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The Forward– and the Equity–Premium Puzzles: Two Symptoms of the Same Illness?

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Agosto de 2007

The Forward- and the Equity-Premium Puzzles: Two Symptoms of the Same Illness?[*]

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August 3, 2007

Abstract

In this paper we revisit the relationship between the equity and the forward premium puzzles. We construct return-based stochastic discount factors under very mild assumptions and check whether they price correctly the equity and the foreign currency risk premia. We avoid log-linearizations by using moments restrictions associated with euler equations to test the capacity of our return-based stochastic discount factors to price returns on the relevant assets. Our main finding is that a pricing kernel constructed only using information on American domestic assets accounts for both domestic and international stylized facts that escape consumption based models. In particular, we fail to reject the null hypothesis that the foreign currency risk premium has zero price when the instrument is the own current value of the forward premium. *JEL Code: G12; G15. Keywords: Equity Premium Puzzle, Forward Premium Puzzle, Return-based Pricing Kernel.*

1 Introduction

The Forward Premium Puzzle – henceforth, FPP – is how one calls the systematic departure from the intuitive proposition that the expected return to speculation in the forward foreign exchange market should be zero, conditional on available information.

One of the most acknowledged puzzles in international finance, the FPP was, in its infancy, investigated by Mark (1985) within the framework of the consumption capital asset pricing model – CCAPM. Perhaps, following Hansen and Singleton's (1982, 1984) earlier method, Mark used a non-linear GMM approach, which revealed the model's inability, in its canonical version, to account for its implicit over-identifying restrictions. The results found by Mark are similar to the ones found by Hansen and Singleton with respect to the equity premium, an idea carried forward by Mehra and Prescott (1985) who went on to propose what they have labelled the Equity Premium Puzzle – henceforth, EPP.

At first sight, it may seem surprising that such similar results were never properly linked, and we may only conjecture why the literature on the FPP and the EPP drifted apart after the work of Mark. We list two alternative explanations. First, the failure of the CCAPM was a great disappointment for the profession, since it meant the absence of a fully specified economic model that could price assets. Such unheartening finding may have lead to a momentary halt in research linking the equity- and the forward-premium puzzles[1]. Second, the existence of a specificity of the FPP with no parallel in the case of the EPP – the predictability of returns based on interest rate differentials[2] – may have led many to believe that even if the CCAPM was capable of accounting for the equity premium it would not solve the FPP; see Engel (1996).

Thinking deeper about these two puzzles, we are forced to conclude that proving that they are related is currently an impossible task, since it requires the existence of

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[1]While pricing failures for the CCAPM were found before, it was only with the work of Mehra and Prescott (1985), published the same year as Mark's work, that it became clear that there was something fundamentally wrong with the CCAPM in its canonical form.

[2]We do not claim that returns on equity are not predictable. In fact we know that dividend-price ratios and other variables "predict" returns. The point is that this empirical regularity was not seen, in the early days of research with the CCAPM, as a defining feature of the EPP. Nowadays, however, an empirically successful model ought to take care of this (and many other) non-trivial aspects of asset behavior.

a consumption model generating a pricing kernel that properly prices assets, showing the shortcomings of previous models. Because we do not have such a proper model today, we cannot relate the EPP and the FPP within a CCAPM framework. This may explain why relating these two puzzles was not tried before and why two distinct research agendas involving them appeared over time.

As is well known, research regarding the FPP is mostly done within the scope of international economics, like in Fama and Farber (1979), Hodrick (1981) and Lucas (1982). It emphasizes international affine term structure models and/or a microstructure approach. Research involving the EPP has focused on adding state variables to standard consumption-based pricing kernels to change its behavior; see Epstein and Zin (1989), Constantinides and Duffie (1996) and Campbell and Cochrane (1999).

In this paper we revisit the FPP and the EPP and ask whether they deserve two distinct agendas, or whether they are but two symptoms of the same illness: the incapacity of existing consumption-based models to generate the implied behavior of *a* pricing kernel that correctly prices asset returns.

Given the limitations on proving that the FPP and the EPP are related, we use an indirect approach. If these two puzzles are solely a symptom of the inappropriateness of existing consumption-based pricing kernels, then they will not be manifest when appropriate pricing kernels are used. Suppose we find a single pricing kernel that is not a function of consumption and is compatible with all regularities in domestic financial markets not accounted for by current consumption-based kernels. At the same time, suppose that this pricing kernel accounts for the behavior of the forward premium. Then, we have good reason to believe that we should not disperse our research effort on two completely different agendas, but rather concentrate on a single one focused on rethinking consumption-based pricing kernels[3] – the common suspect for the two puzzles.

A crucial issue of our approach is to find what an *appropriate* pricing kernel is in this context. Hansen and Jagganathan (1991) have lead the profession towards return-based kernels instead of consumption-based kernels; see also Connor and Korajczyk (1986), Chamberlain and Rothschild (1983), and Bai (2005). The idea is to combine statistical methods with economic theory – the Asset Pricing Equation – to devise pricing-kernel estimates as the unique projection of a stochastic discount factor – henceforth, SDF – on the space of returns: the SDF mimicking portfolio.

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[3]In what follows, we use the terms pricing kernel and stochastic discount factor interchangeably.

The latter can be estimated without any assumptions on a functional form for preferences, despite having a strong footing on theory as a consequence of the use of the Asset Pricing Equation.

One way to rationalize the SDF mimicking portfolio is to realize that it is the projection of a proper consumption model (yet to be written) on the space of payoffs. Thus, the pricing properties of this projection are no worse than those of the proper model – a key insight of Hansen and Jagganathan. An advantage of concentrating on the projection is that we can approximate it arbitrarily well in-sample using statistical methods and asset returns alone. Therefore, using such projection not only circumvents the inexistence of a proper consumption model but is also guaranteed not to underperform such ideal model.

Bearing in mind our stated goal, we extract a time series for the pricing kernel that does not depend on preferences or on consumption data. Two techniques are considered in this paper to estimate the SDF mimicking portfolio: i) Hansen and Jagganathan's mimicking portfolio, which is the projection of any stochastic discount factor on the space of returns and ii) the unconditional linear multifactor model, which is perhaps the dominant model in discrete-time empirical work in Finance.

As noted by Cochrane (2001), SDF estimates are just functions of data. Pricing correctly a specific group of assets can be achieved by building non-parsimonious SDF estimates, i.e., SDF estimates that price arbitrarily well that group of assets in-sample but not necessarily assets outside that group.[4] In order to avoid this critique, we construct SDF mimicking portfolio estimates using domestic (U.S.) returns alone – on the 200 most traded stocks in the NYSE, extracted from the CRSP database.

Assume we had a consumption-based model that did account for the equity premium. Then, the behavior of its projection on the space of domestic-asset returns would have to coincide with that of an SDF mimicking portfolio built from these same assets. We then use this domestic SDF projection to price the forward premium. Tests are implemented for four countries within the G7 group, besides Switzerland, for which there exists a relatively long-span data for spot and future foreign exchange

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[4]If we use all foreign and domestic assets to construct SDF estimates, we should expect to price correctly both the equity and forward premium. If that was the case, we could rule out that explanations based on market imperfections are needed to explain these puzzles, since the existence of an SDF is only guaranteed if the law of one price holds. If we cannot price the equity and forward premium in the exercise, all we can conclude is that the Asset Pricing Equation is inappropriate.

markets. Here, our tests show their out-of-sample character, avoiding Cochrane's critique.

Our tests make intensive use of the Asset Pricing Equation. They are all based on euler equations, exploiting theoretical lack of correlation between discounted risk premia and variables in the conditioning set, or between discounted returns and their respective theoretical means, i.e., we employ *discounted scaled excess-returns* and *discounted scaled returns* in testing. We investigate whether discounted risk premia have mean zero or whether discounted returns have a mean of unity.

Our results are clear cut: return-based pricing kernels using U.S. assets alone account for domestic stylized facts, pricing correctly the equity premium for the U.S. economy – which shows no signs of the EPP – and also pricing most of the Fama-French benchmark factor returns. At the same time, these same pricing-kernel estimates show no signs of the FPP in pricing the expected return to speculation in forward foreign-exchange markets for the widest group possible of developed countries with a long enough span of future exchange-rate data (Canada, Germany, Japan, Switzerland and the U.K.). This evidence raises the question of whether the FPP and the EPP are two symptoms of the same illness.

We summarize our empirical results as follows. First, the null of zero discounted excess returns on equities is not rejected even when potentially interesting forecasting variables are used as instruments. Second, for most countries, the moment restrictions associated with the euler equations (Asset Pricing Equations) are not rejected for excess returns and returns on operations with foreign assets for any of the instruments used. This includes the own current value of the forward premium, which shows no signs of predictability of the expected return to speculation, contradicting one of the defining features of the FPP. Only in the case of British bonds the results are, in some sense, conflicting. In some occasions, we reject the null hypothesis that the foreign currency risk premium has zero price.

Our results can be viewed as new evidence supporting the usefulness of reuniting the research agendas on the EPP and the FPP. Although we cannot claim that a consumption model that did account for the behavior of the equity premium would also price correctly the forward premium, we can claim that its projection on the space of domestic returns would.[5] In our view, this is as far as one can go today in

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[5]Therefore, the model would only have problems in prcing foreign assets if the residual of such projection was correlated with the part of foreign assets return which has zero price. We do believe

showing that these two puzzles are related.

As argued above, we search not for a consumption model of the SDF, but simply for a procedure that identifies the SDF mimicking portfolio circumventing the fact that we still lack a good model for pricing risk or risk premia. Employing SDF mimicking portfolio estimates allows to test directly the pricing of risk or risk premia by using the theoretical restrictions associated with the Asset Pricing Equation. In our context, there is neither the need to specify a full model for preferences (consumption SDF) nor the need to perform a log-linearization of the Asset Pricing Equation in pricing tests. In that sense, we are able to isolate possible causes for rejection of theory (EPP and FPP) not isolated by the previous literature.

The remainder of the paper is organized as follows. Section 2 gives an account of the literature that tries to explain the FPP and is related to our current effort. Section 3 discusses the techniques used to estimate the SDF and the pricing tests are implemented in this paper. Section 4 presents the empirical results obtained in this paper. Concluding remarks are offered in Section 5.

2   A Critical Literature Review

Most studies[6] report the existence of the FPP through the finding that $\widehat{\alpha}_1$ is significantly smaller than zero when running the regression,

$$s_{t+1} - s_t = \alpha_0 + \alpha_1(_t f_{t+1} - s_t) + u_{t+1}, \tag{1}$$

where s_t is the log of the exchange rate at time t, $_t f_{t+1}$ is the log of time t forward exchange rate contract and u_{t+1} is the regression error.[7] Notwithstanding the possible effect of Jensen inequality terms, testing the uncovered interest rate parity (UIP) is equivalent to testing the null $\alpha_1 = 1$ and $\alpha_0 = 0$, along with the uncorrelatedness of residuals from the estimated regression.

Although the null is rejected in almost all studies, it should be noted that α_1 not being equal to one ought not to be viewed as evidence of market failure or some form of irrationality and, *per se*, does not imply the existence of a 'puzzle' since

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that a successful consumption model for domestic markets is very unlikely to present such pattern, but our tests cannot rule it out.

[6] See the comprehensive surveys by Hodrick (1987) and Engel (1996), and the references therein.

[7] In what follows, capital letters are used to represent variables in levels and small letters to represent the logs of these same variables.

the uncovered parity needs only to hold exactly in a world of risk-neutral agents, or if the return on currency speculation is not risky. The probable reason why these findings came to be called a puzzle was the magnitude of the discrepancy from the null: according to Froot (1990), the average value of $\hat{\alpha}_1$ is -0.88 for over 75 published estimates across various exchange rates and time periods. This implies an expected domestic currency appreciation when domestic nominal interest rates exceed foreign interest rates, contrary to what is needed for the UIP to hold.

Log-linear regressions such as (1) have a long tradition in economics. As is well known, getting to (1) from first principles requires stringent assumptions, something that is usually overlooked when hypothesis testing is later performed using it. Next, we shall make explicit how strong the assumptions that underlie the null tested in (1) are. Our departing point is the Pricing Equation,

$$1 = \mathbb{E}_t \left[M_{t+1} R_{i,t+1} \right] \quad \forall i = 1, 2, \cdots, N. \tag{2}$$

where $R_{i,t+1}$ is the return of asset i and M_{t+1} is the pricing kernel or stochastic discount factor, SDF (e.g. Hansen and Jagganathan (1991)), a random variable that discount payoffs in such a way that their price is simply the discounted expected value.

Given free portfolio formation, the law of one price – the fact that two assets with the same payoff in all states of nature must have the same price – is sufficient to guarantee, through Riesz representation theorem, the existence of a SDF, M_{t+1}. Log linearizing (2) makes it is possible to justify regression (1), but not without unduly strong assumptions on the behavior of discounted returns.

Gomes and Issler (2007) criticize the empirical use of the log-linear approximation of the Pricing Equation (2) leading to (1). First, is the usual criticism that any hypothesis test using results of a log-linear regression is a joint test which includes the validity of the log-linearization being performed, i.e., includes an auxiliary hypothesis in testing. Therefore, rejection can happen if the null is true but the log-linearization is inappropriate. Second, they show that it is very hard to find appropriate instruments in estimating log-linear regressions such as (1), since, by construction, lagged variables are correlated with the error term.

