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Hybrid Auctions II: Experimental Evidence*

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Abstract

In this paper we report the results of an experiment designed to examine the properties of a hybrid auction - a Dutch-Vickrey auction, that combines a sealed bid first-price auction with a sealed bid second-price auction. This auction mechanism shares some important features with that used in the sale of the companies constituted through the partial division of the Telebras System - the government-owned Telecom holding in Brazil.

We designed an experiment where individuals participate in a sequence of independent first-price auctions followed by a sequence of hybrid auctions. Several conclusions emerged from this experimental study. First, ex-post efficiency was achieved overwhelmingly by the hybrid auctions. Secondly, although overbidding (with respect to the risk-neutral Bayesian Nash equilibrium) was a regular feature of participants' bidding behavior in the first-price auctions — as it is commonly reported in most experimental studies of first-price auctions, it was less frequent in the hybrid auctions. By calibrating the results to allow for risk-averse behavior we were able to account for a significant part of the overbidding. Finally, we compared the revenue generated by the hybrid auction with that generated by a standard first-price sealed bid auction and the results were ambiguous.

JEL Classification: C72, C91, D44.
Keywords: experiments; hybrid auctions; expected revenue; efficiency.

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1 Introduction

There was a change in the perceived role of government in the last decades of the twentieth century. This change brought about a worldwide privatization process and major restructuring of infrastructure sectors such as electricity, gas distribution, and telecommunications. Auctions were one of the favored sale instruments in the privatization process and auction-like markets were instrumental for the introduction of competition in most restructuring processes.

Although auctions have been used since time immemorial, they have never been used on such a scale before. By using insights from auction theory and known properties of standard auction formats, such as the first-price, English and Dutch auctions, several new auction formats were designed to deal with specific concerns such as how to prevent collusive, predatory and entry deterring behavior, to maximize efficiency (for example by allowing synergies to be realized) or to maximize the number of potential competitors.

As an example, with asymmetric players, a sealed bid auction may attract more bidders than an English auction as weaker bidders have a higher chance of winning in the former than in the latter auction format. English auctions, however, allow information to be revealed about other players' signals. Klemperer [15], for instance, follows along these lines to suggest an Anglo-Dutch auction, a hybrid of the ascending and sealed bid auctions, that may perform better in terms of the traditional concerns of competition policy.

In this paper we examine the properties of another hybrid auction - a Dutch-Vickrey auction, that combines a sealed bid first-price auction with a sealed bid second-price auction. This auction mechanism shares some important features with the Dutch-Anglo mechanism used in the sale of the companies constituted through the partial division of the Telebras System - the government-owned Telecom holding in Brazil. The sale represented a major step towards the restructuring of the telecommunications sector in the country and it raised in excess of

US$20 billion.

The Dutch-Anglo auction works as follows. Each buyer submits a sealed bid. Once the highest bid is known, the bidder who submitted it wins if her bid is higher than the second highest bid by more than a predetermined amount or percentage. If at least one bidder submits a bid sufficiently close to the highest bid (that is, if the difference between this bid and the highest one is smaller than the predetermined amount or percentage) the qualified bidders compete in an open ascending bid auction that has the highest bid of the first stage as the reserve price. The qualified bidders include the one with the highest bid in the first stage of the auction and those who bid close enough to her.

In a related paper Dutra and Menezes [7] develop a model that captures some of the features of this hybrid auction. We model a situation where risk neutral bidders compete in a two stage auction: a first price auction followed by a Vickrey auction as a second stage when there are bids sufficiently close to the highest bid in the first stage. We consider a model in which potential buyers' values have both a private and a common component. Special cases include the independent private values model and the pure common values model. Of course, with private independent values the Vickrey and the Ascending (English) auctions are strategically equivalent and, consequently, so are the Dutch-Vickrey and the Dutch-Anglo auctions.

For the case of a discrete distribution, we show that the hybrid auction generates more revenue than any standard auction. The reason is that one may view this hybrid auction as a Vickrey auction with an endogenously set reserve price at the first stage. Although this hybrid auction generates (weakly) less revenue than the optimal auction, it has the advantage of being ex-post efficient. In contrast, the optimal auction may not be ex-post efficient.

In this paper we report the results of an experiment designed to verify this proposition for the case of independent private values. We conducted an experimental session composed of three parts, with 45 subjects recruited from undergraduate economic courses. During the experiment the subjects were allocated in groups of three participants according to a predetermined rule. Each subject participated in 18 auctions. In the first six auctions the subjects bid for a fictitious commodity sold through a standard first price sealed bid auction. In the next twelve auctions, a hybrid Dutch-Vickrey auction was used.

Several conclusions emerged from this experimental study. Firstly, ex-post efficiency was achieved overwhelmingly by the hybrid auctions. Secondly, overbidding (with respect to the risk-neutral Bayesian Nash equilibrium) was a regular feature of individual behavior in first-price auctions. Overbidding, however, was less prominent in hybrid auctions. Finally, we compared the revenue generated by the hybrid auction with that generated by a standard first-price sealed-bid auction and the results were ambiguous.

The paper is organized as follows. In the next section we describe our theoretical model and predictions of equilibrium behavior in the Dutch-Vickrey auction. Section 3 contains the key points developed in the instructions to subjects, together with other significant design features. Section 4 discusses the results and in section 5 we comment on issues that need further investigation.

2 The Dutch-Vickrey Auction

Suppose that three risk neutral bidders compete for a single object, so that each bidder i receives a signal v_i and her final valuation is equal to $u(v_i) = v_i$. The private values may take one of three values:

$$v_i = \{x_0, x_1, x_2\}, \qquad i = 1, 2, 3.$$

We suppose further that each player i knows her own private value, but knows only that her opponents' values are x_0 with probability p_0, x_1 with probability p_1 and x_2 with probability p_2. This structure is common knowledge among players.

The seller's value for the single object is equal to zero.

