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Equilibrium theory with asymmetric information and infinitely many states

Victor Filipe Martins-da-Rocha, Paulo K. Monteiro, Carlos Hervés-Beloso

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Carlos Hervés-Beloso † V. Filipe Martins-da-Rocha ‡
Paulo K. Monteiro §

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Abstract

Radner (1968) proved the existence of a competitive equilibrium for differential information economies with finitely many states. We extend this result to economies with infinitely many states of nature.

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1 Introduction

For exchange economies under uncertainty, Arrow and Debreu (1954) proved that a competitive equilibrium exists if agents have a complete and symmetric information about a finite set of possible states of nature. This seminal existence result was generalized in several directions.

Asymmetric information was introduced in Radner (1968). Agents arrange contracts at the first period that may be contingent on the realized state of nature at the second period. But after the realization of state, they do not necessarily know which state of nature has actually occurred. Agents have incomplete information and this information may differ across agents (differential information economies). Therefore they are restricted to sign contracts that are

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†Facultad de Económicas, Campus Universitario, 36310 Vigo, Spain. E-mail: cherves@uvigo.es

‡Graduate School of Economics, Getulio Vargas Foundation, Praia de Botafogo 190, 22.250-900 Rio de Janeiro, Brazil. Email: victor.rocha@fgv.br

§Graduate School of Economics, Getulio Vargas Foundation, Praia de Botafogo 190, 22.250-900 Rio de Janeiro, Brazil. Email: paulo.klinger@fgv.br

compatible with their private information. For such an economy Radner defined a notion of competitive equilibrium (Walrasian expectations equilibrium) which is an analogous concept to the Walrasian equilibrium in Arrow–Debreu model with symmetric information. There is an important literature dealing with competitive solutions for differential information exchange economies: Maus (2004) for economies with production and Einy, Moreno and Shitovitz (2001) for economies with a continuum of agents.[1] All these contributions deal with either a finite dimensional commodity space or with a commodity space for which the positive cone has a non-empty interior.

For models with symmetric information, the existence result in Arrow and Debreu (1954) was generalized to economies with infinitely many states. Since the path-breaking papers of Peleg and Yaari (1970) and Bewley (1972), many theorems have been proved on the existence of competitive equilibrium with an infinite dimensional commodity space for which the positive cone may have an empty interior. However, nearly all[2] require that the consumption possibility sets are the positive orthant. These results cannot be applied to models with asymmetric information since informationally constrained consumption sets are in general subsets of strict subspaces of the commodity space.

The main purpose of this paper is to extend the existence result in Arrow and Debreu (1954) by considering both asymmetric information and infinitely many states of nature. Uncertainty is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where Ω represents the possibly infinite set of states of nature. Each agent i's private information is represented by a sub-tribe $\mathcal{F}^i$ of $\mathcal{F}$ and the set of possible consumption plans is the cone $L^p_+(\Omega, \mathcal{F}^i, \mathbb{P})$ of p-integrable ($1 \leqslant p < +\infty$) and $\mathcal{F}^i$-measurable functions from Ω to $\mathbb{R}_+$. When endowed with the norm topology, the cone $L^p_+(\Omega, \mathcal{F}, \mathbb{P})$ may have an empty interior. In the symmetric framework, the Riesz-Kantorovich formula and properness assumptions are a powerful tool (see e.g. Aliprantis, Tourky and Yannelis (2001) and Aliprantis et al. (2004)) to prove existence of equilibrium when the positive cone of the commodity space has an empty interior. However these techniques cannot be directly applied to

[1] See also Herves-Beloso, Moreno-Garcia and Yannelis (2005a), Herves-Beloso, Moreno-Garcia and Yannelis (2005b), Einy, Haimanko, Moreno and Shitovitz (2005), Graziano and Meo (2005), Correia-da-Silva and Hervés-Beloso (2006), Correia-da-Silva and Hervés-Beloso (2007a), Correia-da-Silva and Hervés-Beloso (2007b) and many others. Recently, there has been a resurgent interest on the execution of contracts at the second period. At issue are questions of enforceability. Since information is incomplete, some agents may have incentives to misreport their information and then contracts may not be executable. For the interested readers we refer to Daher, Martins-da-Rocha and Vailakis (2007), Angeloni and Martins-da-Rocha (2007) and Podczeck and Yannelis (2008).

[2] See Bewley (1972), Magill (1981), Aliprantis and Brown (1983), Jones (1984), Mas-Colell (1986), Araujo and Monteiro (1989), Yannelis and Zame (1986), Mas-Colell and Richard (1991), Podczeck (1996), Tourky (1998), Deghdak and Florenzano (1999), Aliprantis, Florenzano and Tourky (2004), Aliprantis, Florenzano and Tourky (2005) and many others. There is a notable exception: in Podczeck and Yannelis (2008) consumption sets need not be the positive orthant of the commodity space.

the asymmetric framework. This was already stressed in Podczeck and Yannelis (2008) where uncertainty is represented by a finite set but for each possible state of nature, an infinite dimensional spot market is considered. We differ from the aforementioned work since we consider the polar case: uncertainty is represented by an infinite set of possible states but for each state there is only one commodity available for consumption.

Even when there is an incomplete and asymmetric information about infinitely many states of nature, it is straightforward to check that every competitive equilibrium is actually a private Edgeworth equilibrium (see Yannelis (1991)). In the symmetric case, properness assumptions on preferences play a crucial role to prove that the converse is true, i.e., every private Edgeworth equilibrium is a competitive equilibrium. Our main contribution is to provide conditions on the information structure that are sufficient for this decentralization result to be still valid when information is asymmetric.[3] We assume that each agent i knows at the first period that he will observe two signals at the second period: a public signal κ and a private signal τ^i. Agent i's information is then represented by the σ-algebra generated by the pair (κ, τ^i). We don't impose any restrictions on the publicly observed signal κ which may take infinitely many values. However, we only provide existence results when the private signal τ^i takes finitely many values, letting as an open question the general case where both the public and the private signals may take infinitely many values. Under suitable continuity conditions on preference relations, we prove existence of a competitive equilibrium with a continuous price in $L^q(\mathcal{F}, \mathbb{P})$.

The paper is organized as follows. Section 2 presents the model and the equilibrium concepts. Conditions on the information structure are imposed in Section 3 and the standard assumptions on preferences and initial endowments are introduced in Section 4. Section 5 addresses existence of an Edgeworth equilibrium and its decentralization as a competitive equilibrium. Finally, in the last section, we discuss an alternative equilibrium concept by allowing for free-disposal.

2 The Model

We consider a pure exchange economy with a finite set I of agents and, for convenience, one good. The economy extends over two periods $t \in \{0, 1\}$ with uncertainty on the realized state of nature in the second one represented by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Each agent i knows at $t = 0$ that at $t = 1$ he will have an incomplete and private information in the sense that he will only observe the outcome of random variables measurable with respect to a sub-tribe $\mathcal{F}^i$ of $\mathcal{F}$. The family $(\mathcal{F}^i)_{i \in I}$ is denoted by $\boldsymbol{\mathcal{F}}$. At $t = 0$, there is an anonymous market

[3] The non-emptiness of the set of Edgeworth equilibria, and then the existence of a competitive equilibrium, follows from standard arguments: see e.g. Florenzano (2003).

for consumption plans (or contingent contracts) in $L^p_+(\mathcal{F}, \mathbb{P})$ where $p \in [1, +\infty)$. Each agent i knows that, contingent to the realization of the state ω, he will have at $t = 1$ an initial endowment $e^i(\omega) \geqslant 0$ of the unique good. The random variable e^i is assumed to belong to $L^p(\mathcal{F}^i, \mathbb{P})$ and the family $(e^i)_{i \in I}$ is denoted by $\boldsymbol{e}$. At $t = 0$, agents make contracts on redistribution of their initial endowments before the state of nature is realized. As in Radner (1968), these contracts have to be consistent with their private information, i.e., we assume that each agent i chooses a consumption plan in subset X^i of $L^p_+(\mathcal{F}^i, P)$. In the second period agents carry out previously made agreements, and consumption takes place. For discussions on the interpretation of this model and on the enforceability of contracts at $t = 1$, we refer to Daher et al. (2007, Section 2), Angeloni and Martins-da-Rocha (2007, Section 6) and Podczeck and Yannelis (2008, Section 4). Agent i's (strict) preference relation on consumption plans is represented by a correspondence P^i from X^i to X^i. The economy is then defined by the collection

$$\mathcal{E} = (\boldsymbol{\mathcal{F}}, \boldsymbol{X}, \boldsymbol{e}, \boldsymbol{P})$$

where $\boldsymbol{X}$ is the family $(X^i)_{i \in I}$ and $\boldsymbol{P}$ is the family $(P^i)_{i \in I}$. The vector subspace of $L^p(\mathcal{F}, \mathbb{P})$ generated by the family $\boldsymbol{X}$ is denoted by $\mathcal{X}$ and the space of linear functionals defined on $\mathcal{X}$ is denoted by $\mathcal{X}^\star$. The space $\mathcal{X}$ represents the commodity space and $\mathcal{X}^\star$ the price space. The set X^i represents the consumption set and a vector $x \in X^i$ represents a possible consumption plan for agent i. If $x \in X^i$ the set $P^i(x) \subset X^i$ represents the set of strictly preferred consumption plans by agent $i \in I$. An allocation $\boldsymbol{x} = (x^i)_{i \in I}$ is a family of consumption plans $x^i \in X^i$. An allocation $\boldsymbol{x}$ is said *feasible* if

$$\sum_{i \in I} x^i = \sum_{i \in I} e^i.$$

The aggregate initial endowment $\sum_{i \in I} e^i$ is denoted by e. We now recall some properties a feasible allocation may satisfy.

