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A Model of Capital Accumulation and Rent-Seeking*

Paulo Barelli† Samuel de Abreu Pessôa‡

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Abstract

A simple model incorporating rent-seeking into the standard neoclassical model of capital accumulation is presented. It embodies the idea that the performance of an economy depends on the efficiency of its institutions. It is shown that welfare is positively affected by the institutional efficiency, although output is not necessarily so. It is also shown that an economy with a monopolistic rent-seeker performs better than one with a competitive rent-seeking industry.

JEL Classification: D23, D74, O40, O41, O47

Key Words: Rent-seeking, Two Sector Model, Capital Accumulation, Productivity

"Economic history may be thought of as a struggle between a propensity for growth and one for rent-seeking, that is, for someone improving his or her position, or a group bettering its position, at the expense of the general welfare. (...) Whenever conditions permitted, that is, when rent-seeking was somehow curbed, growth manifested itself." (Jones, 1988)

"Institutions form the incentive structure of a society, and the political and economic institutions, in consequence, are the underlying determinants of economic performance." (North, 1994)

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1 Introduction

It is not a novelty to claim that the performance of an economy is shaped by its institutions. Douglass North and others have published several books and papers on the subject. The argument goes as follows. Institutions are the rules of the game in an economy. If these rules foster activities that generate high private benefits and low social benefits, then the economy performs poorly. Conversely, if the rules align private and social benefits, economic growth and high social welfare result. Economic activities can, then, be classified according to the relation between private and social benefits generated by each activity. To simplify matters, let us assume that there are only two types of economic activities: those that generate social returns, and those that do not. The institutional background will be considered efficient if it fosters relatively more of the first type of activity. This simplification summarizes the argument set forth in the literature mentioned above.

In this paper a model capturing the ideas above is presented. It is a standard neoclassical capital accumulation model with intertemporal consumers, with the introduction of an additional, rent-seeking, industry. That is, economic activities are classified into two economic sectors: a productive and an unproductive. Both employ productive factors to produce an output, but the second's output is an effort to confiscate what is produced by the first. Such a formulation seems to capture the idea of an activity with private benefits and no social benefits. It is a pure transfer activity, that only redistributes income (does not generate it).[1] The first activity, on its turn, does generate income - its output is the socially valued homogeneous good in the economy.

The introduction of the new industry is made by the use of a function, which we interpret as the 'aggregate rent-seeking technology.' This function translates the output of the unproductive sector into a number between 0 and 1, which represents the fraction of the productive sector's output that is captured by the unproductive sector. This function is the central piece of the model. A set of properties that such a function ought to satisfy is presented,[2] and it is shown that these properties are sufficient to close the model. In particular, no functional form is needed to solve the model. As it is the case with production functions, functional forms are only necessary for some applications (and our function does have a 'Cobb-Douglas-like' counterpart that is used to calibrate the model). We argue, in addition, that such a function is a natural way of incorporating institutions into a macroeconomic model.

The institutional efficiency is captured by the above mentioned function. The fraction of sector

[1] Such an activity is also called a rent-seeking activity, as coined by Krueger (1974), or a directly unproductive profit-seeking (DUP) activity, as coined by Bhagwati (1982).

[2] We assume, in particular, that the shape of the function is one that delivers uniqueness of equilibrium. It is our view that development issues are not to be explained by multiplicity of equilibria and coordination failures. The successes and failures of economies are to be explained by fundamentals, not by expectations.

1's output that sector 2 is able to confiscate depends on how well property rights are enforced. We introduce one institutional parameter that determines the success of sector 2's output in capturing sector 1's output. It can be viewed as a measure of the total factor productivity in that sector. A low value for this parameter makes unproductive activities relatively unsuccessful in their endeavor of capturing sector 1's output, so it represents well defined property rights. A high value for the parameter represents an inefficient institutional background, i.e., poorly defined property rights. This parameter is the main maintened assumption of the model - it is the exogenous variable that eventually drives all results. The view set forth by the model is, then, that of an economic long-run in which the institutional efficiency is held fixed, so that the "long-run" does not include institutional change.

With two sectors there are also two economic decisions: one static and the other dynamic. The static is the factor allocation problem (for a given level of productive factors), and the dynamic is the consumption-investment allocation problem (that is, endogenous levels of reproductive factors of production). In both an equilibrium is defined and shown to exist and be unique. The effect of the institutional efficiency can also be disentangled into a static and a dynamic parts. That is, for a given level of productive factors, the institutional efficiency determines the amount of resources employed in the rent-seeking sector, i.e., employed with a view to capture rents. This is similar to Gordon Tullock's idea, and we will call it the Tullock effect accordingly. Moreover, the institutional efficiency also generates a dynamic effect, that of a distortion in capital accumulation. This is the usual effect of a distortion, and will be called the Harberger effect.[3] These two effects summarize the workings of the model.

There are two central results in this paper. First, welfare is positively related to institutional efficiency. Second, a monopolist rent-seeker is better for the economy than a competitive rent-seeking sector. The first result, although intuitive, is by no means obvious. The long-run comparative statics indicates that there is no monotonic relation between per capita output and institutional efficiency. In particular, if the rent-seeking sector is capital intensive, then worse institutions might be associated with more output in the long-run. The welfare result states that, even when long-run output and consumption do increase, society is negatively affected by a worsening in institutional efficiency. Moreover, the effect of a change in institutional efficiency on welfare can be disentangled

[3]We named the two effects Tullock and Harberger because they resemble the Tullock/Harberger debate of the social costs of monopoly. Harberger pointed out that the cost of the monopoly is the deadweight loss it generates, and found out that this loss is small. Tullock replied saying that the monopolist captures part of consumers' surplus, and hence that real resources would be employed to capture these economic rents, so the cost of a monopoly is much larger than the deadweight loss it generates. Hence, our Tullock effect measures the resources used to capture rents, and our Harberger effect measures the usual cost of inefficient institutions (the dynamic distortion). Posner (1975) evaluated empirically the Harberger and Tullock effects for a monopoly in a partial equilibrium framework.

into two effects, which correspond to the Tullock and Harberger effects mentioned above. The unambiguous result can be interpreted as the Tullock effect dominating the Harberger effect when the latter happens to be of opposite sign (the Tullock effect is always of the same sign: the worse the institutions, the more is captured by the rent-seeking sector. The Harberger effect can be of the opposite sign when rent-seeking is capital intensive). Hence, the fact that productive resources are employed in unproductive activities is the main cause of welfare being reduced because of inefficient institutions.

The second result is important because it qualifies the claim that competition is always to be recommended. Competition in productive sectors does indeed improve welfare. But competition in unproductive sectors generates the opposite effect. As several producers compete for rents, they employ more productive resources and generate more unproductive output than a sole rent-seeker would generate.[4]

This helps explain the events that ensued from three different historical phenomena of the second half of last century: the end of European colonization in the 60's and 70's in many African countries, the end of political regimes based on military dictatorship in many Latin America countries in the 80's, and finally the 'fall of the wall' leading to the end of the communist regimes in east Europe in the late 80's and early 90's. These three recent episodes of the world history share one fundamental characteristic: there is a transition from a centralized political (economic) organization toward a more decentralized system. And such transitions were all accompanied by a period of economic recession. One rationale for that is provided by our second result above. That is, assuming that monopoly in rent-seeking takes place in either a colony (the European imperial power being the monopolist),[5] or in a military dictatorship (the army being the monopolist), or a centralized economy (the communist party being the monopolist), a given level of institutional efficiency is associated with a better economic performance in the more centralized system as compared to a system in which there is free entry into the rent-seeking sector (for two otherwise identical economies, of course). Also, the transition to a more open system of organizing either the politics or the economy means a lifting of the barriers to entry in the rent-seeking sector, so the economy is bound to experience a recession as productive resources are directed to unproductive activities.[6]

[4] Shleifer and Vishny (1993) make this point informally. Bliss and Di Tella (1997) study the link between corruption and competition. Their model differs from our formulation in many aspects: (i) they consider competition in the productivity activity but monopoly in the rent-seeking sector; (ii) they examine a partial equilibrium set up (there is no factor mobility); (iii) they do not consider capital accumulation.

[5] Lucas (1990) considered the case in which the European power is the monopolist in the capital market.

[6] Consequently, prior to this transition, an economy should endure a process of institutional change, in order to generate an institutional background in which competitive rent-seeking is not too harmful. For the level of generality of the model, we can only say that one should aim toward better defined and enforced property rights. Of course, a process of institutional building involves a myriad of complex details. For instance, a market economy relies on a well

Some qualitative results are presented as well. For the static part, an increase in the capital stock leads to an increase (decrease) in sector 2's relative output when sector 1 is capital (labor) intensive. This is the expected result: when the rent-seeking industry is labor intensive, more capital means more output and hence a bigger pool to be robbed. On its turn, a worsening in the institutional set leads unambiguously to an increase in the relative size of the rent-seeking industry as expected. For the long-run, one would expect that when sector 1 is capital intensive, a worsening in the institutional set would lead to a reduction in capital and output. And this is indeed the case. But when sector 2 is capital intensive, the expected results do not necessarily take place. One would expect that a worsening in the institutional set would lead to more capital and less output, but we cannot rule out the cases where capital decreases and/or output increases. The results depend on values of parameters and we argue that, for the empirically relevant values, the expected results do emerge.[7]

Features of the transitional dynamics are also considered. If the rent-seeking sector is labor intensive the model delivers the usual dynamics of the neoclassical model of capital accumulation: both capital stock and output increase. On the other hand, if the rent-seeking sector is capital intensive, capital stock may increase and output may decrease along the transition. Arguably, the rent-seeking sector is labor intensive since it produces a service. However the opposite case can be illustrated by some African countries. For such countries it is reasonable to assume that the rent-seeking sector is the capital intensive one, since the productive sector is mostly agricultural and the rent-seekers are mostly armed bands and the army itself, and indeed these economies present positive investment and a decrease in output. In this way the model offers one rationale for the recent experience of such countries. Observe that such result is similar to the immizerizing growth literature, but not exactly the same. It states that capital accumulation generates less output because capital is mainly employed in unproductive activities. Immizerizing growth, on its turn, comes from capital accumulation generating a deterioration in the terms of trade that more than offsets the positive effects of the former.

Finally, some quantitative implications of the model are considered. First, the model is calibrated using explicit functional forms. We use data on per capita income and a measure of institutional efficiency both from Hall and Jones (1999). The model fits the data quite well. The

functioning legal system that enforces contracts (and also, clearly, on economic relations based on contracts). Every such aspect ought to be considered in the transition. What our model says is that any such institutional change is reflected in our institutional parameter. Indeed, all those changes lead in one way or the other to an improvement in the protection of property rights, and this is captured by our institutional parameter. (See Svejnar (2002) for an exposition of the transition in the former Soviet economies.)

[7] Although the indeterminacy is somewhat counter-intuitive, it shows that there is more to the model than just 'bad institutions causing bad economic performance.'

calibration exercise illustrates that a monopolist rent-seeker is significantly better than a competitive rent-seeking industry to the economy. Second, it is shown that the model can be used to explain income differences among countries based only on incentives. In fact, for this task the model performs quite similarly to the neoclassical model with a high capital share. It is well known that one needs a capital share in excess of $\frac{2}{3}$ for the latter model to explain differences in income (see Lucas (1990), Barro and Sala-i-Martin (1995) and Mankiw (1995)). The model presented here explains those differences with a capital share of $\frac{1}{3}$, which is consistent with the observed one. Since the model is an extension of the neoclassical model, one can argue that it not only introduces rent-seeking in a standard macroeconomic model, it also makes that model more congruent with the observed data. One reason why a high capital share is needed in the neoclassical model is that all factors of production are assumed to be employed in productive activities. Once this assumption is relaxed, there is no need for a high (and unrealistic) capital share. In other words, the model provides a rationale for lower TFP among poorer economies, where the share of production factors allocated in the productive activity can be viewed as an "endogenous TFP."[8]

The paper is organized as follows. The model is presented in section 2. The assumptions behind the aggregate rent-seeking technology are given, and the static and dynamic equilibria are defined. Section 3 shows the existence and uniqueness of these two equilibria. Section 4 presents the comparative statics results, and also the properties of the transition dynamics. The two central results are shown in sections 5 and 6, and in section 7 the quantitative results are presented. Section 8 relates our work with the previous literature and section 9 concludes with some remarks about possible extensions and applications of the model.

2 The Model

The model presented in this paper can be viewed as a simple extension of the neoclassical model of capital accumulation. In that model, there is just one good produced by a constant returns to scale technology employing capital and labor, whose services are rented by a representative consumer

[8] As Prescott (1998) pointed out, a theory for TFP diversity among economies is needed. Many possible explanations have been suggested: Parente and Prescott (2000) argued that lower TFP is prevalent among poorer economies due to monopoly groups or unions which preclude the adoption of newer technology; Parente, Rogerson, and Wright (2000) maintained that the inexistence of home production statistics could do a good job in explaining it, once one acknowledges that home production is much higher in poor economies; Acemoglu and Zilibotti (2001) argued that the lower TFP is caused by the mismatch between technology and the conditions of a poor economy, given that technology is developed in rich economies with different conditions and factor endowments; while Pessoa and Rob (2002) claimed that due to bad incentives poor economies use capital of lower quality, so that a model which takes into consideration both quantity and quality of capital can improve on the standard model. Our model states that TFP is smaller in economies with low institutional efficiency, due to the use of productive factors in unproductive activities. See section 9.

to the firms. The representative consumer makes her intertemporal decision optimally taking into account the income stream she will receive from her renting of those services. Institutions are usually introduced, in a macroeconomic setup, as a wedge between what firms produce and the income they earn. That is, output of the firms is summarized by an aggregate production function, $F(K, L)$, and firms' income is given by a fraction of that output, $(1-\tau)F(K, L)$. The 'tax rate' τ represents any sort of distortion that might characterize the economy, which could be a tax itself. In general, it can be identified with the efficiency of the institutional background of the economy. The simple extension considered here is to give a specific formulation for the 'tax rate' τ.

In particular, it will be assumed that there exists another sector in the economy, called the unproductive sector (also the rent-seeking sector, or sector 2). Like the productive sector (sector 1), it combines capital and labor to produce an output, but this output is not another good. It is a service, a transfer service. That is, an effort to confiscate goods produced in sector 1. The more service is produced, the larger the amount of goods that gets transferred toward sector 2. Calling Y_1 and Y_2 the output levels of sectors 1 and 2 respectively, the idea above can be stated as follows: sector 1 keeps $(1-\tau(Y_2))Y_1$ and sector 2 is able to confiscate $\tau(Y_2)Y_1$ goods from sector 1, where τ is an increasing function of the transfer services, Y_2. Formally, the burden imposed by the rent-seeking sector on the productive sector is a negative externality, which would not emerge if property rights were fully enforced. This is not the case by the very nature of the rent-seeking problem.

The function τ will be fully derived and characterized below (it will be denoted by g to reserve the symbol τ for a bona fide tax rate). This function g is the main analytical contribution of the model presented here. As mentioned above, the standard way of introducing institutions in a macroeconomic model is via something like g, so it seems natural to suggest a characterization of such entity. To the best of our knowledge, though, no such characterization has been provided yet.

In this section the production side of the economy will be presented, including the function g mentioned above. The two-sector structure allows one to define a static equilibrium, the equilibrium allocation of productive factors between the two sectors. This equilibrium is characterized and some interpretations are given. Then the demand side of the economy is presented using the standard intertemporal representative consumer. The long-run equilibrium is then considered. Finally, both equilibria are shown to exist and be unique under the maintained assumptions.