Given symmetry, we can restrict attention to the problem faced by one of the bidders, say bidder 1. Her goal is to choose a bid $b(v_i)$ that maximizes her expected payoff. Let $v^{(t)}$ represent the t^{th} highest signal. Conditional on winning the hybrid auction the expected profit of Bidder 1 who receives signal v_1 and bids b is given by

$$\begin{aligned} \pi(v_1, b(v_1)) &= E_j\left[(v_1 - b(v_1))\, 1_{\{b(v_1)>b(v_j)+z; j\neq 1\}}\right] + \\ &\quad + E_j\left[\left(v_1 - v^{(2)}\right) 1_{\left\{v^{(2)}<b(v_1)<b\left(v^{(2)}\right)+z\right\}}\right]. \end{aligned} \tag{1}$$

where z stands for the cutoff value. If the difference between the two highest bids is less than or equal to the predetermined amount z, the winner of the hybrid auction is chosen in a Vickrey auction that takes place in a second stage. This is expressed in the second expectation term in the right hand side of equation (1).

We focus on the symmetric equilibrium. Given the discrete nature of the model, we characterize a mixed strategy equilibrium for this game; that is, for each individual's possible value, we compute the support and the associated distribution of the equilibrium bidding functions.

Proposition 1 *The symmetric equilibrium bid strategies for the Dutch-Vickrey auction are as follows:*

$$b(v_1) = \begin{cases} \cdot \quad x_0 \quad if \quad v_1 = x_0; \\ \cdot \quad \textit{bid randomly in the interval } \left[x_0, \overline{b}_1\right] \\ \textit{according to the bid distribution function} \\ G_1(b) \quad if \quad v_1 = x_1; \\ \cdot \quad \textit{bid randomly in the interval } \left[\overline{b}_1, \overline{b}_2\right] \\ \textit{according to the bid distribution function} \\ G_2(b) \quad if \quad v_1 = x_2; \end{cases}$$

where the expressions for $G_1(b), G_2(b), \overline{b}_1$ and $\overline{b}_2$ can be found in the appendix. [1]

[1] A complete proof can be found in Dutra and Menezes [7]. The exact values that hold for the experimental session are presented in section 4.

These equilibrium bidding strategies imply the following

Proposition 2 *The Dutch-Vickrey auction generates more expected revenue than any standard auction with a reserve price equal to the seller's valuation.*

Sketch of the Proof. The seller's expected revenue is the difference between the expected social value and the bidder's expected return. The effect of z is to reduce the expected return to the bidders who have a value other than the lowest and, therefore, to increase the seller's expected revenue. The reason is that the hybrid auction may be seen as a Vickrey auction with an endogenously set reserve price. This generates more revenue than any standard auction with a reserve price set at zero (that is, equal to the seller's value). ■

3 The Design of the Experiment

Bidder $i's$ valuation for the object consists of a private value v_i only. At the beginning of the period (auction) the buyer receives an envelope with an enclosed bidding form that contains her private value to the fictitious commodity in the current period. The subject is also informed that in each and every period her opponents' values to the item are independent draws from a fixed discrete distribution. In particular, in each period each agent's private value, v_i, is the result of a lottery that with probability 0.4 is 0, with probability 0.3 is R$3.00 and with probability 0.3 is R$6.00. When bidder i wins the auction, she receives a net amount of $v_i - p$, where p stand for the commodity price at the period. The subjects' profits in the experiment are expressed in monetary values ("reais", the local currency).[2]

The recruiter accepted subscriptions of 50 students in order to make a provision for bankruptcies or no shows. After all subjects arrived, the 45 participants

[2] Kagel [13] points to a procedure called the binary lottery, introduced by Becker, Degroot and Marschak [1], which controls for risk aversion. However, application of the binary lottery procedure to induce risk neutral bidding in first price auctions has met with mixed results in the literature, so we decided to express subjects receipts in monetary units.

were distributed in 15 groups, according to a preestablished order. In order to minimize interactions we implemented a fixed rotation rule such that no individual would be matched with any opponent more than twice in the whole experiment. In each group a graduate student previously selected and trained was responsible for the conduct of the auctions, acting as a monitor.

The instructions were divided into three parts, one for each part of the experiment.[3] After reading the instructions relative to each part, the subjects were required to answer some questions in order to verify their understanding of the experiment rules. Monitors evaluated subjects' understanding of the game rules by checking their answers to these hypothetical questions. Once all subjects finished reading the instructions and all questions were properly answered the experimental session started.

The complete experiment consisted of three parts.

Part 1. In each of the 6 auctions that composed the first part of the experiment a fictitious commodity was sold through a first-price sealed bid auction. This is of course equivalent to a hybrid auction with $z = 0$. At the beginning of a period each subject knew only her value of the commodity - which was printed on the bidding form - and the process by which others' values were generated – that is, through the lottery. The private values signals were independently and identically distributed across subjects and periods and the procedure for generating them was common knowledge.

There were no restrictions on bids values. In order to bid the subjects had a R$10 credit that could be used during the experimental session.[4] Once each participant's cumulative losses surpassed this amount, they would refrain from bidding. During the first part of the experiment the subject who submitted the highest sealed bid won. In the event of a tie, the winner was chosen randomly

[3] Full instructions are available from the authors by request.

[4] For comparison purposes, R$10 is about two times what a typical student would pay for his lunch. It is about the same as the cost of a movie ticket.

with equal probability. At the end of a period, the monitor in charge revealed only the final price and the identity of the winner so that the winner could update his or her earnings in an enclosed worksheet. At the end of the experimental session the subjects received their monetary rewards in cash.

Part 2. The second part of the experiment lasted from period 7 through 12. In every period of this session the commodity was sold through a hybrid of first-price sealed bid auction and Vickrey auction: the subject who submitted the highest bid won if her bid was higher than the second highest bid by more than R$1.00. If one or more subjects submitted a bid smaller than the highest bid by up to $R$$1.00, they were qualified, together with the subject who submitted the highest bid in the first price auction, to compete in a Vickrey auction that had the highest bid of the first price auction as the reserve price. The winner of the Vickrey auction was declared the winner, earning a profit equal to the difference between her value and the second highest bid in the Vickrey auction.[5]

Part 3. The third part of the experiment lasted from period 13 to 18. In this part a second stage occurred if one or more subjects submitted a bid smaller than the highest bid by up to R$1.50. Then the qualified bidders played a Vickrey auction.