To understand this latter point, consider a second-order taylor expansion of the exponential function around x, with increment h,

$$e^{x+h} = e^x + he^x + \frac{h^2 e^{x+\lambda(h)h}}{2}, \text{ with } \lambda(h) : \mathbb{R} \to (0,1). \tag{3}$$

For a generic function, $\lambda(\cdot)$ depends on both x and h, but not for the exponential function. Indeed, dividing (3) by e^x, we get

$$e^h = 1 + h + \frac{h^2 e^{\lambda(h)h}}{2},\qquad(4)$$

showing that $\lambda(\cdot)$ depends only on h.[8] To connect (4) with the Pricing Equation (2), we assume $M_t R_{i,t} > 0$ and let $h = \ln(M_t R_{i,t})$ to obtain[9]

$$M_t R_{i,t} = 1 + \ln(M_t R_{i,t}) + z_{i,t},\qquad(5)$$

where the higher-order term of the expansion is

$$z_{i,t} \equiv \frac{1}{2} \times [\ln(M_t R_{i,t})]^2\, e^{\lambda(\ln(M_t R_{i,t}))\ln(M_t R_{i,t})}.$$

It is important to stress that (5) is not an approximation but an exact relationship. Also, $z_{i,t} \geq 0$. Taking the conditional expectation of both sides of (5), using past information, denoted by $\mathbb{E}_{t-1}(\cdot)$, imposing the Pricing Equation, and rearranging terms, gives:

$$\mathbb{E}_{t-1}\{M_t R_{i,t}\} = 1 + \mathbb{E}_{t-1}\{\ln(M_t R_{i,t})\} + \mathbb{E}_{t-1}(z_{i,t})\text{, or,}\qquad(6)$$
$$\mathbb{E}_{t-1}(z_{i,t}) = -\mathbb{E}_{t-1}\{\ln(M_t R_{i,t})\}.\qquad(7)$$

Equation (7) shows that behavior of the conditional expectation of the higher-order term depends only on that of $\mathbb{E}_{t-1}\{\ln(M_t R_{i,t})\}$. Therefore, in general, it depends on lagged values of $\ln(M_t R_{i,t})$ and on powers of these lagged values. This will turn out to a major problem when estimating (1). To see it, denote by $\varepsilon_{i,t} = \ln(M_t R_{i,t}) - \mathbb{E}_{t-1}\{\ln(M_t R_{i,t})\}$ the innovation of $\ln(M_t R_{i,t})$. Let $\mathbf{R}_t \equiv (R_{1,t}, R_{2,t},\ ...,R_{N,t})'$ and $\boldsymbol{\varepsilon}_t \equiv (\varepsilon_{1,t}, \varepsilon_{2,t},\ ...,\varepsilon_{N,t})'$ stack respectively the returns $R_{i,t}$ and the forecast errors $\varepsilon_{i,t}$. From the definition of $\boldsymbol{\varepsilon}_t$ we have:

$$\ln(M_t \mathbf{R}_t) = \mathbb{E}_{t-1}\{\ln(M_t \mathbf{R}_t)\} + \boldsymbol{\varepsilon}_t.\qquad(8)$$

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[8]The closed-form solution for $\lambda(\cdot)$ is:

$$\lambda(h) = \begin{cases} \frac{1}{h} \times \ln\left[\frac{2\times(e^h - 1 - h)}{h^2}\right], & h \neq 0 \\ 1/3, & h = 0, \end{cases}$$

where $\lambda(\cdot)$ maps from the real line into $(0,1)$.

[9]This is *not* an innocuous assumption. By assuming no arbitrage (stronger than law of one price) we guarantee the existence of a positive M. Uniqueness of M, however, requires complete markets: a very strong assumption. Without uniqueness not all pricing kernels need to be positive.

Denoting $\mathbf{r}_t = \ln(\mathbf{R}_t)$, with elements $r_{i,t}$, and $m_t = \ln(M_t)$ in (8), and using (7) we get

$$m_t = -r_{i,t} - \mathbb{E}_{t-1}(z_{i,t}) + \varepsilon_{i,t}, \qquad \forall i. \tag{9}$$

Starting from (9), the covered, R^C, and the uncovered return, R^U, on foreign government bonds trade are, respectively,

$$R^C_{t+1} = \frac{{}_tF_{t+1}(1+i^*_{t+1})P_t}{S_tP_{t+1}} \qquad \text{and} \qquad R^U_{t+1} = \frac{S_{t+1}(1+i^*_{t+1})P_t}{S_tP_{t+1}}, \tag{10}$$

where ${}_tF_{t+1}$ and S_t are the forward and spot prices of foreign currency in terms of domestic currency, P_t is the dollar price level and i^*_{t+1} represents nominal net return on a foreign asset in terms of the foreign investor's preferences.

Using a forward version of (9) on both assets, and combining results, yields:

$$s_{t+1} - s_t = ({}_tf_{t+1} - s_t) - [\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})] + \varepsilon_{U,t+1} - \varepsilon_{C,t+1}, \tag{11}$$

where the index i in $\mathbb{E}_t(z_{i,t+1})$ and $\varepsilon_{i,t+1}$ in (9) is substituted by either C or U, respectively for the covered and the uncovered return on trading foreign government bonds.

Under $\alpha_1 = 1$ and $\alpha_0 = 0$ in (1), taking into account (11), allows concluding that:

$$u_{t+1} = -[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})] + \varepsilon_{U,t+1} - \varepsilon_{C,t+1}.$$

Hence, by construction, the error term u_{t+1} is serially correlated because it is a function of current and lagged values of observables.[10] However, in most empirical studies, lagged observables are used as instruments to estimate (1) and test the null that $\alpha_1 = 1$, and $\alpha_0 = 0$. In that context, estimates of α_1 are biased and inconsistent, which may explain the finding that the average value of $\widehat{\alpha_1}$ is -0.88 for over 75 published estimates across various exchange rates and time periods. As far as we know, this is the first instance where FPP results are criticized in this fashion.

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[10]Of course, one can get directly to (1) when $\alpha_1 = 1$ and $\alpha_0 = 0$ using (2) under log-Normality and Homoskedasticity of $M_tR_{i,t}$. One can also do it from (2) if $[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})]$ is constant. However, the conditions are very stringent in both cases: there is overwhelming evidence that returns are not log-Normal and homoskedastic, and to think that $[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})]$ is constant can only be justified as an algebraic simplification for expositional purposes.

Even under log-Normality, if returns are heteroskedastic, then the term $[\mathbb{E}_t(z_{U,t+1}) - \mathbb{E}_t(z_{C,t+1})]$ will be replaced by the difference in conditional variances. Again, this is projection on lagged values of observables, and the same problems alluded above are present.

Because our goal is to relate the two puzzles, it is important to rephrase the FPP in the same language as the EPP. Recalling that rational expectations alone does not restrict the behavior of forward rates, since it is always possible to include a risk-premium term that would reconcile the time series behavior of the involved data, e.g., Fama (1984), the rejection of the null that $\alpha_1 = 0$, in favor of $\alpha_1 < 0$, only represents a true puzzle if reasonable risk measures cannot explain the empirical regularities of the data.

Here is where an asset-pricing approach may help, which is our starting point. The relevant question is whether a theoretically sound economic model is able to provide a definition of risk capable of correctly pricing the forward premium.[11] The natural candidate for a theoretically sound model for pricing risk is the CCAPM of Lucas (1978) and Breeden (1979).

Assuming that the economy has an infinitely lived representative consumer, whose preferences are representable by a von Neumann-Morgenstern utility function $u(\cdot)$, the first order conditions for his(ers) optimal portfolio choice yields

$$1 = \beta \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} R_{i,t+1} \right] \quad \forall i, \tag{12}$$

and, consequently,

$$0 = \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} (R_{i,t+1} - R_{j,t+1}) \right] \quad \forall i,j, \tag{13}$$

where $\beta \in (0,1)$ is the discount factor in the representative agent's utility function, $R_{i,t+1}$ and $R_{j,t+1}$ are, respectively, the real gross return on assets i and j at time $t+1$ and, C_t is aggregate consumption at time t. In other words, under the CCAPM, $M_{t+1} = \beta u'(C_{t+1})/u'(C_t)$.

Let the standing representative agent be a U.S. investor who can freely trade domestic and foreign assets.[12] Define the covered and the uncovered return on trading foreign government bonds as in (10) and substitute R^C for R_i and R^U for R_j in (12)

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[11]Frankel (1979) argues that most exchange rate risks are diversifiable, there being no grounds for agents to be rewarded for holding foreign assets.

[12]Here, we are implicitly assuming the absence of short-sale constraints or other frictions in the economy. Our assumption is in contrast with that of Burnside et al. (2006) for whom bid-ask spreads' impact on the profitability of currency speculation plays the main role in generating the FPP.

to get

$$0 = \mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t(1 + i_{t+1}^*)[_t F_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right]. \qquad (14)$$

Assuming that preferences exhibit constant relative risk aversion, Mark (1985) estimated the parameter α in $u(C) = C^{1-\alpha}(1-\alpha)^{-1}$, applying Hansen's (1982) Generalized Method of Moments (GMM) to (14), reporting an estimated coefficient of relative risk aversion, $\widehat{\alpha}$, above 40. He then tested the over-identifying restrictions to assess the validity of the model, rejecting them when the forward premium and its lags are used as instruments. Similar results were reported later by Modjtahedi (1991). Using a different, larger data set, Hodrick (1989) reported estimated values of $\widehat{\alpha}$ above 60, but did not reject the over-identifying restrictions, while Engel (1996) reported some estimated $\widehat{\alpha}$'s in excess of 100. A more recent attempt to use euler equations to account for the FPP is Lustig and Verdelhan (2006 a), where risk aversion in excess of 100 is needed to price the forward premium on portfolios of foreign currency.

Are we to be surprised with these findings? If we recall that the EPP is identified with the failure of consumption-based kernels to explain the excess return of equity over risk-free short term bonds – $R_{i,t+1} = (1 + i_{t+1}^{SP})P_t/P_{t+1}$ and $R_{j,t+1} = (1 + i_{t+1}^b)P_t/P_{t+1}$, in (13), where i_{t+1}^{SP} is nominal return on S&P500 and i_{t+1}^b nominal return on the U.S. Treasury Bill – with reasonable parameters of risk aversion for (13),[13] why should we expect these same consumption-based models not to generate the FPP? Indeed, we should expect the opposite.

The inexistence of a widely accepted model to account for risk is partly to blame for the separation of the research agendas involving the two puzzles. There is, however, an additional reason. Another characteristic of the FPP may have played a role in the separation of these two research agendas: the predictability of returns on currency speculation. Because $\widehat{\alpha_1} < 0$ and significant, given that the auto-correlation of risk premium is very persistent, interest-rate differentials predict excess returns. Although predictability in equity markets has by now been extensively documented, it was *not* viewed as a defining feature of the EPP, back then. It was, however, a defining feature of the FPP, which has lead Engel (1996, p. 155), for example, to write: "International economists face not only the problem that a high degree of risk aversion is needed to account for estimated values of [the risk premium demanded by a rational agent]. There is also the question of why the forward premium is such

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[13]These asset choices follow Hansen and Singleton (1982, 1984) and Mehra and Prescott (1985).

a good predictor of $s_{t+1} -_t f_{t+1}$. There is no evidence that the proposed solutions to the puzzles in domestic financial markets can shed light on this problem."

Predictability is now acknowledged to be present in domestic markets as well, in the context of the equity premium. Dividend-to-price ratio, and other variables are capable of predicting returns, which means that, once again, we should be suspicious that the same underlying forces may account for asset behavior in both markets.

Before describing our strategy it is important to draw attention to the fact that pricing excess returns is crucial, but should not be the sole goal of asset-pricing theory. Returns, and not only excess returns need to be priced, and accomplishing both is a much harder task. To make the point as stark as possible, let us get back to (12). When we substitute R^C for R_i and R^U for R_j, we get

$$1 = \mathbb{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{{}_t F_{t+1}(1 + i^*_{t+1})P_t}{S_t P_{t+1}} \right] \quad \text{and} \quad 1 = \mathbb{E}_t \left[\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{S_{t+1}(1 + i^*_{t+1})P_t}{S_t P_{t+1}} \right].$$
(15)

It turns out that in the canonical model, e.g., Hansen and Singleton (1982, 1983, 1984), the parameter of risk aversion is the inverse of the intertemporal elasticity of substitution, meaning that if one wants to accept a high risk aversion, one generates implausibly high and volatile interest rates.

Accordingly, if one wants to identify the structural parameter β, in an econometric sense, one cannot resort to direct estimation of excess returns (e.g., 14), but rather to joint estimation of the two euler equations for returns (e.g., 15), or to any linear rotation of them. It is, therefore, important to make a distinction between studies that test the over-identifying restrictions jointly implied by returns and those that test the ones implied by excess returns. For the latter no-rejection may be consistent with *any* value for β, including inadmissible ones.[14]

In our view, a successful consumption-based model must account for asset prices everywhere (domestically and abroad), as well as price returns, excess returns, and many new facts recently evidenced in the extensive empirical research that has been in a great deal sparked by the theoretical developments of the late seventies - see, for example, Cochrane (2006).

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[14]This is for example the case of Lustig and Verdelhan (2006 b), as they point out in their footnote 8.

3 Our Strategy

The stated purpose of this paper is to relate the EPP and the FPP. The fact that, as of this moment, no satisfactory consumption-based model derived from the primitives of the economy can account for asset behavior in either market is a hint that it is generating the two puzzles. We argue here in favor of an indirect approach. We do not need a proper consumption model for the pricing kernel to link the two puzzles. All we need is a strategy to extract a proper pricing kernel from return data, showing that it prices both the domestic and the foreign-exchange returns and excess returns. This isolates current consumption-based kernels as the most likely culprits for mispricing these two markets. Of course, a final proof that these puzzles are linked in this fashion can only be obtained when we finally have a proper consumption-based model to price assets. That will explain why current models fail, something we cannot do here.