2.1 Firms

2.1.1 Productive Sector

The productive sector consists of N_1 identical firms[9] operating under the same technology and producing a single commodity, called 'the good.' Firm i ($i \in N_1$) combines capital, K_{1i}, and labor, L_{1i}, according to a constant returns to scale technology F_1, to produce output Y_{1i}. That is, $Y_{1i} \equiv F_1(K_{1i}, L_{1i})$. Part of what this firm produces is captured by the firms operating in the unproductive sector. In other words, from the point of view of the productive sector, the unproductive activity acts like a tax rate τ on its output, so firm i keeps only $(1-\tau)Y_{1i}$ of its output. Under perfect competition, firm i's program is to

$$\max_{K_{1i},\, L_{1i}} (1-\tau)Y_{1i} - r_1 K_{1i} - \mathrm{w}_1 L_{1i},$$

where r_1 and w_1 are the rental and wage rates prevailing in sector 1.

The first order conditions are given by

$$r_1 = (1-\tau) f_1'(k_1), \tag{1}$$

$$\mathrm{w}_1 = (1-\tau)\left[f_1(k_1) - k_1 f_1'(k_1)\right], \tag{2}$$

where $f_1 \equiv F_1(k_1, 1)$ and $k_1 \equiv \frac{K_{1i}}{L_{1i}}$, which are the same for any firm.

The total output of sector 1 is denoted by Y_1 and is given by $Y_1 = \sum_{i \in N_1} Y_{1i}$. The per capita output is $y_1 \equiv \frac{Y_1}{L} = l_1 f_1(k_1)$, where L is the population and $l_1 \equiv \frac{1}{L}\sum_{i \in N_1} L_{1i}$ is sector 1's labor share.

2.1.2 Aggregate Rent-Seeking

From the technological point of view, the major distinction between the productive activity and the unproductive one is that in order to 'produce' unproductive output it is required capital and labor services, and output. The productive activity, on its turn, requires only capital and labor services. Let G be the total amount of output which is extracted from the productive sector by the unproductive sector. We assume that $G = G(\theta Y_2, Y_1)$, where the function G is homogeneous of the first degree, Y_2 is the total output of transfer services, and θ is an institutional variable that describes the quality of the institutional set. A high (low) θ represents a bad (good) institutional background. We view θ as a measure of 'total factor productivity' (TFP) of sector 2, and hence θ

[9] The numbers of firms in both sectors are endogenously determined by the free entry assumption. See below.

enters as an argument of G multiplying Y_2. From the homogeneity of G, one can write

$$G = g\left(\theta \frac{Y_2}{Y_1}\right) Y_1 = g\left(\theta y^{\mathrm{R}}\right) Y_1, \tag{3}$$

where $y_2 \equiv \frac{Y_2}{L}$, $y^{\mathrm{R}} = \frac{y_2}{y_1}$, and $g\left(\theta \frac{Y_2}{Y_1}\right) \equiv G(\frac{\theta Y_2}{Y_1}, 1)$. The function g is the share of the output of the productive sector that is extracted by the unproductive sector.[10] As anticipated above, the share g is precisely the tax rate τ that firms in sector 1 take as given:

$$g\left(\theta y^{\mathrm{R}}\right) = \tau.$$

The formulation states, therefore, that the aggregate rent-seeking technology, g, must be a function of the relative output $\frac{y_2}{y_1}$ multiplied by an institutional variable θ. It is reasonable to assume that $g(0) = 0$ and $g'(x) > 0$, for any $x \geq 0$. In addition, for it to be a share, it will be assumed that $\lim_{x \to \infty}(x) = 1$. Any function g satisfying these properties can be used to introduce rent-seeking in the neoclassical capital accumulation model.[11] In this paper g will be assumed to satisfy the following four axioms as well. Let $\overline{\alpha}_g \in (0, 1)$ be given, and define $\alpha_g(x) \equiv x \frac{g'(x)}{g(x)} \frac{1}{1-g(x)}$. Let also $\alpha_{1\mathrm{L}}$ be sector 1's labor share on income.

Axiom 1 $\lim_{x \to 0} g'(x) = \infty$ *and* $g''(x) < 0$.

Axiom 2 $0 < \alpha_g(x) \leq \overline{\alpha}_g$.

Axiom 3 $\overline{\alpha}_g < \alpha_{1L}$.

Axiom 4 $g(x) = \frac{m(x)}{1+m(x)}$ *for some* m *s.t.* $m'(x) > 0$, $m''(x) < 0$, $m(0) = 0$, $\lim_{x \to 0} m'(x) = \infty$.

Axiom 1 is the standard Inada condition plus strict concavity, and is what ensures uniqueness of equilibrium. Such an assumption reflects our view that development issues are to be explained by differences on the fundamentals among economies, and not by coordination failures.[12] The term

[10] The function G plays, in the context of rent seeking, the role of the matching function in the equilibrium unemployment literature. (See Mortensen and Pissarides, 1994.) There g is the rate that seekers of job position meet vacancies; here g is the rate that the seekers of rents exploit the productive sector. Although one activity, job search, is productive and the other, rent-seeking, is not, the formal properties of the function g are the same.

[11] Observe that the function g can be viewed as a cumulatitive distribution function, and issues of risk aversion could be considered as well. We do not pursue this line of reasoning here as the setup is assumed deterministic.

[12] Of course, multiplicity of equilibria can be introduced by relaxing the strict concavity assumption. This would lead to the issue of indeterminacy of equilibrium and of coordination failures. Such phenomena belong, in our view, to short to medium run macroeconomic theories. In the very long run, what matters is the more fundamental properties of an economy. We would not argue, for instance, that Brazilian GDP is five times smaller than the American one

$\overline{\alpha}_g$ is an upper bound for $\alpha_g(x)$. Axiom 2 states that $\alpha_g(x)$ must be strictly less than one. If $\overline{\alpha}_g = 1$ (< 1), the aggregate rent-seeking technology is said to present constant (decreasing) returns to scale.[13] Observe that if α_g happens to be constant then integrating $\alpha_g(x)$ yields

$$g(x) = \frac{x^{\alpha_g}}{1 + x^{\alpha_g}}, \tag{4}$$

which is one candidate for a functional form[14] for g. Axiom 3 is needed for long-run stability and is only used in that section of the model. It ensures saddle-path stability of the dynamic system.[15] Finally, Axiom 4 ensures uniqueness of equilibrium in the monopoly formulation (section 6), and it is needed only for that section. In particular, the main model (the competitive one) is solved for a generic function g satisfying Axioms 1, 2, and 3.

2.1.3 Unproductive Firm

The unproductive sector consists of N_2 (endogenously determined) identical firms operating under the same technology and producing a single service, called transfer service. Firm i ($i \in N_2$) combines capital, K_{2i}, and labor, L_{2i}, according to a constant returns to scale technology F_2, to produce output $Y_{2i} \equiv F_2(K_{2i}, L_{2i})$. The quantity of goods that this particular firm expropriates from the productive sector is a share of the total booty G in (3). It is assumed that this share is of the additive Contest Success Function (CSF) form,[16] so that it can be written as $\frac{h(\theta Y_{2i})}{\sum_{j \in N_2} h(\theta Y_{2j})}$. That is, firm i will fight for a share of G and the success of such a fight will be determined by the CSF. It is again reasonable to assume that $h(0) = 0$ and $h'(x) > 0$, for any $x \geq 0$. We will make one further assumption.

Axiom 5 *There exists a unique $\bar{x}$ such that* $\arg\max_x \frac{h(x)}{x} = \bar{x}$.

In other words, there exits one, and just one, optimal scale for each firm in sector 2. Observe that Axiom 5 does not posit uniqueness of a point where marginal returns equal average returns, it only asks that there exists just one point that maximizes average returns.

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because of some unlucky choice of equilibrium. The structure of incentives (institutions) in Brazil is clearly less efficient than the American one. (Evidently, coordination failures, history dependence, or political economy issues can help understanding why these bad institutions were adopted in one place and not in other. See for instance Engerman and Sokoloff (1997).)

[13] This denominations are explained in the static equilibrium section.

[14] Observe the analogy with the Cobb-Douglas functional form.

[15] The term $\alpha_g(x)$ is the normalized elasticity of $g(x)$. It was introduced here because $\alpha_g(x) < \alpha_{1\mathrm{L}}$ is the stability condition. The functional form (4) is a useful by-product.

[16] Tullock (1980) introduced the CSF in the theory of rent-seeking. Skaperdas (1996) axiomatized the additive CSF.

The analysis is made on the limit in which there are many firms in each sector so that the Dixit-Stiglitz (1977) assumption of discharging terms that depend on $\frac{1}{N_1}$ or $\frac{1}{N_2}$ from the first order conditions can be made.[17,18] Consequently, firm i's program

$$\max_{K_{2i},\, L_{2i}} \frac{h(\theta Y_{2i})}{\sum_{j\in N_2} h(\theta Y_{2j})} g\left(\theta \frac{\sum_{j\in N_2} Y_{2j}}{Y_1}\right) Y_1 - r_2 K_{2i} - \mathrm{w}_2 L_{2i}, \tag{5}$$

generates the following first order conditions:

$$r_2 = \frac{\theta h'(\theta Y_{2i})}{\sum_{j\in N_2} h(\theta Y_{2j})} g\left(\theta y^{\mathrm{R}}\right) f_2'(k_2) Y_1, \tag{6}$$

$$\mathrm{w}_2 = \frac{\theta h'(\theta Y_{2i})}{\sum_{j\in N_2} h(\theta Y_{2j})} g\left(\theta y^{\mathrm{R}}\right) \left[f_2(k_2) - k_2 f_2'(k_2)\right] Y_1. \tag{7}$$

2.1.4 Free Entry

In keeping with the competitive paradigm, equilibrium within each sector is achieved when each firm makes zero profit. It follows from (1) and (2) that $\pi_{1i} = 0$ for any $i \in N_1$ (hence N_1 is indeterminate). For sector 2, substituting (6) and (7) into (5) yields

$$\pi_{2i} = \frac{h(\theta Y_{2i})}{\sum_{j\in N_2} h(\theta Y_{2j})} g\left(\theta y^{\mathrm{R}}\right) Y_1 \left(1 - \theta Y_{2i} \frac{h'(\theta Y_{2i})}{h(\theta Y_{2i})}\right), \tag{8}$$

which is not necessarily zero. Here is where Axiom 5 plays a role. Setting $Y_{2i} = \frac{\bar{x}}{\theta}$ in (8) yields $\pi_{2i} = 0$ (because $h'(\bar{x}) = \frac{h(\bar{x})}{\bar{x}}$), so this level of Y_{2i} for every firm in sector 2 is an equilibrium with free entry. It is unique by hypothesis.[19]

Substituting the free entry condition $\pi_{2i} = 0$ into (6) and (7), it follows that a symmetric

[17] The attentive reader will have noticed that we have already make this assumption in deriving (1) and (2): since $\tau = g\left(\theta \frac{Y_2}{Y_1}\right)$, to take τ as given amounts to assume away the effect of a particular firm in sector 1 on Y_1.

[18] Consequently, we are assuming that the optimum size of a firm in sector 2, $\bar{x}$, is small enough such that in equilibrium N_2 is large.

[19] It is also a Nash equilibrium for the game played by the firms in sector 2. That is, assume each firm plays $\bar{x}$ and consider firm i contemplating playing $x \neq \bar{x}$ instead. Straightforward computations yield $\pi_{2i} = \frac{h(x)}{(N_2-1)h(\bar{x})+h(x)} g\left(\theta \frac{y_2}{y_1}\right) Y_1 - r_2 K_{2i} - \mathrm{w}_2 L_{2i}$. Substituting (6) and (7) yields $\pi_{2i} = \frac{h(x)}{(N_2-1)h(\bar{x})+h(x)} g\left(\theta \frac{y_2}{y_1}\right) Y_1 \left(1 - x\frac{h^{0}(\bar{x})}{h(x)}\right) < 0$, so firm i will not deviate.

equilibrium[20] is given by

$$r_2 = \frac{g\left(\theta y^{\mathrm{R}}\right)}{y^{\mathrm{R}}} f_2'(k_2), \tag{9}$$

$$\mathrm{w}_2 = \frac{g\left(\theta y^{\mathrm{R}}\right)}{y^{\mathrm{R}}} \left[f_2(k_2) - k_2 f_2'(k_2)\right]. \tag{10}$$

2.2 Static Equilibrium

The static equilibrium is an equilibrium in the allocation of productive factors between the two sectors, for given levels of productive factors and institutional efficiency (k and θ). We use the underlying two-sector structure of the model to define such equilibrium. The idea is that each combination of output levels of both sectors determines a marginal rate of transformation and is in turn determined by the latter. The equilibrium is a fixed point of this mutual determination.

More specifically, each allocation of productive factors generates some output levels y_1 and y_2 and consequently a marginal rate of transformation of $\frac{g\left(\theta y^{\mathrm{R}}\right)}{y^{\mathrm{R}}} \frac{1}{1-g(\theta y^{\mathrm{R}})}$ of goods into transfer services. The term $\frac{g\left(\theta y^{\mathrm{R}}\right)}{y^{\mathrm{R}}}$ represents a change in sector 2's output, and the term $1 - g\left(\theta y^{\mathrm{R}}\right)$ represents a change in sector 1's output. The ratio is a feasible reallocation of productive factors. On the other hand, a marginal rate of transformation (MRT) also defines output levels y_1 and y_2. Indeed, using the underlying two-sector structure, one can write $y_i(p,k)$ as sector i's static supply function,[21] where p is the slope of the PPF, i.e., the MRT. That is, each MRT corresponds to one point on the PPF. The static equilibrium is defined as a p that is self-determining in the above sense, so that it is given by the following fixed-point:

$$p = \frac{g\left(\theta y^{\mathrm{R}}(p,k)\right)}{y^{\mathrm{R}}(p,k)} \frac{1}{1 - g\left(\theta y^{\mathrm{R}}(p,k)\right)} \equiv H(p,k,\theta). \tag{11}$$

2.3 Interpretation

The production side of the economy was presented above. The characterization of the 'tax rate' τ as a function g representing the aggregate rent-seeking technology was made under fairly general conditions. The two-sector structure provides a characterization of the static equilibrium in terms of equation (11).

From now on as an abuse of language $y_i(p,k)$, $i = 1,2$, will be called the supply of goods and unproductive services and p the relative price of the unproductive service in units of goods. Observe

[20] Recall that for a symmetric equilibrium $\frac{h(\theta Y_{2i})}{\sum_{j=1}^{N_2} h(\theta Y_{2j})} = \frac{1}{N_2}$.

[21] Appendices A.1 and A.2 provide a short review of the static two-sector general equilibrium model. See chapter 1 of Kemp (1969) for a more thoroughly presentation.

that in this economy to produce one unit of good does not mean to be the owner of it. There are, therefore, three goods in this two-sector economy: the good, the rent-seeking service, and the good at somebody's hands. The price $p_1 = 1 - g$ is the relative price of one unit of the good in units of goods at somebody's hands, and $p_2 = \frac{g}{y^{\mathrm{R}}}$ is the relative price of one unit of the rent-seeking service in units of goods at somebody's hands. By construction, $y_1 = p_1 y_1 + p_2 y_2$, so one can view $p_1 y_1 + p_2 y_2$ as total output of the economy in units of goods at somebody's hands.