Subjects and Bankruptcies. We conducted the experiment with inexperienced subjects recruited at undergraduate economic courses. The experimental session finished within two hours and subjects earned R$ 6.50 on average plus a R$10.00 show up fee. Their starting capital of R$10.00 provided some buffer against bankruptcies; a subject would become bankrupt if her initial cash balance of R$10.00 was depleted. Only one subject went bankrupt. Even though subjects were told that once their losses surpassed R$10.00 they would not be allowed to bid, we did not enforce this rule.[6] Instead we let the other subjects that were

[5] The reason for ordering the treatiment in the order $z = 0, 1$ and 1.5 was a practical one. Since we wanted to shift people around to avoid repated interaction, this was the most efficient way to design the experiment.

[6] We believe this procedure did not significantly affect our results since only one subject went

matched with the bankrupt subject continue playing (only) to give them the opportunity to make some money. However, the data from these auctions were discarded.

bankrupt by the end of the third part and her opponents did not know this.

4 Results

In this session we report on subjects' bidding behavior on both the first-price and the hybrid auctions. We also provide a comparison of the actual revenue generated by the different auction formats and report on the overwhelming evidence of the ex-post efficiency property of the hybrid auctions.

4.1 Equilibrium

According to our equilibrium predictions subjects with a null private value should bid 0, that is $b(x_0) = \underline{b} = x_0 = 0$ in all auctions mechanisms. Subjects receiving a value x_1 bid in the range $[\underline{b}, \overline{b}_1]$ while subjects with a x_2 value bid in the range $[\overline{b}_1, \overline{b}_2]$. >From now on we designate the hybrid auction with a R\$1.00 cutoff by hybrid-1 and the analogous auction with a R\$1.50 cutoff by hybrid-2. Table 1 presents the exact values for the experimental session.

z	$\underline{b}$	$\overline{b}_1$	$\overline{b}_2$	U_1	U_2	ER
0	0	2.02	4.05	0.48	1.95	2.59
1.0	0	1.35	3.21	0.32	1.79	2.88
1.5	0	1.01	2.79	0.24	1.71	3.02

Table 1: Exact Values

Table 1 also presents the expected payoff per auction under equilibrium play. U_1 designates the expected profit to a winner with a value $x_1 = R\$3.00$ while U_2 is the expected payoff when the winner's value is $R\$6.00$.

Figure 1 presents period-by-period bids for the whole experimental session averaged over groups. When the value is $R\$0.00$, there is occasional bidding above value, a phenomenon reported in the experimental literature. However, looking at the data at an individual level one can verify that this behavior is restricted to a subgroup of subjects: 15% of the subjects occasionally did not bid

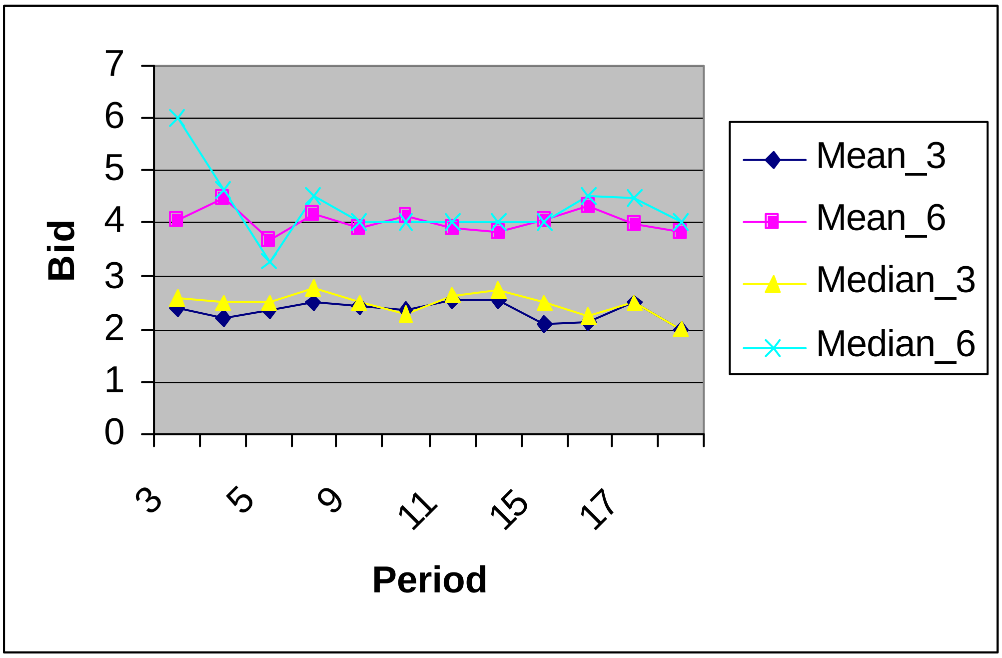

Figure 1: Mean and Median Bids

0, their dominant strategy, when their private value was 0. Nearly half of these subjects bid higher than their dominant strategy only in the first-price sealed bid auctions of the first part.

One concern is the properness of the equilibrium assumptions in the presence of discernible time trends in the actual data. The existence of a time trend could prevent beliefs from "settling down". As suggested by Goeree, Holt and Palfrey [9] we fitted a regression of average bids on time, pooled over all private values to verify the possible existence of time trends in bid data. The time coefficient was insignificant at a 10% level. Using the reciprocal of time, as suggested by Kagel [13], we also estimated the coefficient of $1/t$. We were unable to reject the null hypothesis that mean bids were equal at a 10% significance level. In addition, we estimated separate equations for each private value. The time coefficients were all statistically insignificant.