Following Harrison and Kreps (1979) Hansen and Richard (1987), and Hansen and Jagganathan (1991), we write the system of asset-pricing equations,

$$1 = \mathbb{E}_t \left[M_{t+1} R_{i,t+1} \right], \quad \forall i = 1, 2, \cdots, N, \tag{16}$$

leading to

$$0 = \mathbb{E}_t \left[M_{t+1} \left(R_{i,t+1} - R_{j,t+1} \right) \right], \quad \forall i, j. \tag{17}$$

We combine statistical methods with these Asset Pricing Equations to devise pricing-kernel estimates as projections of SDF's on the space of returns, i.e., the SDF mimicking portfolio, which is unique even under incomplete markets. We denote the latter by M_{t+1}^*. These pricing kernels do not depend on any assumptions about a functional form for preferences, but solely on returns. In this sense, these methods are *preference free*, despite the fact that they have a strong footing on theory as a consequence of the use of the Asset Pricing Equation.

Our exercise consists in exploring a large cross-section of U.S. time-series stock returns to construct return-based pricing kernel estimates satisfying the Pricing Equation (16) for that group of assets. Then, we take these SDF estimates and use them to price assets not used in constructing them. Therefore, we perform a genuine out-of-sample forecasting exercise using SDF mimicking portfolio estimates, avoiding in-sample over-fitting.

We cannot overstress the importance of out-of-sample forecasting for our purposes. Our main point in this paper is to show that the forward- and the equity-premium puzzle are intertwined. Under the law of one price, an SDF exists that prices all assets, necessarily. Thus, an in-sample exercise would only provide evidence that the forward-premium puzzle is not simply a consequence of violations of the law of one price. We aim at showing more: a SDF can be constructed using only domestic assets, i.e., using the same source of information that guides research regarding the equity premium puzzle, and still price foreign assets. It is our view that this SDF is to capture the growth of the marginal utility of consumption in a model yet to be written.[15]

The main rationale for our methodological choice is the fact that, however successful, a consumption-based model will not perform better in pricing tests than its related mimicking portfolio. This is the main trust of Hansen and Jagganathan (1991). The mimicking portfolio, thus, represents an upper bound on the pricing capability of any model. Given our purposes an alternative approach would be to try and relate the EPP and the FPP using a volatile consumption kernel constructed with high aversion values. The trouble with this approach is that the work by Hansen and Singleton (1982, 1983, 1984) generated a consensus that the over-identifying restrictions of traditional consumption models are often rejected when used to price not only excess returns but also returns, i.e., when both the discount factor β and the risk-aversion coefficient γ are identified in an econometric sense in canonical models. We show that this pattern is present in our data set as well.

Before moving on to the description of our methodology it is worth mentioning that, in dispensing with consumption data, our paper parallels those of Hansen and Hodrick (1983), Hodrick and Srivastava (1984), Cumby (1988), Huang (1989), and Lewis (1990), all of which implemented latent variable models that avoid the need for specifying a model for the pricing kernel by treating the return on a benchmark portfolio as a latent variable. [16] Also related is Korajczyk and Viallet (1992). Applying the arbitrage pricing theory – APT – to a large set of assets from many countries,

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[15]Though important in themselves, market imperfections are sometimes invoked to explain the FPP; see Burnside et al. (2006).

[16]Their results met with partial success: all these papers reject the unbiasedness hypothesis but are in conflict with each other with regards to the rejection of restrictions imposed by the latent-variable model. However, contrary to what we do here, this line of research does not try to relate the EPP and the FPP.

they test whether including the factors as the prices of risk reduces the predictive power of the forward premium. They do not perform any out-of-sample exercises and do not try to relate the two puzzles.

Finally, Backus et al. (1995) ask whether a pricing kernel can be found that satisfies, at the same time, log-linearized versions of

$$0 \;=\; \mathbb{E}_t \left[M_{t+1}^* \frac{P_t(1 + i_{t+1}^*)[_t F_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right] \text{ and,} \tag{18}$$

$$R_{t+1}^f \;=\; \frac{1}{\mathbb{E}_t(M_{t+1}^*)}, \tag{19}$$

where R_{t+1}^f is the risk-free rate of return. The nature of the question we implicitly answer is similar to the one posed by Backus et al. (1995), albeit adding a pricing test of the excess return on equity over risk free short term bonds in the U.S.[17]

3.1 Econometric Tests

Assume that we are able to approximate well enough a time series for the pricing kernel, M_{t+1}^*. Next, we show how to use this approximation to implement direct pricing tests for the forward and the equity-premium, in an euler equation framework. In Section 3.2 we discuss how to construct this time series for M_{t+1}^* using asset-return information.

3.1.1 Pricing Test

In the context of the SDF mimicking portfolio, euler equations (16) and (17) must hold for all assets and portfolios. If we had observations on M_t^*, then we would only need return data to test directly whether they held. Of course, M_t^* is a latent variable. Despite that, if we had a consistent estimator for M_t^* based on return data, and a large enough sample, so that M_t^* and their estimators are "close enough," we could still directly test the validity of these euler equations using return data alone: the estimators of M_t^* are a function of return data and returns are also used to verify the Asset Pricing Equation. In this case, we do not have to perform log-linear approximations of these euler equations, nor do we have to impose the stringent restrictions that returns are log-Normal and Homoskedastic to test theory. Because

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[17] Jumping to our results, we should emphasize that we do not reject (19) for any of the instruments, as well, which means that our SDF satisfies both conditions presented by Backus et al. (1995).

we want our tests to be *out-of-sample* the returns used to construct the estimates of M_t^* will not be the ones used directly in the euler equations when testing theory.

Consider z_t to be a vector of instrumental variables, which are all observed up to time t, therefore measurable with respect to $\mathbb{E}_t(\cdot)$. Employing *scaled returns* and *scaled excess-returns* – defined as $R_{i,t+1} \times z_t$ and $(R_{i,t+1} - R_{j,t+1}) \times z_t$, respectively – we are able to test the conditional moment restrictions associated with the euler equations and consequently to derive the implications from the presence of information. This is particularly important for the FPP, since, when the CCAPM is employed, the over-identifying restriction associated with having the own current forward premium as an instrument is usually rejected: a manifestation of its predictive power.

Multiply

$$0 = \mathbb{E}_t \left[M_{t+1}^* \frac{P_t(1 + i_{t+1}^*)[_tF_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right], \text{ and,} \tag{20}$$

$$1 = \mathbb{E}_t \left[M_{t+1}^* \frac{S_{t+1}(1 + i_{t+1}^*)P_t}{S_t P_{t+1}} \right] \tag{21}$$

by z_t and apply the Law-of-Iterated Expectations to get, respectively,

$$0 = \mathbb{E} \left\{ \left[M_{t+1}^* \frac{P_t(1 + i_{t+1}^*)[_tF_{t+1} - S_{t+1}]}{S_t P_{t+1}} \right] \times z_t \right\}, \text{ and,} \tag{22}$$

$$0 = \mathbb{E} \left\{ \left[M_{t+1}^* \frac{S_{t+1}(1 + i_{t+1}^*)P_t}{S_t P_{t+1}} - 1 \right] \times z_t \right\} \tag{23}$$

Equations (22) and (23) form a system of orthogonality restrictions that can be used to assess the pricing behavior of estimates of M_{t+1}^* with respect to the components of the forward premium or any linear rotation of them. Equations in the system can be tested separately or jointly. In testing, we employ a generalized method-of-moment (GMM) perspective, using (22) and (23) as a natural moment restriction to be obeyed. Consider parameters μ_1 and μ_2 in:

$$0 = \mathbb{E} \left\{ \left[M_{t+1}^* \frac{P_t(1 + i_{t+1}^*)[_tF_{t+1} - S_{t+1}]}{S_t P_{t+1}} - \mu_1 \right] \times z_t \right\}, \text{ and,} \tag{24}$$

$$0 = \mathbb{E} \left\{ \left[M_{t+1}^* \frac{S_{t+1}(1 + i_{t+1}^*)P_t}{S_t P_{t+1}} - (1 - \mu_2) \right] \times z_t \right\} \tag{25}$$

We assume that there are enough elements in the vector z_t for μ_1 and μ_2 to be over identified. In order for (22) and (23) to hold, we must have $\mu_1 = 0$ and $\mu_2 = 0$, and the over-identifying restriction $T \times J$ test in Hansen (1982) should not reject

them. This constitutes the econometric testing procedure implemented in this paper to examine whether the FPP holds when return-based pricing kernels are used.

A similar procedure can be implemented for the domestic market equations, in order to investigate domestic stylized facts that escape consumption based models. To analyze the EPP, the system of conditional moment restrictions is given by:

$$0 = \mathbb{E}\left\{\left[M_{t+1}^* \frac{(i_{t+1}^{SP} - i_{t+1}^b)P_t}{P_{t+1}} - \mu_1\right] \times z_t\right\}, \text{ and} \tag{26}$$

$$0 = \mathbb{E}\left\{\left[M_{t+1}^* \frac{(1 + i_{t+1}^{SP})P_t}{P_{t+1}} - (1 - \mu_2)\right] \times z_t\right\}, \tag{27}$$

where i_{t+1}^{SP} and i_{t+1}^b are respectively the returns on the S&P500 and on a U.S. government short-term bond, and we also test whether $\mu_1 = 0$ and $\mu_2 = 0$, and check the appropriateness of the over-identifying restrictions using Hansen's $T \times J$ test[18].

Beyond the high equity Sharp ratio or the reported power of the dividend-price ratio to forecast stock-market returns, the pattern of cross-sectional returns of assets exhibit some "puzzling aspects" as the "size" and the "value" effects – e.g. Fama and French (1996) and Cochrane (2006) –, i.e., the fact that small stocks and of stocks with low market values relative to book values tend to have higher average returns than other stocks.

We follow Fama and French (1993) in using our pricing kernels to try and account for their stock-market factors; zero-cost portfolios which are able to summarize these effects, explaining average returns on stocks and bonds. In this case, the system of conditional moment restrictions is given by:

$$0 = \mathbb{E}\left\{\left[M_{t+1}^*(R_m - R_f)_{t+1} - \mu_1\right] \times z_t\right\}, \tag{28}$$

$$0 = \mathbb{E}\left\{\left[M_{t+1}^* HML_{t+1} - \mu_2\right] \times z_t\right\} \text{ and} \tag{29}$$

$$0 = \mathbb{E}\left\{\left[M_{t+1}^* SMB_{t+1} - \mu_3\right] \times z_t\right\}, \tag{30}$$

where $Rm - Rf$, the excess return on the market, is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate, HML (High Minus Low) is the average return on two value portfolios minus the average return on two growth portfolios and SMB (Small Minus Big) is the average return on three small portfolios minus the average return on three big portfolios.[19] We, again,

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[18]An alternative to the GMM testing procedure described above is to test directly the zero-mean restrictions, instead of estimating by GMM.

[19]See Fama and French (1993) for a complete description of them.

test $\mu_1 = 0$, $\mu_2 = 0$ and $\mu_3 = 0$, and check the appropriateness of the over-identifying restrictions using Hansen's $T \times J$ test.

It worth recalling that we employ an euler-equation framework, something that was missing in the forward-premium literature after Mark (1985). Since the two puzzles are manifest in logs and in levels, by working directly with the Pricing Equation we avoid imposing stringent auxiliary restrictions in hypothesis testing, while keeping the possibility of testing the conditional moments through the use of lagged instruments along the lines of Hansen and Singleton (1982 and 1984) and Mark. We hope to have convinced our readers that the log-linearization of the euler equation is an unnecessary and dangerous detour. Any criticism arising from the use of the log-linear approximation is avoided here.

An important feature of our testing procedure is its out-of-sample character. To preserve the temporal structure of the euler equations, we perform out-of-sample tests in the cross-sectional dimension, i.e., the returns used in estimating M^*_{t+1} exclude the return of the assets appearing directly in our main tests. Therefore, there is no reason for the Asset Pricing Equation to hold for the assets not used in estimating M^*_{t+1}.

3.1.2 Instruments

There seems to be a consensus in the return forecasting literature about the rejection of the time-invariant excess returns hypothesis. However, the question of which variables can be considered as good predictors for returns is still open. Hence, the choice of a representative set of forecasting instruments plays certainly an important role and highlights the relevance of the conditional tests.

Taking into account the fact that expected returns and business cycles are correlated, as documented in Fama and French (1989), for both domestic and international markets, we use the following macroeconomic variables: real consumption instantaneous growth rates, real GDP instantaneous growth rates, and the consumption-GDP ratio. However, since in our exercise, the forecasting variables, the pricing kernels, and the excess returns, are all based on market prices, we also include specific financial variables as instruments, carefully choosing them based on their forecasting potential.

Regarding the FPP, besides using as instruments the past values for the covered and uncovered returns on trading of the respective foreign government bond,

we also use the current value of the forward premium, since the well documented predictability power of this variable is a defining feature of this puzzle.

For the Fama and French (1993) portfolios and the EPP, we use lagged values of the returns on relevant assets as instruments, since one should not omit the possibility that returns could be predictable from past returns for any financial market. Finally, for the EPP we still use the dividend-price ratio, following Campbell and Shiller (1988) and Fama and French (1988), who show evidences of the good performance of this variable as a predictor of stock-market returns.

