FUNDAÇÃO GETULIO VARGAS

EPGE
Escola de Pós-Graduação em Economia

Ensaios Econômicos

Escola de Pós-Graduação em Economia da Fundação Getulio Vargas

N° 367 ISSN 0104-8910

Strategic Behavior in Repeated Trials of Vickrey Auctions

Flavio Marques Menezes, Paulo Klinger Monteiro

Fevereiro de 2000

URL: http://hdl.handle.net/10438/393

Marques Menezes, Flavio
Strategic Behavior in Repeated Trials of Vickrey Auctions/ Flavio Marques Menezes, Paulo Klinger Monteiro - Rio de Janeiro : FGV,EPGE, 2010
(Ensaios Econômicos; 367)

Inclui bibliografia.

CDD-330

Strategic behavior in repeated trials of Vickrey auctions

Flavio M. Menezes and Paulo K. Monteiro*
FGV/EPGE
Praia de Botafogo 190, 11o. andar
Rio de Janeiro, RJ, 22253-900
Brazil

January 13, 2000

Vickrey's insight that in a second-price sealed-bid auction one's dominant strategy is to bid one's valuation — when competing for a single object under the independent private values assumption — is well understood by most economists. Less well understood is individual behavior in repeated auctions. For example, there are several contexts where bidding one's true valuation is not an equilibrium bidding strategy in a second-price auction . Consider, for instance, two identical objects that are sold *sequentially* by second-price sealed-bid auctions and three potential buyers who want one object each. Although it is a dominant strategy to bid one's true valuation in the last auction, in the first auction bidders bid less than their values – in equilibrium participants bid in such way that the individuals with the two highest valuations are indifferent between winning the first or the second object. (See, for example, Weber (1983)).

In this paper we analyze bidding behavior in repeated trials of second-price auctions. This is motivated by several experimental designs involving multiple trials of second-price auctions and a random draw that determines which round counts. For example, among others, Shogren, Shin, Hayes and Kliebenstein (1994) and Coursey and Schulze (1986) use respectively five repetitions and up to ten repetitions of second-price auctions to analyze the disparity between willingness to pay (by buyers) and willingness to accept (by sellers). In the former study, a random draw determined which round would count and in the later study a voting mechanism was used to determine the binding round (up to the tenth round when the game necessarily ended). In these experiments, typically some individuals in a group are endowed with certain objects (candy bars, coffee mugs, pens, etc.) while others are endowed with money. An experiment is then organized where "sellers" submit ask prices and "buyers" submit bid prices. Prices are determined by a Vickrey auction mechanism. If one object is being traded, a second-price auction is used. If three objects are

*We would like to thank Simon Grant for useful comments.

to be traded, then a fourth-price auction is used, and so on and so forth. At the end of each round bidders learn the outcome of the auction (namely, price and allocation) and at the end of the experiment learn which round will be chosen to determine the binding outcome.

It is commonly argued that repeated trials are needed for individuals to learn that bidding one's valuation is a dominant strategy. For example, Coursey, Hovis and Schulze (1987) argue that "an important observation to be drawn from experimental economics is that individuals participating in a Vickrey auction do not initially reveal "true" values. On a purely theoretical economic basis, they should realize that this is their dominant strategy. However, a number of trial iterations are required to allow individuals to learn that revealing "true" values is their best strategy." Shogren, Shin, Hayes and Kliebenstein (1994) also argue that they use five trials, with the binding trial being selected at random, to allow learning on how to play the game[1].

Another example is Kahneman, Knetsch, and Thaler (1990) who ran a series of experiments two of which are reported here. In the first experiment, "induced value tokens " were traded. In such markets all subjects are told how much a token is worth to them, with the amounts varying across subjects. Half the subjects were allocated tokens, the other half were not. Subjects alternated between being a buyer and being a seller in three successive markets, and were assigned a different individual redemption value in each trial. After each market, both the market-clearing price and the number of trades was announced. Subjects were specifically told that "it is in your best interest to answer these questions truthfully. For any question, treat the price as fixed." Indeed, as valuations were independently drawn after each round, there was no room for strategic behavior. The results confirmed the theoretical prediction.