In the analysis of the experimental results we discarded the data relative to

Kolmogorov-Smirnov One Sample Test				
Auction	KS statistic [a] *(v=3)*	N	KS statistic [a] *(v=6)*	N
3	0.71***	15	0.50***	14
4	0.75***	18	0.60***	14
5	0.83***	12	0.26	10
6	0.76***	13	0.53***	14
9	0.54***	16	0.15	9
10	0.38*	13	0.37**	16
11	0.50***	14	0.32*	16
12	0.61***	13	0.32	10
15	0.33**	16	0.31	13
16	0.40**	14	0.42**	12
17	0.44***	13	0.50***	18
18	0.35*	14	0.25	16

[a] *The null hypothesis is that the empirical distribution agrees with the theoretical distribution of bids.*

**** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.*

*** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.*

** indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.*

Figure 2: Analysis of empirical distribution of bids under risk neutrality

the first two auctions from each part as the subjects were inexperienced. For consistency, we also investigated the existence of a time trend in the final four periods of each part. Again we found no evidence of a significant time trend.

An individual bidder observes a private value and then computes a bid to compete against the aggregate bid distribution that contains all the relevant strategic information about her opponents' bids. Using a Kolmogorov-Smirnov statistic we tested the null hypothesis that the data were generated by our proposed theoretical distribution. We decided to aggregate data over groups. For every period we tested the null that for a given private value the empirical distribution of a set of sample values (the observed bids in the period controlled by the subject's value) agrees with the corresponding theoretical distribution, as specified in section 2.

Table 2 presents the results of the Kolmogorov-Smirnov tests applied to the restricted data set.[7] Observe that the fit is relatively worse in the first price auctions and in the hybrid-1 auction when subject's value is equal to $R\$3.00$.

It is a well documented fact in the experimental literature that market prices from first price private value auctions typically exceed the risk neutral Nash Equi-

[7] The critical values are presented in Siegel [17].

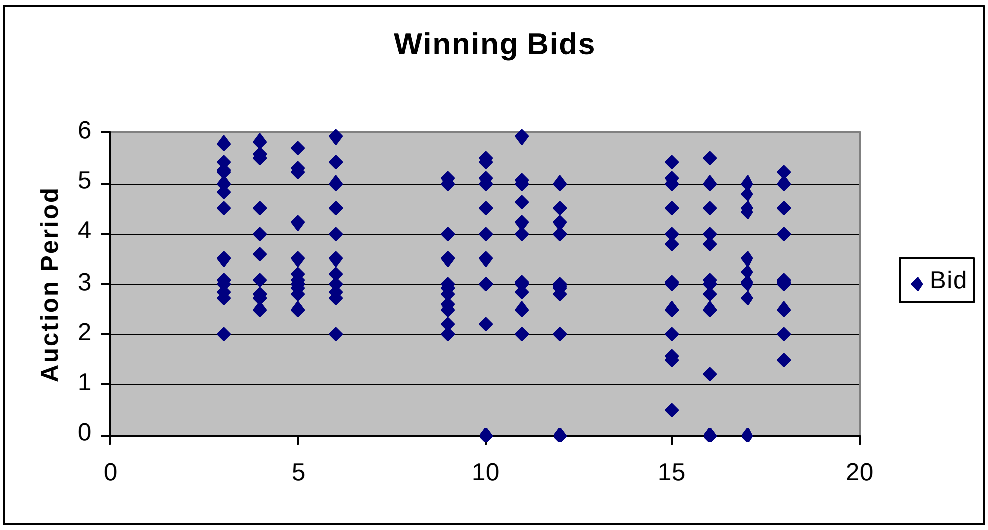

Figure 3: Plot of the Winning Bids

librium (RNNE) prediction irrespective of the number of bidders in the auction or the research group conducting the investigation.[8] A naive plot of the bids points to a general overbidding pattern relative to the equilibrium prediction, detailed in Table 1, as can be seen from Figures 4 and 5. This is consistent with risk averse behavior. Some facts reinforce the explanation of this behavior in terms of risk aversion. Bidding the dominant strategy in second price auctions is independent of attitudes towards risk. Even though in the experimental session subjects were not restricted to bid up to their private value, in general they did. Plots of the bids in the contingent Vickrey stage present some adherence of subjects' bids to the dominant strategy, as can be seen by Figures 6 and 7. Additionally, we were unable to reject the null hypothesis that mean bids were equal (using a simple test of equality of means) at a 5% significance level. This suggests that subjects follow their dominant strategy in the contingent Vickrey stage.

The examination of these figures and of Table 2 allows one to infer that deviations from this dominant strategy behavior are relatively more frequent for

[8]Cox,J.C., B.Roberson and V.L.Smith [4]; Kagel, J.H. and A.E. Roth. [14]; and Goeree, Jacob K., Charles A. Holt and Thomas R. Palfrey, [9].

