$ and $\nabla_4 y(t)$, the sign on the coefficient of the acceleration term is reversed:

$$
\begin{aligned}
&\nabla_4\hat{y}(t) = \underset{(0.001)}{0.006} + \underset{(0.43)}{0.689}\nabla_4\hat{x}(t) + \underset{(0.169)}{1.055}\nabla\nabla_4\hat{x}(t) + e(t) \\
&R^2 = 0.87.
\end{aligned}
\tag{15}
$$

The explanation of this anomaly must lie in the nature of the gain of the four-period difference filter $\nabla_4 = I - L_4$, which is represented in Figure 2.

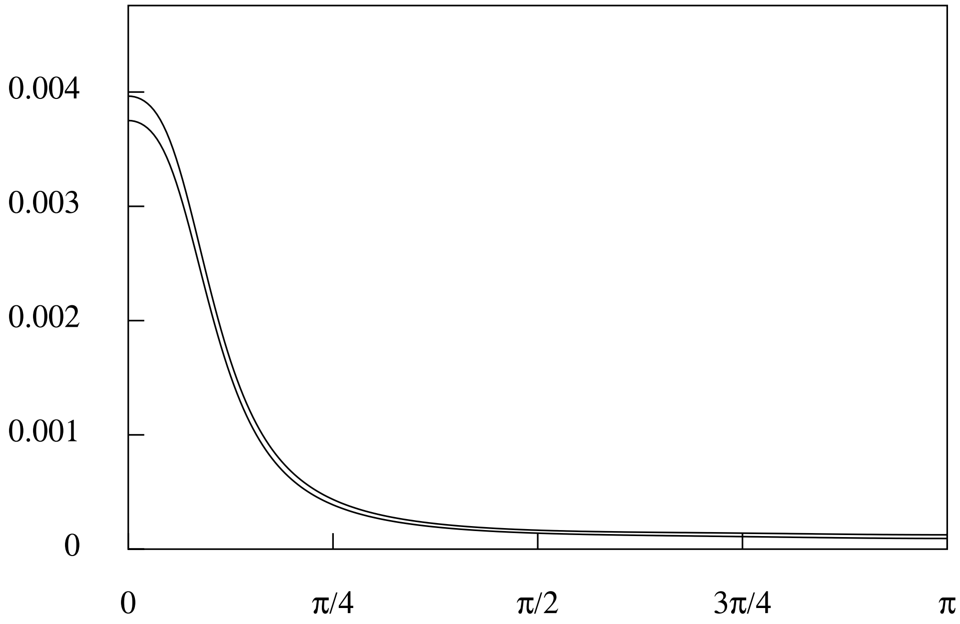

Figure 9. The spectrum of the consumption growth sequence $\nabla_4 y(t)$ (the outer envelope) and that of its auto-innovation component $\{\alpha(L)/\pi(L)\}\varepsilon(t)$ (the inner envelope).