Definition 2.1. A feasible allocation $\boldsymbol{x}$ is:

1. *weakly Pareto optimal* if there is no feasible allocation $\boldsymbol{y}$ satisfying $y^i \in P^i(x^i)$ for each $i \in I$;

2. *a core allocation*, if it cannot be blocked by any coalition in the sense that there is no coalition $S \subseteq I$ and some $(y^i)_{i \in S} \in \prod_{i \in S} P^i(x^i)$ such that $\sum_{i \in S} y^i = \sum_{i \in S} e^i$;

3. an *Edgeworth equilibrium* if there is no $0 \neq \boldsymbol{\lambda} \in (\mathbb{Q} \cap [0,1])^I$ and some allocation $\boldsymbol{y}$ such that $y^i \in P^i(x^i)$ for each $i \in I$ with $\lambda^i > 0$ and $\sum_{i \in I} \lambda^i y^i = \sum_{i \in I} \lambda^i e^i$;

4. an *Aubin equilibrium* if there is no $0 \neq \boldsymbol{\lambda} \in [0,1]^I$ and some allocation $\boldsymbol{y}$ such that $y^i \in P^i(x^i)$ for each $i \in I$ with $\lambda^i > 0$ and $\sum_{i \in I} \lambda^i y^i = \sum_{i \in I} \lambda^i e^i$.

Remark 2.1. The reader should observe that these concepts are "price free" in the sense that they are intrinsic property of the commodity space. It is proved in Florenzano (2003, Propositions 4.2.6) that the set of Aubin equilibria and the set of Edgeworth equilibria coincide provided that for each $i \in I$, the set $P^i(x^i)$ is open[4] in X^i or $P^i(x^i) = \{y \in X^i \colon U^i(y) > U^i(x^i)\}$ for a concave utility function U^i.

We denote by $\|\cdot\|_p$ the standard norm in $L^p(\mathcal{F}, \mathbb{P})$ defined by

$$\forall y \in L^p(\mathcal{F}, \mathbb{P}), \quad \|y\|_p := \left[\int_\Omega \mid y(\omega) \mid^p \mathbb{P}(d\omega)\right]^{\frac{1}{p}},$$

and let q be the (extended) real number in $(0, \infty]$ satisfying $\frac{1}{q} + \frac{1}{p} = 1$.

We now recall the concept of competitive (or Walrasian expectations) equilibrium.

Definition 2.2. A couple $(\boldsymbol{x}, p)$ is said to be a *competitive equilibrium* if $\boldsymbol{x}$ is a feasible allocation and $p \in \mathcal{X}^\star$ is a price such that $p(x^i) = p(e^i)$ and if $y^i \in P^i(x^i)$ then $p(y^i) > p(e^i)$. If a function $\psi \in L^q(\mathcal{F}, \mathbb{P})$ representing the price p, in the sense that

$$\forall x \in \mathcal{X}, \quad p(x) = \langle \psi, x \rangle = \mathbb{E}[\psi x]$$

exists, then $(\boldsymbol{x}, p)$ is said to be a continuous competitive equilibrium.

3 The information structure

The commodity space $\mathcal{X}$ is a subspace of $L^p(\vee_{i \in I} \mathcal{F}^i, \mathbb{P})$, where $\vee_{i \in I} \mathcal{F}^i$ is the coarsest tribe containing each $\mathcal{F}^i$. Therefore, without any loss of generality, we may assume that

Assumption (I). *The tribe $\mathcal{F}$ coincides with $\vee_{i \in I} \mathcal{F}^i$.*

We denote by $\mathcal{F}^c$ the common knowledge information, i.e., $\mathcal{F}^c$ is the meet of the family $(\mathcal{F}^i)_{i \in I}$:

$$\mathcal{F}^c = \{A \in \mathcal{F} \,:\, \forall i \in I, \quad A \in \mathcal{F}^i\}.$$

For each $x \in L^p(\mathcal{F}, \mathbb{P})$, we write $x \geqslant 0$ if $x \in L^p_+(\mathcal{F}, \mathbb{P})$, we write $x > 0$ if $x \geqslant 0$ and $x \neq 0$, and we write $x \gg 0$ if $\mathbb{P}\{x > 0\} = 1$. A vector $x \gg 0$ is said strictly positive.

As in Radner (1968) and Mas-Colell (1986), we don't allow for restrictions on possible consumption bundles.

Assumption (II). *For each $i \in I$, the consumption set X^i coincides with $L^p_+(\mathcal{F}^i, \mathbb{P})$.*

[4] In a linear topology on $\mathcal{X}$.

Under Assumptions I and II, the commodity space $\mathcal{X}$ coincides with the space

$$\Sigma := \sum_{i \in I} L^p(\mathcal{F}^i, \mathbb{P}).$$

We now introduce the two main restrictions on the information structure $\boldsymbol{\mathcal{F}}$.

Assumption (III)**.** *There exist*

- *a measurable space $(S, \mathcal{S})$ and a measurable mapping $\kappa : (\Omega, \mathcal{F}) \longrightarrow (S, \mathcal{S})$,*
- *for each i, a finite set T^i and a measurable mapping $\tau^i : (\Omega, \mathcal{F}) \longrightarrow T^i$,*

such that the information available for each agent i comes from the observation of κ and τ^i, i.e.

$$\mathcal{F}^i = \sigma(\kappa, \tau^i)$$

in the sense that $\mathcal{F}^i$ is the coarsest tribe containing $\sigma(\kappa) = \{\kappa^{-1}(A)\colon A \in \mathcal{S}\}$ and $\sigma(\tau^i) = \{(\tau^i)^{-1}(C)\colon C \subset T^i\}$.

The set 2^T of subsets of $T = \prod_{i \in I} T^i$ is denoted by $\mathcal{T}$. We denote by $\mathbb{P}^{\kappa \times \boldsymbol{\tau}}$ the probability on $\mathcal{S} \otimes \mathcal{T}$ defined by

$$\forall (A, B) \in \mathcal{S} \times \mathcal{T}, \quad \mathbb{P}^{\kappa \times \boldsymbol{\tau}}(A \times B) = \mathbb{P}(\{\kappa \in A\} \cap \{\boldsymbol{\tau} \in B\})$$

where $\boldsymbol{\tau}$ is the measurable mapping from $(\Omega, \mathcal{F})$ to T defined by $\boldsymbol{\tau}(\omega) = (\tau^i(\omega))_{i \in I}$. We let $\mathbb{P}^\kappa$ and $\mathbb{P}^{\boldsymbol{\tau}}$ be the marginal probabilities defined on $\mathcal{S}$ and $\mathcal{T}$ by

$$\forall A \in \mathcal{S}, \quad \mathbb{P}^\kappa(A) = \mathbb{P}\{\kappa \in A\} \quad \text{and} \quad \forall B \in \mathcal{T}, \quad \mathbb{P}^{\boldsymbol{\tau}}(B) = \mathbb{P}\{\boldsymbol{\tau} \in B\}.$$

Observe that if $\mathbb{P}^\kappa(A)\mathbb{P}^{\boldsymbol{\tau}}\{t\} = 0$ then $\mathbb{P}^{\kappa \times \boldsymbol{\tau}}(A \times \{t\}) = 0$. This implies that given $t \in T$, the measure

$$\mathbb{P}^{\kappa \times \boldsymbol{\tau}}(., t) : A \longmapsto \mathbb{P}^{\kappa \times \boldsymbol{\tau}}(A \times \{t\})$$

defined on $\mathcal{S}$ is absolutely continuous with respect to the measure $\mathbb{P}^{\boldsymbol{\tau}}\{t\}\mathbb{P}^\kappa$. In particular there exists a $\mathbb{P}^\kappa$-integrable and strictly positive function $\psi(., t) : S \to (0, \infty)$ such that

$$\forall A \in \mathcal{S}, \quad \mathbb{P}^{\kappa \times \boldsymbol{\tau}}(A \times \{t\}) = \int_A \psi(s, t)\mathbb{P}^{\boldsymbol{\tau}}\{t\}\mathbb{P}^\kappa(ds).$$

Assumption (IV)**.** *There exists $\varepsilon > 0$ such that*

$$\frac{d\mathbb{P}^{\kappa \times \boldsymbol{\tau}}}{d\mathbb{P}^\kappa \otimes d\mathbb{P}^{\boldsymbol{\tau}}} \geqslant \varepsilon$$

or equivalently $\psi(s, t) \geqslant \varepsilon$ for $\mathbb{P}^\kappa \otimes \mathbb{P}^{\boldsymbol{\tau}}$-a.e. (s, t).