3.2 Return-Based Pricing Kernels and the SDF Mimicking Portfolio

The basic idea behind estimating return-based pricing kernels with asymptotic techniques is that asset prices (or returns) convey information about the intertemporal marginal rate of substitution in consumption. If the Asset Pricing Equation holds, all returns must have a common factor that can be removed by subtracting any two returns. A common factor is the SDF mimicking portfolio M_{t+1}^*. Because every asset return contains "a piece" of M_{t+1}^*, if we combine a large enough number of returns, the average idiosyncratic component of returns will vanish in limit. Then, if we choose our weights properly, we may end up with the common component of returns, i.e., the SDF mimicking portfolio.

Although the existence of a strictly positive SDF can be proved under no arbitrage, uniqueness of the SDF is harder to obtain, since under incomplete markets there is, in general, a continuum of SDF's pricing all traded securities. However, each M_{t+1} can be written as $M_{t+1} = M_{t+1}^* + \nu_{t+1}$ for some ν_{t+1} obeying $\mathbb{E}_t\left[\nu_{t+1}R_{i,t+1}\right] = 0$ $\forall i$. Since the economic environment we deal with is that of incomplete markets, it only makes sense to devise econometric techniques to estimate the unique SDF mimicking portfolio – M_{t+1}^*.

There are two basic techniques employed here to estimate M_{t+1}^*. The first one uses principal-component and factor analyses. It can be traced back to the work of Ross (1976), developed further by Chamberlain and Rothschild (1983), and Connor and Korajczyk (1986, 1993). A recent additional reference is Bai (2005). This method is asymptotic: either $N \to \infty$ or $N, T \to \infty$, relying on weak law-of-large-numbers to provide consistent estimators of the SDF mimicking portfolio – the unique systematic

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portion of asset returns. An alternative to this asymptotic method is to use a method-of-moment approach, constructing algebraically the unique projection of any SDF on the space of returns. This can be achieved through the following linear combination of traded returns:

$$M^*_{t+1} \equiv \mathbf{1}' \left[\mathbb{E}(\mathbf{R}_{t+1}\mathbf{R}'_{t+1}) \right]^{-1} \mathbf{R}_{t+1},$$

where $\mathbf{1}$ and $\mathbf{R}_{t+1}$ are $N \times 1$ vectors of ones and of traded returns respectively. This technique was proposed by Hansen and Jagganathan (1991) to estimate the SDF mimicking portfolio using the Pricing Equation. For sake of completeness, we present a summary account of these the first method in section 3.2.1, as well as a more complete description of both of them in the Appendix.

3.2.1   Multifactor Models

Factor models summarize the systematic variation of the N elements of the vector $\mathbf{R}_t = (R_{1,t}, R_{2,t}, ..., R_{N,t})'$ using a reduced number of K factors, $K < N$. Consider a K-factor model in $R_{i,t}$:

$$R_{i,t} = a_i + \sum_{k=1}^{K} \beta_{i,k} f_{k,t} + \eta_{it}, \tag{31}$$

where $f_{k,t}$ are zero-mean pervasive factors and, as is usual in factor analysis and

$$\operatorname*{plim}_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \eta_{i,t} = 0.$$

Denote by $\Sigma_{\mathbf{r}} = \mathbb{E}\left(\mathbf{R}_t \mathbf{R}'_t\right) - \mathbb{E}\left(\mathbf{R}_t\right)\mathbb{E}\left(\mathbf{R}'_t\right)$ the variance-covariance matrix of returns. The first principal component of the elements of $\mathbf{R}_t$ is a linear combination $\theta'\mathbf{R}_t$ with maximal variance subject to the normalization that θ has unit norm, i.e., $\theta'\theta = 1$. Subsequent principal components are identified if they are all orthogonal to the previous ones and are subject to the same normalization. The first K principal components of $\mathbf{R}_t$ are consistent estimates of the $f_{k,t}$'s. Factor loadings can be estimated consistently by simple OLS regressions of the form (31).

It is straightforward to connect principal-component and factor analyses with the Pricing Equation, delivering a consistent estimator for M^*_t. Given estimates of a_i, $\beta_{i,k}$, and $f_{k,t}$ in (31), one can write their respective expected-beta return expression:

$$\mathbb{E}(R_i) = \gamma + \sum_{k=1}^{K} \beta_{i,k}\lambda_k, i = 1, 2, ..., N$$

where λ_k is interpreted as the price of the k-th risk factor. The fact that the zero-mean factors $f \equiv \tilde{f} - \mathbb{E}(\tilde{f})$ are such that $\tilde{f}$ are returns with unitary price allows us to measure the λ coefficients directly by

$$\lambda = \mathbb{E}(\tilde{f}) - \gamma$$

and consequently to estimate only γ via a cross-sectional regression[20]. Given these coefficients, one can easily get an estimate of M_t^*,

$$\widetilde{M_t^*} \equiv a + \sum_{k=1}^{K} b_k f_{k,t}$$

where (a, b) is related to (λ, γ) through

$$a \equiv \frac{1}{\gamma} \text{ and } b \equiv -\gamma \left[cov(ff') \right]^{-1} \lambda,$$

It is easy then to see the equivalence between the beta pricing model and the linear model for the SDF. More, it is immediate that

$$\mathbb{E}(\widetilde{M_t^*} R_{i,t}) = 1, \quad i = 1, 2, ..., N.$$

The number of factors used in the empirical analysis is an important issue. We expect K to be rather small. We followed Lehmann and Modest (1988) and Connor and Korajczyk (1988), taking the pragmatic view whereby increasing K until the estimate of M_t^* changed very little due to the last increment in the number of factors.

4   Empirical Results

4.1   Data and Summary Statistics

In principle, whenever econometric or statistical tests are performed, it is preferable to employ a large data set either in the time-series (T) or in the cross-sectional dimension (N). Regarding the FPP, the main limitation is the fact that the Chicago Mercantile Exchange, the pioneer of the financial-futures market, only launched currency futures in 1972. In addition to that, only futures data for a few developed countries are available since then. In order to have a common sample for the largest set of

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[20]Note that we are not assuming the existence of a risk free rate.

countries possible, we considered here U.S. foreign-exchange data for Canada, Germany, Japan, Switzerland and the U.K., covering the period from 1990:1 to 2004:3, on a quarterly frequency. In order to extend the time span of used here, keeping a common sample for all countries, we would have to accept a drastic reduction in the number of countries, which we regard as an inferior choice.

Spot and forward exchange-rate returns were transformed into U.S.\$ real returns using the consumer price index in the U.S. The forward-rate series were extracted from the Chicago Mercantile of Exchange database, while the spot-rate series were extracted from Bank of England database. To study the EPP we used the U.S.\$ real returns on the S&P500 and on 90-day T-Bill. Real returns were obtained using the consumer price index in the U.S.

A second ingredient for testing these two puzzles is to estimate return-based pricing kernels. Again, in choosing return data, we had to deal with the trade-off between N and T. In order to get a larger N, one must accept a reduction in T: disaggregated returns are only available for smaller time spans than aggregated returns. The database used here to estimate the SDF is comprised of U.S.\$ real returns on two hundred U.S. stocks – those with the 200 largest volumes according to CRSP database. Therefore, it is completely U.S. based and available at a very disaggregated level. Our choice of returns to estimate the SDF mimicking portfolio is a direct response to Cochrane's (2001) criticism of in-sample over-fitting: the return data used to construct SDF estimates is not the same used to construct excess returns in foreign markets. Hence, our pricing tests are out-of-sample in the cross-sectional dimension (assets).

All macroeconomic variables used in econometric tests were extracted from FED's FRED database. We also employed additional forecasting financial variables that are specific to each test performed, and are listed in the appropriate tables of results. The Fama-French benchmark factors series were extracted from the French data library. In terms of the notation used in the tables below, we adopted the following: the estimate of M_t^* using multi-factor models is labelled $\widetilde{M_t^*}$, while that using the projection in Hansen and Jagganathan (1991) is labelled $\overline{M_t^*}$.

Table 1 presents a summary statistic of our database over the period 1990:1 to 2004:3. The average real return on the covered trading of foreign government bonds range from 0.91% to 1.78% a year, while that of uncovered trading range from 1.69% to 4.91%. The real return on the S&P500 is 8.78% at an annual rate, while that of

the 90-day T-Bill is 1.42%, with a resulting excess return of 7.28%. As expected, real stock returns are much more volatile than the U.S. Treasury Bill return – annualized standard deviations of 16.82% and 1.10% respectively. Over the same period, except for the Swiss case, the real return on covered trading of foreign bonds show means and standard deviations quite similar to that of the U.S. Treasury Bill. Regarding the return on uncovered trading, means range from 1.69% to 4.91%, while standard deviations range from 5.52% to 15.50%.

We computed the Sharpe ratio for the U.S. stock market to be 0.44, while the Sharpe ratio of the uncovered trading of foreign bonds ranges from −0.01 to 0.38. According to Shiller (1982), Hansen and Jagganathan (1991), and Cochrane and Hansen (1992), an extremely volatile SDF is required to match the high equity Sharpe ratio of the U.S. Hence, the smoothness of aggregate consumption growth is the main reason behind the EPP. Since the higher the Sharpe ratio, the tighter the lower bound on the volatility of the pricing kernel, a natural question that arises is the following: may we regard this fact as evidence that a kernel that prices correctly the equity premium would also price correctly the forward premium? We will try to answer this question here in an indirect way.

4.2   SDF Estimates

In constructing $\widetilde{M_t^*}$, from the real returns on the two hundred most traded (volume) U.S. stocks, we must first choose the number of factors, i.e., how many pervasive factors are needed to explain reasonably well the variation of these 200 stock returns? Following Lehmann and Modest (1988), Connor and Korajczyk (1988), and most of the empirical literature, we took a pragmatic view, increasing the number of factors until $\widetilde{M_t^*}$ changed very little with respect to choosing an additional factor. We concluded that 6 factors are needed to account for the variation of our 200 stock returns: starting from 3 factors, increasing this number up to 6, implies very different estimates of M_t^*. However, starting from 6 factors, increasing this number up to 8, implies practically the same estimate of M_t^*. Our choice (6 factors) is identical to that of Connor and Korajczyk (1993), who examined returns from stocks listed on the New York Stock Exchange and the American Stock Exchange.

Looking at the final linear combination of returns that comprise $\widetilde{M_t^*}$, we list the following most relevant stocks in its composition: Informix Corporation (13th

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largest volume), AMR Corporation DEL (64th), Emulex Corporation (98th), Erics-son L M Telephone Corporation (99th), Iomega Corporation (118th), LSI Corpora-tion (124th), Lam Resch Corporation (125th), Advanced Micro Services Inc. (154th) and 3-Com Corporation (193th).

In constructing $\overline{M_t^*}$, a practical numerical problem had to be faced, which is how to invert the second-moment matrix $\mathbb{E}\left(\mathbf{R}_{t+1}\mathbf{R}_{t+1}'\right)$ – a square matrix of order 200. Standard inversion algorithms broke down and we had to resort to the Moore-Penrose generalized inverse technique.

The estimates of M_t^* – $\overline{M_t^*}$ and $\widetilde{M_t^*}$ – are plotted in Figure 1, which also includes their summary statistics. Their means are slightly below unity, 0.962 and 0.977 respectively. Moreover, the mimicking portfolio estimate is about twice more volatile than the multi-factor model estimate. The correlation coefficient between $\overline{M_t^*}$ and $\widetilde{M_t^*}$ is 0.407.

4.3 Pricing-Test Results

Table 2 presents results of the over-identifying-restriction tests when consumption-based kernels are employed and excess returns are represented by the equity premium in the U.S. These results will be later compared to those using return-based kernels. We considered three types of preference representations here: standard CRRA, fol-lowing Hansen and Singleton (1982, 1983, 1984), Kreps-Porteus, following Epstein and Zin (1991), and External Habit, following Abel (1990). Tests are conducted separately for the euler equation for excess returns and for the two euler equations for returns. In the former case, the discount rate β is not identified, which is not a feature of the latter.

The top portion of Table 2 presents test results when excess returns (U.S. equity versus U.S. government bonds) are considered, i.e., when β is not identified. In this case, the over-identifying-restriction test does not reject the null at 5% significance regardless of the type of the utility function we considered. Since β is not identified, the euler equation for excess returns is consistent with any arbitrary value of β. Notice that estimates of the constant relative-risk aversion coefficient are in excess of 160 for all preference specifications used, which is similar to the results obtained by Lustig and Verdelhan (2006 a).[21]

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[21]It is straightforward to understand why we obtain these results for CRRA utility. In this case,

The lower portion of Table 2 presents test results for a system of two euler equations for U.S. equities and government bonds. Here, both the discount rate β and the risk-aversion coefficient α are identified. A completely different result with respect to Table 2 emerges in this case: with very high confidence, the over-identifying-restriction test rejects the null regardless of the preference-specification being considered; estimates of α are relatively small, but significant; and β estimates are close to unity and significant as well. Because of the overwhelming rejection of the over-identifying-restriction test, we conclude that the EPP is a feature of our data set: we cannot reconcile data and theory using standard econometric tests and at the same time obtain "reasonable" parameter estimates. When testing did not reject the over-identifying-restrictions – results in Table 2 – β was not identified, and estimates of α were in excess of 160, which are far from what we may call reasonable.

Table 3 presents single-equation equity-premium test results when return-based pricing kernel estimates are used in place of consumption-based kernels. A variety of macroeconomic and financial instruments, including up to their own two lags, are employed in testing. At the 5% significance level, when $\widetilde{M_t^*}$ is used, there is only one instance when the $T \times J$ statistic rejects the null. This happens when the dividend-price ratio $\frac{D_t}{P_t}$ (up to its own two lags) is used as an instrument. When the mimicking portfolio is estimated using $\overline{M_t^*}$, there is no rejection of the $T \times J$ statistic at the 5% significance level. Also, in all instances, estimates of μ_1 and μ_2 obey $\mu_1 = 0$ and $\mu_2 = 0$ at the usual confidence levels.