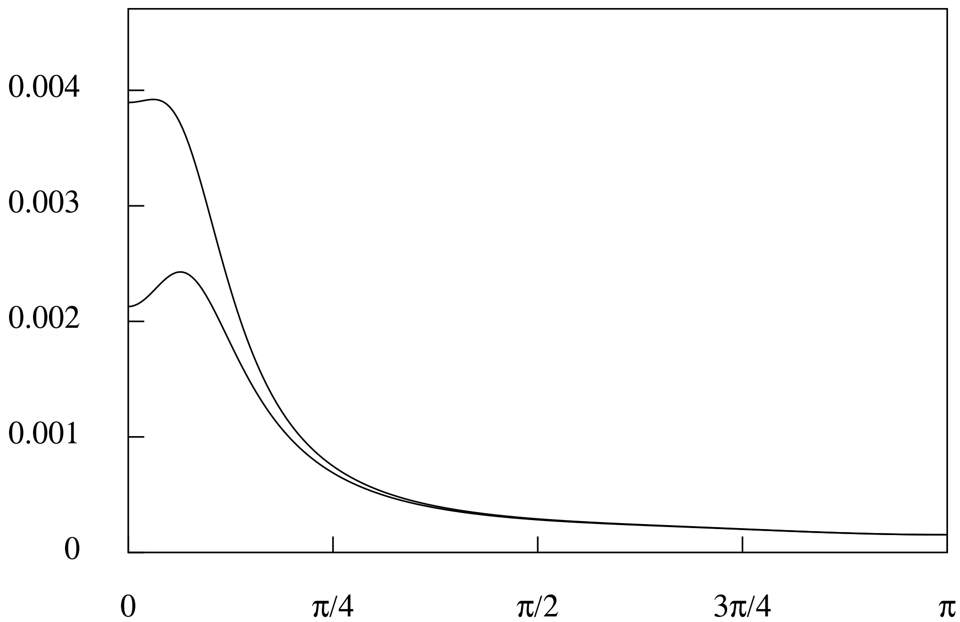

Figure 10. The spectrum of the income growth sequence $\nabla_4 x(t)$ (the outer envelope) and that of its auto-innovation component $\{\gamma(L)/\pi(L)\}\eta(t)$ (the inner envelope).

The effect of the filter is to amplify some of the minor components of the data that lie in the dead spaces of the periodogram of Figure 5 on either side of the frequencies $\pi/4$ and $3\pi/4$. Thus it can be concluded that, notwithstanding its specious justification, the negative acceleration term is an artifact of the differencing filter. This finding conflicts with the belief that consumption responds in a laggardly fashion to rapid changes in income.

The perception that the series of the annual growth rate in consumption is leading the corresponding series in income can be reaffirmed within the context of a bivariate vector autoregressive model. The model must be applied to the unsmoothed growth rates obtained by taking the four-period differences of the logarithms of the two series. It cannot be applied directly to the smoothed growth-rate series of Figure 8, which have band-limited spectra. The reason is that an autoregressive model presupposes a spectral density function that is nonzero everywhere in the frequency range except on a set of measure zero.

The bivariate vector autoregressive model takes the form of

$$\nabla_4 y(t) = c_y + \sum_{i=1}^{p} \phi_i \nabla_4 y(t-i) + \sum_{i=1}^{p} \beta_i \nabla_4 x(t-i) + \varepsilon(t), \tag{16}$$

$$\nabla_4 x(t) = c_x + \sum_{i=1}^{p} \psi_i \nabla_4 x(t-i) + \sum_{i=1}^{p} \delta_i \nabla_4 y(t-i) + \eta(t). \tag{17}$$

The terms c_y and c_y stand for small constants, which are eliminated from the model when the differenced series are replaced by deviations about their mean values. The deviations may be denoted by $\tilde{y}(t) = \nabla_4 y(t) - E\{\nabla_4 y(t)\}$ and $\tilde{x}(t) = \nabla_4 x(t) - E\{\nabla_4 x(t)\}$. The expected values can be represented by the corresponding sample means.

In the case of $p = 2$, the estimated equations are

$$\underset{(18)}{} \quad \tilde{y}(t) = \underset{(0.85)}{0.51}\tilde{y}(t-1) + \underset{(0.86)}{0.34}\tilde{y}(t-2) + \underset{(0.72)}{0.27}\tilde{x}(t-1) - \underset{(0.71)}{0.38}\tilde{x}(t-2) + e(t),$$

$$\underset{(19)}{} \quad \tilde{x}(t) = \underset{(0.92)}{0.52}\tilde{x}(t-1) - \underset{(0.91)}{0.10}\tilde{x}(t-2) + \underset{(0.10)}{0.16}\tilde{y}(t-1) + \underset{(0.10)}{0.25}\tilde{y}(t-2) + h(t).$$

To facilitate the analysis of the model, it is helpful to write the equations (16) and (17) in a more summary notation that uses polynomials in the lag operator to represent the various sums. Thus

$$\phi(L)\tilde{y}(t) - \beta(L)\tilde{x}(t) = \varepsilon(t), \tag{20}$$

$$-\delta(L)\tilde{y}(t) + \psi(L)\tilde{x}(t) = \eta(t), \tag{21}$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\beta(L) = \beta_1 L + \cdots + \beta_p L^p$, $\psi(L) = 1 - \psi_1 L - \cdots - \psi_p L^p$ and $\delta(L) = \delta_1 L + \cdots + \delta_p L^p$.