Remark 3.1. If the public information is independent of the private information, i.e., the mappings κ and $\boldsymbol{\tau}$ are $\mathbb{P}$-independent, then Assumption IV is automatically satisfied since we have $d\mathbb{P}^{\kappa\times\boldsymbol{\tau}} = d\mathbb{P}^{\kappa} \otimes d\mathbb{P}^{\boldsymbol{\tau}}$. Note that we do not assume that the family of private signal functions $(\tau^i)_{i\in I}$ is pairwise independent. Two different agents $i \neq j$ may have the same information $\mathcal{F}^i = \mathcal{F}^j$. It then follows that $\sigma(\kappa)$ is a subtribe of $\mathcal{F}^c$ but the inclusion may be strict (e.g. if $\tau^i = \tau^j$ for every pair (i, j)).

We let $L^0(\mathcal{F}, \mathbb{P})$ be the space (of $\mathbb{P}$-equivalent classes of) real valued and $\mathcal{F}$-measurable functions. If $x \in L^0(\mathcal{F}, \mathbb{P})$ then from Assumption I, there exists a unique (up to $\mathbb{P}^{\kappa\times\boldsymbol{\tau}}$-equivalent classes) $\mathcal{S} \otimes \mathcal{T}$-measurable function

$$f_x : S \times T \longrightarrow \mathbb{R}$$

such that

$$x(\omega) = f_x(\kappa(\omega), \boldsymbol{\tau}(\omega)) \quad \text{for } \mathbb{P}\text{–a.e. } \omega \in \Omega.$$

We denote by $\mathfrak{F} : x \mapsto \mathfrak{F}x$ the mapping from $L^0(\mathcal{F}, \mathbb{P})$ to $L^0(\mathcal{S}\otimes\mathcal{T}, \mathbb{P}^{\kappa\times\boldsymbol{\tau}})$ defined by $\mathfrak{F}x := f_x$. Observe that if x belongs to $L^p(\mathcal{F}, \mathbb{P})$ then

$$\int_{\Omega} |x(\omega)|^p \mathbb{P}(d\omega) = \int_{S\times T} |\mathfrak{F}x(s, t)|^p \psi(s, t)\mathbb{P}^{\kappa}(ds)\mathbb{P}^{\boldsymbol{\tau}}(dt).$$

4 Assumptions

It is straightforward to check that every competitive equilibrium is an Edgeworth equilibrium. In order to prove the converse, we consider the following list of assumptions that an economy may satisfy.

Definition 4.1. A differential information economy is said *standard* if Assumptions I and II and the following Assumptions C and P are satisfied.

Assumption (C)**.** *There exists a strictly positive function a in $L^p_+(\mathcal{F}^c, \mathbb{P})$ such that for each $i \in I$,*

C.1 *the preference P^i is irreflexive,*[5] *strictly monotone*[6]*, with weakly-open lower sections,*[7]*and $\|\cdot\|_p$-open convex upper sections;*[8]

C.2 *there exists $\mu > 0$ such that $e \leqslant \mu a$;*

C.3 *there exists $b^i \in L^p_+(\mathcal{F}^c, \mathbb{P})$ such that $0 \neq b^i \leqslant e^i$ and $a = \sum_{i\in I} b^i$.*

[5]In the sense that for each $x^i \in X^i$, $x^i \notin P^i(x^i)$.

[6]In the sense that for each $x \in X^i$, $x + L^p_+(\mathcal{F}^i, \mathbb{P}) \subset P^i(x) \cup \{x\}$.

[7]In the sense that for each $y \in X^i$, the set $P^{-1}(y) = \{x \in X^i : y \in P^i(x)\}$ is σ-open in X^i, where σ is the weak topology $\sigma(L^p(\mathcal{F}, \mathbb{P}), L^q(\mathcal{F}, \mathbb{P}))$.

[8]In the sense that for each $x \in X^i$, the set $P^i(x)$ is convex and open for the $\|.\|_p$-topolgy.

Remark 4.1. Observe that under Assumptions C.2 and C.3, the aggregate initial endowment e belongs to the order interval $[a, \mu a]$.[9] When the information is symmetric, i.e., $\mathcal{F}^i = \mathcal{F}$ for every $i \in I$, then Assumptions C.2 and C.3 are automatically satisfied if for every $i \in I$, the initial endowment e^i is not zero. When $\mathcal{F}$ has finitely many atoms (e.g. if the state space Ω is finite) then Assumptions C.2 and C.3 are automatically satisfied if for every $i \in I$, the initial endowment e^i is strictly positive.

Assumption (P). *For each feasible allocation $\boldsymbol{x}$ and for each $i \in I$, there exists a convex set $\widehat{P}^i(x^i) \subset L^p(\mathcal{F}^i, \mathbb{P})$ with a non-empty $\|\cdot\|_p$-interior in $L^p(\mathcal{F}^i, \mathbb{P})$ such that*

$$\widehat{P}^i(x^i) \cap A_{x^i} \cap L^p_+(\mathcal{F}^i, \mathbb{P}) \subset P^i(x^i)$$

for some subset $A_{x^i} \subset L^p(\mathcal{F}^i, \mathbb{P})$ radial at x^i and such that

$$\forall y \in \widehat{P}^i(x^i), \quad \forall \alpha \in (0, 1], \quad \alpha y + (1 - \alpha)x^i \in \widehat{P}^i(x^i).$$

Assumption P is taken from Podczeck (1996) and related to properness conditions introduced by Mas-Colell (1986).

Remark 4.2. When $\mathcal{F}$ has finitely many atoms, Assumption P is automatically satisfied. Indeed, it is sufficient to pose

$$\widehat{P}^i(x^i) = x^i + L^p_+(\mathcal{F}^i, \mathbb{P}) \setminus \{0\}.$$

We consider now preference relations defined by utility functions. Consider the following conditions on utility functions.

Assumption (U). *For each $i \in I$ there exists a function $U^i : X^i \to \mathbb{R}$ such that*

$$\forall x^i \in X^i, \quad P^i(x^i) = \{y^i \in X^i \ : \ U^i(y^i) > U^i(x^i)\}.$$

Moreover there exists a strictly positive function a in $L^p_+(\mathcal{F}^c, \mathbb{P})$ such that for each $i \in I$,

U.1 *the function U^i is continuous for the $\|.\|_p$-topology, quasi-concave and strictly increasing;*[10]

U.2 *there exists $\mu > 0$ such that $e \leqslant \mu a$;*

U.3 *there exists $b^i \in L^p_+(\mathcal{F}^c, \mathbb{P})$ such that $0 \neq b^i \leqslant e^i$ and $a = \sum_{i \in I} b^i$;*

U.4 *for each $x^i \in X^i$, there exists a vector $\nabla U^i(x^i) \neq 0$ in $L^q(\mathcal{F}^i, \mathbb{P})$ such that*

$$\forall v \in S^i(x^i), \quad \lim_{t \downarrow 0} \frac{U^i(x^i + tv) - U^i(x^i)}{t} = \langle \nabla U^i(x^i), v \rangle$$

where $S^i(x^i) = \{v \in L^p(\mathcal{F}^i, \mathbb{P}) \colon x^i + tv \in X^i$ for some $t > 0\}$.

[9] If a and b are two vectors in $L^p(\mathcal{F}, \mathbb{P})$ then the order interval $[a, b]$ is the set of all vectors x in $L^p(\mathcal{F}, \mathbb{P})$ satisfying $a \leqslant x \leqslant b$.

[10] That is for each x, y in X^i, if $y > x$ then $U^i(y) > U^i(x)$.

Remark 4.3. Note that since U^i is increasing then $\nabla U^i(x^i)$ belongs to $L^q_+(\mathcal{F}^i, \mathbb{P})$. Observe that Assumptions U.2 and U.3 are just repetition of C.2 and C.3.

We claim that Assumption U implies Assumptions C and P.

Proposition 4.1. *If an economy satisfies Assumption U then it satisfies Assumptions C and P.*

Proof. Consider an economy satisfying Assumption U. It is straightforward to check that Assumptions U.1 to U.3 imply Assumption C. Now fix $i \in I$, $x^i \in X^i$ and consider the following set

$$\widehat{P}^i(x^i) = \{y^i \in E^i \ : \ \langle \nabla U^i(x^i), y^i - x^i \rangle > 0\}.$$

This set is convex, non-empty and $\|\cdot\|_p$-open. It is now straightforward to prove that Assumption U4 implies Assumption P. Q.E.D

We consider hereafter the special case of separable utility functions.