The static equilibrium can be understood as a consequence of factor mobility. With mobility and interior solution, it must be that $r_1 = r_2$ and $\mathrm{w}_1 = \mathrm{w}_2$, otherwise all productive factors would be allocated in just one of the industries. From the two-sector general equilibrium model, we know that

$$r_i = p_i f_i'(k_i) \tag{12}$$

$$\mathrm{w}_i = p_i \left[f_i(k_i) - k_i f_i'(k_i) \right], \tag{13}$$

where p_i is the price of sector i's output, $i = 1, 2$. It follows from comparing (1), (2), (9), and (10) to (12) and (13), that the static equilibrium is given by $p_1 = 1 - g$ and $p_2 = \frac{y_1}{y_2} g$, which is what is expressed in (11). If $p > H(p, k, \theta)$ ($p < H(p, k, \theta)$) then factors will move towards sector 2 (sector 1), reducing (increasing) p and increasing (reducing) H because sector 2 (sector 1) pays relatively more. In equilibrium, factor prices are equalized and there is no further factor reallocation.

The static equilibrium condition (11) establishes the allocation at each point in time of capital and labor between the two sectors. It can be written as

$$g(\theta \frac{y_2}{y_1}) = \frac{p_2 y_2}{p_1 y_1 + p_2 y_2}. \tag{14}$$

The LHS of (14) can be written as $\frac{g y_1}{g y_1 + (1-g) y_1}$, which is the ratio between the 'output' of the unproductive sector, $g y_1$, and the total output, y_1. The RHS of (14) can be written as $\frac{(r k_2 + \mathrm{w}) l_2}{r k + \mathrm{w}}$, which is the ratio between the remuneration of the factors employed in the unproductive sector and the total factor remuneration. The short-run equilibrium is the allocation that equalizes the relative output of the unproductive sector with its relative income. In other words, given that there is free entry in both industries, the equilibrium condition is that average benefit equals average cost.

2.4 Consumers

At a point in time, that is, for given values for k and θ, the static model is solved yielding p, $y_i(p, k)$, and the factor prices r and w. The representative household rents her capital and labor services to the firms. She chooses a consumption plan that solves

$$\max_{c(t)} \int_0^\infty e^{-\rho t} u\left(c(t)\right) \mathrm{d}t$$

$$\text{s.t. } \dot{k}(t) = \left(r(t) - \delta\right) k(t) + \mathrm{w}(t) - c(t),$$

given $k(0)$, where ρ is the intertemporal discount rate, and δ is the physical depreciation rate.

This is the standard Ramsey problem that yields the following Euler equation

$$\dot{c}(t) = c(t)\gamma(c(t))\left(r(t) - \rho - \delta\right), \tag{15}$$

where γ is the intertemporal elasticity of substitution, and $r = (1 - g(\theta y^{\mathrm{R}}(p,k))f_1'(k_1(p))$ as derived before.

From the two-sector model it is known that per capita income equals per capital output, i.e., that

$$r(t)k(t) + \mathrm{w}(t) = p_1(t)y_1(t) + p_2(t)y_2(t) = y_1(t),$$

so that the dynamics are represented by the following dynamic system[22]

$$\begin{cases} \dot{k} = y_1(p,k) - c - \delta k \\ \dot{c} = c\gamma(c)\left[\underbrace{\left(1 - g(\theta y^{\mathrm{R}}(p,k))\right) f_1'(k_1(p))}_{r} - \rho - \delta\right], \end{cases} \tag{16}$$

together with the initial condition for capital, $k(0)$, and the terminal condition $\lim_{t\to\infty} e^{-\rho t} u'\left(c(t)\right) k\left(t\right) = 0$, where $p = p(k,\theta)$ is the short-run equilibrium,

The condition for saddle point stability of (16) is that the Jacobian of the linearized system be negative, and this boils down to

$$\left.\frac{k}{r}\frac{\mathrm{d}r}{\mathrm{d}k}\right|_\theta = \left(g\frac{\alpha_g}{1-\alpha_g} - \frac{1}{\alpha_{2K}}\frac{k_2}{k_1 - k_2}\right)\left.\frac{k}{p}\frac{\partial p}{\partial k}\right|_\theta < 0. \tag{17}$$

Appendix B.1 shows that Axiom 3 is a sufficient condition for the inequality above to hold.

[22] The variable t is omitted whenever the understanding is clear.

2.5 Long Run Equilibrium

The long-run equilibrium is given by a capital stock and a relative price that satisfy the conditions of a steady-state of the dynamic system (16). In other words, the following system of equations must hold in the long-run:

$$\begin{cases} \psi_1(p,k) = \frac{g\left(\theta y^{\mathrm{R}}(p,k)\right)}{y^{\mathrm{R}}(p,k)} \frac{1}{1-g(\theta y^{\mathrm{R}}(p,k))} - p = 0 \\ \psi_2(p,k) = \left(1 - g(\theta y^{\mathrm{R}}(p,k))\right) f_1'(k_1(p)) - (\rho + \delta) = 0. \end{cases} \tag{18}$$

3 Existence and Uniqueness

In this section it is shown that both static and long-run equilibria exist and are unique. Such results complete the set up of the model. The properties and applications of the model will be presented in the following sections. Let $\underline{p}$ and $\overline{p}$ be the prices under which the economy is specialized in sector 1 and 2 respectively.[23]

Proposition 1 *The short-run equilibrium exists and is unique.*

Proof. *Let $H : [\underline{p}, \overline{p}] \to R_+$ be the mapping defined by $H(p) \equiv H(p,k,\theta)$ for given (k,θ), so that the short-run equilibrium $p(k,\theta)$ is given by a fixed point of H. Taking derivatives and rearranging:*

$$\frac{p}{H(p)} \frac{dH(p)}{dp} = -(1-\alpha_g) \frac{p}{y^R(p)} \frac{dy^R(p)}{dp} < 0,$$

and the inequality follows from $\alpha_g < 1$. From Axiom 1 we have $\lim_{p \to \underline{p}^+} H(p) = \lim_{y^R \to 0} \frac{g\left(\theta y^R\right)}{y^R} \geq \lim_{y^R \to 0} g'\left(\theta y^R\right) = \infty$. *From Axiom 2, integrating α_g gives $g(x) \leq \frac{x^{\overline{\alpha}_g}}{1+x^{\overline{\alpha}_g}}$. Hence* $\lim_{x\to\infty} x\left(1-g(x)\right) \geq \lim_{x\to\infty} \frac{x}{1+x^{\overline{\alpha}_g}} = \infty$, *which ensures that* $\lim_{p\to\overline{p}} H(p) = \lim_{y^R\to\infty} \frac{g(\theta y^R)}{y^R} \frac{1}{1-g(\theta y^R)} = 0$. ■

Observe that if instead of $\alpha_g < 1$ one had $\alpha_g = 1$, then if one point on the PPF is an equilibrium any other one is also an equilibrium. That is why this case was called constant returns to scale on aggregate rent-seeking. Finally, if $\alpha_g > 1$ there is still a unique interior equilibria, which is unstable (the two corners become stable).

Proposition 2 *The long-run equilibrium exists and is unique.*

[23] For a given level of factor endowment k, $p \geq \overline{p}(k)$ ($p \leq \underline{p}(k)$) means that the economy is specialized in the production of rent-seeking services (sector 1's good). Note that $\overline{p}'(k) \gtrless 0$ and $\underline{p}'(k) \gtrless 0$ as $k_1 \gtrless k_2$.

Proof. *Let* $f_1'(k_{\rho+\delta}) = \rho + \delta$ *and* $\psi_1(p_{\rho+\delta}, k_{\rho+\delta}) = 0$. *Given that on* $\underline{p}(k)$ *the economy is specialized in the production of the productive good,*

$$r|_{(\underline{p}(k_{\rho+\delta}),k_{\rho+\delta})} = \rho + \delta \Rightarrow \left(\underline{p}(k_{\rho+\delta}), k_{\rho+\delta}\right) \in [\psi_2 = 0] \,.$$

Additionally, we know that $r = (1-g)\, f_1'|_{(p_{\rho+\delta},k_{\rho+\delta})\in\psi_1} < \rho+\delta$. *In order to show that there is a point on* $\psi_1 = 0$ *such that* $r = \rho + \delta$ *we show that* $\lim_{p\to 0} r = \infty$. *Given Axiom 2, write* $\alpha_g(x) \le \overline{\alpha}_g - \varepsilon$, *for some* $\varepsilon > 0$. *From (17)*

$$\frac{p}{r}\frac{dr}{dp} = g\frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1L}}{1-\alpha_{2K}-\alpha_{1L}} \le \frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1L}}{1-\alpha_{1L}} \le -\beta,$$

where $\beta \equiv \frac{\varepsilon}{(1-\alpha_{1L})(1-\alpha_{1L}+\varepsilon)} > 0$. *Let* $(p_0, k_0) \in \psi_1$, *and let* $r_0 = (1-g)\, f_1'|_{(p_0,k_0)}$. *Integrating the last inequality yields*

$$\frac{r}{r_0} \ge \left(\frac{p}{p_0}\right)^{-\beta},$$

for any $p \le p_0$. *This shows existence. For uniqueness, notice that since* $0 > \frac{dr}{dk}\Big|_\theta = \frac{\partial r}{\partial k}\Big|_{p,\theta} + \frac{\partial r}{\partial p}\Big|_{k,\theta} \frac{dp}{dk}\Big|_{\psi_1,*}$, *and* $\frac{dp}{dk}\Big|_{\psi_2,*} = -\frac{\frac{\partial r}{\partial k}\big|_{p,\theta}}{\frac{\partial r}{\partial p}\big|_{k,\theta}}$, *we have*

$$\frac{dp}{dk}\Big|_{\psi_2,*} = \frac{\frac{\partial r}{\partial k}\big|_{p,\theta}}{\frac{\partial r}{\partial k}\big|_{p,\theta} - \frac{dr}{dk}\big|_\theta} \frac{dp}{dk}\Big|_{\psi_1,*} \lesseqgtr \frac{dp}{dk}\Big|_{\psi_1,*} \quad \text{as } k_1 \gtrless k_2,$$

so the curves intersect only once.[24] ■

The idea of the proof is shown by Figures 1 and 2 below. They represent the system $\psi_1(p,k) = 0$ and $\psi_2(p,k) = 0$ when the production functions are Cobb-Douglas and the aggregate rent-seeking function is given by (4). (The curve $\psi_1^{\mathrm{M}} = 0$ refers to the monopoly solution of the models. See section 6.) Figure 1 considers sector 1 as capital intensive (the parameter values are $\{\alpha_1, \alpha_2, \alpha_g, \theta, \delta, \rho\} = \{1/3, 1/6, 1/8, 1, \log(1.066), \log(1.03)\}$) and in Figure 2 sector 1 is labor intensive (the parameters are $\{1/6, 1/3, 1/8, 10, \log(1.066), \log(1.03)\}$). As it is clear from the figures, $\psi_1(p,k) = 0$ and $\psi_2(p,k) = 0$ must intersect once, and only once.

[24] *Recall that* $\frac{dp}{dk}\big|_{\psi_1,*} \gtrless 0$ *as* $k_1 \gtrless k_2$.

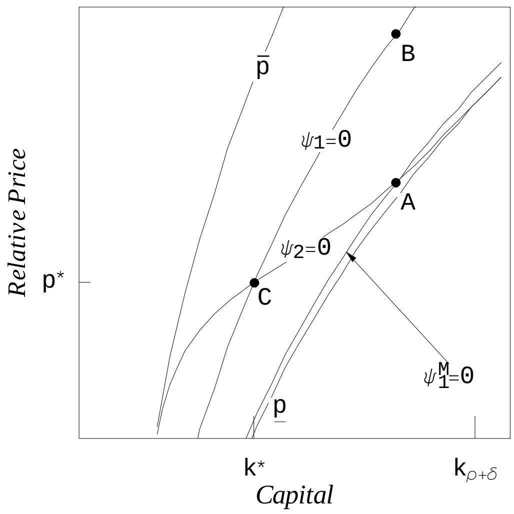

Figure 1: Sector 1 capital intensive

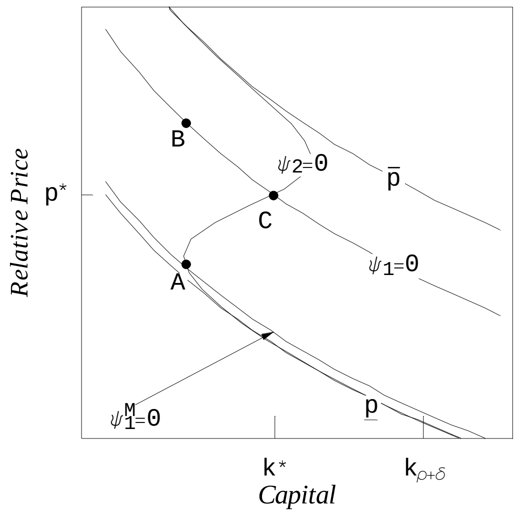

Figure 2: Sector 1 labor intensive

4 Properties of the Model

4.1 Comparative Statics

4.1.1 Short-Run

From (11), the effects of k and θ on the static equilibrium p are given by

$$\left.\frac{k}{p}\frac{\partial p}{\partial k}\right|_{\theta} = -\frac{(1-\alpha_g)\left.\frac{k}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_{p}}{1+(1-\alpha_g)\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_{k}} \gtrless 0 \text{ as } k_1 \gtrless k_2, \tag{19}$$

$$\left.\frac{\theta}{p}\frac{\partial p}{\partial \theta}\right|_{k} = \frac{\alpha_g}{1+(1-\alpha_g)\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_{k}} > 0. \tag{20}$$

Hence the comparative statics in the short-run are as follows: an increase in the per capita capital stock leads to more unproductive activity when sector 1 is capital intensive, and to less unproductive activity when sector 1 is labor intensive. The former case can be thought of as the case in which capital is a valuable resource in the economy (a good). The more of it, the better for both sectors. Sector 1 produces more and sector 2 is able to appropriate more (the "pie" increases, so more to everybody.) For the latter case ($k_1 < k_2$), capital is less valued than labor (it is a bad). An increase in k is effectively a reduction in the relative supply of labor, so it is harmful for sector 1 (and for sector 2 consequently). The effect of the institutional variable does not depend on the technologies: the worse the institutional background, the bigger the unproductive sector.

4.1.2 Long-Run

In the long-run capital is endogenous and given by (18). The only exogenous variable is θ, the variable that captures the efficiency of the institutional background. When sector 1 is capital intensive the results are intuitive: a worsening in institutional efficiency generates less capital and output in the long-run. But when the rent-seeking sector is capital intensive, then the reverse result cannot be ruled out. That is, it can be that as the institutional background becomes worse, long-run income increases. Since this counter-intuitive effect does not seem to correspond to any relevant empirical evidence, it is qualified in terms of parameter values. In particular, for it to happen, it is necessary that sector 2 be significantly more capital intensive than sector 1. In particular, Appendix C.5 shows that

$$\alpha_{2\mathrm{K}} \leq \min\left\{\sqrt{l_1}, (1+\sigma_1)\,\alpha_{1\mathrm{K}}\right\} \tag{21}$$

is a sufficient condition to rule out a counter-intuitive effect of θ on y_1. Observe that (21) is likely to take place. There is no available data for $\alpha_{2\mathrm{K}}$, but σ_1 and $\alpha_{1\mathrm{K}}$ are known to be close to 1 and $\frac{1}{3}$ respectively. l_1 is the share of the labor force employed in the productive sector, which is also something not available. Tentatively, let us say that l_1 is close to $\frac{1}{4}$, so that three fourths of the workers are employed as rent-seekers. Even in this case, $\alpha_{2\mathrm{K}}$ would have to be bigger that $\frac{1}{2}$, which seems unlikely since the overall capital share is close to $\frac{1}{3}$.

4.2 Features of the Dynamics

If the economy is not at its long-run equilibrium, it is at a transition path of capital accumulation. In what follows, it is shown that an economy might be in a dynamic path of capital accumulation with a decreasing level of output.