The notion that the sequence $\tilde{y}(t)$ is driving the sequence $\tilde{x}(t)$ would be substantiated if the influence of the innovations sequence $\varepsilon(t)$ upon $\tilde{y}(t)$ were found to be stronger than the influence of $\eta(t)$ upon the corresponding sequence $\tilde{x}(t)$. The matter can be investigated via the moving-average forms of the equations, which express $\tilde{x}(t)$ and $\tilde{y}(t)$ as functions only of the innovations sequences $\varepsilon(t)$ and $\eta(t)$. The moving-average equations, which are obtained by inverting equations (20) and (21) jointly, are

$$\tilde{y}(t) = \frac{\psi(L)}{\pi(L)}\varepsilon(t) + \frac{\beta(L)}{\pi(L)}\eta(t), \tag{22}$$

$$\tilde{x}(t) = \frac{\delta(L)}{\pi(L)}\varepsilon(t) + \frac{\phi(L)}{\pi(L)}\eta(t), \tag{23}$$

where $\pi(L) = \phi(L)\psi(L) - \beta(L)\delta(L)$.

Since there is liable to be a degree of contemporaneous correlation between innovations sequences, the variance of the observable sequences $\tilde{y}(t)$ and $\tilde{x}(t)$ will not equal the sum of the variances of the components in $\varepsilon(t)$ and $\eta(t)$ on the RHS. The problem can be overcome by reparametrising the two equations so that each is expressed in terms of a pair of uncorrelated innovations. Such a procedure has been adopted by Geweke (1982), for example.

Consider the innovation sequence $\eta(t)$ within the context of equation (22), which is for $\tilde{y}(t)$. We may decompose $\eta(t)$ into a component that lies in the space spanned by $\varepsilon(t)$ and a component $\zeta(t)$ that is in the orthogonal complement of the space. Thus

$$\begin{aligned}\eta(t) &= \frac{\sigma_{\eta\varepsilon}}{\sigma_{\varepsilon}^2}\varepsilon(t) + \left\{\eta(t) - \frac{\sigma_{\eta\varepsilon}}{\sigma_{\varepsilon}^2}\varepsilon(t)\right\}\\ &= \frac{\sigma_{\eta\varepsilon}}{\sigma_{\varepsilon}^2}\varepsilon(t) + \zeta(t),\end{aligned} \tag{24}$$

where $\sigma_{\varepsilon}^2 = V\{\varepsilon(t)\}$ is the variance of the consumption innovations and $\sigma_{\varepsilon\eta}^2 = C\{\varepsilon(t), \eta(t)\}$ is the covariance of the consumption and income innovations. Substituting (24) in equation (22) and combining the terms in $\varepsilon(t)$ gives

$$\tilde{y}(t) = \frac{\alpha(L)}{\pi(L)}\varepsilon(t) + \frac{\beta(L)}{\pi(L)}\zeta(t), \tag{25}$$

where

$$\alpha(L) = \psi(L) + \frac{\sigma_{\eta\varepsilon}}{\sigma_{\varepsilon}^2}\beta(L). \tag{26}$$

We may describe the sequence $\varepsilon(t)$ as the auto-innovations of $\tilde{y}(t)$ and $\zeta(t)$ as the allo-innovations.