In the second experiment, which was ran immediately after the first experiment, subjects on alternating seats were given coffee mugs from Cornell University (available for $6,00 at the local bookshop). Subjects were informed that four markets for mugs would be conducted using the same procedures as the prior induced value tokens markets except that only one of the four market trials would subsequently be selected at random as the binding trial. Moreover, all trades would be implemented on the binding market trial, unlike the subset implemented in the induced value tokens markets. The initial assignment of buyer and seller roles was maintained for all four trading periods. The clearing price and the number of trades were announced after each period. The binding trial was indicated after the fourth period, and transactions were executed immediately. If these markets were independent, the theory would predict that half of the mugs should trade. In the four mug markets the number of trades was 4, 1 , 2 and 2 respectively. However, given that transactions were not independent, unlike in the first experiment, there is no clear theoretical prediction —

[1]It is worth noting that Cox, Roberson, and Smith (1982) is usually cited as supporting this view that repeated trials are needed for learning how to play. However, in Cox, Roberson and Smith each trial was binding and genuinely independent of each other so that strategic behavior by bidders between rounds was very unlikely and nonexistent from a theoretical viewpoint.

except for the last trial where there is indeed no incentive for misrepresentation of values.[2] It is our contention that there may be strategic reasons for buyers to understate their true valuations on earlier trials. This is specially relevant in an environment where sellers have the opportunity to behave strategically and to use information revealed by potential buyers on earlier rounds as in the experimental design of Kahneman, Knetsch and Thaler (1990).

In this short paper we demonstrate that it is not a bidder's dominant strategy – indeed it is not even an equilibrium – to bid one's true valuation in the first of two rounds where the binding outcome is chosen by a random device. In our example, the seller's behavior is fixed; he uses the information revealed by bidders in the first round to set the reserve price for the second auction. In particular, the seller sets the second period reserve price equal to the highest bid in the first auction. Indeed we show that when buyers believe that the seller follows this second-period reserve-price policy and the seller is constrained to set a reserve price no greater than the highest bid in the first auction, then it is an equilibrium for the seller to set the reserve price at the highest first-period bid.

Of course, if the seller is free to set any reserve price he likes and if there is a symmetric equilibrium bidding strategy in the first auction that is increasing in values, then the seller will invert first-period bidding strategies, compute the highest valuation and set the second-period reserve price at the highest valuation. However, we show that with a fully strategic seller there is no increasing symmetric equilibrium bidding strategy in the first period. In particular, bidding one's true valuation is not a symmetric equilibrium bidding strategy in the first period even with a fully strategic seller.

This note is meant to illustrate one of several reasons for buyers not to reveal their true valuations in repeated interactions. The possibility of strategic behavior had already been pointed out by Gregory and Furby (1987), who emphasize – in the context of Coursey, Hovis and Schulze (1987), Coursey and Schulze (1986) and related research – that "more sophisticated individuals might use the four non-binding trials strategically, creating bargaining positions to be used in the later (binding) rounds." Here we explore a specific reason for buyers to misrepresent their valuations on earlier rounds.

This provides an alternative explanation for the disparity between willingness to pay and willingness to accept in addition to existing explanations such as the endowment effect (Kahneman and Tversky (1979)) — individuals value the good more highly after they endow them; Hanemann's (1991) substitutability argument — differences between willingness to pay and willingness to accept relate to movements along different indifference curves; or different segments of the same curve; learning about values (Shogren et al. (1994) and Morrison (1997-a,b)) and imprecise preferences (Dubourg, Jones-Lee and Loomes (1994) and Morrison (1998)).

[2] Kahneman, Knetsch and Thaler further test for strategic behavior by selecting the market price by a random draw. They find similar results to the second experiment described above.

1 Bidding Behavior when the seller's strategy is fixed

We assume that there is one object for sale and there are n bidders with independent private values $v_i \in [0,1]$ drawn from a distribution F with density $f(\cdot) > 0$. The auction is organized as follows:

Period 1: Bidders submit bids in a second-price sealed-bid auction and the seller sets a reserve price $R_1 = 0$;

Period 2: Bidders submit new bids in another second-price auction knowing the result of the first auction (price and allocation) and the seller sets a new reserve price $R_2 =$ "winning bid in the first auction";

Period 3: A random device is used to determine which outcome (that is, price and allocation) will be binding. The device assigns probability $\frac{1}{2}$ to each of the auctions.