In Appendix B.2 it is shown that

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta = \left.\frac{\partial y_1}{\partial k}\right|_p + \left.\frac{\partial y_1}{\partial p}\right|_k \left.\frac{\partial p}{\partial k}\right|_\theta > 0 \tag{22}$$

if $k_1 \geq k_2$. That is, one can only guarantee that output is increasing along the transition if sector 1 is capital intensive. Analogously, it is also possible to show that $\left.\frac{\mathrm{d}y_2}{\mathrm{d}k}\right|_\theta > 0$ if $k_1 \leq k_2$. More specifically, in Appendix B.2 it is shown that $\left.\frac{\mathrm{d}y^{\mathrm{R}}}{\mathrm{d}k}\right|_\theta \lesseqgtr 0$ as $k_1 \gtreqless k_2$, i.e., that the ratio $\frac{y_2}{y_1}$ is monotone in k: increasing if rent-seeking is capital intensive and decreasing otherwise. Also, the same pattern is followed by the relative value of sector 2's output, $\frac{py_2}{y_1+py_2}$.

An important consequence of (22) is that there can be a situation where total output of the economy decreases while the capital stock increases. A necessary condition for it is that the rent-seeking sector is capital intensive. Although this configuration is uncommon - the rent-seeking firm produces a service and services are usually labor intensive - it is not only a theoretical possibility. Take a very underdeveloped economy (from sub-Saharan Africa for instance). Its productive sector is the agricultural sector. Its unproductive sector is the army and armed bands. It makes sense, then, to consider the rent-seeking sector as the capital intensive sector for this economy. Many sub-Saharan countries have been experiencing negative growth rates and positive investment. One way of explaining it is that investment has been directed mainly to unproductive activities. As an example, in Appendix B.2 it is shown that, for the extreme case that the productive sector only employs labor ($\alpha_{1\mathrm{K}} = 0$), the condition $1 > (1 - \alpha_g)\,\sigma_2$ is sufficient to ensure that $\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta} < 0$, and this is the case as long as $0 \leq \sigma_2 \leq 1$, which is by no means a strong assumption. Hence, if an economy can be characterized by the above parameters, the condition ensuring positive investment and negative growth is that the elasticity of substitution in sector 2 be not larger than 1.

In the next section a welfare analysis will be presented. It is the analysis of the impact of θ on welfare, and not the effect of capital accumulation on welfare. While the latter can be negative (immizerizing), it is shown that the former cannot.

5 Welfare Analysis

The results above show that properties of the model capture a variety of possible phenomena. The fact that the long-run comparative statics depend on the underlying factor intensity is viewed as a positive feature of the model, since one can, then, use the model to explain different phenomena. But this dependency on factor intensity might be viewed as an indeterminacy. Such is not a concern when the welfare analysis is considered. There is a monotone relation between institutional efficiency and overall welfare in the economy. The worse the institutional background, the lower the welfare enjoyed by the representative consumer. The relevant criterion for evaluating economic performance is welfare, and under such criterion the variable θ does represent the "underlying determinants of economic performance," as North would put it.

A worsening in the institutional set of the economy generates two effects. First, an increase in θ increases p (see (20)) producing an inflow of factors toward the rent-seeking sector and a reduction on the productive sector's output. This is the called the Tullock effect. Second, from (1), an increase in θ increases the distortion to capital accumulation. This is called the Harberger effect. Under the assumption that initially the economy is in long-run equilibrium, it is shown that (i) it is possible to disentangle the welfare effect in two components, which are the two above mentioned effects, (ii) the marginal impact of a reduction on institutional efficiency is a reduction in welfare, and (iii) if both sectors operate under the same technology the Harberger effect is zero.

Given that the economy is a representative agent economy, the intertemporal utility is the social welfare function. In order to evaluate the welfare impact of a marginal increase in θ, taken into consideration the transitional dynamics, a technic developed by Judd (1982 and 1987) is employed. Let $W = \int_0^\infty e^{-\rho t} u\left(c(t)\right) \mathrm{d}t$ be the welfare index. The impact of θ on W at steady state (denoted by an *) is:

$$\left.\frac{\mathrm{d}W}{\mathrm{d}\theta}\right|_* = \int_0^\infty e^{-\rho t}\, u'(c(t))\big|_* \frac{\mathrm{d}c(t)}{\mathrm{d}\theta}\mathrm{d}t = u'(c^*) \int_0^\infty e^{-\rho t} \frac{\mathrm{d}c(t)}{\mathrm{d}\theta}\mathrm{d}t = u'(c^*) C_\theta(\rho),$$

where $X_\theta(\vartheta) \equiv \int_0^\infty e^{-\vartheta t} \frac{\mathrm{d}x(t)}{\mathrm{d}\theta}\mathrm{d}t$ is the Laplace transform of $\frac{\mathrm{d}x(t)}{\mathrm{d}\theta}$ for any function $x(t)$.

Hence, the effect on welfare is given by the Laplace transform ($C_\theta(\rho)$) of $\frac{\mathrm{d}c(t)}{\mathrm{d}\theta}$ multiplied by marginal utility evaluated at c^*. In appendix C.1 it is shown that

$$\rho C_\theta(\rho) = \underbrace{\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*}}_{\text{Tullock Effect}} + \underbrace{\frac{\mu-\rho}{\mu} \frac{\frac{c^*\gamma(c^*)}{\rho} \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}{\rho + \delta - \left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} + \frac{c^*\gamma(c^*)}{\rho} \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}} \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \rho - \delta \right) \left.\frac{\mathrm{d}k}{\mathrm{d}\theta}\right|_*}_{\text{Haberger Effect}}, \tag{23}$$

where μ is the positive eigenvalue associated with the matrix of the linearized dynamic system. Hence, the impact of θ on welfare is given by the sum of two terms that are identified as the Tullock and Harberger effects.

The Tullock effect is given by the instantaneous reduction on output, and consequently on consumption, resulting from the deterioration of the institutional set and the corresponding increase in the relative size of the rent-seeking industry. Under any configuration the Tullock effect is negative (see (20)).

The Harberger effect is the composition of two terms. One is the net marginal impact of capital accumulation on output (net of physical depreciation and of the intertemporal opportunity cost of investment),

$$\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \rho - \delta \right) \left.\frac{\mathrm{d}k}{\mathrm{d}\theta}\right|_*. \tag{24}$$

The other is the attenuation factor (AF), which translates a change in output due to capital accumulation into a change in welfare taking into consideration the transitional dynamics,

$$\mathrm{AF} = \frac{\mu-\rho}{\mu} \frac{\frac{c^*\gamma(c^*)}{\rho} \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}{\rho + \delta - \left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} + \frac{c^*\gamma(c^*)}{\rho} \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}.$$

The attenuation factor is smaller the smaller is the intertemporal elasticity of substitution $\gamma(c)$.

Note that $\lim_{\gamma\to 0}\text{AF} = 0$ and $\lim_{\gamma\to\infty}\text{AF} = \lim_{\gamma\to\infty}\frac{\mu-\rho}{\mu} = 1$. In Appendix C.2 it is shown that $0 \leq \text{AF} \leq 1$. Appendix C.4 shows that the Tullock effect is larger than the net marginal impact of capital accumulation on output (24). Consequently,

$$\left.\frac{\mathrm{d}W}{\mathrm{d}\theta}\right|_{*} < 0,$$

i.e., the effect on welfare of a change in the institutional background is unambiguous: welfare is reduced when institutions get less efficient.[25]

This is the main qualitative result of the model. It makes a case for improving efficiency of institutions of property rights enforcement based on welfare grounds. Alternatively, it states that the main problem of an unproductive activity is that it employs productive resources that could have been employed socially valued activities. This is the main driving force behind the result that welfare depends positively on institutional efficiency.

Remark 1 *Appendix C.3 shows that, even when the economy is not initially at a steady state postition, we still have $\frac{dW}{d\theta} < 0$. Better still, the impact of θ on welfare can always be decomposed into Tullock and Harberger effects, and these two effects combined are always negative.*

Finally, Appendix B.2.2 shows that

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \rho - \delta \gtreqless 0 \text{ as } k_1 \gtreqless k_2,$$

so, from (24), the Harberger effect is zero if $k_1 = k_2$.[26]

6 Monopoly

The model presented in this paper assumes that the rent-seeking sector is a competitive industry. The free entry condition and the assumption of an optimum plant size for each firm in sector 2 guarantee that rents are fully dissipated in equilibrium. It seems reasonable to assume that profit

[25] Observe an analogy of (23) with the Slutsky equation of consumer theory: a change in θ may be viewed as a change in the price of the consumption good. The total effect is then separated into the substitution effect (the Tullock effect, always of the right sign) and the income effect (the Harberger effect, which has ambiguous sign). What is shown is that the substitution effect always dominates the income effect making the consumption good an "ordinary" good.

[26] Under this configuration ($k_1 = k_2$) it follows from the short-run equilibrium condition (14) that the share of workers in the rent-seeking sector, l_2, is equal to the share of output extracted by the rent-seeking sector, g. Consequently, the marginal impact of capital on output, $l_1 f'(k)$, is equal to the market interest rate, $(1-g)f'(k)$, and the social value of capital is equal to the private one.

opportunities will be taken up by someone in a society, so the assumption of a large number of rent-seekers has its appeal. It is plain that different forms of market organization could be considered. We decided to stick to the competitive paradigm because we view it as the relevant scenario for a market economy, especially in the long-run. In this section, the model with just one firm operating in sector 2 is considered.[27] With such a model, one can compare the results of the previous model and also, as was argued in the Introduction, compare the effects of rent-seeking in open societies with rent-seeking in more closed societies.

It is possible to imagine a situation in which there is a central organization that gives right to a unique firm to practice rent-seeking.[28] Assume that this central organization does exist and that it is able to enforce this right.[29] The monopolist uses its market power to make positive profits[30] and generates less rent-seeking than a competitive industry does.

The monopolist problem is to solve

$$\max_{K_2,\, L_2} g\left(\theta \frac{L_2 f_2(k_2)}{Y_1}\right) Y_1 - r_2 K_2 - \mathrm{w}_2 L_2,$$

which yields

$$\begin{aligned} r_2 &= \theta g'\left(\theta y^{\mathrm{R}}\right) f_2'(k_2) \\ \mathrm{w}_2 &= \theta g'\left(\theta y^{\mathrm{R}}\right) \left[f_2(k_2) - k_2 f_2'(k_2)\right]. \end{aligned}$$

These two equations replace (9) and (10) for the competitive economy.

The argument to characterize the static equilibrium is the same as before. The marginal rate of transformation determined by y^{R} is now $\frac{\theta g'\left(\theta y^{\mathrm{R}}\right)}{1-g\left(\theta y^{\mathrm{R}}\right)}$ and, for each given MRT, the relative price p^{M} determines the relative supply y^{R}, so that the equilibrium is a fixed point as before[31]

$$H^{\mathrm{M}}(p^{\mathrm{M}}) \equiv \frac{\theta g'\left(\theta y^{\mathrm{R}}\left(p^{\mathrm{M}}\right)\right)}{1 - g\left(\theta y^{\mathrm{R}}\left(p^{\mathrm{M}}\right)\right)} = p^{\mathrm{M}}.$$

[27] By doing that we analyze the two polar cases of market organization. Other market structures are likely to generate conclusions lying somewhere in between the two extreme cases.

[28] As was argued in the Introduction, one can imagine that this monopolist is an Imperial European power, or a military government, or the communist party.

[29] We can think, instead, that there are many firms which are working as a cartel, maximizing jointly their profit. The key hypothesis here is limited entry in the rent-seeking sector.

[30] In order to close the model in general equilibrium we can think that each individual in the society is the owner of an equal share of the rent-seeking firm, such that the profit is redistributed back to the household in a lump-sum fashion.

[31] Clearly, the interpretation in terms of factor mobility is still valid.

Strict concavity of g implies that, for a given level of θy^{R}, $g'\left(\theta y^{\mathrm{R}}\right) < \frac{g\left(\theta y^{\mathrm{R}}\right)}{\theta y^{\mathrm{R}}}$, or that $H^{\mathrm{M}}(p) < H(p)$. Under Axiom 4 it is straightforward to show that $\left.\frac{\partial H^{\mathrm{M}}}{\partial p}\right|_{k} < 0$. Given that $\left.H^{\mathrm{M}}(p^{\mathrm{M}})\right|_{\psi_1=0} < 0$ it follows that $H^{\mathrm{M}}(p^{\mathrm{M}}) - p^{\mathrm{M}} = 0$ lies somewhere in the middle of the stripe connecting $\psi_1 = 0$ and $\underline{p}(k)$ (see Figures 1 and 2, where is it depicted as the curve $\psi_1^{\mathrm{M}} = 0$). Consequently, a fixed point p^{M} of H^{M} must be smaller that the equilibrium price of the competitive model: $p^{\mathrm{M}} < p$. This implies that

$$y_1\left(p^{\mathrm{M}}(\theta, k), k\right) > y_1\left(p(\theta, k), k\right).$$

Hence, for given values of k and θ (that is, in the static part), a monopoly in sector 2 is better for sector 1. Less output gets confiscated by sector 2. In other words, monopoly in the rent-seeking sector is better for the economy than competition in that sector. Competition improves welfare as long as it is employed in productive sectors of an economy. In an unproductive sector, given that competitors do not internalize the reduction in aggregate output due to their action, competition means too much production of transfer services, and consequently too much taken way from the productive sector and too much productive factors allocated in unproductive activities.

The long-run part is as before. The same intertemporal decision leads to a dynamic system like (16) with p^{M} instead of p. Consequently, the long-run equilibrium is described by the crossing of $\psi_2 = 0$ and $\psi_1^{\mathrm{M}} = 0$ in Figures 1 and 2. Given that $\psi_2 = 0$ crosses $\psi_1 = 0$ and intersects $\underline{p}(k)$ at $k_{\rho+\delta}$, existence follows from the same argument as before. Appendix D shows that even taking into consideration the long-run endogenous capital adjustment we still have that monopoly is better:

$$y_1^{\mathrm{M}}(\theta) > y_1(\theta).$$

This result is far from immediate. The monopoly solution also delivers ambiguous results in the long-run comparative statics with respect to θ, so one could expect that the ambiguity would carry on to the comparison between y_1^{M} and y_1. It turns out that this is not the case, and that the short-run result holds in the long-run as well. The relatively smaller monopoly's output is a benefit for the economy as a whole, as overall output (GDP) is larger.

At this point it is possible to analyze the transition from centralized political or economic systems to more open systems.[32] Consider an economy in long-run equilibrium with monopoly in sector 2 (point A at Figure 1 or 2). Following a change of system, the economy jumps to B, where it begins a dynamic path toward C, the long-run equilibrium for competitive rent-seeking and the same θ.[33] The jump from A to B represents an unambiguous decrease in output. Given that

[32] See the discussion in the Introduction.

[33] Notice that A might lie to the right of point C when sector 2 is capital intensive.

output in C is lower than output in A and that along the path from B toward C output is lower than output in A it follows that welfare reduces after the introduction of competitive rent-seeking.

7 Quantitative Implications

7.1 A Calibration Exercise

In this section the model is solved with particular functional forms and observed data is used to compute the implied values for the relevant parameters. The idea is to show how well the model fits the data and also to illustrate the difference between the competitive and monopoly solutions. We assume that the two sectors operate under the same technology (Cobb-Douglas with parameter α) and that the aggregate rent-seeking technology is given by the "Cobb-Douglas-like" form (4). With equal technologies it follows that $y^{\mathrm{R}} = \frac{l_2 f(k)}{l_1 f(k)} \equiv l^{\mathrm{R}}$. The long-run equilibrium condition becomes $\frac{\alpha k^{\alpha-1}}{1+(\theta l^{\mathrm{R}})^{\alpha_g}} = \rho+\delta$. The short-run condition for the competitive economy is given by $\theta^{\alpha_g}\left(l^{\mathrm{R}}\right)^{\alpha_g-1} = 1$, and for the monopoly is given by $\theta\frac{\alpha_g\left(\theta l^{\mathrm{R}}\right)^{\alpha_g-1}}{1+(\theta l^{\mathrm{R}})^{\alpha_g}} = 1$.