Table 4 presents the first set of results regarding the forward-premium puzzle, in a single-equation context, where $\widetilde{M_t^*}$ and $\overline{M_t^*}$, as estimators of M_t^*, are used to price the excess return of uncovered over covered trading with foreign government bonds, while Table 5 presents tests, where $\widetilde{M_t^*}$ and $\overline{M_t^*}$ are used to price the return on uncovered trading with foreign government bonds. In both tables, at the 5% significance level, there is not one single rejection either in mean tests for μ_1 and μ_2 or for the $T \times J$ statistic.

Table 6 presents equity-premium tests for systems, when $\widetilde{M_t^*}$ and $\overline{M_t^*}$ are used. No theoretical restriction is rejected here either by individual tests for μ_1 and μ_2 or for

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$M_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}$, and GMM chooses α as to make a quadratic form using $M_{t+1} \frac{(i_{t+1}^{SP} - i_{t+1}^b) P_t}{P_{t+1}} \times z_t$ as close to zero as possible. A large value for α makes $\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha} = \left(\frac{C_t}{C_{t+1}}\right)^{\alpha}$ close to zero, since $\frac{C_t}{C_{t+1}}$ is usually smaller than unity.

the $T \times J$ statistic. Table 7 presents system tests for the Fama-French portfolios. We do not reject the null for the market excess return $(R_m - R_f)$ and for the small/big (SMB) portfolio, but not for the high/low (HML) one. It is worth stressing that, even then, the over-identifying-restriction test does not reject the null.

Finally, Table 8 presents system tests for the FPP when $\widetilde{M_t^*}$ and $\overline{M_t^*}$ are used respectively. At 5% significance, there is a single rejection (out of 10) of the over-identifying-restriction test for German bonds with $\overline{M_t^*}$. Also, for British bonds, there is evidence that $\mu_1 \neq 0$, although the $T \times J$ statistic does not reject the null.

4.3.1 Discussion

In this paper, we first questioned the standard testing procedure of the FPP, relying on estimates $\widehat{\alpha_0}$ and $\widehat{\alpha_1}$, obtained from running

$$s_{t+1} - s_t = \alpha_0 + \alpha_1 (_t f_{t+1} - s_t) + u_{t+1},$$

from a theoretical point of view. Our key point is that $_t f_{t+1} - s_t$ and u_{t+1} are correlated, and that lagged observables are not valid instruments. Since these are exactly the instrumental variables used in obtaining $\widehat{\alpha_0}$ and $\widehat{\alpha_1}$, tests of the FPP relying on such estimates are biased and inconsistent.

Next, we show evidence of the EPP in our data, if one takes the EPP to mean "the failure of consumption-based kernels to explain the excess return of equity over risk-free short term bonds with reasonable parameters values for risk aversion." This is a consequence of the results obtained in Table 2, where system tests involving the returns of these two assets overwhelmingly rejected the implied over-identifying restrictions, and single-equation estimates of the risk-aversion coefficient were in excess of 160, regardless of the preference specification employed. A risk-aversion coefficient greater than 160 cannot be called "reasonable" under any circumstance, especially if it is obtained only in models where the discount rate coefficient β is not identified in the econometric sense. Recent work by Lustig and Verdelhan (2006a) estimates the coefficient of relative risk aversion to fit an euler equation under more general preferences than those used in Mark (1985) and Hodrick (1987) capable of pricing the returns on eight different portfolios of foreign currencies. Because they do not try to price returns on these portfolios, one cannot fully assess the adequacy of the pricing kernels they generate.

Next, we showed that, using return-based pricing kernels, in an euler-equation setup, we are able to properly price returns and excess returns of assets that comprise the equity premium and the forward premium puzzles; see results presented in Tables 3 through 8, where several econometric tests were performed, either in single equations or in systems, and using two distinct estimates of $M_t^* - \overline{M_t^*}$ and $\widetilde{M_t^*}$. These test results are very informative for at least two reasons. First, even if we do not have a consumption model that delivers a proper pricing kernel, the mimicking portfolio is a valid kernel. Second, our tests have an out-of-sample character in the cross-sectional dimension.

One important element of our testing procedure is that, when the ratio $_tF_{t+1}/S_t$ is used as an instrument, the theoretical restrictions tested were not rejected, leading to the conclusion that $_tF_{t+1}/S_t$ has no predictive power for $M_{t+1}^* \frac{P_t(1+i_{t+1}^*)}{P_{t+1}} \frac{_tF_{t+1}-S_{t+1}}{S_t}$. Hence, although the excess returns on uncovered over covered trading with foreign bonds are predictable, "risk adjusted" excess returns are not. This raises the question that predictability results using (1) may just be an artifact of the log-linear approximation of the euler equation for excess returns. This is a very important result, since predictability of $s_{t+1} - s_t$ in (1) is a defining feature of the forward-premium puzzle.

Given that Mark (1985) found evidence of the FPP in proper econometric tests, also found by Hansen and Singleton (1982, 1984) regarding the EPP[22], and that we show enough evidence that proper estimates of M_t^* do not misprice returns comprising the equity premium and the forward premium, we conclude that the EPP and the FPP are "two symptoms of the same illness" – the poor (although steadily improving) performance of current consumption-based pricing kernels to price asset returns or excess returns. Our result takes the correlation with the pricing kernel as the appropriate measure of risk, and adds to the body of evidence that "explains" the forward premium as a risk premium. The reason why we quote the term explains is because, we have not tried to advance on the explanation of either puzzle, but simply to show that the two are related.

Even though we believe that we have elements to give a positive answer to the question posed in the title of this paper, a different question we may ask is: should the forward premium be regarded as a reward for risk taking? If we take the covariance with M_{t+1}^* as the relevant measure of risk, then our answer is yes. In this sense we side with the position implicit in Brandt et al. (2006), where the behavior of

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[22]Corroborated by the evidence also shown in Table 2.

the SDF is viewed as being equal to that of the marginal rate of substitution for a model of preferences and/or market structure yet to be written. However, this is not without controversy. Citing Engel (1996, p. 162): "If the [CAPM] model were found to provide a good description of excess returns in foreign exchange markets, there would be some ambiguity about whether these predicted excess returns actually represent premiums."

5   Conclusion

Previous research has cast doubt on whether consumption-based pricing kernels were capable of correctly pricing the equity and the forward premium,[23] generating respectively the EPP and the FPP, with two different literatures. Here, we propose a fresh look into the relationship between these well known puzzles. We employ an asset-pricing approach. Our starting point is the Asset Pricing Equation, coupled with the use of consistent estimators of the SDF mimicking portfolio. They are a function of return data alone and do not depend on aggregate consumption or on any parametric representation for preferences. In this context, we first show that, our estimated return-based kernels price correctly the equity and the forward premium, as well as the individual returns that compose them. Our estimates are constructed using domestic (U.S.) returns alone.

Based on our empirical results, we go one step further and ask whether the EPP and the FPP are but two symptoms of the same illness – the inability of standard (and augmented) consumption-based pricing kernels to price asset returns or excess-returns. Given the tendency in the profession of generating new research agendas whenever a new empirical regularity that cannot be accounted for by our models is discovered, it is important to always ask whether these are distinct phenomena or if they are but two manifestations of a same problem. Otherwise, in the limit, we could find as many puzzles as there were assets. Since the number of assets in any real economy is large, would it make any sense to investigate all of them separately? Obviously not. What we are able to show is that, indeed, regarding the EPP and the FPP, finding a model that does account for either puzzle is bound to double its prize by accounting for the other.

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[23]A more optimistic viewpoint is offered by Cochrane (2006), who contrast the disheartening results of the first years of this research with the recent success stories.

Our empirical tests are robust to important sources of misspecification incurred by the previous literature: inappropriate log-linear approximation of the euler equation for returns and inappropriate models for consumption-based kernels.[24]

Our return-based pricing kernels, constructed using only U.S.-stock returns, are orthogonal to past information that is usually known to forecast undiscounted excess returns. Moreover, they are also able to price the equity premium for the U.S. and the exchange-rate forward premium for five distinct developed economies: United Kingdom, Canada, Germany, Japan and Switzerland. In our tests, we found that the *ex-ante* forward premium is not a predictor of discounted excess returns, as is usually found when a log-linear approximation of the Asset Pricing Equation is employed in testing. This evidence, coupled with our theoretical discussion on the log-linear approximation of the euler equation, cast doubt on the predictability of exchange-rate changes. In our opinion predictability may be a consequence of the inappropriateness of the log-linear approximations previously employed in testing theory.

In our tests, although consumption-based kernels are not able to correctly price returns, return-based kernels are. This provides the basis for believing that the two puzzles are two symptoms of the same illness, being therefore more profitable to concentrate efforts on a single research agenda, focused on rethinking consumption-based pricing kernels. As stressed before, the final proof that these puzzles are linked can only be obtained when we finally have a proper consumption-based model to price assets. That will explain why current models failed, something we cannot do today.

References

[1] **Abel, A.** (1990), "Asset prices under habit formation and catching up with the Joneses," *American Economic Review Papers and Proceedings* 80, 38 - 42.

[2] **Bai, J.** (2005), "Panel data models with interactive fixed effects," Working Paper: New York University.

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[24]There is obviously a problem with current consumption models for the pricing kernel. Therefore, the profession can only claim to have solved the puzzles when we are finally able to write down a proper working model for aggregate consumption.

[3] **Backus, D., Foresi, S. and Telmer, C.** (1995), "Interpreting the forward premium anomaly," *The Canadian Journal of Economics* 28, 108 - 119.

[4] **Brandt, M. W., Cochrane, J. H. and Santa-Clara, P.** (2006), "International risk sharing is better than you think, or exchange rates are too smooth," *Journal of Monetary Economics* 53, 671 - 698.

[5] **Breeden, D.** (1979), "An intertemporal asset pricing model with stochastic consumption and investment opportunities," *Journal of Financial Economics* 7, 265 - 296.

[6] **Burnside, C. , Eichenbaum, M. Kleshchlski, I. and Rebelo, S.** (2006), "The returns to currency speculation," NBER Working paper #12489.

[7] **Campbell, J. Y. and Cochrane, J. H.** (1999), "By force of habit: a consumption-based explanation of aggregate market behavior," *Journal of Political Economy* 107, 205 - 251.

[8] **Campbell, J. Y. and Shiller, R. J.** (1988), "The dividend-price ratio and the expectations of future dividends and discount factors," *Review of Financial Studies* 1, 195 - 228.

[9] **Chamberlain, Gary, and Rothschild, M.** (1983), "Arbitrage, factor structure, and mean-variance analysis on large asset markets," *Econometrica*, vol. 51(5), pp. 1281-1304.

[10] **Cochrane, J.H.** (2001), "Asset pricing," Princeton Univerity Press.

[11] **Cochrane, J.H.** (2006), "Financial markets and the real economy," Mimeo.

[12] **Cochrane, J.H. and Hansen, L.** (1992), "Asset pricing explorations for macroeconomics," NBER Working paper # 12026.

[13] **Connor, G., and R. Korajzcyk** (1986), "Performance measurement with the arbitrage pricing theory: A New Framework for Analysis," *Journal of Financial Economics*, 15, 373-394.

[14] **Connor, G., and R. Korajzcyk** (1988), "Risk and return in an equilibrium APT: application of a new test methodology," *Journal of Financial Economics*, 21, 255-290.

[15] **Connor, G. and Korajczyk, R.** (1993), "A test for the number of factors in am approximate factor structure", *Journal of Finance* 48, 1263 - 1291.

[16] **Constantinides, G. M. and Duffie, D.** (1996), "Asset pricing with heterogeneous consumers," *Journal of Political Economy* 104, 219 - 240.

[17] **Cumby, R. E.** (1988), "Is it risk? Explaining deviations from uncovered interest parity," *Journal of Monetary Economics* 22, 279 - 299.

[18] **Dhrymes, P. J.** (1974), "Econometrics: statistical foundations and applications," New York, Springer-Verlag.

[19] **Engel, C.** (1996), "The forward discount anomaly and the risk premium: a survey of recent evidence," *Journal of Empirical Finance* 3, 123 - 192.

[20] **Epstein, L. and Zin, S.** (1989), "Substitution, risk aversion and the temporal behavior of consumption and asset returns: a theoretical framework," *Econometrica* 57, 937 - 969.

[21] **Epstein, L. and Zin, S.** (1991), "Substitution, risk aversion and the temporal behavior of consumption and asset returns: an empirical investigation," *Journal of Political Economy* 99, 263 - 286.

[22] **Fama, E. F.** (1984), "Term premiums in bond returns," *Journal of Financial Economics*, 13, 529 - 546.

[23] **Fama, E. F. and Farber, A.** (1979), "Money, bonds and foreign exchange," *American Economic Review* 69, 269 - 282.

[24] **Fama, E. F. and French, K, R.** (1988), "Dividend yields and expected stock returns," *Journal of Financial Economics* 22, 3 - 27.

[25] **Fama, E. F. and French, K, R.** (1989), "Business conditions and the expected stock returns," *Journal of Financial Economics* 25, 23 - 49.

[26] **Fama, E. F. and French, K, R.** (1993), "Common risk factors in the returns on stocks and bonds," *Journal of Financial Economics* 33, 3 - 56.

[27] **Fama, E. F. and French, K, R.** (1996), "Multifactors explanations of asset-pricing anomalies," *Journal of Financial Economics* 51, 55 - 84.

