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The Diversity of Diversity: further methodological considerations on the use of the concept in cultural economics

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Abstract

The implications of diversity still raise confusion in the cultural debate. We address them from both a formal and conceptual viewpoints, putting in check the validity of some arguments. We conclude that: measuring diversity demands key decisions and careful statistical procedures; ignorance on optimal diversity levels and on ways to generate them is widespread in the cultural field; there is no support for cultural diversity as something associated to fair economic and political systems; restriction to sheer economics requires the establishment of links between diversity and measurable properties – something rather incipient. Diversity, as a social choice, should be distinguished from it as an economic value.

Key words: culture, diversity, historical periods, indexes, political systems

1. Introduction.

Though so widely used nowadays, the idea of diversity still raises a reasonable amount of confusion in the cultural debate, its economic implications being far from clarified. This paper addresses the issue from two viewpoints: first, a formal, methodological appraisal is made, second, more conceptual questions are posed, putting in check the validity of some nowadays frequent arguments.

Formal aspects are treated in the next section, where measurement questions are also discussed. Sections 3 and 4 deal with conceptual issues, the former draws on history and biology, the latter on history and economics, or rather, on a less cared-for point in economic history. In these two sections I'm fully aware that I'm *raising* points, not proving anything. It would be nothing but ridiculous and naïvely preposterous, in the limited space of this paper, to have a clear and definitive statement on questions like the optimal (ecological) diversity level for the development of sustainable civilizations and cultures (section 3). Such a theme, for instance, at least since the monumental and controversial work by the British historian Arnold J. Toynbee[1], has, under different guises, drawn the attention of many a serious researcher. Nevertheless, I hope to have composed a minimally sustainable argument for justifying the questions, puzzles and inferences I make in both sections.

My final conclusion, in section 5, cannot but call for further studies. A bit disappointing, perhaps, but my pledge is for *applied, empirical* studies. We seem to be well served of basic theoretical constructs and digressions for the time being; while the conduct of more, well designed empirical analyses can uncover situations where consistent causal links – in broader, not only statistical terms – may be of crucial value. Moreover, re-stressing a point raised in other related work, Flôres (2006), I call for a clearer disentanglement of diversity from the manifold confounding realms it

[1] Arnold J. Toynbee (1889; 1975) – a once very fashionable name - started work in his twelve-volume *A Study in History* in 1922, the last volume of which being published in 1961. In the *Study* he analyses 26 civilisations, trying to understand their rise & fall process. The leadership of creative elites, together with spiritual rather than economic forces, plays for him the foremost explanatory role, less emphasis being given, for instance, on the environment (see section 3, in the paper).

has been attached to. Beyond economics, and even many aspects of culture, diversity turns out to be a social choice.

2. Measuring diversity.

The present requirements on the (diversity) concept have made it mandatory to have a straightforward way of measuring the very diversity in concrete situations or markets. This led to the search of indicators or indexes to assess diversity. Supposing a suitably characterised context is given, basic elements for the construction of such indexes are a well-defined set of objects, outcomes or types, say 1, 2, …, n, and an associated frequency (or probability) distribution p_i , $1 \leq i \leq n$.

A common mistake, still present in many studies and arguments, is to associate diversity with the sheer multiplicity of types, forgetting that their relative frequencies are also crucial for defining "the amount of diversity"[2]. In spite of different options duly taking into account the two basic constituents above, the Shannon-Wiener entropy index seems to be most favoured and, to many a number of viewpoints, the best candidate. Indeed, since Shannon (1948), several proofs of optimality of the entropy index have been produced. Its definition, as known, is:

$$H_{SW} = - \sum_i p_i \ln p_i \ , \qquad (1)$$

where, though in the theoretical developments the logs are assumed to be neperian, in practical applications they are often taken base 2.