At this stage, two parameters are not observable in principle, θ and α_g. Instead of calibrating the first, we use a proxy for it. Given that it is meant to be a measure of the quality of the institutions of an economy, there is an observed variable that measures such a parameter. It is the variable SI (social infrastructure) created by Hall and Jones (1999), which is an index of institutional efficiency ranging from 0 to 1, 1 being the highest degree of institutional efficiency. We assume that

$$\theta = B\frac{1-\mathrm{SI}}{\mathrm{SI}},$$

where B has to be calibrated. In other words the observable counterpart for θ is an increasing mapping on $\mathsf{R}^{[0,1]}$ of the observable SI with an adjustable Jacobian given by B. So now the set $\{B, \alpha_g\}$ of parameters has to be calibrated, and for that two observables are needed.

The first observable comes from Anderson (1999). He reports that the aggregate burden of crime in the US is around 10% of GDP. Under the extreme assumption that rent-seeking in the US is mainly crime (US is indeed one of the most efficient economies in the world, so one might think of that assumption as a normalizing assumption that sets US's institutional inefficiency outside crime to zero) we consider $l^{\mathrm{R,US}} = \frac{0.1}{0.9} = \frac{1}{9}$, or $\frac{l_1^{\mathrm{US}}}{l_1^{\mathrm{NN}}} = \frac{9}{10}$, where the superscript NN stands for 'neoclassical nirvana,' which is the situation with $\theta = 0$. According to the data set of Hall and Jones (1999), $\mathrm{SI}_{\mathrm{US}} = 0.973$. Given that the long-run solution of the competitive model is $\frac{y^{\mathrm{US}}}{y^{\mathrm{NN}}} = \left(1+\theta^{\frac{\alpha_g}{1-\alpha_g}}\right)^{\frac{1}{\alpha-1}}$, and assuming $\alpha = \frac{1}{3}$, we get

$$B = \frac{0.973}{1 - 0.973} \left[(0.9)^{-\frac{2}{3}} - 1 \right]^{\frac{1-\alpha_g}{\alpha_g}} . \tag{25}$$

A second observable is needed to match the curvature parameter α_g. Hall and Jones (1999) estimated the equation $\log y_i = \beta_0 + \beta_1 \mathrm{SI}_i + \epsilon_i$. Their estimate for β_1 is 5.14. We therefore consider α_g as the solution of the following problem:

$$\min_{\alpha_g,\zeta} \int_0^1 \left\{ \zeta + 5.14\mathrm{SI} - \log \left[1 + \left(B \frac{1 - \mathrm{SI}}{\mathrm{SI}} \right)^{\frac{\alpha_g}{1-\alpha_g}} \right]^{\frac{1}{\alpha - 1}} \right\}^2 \mathrm{d}\,(\mathrm{SI}) \,,$$

where B is given by (25).[34] That is, we consider α_g that minimizes the distance between $\zeta + 5.14\mathrm{SI}$ and the logarithm of the income when θ is given by $B\frac{1-\mathrm{SI}}{\mathrm{SI}}$. The solution is $\alpha_g = 0.506$, which is well inside the stability region of $\alpha_g < 1 - \alpha = \frac{2}{3}$.

Figure 3 below is the scatter diagram of the Hall and Jones data set for SI and per capita income,[35] together with the long-run solution of the model with $\alpha_g = 0.506$ and $B = 1.391$. It is apparent that the pattern of the data is reproduced by the model. There is a negative relation between institutional inefficiency and output as expected. The fact that the fit is a good one shows that the model can potentially explain the data.

Figure 4 displays the prediction of the model under the two configurations: competitive rent-seeking and monopoly. It is apparent that the competitive formulation generates much more rent-seeking for each level of θ (or SI). This is in line with the interpretation that the monopoly solution is better for the rest of the economy.[36]

[34]The constant ζ is an unimportant level parameter. Notice that we could have calibrated α_g directly from the data but we decided to use Hall and Jones's regression because they controlled for endogeneity of SI.

[35]The values of per capita income are net of mining activities as a way of controlling for natural resources and refers to the year of 1988. Further data information can be found in their paper.

[36]In Figure 5 we assumed that the monopoly economy is as good as the competitive rent-seeking economy in producing the good (that is, we assumed that the TFP of the first sector is the same in both models). Although this is not true for either a centralized economy or a colony under European rule or a Latin American military dictatorship, we assumed it in order to isolate the differences in economic performance caused only by the form of market organization in the rent-seeking sector.

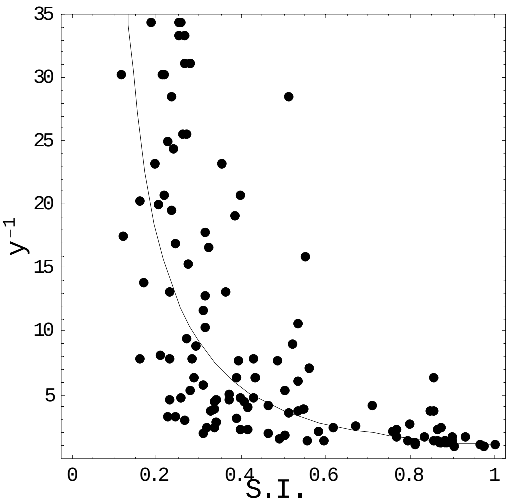

Figure 3

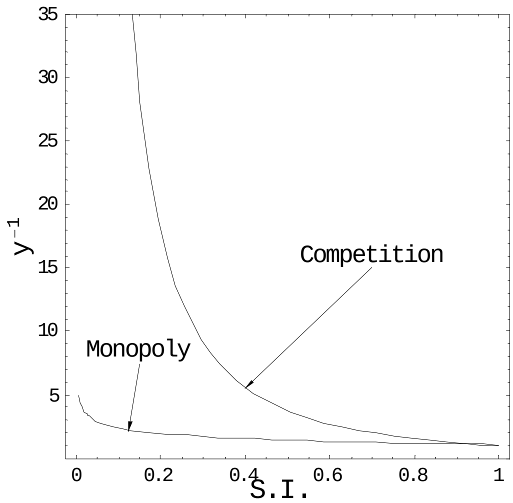

Figure 4

7.2 Explaining Income Differences Using Incentives

The aim of this section is to show that the model presented here is capable of explaining the observed income inequality among economies. The driving force is, of course, the rent-seeking formulation, that incorporates in the same framework a distortion to capital accumulation and a displacement of productive factors away from the productive sector (which emulates a reduction in the TFP). That is, the Harberger and Tullock effects. The relevance of such capability stems from the fact that the standard neoclassical model is not able to explain the observed income inequality unless one makes some questionable assumptions. In particular, usually the capital share is assumed to be in excess of $\frac{2}{3}$ (although it is a well established fact that such a share is close to $\frac{1}{3}$). This model[37] will be taken as the benchmark. It is characterized by $y_1 = f_1(k)$ and $y_2 = 0$, which mean that there is a link between capital and output:

$$\frac{\mathrm{d}y_1}{y_1} = \alpha_{1\mathrm{K}} \frac{\mathrm{d}k}{k}. \tag{26}$$

In a long-run situation such that, after allowing for risk, taxation, and other distortions, one has equalization of interest rates among economies, it is the case that $r = Tf_1'(k) = \text{constant}$, where T stands for any distortion to capital accumulation as before. Consequently,

$$\frac{\mathrm{d}r}{r} = \frac{\mathrm{d}T}{T} - \frac{1-\alpha_{1\mathrm{K}}}{\sigma_1}\frac{\mathrm{d}k}{k} = 0. \tag{27}$$

[37] See Mankiw (1995).

Substituting (27) into (26), and assuming a Cobb-Douglas production function, yields $\frac{\mathrm{d}y_1}{y_1} = \frac{\alpha_{1K}}{1-\alpha_{1K}}\frac{\mathrm{d}T}{T}$, which means that

$$y_1 \sim T^{\frac{\alpha_{1K}}{1-\alpha_{1K}}} = \begin{cases} T^{\frac{1}{2}} \text{ if } \alpha_{1K} = \frac{1}{3} \\ T^2 \text{ if } \alpha_{1K} = \frac{2}{3}. \end{cases}$$

In order to make the neoclassical model capable of explaining the observed income inequality using only incentives one has to assume a capital share of $\frac{2}{3}$, which is definitely not in line with the empirical observation. For a reasonable value for capital share, $\frac{1}{3}$, a difference on incentives to capital accumulation of two orders of magnitude produces an income inequality of one order of magnitude. This result can be improved when education is considered. To do that, let us use a Mincerian specification, $y_1 = f_1(k, e^{\phi S}) = e^{\phi S} f_1\left(\frac{k}{e^{\phi S}}\right)$ and $r = T f_1'\left(\frac{k}{e^{\phi S}}\right) = \text{constant}$, where S is the average years of education of the active population and ϕ is the average return of education (around 7% a year). Repeating the computations above yields

$$y_1 \sim e^{\phi S} T^{\frac{\alpha_{1K}}{1-\alpha_{1K}}} \sim 2T^{\frac{1}{2}},$$

when one considers the difference of education between a rich economy and a poor economy to be around 10 years. The observed income differences are now accounted for by only $2T^{\frac{1}{2}}$, which is better than the $T^{\frac{1}{2}}$ found above, but still not as good as T^2.

For the rent-seeking model, assuming the same technology for both sectors, one has $y_1 = l_1 f(k, e^{\phi S}) = l_1 e^{\phi S} f\left(\frac{k}{e^{\phi S}}\right)$ which implies

$$\frac{\mathrm{d}y_1}{y_1} = \frac{\mathrm{d}l_1}{l_1} + \alpha_{1K}\frac{\mathrm{d}k}{k} + (1-\alpha_{1K})\frac{\mathrm{d}e^{\phi S}}{e^{\phi S}}.$$

What is new is that the reduction on productivity brought about by the rent-seeking activity, $l_1 < 1$, is also linked to the intertemporal distortion by the Harberger effect. Here l_1 plays the role of T.[38] For the interest rate one has

$$\frac{\mathrm{d}r}{r} = \frac{\mathrm{d}l_1}{l_1} - \frac{1-\alpha_{1K}}{\sigma_1}\left(\frac{\mathrm{d}k}{k} - \frac{\mathrm{d}e^{\phi S}}{e^{\phi S}}\right) = 0.$$

[38] We make the extreme assumption that every source of distortion is at the same time a source of diversion of production factors away from the productive activity. Evidently, this is a limit case. It shows the potentiality of the model and, on the other hand, allows us to perform the quantitative analysis without specifying a functional form for g.

Substituting this last equation for $\frac{\mathrm{d}k}{k}$ on $\frac{\mathrm{d}y_1}{y_1}$, and recalling that $l_1 = T$, yields

$$\frac{\mathrm{d}y_1}{y_1} = \frac{\mathrm{d}T}{T} + \sigma_1 \frac{\alpha_{1\mathrm{K}}}{1-\alpha_{1\mathrm{K}}} \frac{\mathrm{d}T}{T} + \frac{\mathrm{d}e^{\phi S}}{e^{\phi S}},$$

which means that

$$y_1 \sim e^{\phi S} T^{\frac{1}{1-\alpha_{1\mathrm{K}}}} \sim 2T^{\frac{3}{2}},$$

which is much closer to T^2. In fact, Figure 5 shows that the neoclassical model with a large capital share ($\alpha_{1\mathrm{K}} = \frac{2}{3}$) and the rent-seeking model with education and a reasonable capital share ($\alpha_{1\mathrm{K}} = \frac{1}{3}$) generate similar results for the impact of distortions on the long-run income. The distortion T is shown on the x-axis and the reciprocal of income is shown on the y-axis.

One way of understanding the result above is as follows: without rent-seeking, $y_1 \sim 2T^{\frac{1}{2}}$, and with it, $y_1 \sim 2T^{\frac{3}{2}} = (2T^{\frac{1}{2}})T$, so rent-seeking enters as a linear factor in the computation. Given that rent-seeking can be viewed as a reduction in TFP (see section 9.1), which is one-to-one when the technologies are the same in both sectors, this reduction is expressed as the linear factor above.

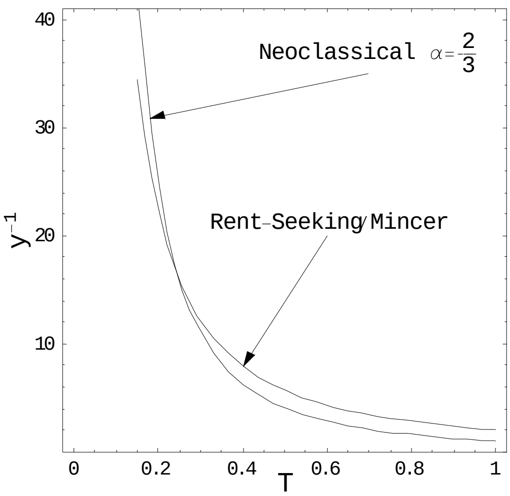

Figure 5

8 Relation to Other Works

Murphy, Shleifer and Vishny (1993) and Acemoglu (1995) build simple static models of factor allocation in presence of competitive rent-seeking. Their models produce multiple equilibria which should be contrasted with our uniqueness result. Our assumptions on the aggregate rent-seeking technology guarantee that an inflow of productive factors into the rent-seeking activity reduces this activity's rentability more than the reduction on the rentability of the productive activity. As we argued above, this choice reflects our belief on the relative importance of fundamental (including the institutional set) vis-a-vis initial conditions/coordination failures in understanding issues of economic development.[39]

Tullock (1980), Skaperdas (1992), Hirshleifer (1995), and Grossman and Kim (1995) present models in which a fixed number of individual or bands or groups (usually 2) fight for a slice of a pie, which in some cases is endogenously determined by the production decision of the contenders.[40] The lack of free entry in the rent-seeking activity means that in equilibrium rents are not fully dissipated. As it was the case with Murphy, Shleifer and Vishny (1993) and Acemoglu (1995), these contributions assume one productive factor, usually a linear production function, and do not consider capital accumulation.

Krueger (1974) and Bhagwati and Srinivasan (1980) present models of rent-seeking in a international trade economic environment (some form of the H.O.V. model). In this literature many of the results rest on the specific interaction among the unproductive activity and other distortions related to international trade. In particular, the unproductive activity might be welfare improving, which is not the case in our setup.

Tornell and Velasco (1992), Benhabib and Rustichini (1996), and Grossman and Kim (1996) present models in which capital accumulation takes place in a dynamic game framework. A given number of agents (two or more) face the strategic choice of how much to appropriate of what is produced, which might generate less incentives to production and capital accumulation. The explicit game-theoretic formulation used in those papers is to be contrasted with our assumption of perfect competition, where a large number of rent-seeking firms compete for the appropriation of sector 1's output. The strategic considerations are summarized by the aggregate rent-seeking technology, the Contest Success Function, and by the free entry condition (see section 2). As a result, our formulation is simpler, and, since those models use a single factor technology with constant returns to scale (basically a variant of the AK model), it seems to us that our model is the first attempt to incorporate rent-seeking in the neoclassical model of capital accumulation. Moreover, the first two models do not take into consideration the resources employed in the appropriation of sector 1's ouput (the Tullock effect), which turns out to be the most important effect of the model (see section 5).