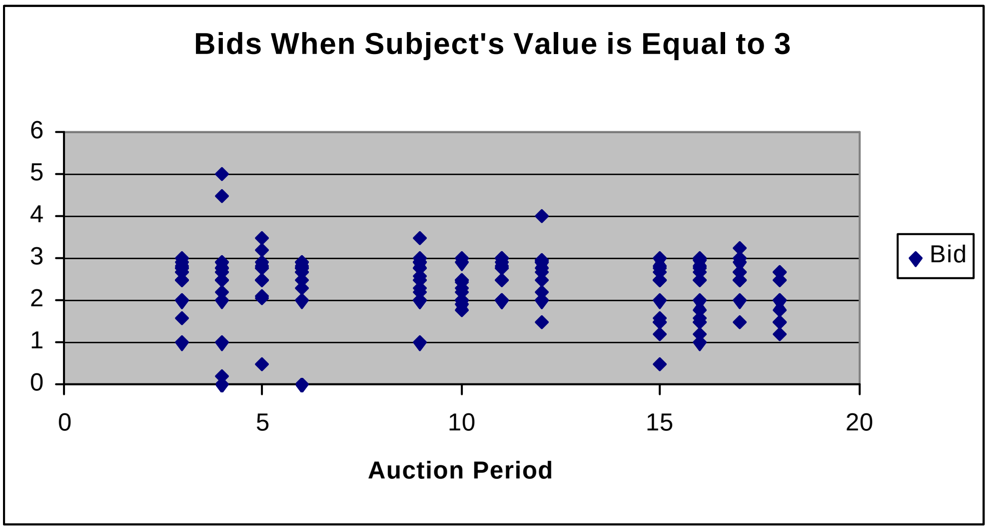

Figure 4: First-price auction bids when subject's value equals R$3.00

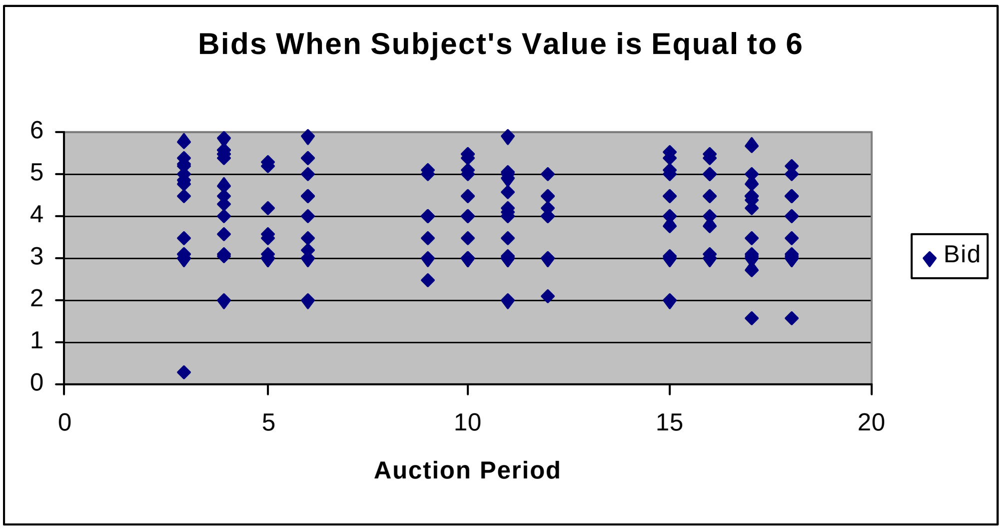

Figure 5: First-price auction bids when subject's value equals R$6.00

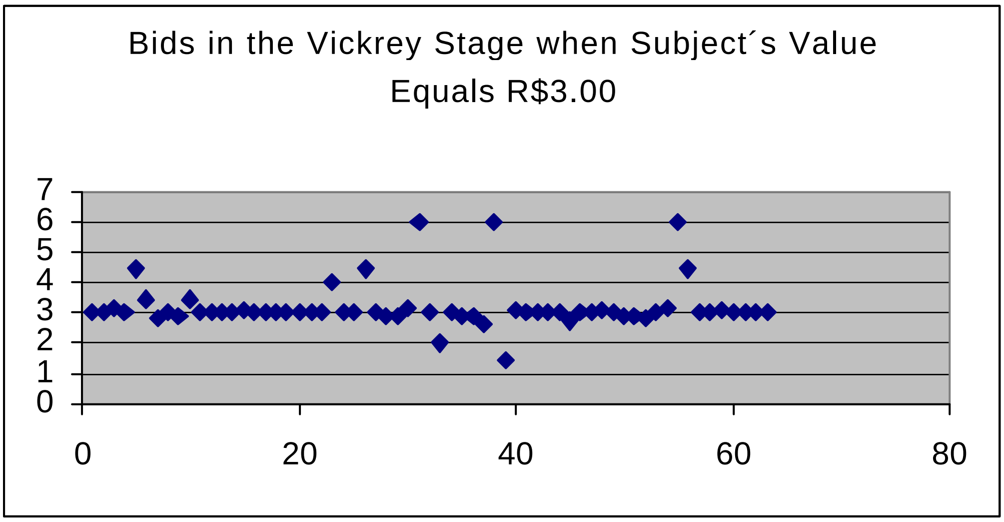

Figure 6: Bids in the Vickrey stage when subject's value equals R$3.00

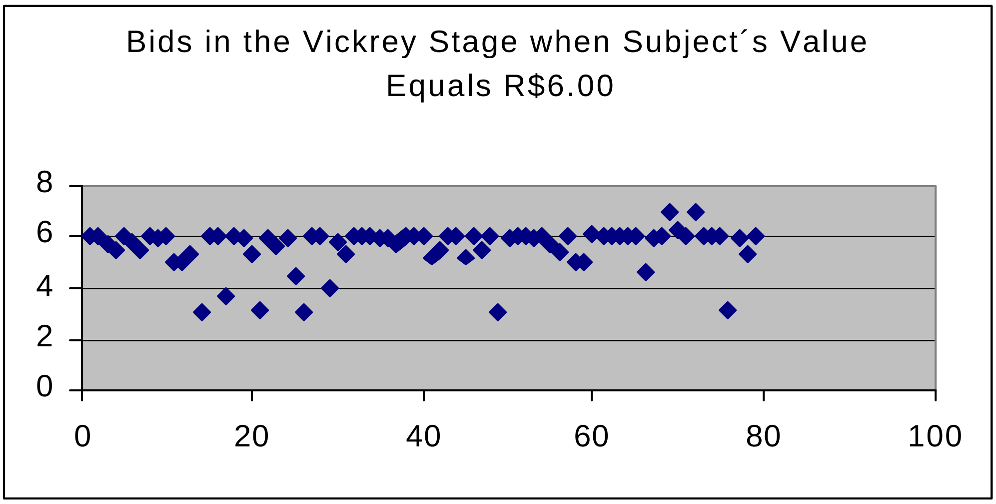

Figure 7: Bids in the Vickrey stage when subject's value equals R$6.00

low value as compared to high value bidders. This supports Kagel and Roth's [14] examination of Harrison's critique. Harrison [10] presents a methodological critique of the evidence Cox, Smith and Walker [6] employ to reject RNNE bidding theory. He argues that under the typical payoff values employed the expected cost of deviations from the RNNE is quite small, so that in terms of expected monetary payoffs many subjects had little to loose by deviating from the RNNE strategy (that is, the payoff function around the maximum is flat).