Inspired perhaps in the works by Weitzman (1992, 1993), Stirling (1999) proposed a different index, taking into account the degree of similarity between any given pair of objects or types. Stirling's proposal introduces a new element in the set of basic constituents, where objects, till then, were considered uniquely, and intrinsically, distinguished, no differences in their (relative) *proximities* being at stake. However, even ecologists are aware of the 'redundancy hypothesis', related to

[2] I've risen and more fully discussed this point elsewhere, Flôres (2005).

different species with similar characteristics and, more importantly, similar functional roles. Systems with distinct H_{SW} would then show a nearly equivalent behaviour, as long as representatives of the same functional groups were present in both.

Cutting through more careful discussions on the problems raised by imposing a metric in objects' space, Stirling assumes the existence of a distance function d_{ij} , well-defined for all pairs (i,j). In this, we may also see an implicit influence of Lancaster (1966)'s early ideas – pioneered, in their turn, by Gorman (1953, 1956 and 1961) - to incorporate quality in consumer theory, where products – i.e., types – are defined by transformations of an original attribute's space[3]. In this way, a Euclidean distance can be naturally computed between products.

In the light of these assumptions, Stirling's proposal is:

$$H_{St} = \sum_{i,j} d_{ij} \, p_i \, p_j \quad . \tag{2}$$

In its original formulation, the index is dependent on the measurement unit adopted for the distances, so that I prefer to impose a normalisation by setting the smallest distance, say d_{12} , equal to 1, and defining $d_{ij}^* = d_{ij} / d_{12}$, so that

$$H_{St}^* = \sum_{i,j} d_{ij}^* \, p_i \, p_j \quad , \quad d_{12}^* \equiv 1. \tag{3}$$

Written as above, the index is invariant to linear transformations on the set of distances, though, annoyingly, it continues not to be invariant to other classes of transformations, even affine ones. Indeed, in spite of the fact that, in the cultural context, the more or less similarity among objects makes sense, use of (2), or (3), instead of (1) poses a few questions. While (1) enjoys important properties that aid in the interpretation of practical results, Stirling's idea presents a confusing behaviour.

[3] As known, purely economic approaches to diversity can differ. Rosen (2004), for instance, is an example of another independent line, though based on standard ideas on product differentiation and imperfect competition.

The first problem has to do with the range of both indexes for a fixed number n of objects – and fixed distances between them -, the probability distribution being allowed to vary. The Shannon-Wiener entropy in these circumstances, as known, achieves its maximum for the uniform distribution, a result which has a strong intuitive appeal, as well as many practical and theoretical implications. Something equivalent, unfortunately, cannot be stated for Stirling's. Proposition 1 summarises the point:

Proposition 1. If the number of types is held constant:

a) under n=2 or $d_{ij} = d$, for all (i,j),

$$\arg\max_{\{pi\}} H_{SW} = \arg\max_{\{pi\}} H_{St}$$

b) outside case a), H_{St} 's maximum is difficult to interpret, given its dependence on the set of distances.

Proof. a) Immediate, as in any of the cases the maximum is attained for the uniform distribution (see the Appendix for the proof of the H_{St} case).

b) The Appendix shows the H_{St} maximum for n=3; it is easy to see that, as perhaps expected, by varying the distances, a wide spectrum of values can be attained by the index. Moreover, for a given set of distances, the optimal probabilities may either be a corner solution, in the boundary of the simplex $\{ (p_1 , p_2 , p_3) \geq 0 \mid p_1 + p_2 + p_3 = 1 \}$, or indeed characterise a global maximum. Though in the latter case the relative values of the probabilities follow some expected patterns, in both instances – particularly the former one - the solution bears a much less intuitive meaning.

Though nearly elementary, Proposition 1 has disturbing consequences, as an important *and easy* reference value for assessing the diversity of a market or community with n types is lost. If we add the fact that the imposition of distances will be usually fraught with error, the meaning of the optimal frequencies for (2) (or (3)) becomes even shakier.