These rules are common knowledge prior to the start of the bidding process. In particular, the seller's policy on setting the reserve price is common knowledge although the specific value of R_2 is only known at the end of period 1.

We will now compute the equilibrium bidding strategies in both rounds. Suppose $(b_i, c_i)_{i=1}^n$ is an equilibrium of this auction. In the second auction it is a dominant strategy to bid $c_i(v_i) = v_i$ — the second auction is simply a one-shot second-price sealed-bid auction. In the first auction, however, it is not an equilibrium to bid one's true valuation as we demonstrate next!

Theorem 1 *Buyers bid less than their valuations in the first auction symmetric equilibrium when the seller's strategy is fixed.*

Proof. To characterize the symmetric equilibrium bidding strategy $b_i(\cdot) = b_1(\cdot), 2 \leq i \leq n$, we assume that $b(\cdot)$ is a differentiable increasing strategy and find a differential equation that $b(\cdot)$ satisfies. Let's define $Z^1 = \max\{v_i; i \geq 2\}$. If bidders $i = 2, \ldots, n$ bid accordingly to $b(\cdot)$, Bidder 1 has a signal v and submits a bid $x \geq 0$ his expected utility is given by

$$H(x) = E\left[\frac{1}{2}\chi_{x \geq b(Z^1)}\left(v - b\left(Z^1\right)\right) + \frac{1}{2}\left(v - \max\left\{x, b\left(Z^1\right), Z^1\right\}\right)^+\right].$$

Note that $\max\left\{x, b\left(Z^1\right)\right\}$ is the seller´s reserve value for the second auction. $(w)^+$ is the standard notation for $max\{0, w\}$. To maximize H Bidder 1 never chooses $x > v$ since he will never win the second auction and will win the first auction only when either $x > b\left(Z^1\right) > v$ – losing $v - b\left(Z^1\right)$ – or if $v \geq b\left(Z^1\right)$ when by bidding $x = v$ he would have won as well. Therefore we may suppose without loss of generality that $x \leq v$. As $b(\cdot)$ is an equilibrium, we have $b(v) \leq v$. Bidder 1's expected utility can therefore be rewritten as:

$$\begin{aligned} 2H(x) &= E\left[\chi_{x \geq b(Z^1)}\left(v - b\left(Z^1\right)\right) + \left(v - \max\left\{x, Z^1\right\}\right)^+\right] = \\ & \int_0^{b^{-1}(x)} (v - b(z)) f_{Z^1}(z)\, dz + \int_0^1 (v - \max\{x, z\})^+ f_{Z^1}(z)\, dz = \\ & \int_0^{b^{-1}(x)} (v - b(z)) f_{Z^1}(z)\, dz + (v - x) F_{Z^1}(x) + \int_x^1 (v - z)^+ f_{Z^1}(z)\, dz. \end{aligned}$$

The first order condition is

$$2H^{'}(x) =$$

$$\left(b^{-1}(x)\right)^{'} (v - x) f_{Z^1}\left(b^{-1}(x)\right) - F_{Z^1}(x) + (v - x) f_{Z^1}(x) - (v - x) f_{Z^1}(x) = 0.$$

After cancelling the last two terms we obtain

$$\left(b^{-1}(x)\right)^{'} (v - x) f_{Z^1}\left(b^{-1}(x)\right) - F_{Z^1}(x) = 0.$$