The original contribution of Tullock (1967) is taken as the background of our formalization.

[39] Evidently, we are not ruling out that initial conditions and/or coordination problems might explain the choice of a bad institutional set.

[40] Grossman and Kim (1995) considered that in addition to the two usual activities in this literature - production and predation - there is another activity, protection.

Eric Jones (1988) provided a very illuminating account of the world economic history in terms of a struggle between rent-seeking and productive activities. North (1990, 1994), Baumol (1990), Olson (1992), Murphy, Shleifer and Vishny (1993) among others were also instrumental in shaping the hypothesis that institutions form the fundamental structure of incentives that eventually drives all results in a market economy. One of the main purposes of setting up the present model was to provide a formalization of these ideas under a general and standard macroeconomic framework.

9 Concluding Remarks

A simple model introducing rent-seeking in the standard neoclassical model of capital accumulation was presented. The introduction was made through the use of an aggregate rent-seeking technology which determines the success of the rent-seeking activities. It resembles a tax on the productive sector, which is a standard way of introducing institutions in macroeconomic models. The model can be considered as a formalization of the ideas presented by the new institutional literature[41] in conjunction with the rent-seeking literature.

In addition to the results and applications considered above, we point out that the model can be applied to a variety of interesting economic questions. Here we consider briefly two of them.

9.1 Endogenous TFP

As mentioned before, the share of productive factors allocated in sector 1 can be viewed as an endogenous part of the TFP. The output of an economy increases (decreases) as productive factors move from sector 2 (1) to sector 1 (2), and this happens for a given level of productive factors. So this is not accounted as a change in output due to a change in productive factors, but due to a change in the productivity of the existing factors, i.e., a change in TFP. More formally, consider first the case with $k_1 = k_2$. Then $y_1 = l_1 f(k) = (1 - l_2) f(k)$, and the term $1 - l_2$ can be viewed as (part of) the TFP. The bigger the rent-seeking sector, the less productive the economy. That is, for a given k, an increase in l_2 represents a decrease in TFP, since less output is produced by the same level of productive factors, k. The same intuition is also valid for generic values of k_1 and k_2.

In other words, let us assume that our model describes well two economies that are identical in every aspect but differ in the parameter θ. Then an observer looking at these economies from the viewpoint of an one-sector aggregative model would conclude that the economy with the smaller θ is the economy with higher TPF, although both economies operate under the same technology

[41] See Rutherford (1996) for an account of institutionalism in economics.

by assumption. In this sense, TFP (or part of it) is endogenously determined by the institutional efficiency.

Moreover, there is also a dynamic issue in this endogenous TFP. An once and for all change in institutional efficiency is given by a discrete jump in θ.[42] This generates an immediate reallocation of factors between the sectors, so an immediate change in TFP. But the economy enters in a transitory dynamic path towards its new steady state, and along this path further reallocations of factors take place. That is, along this path the share of the labor force allocated in the productive sector keeps changing and this is observationally equivalent to a continuous change of the TFP if an one-sector economy perspective is considered. Such dynamic behavior is to be contrasted with the usual exercises in the literature of considering once and for all changes in TFP itself: the resulting transitory dynamics of capital accumulation does not include an associated transitory dynamics of TFP. Also, there is some evidence that TFP is indeed not constant. A simple look at the Summers and Heston data set reveals that several countries, like Venezuela, endured a process of reduction in TFP for the period of 1960 to 1990. Other countries, like Japan, endured the opposite process. The model provides an immediate rationale for such facts, as one can easily argue that Venezuela and Japan witnessed changes in institutional efficiency prior to (or during) the period in question. The issue of endogeneity of TFP is of great interest (see Prescott (1998)) and ought to be studied further.

9.2 Foreign Aid and Rent-Seeking

Consider now the issue of foreign aid in the presence of rent-seeking. There is a concern that aid, if the recipient economy does not have a good institutional set, is a waste of resources: it ends up as consumption, without any effect on the productive capacity of the economy (Burnside and Dollar (2000)). The model shows that things might be even worse when rent-seeking is considered: the aid generates an increase in rent-seeking activities, so that the society would be better off if the aid was given directly as consumption goods to the households. To see this consider the aid as a permanent flow, A, of resources per capita, so that per capita output becomes $y_1 + A$. The short-run equilibrium condition (11) is still valid, now with $y^{\mathrm{R}} = \frac{y_2}{y_1+A}$. Consequently, in the short-run an increase in A increases the relative price according to:

$$\left.\frac{A}{p}\frac{\partial p}{\partial A}\right|_k = -\frac{(1-\alpha_g)\left.\frac{A}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial A}\right|_p}{1+(1-\alpha_g)\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_A} > 0,$$

[42] Rodrik (1999) suggested one possible mechanism that can produce such change.

meaning that factors move into the rent-seeking sector. Given the increase in the "pie", there are more resources to be stolen, and hence the transfer efforts increase. In the long-run an increase in aid leads to an increase in y_2 and a decrease in y_1. Hence there is an unambiguous increase in rent-seeking activities generated by the foreign aid (that's Tornell and Velasco (1992)'s 'voracity effect').

Moreover, following the same steps of section 5 it is possible to calculate the impact of aid on welfare, and the analogous of equation (23) is

$$\rho C_A(\rho) = 1 + \left.\frac{\mathrm{d}y_1}{\mathrm{d}A}\right|_{k,*} + \mathrm{AF}\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \rho - \delta\right)\left.\frac{\mathrm{d}k}{\mathrm{d}A}\right|_{*}.$$

It has now three terms: (i) the direct effect of the aid on income, (ii) the direct effect of aid on output of the productive sector (Tullock), and (iii) the indirect effect of aid on output through capital accumulation (Harberger). It is possible to show that the last two are negative,[43] so the direct effect of the aid is reduced by the presence of rent-seeking. The households would be better off if the aid was given directly to them.

Notice also that the effect generated by an increase in A above could be interpreted as a permanent improvement in the terms of trade or the discovery of natural resources (or the valorization of existing reserves). Hence, episodes like the Dutch Disease can also be explained by the model.

A The Static Model: Existence and Comparative Statics Properties

A.1 The Two Sector Model of Production

The following equations describe the 2×2 static model:

$$y_i = \frac{L_i}{L} f_i\left(\frac{K_i}{L_i}\right) = l_i f_i\left(k_i\right) \quad i = 1, 2, \tag{28}$$

$$l_1 + l_2 = 1, \tag{29}$$

$$k_1 l_1 + k_2 l_2 = k, \tag{30}$$

$$\mathrm{w} = p_i\left(f_i - k_i f_i'\right), \tag{31}$$

$$r = p_i f_i', \tag{32}$$

[43] Due to linearity of aid there is no reverse Haberger effect.

where

$$f_i(0) = 0,\ f_i'(k_i) > 0,\ \lim_{k_i \to 0} f_i'(k_i) = \infty,\ \lim_{k_i \to \infty} f_i'(k_i) = \infty,\ f_i''(k_i) < 0.$$

From (31) and (32) we get

$$\omega \equiv \frac{\mathrm{w}}{r} = \frac{f_i}{f_i'} - k_i, \tag{33}$$

or

$$k_i = k_i(\omega), \ \frac{\mathrm{d}k_i}{\mathrm{d}\omega} = -\frac{(f_i')^2}{f_i f_i''} > 0. \tag{34}$$

The relative price is the ratio of average costs:

$$p \equiv \frac{p_2}{p_1} = \frac{\frac{\mathrm{w}L_2 + rK_2}{L_2 f_2}}{\frac{\mathrm{w}L_1 + rK_1}{L_1 f_1}} = \frac{\omega + k_2}{\omega + k_1}\frac{f_1}{f_2},$$

which is solved as

$$\omega = \omega(p), \ \frac{p}{\omega}\frac{\mathrm{d}\omega}{\mathrm{d}p} = \frac{(\omega + k_1)(\omega + k_2)}{\omega(k_1 - k_2)} \gtrless 0 \text{ as } k_1 \gtrless k_2. \tag{35}$$

Solving (29) and (30) for l_i, after substituting into (28) we get the supply functions:

$$y_1(p,k) = l_1 f_1(k_1) = \frac{k_2(\omega(p)) - k}{k_2(\omega(p)) - k_1(\omega(p))} f_1(k_1(\omega(p))), \tag{36}$$

$$y_2(p,k) = l_2 f_2(k_2) = \frac{k - k_1(\omega(p))}{k_2(\omega(p)) - k_1(\omega(p))} f_2(k_2(\omega(p))). \tag{37}$$

Usually we will write simply $k_i(\omega(p)) = k_i(p)$.

Finally, for a given factor endowment, there is a price $\overline{p}(k)$ such that the economy is specialized in the production of the rent-seeking service if $p \geq \overline{p}(k)$, and there is a price $\underline{p}(k)$ such that the economy is specialized in the production of the first sector good if $p \leq \underline{p}(k)$. Note that $\overline{p}'(k) \gtrless 0$ and $\underline{p}'(k) \gtrless 0$ as $k_1 \gtrless k_2$.[44]

[44] See Kemp (1969), chapter 1.

A.2 Comparative Statics

The following notation is employed from now on:

$$\alpha_{i\mathrm{K}} \equiv 1-\alpha_{i\mathrm{L}} = k_i \frac{f_i'}{f_i}, \tag{38}$$

$$\sigma_i \equiv \frac{\omega}{k_i}\frac{\mathrm{d}k_i}{\mathrm{d}\omega} = \frac{\alpha_{i\mathrm{L}}}{\alpha_{i\mathrm{K}}}\frac{\mathrm{d}k_i}{\mathrm{d}\omega} = -\frac{\alpha_{i\mathrm{L}}}{\alpha_{i\mathrm{K}}}\frac{(f_i')^2}{f_i f_i''}. \tag{39}$$

An important consequence of (33) is that

$$\frac{k_1}{k_2} = \frac{1-\frac{f_1}{k_1 f_1'}}{1-\frac{f_2}{k_2 f_2'}} = \frac{\alpha_{2\mathrm{L}}}{\alpha_{1\mathrm{L}}}\frac{\alpha_{1\mathrm{K}}}{\alpha_{2\mathrm{K}}} = \frac{\alpha_{1\mathrm{K}}}{1-\alpha_{1\mathrm{K}}}\frac{1-\alpha_{2\mathrm{K}}}{\alpha_{2\mathrm{K}}} \gtrless 1 \text{ whether } \alpha_{1\mathrm{K}} \gtrless \alpha_{2\mathrm{K}}. \tag{40}$$

The comparative statics for (36) and (37) are:

$$\left.\frac{k}{y_1}\frac{\partial y_1}{\partial k}\right|_p = \frac{k}{k-k_2} = \frac{1}{l_1}\frac{k}{k_1-k_2}, \tag{41}$$

$$\left.\frac{k}{y_2}\frac{\partial y_2}{\partial k}\right|_p = \frac{k}{k-k_1} = -\frac{1}{l_2}\frac{k}{k_1-k_2} = -\frac{1}{l^{\mathrm{R}}}\left.\frac{k}{y_1}\frac{\partial y_1}{\partial k}\right|_p, \tag{42}$$

and

$$\left.\frac{p}{y_1}\frac{\partial y_1}{\partial p}\right|_k = -\frac{\alpha_{1\mathrm{L}}}{(\alpha_{1\mathrm{K}}-\alpha_{2\mathrm{K}})^2}\left(\sigma_2\alpha_{2\mathrm{K}} l^{\mathrm{R}} + \sigma_1\alpha_{1\mathrm{K}}\right), \tag{43}$$

$$\left.\frac{p}{y_2}\frac{\partial y_2}{\partial p}\right|_k = \frac{\alpha_{2\mathrm{L}}}{(\alpha_{1\mathrm{K}}-\alpha_{2\mathrm{K}})^2}\left(\sigma_1\alpha_{1\mathrm{K}}\frac{1}{l^{\mathrm{R}}} + \sigma_2\alpha_{2\mathrm{K}}\right) = -\frac{1}{l^{\mathrm{R}}}\frac{\alpha_{2\mathrm{L}}}{\alpha_{1\mathrm{L}}}\left.\frac{p}{y_1}\frac{\partial y_1}{\partial p}\right|_k, \tag{44}$$

where $l^{\mathrm{R}} \equiv \frac{l_2}{l_1}$. In deriving this last two equations we employed (30), (33)-(35), (38)-(40). Two important results that follow from (41) and (42) are:

$$\left.\frac{1-g}{r}\frac{\partial y_1}{\partial k}\right|_p = \frac{\alpha_{2\mathrm{L}}}{\alpha_{1\mathrm{K}}-\alpha_{2\mathrm{K}}}, \tag{45}$$

$$\left.\frac{1-g}{r}\frac{\partial y_2}{\partial k}\right|_p = -\frac{y^{\mathrm{R}}}{l^{\mathrm{R}}}\frac{\alpha_{2\mathrm{L}}}{\alpha_{1\mathrm{K}}-\alpha_{2\mathrm{K}}} = -\frac{1}{p}\frac{\alpha_{1\mathrm{L}}}{\alpha_{1\mathrm{K}}-\alpha_{2\mathrm{K}}}, \tag{46}$$

where in the last equation we substitute (11) and (20).

There are two results from the two-sectors general equilibrium model of production that we can use:

$$\left.\frac{\partial y_1}{\partial p}\right|_k + p \left.\frac{\partial y_2}{\partial p}\right|_k = 0, \tag{47}$$

$$\left.\frac{\partial}{\partial k}\left(y_1 + p y_2\right)\right|_p = \left.\frac{\partial y_1}{\partial k}\right|_p + p \left.\frac{\partial y_2}{\partial k}\right|_p = \frac{r}{1-g}. \tag{48}$$

(The reader can try to show directly these two results. (47) follows from (43) and (44) recalling the short-run equilibrium condition. (48) follows from (45) and (46).)