The second point has to do with another important operation regarding diversity assessments. We shall call a hierarchical structure of objects a structure informally defined as follows. We start with n_1 types and an associated distribution $\{p_{i1}, 1 \le i_1 \le n_1\}$; then, some of the original types are "opened" or "disaggregated" giving birth to a subset of types. The new structure $\{p_{i2}, 1 \le i_2 \le n_2 , n_2 > n_1\}$ is such that, if $i_1{*}$ is a type of the first structure which was disaggregated into m new types 1, 2, …, j_2, …, m:

$$p_{i1*} = \Sigma_{j2}\, p_{j2} \quad ,$$

so that a tree structure is generated, with probabilities adding-up to the higher nodes.

Situations as such are quite common when, in applied work, one moves from a more aggregate to a finer classification, preserving the correspondence - from subsets of the latter to elements of the first - with the original one. Movies can first be studied by branch of drama, next each branch may be opened by country of production, etc. It is expected that, with more types related to the previous ones in the above way, the diversity will, at least, not decrease. This is indeed the case with the Shannon-Wiener proposal but, again, fails to happen with Stirling's, as stated in

Proposition 2. In a hierarchical structure of types and associated frequencies (informally, a tree structure),

H_{SW} is strictly increasing when one moves *down* the tree, but

under reasonable hypotheses on the set of distances, H_{St} can display any behaviour.

Proof. See the Appendix for the proof of the H_{SW} case and an example of H_{St}'s odd behaviour, for $n_1 = 2$ and $n_2 = 3$.

The non-intuitive behaviour of (2) *vis-à-vis* (1) is not the only issue to weight on the choice of a practical diversity measure. Both indexes suffer from serious reliability drawbacks. First, the probability distribution needed for their computation must be inferred from a practical sample where observed (relative) frequencies will be computed. This means that what one gets is not the set $\{p_i, 1 \le i \le n\}$ but rather an estimate $\{f_i = p_i \pm \varepsilon_i , 1 \le i \le n\}$ where the stochastic errors ε_i, beyond bearing a relationship among them, should hopefully be small. By the same token, it is only fair to admit that the distances needed for (2) will carry intrinsic errors, one eventually disposing of a set $\{d_{ij} \pm \nu_{ij} , 1 \le i,j \le n \}$ where, again, errors should ideally be negligible.

These considerations amount to a warning on the mandatory calculation of confidence bands – as well as proper statistical inference procedures - for any of the two indexes. Without this, evaluations of changes in diversity or use of the index as a free-from-errors variable in a (stochastic, say linear) model, to explain further behaviour related to diversity, may lead to serious distortions and false conclusions.

Combining all previous observations, it becomes evident that the conduct of empirical studies on diversity requires careful design and an as-clear-as-possible idea of the different issues at stake, in order that, starting from the proper way of measuring diversity, a minimal reliability can be assured of the results.

3. Biology and history still have a say.

It is no wonder any more, in the field of cultural economics, that the concept of diversity has important and insightful roots in the biological sciences, particularly in the ecological discourse. In broad terms, diversity is irrevocably associated to life itself, be it in the context of an eco-system or in the microcosm of a cell. It is a key element for the robustness of any of these systems, allowing them to adequately resist to exogenous (environmental) shocks that would otherwise extinguish "their" life. The higher the system's diversity, the greater its survival and evolution possibilities.

A logical consequence of the previous lines might seem the point that, in general terms, the higher the diversity the better. However, if we couple the different diversities found in our planet with the facts from the history of civilisation a somewhat different lesson emerges.

Let's imagine, for a moment, an oversimplified (continuous) scale of actual eco-systems in order of increasing diversity. In the lower end of the scale we find the deserts, considered as very low diversity systems into which – unfortunately – many previously flourishing areas of our planet are turning. At the opposite end we can place the Amazon Forest, perhaps the most diversified ecological system on earth. Direct use of the "higher/better" diversity reasoning would imply that the Amazon system is the most preferable one. However, to which purpose ?