Kagel and Roth [14] have examined some implications of Harrison's critique. They present evidence that the greatest proportionate deviations from the RNNE predictions are made by bidders who draw the lowest valuations by computing the simple correlation coefficient between private values and the absolute value size of the deviation from RNNE bidding relative to the underlying private valuation. As these subjects have relatively smaller chances of winning the auction, their expected cost of deviating from the Nash equilibrium is lower either if they are risk neutral or risk averse. Our results are compatible with deviations from the Nash equilibrium guided by risk aversion and low expected costs of deviating in terms of expected payoff.[9]

4.2 Revenue

Table 8 presents the expected payoff under equilibrium play as well as the mean payoff observed in the experimental session controlled by the group private values' configuration. Given that values are drawn independently in each period and, therefore, the composition of people having the same configuration of values is changing at each round, there is no *a priori* reason to suspect that the independence hypothesis has been violated.

Simple tests of equality of means were conducted in the data to verify the

[9] By stating that our results are consistent with Harrison's critique we open the question of whether the incentives offered to the bidders were sufficiently strong. However, the evidence we have is that incentives were appropriate. We delay this discussion concerning the possible motivations to the observed bidding pattern in the first-price auctions to subsection 4.3.

Expected Revenue						
Format	First Price		Hybrid-1		Hybrid-2	
Private Values	Observed (P-values)	Predicted	Observed (P-values)	Predicted	Observed (P-values)	Predicted
(6,6,6)	5.40 (n.a.)	4.05	5.90 (n.a.)	4.21	5.45 (n.a.)	4.29
(6,6,3)	5.67 (0.00)	4.05	4.85 (0.43)	4.21	5.46 (0.00)	4.29
(6,6,0)	5.24 (0.00)	4.05	4.54 (0.31)	4.21	5.57 (0.00)	4.29
(6,3,3)	5.10 (0.02)	4.05	3.83 (0.40)	4.21	3.90 (0.52)	4.29
(6,3,0)	3.95 (0.65)	4.05	4.01 (0.38)	4.21	4.13 (0.49)	4.29
(6,0,0)	4.13 (0.86)	4.05	4.13 (0.87)	4.21	4.26 (0.93)	4.29
(3,3,3)	2.65 (0.55)	2.52	3.00 (n.a.)	2.68	3.01 (n.a.)	2.76
(3,3,0)	2.40 (0.63)	2.52	3.00 (n.a.)	2.68	3.00 (0.00)	2.76
(3,0,0)	2.73 (0.06)	2.52	2.36 (0.02)	2.68	2.00 (0.03)	2.76
(0,0,0)	0.10	0.00	0.19	0.00	0.30	0.00

Figure 8: Observed and predicted revenue under risk neutrality

adherence of observed to predicted revenue. These results are also presented in Table 8. As can be inferred from the data, in going from the second to the third part, that is from the hybrid-1 to the hybrid-2 auction format, the observed mean revenue increases, as expected. However, in the first price auctions the mean observed revenue is higher than predicted for some values' configurations. One could argue that in the initial periods subjects could be bidding too high due to their inexperience. According to Kagel [13], one must find an explanation that works consistently across different treatment conditions before one could relate the overbidding pattern in the first price auctions to risk aversion. We have already verified that in the second price auctions of the contingent second stage the bidding pattern was consistent with risk aversion. In the next subsection we investigate the explanatory power of the risk aversion assumption in the alternative conditions of our experiment.

4.3 Risk Aversion

As the tendency for subjects to "overbid" in first price auctions is commonly rationalized in terms of risk aversion (as in Cox, Smith and Walker, [5]), we extended the hybrid auction model to the case in which a buyer i who values the commodity at v_i and purchases a single item at a price p receives a Von Neumann utility $u(v_i - p)$, normalized so that $u(0) = 0$. Conditional on winning the auction, the expected payoff of Bidder 1 who observes a private value v_1 and bids b in the first price auction of the first stage is given by:

$$\begin{aligned} \pi(v_1, b(v_1)) &= E_j \left[u(v_1 - b(v_1)) \, 1_{\{b(v_1) > b(v_j) + z; j \neq 1\}} \right] + \\ & \quad E_j \left[u\left(v_1 - v^{(2)}\right) 1_{\left\{v^{(2)} < b(v_1) < b\left(v^{(2)}\right) + z\right\}} \right] \end{aligned} \tag{2}$$

considering that in the contingent Vickrey Stage bidding one's value remains a bidder's best response even under risk aversion. The property of an increase in the expected revenue relative to standard auction institutions extends to the present setting.[10]

It is a well known fact in the auction literature that under risk aversion the expected revenue in the first price auction is higher than in the oral auction in the private values case. It turns out that under risk aversion the equilibrium bids distributions in the hybrid auction stochastically dominate the corresponding distributions in the risk neutral case.[11] Intuitively, in the present setting risk aversion induces a bidder to bid higher in the first price auction stage. Given our interpretation that the hybrid auction amounts to a Vickrey auction with a reserve price endogenously set, the effect of risk aversion is to increase this reserve price. Comparing Tables 8 and 9 one can see that risk aversion implies a larger increase in the expected revenue from the first price auction relative to the hybrid auction.