Definition 4.2. A family $\boldsymbol{U} = (U^i)_{i \in I}$ of utility functions from X^i to $\mathbb{R}$ is said separable if for each $i \in I$ there exists $V^i : \Omega \times \mathbb{R}_+ \to \mathbb{R}_+$ such that

(a) the function V^i is $\mathcal{F}^i \otimes \mathcal{B}(\mathbb{R}_+)$-measurable;

(b) for almost every $\omega \in \Omega$, $V^i(\omega, .) : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, concave and strictly increasing;[11]

(c) for every $x \in L^p_+(\mathcal{F}^i, \mathbb{P})$, the function $\omega \mapsto V^i(\omega, x(\omega))$ belongs to $L^1(\mathcal{F}^i, \mathbb{P})$ and

$$U^i(x) = \int_\Omega V^i(\omega, x(\omega)) \mathbb{P}(d\omega).$$

The function V^i is called the kernel of U^i. The left derivative of $V^i(\omega, .)$ in $t > 0$ is denoted by $V^i_-(\omega, t)$ and the right derivative is denoted by $V^i_+(\omega, t)$. We denote by $V^i_+(\omega, 0)$ the extended real number $\lim_{t \to 0} V^i_+(\omega, t)$ (we may have $V^i_+(\omega, 0) = \infty$). If $x \in L^p_+(\mathcal{F}^i, \mathbb{P})$ then we denote by $V^i_-(x)$ the function in $L^0(\mathcal{F}^i, \mathbb{P})$ defined by

$$V^i_-(x) : \omega \longmapsto V_-(\omega, x(\omega))$$

and $V^i_+(x)$ the function in $L^0(\mathcal{F}^i, \mathbb{P})$ defined by

$$V^i_+(x) : \omega \longmapsto V_+(\omega, x(\omega)).$$

Proposition 4.2. *If $\boldsymbol{U}$ is a family of separable utility functions such that*

$$\forall x \in X^i, \quad V^i_+(x) \in L^q(\mathcal{F}^i, \mathbb{P}) \quad \textit{and} \quad V^i_-(x) \in L^q(\mathcal{F}^i, \mathbb{P})$$

then Assumption U.4 is satisfied with

$$\forall x \in X^i, \quad \forall h \in L^p(\mathcal{F}^i, \mathbb{P}), \quad \langle \nabla V^i(x), h \rangle = \mathbb{E}[V^i_+(x) h_+ - V^i_-(x) h_-].$$

For related results about properness of separable utility functions, we refer to Le Van (1996) and Aliprantis (1997).

[11] A function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing if for each $x, y \in \mathbb{R}_+$, whenever $x > y$ implies $f(x) > f(y)$.

5 Decentralizing Edgeworth equilibrium allocations

As a consequence of Proposition 5.2.2 in Florenzano (2003), we get the following non-emptiness result.

Proposition 5.1. *For every standard differential information economy, the set of Edgeworth equilibria is non-empty.*

Proof. We let

$$\widehat{X} = \left\{ \boldsymbol{x} \in \prod_{i \in I} X^i : \sum_{i \in I} x^i = e \right\}$$

be the set of attainable allocations. In order to apply Proposition 5.2.2 in Florenzano (2003), it is sufficient to prove that the set $\widehat{X}$ is compact for the weak-topology.[12] Denote that

$$\widehat{X} \subset \prod_{i \in I} [0, e] \cap L^p_+(\mathcal{F}^i, \mathbb{P}).$$

Since $[0, e]$ is a weakly compact subset of $L^p(\mathcal{F}, \mathbb{P})$ and $L^p_+(\mathcal{F}^i, \mathbb{P})$ is weakly closed in $L^p(\mathcal{F}, \mathbb{P})$, we get the desired result. Q.E.D

The main result of the paper is the following.

Theorem 5.1. *Consider a standard differential information economy. Assume that Assumptions III and IV are satisfied, then for every Edgeworth equilibrium $\boldsymbol{x}$ there exists $\psi \in L^q(\mathcal{F}, \mathbb{P})$ such that $(\boldsymbol{x}, \langle \psi, \cdot \rangle)$ is a competitive equilibrium with a continuous price.*

The proof of Theorem 5.1 follows from Propositions 5.2 and 5.3 below.

Remark 5.1. When $(\mathcal{F}, \mathbb{P})$ has finitely many atoms, we get as a corollary of Theorem 5.1 the existence result in Radner (1968). When the information is symmetric, i.e., when $\mathcal{F}^i = \mathcal{F}$ for each $i \in I$, we get as a corollary of Theorem 5.1 the existence result in Araujo and Monteiro (1989).

For technical reasons, we consider the following concept of competitive quasi-equilibrium.

Definition 5.1. A couple $(\boldsymbol{x}, p)$ is said to be a (non-trivial) *competitive quasi-equilibrium* if $\boldsymbol{x}$ is a feasible allocation and $p \in \mathcal{X}^\star$ is a price with $p(e) > 0$ and such that $p(x^i) = p(e^i)$ and if $y^i \in P^i(x^i)$ then $p(y^i) \geqslant p(e^i)$. If there exists $\psi \in L^q(\mathcal{F}, \mathbb{P})$ representing the price p, i.e., $p = \langle \psi, \cdot \rangle$ then $(\boldsymbol{x}, p)$ is said to be a competitive quasi-equilibrium with a continuous price.

Obviously a competitive quasi-equilibrium is a competitive equilibrium. We propose hereafter conditions under which the converse is true.

[12] The weak topology on $L^p(\mathcal{F}, \mathbb{P})$ is the topology $\sigma(L^p(\mathcal{F}, \mathbb{P}), L^q(\mathcal{F}, \mathbb{P}))$.

Proposition 5.2. *Consider a standard economy, then every competitive quasi-equilibrium is actually a competitive equilibrium.*

Proof. Let $(\boldsymbol{x}, p)$ be a competitive quasi-equilibrium of a standard economy. In particular we have that for each $i \in I$,

$$p(x^i) = p(e^i) \quad \text{and} \quad \forall y^i \in P^i(x^i), \quad p(y^i) \geqslant p(e^i).$$

Since preferences are strictly monotone, we have that $p(z) \geqslant 0$ for each $z \in L^p_+(\mathcal{F}^i, \mathbb{P})$. Now we know that $p(e) = \sum_i p(e^i) > 0$. Therefore there exists $j \in I$ such that $p(e^j) > 0$. We first prove that if $y^j \in P^i(x^j)$ then $p(y^j) > p(e^j)$. Assume by way of contradiction that $p(y^j) = p(e^j)$. From Assumption C.1, there exists $\alpha \in (0,1)$ such that αy^j still lies in $P^j(x^j)$. Therefore $p(\alpha y^j) = \alpha p(y^j) \geqslant p(e^j)$: contradiction. Therefore for every $0 \neq z \in L^p_+(\mathcal{F}^j, \mathbb{P})$, we have $p(z) > 0$. In particular, since for each $i \in I$ the vector b^i belongs to $L^p_+(\mathcal{F}^c, \mathbb{P}) \setminus \{0\}$, we have $p(e^i) \geqslant p(b^i) > 0$ for each $i \in I$. Following the previous argument, we can prove that for every $i \in I$, if $y^i \in P^i(x^i)$ then $p(y^i) > p(e^i)$. Q.E.D

We say that any economy is quasi-standard if it satisfies Assumptions I, II, P, C.1 and C.2 together with C.3' defined by

C.3' there exists $\boldsymbol{b} = (b^i)_{i \in I} \in \prod_{i \in I} L^p_+(\mathcal{F}^i, \mathbb{P})$ such that $b^i \leqslant e^i$, $a = \sum_{i \in I} b^i$ and for some $j \in I$, $e^j > 0$.

We present hereafter the main technical result of the paper.

Proposition 5.3. *Consider a quasi-standard economy satisfying Assumptions III and IV. For every Edgeworth equilibrium $\boldsymbol{x}$ there exists $\psi \in L^q(\mathcal{F}^i, \mathbb{P})$ such that $(\boldsymbol{x}, \langle \psi, \cdot \rangle)$ is a competitive quasi-equilibrium with a continuous price.*

Proof. For notational convenience, we denote the spaces $L(\mathcal{F}, \mathbb{P})$, $L(\mathcal{F}^i, \mathbb{P})$ and $L^p(\mathcal{F}^c, \mathbb{P})$ by L, L^i and L^c. Let $\boldsymbol{x}$ be an Edgeworth equilibrium of a quasi-standard differential information economy. From Proposition 4.2.6 in Florenzano (2003), the allocation $\boldsymbol{x}$ is an Aubin equilibrium and thus

$$0 \notin G(\boldsymbol{x}) := \operatorname{co} \bigcup_{i \in I} \left[P^i(x^i) - e^i \right].$$

Let a be the strictly positive function in L^c_+ satisfying Assumption C. For each $i \in I$, we let $L^i(a)$ be the subspace of L^i defined by

$$L^i(a) := L(a) \cap L^i = \bigcup_{\lambda > 0} \lambda[-a, a] \cap L^i.$$

Observe that from Assumption C.2, for each $i \in I$, e^i and x^i belong to $L^i_+(a)$. Let $\Sigma(a) := \sum_{i \in I} L^i(a)$, we endow $\Sigma(a)$ with the topology σ for which a base of 0-neighborhoods is

$$\left\{ \sum_{i \in I} \alpha^i [-a, a] \cap L^i \ : \ \alpha^i \in (0, 1], \quad \forall i \in I \right\}.$$

Observe that the topology σ is Hausdorff and locally convex. From Assumption C we have

$$x^i + a + a + [-a, a] \cap L^i \subset x^i + a + L^i_+ \subset P^i(x^i).$$

Therefore

$$2a + \frac{1}{\#I} \sum_{i \in I} [-a, a] \cap L^i \subset G(\boldsymbol{x}).$$

We have proved that $G(\boldsymbol{x}) \cap \Sigma(a)$ is a non-empty convex subset of $\Sigma(a)$ such that $2a$ belongs to its σ-interior. It then follows from a classical separation theorem that there exists $p \in (\Sigma(a), \sigma)'$ such that $p(a) > 0$ and satisfying

$$\forall i \in I, \quad \forall y^i \in P^i(x^i) \cap L^i(a), \quad p(y^i) \geqslant p(e^i). \tag{1}$$

Applying Assumption C, we get that $p(x^i) \geqslant p(e^i)$ for every $i \in I$. Since $\boldsymbol{x}$ is feasible, this implies that

$$\forall i \in I, \quad p(x^i) = p(e^i). \tag{2}$$

Moreover, from strict monotonicity of preferences we have $p(z) \geqslant 0$ for every $z \in L^i_+(a)$.