[28] **Frankel, J. A.** (1979), "The diversifiability of exchange risk," *Journal of International Economics* 9, 379-393.

[29] **Froot, K. A.** (1990), "Short rates and expected asset returns," NBER Working paper # 3247.

[30] **Gomes, F.A. R. and Issler, J.V.** (2006), "A General test for rule-of-thumb behavior." Paper presented at the European Econometric Society Meeting of Vienna, downloadable form http://www.eea-esem.com/EEA-ESEM/2006/prog/viewpaper.asp?pid=1764

[31] **Hansen, L. P.** (1982), "Large sample properties of generalized method of moments estimators," *Econometrica* 50, 1029 - 1054.

[32] **Hansen, L. P. and Hodrick, R. J.** (1983), "Risk averse speculation in the forward foreign exchange market: An econometric analysis of linear models," *J.A. Frenkel, ed., Exchange rates and international macroeconomics (University of Chicago Press, Chicago, IL)*, 113 - 142.

[33] **Hansen, L. P. and Jagannathan, R.** (1991), "Restrictions on the intertemporal marginal rates of substitution implied by asset returns," *Journal of Political Economy* 99, 225 - 262.

[34] **Hansen, L. P. and Richard, S. F.** (1987), "The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models," *Econometrica* 55, 587 - 613.

[35] **Hansen, L. and Singleton, K.** (1982), "Generalized instrumental variables estimation of nonlinear expectations models," *Econometrica* 50(5), pp. 1269-1286.

[36] **Hansen, L. and Singleton, K.** (1983), "Stochastic consumption, risk aversion and the temporal behavior of asset returns". *Journal of Political Economy* 91(2), 249-265.

[37] **Hansen, L. and Singleton, K.** (1984), "Erratum of the article: "Generalized instrumental variables estimation of nonlinear expectations models," *Econometrica* 52(1), pp. 267-268.

[38] **Harrison, J. M. and Kreps, D. M.** (1979), "Martingales and arbitrage in multiperiod securities markets," *Journal of Economic Theory* 20, 249 - 268.

[39] **Hodrick, R. J.** (1981), "International asset pricing with time-varying risk premiums," *Journal of International Economics* 11, 573 - 587.

[40] **Hodrick, R. J.** (1987), "The empirical evidence on the efficiency of forward and futures foreign exchange markets" (Harwood, Chur).

[41] **Hodrick, R. J.** (1989), "US international capital flows: perspectives from rational maximizing models," Carnegie-Rochester Conference Series on Public Policy 30, 1 - 29.

[42] **Hodrick, R. J. and Srivastava, S.** (1984), "An investigation of risk premiums and expected future spot rates," *Journal of International Money and Finance* 5, 5 - 21.

[43] **Huang, R. D.** (1989), "An analysis of intertemporal pricing for forward exchange contracts," *Journal of Finance* 44, 183 - 194.

[44] **Korajczyk, R. A. and Viallet, C. J.** (1992), "Equity risk premia and the pricing of foreign exchange risk," *Journal of International Economics* 33, 199 - 219.

[45] **Lehmann, B. and Modest, D.** (1988), "The empirical foundations of the arbitrage pricing theory," *Journal of Financial Economics*, 21, 213-254.

[46] **Lewis, K. K.** (1990), "The behavior of Eurocurrency returns across different holding periods and regimes," *Journal of Finance* 45, 1211 - 1236.

[47] **Lucas, R. E.** (1978), "Asset pricing in an exchange economy," *Econometrica*, vol. 46, pp. 1429 - 1445.

[48] **Lucas, R. E.** (1982), "Interest rates and currency prices in a two-country world," *Journal of Monetary Economics* 10, 335 - 360.

[49] **Lustig, H. and Verdelhan, A.** (2006 a), "The cross-section of foreign currency risk premia and consumption growth risk," *American Economic Review*, forthcoming.

[50] **Lustig, H. and Verdelhan, A.** (2006 b) "Investing in foreign currency is like betting on your intertemporal marginal rate of substitution", *Journal of the European Economic Association*, 4:644-655.

[51] **Mark, N.** (1985), "On time varying risk premiums in the foreign exchange market: an econometric analysis," *Journal of Monetary Economics* 16, 3 - 18.

[52] **Mehra, R. and Prescott, E.** (1985), "The equity premium: a puzzle," *Journal of Monetary Economics* 15, 627 - 636.

[53] **Modjtahedi, B.** (1991), "Multiple maturities and time-varying risk premia in forward exchange markets: An econometric analysis," *Journal of International Economics* 30, 69-86.

[54] **Ross, S. A.** (1976), "The arbitrage theory of capital asset pricing," *Journal of Economic Theory*, 13, pp. 341-360.

[55] **Shiller R. J.** (1982), "Consumption, asset markets and economic fluctuations," *Carnegie-Rochester Conference on Public Policy*, 17, pp. 203-238.

A Return-Based Estimates of the SDF Mimicking Portfolio

A.1 The Approach of Hansen and Jagganathan (1991)

Given a set of N traded returns stacked in a vector $\mathbf{R}_{t+1}$, Hansen and Jagganathan construct algebraically the unique projection of any SDF on the space of returns. It is given by the following linear combination of traded returns:

$$\overline{M_{t+1}^*} \equiv \mathbf{1}' \left[\mathbb{E}(\mathbf{R}_{t+1}\mathbf{R}_{t+1}') \right]^{-1} \mathbf{R}_{t+1}, \tag{32}$$

where $\mathbf{1}$ is $N \times 1$ a vector of ones. It is straightforward to verify that the estimator of M_{t+1}^* will obey a vector version of the Pricing Equation, since:

$$\mathbb{E}\left\{ \overline{M_{t+1}^*}\mathbf{R}_{t+1}' \right\} = \left\{ \mathbf{1}' \left[\mathbb{E}(\mathbf{R}_{t+1}\mathbf{R}_{t+1}') \right]^{-1} \mathbb{E} \left[\mathbf{R}_{t+1}\mathbf{R}_{t+1}' \right] \right\} = \mathbf{1}'.$$

Even is all returns are non-negative, it is possible that the projection of the SDF on the space of returns to be negative for some t, although this is not very common empirically.

A.2 The Multifactor-Model Approach

Principal components are simply linear combinations of returns. They are constructed to be orthogonal to each other, to be normalized to have a unit length and to deal with the problem of redundant returns, which is very common when a large number of assets is considered. They are ordered so that the first principal component explains the largest portion of the sample variance-covariance matrix of returns, the second one explains the next largest portion, and so on.

Factor models summarize the systematic variation of the N elements of the vector $\mathbf{R}_t = (R_{1,t}, R_{2,t}, \ ..., R_{N,t})'$ using a reduced number of K factors, $K < N$. Consider a K-factor model in R_t^i:

$$R_{i,t} = a_i + \sum_{k=1}^{K} \beta_{i,k} f_{k,t} + \eta_{it} \tag{33}$$

where $f_{k,t}$ are zero-mean pervasive factors and, as is usual in factor analysis,

$$\text{plim} \frac{1}{N} \sum_{i=1}^{N} \eta_{i,t} = 0.$$

Denote by $\Sigma_{\mathbf{r}} = \mathbb{E}\left(\mathbf{R}_t \mathbf{R}_t'\right) - \mathbb{E}\left(\mathbf{R}_t\right) \mathbb{E}\left(\mathbf{R}_t'\right)$ the variance-covariance matrix of returns. The first principal component of the elements of $\mathbf{R}_t$ is a linear combination $\theta' \mathbf{R}_t$ with maximal variance subject to the normalization that θ has unit norm, i.e., $\theta' \theta = 1$. Subsequent principal components are identified if they are all orthogonal to the previous ones and are subject to the same normalization. The first K principal components of $\mathbf{R}_t$ are consistent estimates of the $f_{k,t}$'s. Factor loadings can be estimated consistently by simple OLS regressions of the form (33).

To understand why we need the unit-norm condition, recall that the variance of the first principal component $\theta' \mathbf{R}_t$ is $\theta' \Sigma_{\mathbf{r}} \theta$. As discussed in Dhrymes (1974), because we want to maximize this variance, the problem has no unique solution – we can make the variance as large as we want by multiplying θ by a constant $\kappa > 1$. Indeed, we are facing a *scale* problem, which is solved by imposing unit norm, i.e., $\theta' \theta = 1$.

When a multi-factor approach is used, the first step is to specify the number of factors by means of a statistical or theoretical method and then to consider the estimation of the model with known factors. Here, we are not particularly concerned about identifying the factors themselves. Rather, we are interested in constructing

an estimate of the unique SDF mimicking portfolio M_t^*. For that, we shall rely on a purely statistical model, combined with the Pricing Equation.

Given estimates of the factor model of R_t^i in (33), one can write the respective expected-beta return expression

$$\mathbb{E}(R_i) = \gamma + \sum_{k=1}^{K} \beta_{i,k}\lambda_k, \qquad i = 1, 2, ..., N$$

where λ_k is interpreted as the price of the k-th risk factor. The fact that the zero-mean factors $f \equiv \tilde{f} - \mathbb{E}(\tilde{f})$ are such that $\tilde{f}$ are returns with unitary price allows us to measure the λ coefficients directly by

$$\lambda = \mathbb{E}(\tilde{f}) - \gamma$$

and consequently to estimate only γ via a cross-sectional regression[25].

Given these coefficients, one can easily get an estimate of M_t^*,

$$\widetilde{M_t^*} \equiv a + \sum_{k=1}^{K} b_k f_{k,t}$$

where (a, b) is related to (λ, γ) through

$$a \equiv \frac{1}{\gamma} \text{ and } b \equiv -\gamma \left[cov(ff') \right]^{-1} \lambda,$$

It is easy then to see the equivalence between the beta pricing model and the linear model for the SDF. More, it is immediate that

$$\mathbb{E}(\widetilde{M_t^*} R_{i,t}) = 1, \quad i = 1, 2, ..., N.$$

It is important to stress that we need to impose a scale whenever a factor-model is used. Here, it is implicitly imposed that $VAR\left(\widetilde{M_t^*}\right) = 1$; see Cochrane (2001).

The number of factors used in the empirical analysis is an important issue. We expect K to be rather small, but have some flexibility for this choice.[26] We followed Lehmann and Modest (1988) and Connor and Korajczyk (1988), taking the pragmatic view whereby increasing K until the estimate of M_t^* changed very little due to the last increment in the number of factors. We also performed a robustness analysis for the results of all of our statistical tests using different estimates of M_t^* associated with different K's. Results changed very little around our choice of K.

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[25]One should note that we are not assuming the existence of a risk free rate. If this is the case, rather then estimate the intercept γ, one set it equal to the real return of the risk free rate.

[26]Despite this relevance, the pure number of factors is not a meaninful question.

B Tables and Figures

Table 1: Data summary statistics

International quarterly data: observations from 1990:I to 2004:III

Government bonds	US$ real net return on the covered trading of foreign government bonds		US$ real net return on the uncovered trading of foreign government bonds		US$ real excess return on the uncovered over the covered trading of foreign government bonds	
	Sample Mean (% per year)	Sample S.D. (% per year)	Sample Mean (% per year)	Sample S.D. (% per year)	Sample Mean (% per year)	Sample S.D. (% per year)
British	1.334	1.681	4.911	9.857	3.541	9.421
Canadian	1.746	1.122	1.956	5.517	0.207	5.605
German	0.913	2.824	3.123	11.138	2.195	11.818
Japanese	1.777	1.342	1.691	12.501	-0.084	12.590
Swiss	1.750	1.450	3.049	12.196	1.282	12.344

US quarterly data: observations from 1990:I to 2004:III

US$ real net return on 90-day Treasury-Bill		US$ real net return on S&P500		US$ real excess return on S&P500 over 90-day Treasury-Bill	
Sample Mean (% per year)	Sample S.D. (% per year)	Sample Mean (% per year)	Sample S.D. (% per year)	Sample Mean (% per year)	Sample S.D. (% per year)
1.422	1.098	8.784	16.815	7.285	16.409

Table 2: Testing overidentifying restrictions of consumption models

Testing consumption models taking into account only excess return of S&P500 over 90 day Treasury-Bill

| CRRA Preference | | | Epstein and Zin (1991) Preference | | | | Abel (1990) Preference | | | |

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\left(R_{t+1}^{SP}-R_{t+1}^b\right)\right]=0$$

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\rho(\alpha-1)/(1-\rho)}R_{M,t+1}^{(\rho-\alpha)/(1-\rho)}\left(R_{t+1}^{SP}-R_{t+1}^b\right)\right]=0$$

$$E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\left(\frac{C_t}{C_{t-1}}\right)^{-\kappa(\alpha-1)}\left(R_{t+1}^{SP}-R_{t+1}^b\right)\right]=0$$

α	J-statistic		α	ρ	J-statistic		α	κ	J-statistic
(p-value)	(p-value)		(p-value)	(p-value)	(p-value)		(p-value)	(p-value)	(p-value)
160.252	0.157		181.205	84.562	0.168		164.622	-0.198	0.147
(0.003)	(0.181)		(0.015)	(0.248)	(0.086)		(0.013)	(0.699)	(0.091)

Testing consumption models taking into account both return on S&P500 and excess return of S&P500 over 90 day Treasury-Bill

| CRRA Preference | | | | Epstein and Zin (1991) Preference | | | | | Abel (1990) Preference | | | |

$$\left\{\begin{array}{l}E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\left(R_{t+1}^{SP}-R_{t+1}^b\right)\right]=0\\[2ex]E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}R_{t+1}^{SP}\right]=1\end{array}\right.$$

$$\left\{\begin{array}{l}E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\rho(\alpha-1)/(1-\rho)}R_{M,t+1}^{(\rho-\alpha)/(1-\rho)}\left(R_{t+1}^{SP}-R_{t+1}^b\right)\right]=0\\[2ex]E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho(\alpha-1)/(1-\rho)}R_{M,t+1}^{(\rho-\alpha)/(1-\rho)}R_{t+1}^{SP}\right]=1\end{array}\right.$$

$$\left\{\begin{array}{l}E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\left(\frac{C_t}{C_{t-1}}\right)^{-\kappa(\alpha-1)}\left(R_{t+1}^{SP}-R_{t+1}^b\right)\right]=0\\[2ex]E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\alpha}\left(\frac{C_t}{C_{t-1}}\right)^{-\kappa(\alpha-1)}R_{t+1}^{SP}\right]=1\end{array}\right.$$

β	α	J-statistic		β	α	ρ	J-statistic		β	α	κ	J-statistic
(p-value)	(p-value)	(p-value)		(p-value)	(p-value)	(p-value)	(p-value)		(p-value)	(p-value)	(p-value)	(p-value)
1.009	7.347	0.467		1.011	3.332	577.616	0.469		1.009	9.681	0.289	0.418
(0.000)	(0.000)	(0.007)		(0.000)	(0.000)	(0.855)	(0.004)		(0.000)	(0.000)	(0.043)	(0.012)

Notes: Hansen`s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters over the period from 1990:1 to 2004:3.