B Dynamics and Long-Run Equilibrium

B.1 Stability

The saddle point stability for the dynamic system (16) requires

$$\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_\theta = \left.\frac{\mathrm{d}}{\mathrm{d}k}\left(1 - g\left(\theta y^{\mathrm{R}}\left(p,k\right)\right)\right) f_1'\left(k_1\left(p\right)\right)\right|_\theta < 0.$$

But

$$\left.\frac{k}{y^{\mathrm{R}}}\frac{\mathrm{d}y^{\mathrm{R}}}{\mathrm{d}k}\right|_\theta = \left.\frac{k}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p + \left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k \left.\frac{k}{p}\frac{\partial p}{\partial k}\right|_\theta = -\frac{1}{1-\alpha_g}\left.\frac{k}{p}\frac{\partial p}{\partial k}\right|_\theta, \tag{49}$$

where in the last equality (19) was used. Consequently, using the definition of α_g, (38) and (39), we get

$$\left.\frac{k}{r}\frac{\mathrm{d}r}{\mathrm{d}k}\right|_\theta = \left(g\frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1\mathrm{L}}}{\sigma_1}\frac{p}{k_1}\frac{\mathrm{d}k_1}{\mathrm{d}p}\right)\left.\frac{k}{p}\frac{\partial p}{\partial k}\right|_\theta.$$

After substituting (33)-(35) we get

$$\left.\frac{k}{r}\frac{\mathrm{d}r}{\mathrm{d}k}\right|_\theta = \left(g\frac{\alpha_g}{1-\alpha_g} - \frac{1}{\alpha_{2\mathrm{K}}}\frac{k_2}{k_1-k_2}\right)\left.\frac{k}{p}\frac{\partial p}{\partial k}\right|_\theta = \left(g\frac{\alpha_g}{1-\alpha_g} - \frac{\alpha_{1\mathrm{L}}}{1-\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{L}}}\right)\left.\frac{k}{p}\frac{\partial p}{\partial k}\right|_\theta, \tag{50}$$

where in this last equality we used (40). A sufficient condition for $\left.\frac{k}{r}\frac{\mathrm{d}r}{\mathrm{d}k}\right|_\theta < 0$ when $\alpha_{1\mathrm{K}} > \alpha_{2\mathrm{K}}$ is $\alpha_{1\mathrm{L}} > \alpha_g$, because

$$\frac{\alpha_{1\mathrm{L}}}{1-\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{L}}} \geq \frac{\alpha_{1\mathrm{L}}}{1-\alpha_{1\mathrm{L}}} \geq \frac{\alpha_g}{1-\alpha_g}.$$

B.2 Features of the Dynamics

B.2.1 Behavior of y_1 and y_2

After substituting (19) we get

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta = \left.\frac{\partial y_1}{\partial k}\right|_p + \left.\frac{\partial y_1}{\partial p}\right|_k \left.\frac{\partial p}{\partial k}\right|_\theta = \left.\frac{\partial y_1}{\partial k}\right|_p - \left.\frac{\partial y_1}{\partial p}\right|_k \frac{p}{k} \frac{(1-\alpha_g)\, \frac{k}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p}{1+(1-\alpha_g)\, \frac{p}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k}$$

$$= \frac{1}{\Delta}\left\{ \left.\frac{\partial y_1}{\partial k}\right|_p \left[1+(1-\alpha_g)\, \frac{p}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k\right] - (1-\alpha_g) \left.\frac{\partial y_1}{\partial p}\right|_k \frac{p}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p \right\},$$

where

$$\Delta \equiv 1+(1-\alpha_g)\, \frac{p}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k .$$

After employing (47) and (48) we get:

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta = r \frac{\frac{1}{r} \left.\frac{\partial y_1}{\partial k}\right|_p + \frac{1-\alpha_g}{1-g}\, \frac{p}{y_2} \left.\frac{\partial y_2}{\partial p}\right|_k}{1+(1-\alpha_g)\, \frac{p}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k} > 0 \tag{51}$$

if $k_1 \geq k_2$.

Analogously it is possible to show that

$$\left.\frac{\mathrm{d}y_2}{\mathrm{d}k}\right|_\theta = r \frac{\frac{1}{r} \left.\frac{\partial y_2}{\partial k}\right|_p - \frac{1-\alpha_g}{1-g}\, \frac{1}{y_1} \left.\frac{\partial y_1}{\partial p}\right|_k}{1+(1-\alpha_g)\, \frac{p}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k} > 0 \tag{52}$$

if $k_2 \geq k_1$.

Finally, it follows from (51) that

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta \gtrless 0 \text{ as } \frac{1}{r} \left.\frac{\partial y_1}{\partial k}\right|_p + (1-\alpha_g)\, \frac{p}{y_2} \left.\frac{\partial y_2}{\partial p}\right|_k \frac{1}{1-g} \gtrless 0.$$

After substituting (41) and (44) in this last expression we get

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta \gtrless 0 \text{ as } -1 + \frac{1-\alpha_g}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} \left(\sigma_1 \alpha_{1\mathrm{K}} \frac{1}{l^{\mathrm{R}}} + \sigma_2 \alpha_{2\mathrm{K}}\right) \gtrless 0.$$

B.2.2 Social Value of Capital

It follows from the short-run equilibrium condition, expression (14) and from (47) that

$$(1-g)\, \frac{p}{y_1} \left.\frac{\partial y_1}{\partial p}\right|_k + g\, \frac{p}{y_2} \left.\frac{\partial y_2}{\partial p}\right|_k = 0.$$

With the help of this last equation and (51) one can show that

$$\frac{k}{y_1}\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta}-r\right)=-\frac{g}{\Delta}\left.\frac{k}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_{p}\gtreqless 0 \text{ as } k_1\gtreqless k_2.$$

B.2.3 Behavior of $y^{\mathbf{R}}$

Note that a consequence of these last two results is that, after employing (47)

$$\begin{aligned}\left.\frac{k}{y^{\mathrm{R}}}\frac{\mathrm{d}y^{\mathrm{R}}}{\mathrm{d}k}\right|_{\theta}&=\left.\frac{k}{y_2}\frac{\mathrm{d}y_2}{\mathrm{d}k}\right|_{\theta}-\left.\frac{k}{y_1}\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta}=\frac{\left.\frac{k}{y_2}\frac{\partial y_2}{\partial k}\right|_p-\left.\frac{k}{y_1}\frac{\partial y_1}{\partial k}\right|_p}{1+(1-\alpha_g)\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k}\\&=\frac{1}{\Delta}\left.\frac{k}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_{p}\lesseqgtr 0 \text{ as } k_1\gtreqless k_2.\end{aligned}$$

B.3 Long-Run Capital Stock

Given that in the long-run the interest rate is fixed at $\rho+\delta$, the effect of θ on k is given by $\frac{\theta}{k}\frac{\mathrm{d}k}{\mathrm{d}\theta}=-\frac{\left.\frac{\theta}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_k}{\left.\frac{k}{r}\frac{\mathrm{d}r}{\mathrm{d}k}\right|_\theta}$. It follows from the stability, condition (17), that

$$\frac{\mathrm{d}k}{\mathrm{d}\theta}\gtreqless 0 \text{ as } \left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_k\gtreqless 0.$$

Following the same steps taken in Appendix B.1, recalling that (20) implies that

$$g\alpha_g\left(1+\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k\left.\frac{\theta}{p}\frac{\partial p}{\partial\theta}\right|_k\right)=g\left(1+\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k\right)\left.\frac{\theta}{p}\frac{\partial p}{\partial\theta}\right|_k,$$

we get

$$\left.\frac{\theta}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_k=-\left\{g\left(1+\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k\right)+\frac{1}{\alpha_{2\mathrm{K}}}\frac{k_2}{k_1-k_2}\right\}\left.\frac{\theta}{p}\frac{\partial p}{\partial\theta}\right|_k>0$$

if $k_1\geq k_2$.

From (43) and (44), after substituting (20), we get

$$-g\left.\frac{p}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k=\left.\frac{p}{y_1}\frac{\partial y_1}{\partial p}\right|_k, \tag{53}$$

which, substituting in the previous equation, after some manipulations, follows

$$\left.\frac{\theta}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_k=\left(-g+\left.\frac{p}{y_1}\frac{\partial y_1}{\partial p}\right|_k+\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\right)\left.\frac{\theta}{p}\frac{\partial p}{\partial\theta}\right|_k. \tag{54}$$

Substituting (43) and (20) we get:

$$\frac{\theta}{k}\frac{\mathrm{d}k}{\mathrm{d}\theta} \gtrless 0 \text{ as } -g + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} - \frac{\alpha_{1\mathrm{L}}}{(\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}})^2}\left(\sigma_1\alpha_{1\mathrm{K}} + \sigma_2\alpha_{2\mathrm{K}} l^{\mathrm{R}}\right) \gtrless 0. \tag{55}$$

C Welfare

C.1 Computation of $C_\theta(\rho)$

Differentiating (16) yields

$$\begin{cases} \frac{\mathrm{d}\dot{k}}{\mathrm{d}\theta} = \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)\frac{\mathrm{d}k}{\mathrm{d}\theta} - \frac{\mathrm{d}c}{\mathrm{d}\theta} + \left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*} \\ \frac{\mathrm{d}\dot{c}}{\mathrm{d}\theta} = c^*\gamma(c^*)\left(\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}\frac{\mathrm{d}k}{\mathrm{d}\theta} + \left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*}\right). \end{cases}$$

Given that

$$\int_0^\infty e^{-\vartheta t}\frac{\mathrm{d}\dot{x}(t)}{\mathrm{d}\theta}\mathrm{d}t = \int_0^\infty e^{-\vartheta t}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}x(t)}{\mathrm{d}\theta}\mathrm{d}t = -\Delta x(0) + \vartheta X_\theta(\vartheta),$$

where $\Delta x(0)$ is the jump in the variable x right after the change in[45] θ, we can write the system above as

$$\begin{cases} \vartheta K_\theta(\vartheta) = \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)K_\theta(\vartheta) - C_\theta(\vartheta) + \frac{1}{\vartheta}\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*} \\ \vartheta C_\theta(\vartheta) = c^*\gamma(c^*)\left(\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}K_\theta(\vartheta) + \frac{1}{\vartheta}\left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*}\right) + \Delta c(0). \end{cases} \tag{56}$$

Where in the first line $\Delta k(0) = 0$ was used (capital is the state variable). Rearranging,

$$\begin{bmatrix} \vartheta - \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right) & 1 \\ -c^*\gamma(c^*)\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*} & \vartheta \end{bmatrix}\begin{bmatrix} K_\theta(\vartheta) \\ C_\theta(\vartheta) \end{bmatrix} = \begin{bmatrix} \frac{1}{\vartheta}\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*} \\ c^*\gamma(c^*)\frac{1}{\vartheta}\left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*} + \Delta c(0) \end{bmatrix},$$

whose solutions are

$$C_\theta(\vartheta) = \frac{c^*\gamma(c^*)\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}\frac{1}{\vartheta}\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*} + \left[\vartheta - \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)\right]\left[\frac{c^*\gamma(c^*)}{\vartheta}\left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*} + \Delta c(0)\right]}{\vartheta\left[\vartheta - \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)\right] + c^*\gamma(c^*)\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}$$

$$K_\theta(\vartheta) = \frac{\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*} - \Delta c(0) - \frac{c^*\gamma(c^*)}{\vartheta}\left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*}}{\vartheta\left[\vartheta - \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)\right] + c^*\gamma(c^*)\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}.$$

[45] That is, $\Delta x(0) = \lim_{t\to 0^+}\frac{\mathrm{d}x(t)}{\mathrm{d}\theta}$.

The initial jump in consumption $\Delta c(0)$ still has to be computed. From the second equation in (56)

$$\Delta c(0) = -c^*\gamma(c^*) \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*} K_\theta(\mu) + \mu C_\theta(\mu) - c^*\gamma(c^*) \left.\frac{1}{\mu}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*},$$

where μ is the positive eigenvalue of the linearized matrix. It must satisfy the characteristic equation:

$$\mu\left[\mu - \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)\right] = -c^*\gamma(c^*) \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}. \tag{57}$$

Hence, from the first equation in (56)

$$\begin{aligned}\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*} &= \mu\left[\mu - \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)\right] K_\theta(\mu) + \mu C_\theta(\mu) \\ &= -c^*\gamma(c^*) \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*} K_\theta(\mu) + \mu C_\theta(\mu)\end{aligned}$$

so that

$$\Delta c(0) = \left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*} - c^*\gamma(c^*) \left.\frac{1}{\mu}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*}.$$

Substituting for $\Delta c(0)$ into the expression for $C_\theta(\vartheta)$, and rearranging terms yields

$$\rho C_\theta(\rho) = \underbrace{\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_{k,*}}_{\text{Tullock Effect}} + \underbrace{\frac{\mu-\rho}{\mu}\frac{\frac{c^*\gamma(c^*)}{\rho}\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}{\rho+\delta-\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}+\frac{c^*\gamma(c^*)}{\rho}\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \rho - \delta\right)\left.\frac{\mathrm{d}k}{\mathrm{d}\theta}\right|_{*}}_{\text{Haberger Effect}},$$

where $\left.\frac{\mathrm{d}k}{\mathrm{d}\theta}\right|_{*} = -\frac{\left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_{k,*}}{\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}$ was used.

C.2 The Attenuation Factor

In this Appendix we show that the Attenuation Factor satisfies $0 \leq \mathrm{AF} \leq 1$. Recalling that

$$\mathrm{AF} = \frac{\mu-\rho}{\mu} \frac{\frac{c^*\gamma(c^*)}{\rho} \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}{\rho+\delta-\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}+\frac{c^*\gamma(c^*)}{\rho} \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}$$

$$\frac{\frac{c^*\gamma(c^*)}{\rho} \left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}{\mu} < 0$$

$$\mu = \frac{1}{2}\left\{\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta + \sqrt{\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - \delta\right)^2 - c^*\gamma(c^*)\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*}}\right\},$$

it follows that $\mathrm{AF} > 0$ because $\mu - \rho \gtreqless 0$ iff

$$\rho+\delta-\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}+\frac{c^*\gamma(c^*)}{\rho}\left.\frac{\mathrm{d}r}{\mathrm{d}k}\right|_{\theta,*} \lesseqgtr 0.$$

If $\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - (\rho+\delta) \geq 0$ it is a direct consequence of the definition of AF that $\mathrm{AF} < 1$. If $\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*} - (\rho+\delta) < 0$, using the characteristic equation (57), we can write AF as

$$\mathrm{AF} = \frac{\mu-\rho}{\mu} \frac{\frac{\mu}{\rho}\left[\mu-\rho-\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right)\right]}{\frac{\mu}{\rho}\left[\mu-\rho-\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right)\right]+\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)}$$

$$= \frac{\mu-\rho}{\mu} \frac{\frac{\mu}{\rho}\left[\mu-\rho-\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right)\right]}{\frac{\mu}{\rho}(\mu-\rho)-\frac{\mu-\rho}{\rho}\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right)}$$

$$= \frac{\mu-\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right)-\rho}{\mu-\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right)} < 1.$$

(Recall that $\mu-\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right)-\rho > 0$ because $\mathrm{AF} > 0$ and $\mu-\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta)\right) > 0$ when $\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,*}-(\rho+\delta) < 0$.)