Claim 5.1. *For each $i \in I$, there exists $\pi^i \in (L^i, \|\cdot\|_p)'$ such that*

$$\forall z \in L^i_+(a), \quad \pi^i(z) \leqslant p(z) \quad \textit{and} \quad \forall z \in L^i_+(x^i), \quad \pi^i(z) = p(z). \tag{3}$$

The proof of this claim is standard (for a similar result we refer, among others, to Podczeck (1996) and Deghdak and Florenzano (1999)) and is postponed to Appendix A.2. Note that from Assumption C.2, the ideal $L^i(x^i) = \cup_{\lambda \geqslant 0} \lambda[-x^i, x^i]$ is a subspace of $L^i(a)$. For each $i \in I$, we let M^i be defined by[13]

$$M^i = \sup\{|\pi^i(z)| \ : \ z \in L^i \quad \text{and} \quad \|z\|_p \leqslant 1\}.$$

We propose now to prove that for each $i \in I$, the functional p is $\|\cdot\|_p$-continuous on $L^i(a)$.

Claim 5.2. *There exists $M > 0$ such that for each $i \in I$,*

$$\forall x \in L^i(a), \quad |p(x)| \leqslant M \|x\|_p \, . \tag{4}$$

Proof. For each $i \in I$, we let $\Omega^i := \{\omega \in \Omega \ : \ x^i(\omega) \geqslant (1/\#I)a(\omega)\}$. The set Ω^i belongs to $\mathcal{F}^i$ and since

$$\sum_{i \in I} x^i = \sum_{i \in I} e^i \geqslant \sum_{i \in I} b^i = a,$$

[13]Since π^i belongs to $(L^i, \|\cdot\|_p)'$, it can be represented by a vector ψ in $L^q(\mathcal{F}^i, \mathbb{P})$. The real number M^i coincides with $\|\psi\|_q$.

we have $\bigcup_{i\in I}\Omega^i = \Omega$. Let $h \in L^c_+(a) = L(a) \cap L^p_+(\mathcal{F}^c, \mathbb{P})$, then for each $i \in I$, the vector $h\mathbf{1}_{\Omega^i}$ belongs to $L^i_+(x^i)$. Indeed, since h belongs to $L^c_+(a)$, there exists $\lambda > 0$ such that $0 \leqslant h \leqslant \lambda a$, implying that $h\mathbf{1}_{\Omega^i} \leqslant (\#I)\lambda x^i$. It then follows from (3) that

$$
\begin{aligned}
p(h) \leqslant \sum_{i\in I} p(h\mathbf{1}_{\Omega^i}) &= \sum_{i\in I} \pi^i(h\mathbf{1}_{\Omega^i}) \\
&\leqslant \left[\sup_i M^i\right] \sum_{i\in I} \|h\mathbf{1}_{\Omega^i}\| \\
&\leqslant \|h\|_p (\#I) \left[\sup_i M^i\right]. \qquad (5)
\end{aligned}
$$

Now fix $i \in I$ and $x \in L^i(a)$. There exists $\mu > 0$ such that $|x| \leqslant \mu a$. Following Proposition A.1, there exists $y \in L^c_+$ such that $|x| \leqslant y$. It follows that $|x| \leqslant (\mu a) \wedge y$. This implies from (5) that

$$
|p(x)| \leqslant p(|x|) \leqslant p((\mu a)\wedge y) \leqslant (\#I)\left[\sup_i M^i\right] \|(\mu a) \wedge y\|_p \leqslant (\#I)\left[\sup_i M^i\right] \|y\|_p .
$$

It follows that

$$
|p(x)| \leqslant (\#I)\left[\sup_i M^i\right] \rho(x) \quad \text{where} \quad \rho(x) := \inf\{\|y\|_p \ : \ y \in L^c_+ \text{ and } |x| \leqslant y\}
$$

Applying Proposition A.1, if we let

$$
M := \frac{1}{[\varepsilon \inf \mathbb{P}^{\boldsymbol{\tau}}(T_0)]^{\frac{1}{p}}} (\#I)\left[\sup_i M^i\right]
$$

then $|p(x)| \leqslant M\,\|x\|_p$. Q.E.D

As a consequence of the previous claim, we can prove that the linear functional p is η-continuous on $\Sigma(a)$ where η is the norm defined on $\Sigma(a)$ by

$$
\forall x \in \Sigma(a), \quad \eta(x) = \inf\left\{\sum_{i\in I}\|x^i\|_p \ : \ (x^i)_{i\in I} \in \prod_{i\in I} L^i(a) \quad \text{and} \quad \sum_{i\in I} x^i = x\right\}.
$$

Indeed, if $x \in \Sigma(a)$ then for every sum decomposition $x = \sum_{i\in I} x^i$ with $x^i \in L^i(a)$ for each i, we have

$$
|p(x)| \leqslant \sum_{i\in I} |p(x^i)| \leqslant M \sum_{i\in I}\|x^i\|_p .
$$

It then follows that $|p(x)| \leqslant M\eta(x)$, i.e., p is η-continuous on $\Sigma(a)$. From Proposition A.4, we know that $x \mapsto \eta(x)$ is $\|\cdot\|_p$-continuous on $\Sigma(a)$, implying that p is actually $\|\cdot\|_p$-continuous on $\Sigma(a)$.

Since a is strictly positive, the space $L^i(a)$ is $\|\cdot\|_p$-dense in L^i. This implies that the space $\Sigma(a)$ is $\|\cdot\|_p$-dense in Σ. Indeed, let $x \in \Sigma$. There exists a sum decomposition $x = \sum_{i\in I} x^i$ where $x^i \in L^i$ for each i. The space $L^i(a)$ is $\|\cdot\|_p$-dense in L^i. Therefore there exists a sequence $(x^i_n)_{n\in\mathbb{N}}$ of vectors in $L^i(a)$ which $\|\cdot\|_p$-converges to x^i. Let x_n be the vector in Σ defined by $x_n = \sum_{i\in I} x^i_n$. Since

$$\|x - x_n\|_p \leqslant \sum_{i\in I} \left\|x^i - x^i_n\right\|_p$$

we get that the sequence $(x_n)_{n\in\mathbb{N}}$ is $\|\cdot\|_p$-converging to x.

The linear functional p is $\|\cdot\|_p$-continuous on $\Sigma(a)$ which is a subspace of $L^p(\mathcal{F}, \mathbb{P})$. We directly obtain the following claim.[14]

Claim 5.3. *There exists a $\|\cdot\|_p$-continuous linear functional $\pi \in (L^p(\mathcal{F}, \mathbb{P}), \|\cdot\|_p)'$ which extends p.*

We claim that $(\boldsymbol{x}, \pi)$ is a competitive quasi-equilibrium with a continuous price. Fix $i \in I$ and $y \in P^i(x^i)$. There exists a sequence $(y_n)_{n\in\mathbb{N}}$ in $L^i(a)$ which is $\|\cdot\|_p$-converging to y. The correspondence P^i has $\|\cdot\|_p$-open upper sections. It follows that for every n large enough, we have $y_n \in P^i(x^i) \cap L^i(a)$. Applying (1), we have $p(y_n) \geqslant p(e^i)$ and passing to the limit, we get $\pi(y) \geqslant \pi(e^i) = p(e^i)$. Now since $a = \sum_{i\in I} b^i$, there exists $j \in J$ such that $p(b^j) > 0$, which implies by Assumption C.3 that $\pi(e^j) = p(e^j) > 0$. Q.E.D

6 Competitive equilibrium with free disposal

In the literature of asymmetric information, it is quite common to use the concept of competitive equilibrium with free disposal.

Definition 6.1. A couple $(\boldsymbol{x}, p)$ is said to be a competitive equilibrium with free disposal if $\boldsymbol{x}$ is an allocation satisfying

$$\sum_{i\in I} x^i \leqslant \sum_{i\in I} e^i$$

and if $p \in \mathcal{X}^\star$ is a price such that $p(x^i) = p(e^i)$ and if $y^i \in P^i(x^i)$ then $p(y^i) > p(e^i)$.

Remark 6.1. Obviously a competitive equilibrium is a competitive equilibrium with free disposal. Note that markets may not clear but the value of the disposal $\sum_{i\in I} e^i - x^i$ under the price p is zero.

[14]See Lemma 6.13 in Aliprantis and Border (1999).

Remark 6.2. There is no measurability constraint on the disposal $\sum_{i \in I} e^i - x^i$. This assumption may be problematic in the context of asymmetric information. Indeed, as it was shown in Glycopantis, Muir and Yannelis (2003), the free disposal assumption may destroy the incentive compatibility of the competitive equilibrium and thus the resulting trades (contracts) need not be enforceable (see also Angeloni and Martins-da-Rocha (2007)).

As a corollary of Proposition 5.3, we get the following existence result.