Instrument set: real consumption and GDP instantaneous growth rates, consumption/GDP ratio, lagged value of the returns in question and dividend/price ratio and a constant.

Parameters: β is is the one-period discount rate, α is the relative risk aversion coefficient, κ is the time-separability parameter and ρ equals the inverse elasticity of intertemporal substitution.

Figure 1: Pricing kernels with US domestic financial market (1990:I - 2004:III): $\tilde{M}^*_{t+1}$ and $\overline{M}^*_{t+1}$

Legend: Multifactor model — Mimicking portfolio
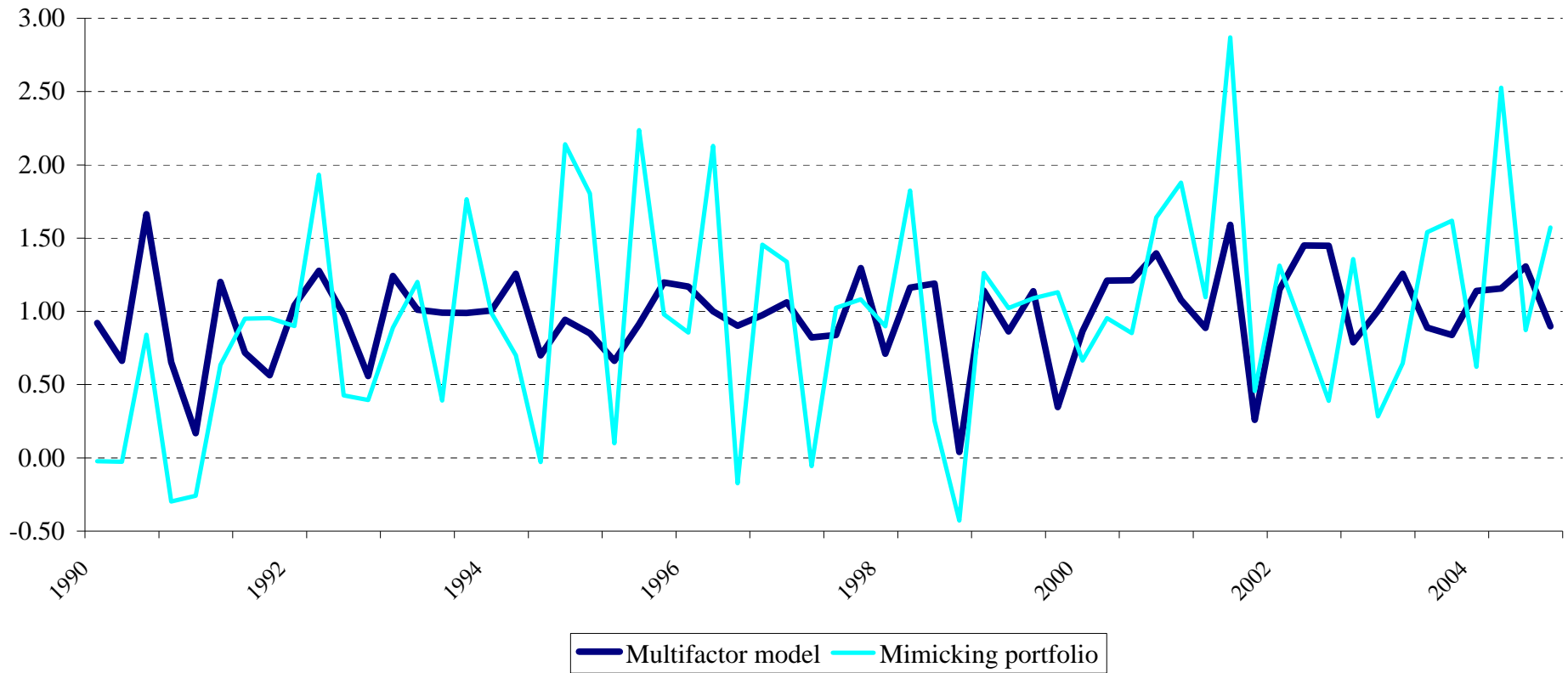
Pricing kernel estimators summary statistics (quarterly)

	Mean	S.D.	Max.	Min.
Multifactor model	0.977	0.318	1.664	0.041
Mimicking portfolio	0.962	0.725	2.871	-0.428

Table 3: Equity-Premium Puzzle tests (single-equation)

Testing the capacity of our return-based pricing kernels to price excess return of S&P500 over 90-day Treasury-Bill

Instrument sets:	$\frac{C_t}{C_{t-1}};\frac{C_{t-1}}{C_{t-2}};\frac{C_{t-2}}{C_{t-3}};const$		$\frac{Y_t}{Y_{t-1}};\frac{Y_{t-1}}{Y_{t-2}};\frac{Y_{t-2}}{Y_{t-3}};const$		$\frac{C_t}{Y_t};\frac{C_{t-1}}{Y_{t-1}};\frac{C_{t-2}}{Y_{t-2}};const$		$R_t^b;R_{t-1}^b;R_{t-2}^b;const.$		$R_t^{SP};R_{t-1}^{SP};R_{t-2}^{SP};const.$		$(D/P)_t; (D/P)_{t-1}; (D/P)_{t-2}; const$	
	$\mu_1 \times 100$	J-stat.	$\mu_1 \times 100$	J-stat.	$\mu_1 \times 100$	J-stat.	$\mu_1 \times 100$	J-stat.	$\mu_1 \times 100$	J-stat.	$\mu_1 \times 100$	J-stat.
Euler equations:	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)
$E_t\left[\tilde{M}^*_{t+1}\dfrac{P_t(i^{SP}_{t+1}-i^b_{t+1})}{P_{t+1}}-\mu_1\right]=0$	0.385 (0.724)	0.028 (0.673)	0.385 (0.750)	0.069 (0.285)	-0.065 (0.963)	0.069 (0.275)	0.385 (0.768)	0.047 (0.456)	0.385 (0.747)	0.056 (0.382)	0.385 (0.686)	0.130 (0.063)
$E_t\left[\bar{M}^*_{t+1}\dfrac{P_t(i^{SP}_{t+1}-i^b_{t+1})}{P_{t+1}}-\mu_1\right]=0$	0.659 (0.571)	0.089 (0.189)	0.659 (0.495)	0.087 (0.197)	0.424 (0.715)	0.089 (0.183)	0.659 (0.640)	0.070 (0.267)	0.659 (0.542)	0.075 (0.238)	0.659 (0.516)	0.075 (0.238)

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Testing the capacity of our return-based pricing kernels to price return on S&P500

| Instrument sets:	$\frac{C_t}{C_{t-1}};\frac{C_{t-1}}{C_{t-2}};\frac{C_{t-2}}{C_{t-3}};const$		$\frac{Y_t}{Y_{t-1}};\frac{Y_{t-1}}{Y_{t-2}};\frac{Y_{t-2}}{Y_{t-3}};const$		$\frac{C_t}{Y_t};\frac{C_{t-1}}{Y_{t-1}};\frac{C_{t-2}}{Y_{t-2}};const$		$R_t^b;R_{t-1}^b;R_{t-2}^b;const.$		$R_t^{SP};R_{t-1}^{SP};R_{t-2}^{SP};const.$		$(D/P)_t; (D/P)_{t-1}; (D/P)_{t-2}; const$	
	$\mu_2 \times 100$	J-stat.	$\mu_2 \times 100$	J-stat.	$\mu_2 \times 100$	J-stat.	$\mu_2 \times 100$	J-stat.	$\mu_2 \times 100$	J-stat.	$\mu_2 \times 100$	J-stat.
Euler equations:	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)
$E_t\left[\tilde{M}^*_{t+1}\dfrac{P_t(1+i^{SP}_{t+1})}{P_{t+1}}-(1-\mu_2)\right]=0$	-2.014 (0.403)	0.071 (0.269)	-2.014 (0.489)	0.043 (0.495)	-1.272 (0.680)	0.006 (0.986)	-2.014 (0.490)	0.044 (0.480)	-2.014 (0.470)	0.111 (0.098)	-2.014 (0.470)	0.144 (0.044)
$E_t\left[\bar{M}^*_{t+1}\dfrac{P_t(1+i^{SP}_{t+1})}{P_{t+1}}-(1-\mu_2)\right]=0$	-0.891 (0.875)	0.094 (0.169)	-0.891 (0.879)	0.046 (0.470)	-0.364 (0.959)	0.010 (0.903)	-0.891 (0.905)	0.117 (0.087)	-0.891 (0.892)	0.053 (0.407)	-0.891 (0.902)	0.087 (0.191)

Notes: Hansens`s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters over the period from 1990:1 to 2004:3.

Table 4: Forward-Premium Puzzle tests (single-equation)

Testing the capacity of our return-based pricing kernels to price excess return of uncovered over covered trading of foreign government bonds

$$E_t\left[\tilde{M}_{t+1}^*\,\frac{P_t(1+i_{t+1}^*)}{P_{t+1}}\,\frac{(S_{t+1}-{}_tF_{t+1})}{S_t}-\mu_1\right]=0 \qquad\qquad E_t\left[\bar{M}_{t+1}^*\,\frac{P_t(1+i_{t+1}^*)}{P_{t+1}}\,\frac{(S_{t+1}-{}_tF_{t+1})}{S_t}-\mu_1\right]=0$$

Instrument sets:	British bonds $\mu_1\times100$ (p-value)	British bonds J-stat. (p-value)	Canadian bonds $\mu_1\times100$ (p-value)	Canadian bonds J-stat. (p-value)	German bonds $\mu_1\times100$ (p-value)	German bonds J-stat. (p-value)	Japanese bonds $\mu_1\times100$ (p-value)	Japanese bonds J-stat. (p-value)	Swiss bonds $\mu_1\times100$ (p-value)	Swiss bonds J-stat. (p-value)	British bonds $\mu_1\times100$ (p-value)	British bonds J-stat. (p-value)	Canadian bonds $\mu_1\times100$ (p-value)	Canadian bonds J-stat. (p-value)	German bonds $\mu_1\times100$ (p-value)	German bonds J-stat. (p-value)	Japanese bonds $\mu_1\times100$ (p-value)	Japanese bonds J-stat. (p-value)	Swiss bonds $\mu_1\times100$ (p-value)	Swiss bonds J-stat. (p-value)
$\frac{C_t}{C_{t-1}};\frac{C_{t-1}}{C_{t-2}};\frac{C_{t-2}}{C_{t-3}};const$	0.694 (0.182)	0.055 (0.398)	-0.019 (0.964)	0.044 (0.486)	0.686 (0.493)	0.080 (0.224)	-0.306 (0.708)	0.081 (0.220)	0.308 (0.703)	0.077 (0.265)	0.840 (0.055)	0.048 (0.454)	-0.442 (0.233)	0.017 (0.812)	0.575 (0.569)	0.056 (0.390)	-1.285 (0.216)	0.057 (0.382)	0.212 (0.816)	0.080 (0.246)
$\frac{Y_t}{Y_{t-1}};\frac{Y_{t-1}}{Y_{t-2}};\frac{Y_{t-2}}{Y_{t-3}};const$	0.694 (0.120)	0.030 (0.649)	-0.019 (0.962)	0.030 (0.649)	0.686 (0.428)	0.038 (0.552)	-0.306 (0.715)	0.039 (0.540)	0.308 (0.617)	0.030 (0.675)	0.840 (0.067)	0.050 (0.438)	-0.442 (0.234)	0.018 (0.798)	0.575 (0.549)	0.061 (0.350)	-1.285 (0.287)	0.047 (0.462)	0.212 (0.798)	0.070 (0.318)
$\frac{C_t}{Y_t};\frac{C_{t-1}}{Y_{t-1}};\frac{C_{t-2}}{Y_{t-2}};const$	1.007 (0.050)	0.025 (0.704)	0.057 (0.884)	0.040 (0.519)	0.879 (0.302)	0.075 (0.238)	0.038 (0.965)	0.069 (0.275)	0.568 (0.379)	0.081 (0.237)	0.990 (0.090)	0.051 (0.423)	-0.396 (0.274)	0.023 (0.728)	0.669 (0.479)	0.099 (0.143)	-1.092 (0.353)	0.097 (0.151)	0.343 (0.670)	0.106 (0.144)
$R_t^C;R_{t-1}^C;R_{t-2}^C;const$	0.694 (0.140)	0.016 (0.821)	-0.019 (0.950)	0.054 (0.398)	0.686 (0.472)	0.071 (0.259)	-0.306 (0.721)	0.113 (0.094)	0.308 (0.677)	0.100 (0.167)	0.840 (0.106)	0.023 (0.728)	-0.442 (0.115)	0.017 (0.808)	0.575 (0.496)	0.043 (0.488)	-1.285 (0.211)	0.103 (0.127)	0.212 (0.791)	0.091 (0.200)
$R_t^U;R_{t-1}^U;R_{t-2}^U;const$	0.694 (0.089)	0.027 (0.679)	-0.019 (0.958)	0.033 (0.605)	0.686 (0.464)	0.093 (0.167)	-0.306 (0.720)	0.092 (0.171)	0.308 (0.640)	0.048 (0.475)	0.840 (0.067)	0.101 (0.135)	-0.442 (0.196)	0.039 (0.532)	0.575 (0.537)	0.079 (0.222)	-1.285 (0.271)	0.111 (0.098)	0.212 (0.805)	0.078 (0.248)
$\frac{{}_tF_{t+1}}{S_t};\frac{{}_{t-1}F_t}{S_{t-1}};\frac{{}_{t-2}F_{t-1}}{S_{t-2}};const$	1.007 (0.076)	0.029 (0.655)	0.057 (0.878)	0.114 (0.092)	0.879 (0.240)	0.066 (0.300)	0.038 (0.966)	0.131 (0.061)	0.568 (0.376)	0.085 (0.222)	0.990 (0.090)	0.051 (0.423)	-0.396 (0.254)	0.108 (0.107)	0.669 (0.473)	0.079 (0.222)	-1.092 (0.250)	0.121 (0.079)	0.343 (0.628)	0.090 (0.204)

Notes: Hansen`s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters over the period from 1990:1 to 2004:3.