We fitted alternative models corresponding to CARA (constant absolute risk aversion) and CRRA (constant relative risk aversion) utility functions with dif-

[10] See Dutra and Menezes [7] for details.
[11] See Dutra and Menezes [7] for details.

ferent risk aversion parameter values. The best fit corresponds to the utility function $u(x) = x^{1-s}$, when $s = 0.45$.[12] The fact that the overbidding is relatively more pronounced between low value bidders corroborates our choice of a utility function showing decreasing absolute risk aversion. This is compatible with the experimental literature.

According to McAfee and Vincent [16] there is a general acceptance that increasing absolute risk aversion is an unsatisfactory characterization of attitudes towards risk. Goeree, Holt and Palfrey [9] report a risk aversion parameter estimate of 0.52 to their data set. Harrison [11] presents a risk aversion parameter estimate of $r_i = 0.45$ for the utility function $u_i(y_i) = y_i^r$ that implies "virtual equality" between observed and predicted behavior in the data from a four subjects experiment from Cox, Roberson and Smith [4].

Goeree, Holt and Palfrey [9] argue that there is some consistency in bidding behavior across many auctions with varying number of bidders, and value structures, and that behavior is consistent with a simple model of risk aversion.[13]

Table 9 presents the expected payoff under equilibrium play for the preferred specification. The adherence of observed to predicted revenue is examined using tests of equality of mean. Table 10 reports the Kolmogorov Smirnov statistic for the model with risk averse bidders. The agreement between the empirical distribution of bids and the equilibrium bids distribution denotes some improvement when confronted to the risk neutral case. That is, the evidence we have is that the model with risk aversion has a better fit to the experimental data. However, like Goeree, Holt and Palfrey [9], we share Friedman's [8] belief that one has to be careful when specifying a utility function parameter that has the effect of pushing predictions in the direction of the observed data. Our results imply only

[12] We fitted the model with the utility functions $u(x) = 1 - \exp[-ax]$, $u(x) = (1/(1-a))(1+x)^{1-a}$ and $u(x) = x^{1-s}$ for different values of the a and s parameters. The results can be obtained upon request.

[13] Further references can be found in Chen and Plott [2], Cox and Oaxaca [3] and Holt and Sherman [12].

Expected Revenue						
Format	First Price		Hybrid-1		Hybrid-2	
Private Values	Observed (P-values)	Predicted	Observed (P-values)	Predicted	Observed (P-values)	Predicted
(6,6,6)	5.40 (n.a.)	5.04	5.90 (n.a.)	5.06	5.45 (n.a.)	5.07
(6,6,3)	5.67 (0.03)	5.04	4.85 (0.78)	5.06	5.46 (0.06)	5.07
(6,6,0)	5.24 (0.23)	5.04	4.54 (0.12)	5.06	5.57 (0.08)	5.07
(6,3,3)	5.10 (0.83)	5.04	3.83 (0.04)	5.06	3.90 (0.12)	5.07
(6,3,0)	3.95 (0.00)	5.04	4.01 (0.00)	5.06	4.13 (0.00)	5.07
(6,0,0)	4.13 (0.06)	5.04	4.13 (0.12)	5.06	4.26 (0.20)	5.07
(3,3,3)	2.65 (0.76)	2.71	3.00 (n.a.)	2.77	3.01 (n.a.)	2.80
(3,3,0)	2.40 (0.28)	2.71	3.00 (n.a.)	2.77	3.00 (0.01)	2.80
(3,0,0)	2.73 (0.83)	2.71	2.36 (0.00)	2.77	2.00 (0.02)	2.80
(0,0,0)	0.10	0.00	0.19	0.00	0.30	0.00

Figure 9: Observed and Predicted Revenue under Risk Aversion - $(u(w) = w^{0.55})$

that in the experimental session subjects behaved "as if" they were risk averse.

5 Hybrid Auctions

>From auctions 7 to 18 a hybrid auction was played in each group in every period, totalling 180 auctions. Of these, 84 required a Vickrey stage. In 27 of these auctions the winner in the Vickrey stage was not the subject who submitted the highest bid in the first price auction of the first stage. We suspect that one type of behavior that could emerge in hybrid auctions is that, motivated by a conservative bid from the high value subject, a bidder with a value different from the highest value may have incentives to outbid a high value subject, hereby winning the auction in the first stage. This happened in only 7 auctions. Within these 7 auctions, 5 showed the highest value bidder winning the auction - as predicted - in the second stage. In all 27 Vickrey stage auctions, the second stage implied an increase in the seller's revenue. Table 11 presents the total number of auctions and the frequency with which a Vickrey stage took place, for periods 9 to 12 and 15 through 18, grouped according to private values' configurations .

Kolmogorov-Smirnov One Sample Test				
Auction	KS statistic (*v=3*)	N	KS statistic (*v=6*)	N
3	0.40**	15	0.25	14
4	0.39***	18	0.36**	14
5	0.43**	12	0.47**	10
6	0.46***	13	0.22	14
9	0.31*	16	0.44**	9
10	0.23	13	0.25	16
11	0.35**	14	0.31*	16
12	0.46***	13	0.34	10
15	0.35**	16	0.26	13
16	0.34*	14	0.19	12
17	0.15	13	0.28*	18
18	0.49***	14	0.33**	16

[a] *The null hypothesis is that the empirical distribution agrees with the theoretical distribution of bids.*

**** indicates that the null hypothesis can be rejected with only a 0.01 chance of committing a type one error.*

*** indicates that the null hypothesis can be rejected with only a 0.05 chance of committing a type one error.*

** indicates that the null hypothesis can be rejected with only a 0.10 chance of committing a type one error.*

Figure 10: Analysis of empirical distribution of bids under risk aversion

Traditional concerns of competition policy, such as entry deterring behavior, are among the most prominent questions of auction design. There is a strong presumption in ascending auctions that the bidder who values the commodity the most will win the auction. Even if she is outbid in early stages, she can eventually reverse this scenario. So other bidders, who believe themselves not having a chance to win the object, might have reduced incentives to enter the bidding.