Theorem 6.1. *Consider a standard economy satisfying Assumptions III and IV. There exists a competitive equilibrium with free disposal $(\boldsymbol{x}, p)$ such that the price p can be represented by a non-negative functional ψ in $L^q(\mathcal{F}, \mathbb{P})$, i.e., $p = \langle \psi, \cdot \rangle$.*

Proof. Let $\mathcal{E} = (\mathcal{F}^i, X^i, e^i, P^i)_{i \in I}$ be a standard economy. Fix $\ell \notin I$ and consider $\mathcal{E}^\ell$ the economy defined by

$$\mathcal{E}^\ell = (\mathcal{F}^j, X^j, e^j, P^j)_{j \in J}$$

where $J = I \cup \{\ell\}$, $\mathcal{F}^\ell = \mathcal{F}$, $X^\ell = L^p_+(\mathcal{F}, \mathbb{P})$, $e^\ell = 0$ and

$$\forall x^\ell \in L^p_+(\mathcal{F}, \mathbb{P}), \quad P^\ell(x^\ell) = \{y \in L^p_+(\mathcal{F}, \mathbb{P}) : \mathbb{E}[y] > \mathbb{E}[x]\}.$$

It is straightforward to check that the economy $\mathcal{E}^\ell$ is quasi-standard. Applying Proposition 5.3 there exists $((x^j)_{j \in J}, p)$ which is a competitive quasi-equilibrium of $\mathcal{E}^\ell$ where p is a continuous price represented by a vector $\psi \in L^q(\mathcal{F}, \mathbb{P})$. Note first that

$$\sum_{i \in I} x^i \leqslant x^\ell + \sum_{i \in I} x^i = \sum_{j \in J} e^j = e^\ell + \sum_{i \in I} e^i = \sum_{i \in I} e^i.$$

We already know that for every $j \in J$

$$p(x^j) = p(e^j) \quad \text{and} \quad y^j \in P^j(x^j) \Longrightarrow p(y^j) \geqslant p(e^j).$$

Since $x^\ell + L^p_+(\mathcal{F}, \mathbb{P}) \setminus \{0\} \subset P^\ell(x^\ell)$ we have $p(z) \geqslant 0$ for every $z \in L^p_+(\mathcal{F}, \mathbb{P})$, implying that ψ is actually non-negative. Since $p(x^\ell) = p(e^\ell) = 0$, the value of the excess $\sum_{i \in I} e^i - x^i$ is zero. Since $((x^j)_{j \in J}, p)$ is a competitive quasi-equilibrium there exists $k \in I$ such that $p(e^k) > 0$. It is now straightforward to prove that $((x^i)_{i \in I}, p)$ is a competitive equilibrium with free-disposal of the economy $\mathcal{E}$. Q.E.D

A Appendix

We denote by E the subspace of all vectors $x \in L^p(\mathcal{F}, \mathbb{P})$ such that there exists $y \in L^p_+(\mathcal{F}^c, \mathbb{P})$ satisfying $|x| \leqslant y$, i.e.

$$E := \{x \in L^p(\mathcal{F}, \mathbb{P}) : \exists y \in L^p_+(\mathcal{F}^c, \mathbb{P}), \quad |x(\omega)| \leqslant y(\omega) \quad \text{for } \mathbb{P}\text{–a.e. } \omega \in \Omega\}.$$

We endow E with the norm ρ defined by

$$\forall x \in E, \quad \rho(x) := \inf\{\|y\|_p \ : \ y \in L^p_+(\mathcal{F}^c, \mathbb{P}) \text{ and } |x| \leqslant y\}.$$

It is straightforward to check that the ρ-topology is stronger than the $\|.\|_p$-topology restricted to E, more precisely

$$\forall x \in E, \quad \|x\|_p \leqslant \rho(x).$$

Moreover, the ρ-topology and the $\|.\|_p$-topology coincide in $L^p(\mathcal{F}^c, \mathbb{P})$, more precisely

$$\forall x \in L^p(\mathcal{F}^c, \mathbb{P}), \quad \|x\|_p = \rho(x).$$

Proposition A.1. *Under Assumptions I–IV, the topological spaces* $(L^p(\mathcal{F}, \mathbb{P}), \|.\|_p)$ *and* (E, ρ) *coincide, more precisely*

$$\forall x \in L^p(\mathcal{F}, \mathbb{P}), \quad [\varepsilon \inf \mathbb{P}^{\boldsymbol{\tau}}(T_0)]^{\frac{1}{p}} \rho(x) \leqslant \|x\|_p \leqslant \rho(x),$$

where

$$\inf \mathbb{P}^{\boldsymbol{\tau}}(T_0) = \inf\{\mathbb{P}\{\boldsymbol{\tau} = t\} \colon t \in T_0\} \quad \textit{with} \quad T_0 = \{t \in T \colon \mathbb{P}\{\boldsymbol{\tau} = t\} > 0\}.$$

Proof. We have

$$\begin{aligned} \int_{S \times T} |\mathfrak{F}x(s,t)|^p \psi(s,t) \mathbb{P}^\kappa(ds) \mathbb{P}^{\boldsymbol{\tau}}(dt) &= \int_S \mathbb{P}^\kappa(ds) \int_T |\mathfrak{F}x(s,t)|^p \psi(s,t) \mathbb{P}^{\boldsymbol{\tau}}(dt) \\ &\geqslant \varepsilon \int_S \mathbb{P}^\kappa(ds) \sum_{t \in T_0} |\mathfrak{F}x(s,t)|^p \mathbb{P}\{\boldsymbol{\tau} = t\} \\ &\geqslant \varepsilon \inf \mathbb{P}^{\boldsymbol{\tau}}(T_0) \int_S \mathbb{P}^\kappa(ds) \left\{ \max_{t \in T_0} |\mathfrak{F}x(s,t)| \right\}^p. \end{aligned}$$

If we let y be the function defined by $y(\omega) := \max_{t \in T_0} |\mathfrak{F}x(\kappa(\omega), t)|$ for every $\omega \in \Omega$, then y belongs to $L^p_+(\sigma(\kappa), \mathbb{P}) \subset L^p_+(\mathcal{F}^c, \mathbb{P})$ and satisfies

$$|x| \leqslant y \quad \text{and} \quad \|x\|_p \geqslant [\varepsilon \inf \mathbb{P}^{\boldsymbol{\tau}}(T_0)]^{\frac{1}{p}} \|y\|_p.$$

We then get the desired result. Q.E.D

We introduce on $\Sigma = \sum_{i \in I} L^p(\mathcal{F}^i, \mathbb{P})$ the following norm χ:

$$\chi(x) = \inf \left\{ \sum_{i \in I} \left\|x^i\right\|_p \ : \ (x^i)_{i \in I} \in \prod_{i \in I} L^p(\mathcal{F}^i, \mathbb{P}) \quad \text{and} \quad \sum_{i \in I} x^i = x \right\}.$$

It is straightforward to check that the χ-topology is stronger that the $\|\cdot\|_p$-topology restricted to Σ, more precisely

$$\forall x \in \sum_{i \in I} L^p(\mathcal{F}^i, \mathbb{P}), \quad \|x\|_p \leqslant \chi(x).$$

Moreover, the χ-topology and the $\|\cdot\|_p$-topology coincide in $L^p(\mathcal{F}^c, \mathbb{P})$ since

$$\forall x \in L^p(\mathcal{F}^c, \mathbb{P}), \quad \|x\|_p = \chi(x).$$

We propose hereafter a description of χ-continuous linear functionals defined on the space Σ.

Proposition A.2. *A linear functional $\pi \in \Sigma^\star$ is χ-continuous if and only if there exists a family $(\psi^i)_{i\in I}$ with $\psi^i \in L^q(\mathcal{F}^i, \mathbb{P})$ such that*

$$\forall x \in X^i, \quad \pi(x) = \langle \psi^i, x\rangle = \mathbb{E}[\psi^i x]$$

and such that the family $(\psi^i)_{i\in I}$ is consistent in the sense that

$$\forall (i,k) \in I \times I, \quad \mathbb{E}[\psi^i \colon \mathcal{F}^c] = \mathbb{E}[\psi^k \colon \mathcal{F}^c].$$

Proof. Let $\pi \in \Sigma^\star$ be a η-continuous linear functional on Σ. Denote by π^i the restriction of π to the space $L^p(\mathcal{F}^i, \mathbb{P})$. Since $\chi(x) = \|x\|_p$ for every $x \in L^p(\mathcal{F}^i, \mathbb{P})$, the linear functional π^i is $\|\cdot\|_p$-continuous and there exists $\psi^i \in L^q(\mathcal{F}^i, \mathbb{P})$ representing π^i in the sense that

$$\forall x \in L^p(\mathcal{F}^i, \mathbb{P}), \quad \pi^i(x) = \langle \psi^i, x\rangle = \mathbb{E}[\psi^i x].$$

Consider two agents i and k. The restrictions of π^i and π^k to $L^p(\mathcal{F}^c, \mathbb{P})$ coincide with the restriction of π to the same space. It follows that

$$\forall z \in L^p(\mathcal{F}^c, \mathbb{P}), \quad 0 = \mathbb{E}[(\psi^i - \psi^k)z] = \mathbb{E}[z\mathbb{E}[\psi^i - \psi^k \colon \mathcal{F}^c]]$$

implying that $\mathbb{E}[\psi^i - \psi^k \colon \mathcal{F}^c] = 0$.