In equilibrium $x = b(v)$ and thus we obtain

$$\frac{(v - b(v)) f_{Z^1}(v)}{b^{'}(v)} - F_{Z^1}(b(v)) = 0.$$

Rearranging we obtain the differential equation that characterizes $b(\cdot)$ as follows

$$b^{'}(v) = \frac{(v - b(v)) f_{Z^1}(v))}{F_{Z^1}(b(v))}. \tag{1}$$

The distribution of Z^1 is $F_{Z^1}(u) = F^{n-1}(u)$ and its associated density is $f_{Z^1}(u) = (n-1) F^{n-2}(u) f(u)$. Substituting these expressions into (1) we obtain

$$b^{'}(v) = \frac{(v - b(v))(n-1) F^{n-2}(v) f(v)}{F^{n-1}(b(v))}. \tag{2}$$

Note that $b(v) = v$ is not a solution of (2). ■

For the uniform distribution, for example, expression (2) reduces to:

$$b^{'}(v) = \frac{v - b(v))(n-1) v^{n-2}}{b^{n-1}(v)}.$$

It follows that the function $b(v) = \lambda v$ solves the differential equation if

$$g(\lambda) = \lambda^n - (n-1)(1-\lambda) = 0.$$

Since $g(0) = -n + 1$ and $g(1) = 1$ this equation has a solution $r \in (0, 1)$. Moreover since $g(\cdot)$ is strictly increasing it is the unique positive solution. If $n = 2$ we have that $r = \frac{-1+\sqrt{5}}{2} \approx 0.62$. (We omit the proof that the solution of (2) is really an equilibrium.) That is, bidding one's true valuation is not an equilibrium in the first round. The reason is that by revealing one's true valuation in the first round, one could become pivotal in the second auction as the price in the second auction is simply the maximum of R_2 and the second highest bid in the second auction.

Note that if buyers believe that the seller sets $R_2 = b(v^{(1)})$ and the seller is constrained to choose a reserve price $R_2 \leq b(v^{(1)})$, where $v^{(1)}$ is the expected value of the highest valuation, then his expected profits are

$$\frac{1}{2}b(v^{(2)}) + \frac{1}{2}\max\left\{v^{(2)}, R_2\right\}$$

Clearly, setting $R_2 = b(v^{(1)})$ maximizes the seller's expected profits and therefore we have fully described an equilibrium for these repeated trials of Vickrey auctions with constrained sellers.[3] In the next section we examine the case where the seller may select any reserve price.

2 Fully Strategic Seller

In this section we allow the seller freedom in setting any reserve price he likes. Clearly, if there is a symmetric equilibrium bidding strategy in the first auction that is increasing in values, then the seller will invert first-period bidding strategies, compute the highest valuation and set the second-period reserve price at the highest valuation. However, we show next that with a fully strategic seller, there is no increasing symmetric equilibrium bidding strategy in the first period. In particular, bidding one's true valuation is not a symmetric bidding strategy in the first period with a fully strategic seller either. For simplicity we assume that there are only two buyers and one seller.

Theorem 2 *There is no increasing symmetric equilibrium bidding strategy in the first period with a strategic seller.*

Proof. To obtain a contradiction suppose that there is a symmetric equilibrium strategy $b(\cdot)$ that is increasing in values. The seller's reserve price for the second auction will be $R(b(v_1), b(v_2)) = \max\{v_1, v_2\}$. To find the best response of bidder 1 when bidder 2 plays $b(\cdot)$ suppose bidder 1 bids $x \geq 0$ when

[3] Allowing the seller to set the first-period reserve price does not modify the problem fundamentally as first-period bidding strategies remain the same for those bidders with valuations above the reserve price and become zero otherwise. Without loss of generality, we assume that the seller sets the first-period reserve price at $R_1 = 0$.

his value is v. His expected utility is

$$\begin{aligned} h(x) &= \frac{1}{2}E\left[(v - b(v_2))\chi_{x \geq b(v_2)}\right] + \frac{1}{2}E\left[(v - R(x, b(v_2)))^{+}\right] = \\ & \frac{\int_0^{b^{-1}(x)} (v - b(v_2))\, dv_2}{2} + \frac{E\left[\left(v - \max\left\{b^{-1}(x), v_2\right\}\right)^{+}\right]}{2}. \end{aligned}$$