C.3 Outside the Steady State

The impact of θ on welfare at any point in time is given by

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} = \int_0^\infty e^{-\rho t} u'(c(t)) \frac{\mathrm{d}c(t)}{\mathrm{d}\theta} \mathrm{d}t. \tag{58}$$

From the first equation in (16),

$$\frac{\mathrm{d}\dot{k}(t)}{\mathrm{d}\theta} = \left.\frac{\partial y_1}{\partial p}\right|_k \left.\frac{\partial p(t)}{\partial \theta}\right|_k + \left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta \frac{\mathrm{d}k(t)}{\mathrm{d}\theta} - \frac{\mathrm{d}c(t)}{\mathrm{d}\theta} - \delta \frac{\mathrm{d}k(t)}{\mathrm{d}\theta},$$

and hence

$$0 = \int_0^\infty e^{-\rho t} u'(c(t)) \left\{ \left.\frac{\partial y_1}{\partial p}\right|_k \left.\frac{\partial p(t)}{\partial \theta}\right|_k + \left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta \frac{\mathrm{d}k(t)}{\mathrm{d}\theta} - \frac{\mathrm{d}c(t)}{\mathrm{d}\theta} - \delta \frac{\mathrm{d}k(t)}{\mathrm{d}\theta} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}k(t)}{\mathrm{d}\theta} \right\} \mathrm{d}t. \tag{59}$$

But

$$\int_0^\infty e^{-\rho t} u'(c(t)) \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}k(t)}{\mathrm{d}\theta} \mathrm{d}t = \int_0^\infty e^{-\rho t} u'(c(t)) \left(\rho + \frac{\dot{c}(t)}{\gamma\left(c\left(t\right)\right) c\left(t\right)} \right) \frac{\mathrm{d}k(t)}{\mathrm{d}\theta} \mathrm{d}t, \tag{60}$$

so substituting (60) and (59) in (58) gives:

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} = \int_0^\infty e^{-\rho t} u'(c(t)) \frac{\mathrm{d}c(t)}{\mathrm{d}\theta} \left\{ \left.\frac{\partial y_1}{\partial p}\right|_k \left.\frac{\partial p(t)}{\partial \theta}\right|_k - \frac{\mathrm{d}c(t)}{\mathrm{d}\theta} + \left[\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta - \delta - \rho - \frac{\dot{c}(t)}{\gamma\left(c\left(t\right)\right) c\left(t\right)} \right] \frac{\mathrm{d}k(t)}{\mathrm{d}\theta} \right\} \mathrm{d}t.$$

Using the Euler equation (15) we can write the last result as

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} = \int_0^\infty e^{-\rho t} u'(c(t)) \left\{ \underbrace{\left.\frac{\mathrm{d}y_1\left(t\right)}{\mathrm{d}\theta}\right|_k}_{\text{Tullock Effect}} + \underbrace{\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta - r \right) \frac{\mathrm{d}k(t)}{\mathrm{d}\theta}}_{\text{Haberger Effect}} \right\} \mathrm{d}t,$$

where

$$\left.\frac{\mathrm{d}y_1\left(t\right)}{\mathrm{d}\theta}\right|_k = \left.\frac{\partial y_1}{\partial p}\right|_k \left.\frac{\partial p(t)}{\partial \theta}\right|_k.$$

Now, given that $\left.\frac{\mathrm{d}y_1(t)}{\mathrm{d}\theta}\right|_k = \left.\frac{\mathrm{d}y_1(0)}{\mathrm{d}\theta}\right|_k$, and that $\left|\frac{\mathrm{d}k(t)}{\mathrm{d}\theta}\right| \leq \lim_{t\to\infty} \left|\frac{\mathrm{d}k(t)}{\mathrm{d}\theta}\right|$, and given that

$$\left| \left.\frac{\mathrm{d}y_1\left(t\right)}{\mathrm{d}\theta}\right|_k \right| > \left| \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta - r \right) \frac{\mathrm{d}k(t)}{\mathrm{d}\theta} \right|$$

when they happen to have opposite signs, it follows that

$$\left.\frac{\mathrm{d}y_1(t)}{\mathrm{d}\theta}\right|_k + \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta - r\right)\frac{\mathrm{d}k(t)}{\mathrm{d}\theta} < 0 \text{ for any } t,$$

and, consequently,

$$\frac{\mathrm{d}W}{\mathrm{d}\theta} < 0.$$

C.4 Comparing $\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_k$ with $\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,r} - r\right)\frac{\mathrm{d}k}{\mathrm{d}\theta}$ when $\frac{\mathrm{d}k}{\mathrm{d}\theta} < 0$ and $k_1 < k_2$

In this subsection we show that

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_k < \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,r} - r\right)\frac{\mathrm{d}k}{\mathrm{d}\theta}.$$

Substituting (20) we have that:

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_k = \left.\frac{\partial y_1}{\partial p}\right|_k \left.\frac{\partial p}{\partial \theta}\right|_k = \left.\frac{\partial y_1}{\partial p}\right|_k \frac{p}{\theta}\frac{\alpha_g}{\Delta}.$$

It follows from $\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_\theta - r = \frac{1}{\Delta}\left(\left.\frac{\partial y_1}{\partial k}\right|_p - r\right)$ that

$$\left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,r} - r\right)\frac{\mathrm{d}k}{\mathrm{d}\theta} = \frac{1}{\Delta}\left(\left.\frac{\partial y_1}{\partial k}\right|_p - r\right)\frac{\mathrm{d}k}{\mathrm{d}\theta}.$$

From (50) and (54) it follows that

$$\begin{aligned}\frac{\mathrm{d}k}{\mathrm{d}\theta} &= -\frac{k}{\theta}\frac{-g + \frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}}{g\frac{\alpha_g}{1-\alpha_g} + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}}\frac{\frac{\theta}{p}\left.\frac{\partial p}{\partial \theta}\right|_k}{\frac{k}{p}\left.\frac{\partial p}{\partial k}\right|_\theta} \\ &= \frac{-g + \frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}}{g\frac{\alpha_g}{1-\alpha_g} + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}}\frac{\alpha_g}{(1-\alpha_g)\frac{k}{y^{\mathrm{R}}}\left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p},\end{aligned}$$

where we employed (19) and (20).

Consequently, given that $\Delta > 0$ and that $\frac{k}{y^{\mathrm{R}}}\left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p > 0$ when $k_1 < k_2$ we can write

$$\left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_k \gtrless \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,r} - r\right)\frac{\mathrm{d}k}{\mathrm{d}\theta}$$

whether

$$\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k \frac{k}{y^{\mathrm{R}}}\left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p \gtreqless \left(\frac{1-g}{r}\left.\frac{\partial y_1}{\partial k}\right|_p - (1-g)\right)\frac{1}{1-g}\frac{rk}{y_1}\frac{-g+\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}}{g\alpha_g + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}(1-\alpha_g)}.$$

Note that

$$\frac{1}{1-g}\frac{rk}{y_1} = \frac{k}{k_1 l_1}\frac{f_1'}{f_1}k_1 = \frac{k}{k_1 l_1}\alpha_{1\mathrm{K}},$$

from (45) we have that

$$\frac{1-g}{r}\left.\frac{\partial y_1}{\partial k}\right|_p = \frac{\alpha_{2\mathrm{L}}}{\alpha_{1\mathrm{K}}-\alpha_{2\mathrm{K}}},$$

and, after recalling (41) and (42), that

$$\frac{k}{y^{\mathrm{R}}}\left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p = -\frac{k}{k_1 l_1}\frac{k_1}{k_1-k_2}\left(\frac{1}{l^{\mathrm{R}}}+1\right) = \frac{k}{k_1 l_1}\alpha_{1\mathrm{K}}\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\left(\frac{1}{l^{\mathrm{R}}}+1\right).$$

Consequently, it follows that

$$\begin{aligned}&\left[g\alpha_g + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}(1-\alpha_g)\right]\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k \frac{1+l^{\mathrm{R}}}{l^{\mathrm{R}}}\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\\ &\gtreqless -\left(\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}+1-g\right)\left(-g+\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\right).\end{aligned}$$

From (20) we can write

$$\frac{1+l^{\mathrm{R}}}{l^{\mathrm{R}}}\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}} = \left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{L}}}\frac{1}{g}+\frac{\alpha_{1\mathrm{K}}-\alpha_{2\mathrm{K}}}{\alpha_{2\mathrm{L}}}\right)\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}} = \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\frac{1}{g}-1.$$

Or,

$$\begin{aligned}&-\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k\left\{\left[g\alpha_g+\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}(1-\alpha_g)\right]\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\frac{1}{g}-1\right)+\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}+1-g\right\}\\ &\lesseqgtr \left(\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}+1-g\right)\left(-g+\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\right).\end{aligned}$$

If $\frac{\mathrm{d}k}{\mathrm{d}\theta}<0$ we know that $\left.\frac{\mathrm{d}r}{\mathrm{d}\theta}\right|_k<0$ and, consequently that

$$-\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k > -g+\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}.$$

Because

$$\left[g\alpha_g + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}(1-\alpha_g)\right]\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}\frac{1}{g} - 1\right) > 0$$

it follows the result.

C.5 Studying $\frac{\mathrm{d}y_1}{\mathrm{d}\theta}$ when $\frac{\mathrm{d}k}{\mathrm{d}\theta} < 0$ and $k_1 < k_2$

In this subsection we study

$$\frac{\mathrm{d}y_1}{\mathrm{d}\theta} = \left.\frac{\mathrm{d}y_1}{\mathrm{d}\theta}\right|_k + \left(\left.\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\theta,r} - r\right)\frac{\mathrm{d}k}{\mathrm{d}\theta} + r\frac{\mathrm{d}k}{\mathrm{d}\theta}.$$

From the last subsection we know that $\frac{\mathrm{d}y_1}{\mathrm{d}\theta} \gtreqless 0$ as

$$\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k \frac{k}{y^{\mathrm{R}}}\left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p \gtreqless \left(\frac{1-g}{r}\left.\frac{\partial y_1}{\partial k}\right|_p - (1-g)\right)\frac{1}{1-g}\frac{rk}{y_1}\frac{-g + \frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}}{g\alpha_g + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}(1-\alpha_g)} +$$
$$+ (1-g)\left(1 + (1-\alpha_g)\frac{p}{y^{\mathrm{R}}}\left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k\right)\frac{1}{1-g}\frac{rk}{y_1}\frac{-g + \frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}}{g\alpha_g + \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}(1-\alpha_g)}.$$

Following the same steps of last subsection, after using (53) we get:

$$-\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k \left\{\left[g\alpha_g + (1-\alpha_g)\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} - \alpha_{1\mathrm{K}} - \frac{g}{1-g}\alpha_{2\mathrm{K}}\right)\right] + \frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}\right\}\times$$
$$\times\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}\frac{1}{g} - 1\right) \lesseqgtr \left(\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} + \frac{1-\alpha_g}{g}\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|\right)\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} - g\right).$$

Or

$$-\frac{p}{y_1}\left.\frac{\partial y_1}{\partial p}\right|_k \left\{\left[g\alpha_g + (1-\alpha_g)\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} - \alpha_{1\mathrm{K}} - \frac{g}{1-g}\alpha_{2\mathrm{K}} + 1\right)\right] + \frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}\right\}\times$$
$$\times\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}\frac{1}{g} - 1\right) \lesseqgtr \frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} - g\right).$$

A sufficient condition for $\frac{\mathrm{d}y_1}{\mathrm{d}\theta} < 0$ is that

$$g\alpha_g + (1-\alpha_g)\left(\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}} - \alpha_{1\mathrm{K}} - \frac{g}{1-g}\alpha_{2\mathrm{K}} + 1\right) \geq 0$$

or

$$\alpha_{1\mathrm{L}}\left(1+\frac{1}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\right)\geq\frac{g}{1-g}\alpha_{2\mathrm{K}}=\frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{L}}}l^{\mathrm{R}}\alpha_{2\mathrm{K}}.$$

Consequently

$$l^{\mathrm{R}}\leq\frac{\alpha_{2\mathrm{L}}}{\alpha_{2\mathrm{K}}}\left(1+\frac{1}{\alpha_{2\mathrm{K}}-\alpha_{1\mathrm{K}}}\right)$$

is a sufficient condition. For example, taking the extreme case in which $\alpha_{1\mathrm{K}}=0$ we have:

$$l^{\mathrm{R}}\leq\frac{1-\alpha_{2\mathrm{K}}}{\alpha_{2\mathrm{K}}}\frac{1+\alpha_{2\mathrm{K}}}{\alpha_{2\mathrm{K}}}.$$

Recalling that in this extreme case it makes sense assuming $\alpha_{2\mathrm{K}}\geq\frac{1}{3}$, we have

$$l^{\mathrm{R}}\leq 8\Rightarrow l_2\leq\frac{8}{9}\text{ or }l_1\geq\frac{1}{9}.$$

The conclusion is that only in the situation in which the majority of the population is working in the rent-seeking sector it is possible to generate an increase in output after a marginal deterioration on the institutional set.

D Comparing Competitive Rent-Seeking and Monopoly

D.1 Sector 1 Capital Intensive

We show that (see Figure 1)

$$\left.\frac{k}{y_1}\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\psi_2=0}=\left.\frac{k}{y_1}\frac{\partial y_1}{\partial k}\right|_{p}+\left.\frac{p}{y_1}\frac{\partial y_1}{\partial p}\right|_{k}\left.\frac{k}{p}\frac{\mathrm{d}p}{\mathrm{d}k}\right|_{\psi_2=0}>0,\tag{61}$$

where

$$\left.\frac{k}{p}\frac{\mathrm{d}p}{\mathrm{d}k}\right|_{\psi_2=0}=-\frac{\left.\frac{k}{r}\frac{\partial r}{\partial k}\right|_{p}}{\left.\frac{p}{r}\frac{\partial r}{\partial p}\right|_{k}}=-\frac{\alpha_g g\left(\theta y^{\mathrm{R}}\right)\left.\frac{k}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_{p}}{\alpha_g g\left(\theta y^{\mathrm{R}}\right)\left.\frac{k}{y^{\mathrm{R}}}\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_{k}-\frac{k_1}{f_1'}f_1''\frac{\omega}{k_1}k_1'\frac{p}{\omega}\frac{\mathrm{d}\omega}{\mathrm{d}p}},$$

and the last equality comes from the expression for the interest rate,

$$r=\left(1-g\left(\theta y^{\mathrm{R}}\left(p,k\right)\right)\right)f_1'\left(k_1\left(\omega\left(p\right)\right)\right).$$

After substituting from (34), (35), (38), (39), and (40) it follows that:

$$\left.\frac{k}{p}\frac{\mathrm{d}p}{\mathrm{d}k}\right|_{\psi_2=0} = -\frac{\alpha_g g\left(\theta y^{\mathrm{R}}\right) \frac{k}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial k}\right|_p}{\alpha_g g\left(\theta y^{\mathrm{R}}\right) \frac{k}{y^{\mathrm{R}}} \left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{1\mathrm{K}} - \alpha_{2\mathrm{K}}}}. \tag{62}$$

Hence (61) can be simplified (using (47)) to

$$\left.\frac{k}{y_1}\frac{\mathrm{d}y_1}{\mathrm{d}k}\right|_{\psi_2=0} = \frac{1}{\Gamma}\frac{k}{y_1}\left[-\alpha_g g\left(\theta y^{\mathrm{R}}\right)\frac{1}{y_2}\left.\frac{p}{y_1}\frac{\partial y_1}{\partial p}\right|_k \left(\left.\frac{\partial y_1}{\partial k}\right|_p + p\left.\frac{\partial y_2}{\partial k}\right|_p\right) + \frac{\alpha_{1\mathrm{L}}}{\alpha_{1\mathrm{K}} - \alpha_{2\mathrm{K}}}\left.\frac{\partial y_1}{\partial k}\right|_p\right],$$

where

$$\Gamma \equiv \alpha_g g\left(\theta y^{\mathrm{R}}\right) \frac{k}{y^{\mathrm{R}}}\left.\frac{\partial y^{\mathrm{R}}}{\partial p}\right|_k + \frac{\alpha_{1\mathrm{L}}}{\alpha_{1\mathrm{K}} - \alpha_{2\mathrm{K}}} > 0 \text{ if } k_1 > k_2.$$

The inequality in (61) follows from (48).

D.2 Sector 1 Labor Intensive

Note that if $k_1 < k_2$ then

$$\alpha_g g\left(\theta y^{\mathrm{R}}\right)_k \frac{p}{y_2}\left(\left.\frac{\partial y_1}{\partial k}\right|_p + p\left.\frac{\partial y_2}{\partial k}\right|_p\right) > \frac{\alpha_{1\mathrm{L}}}{\alpha_{2\mathrm{K}} - \alpha_{1\mathrm{K}}}\left.\frac{\partial y_1}{\partial k}\right|_p.$$

Consequently, using (48) and (47) in (62) we have

$$\left.\frac{k}{p}\frac{\mathrm{d}p}{\mathrm{d}k}\right|_{\psi_2=0} \lesseqgtr -\frac{\left.\frac{k}{y_1}\frac{\partial y_1}{\partial k}\right|_p}{\left.\frac{p}{y_1}\frac{\partial y_1}{\partial p}\right|_k} = \left.\frac{k}{p}\frac{\mathrm{d}p}{\mathrm{d}k}\right|_{y_1} \quad \text{as } \Gamma \gtreqless 0 \text{, or as } \left.\frac{k}{p}\frac{\mathrm{d}p}{\mathrm{d}k}\right|_{\psi_2=0} \lesseqgtr 0.$$

The fact that

$$\left.\frac{k}{p}\frac{\mathrm{d}p}{\mathrm{d}k}\right|_{\psi_2=0} \neq 0$$

guarantees that $y_1 =$ constant and $\psi_2 = 0$ intersect only once (see Figure 2.)

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