Historians have long taught us that civilisation was born neither in an extremely low nor a too high diversity eco-system, but rather in a place somewhere in-between, the well-known Fertile Crescent, where diversity conditions were optimal for the development of patterns, techniques and, in the limit, cultures evolving into modern civilisation. This point – which owns much to the important excavations led by the US archaeologist James H. Breasted (1865; 1935) - has recently been (re)elaborated by Diamond (1997), in an argument plenty of stimulating examples and additional considerations.

This crossing of two knowledges not only puts a question mark on the "higher/better" logic, it raises an interesting and fundamental question to our purposes. I shall call it

Question 1. What is an (the) optimal diversity level ?

We have been struggling for 'higher diversity' in different contexts – in the movie and music markets, in the printing media, in the expression of local cultures/communities, in the right of any kind of content in the web – but have no idea whatsoever on the desired levels of such very diversity. I'm not questioning the positive feeling most experience in a 'reasonably diversified environment' in all the previously mentioned instances, but I do question the surprisingly ignorance we have on the actual *level* desired for such diversity. A disregard that must urgently be addressed.

But a second question emerges from the biology-history interplay. Through trial and error, careful analyses and scientific tests, the Fertile Crescent stood up as a sort of ideal cradle for civilisation. Other eco-systems, however, gave also birth to specific cultures, as illustrated in Fernández-Armesto (2001). Explanations on the *why* of these different processes – with the tremendous advantage of ex-post rationalisations, and perhaps always somewhat partial – can be produced: a reliable and adequate water supply, a moderate climate, the existence of basic crops, animal species suitable to domestication, etc, etc. Changing such 'friendly environments' for the abundance of the Amazon Forest, for instance, makes for the appearance of "too many" animal and vegetal species. This entails an intense and generalised competition, with felines and other aggressive beasts chasing the tamed ones and an infrequent occurrence of edible plants. Fertile soils are rare and organic layers thin. Ironically, the luxuriant environment, with an overflow of so much fauna and flora, becomes acutely hostile to human settlement. How these so different systems evolved, why they are what they are ? This composes my

Question 2. How to generate/motivate/enhance diversity ?

If in the ecological context the answer to this question is already incomplete, the benefit of hindsight (again) contributing a lot to some existing 'genetic' explanations and models[4], translation of the very same question to the cultural economics context reduces even more the already debatable set of explanations. To make things worse, the possible answers one may find, resorting to economics and, specially, economic history, raise further uncomfortable questions, as seen next.

4. But history and economics add a sizeable grain of salt.

One way of looking for answers to *Question 2* in the context of cultural economics is to search for historical periods where a given diversity surge was apparent – of

[4] See, as an example, Wootton (2001). Of course, knowledge on how to destroy ecological diversity is, unfortunately, abundant.

course, in a specific, well-defined domain – and try to find similarities or common patterns that could give way to a genetic, even if partial, explanation.

Cowley (2002), for backing the main argument of his text, describes many interesting examples of flourishing diversity in contexts ranging from Cuban music to Persian rugs. Notwithstanding, a curious, conspicuous point stands out in most of them, a point common to a great majority of other examples that could be found since, at least, the Roman Empire. I shall call this point the

Puzzle. In *most* interesting cultural/artistic periods, where flourishing diversity can be identified in one or several fields, a great concentration of wealth is found, associated to heightened social inequality and, usually, social unfairness in general.

I have stressed the word 'most', as my puzzle doesn't aim at a universally true statement, but as a qualification of the apparently prevailing association between a high cultural diversity and the low socio-economic fairness of the historically related political system.

It is well known that the money and 'night-life expenditures' that made room for the variety of music spots and related musicians, composers and performers in La Habana, Cuba, came from a corrupt elite, mixed with speculators and adventurers, who sustained Dictator Fulgencio Batista's infamous rule over the island, for around twenty-five years[5]. Climaxes of Persian rugs craftwork are associated to the splendour of the Ottoman court, something in itself – in spite of the wisdom showed by many Ottoman rulers and civil administrators – not a model of social justice and income distribution.