Bidder 1 will never bid $x > b(v)$ since $h(x)$ is decreasing in this region. Therefore, we can suppose without loss of generality that $x \leq b(v)$. Thus

$$\begin{aligned} h(x) &= \frac{\int_0^{b^{-1}(x)} (v - b(v_2))\, dv_2}{2} + \frac{\int_0^{v} \left(v - \max\left\{b^{-1}(x), v_2\right\}\right)^{+}}{2} = \\ & \frac{\int_0^{b^{-1}(x)} (v - b(v_2))\, dv_2}{2} + \frac{b^{-1}(x)\left(v - b^{-1}(x)\right) + \int_{b^{-1}(x)}^{v} (v - v_2)\, dv_2.}{2}. \end{aligned}$$

If we define $\omega = b^{-1}(x)$, Bidder 1's problem is to find ω that maximizes

$$2g(\omega) = \int_0^{\omega} (v - b(v_2))\, dv_2 + \omega(v - \omega) + \int_{\omega}^{v} (v - v_2)\, dv_2.$$

The first-order condition is

$$2g'(\omega) = v - b(\omega) + v - 2\omega - (v - \omega) = v - \omega - b(\omega) = 0.$$

Since $g'(v) = -b(v) < 0$ we see that the optimal $\omega < v$ and the optimal $x < b(v)$. ■

Remark 3 *For example if $b(v) = v$ then the best reply of bidder 1 is $x = \frac{v}{2}$.*

3 Conclusion

In this short paper we provide a simple example of repeated trials of Vickrey auctions where it is not an equilibrium for a bidder to bid his valuation in every trial. This example illustrates that any "convergence " between willingness to pay and willingness to accept might not be the consequence of learning about one's valuation – as suggested by some economists — but rather the result of strategic behavior. This suggests that perhaps only the existence of a wedge between willingness to pay and willingness to accept **at the last trial** should be seen as an "anomaly" and not at earlier trials where both buyers and sellers may be gaming.

References

[1] Coursey, D. L., J. L. Hovis and W. D. Schulze, 1987, "The Disparity Between Willingness to Accept and Willingness to Pay Measures of Value," *Quarterly Journal of Economics 102(3)*, 679-90.

[2] Coursey, D. L. and W. Schulze, 1986, "The Application of Laboratory Experimental Economics to the Contingent Valuation of Public Goods. *Public Choice 49,* 47-68.

[3] Cox, J. L., B. Roberson, and V. L. Smith, 1982, "Theory and Behavior of Single Price Auctions," V. L. Smith, ed., *Research in Experimental Economics II.* Greenwich, CT: JAI Press.

[4] Dubourg, W. R., M. W. Jones-Lee, G. Loomes, 1994, "Imprecise Preferences and the WTP-WTA Disparity, " *Journal of Risk and Uncertainty 9*, 115-133.

[5] Gregory, R. and L. Furby, 1987, "Auctions, Experiments and Contingent Valuation," *Public Choice 55*, 273-289.

[6] Hanemann, W. M., 1991, "Willingness to Pay and Willingness to Accept: How Much Can They Differ?" *American Economic Review 81(3)*, 635-647.

[7] Kahneman, D., J. L. Knetsch and R. H. Thaler, "Experimental Tests of the Endowment Effect and the Coase Theorem," *Journal of Political Economy 98(6)*, 1325-1348.

[8] Kahneman, D. and A. Tversky, 1979, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica 47(2)*, 263-291.

[9] Morrison, G. C., 1998, "Understanding the disparity between WTP and WTA: Endowment Effect, Substitutability, or Imprecise Preferences?" *Economics Letters 59*, 189-194.

[10] Morrison, G. C., 1997-a, "Resolving Differences in Willingness to Pay and Willingness to Accept: Comment, " *American Economic Review 87(1)*, 236-240.

[11] Morrison, G. C., 1997-b, "Willingness to Pa and Willingness to Accept: Some Evidence of an Endowment Effect, " *Applied Economics 29*, 411-417.

[12] Shogren, J. F., S. Y. Shin, D. J. Hayes and J. B. Kliebenstein, 1994, "Resolving Differences in Willingness to Pay and Willingness to Accept," *American Economic Review 84(1)*, 255-270.

[13] Weber, R. J., 1983, "Multi-Object Auctions, " in R. Engelbrecht-Wiggans, M. Shubik, and R. M. Stark, (eds.), *Auction, Bidding, and Contracting: Uses and Theory* (165-194). New York: New York University Press.