By a similar reparametrisation, the equation (23) in $\tilde{x}(t)$ becomes

$$\tilde{x}(t) = \frac{\gamma(L)}{\pi(L)}\eta(t) + \frac{\delta(L)}{\pi(L)}\xi(t), \tag{27}$$

where

$$\gamma(L) = \phi(L) + \frac{\sigma_{\eta\varepsilon}}{\sigma^2_\eta}\delta(L), \qquad (28)$$
$$\xi(t) = \varepsilon(t) - \frac{\sigma_{\eta\varepsilon}}{\sigma^2_\eta}\eta(t),$$

and where $\eta(t)$ and $\xi(t)$ are mutually uncorrelated. These are, respectively, the auto-innovations and the allo-innovations of $\tilde{x}(t)$.

The relative influences of $\varepsilon(t)$ on $\tilde{y}(t)$ and of $\eta(t)$ on $\tilde{x}(t)$ can now be assessed by an analysis of the corresponding spectral density functions. Figure 9 shows the spectrum of $\tilde{y}(t)$ together with that of its auto-innovation component $\{\alpha(L)/\pi(L)\}\varepsilon(t)$, which is the lower envelope. Figure 10 shows the spectrum of $\tilde{x}(t)$ together with that of its auto-innovation component $\{\gamma(L)/\pi(L)\}\eta(t)$.

From a comparison of the figures, it is clear that the innovation sequence $\varepsilon(t)$ accounts for a much larger proportion of $\tilde{y}(t)$ than $\eta(t)$ does of $\tilde{x}(t)$. Thus, the consumption growth series appears to be driven largely by its auto innovations. These innovations also enter the income growth series to the extent that the latter is not accounted for by its auto innovations. Figure 10 shows that the extent is considerable.

The fact the consumption innovations play a large part in driving the bivariate system implies that the consumption function of Davidson *et al.* (1978), which is equation (2), cannot be properly construed as a structural econometric relationship. For it implies that the estimates are bound to suffer from a simultaneous-equations bias. Nevertheless, in so far as the mechanisms generating the data remain unchanged, the above-mentioned function will retain its status as an excellent predictor of the growth rate of consumption that is based on a parsimonious information set.

7. Conclusions

The traditional macroeconomic consumption function depicts a delayed response of consumption spending to changes in income; and many analysts would expect this relationship to be readily discernible in the macroeconomic data. Instead, the data seem to reflect a delayed response of aggregate income to autonomous changes in consumption. Although the two responses can easily coexist, it is the dominant response that is liable to be discerned in the data at first sight.

A crucial question is whether both responses can be successfully disentangled from the macroeconomics data. The construction of a bivariate autoregressive model is the first step in the process of their disentanglement. However, given the paucity of the information contained in the data, one is inclined to doubt whether the process can be carried much further. Indeed, the efforts that have been devoted to the microeconomic analysis of consumer behaviour in the last twenty years can be construed as a reaction to limited prospects facing macroeconomic investigations.

Much has already been accomplished in the microeconomic analysis of consumer behaviour; and an excellent account of some of the numerous influences that affect consumer behaviour directly has been provided recently by Muellbauer and Latimore (1995). However, what is lacking is a methodology that would enable the consumption behaviour of identifiable social and economic groups to be aggregated into a macroeconomic consumption function.

We have found that, within a bivariate autoregressive system designed to explain the growth rates on income and consumption, the innovations sequence of the consumption equation dominates the corresponding innovations sequence of the income equation. Thus the fluctuations in the growth rate of consumption have been depicted mainly as the result of autonomous influences.

Although the innovations sequences are an artifact of the statistical analysis, they are not entirely devoid of worldly connotations. By a detailed study of the historical circumstances, we should be able to relate the consumption innovations to the fiscal policies of the central governments, the state of the financial markets, the rate of inflation, the political and social climate, and to much else besides. Although some of these influences have been included in macroeconomic consumption functions, it seems that, in the main, there has been a remarkable oversight of the circumstantial details in most attempts at explaining the aggregate level of consumption. The present analysis is, regrettably, no exception.

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