I shall not dwell on the fine analysis of the manifold cases, as, of course, this would be a matter for plenty of other papers. If recent, voluminous works like Sassoon (2006)'s seem to support my point, providing evidence on how (flourishing & diversified) culture in Europe had little impact on unfair and unfortunate political systems and decisions, renewed interest on the long life of the Habsburg Empire, as

[5] See Gott (2004) for a basic account of this period.

seen, for instance, in Grassl and Smith (1986), may shed a better light on the interactions between the political, institutional and economic context and different aspects of diversity. Nevertheless, the point in the *Puzzle* stands forth. Diversity usually requires wealth of some sort, sometimes in considerable amounts, and this wealth has often been the result of strong social inequalities.

Moreover, there are two important consequences of the *Puzzle,* even if one is diffident about it. The first is that cultural diversity cannot be dissociated from the socio-economic context that gave birth to it. Through this reasoning, we rejoin Question 2 in the previous section, seeing as nearly mandatory the inclusion of the socio-economic dimension in the 'genetic' studies of diversity.

The second is a better understanding of the possible links between cultural diversity and economics. The former *may* be an interesting property, that should in principle (and often ?) be preserved, even encouraged, but as a basic value in itself, as a social or ethical choice. This choice can even have little to do with a strict or specific cultural value, but rather come out of broader concerns[6].

Notwithstanding, if one wants to couple such a choice with the economic dimension, it is hard to sustain a non-utilitarian position, i.e., the diversity benefits entail unavoidable costs that must enter into the decision-making process. As Brock and Xepapadeas (2003) rightfully argue, if diversity is economically desirable it should be so due to its association with useful characteristics or services it either has or provides. This awareness is becoming increasingly widespread in the context of biodiversity, notably in its association with pharmaceutical research (see, for instance, Craft and Simpson (2001), Goulder and Kennedy (1997) and Simpson et al. (1996)), but seems to be lacking in our field. We must by all means go further, and create endogenous measures of the value of diversity, associated in someway to the descriptive indexes discussed in section 2. In the limit, one could conceive (or dream)

[6] I'm unable to resist the temptation to remind that, in the biblical episode of the Tower of Babel (Genesis 11:1-9), diversity is used as a *punishment.* God introduces linguistic diversity among men, which eventually renders impossible building the Tower, as a way to punish them for their unlimited pride in wanting to reach the heavens through it.

of incorporating diversity in a general equilibrium framework, where the assessment of welfare would be more 'natural'.

At the side of these concrete technical pursuits, the search for periods of relative economic fairness coupled with a significant diversity seems an important line of research which hasn't received due attention yet.

5. Conclusion: the diversity of diversity.

Is diversity doomed to remain a merely descriptive tool, with no further use beyond its (celebrated) meanings and applications in biology?

In spite of a nowadays fairly substantial theoretical tradition on the diversity discourse, many more rigorous empirical studies are needed in the cultural field. Cause and effect relationships, comparisons and evaluations of different contexts and case studies, better grasp on the interactions with other variables as well as useful mechanisms are a few of the alleys to be explored. In fact, we know too little about the *diverse* answers to all these questions.

We have, in particular, highlighted the following points:

i) measuring cultural diversity is less easy than it may seem, demanding key conceptual decisions and careful statistical procedures;

ii) ignorance on optimal diversity levels is widespread in the cultural field, as well as on ways to generate, enhance or sustain diversity;

iii) there is a poor, if non-existent, historical support for cultural diversity, if one expects such diversity to be associated to equitable, fair or just economic and political systems. In particular, relations with the latter often move in opposite directions;

iv) restricting the issues to an economic decision requires, for an enlightened cost-benefit analysis, the establishment of links between cultural diversity and measurable (and desirable) properties or services, broadly, a welfare indicator – something, again, still rather incipient.

Finally, culture, be it understood as an anthropological, sociological or historical phenomenon, doesn't necessarily need the concept of diversity, though both may interact in complex and *diversified* ways. In any case, diversity, as a social choice, should be clearly distinguished from its potential economic value.