Table 5: Forward-Premium Puzzle tests (single-equation)

Testing the capacity of our return-based pricing kernels to price return on uncovered trading of foreign government bonds

$$E_t\left[\tilde{M}^*_{t+1}\frac{P_t(1+i^*_{t+1})}{P_{t+1}}\frac{S_{t+1}}{S_t}-(1-\mu_2)\right]=0 \qquad\qquad E_t\left[\overline{M}^*_{t+1}\frac{P_t(1+i^*_{t+1})}{P_{t+1}}\frac{S_{t+1}}{S_t}-(1-\mu_2)\right]=0$$

	British bonds		Canadian bonds		German bonds		Japanese bonds		Swiss bonds		British bonds		Canadian bonds		German bonds		Japanese bonds		Swiss bonds	
Instrument sets:	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)	$\mu_2\times100$ (p-value)	J-stat. (p-value)
$\frac{C_t}{C_{t-1}};\frac{C_{t-1}}{C_{t-2}};\frac{C_{t-2}}{C_{t-3}};const$	1.703 (0.524)	0.062 (0.342)	2.336 (0.349)	0.063 (0.334)	1.926 (0.487)	0.059 (0.366)	2.625 (0.299)	0.057 (0.382)	2.132 (0.473)	0.068 (0.332)	-1.085 (0.853)	0.089 (0.189)	0.123 (0.983)	0.091 (0.181)	-0.807 (0.893)	0.089 (0.189)	1.002 (0.854)	0.088 (0.193)	-0.202 (0.976)	0.061 (0.385)
$\frac{Y_t}{Y_{t-1}};\frac{Y_{t-1}}{Y_{t-2}};\frac{Y_{t-2}}{Y_{t-3}};const$	1.703 (0.610)	0.037 (0.564)	2.336 (0.463)	0.029 (0.661)	1.926 (0.573)	0.032 (0.625)	2.625 (0.402)	0.042 (0.504)	2.132 (0.561)	0.058 (0.407)	-1.085 (0.862)	0.039 (0.540)	0.123 (0.984)	0.045 (0.478)	-0.807 (0.900)	0.045 (0.478)	1.002 (0.868)	0.050 (0.438)	-0.202 (0.980)	0.038 (0.585)
$\frac{C_t}{Y_t};\frac{C_{t-1}}{Y_{t-1}};\frac{C_{t-2}}{Y_{t-2}};const$	0.197 (0.961)	0.017 (0.808)	1.069 (0.778)	0.018 (0.794)	0.525 (0.900)	0.015 (0.835)	1.086 (0.779)	0.013 (0.862)	0.579 (0.898)	0.027 (0.702)	-0.943 (0.901)	0.011 (0.890)	0.370 (0.960)	0.013 (0.862)	-0.615 (0.938)	0.007 (0.940)	1.099 (0.883)	0.005 (0.962)	-0.028 (0.997)	0.007 (0.946)
$R_t^C;R_{t-1}^C;R_{t-2}^C;const$	1.703 (0.594)	0.062 (0.333)	2.336 (0.450)	0.042 (0.497)	1.926 (0.598)	0.053 (0.406)	2.625 (0.377)	0.062 (0.333)	2.132 (0.577)	0.082 (0.233)	-1.085 (0.883)	0.060 (0.349)	0.123 (0.987)	0.113 (0.094)	-0.807 (0.867)	0.029 (0.655)	1.002 (0.891)	0.071 (0.259)	-0.202 (0.979)	0.146 (0.052)
$R_t^U;R_{t-1}^U;R_{t-2}^U;const$	1.703 (0.588)	0.105 (0.119)	2.336 (0.456)	0.049 (0.439)	1.926 (0.559)	0.099 (0.143)	2.625 (0.391)	0.101 (0.135)	2.132 (0.606)	0.097 (0.178)	-1.085 (0.887)	0.082 (0.210)	0.123 (0.986)	0.031 (0.630)	-0.807 (0.915)	0.104 (0.123)	1.002 (0.882)	0.083 (0.206)	-0.202 (0.981)	0.046 (0.490)
$\frac{_tF_{t+1}}{S_t};\frac{_{t-1}F_t}{S_{t-1}};\frac{_{t-2}F_{t-1}}{S_{t-2}};const$	0.197 (0.960)	0.080 (0.218)	1.069 (0.616)	0.059 (0.357)	0.525 (0.869)	0.044 (0.480)	1.086 (0.741)	0.016 (0.821)	0.579 (0.878)	0.038 (0.577)	-0.943 (0.911)	0.048 (0.447)	0.370 (0.962)	0.071 (0.259)	-0.615 (0.924)	0.056 (0.382)	1.099 (0.881)	0.003 (0.982)	-0.028 (0.997)	0.071 (0.300)

Notes: Hansens`s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters over the period from 1990:1 to 2004:3.

Table 6: Equity-Premium Puzzle tests (system)

Testing the capacity of $\tilde{M}^{*}_{t+1}$ and $\overline{M}^{*}_{t+1}$ to price jointly: return on S&P500 and also excess return of S&P500 over risk-free short-term bonds

$$E_t\left[\tilde{M}^{*}_{t+1}\frac{P_t(i^{SP}_{t+1}-i^{b}_{t+1})}{P_{t+1}}-\mu_1\right]=0 \quad \text{and} \quad E_t\left[\tilde{M}^{*}_{t+1}\frac{P_t(1+i^{SP}_{t+1})}{P_{t+1}}-(1-\mu_2)\right]=0 \qquad E_t\left[\overline{M}^{*}_{t+1}\frac{P_t(i^{SP}_{t+1}-i^{b}_{t+1})}{P_{t+1}}-\mu_1\right]=0 \quad \text{and} \quad E_t\left[\overline{M}^{*}_{t+1}\frac{P_t(1+i^{SP}_{t+1})}{P_{t+1}}-(1-\mu_2)\right]=0$$

Instrument set:	$\mu_1 \times 100$ (p-value)	$\mu_2 \times 100$ (p-value)	J-stat. (p-value)		$\mu_1 \times 100$ (p-value)	$\mu_2 \times 100$ (p-value)	J-stat. (p-value)
$\dfrac{C_t}{C_{t-1}};\dfrac{Y_t}{Y_{t-1}};\dfrac{C_t}{Y_t};R^{SP}_t;$	-0.016	1.775	0.241		-0.414	1.413	0.245
$R^{b}_t;\left(\dfrac{D}{P}\right)_t;const.$	(0.985)	(0.438)	(0.309)		(0.587)	(0.796)	(0.292)

Notes: Hansens`s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters over the period from 1990:1 to 2004:3.

Table 7: Fama-French portfolios pricing tests (system)

Testing the capacity of $\tilde{M}^{*}_{t+1}$ and $\overline{M}^{*}_{t+1}$ to price jointly the Fama-French zero-cost portfolios

$$E_t\left[\tilde{M}^{*}_{t+1}(R_m-R_f)_{t+1}-\mu_1\right]=0,\ E_t\left[\tilde{M}^{*}_{t+1}HML_{t+1}-\mu_2\right]=0 \quad \text{and}\ E_t\left[\tilde{M}^{*}_{t+1}SMB_{t+1}-\mu_3\right]=0 \qquad E_t\left[\overline{M}^{*}_{t+1}(R_m-R_f)_{t+1}-\mu_1\right]=0,\ E_t\left[\overline{M}^{*}_{t+1}HML_{t+1}-\mu_2\right]=0 \quad \text{and}\ E_t\left[\overline{M}^{*}_{t+1}SMB_{t+1}-\mu_3\right]=0$$

Instrument set:	$\mu_1 \times 100$ (p-value)	$\mu_2 \times 100$ (p-value)	$\mu_3 \times 100$ (p-value)	J-stat. (p-value)		$\mu_1 \times 100$ (p-value)	$\mu_2 \times 100$ (p-value)	$\mu_3 \times 100$ (p-value)	J-stat. (p-value)
$\dfrac{C_t}{C_{t-1}};\dfrac{Y_t}{Y_{t-1}};\dfrac{C_t}{Y_t};(R_m-R_f)_t;$	-1.167	1.900	-0.054	0.293		-0.702	2.166	0.199	0.253
$HML_t;SMB_t;const$	(0.165)	(0.003)	(0.925)	(0.707)		(0.326)	(0.000)	(0.684)	(0.829)

Notes: Hansens`s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters over the period from 1990:1 to 2004:3.

Fama-French portfolios: $(R_m - R_f)$ is the excess return on the market, HML is the average return on two value portfolios minus the average return on two growth portfolios and SMB is the average return on three small portfolios minus the average return on three big portfolios.

Table 8: Forward-Premium Puzzle tests (system)

Testing the capacity of $\widetilde{M}^*_{t+1}$ to price jointly: return on uncovered trading of foreign government bonds and also excess return of uncovered over covered trading of foreign government bonds

$$E_t\left[\widetilde{M}^*_{t+1}\frac{P_t(1+i^*_{t+1})}{P_{t+1}}\frac{(S_{t+1}-{}_tF_{t+1})}{S_t}-\mu_1\right]=0 \qquad \text{and} \qquad E_t\left[\widetilde{M}^*_{t+1}\frac{P_t(1+i^*_{t+1})}{P_{t+1}}\frac{S_{t+1}}{S_t}-(1-\mu_2)\right]=0$$

Instrument sets	British bonds			Canadian bonds			German bonds			Japanese bonds			Swiss bonds		
	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.
	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)
$\frac{C_t}{C_{t-1}}$; $\frac{Y_t}{Y_{t-1}}$; $\frac{C_t}{Y_t}$; R^C_t ; R^U_t ; ${}_t\frac{F_{t+1}}{S_t}$; $const.$	1.101	0.657	0.184	0.075	1.606	0.242	0.882	1.072	0.281	0.038	1.642	0.259	0.649	1.093	0.241
	(0.014)	(0.831)	(0.555)	(0.787)	(0.522)	(0.305)	(0.151)	(0.728)	(0.189)	(0.951)	(0.565)	(0.241)	(0.315)	(0.734)	(0.378)

Testing the capacity of $\overline{M}^*_{t+1}$ to price jointly: return on uncovered trading of foreign government bonds and also excess return of uncovered over covered trading of foreign government bonds

$$E_t\left[\overline{M}^*_{t+1}\frac{P_t(1+i^*_{t+1})}{P_{t+1}}\frac{(S_{t+1}-{}_tF_{t+1})}{S_t}-\mu_1\right]=0 \qquad \text{and} \qquad E_t\left[\overline{M}^*_{t+1}\frac{P_t(1+i^*_{t+1})}{P_{t+1}}\frac{S_{t+1}}{S_t}-(1-\mu_2)\right]=0$$

Instrument sets	British bonds			Canadian bonds			German bonds			Japanese bonds			Swiss bonds		
	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.	$\mu_1\times100$	$\mu_2\times100$	J-stat.
	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)	(p-value)
$\frac{C_t}{C_{t-1}}$; $\frac{Y_t}{Y_{t-1}}$; $\frac{C_t}{Y_t}$; R^C_t ; R^U_t ; ${}_t\frac{F_{t+1}}{S_t}$; $const.$	0.969	0.847	0.202	-0.390	2.134	0.187	0.657	1.166	0.380	1.073	2.849	0.247	0.333	1.877	0.232
	(0.016)	(0.878)	(0.470)	(0.146)	(0.693)	(0.540)	(0.334)	(0.824)	(0.039)	(0.176)	(0.656)	(0.284)	(0.425)	(0.695)	(0.412)

Notes: Hansens`s (1982) Generalized Method of Moments (GMM) technique is used to test Euler equations and estimate the model parameters over the period from 1990:1 to 2004:3.