We now prove that the converse is true. Let $\pi \in \Sigma^\star$ be a linear functional such that for each i there exists $\psi^i \in L^q(\mathcal{F}^i, \mathbb{P})$ representing π on $L^p(\mathcal{F}^i, \mathbb{P})$. Let $x \in \Sigma$. For any sum decomposition $x = \sum_{i\in I} x^i$ with $x^i \in L^p(\mathcal{F}^i, \mathbb{P})$ we have

$$|\pi(x)| \leqslant \sum_{i\in I} |\pi(x)| \leqslant \sum_{i\in I} \left\|\psi^i\right\|_q \left\|x^i\right\|_p.$$

It then follows that

$$|\pi(x)| \leqslant \left[\max_{i\in I} \left\|\psi^i\right\|_q\right] \chi(x).$$

We have thus proved that π is χ-continuous. Q.E.D

Let π be a χ-continuous linear functional defined on Σ. We know that it is possible to represent the restriction of π to $L^p(\mathcal{F}^i, \mathbb{P})$ by a vector $\psi^i \in L^q(\mathcal{F}^i, \mathbb{P})$. There is a natural question to ask: Is it possible to find a common representation $\psi \in L^q(\mathcal{F}, \mathbb{P})$ of the linear functional π when defined on the whole space $L^q(\mathcal{F}, \mathbb{P})$? The answer is trivially yes if one of the following conditions is satisfied:

(a) the σ-algebra $\mathcal{F}$ is a finite algebra;

(b) the union $\cup_{i\in I}\mathcal{F}^i$ coincides with $\mathcal{F}$, i.e. for every event $A \in \mathcal{F}$, there exists at least one agent that can discern this event;

(c) the information structure is conditionally independent (see Daher et al. (2007) for details).[15]

Actually the answer is also yes under Assumptions III and IV. In order to prove this result, we first provide a sufficient condition for the $\|\cdot\|$-continuity on Σ of the function χ.

Proposition A.3. *Under Assumption III, the mapping $x \mapsto \chi(x)$ is $\|\cdot\|_p$-continuous on Σ provided that there exists $\beta > 0$ such that*

$$\max_{t\in T_0} \psi(s,t) \leqslant \beta \min_{t\in T_0} \psi(s,t), \quad \textit{for } \mathbb{P}^{\kappa}\textit{-a.e. } s. \tag{6}$$

Proof of Proposition A.3. [16] Let x be a vector in Σ and denote by $\mathfrak{F}x$ the function defined on $S \times T$ by

$$\mathfrak{F}x(s,t) = \begin{cases} x\left((\kappa \times \boldsymbol{\tau})^{-1}(s,t)\right) & \text{if} \quad (s,t) \in \mathrm{Im}(\kappa \times \boldsymbol{\tau}) \\ \\ 0 & \text{if} \quad (s,t) \notin \mathrm{Im}(\kappa \times \boldsymbol{\tau}) \end{cases}$$

where

$$\mathrm{Im}(\kappa \times \boldsymbol{\tau}) = \{(s,t) \in S \times T \,:\, \exists \omega \in \Omega, \quad s = \kappa(\omega) \quad \text{and} \quad t = \boldsymbol{\tau}(\omega)\}.$$

Since x is $\sigma(\kappa, \boldsymbol{\tau})$-measurable, this function is well defined and is $\mathcal{S}\otimes\mathcal{T}$-measurable. Moreover, since x belongs to $L^p(\mathcal{F}, \mathbb{P})$ the function $\mathfrak{F}x$ belongs to $L^p(\mathcal{S}\otimes\mathcal{T}, \mathbb{P}^{\kappa\times\boldsymbol{\tau}})$. More precisely, we have

$$\|x\|_p^p = \int_{\Omega} |x(\omega)|^p \mathbb{P}(d\omega) = \int_{S\times T} |\mathfrak{F}x(s,t)|^p \mathbb{P}^{\kappa\times\boldsymbol{\tau}}(ds \times dt).$$

Applying Assumption III, we obtain

$$\|x\|_p^p = \int_S \mathbb{P}^{\kappa}(ds) \sum_{t\in T} |\mathfrak{F}x(s,t)|^p \psi(s,t) \mathbb{P}^{\boldsymbol{\tau}}\{t\}.$$

[15]The information structure $(\mathcal{F}^i)_{i\in I}$ is conditionally independent if for each pair (i,k) of agents with $i \neq k$ and for every pair of events $A^i \in \mathcal{F}^i$ and $A^k \in \mathcal{F}^k$, we have that $\mathbb{P}(A^i \cap A^k \colon \mathcal{F}^c) = \mathbb{P}(A^i \colon \mathcal{F}^c)\mathbb{P}(A^k \colon \mathcal{F}^c)$ almost everywhere. When the information structure is conditionally independent we can prove (see Daher et al. (2007)) that the vector $\psi \in L^q(\mathcal{F}, \mathbb{P})$ represents π on the whole space $L^p(\mathcal{F}, \mathbb{P})$ where ψ is defined by

$$\psi = \psi^c + \sum_{i\in I} \left[\psi^i - \psi^c\right]$$

where $\psi^c = \mathbb{E}[\psi^i \colon \mathcal{F}^c]$ for any i.

[16]An important part of the arguments of the proof are inspired by those used in Podczeck and Yannelis (2008).

We denote by $\mathcal{T}_0$ the trace of the σ-algebra $\mathcal{T}$ on T_0 and for each i, we denote by $\mathcal{T}_0^i$ the sub σ-algebra of T_0 generated by the projection mapping $t \mapsto t^i$. The vector space $\sum_{i\in I} L^0(\mathcal{T}^i, \mathbb{P})$ is generated by the family

$$\left\{ \mathbf{1}_A \ : \ A \in \bigcup_{i\in I} \mathcal{T}_0^i \right\}.$$

There exists a sub-family $\mathcal{A}$ of $\cup_{i\in I}\mathcal{T}_0^i$ such that the family

$$\{\mathbf{1}_A \ : \ A \in \mathcal{A}\}$$

is a minimal generating family of $\sum_{i\in I} L^0(\mathcal{T}^i, \mathbb{P}^{\boldsymbol{\tau}})$, in other words the family $\{\mathbf{1}_A \ : \ A \in \mathcal{A}\}$ is generating and linearly independent. Since the mapping

$$\begin{array}{ccc} \mathbb{R}^{\mathcal{A}} & \longrightarrow & \sum_{i\in I} L^0(\mathcal{T}_0^i, \mathbb{P}^{\boldsymbol{\tau}}) \\ (\alpha_A)_{A\in\mathcal{A}} & \longmapsto & \sum_{A\in\mathcal{A}} \alpha_A \mathbf{1}_A \end{array}$$

is a linear bijection, then it is continuous whatever the norms we consider on each space. It follows that there exists $0 < m < M < \infty$ such that

$$\forall \alpha \in \mathbb{R}^{\mathcal{A}}, \quad m^p \int_T \left| \sum_{A\in\mathcal{A}} \alpha_A \mathbf{1}_A(t) \right|^p \mathbb{P}^{\boldsymbol{\tau}}(dt) \leqslant \sum_{A\in\mathcal{A}} |\alpha_A|^p \leqslant M^p \int_T \left| \sum_{A\in\mathcal{A}} \alpha_A \mathbf{1}_A(t) \right|^p \mathbb{P}^{\boldsymbol{\tau}}(dt).$$

For each $s \in S$, the mapping $t \mapsto \mathfrak{F}x(s,t)$ belongs to $\sum_{i\in I} L^0(\mathcal{T}^i, \mathbb{P}^{\tau})$, implying that there exits $\alpha(s) \in \mathbb{R}^{\mathcal{A}}$ such that

$$\forall (s,t) \in S \times T, \quad \mathfrak{F}x(s,t) = \sum_{A\in\mathcal{A}} \alpha_A(s)\mathbf{1}_A(t).$$

The set $\mathcal{A}$ can be decomposed in a partition $(\mathcal{A}^i)_{i\in I}$ where $\mathcal{A}^i \subset \mathcal{T}_0^i$ for each i. It then follows that

$$\mathfrak{F}x(s,t) = \sum_{i\in I} \left[\sum_{A\in\mathcal{A}^i} \alpha_A(s)\mathbf{1}_A(t) \right].$$

We denote by x^i the function defined on Ω by

$$\forall \omega \in \Omega, \quad x^i(\omega) = \sum_{A\in\mathcal{A}^i} \alpha_A(\kappa(\omega))\mathbf{1}_A(\tau(\omega)).$$

It is straightforward to check that

$$x = \sum_{i\in I} x^i \quad \text{and} \quad x^i \in L^p(\mathcal{F}^i, \mathbb{P}), \quad \forall i \in I.$$

We denote by $\overline{\psi}$ and $\underline{\psi}$ the functions defined on S by

$$\forall s \in S, \quad \overline{\psi}(s) = \max_{t \in T_0} \psi(s,t) \quad \text{and} \quad \underline{\psi}(s) = \min_{t \in T_0} \psi(s,t).$$

Observe that

$$\begin{aligned}
\left\| x^i \right\|_p^p &= \int_S \mathbb{P}^\kappa(ds) \int_T \left| \sum_{A \in \mathcal{A}^i} \alpha_A(s) \mathbf{1}_A(t) \right|^p \psi(s,t) \mathbb{P}^{\boldsymbol{\tau}}(dt) \\
&\leqslant \frac{1}{m} \int_S \mathbb{P}^\kappa(ds) \overline{\psi}(s) \sum_{A \in \mathcal{A}^i} |\alpha_A(s)|^p \\
&\leqslant \frac{1}{m} \int_S \mathbb{P}^\kappa(ds) \overline{\psi}(s) \sum_{A \in \mathcal{A}} |\alpha_A(s)|^p \\
&\leqslant \frac{M}{m} \int_S \mathbb{P}^\kappa(ds) \overline{\psi}(s) \int_T \left| \sum_{A \in \mathcal{A}} \alpha_A(s) \mathbf{1}_A(t) \right|^p \mathbb{P}^{\boldsymbol{\tau}}(dt) \\
&\leqslant \beta \frac{M}{m} \int_S \mathbb{P}^\kappa(ds) \underline{\psi}(s) \int_T \left| \sum_{A \in \mathcal{A}} \alpha_A(s) \mathbf{1}_A(t) \right|^p \mathbb{P}^{\boldsymbol{\tau}}(dt) \\
&\leqslant \beta \frac{M}{m} \left\| x \right\|_p^p.
\end{aligned}$$

It then follows that

$$\chi(x) \leqslant \sum_{i \in I} \left\| x^i \right\|_p \leqslant (\#I) \left[\beta \frac{M}{m} \right]^{1/p} \left\| x \right\|_p.$$

Q.E.D

Since the set T is finite, we can prove that the function ψ is uniformly bounded.