On the other hand, sealed bid auctions may attract a larger number of bidders as the chances of winning might be higher. However, there is a trade-off in terms of efficiency: a sealed bid auction has a relatively smaller ability to give the commodity to the high value bidder as compared to open ascending bid auctions. The experimental data indicated that this phenomenon did not occur in the hybrid auction mechanism.

5.1 Efficiency

One strategic complication introduced by a hybrid auction is the possibility that subjects might reduce their bids in the first stage, as can be inferred from the

Private Values	Auction Format Auction	Hybrid -1				Total	Hybrid 2				Total
		9	10	11	12		15	16	17	18	
(6,6,6)	Vickrey Stage			1		1			1	1	2
	Number of Auctions			1		1			1	1	2
(6,6,3)	Vickrey Stage		2	1		3	1	2	2	1	6
	Number of Auctions		2	1		3	1	2	3	1	7
(6,6,0)	Vickrey Stage	1	1	2	1	5	0	1	1	2	4
	Number of Auctions	3	2	3	2	10	2	1	2	2	7
(6,3,3)	Vickrey Stage	0	2	0	1	3	1		1	1	3
	Number of Auctions	1	2	1	1	5	2		2	1	5
(6,3,0)	Vickrey Stage	0	1	1	0	2	3	2	1	2	8
	Number of Auctions	2	5	3	3	13	4	4	2	6	16
(3,3,3)	Vickrey Stage			1		1		2			2
	Number of Auctions			1		1		2			2
(3,3,0)	Vickrey Stage	3	1	1	2	7	2	3	0	1	6
	Number of Auctions	3	1	1	3	8	2	3	1	1	7
(6,0,0)	Vickrey Stage		0	0	0	0	0		0		0
	Number of Auctions		1	1	2	4	1		1		2
(3,0,0)	Vickrey Stage	1		0	0	1	1		0	0	1
	Number of Auctions	6		3	2	11	3		2	3	8
(0,0,0)	Vickrey Stage		2		2	4		3	1		4
	Number of Auctions		2		2	4		3	1		4
	Vickrey Stage	5	9	7	6	27	8	13	7	8	36
	Number of Auctions	15	15	15	15	60	15	15	15	15	60

Figure 11: Frequency of Ocurrence of Vickrey Stage

Median Bid and Prices by Auction Format				
Private Values		FPA	Hybrid-1	Hybrid-2
(6,6,6)	Bid	5.50	5.90	3.03
	Price	5.50	5.90	5.27
(6,6,3)	Bid	5.60	4.50	4.51
	Price	5.60	5.00	5.62
(6,6,0)	Bid	5.35	4.35	4.51
	Price	5.35	4.80	5.90
(6,3,3)	Bid	5.25	3.50	3.01
	Price	5.25	3.50	3.76
(6,3,0)	Bid	3.55	4.01	3.90
	Price	3.55	4.01	4.50
(6,0,0)	Bid	3.5	4.25	4.26
	Price	3.5	4.25	4.26
(3,3,3)	Bid	2.65	2.85	2.50
	Price	2.65	3.00	3.01
(3,3,0)	Bid	2.51	2.90	2.51
	Price	2.51	2.95	2.99
(3,0,0)	Bid	2.80	2.20	2.25
	Price	2.80	2.02	2.25

Figure 12: Median Bids and Prices

equilibrium predictions presented in Table 1. This might result in an inefficient outcome: a subject with a private value distinct from the highest one could have incentives to outbid a higher value bidder winning the auction in the first stage.

Out of the 270 auctions conducted in our experimental session, 9 resulted in an inefficient outcome: 4 first-price auctions, 1 hybrid-1 auction and 4 hybrid-2 auctions. In three of the four hybrid-2 inefficient auctions the winner submitted a bid higher than his value in the Vickrey stage. In the other hybrid-2 inefficient auction and the hybrid-1 inefficient auction, a bid higher than the subject's value was submitted in the first price auction. In 8 out of these 9 inefficient auctions the winner incurred in loss. So only one auction was characterized by an inefficient outcome that can not be interpreted as a mistake. The achieved efficiency level of the experimental session is higher than 96%.

6 Conclusion and Directions of Further Research

In this paper we developed and implemented an experimental design to investigate a hybrid auction, similar to that used in the sale of the Brazilian Telecom Company. This hybrid format has achieved efficient outcomes and induced less overbidding (with respect to the risk-neutral Bayesian Nash equilibrium) than first-price auctions. Calibrating the experimental results for risk aversion seems to explain a significant part of the overbidding. Finally, the revenue comparison between the hybrid auction and the standard-first price auctions has proven ambiguous, although an increase in the parameter z has shifted the expected revenue in the predicted direction. These conclusions indicate several questions that need further investigation. Natural extensions include the introduction of a continuous distribution of types, the use of an open auction in the second stage when valuations are not independent, the introduction of asymmetric players, and the reversal of the auction order.

.1 Equilibrium expressions for the experimental session.

$$\bar{b}_1 = \left((x_1 - z)(p_0 + p_1)^2 - (x_1 - x_0 - z)\, p_0^2\right)(p_0 + p_1)^{-2}$$

$$G_1(b) = + \Big[\left((2(x_1 - b)\, p_0 p_1)^2 + 4(x_1 - b)(b - x_0 - z)(p_0 p_1)^2\right)^{0.5} \\ -2(x_1 - b)\, p_0 p_1\Big]\left[2(x_1 - b)\, p_1^2\right]^{-1}$$

$$\bar{b}_2 = (x_2 - z) - \left(x_2 - \bar{b}_1 - z\right)(p_0 + p_1)^2$$

$$G_2(b) = \left(2(x_2 - b)\, p_2^2\right)^{-1}\Big[-(2(x_2 - b)(p_0 + p_1)\, p_2) \\ \left[(2(x_2 - b)(p_0 + p_1)\, p_2)^2 - 4(x_2 - b)\, p_2^2\left[(x_2 - b) - \left(x_2 - \bar{b}_1 - z\right)\right](p_0 + p_1)^2\right]^{0.5}\Big]$$

For further references see [7].

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