Annex. Proof of Propositions 1 and 2.

Proof of Proposition 1. a) the result that the uniform distribution $p_i = 1/n$, $1 \le i \le n$, maximises H_{SW}, for fixed n, is known, as mentioned in the text, since Shannon (1948). For the H_{St} case, under any of the hypotheses, it suffices to look for the maximum of $\sum_{i,j} p_i\, p_j$. Calling λ the Lagrange multiplier of the probability distribution restriction, first order conditions yield:

$$1 - p_i = \lambda \ , \ 1 \le i \le n \ ,$$

showing already that all p_i must be equal. It is straightforward to convince oneself that $\lambda^* = (n-1)/n$, the uniform distribution giving the optimal solution.

It is interesting to compare the two optimal values:

$$H_{SW}{}^{opt} = \log n \qquad , \qquad H_{St}{}^{opt} = d/n \quad . \qquad \text{(A.1)}$$

The Shannon-Wiener result seems to have more appeal.

b) We solve for n=3, with normalised distances, the maximum for the H_{St} as written in (3). Calling again λ the Lagrange multiplier of the probability distribution restriction, first order conditions produce the following linear system:

$$\begin{aligned} p_2 + d_{13}^*\, p_3 - \lambda &= 0 \\ p_1 \qquad + d_{23}^*\, p_3 - \lambda &= 0 \\ d_{13}^*\, p_1 + d_{23}^*\, p_2 \qquad - \lambda &= 0 \\ p_1 + \quad p_2 + \quad p_3 \qquad &= 1 \end{aligned} \qquad \text{(A.2)}$$

Direct use of Cramer's rule gives:

$$p_i = d_{j3}^*\, (1 + d_{i3}^* - d_{j3}^*) / \Delta \quad , \ i=1, 2 \text{ and } j=2,1, \text{ correspondingly,}$$

$$p_3 = (d_{13}^* + d_{23}^* - 1) / \Delta \qquad , \qquad \text{(A.3)}$$

where $\Delta = 2\,(d_{13}^* + d_{23}^*) - (d_{13}^* - d_{23}^*)^2 - 1$, is the determinant of the system.

First notice that, from (A.3), existence of a global maximum within the simplex

$$\{ (p_1 , p_2 , p_3) \geq 0 \mid p_1 + p_2 + p_3 = 1 \} \quad .$$

entails that

$$d_{13}^* \geq d_{23}^* - 1 \quad \text{and} \quad d_{23}^* \geq d_{13}^* - 1 ,$$

so that *admissible pairs* of distances (d_{13}^*, d_{23}^*) lie in a region in the positive (first) quadrant bounded by two parallel 45 degrees lines passing, respectively, through the points (0,1) and (1,0), and - because the normalisation in (3) implies that

$$d_{13}^*, d_{23}^* \geq 1 \qquad \text{(A.4)}$$

- two line segments, one horizontal and the other vertical, passing, respectively, through the same points. Outside this band, the global maximum (as regards the sole restriction $p_1 + p_2 + p_3 = 1$) will be related to at least one negative value in (A.3), and a local optimum must be searched in the boundary of the band[7]. The optimal, in this case, bears no clear interpretation.

If the maximum lies in the simplex, the three probabilities, forgetting the common denominator Δ, are always equal to the product of the distance of the "other two objects" by the algebraic sum of all distances, where that of the "other two objects" is affected by a negative sign. Within this pattern, keeping in mind (A.4), and supposing that

$$d_{23}^* \geq d_{13}^* \; ,$$

it's not difficult to prove that $p_1 < p_2 < p_3$. The first inequality is immediate, as for the second, suppose it's not valid. This entails that

$$d_{23}^* - 1 \leq d_{13}^* (d_{23}^* - d_{13}^*) \quad \text{, or}$$

$$d_{13}^* + 1 \leq d_{23}^* \quad ,$$

but this is a contradiction because for any pair of distances defining a point in the band, it's easy to convince oneself that,

$$d_{23}^* - d_{13}^* \leq 1 \quad .$$

[7] All this can be carefully verified with the use of the Kuhn-Tucker conditions which, by simplicity, we avoid here. It is also easy to prove that, within the band, the determinant of the system, Δ, will be always strictly positive.