Lemma A.1. *For every $t \in T_0$ we have $\psi(s,t)\mathbb{P}^\tau\{t\} \leqslant 1$ for $\mathbb{P}^\kappa$-a.e. $s \in S$. In other words,*

$$\forall t \in T_0, \quad \mathbb{P}^\kappa\{\psi(\cdot,t)\mathbb{P}^\tau\{t\} > 1\} = 0. \tag{7}$$

Proof of Lemma A.1. Fix $t \in T_0$ and let A_t be the set in $\mathcal{S}$ defined by

$$A_t = \{s \in S : \psi(s,t)\mathbb{P}^\tau\{t\} > 1\}. \tag{8}$$

Assume by way of contradiction that $\mathbb{P}^\kappa(A_t) > 0$. It then follows that

$$\begin{aligned}
\mathbb{P}^{\kappa \times \tau}(A_t \times \{t\}) &= \int_{A_t} \psi(s,t)\mathbb{P}^\tau\{t\}\mathbb{P}^\kappa(ds) \\
&> \int_{A_t} \mathbb{P}^\kappa(ds) \\
&> \mathbb{P}^\kappa(A_t) = \mathbb{P}^{\kappa \times \tau}(A_t \times T).
\end{aligned} \tag{9}$$

We thus obtain the following contradiction

$$\mathbb{P}^{\kappa\times\tau}(A_t \times \{t\}) > \mathbb{P}^{\kappa\times\tau}(A_t \times T). \tag{10}$$

Q.E.D

Combining Lemma A.1 and Proposition A.3, it is straightforward to prove the following equivalence result.

Corollary 1. *Under Assumptions III and IV, the two norms $\|\cdot\|_p$ and χ are equivalent in Σ, i.e., they define the same topology on Σ.*

A.1 Ideals

If a belongs to $L^p_+(\mathcal{F}^c, \mathbb{P})$ we recall that $L(a)$ is the vector subspace of $L^p(\mathcal{F}, \mathbb{P})$ defined by

$$L(a) := \{x \in E : \exists \mu > 0, \quad |x(\omega)| \leqslant \mu a(\omega) \quad \text{for } \mathbb{P}\text{–a.e. } \omega \in \Omega\}.$$

For each $i \in I$, the space $L^p(\mathcal{F}^i, \mathbb{P})$ is denoted by $L^i(a)$. On $\Sigma(a) = \sum_{i\in I} L^i(a)$, we let η be the norm defined by

$$\forall x \in \Sigma(a), \quad \eta(x) = \inf\left\{\sum_{i\in I} \left\|x^i\right\|_p \ : \ (x^i)_{i\in I} \in \prod_{i\in I} L^i(a) \quad \text{and} \quad \sum_{i\in I} x^i = x\right\}.$$

We obviously have

$$\forall x \in \Sigma(a), \quad \|x\|_p \leqslant \chi(x) \leqslant \eta(x).$$

Actually these three norms define the same topology on $\Sigma(a)$.

Proposition A.4. *Under Assumptions III and IV, the two norms $\|\cdot\|_p$ and η are equivalent in $\Sigma(a)$, i.e., they define the same topology on $\Sigma(a)$.*

Proof of Proposition A.4. The arguments are very similar to those used to prove Corollary 1. Only the proof of Proposition A.3 deserves some attention. Let x be a vector in $\Sigma(a)$. Since a belongs to $L^p(\mathcal{F}^c, \mathbb{P})_+$, there exists a function $h \in L^p_+(\mathcal{S}, \mathbb{P}^\kappa)$ such that $\mathfrak{F}a(s,t) \leqslant h(s)$ for every (s,t). Following the notations in the proof of Proposition A.3, the mapping

$$\begin{array}{ccc} \mathbb{R}^{\mathcal{A}} & \longrightarrow & \sum_{i\in I} L^0(\mathcal{T}^i_0, \mathbb{P}^{\boldsymbol{\tau}}) \\ (\alpha_A)_{A\in\mathcal{A}} & \longmapsto & \sum_{A\in\mathcal{A}} \alpha_A \mathbf{1}_A \end{array}$$

is a linear bijection. Therefore, it is continuous whatever the norms we consider in each space. It follows that there exists $0 < \Xi < \infty$ such that

$$\forall \alpha \in \mathbb{R}^{\mathcal{A}}, \quad \max_{A\in\mathcal{A}} |\alpha_A| \leqslant \Xi \sup_{t\in T_0} \left|\sum_{A\in\mathcal{A}} \alpha_A \mathbf{1}_A(t)\right|.$$

For each $s \in S$, the mapping $t \mapsto \mathfrak{F}x(s,t)$ belongs to $\sum_{i\in I} L^0(\mathcal{T}^i, \mathbb{P}^\tau)$, implying that there exits $\alpha(s) \in \mathbb{R}^{\mathcal{A}}$ such that

$$\forall (s,t) \in S \times T, \quad \mathfrak{F}x(s,t) = \sum_{A\in\mathcal{A}} \alpha_A(s)\mathbf{1}_A(t).$$

Since there exists $\mu > 0$ such that $|\mathfrak{F}x(s,t)| \leqslant \mu h(s)$ for every $t \in T_0$, we deduce that

$$\forall A \in \mathcal{A}, \quad \alpha_A(s) \leqslant \mu \Xi h(s).$$

The set $\mathcal{A}$ can be decomposed in a partition $(\mathcal{A}^i)_{i\in I}$ where $\mathcal{A}^i \subset \mathcal{T}_0^i$ for each i. It then follows that

$$\mathfrak{F}x(s,t) = \sum_{i\in I}\left[\sum_{A\in\mathcal{A}^i} \alpha_A(s)\mathbf{1}_A(t)\right].$$

We denote by x^i the function defined on Ω by

$$\forall \omega \in \Omega, \quad x^i(\omega) = \sum_{A\in\mathcal{A}^i} \alpha_A(\kappa(\omega))\mathbf{1}_A(\tau(\omega)).$$

Since $\alpha_A(s) \leqslant \mu\Xi h(s)$ for every $A \in \mathcal{A}$ and each $s \in S$, we get that $|x^i| \leqslant \mu\Xi a$. It is then straightforward to check that

$$x = \sum_{i\in I} x^i \quad \text{and} \quad x^i \in L^i(a), \quad \forall i \in I.$$

The rest of the proof follows almost verbatim. Q.E.D

A.2 Proof of Claim 5.1

In order to prove Claim 5.1, we will need the following convexity result due to Podczeck (1996). We also refer to Aliprantis et al. (2004, Lemma 4.3) for a proof.

Lemma A.2. *Let (Z,τ) be an ordered topological vector space, let M be a vector subspace of Z (endowed with the induced order), let Y be an open and convex subset of Z such that $Y \cap M_+ \neq \emptyset$ and let $z \in \mathrm{cl}Y \cap M_+$. If p is a linear functional on M satisfying*

$$\forall y \in Y \cap M_+, \quad p(y) \geqslant p(z)$$

then there exists some $\pi \in (Z,\tau)'$ such that

$$\forall m \in M_+, \quad \pi(m) \leqslant p(m) \quad \textit{and} \quad p(z) = \pi(z).$$

Proof of Claim 5.1. We proved the existence of a linear functional $p \in (\Sigma,\sigma)'$ with $p(a) > 0$ and satisfying[17]

$$\forall i \in I, \quad p(x^i) = p(e^i) \quad \text{and} \quad \forall y^i \in P^i(x^i) \cap L^i(a), \quad p(y^i) \geqslant p(e^i).$$

[17] See (1) and (2).

Fix $y^i \in \widehat{P}^i(x^i) \cap L^i(a)_+$, then for $\alpha > 0$ small enough

$$\alpha y^i + (1-\alpha)x^i \in \widehat{P}^i(x^i) \cap A_{x^i} \cap L^i_+(a) \subset P^i(x^i) \cap L^i(a),$$

in particular $p(y^i) \geqslant p(x^i)$. Applying Lemma A.2 with $(Z,\tau) = (L^i, \|\cdot\|_p)$, $M = L^i(a)$, $Y = \widehat{P}^i(x^i)$ and $z = x^i$, we get Claim 5.1. Q.E.D

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