With the above inequalities, the highest probability is assigned to the farthest object, namely the third one, and it is then straightforward to prove that the third object gives the bigger contribution to the diversity value. This is in line with a point in Weitzman (1992), who reckons the most valuable species as the farthest one. In spite of this, even in this case, the result is much less intuitive.

Proof of Proposition 2. For the H_{SW} the proof is again self contained in a result in Shannon (1948). Indeed, supposing, for the sake of simplicity, that the new structure $\{p_{i2}, 1 \le i_2 \le n_2 , n_2 > n_1\}$ is such that only one $i_1 = i_1^*$ was disaggregated into m new types 1, 2, …, j_2, …, m, with

$$p_{i*1} = \sum_{j2} p_{j2} \quad ,$$

Shannon (1948) states that

$$H_{SW}(\{p_{i2}, 1 \le i_2 \le n_2\}) = H_{SW}(\{p_{i1}, 1 \le i_1 \le n_1\}) + p_{i*1} H_{SW}(\{p_{j2}/ p_{i*1} , 1 \le j \le m\})$$

being evident that

$$H_{SW}(\{p_{i2}, 1 \le i_2 \le n_2\}) > H_{SW}(\{p_{i1}, 1 \le i_1 \le n_1\}) .$$

The generalisation to more than one disaggregation can proceed by induction.

For the H_{St} it suffices to start with a two types' space in which the second object is disaggregated into two new ones, denoted 21 and 22, with probabilities p_{2i}, i=1, 2, such that

$$p_{21} + p_{22} = p_2 . \tag{A.5}$$

Working with non-normalised distances, we want to exhibit an example of the validity of

$$d_{1,21}\, p_1 p_{21} + d_{1,22}\, p_1 p_{22} + d_{21,22}\, p_{21} p_{22} - d_{12}\, p_1 p_2 < 0 \quad , \tag{A.6}$$

where the notation used for the distances seems self-explanatory.

It is reasonable to suppose that, for instance,

$$d_{1,21} > d_{12} > d_{1,22} \quad . \tag{A.7}$$

Rearranging (A.6), and using once (A.5), we have

$$(d_{1,21} - d_{1,22}) + d_{21,22}\,(p_{22}/p_1) < (p_2/p_{21})\,(d_{12} - d_{1,22}) \quad , \tag{A.8}$$

all terms being positive.

Admitting, for simplicity, that we are working with Euclidean distances, the key to produce an example that (A.8) is perfectly feasible, is to lower as much as possible the ratio of probabilities in the l.h.s. of the inequality, while setting the one at the r.h.s. at a fixed 'high' value. Let then,

$p_{21} = \delta$, $p_{22} = 4\delta$, so that $p_2 = 5\delta$ and $p_1 = 1 - 5\delta$.

This implies that $p_{22} / p_1 = 4\delta / 1-5\delta$ and $p_2/p_{21} = 5$.

Moving δ in the interval (0 ; 1/5) towards the neighbourhood of 0, we can make the ratio p_{22} / p_1 smaller than any given desired $\varepsilon>0$.

Now, it is not very hard to imagine, in Euclidean two-dimensional space, for instance, three points '21', '2' and '22', well far from a fourth point '1', in such a way that, beyond satisfying (A.7),

$$d_{1,21} - d_{1,22} \approx d_{12} - d_{1,22} \quad .$$

In this case, irrespectively of the value of $d_{21,22}$, one can find a suitable δ that will render (A.8)'s new version

$$(d_{1,21} - d_{1,22}) + d_{21,22} \,.\, \varepsilon < 5\,(d_{12} - d_{1,22}) \quad , \qquad \text{(A.9)}$$

a true inequality.

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