

F U N D A Ç Ã O  
GETULIO VARGAS

**EPGE**

Escola de Pós-Graduação  
em Economia

Ensaaios Econômicos

Escola de

Pós-Graduação

em Economia

da Fundação

Getulio Vargas

Nº 486

ISSN 0104-8910

## Forecasting Electricity Demand Using Generalized Long Memory

Lacir Jorge Soares, Leonardo Rocha Souza

Junho de 2003

URL: <http://hdl.handle.net/10438/825>

Os artigos publicados são de inteira responsabilidade de seus autores. As opiniões neles emitidas não exprimem, necessariamente, o ponto de vista da Fundação Getulio Vargas.

ESCOLA DE PÓS-GRADUAÇÃO EM ECONOMIA

Diretor Geral: Renato Fragelli Cardoso

Diretor de Ensino: Luis Henrique Bertolino Braido

Diretor de Pesquisa: João Victor Issler

Diretor de Publicações Científicas: Ricardo de Oliveira Cavalcanti

Jorge Soares, Lacir  
Forecasting Electricity Demand Using Generalized Long  
Memory/ Lacir Jorge Soares, Leonardo Rocha Souza - Rio de  
Janeiro : FGV,EPGE, 2010  
(Ensaio Econômico; 486)

Inclui bibliografia.

CDD-330

**Nº 486**

ISSN 0104-8910

***Forecasting electricity demand using generalized long memory***

***Lacir Jorge Soares  
Leonardo Rocha Souza***

*Junho de 2003*

# Forecasting Electricity Demand Using Generalized Long Memory

Lacir Jorge Soares

Department of Electrical Engineering  
Catholic University of Rio de Janeiro

Leonardo Rocha Souza

Graduate School of Economics  
Fundação Getúlio Vargas

June 2003

**Abstract.** This paper studies the electricity hourly load demand in the area covered by a utility situated in the southeast of Brazil. We propose a stochastic model which employs generalized long memory (by means of Gegenbauer processes) to model the seasonal behavior of the load. The model is proposed for sectional data, that is, each hour's load is studied separately as a single series. This approach avoids modeling the intricate intra-day pattern (load profile) displayed by the load, which varies throughout days of the week and seasons. The forecasting performance of the model is compared with a SARIMA benchmark using the years of 1999 and 2000 as the out-of-sample. The model clearly outperforms the benchmark. We conclude for general long memory in the series.

**Keywords.** Long Memory, Generalized Long Memory, Load Forecasting.

## **1 – Introduction**

Managing electricity supply is a complex task. The system operator is responsible for the hourly scheduling of the generators and aims foremost at balancing power production and demand. After this requirement is satisfied, it aims at minimizing production costs, including those of starting and stopping power generating devices, taking into account technical restrictions of electricity centrals. Finally, there must always be a production surplus, so that local failures do not affect dramatically the whole system. It is relevant for electric systems optimization, thus, to develop a scheduling algorithm for the hourly generation and transmission of electricity. Amongst the main inputs to this algorithm are hourly load forecasts for different time horizons. Electricity load forecasting is thus an important topic, since accurate forecasts can avoid wasting energy and prevent system failure. The former when there is no need to generate power above a certain level and the latter when normal operation is unable to withstand a heavy load. The importance of good forecasts for the operation of electric systems is exemplified by many works in Bunn and Farmer (1985) with the figure that a 1% increase in the forecasting error would cause an increase of £10 M in the operating costs per year in the UK.

There is also a utility-level reason for producing good load forecasts. Nowadays they are able to buy and sell energy in the specific market whether there is shortfalls or excess of energy, respectively. Accurately forecasting the electricity load demand can lead to better contracts. Among the most important time horizons for forecasting hourly loads we can cite: one to 168 hours ahead. This paper deals with forecasts of load demand one to seven days (24 – 168 hours) ahead for a Brazilian utility situated in the southeast of the country.

The data in study cover the years from 1990 to 2000 and consist of hourly load demands. We work with sectional data, treating each hour as different time series, so that 24 different models are estimated, one for each hour of the day. All models, however, have the same structure. Ramanathan et al. (1997) won a load forecasting competition at Puget Sound Power and Light Company using models individually tailored for each hour. They also cite the use of hour-by-hour models at the Virginia Electric Power Company. However, our approach differs from theirs in various ways, including that we use only sectional data in each model. Using hour-by-hour models avoids modeling the daily load profile, which varies for different days of the week and different parts of the year, increasing model complexity much more intensely than

allowing improvements in the forecast accuracy. Although temperature is an influential variable to hourly loads, temperature records for the region in study are very hard to obtain and we focus our work on univariate modeling.

Long memory in stationary processes has traditionally two alternative definitions, one in the frequency-domain and the other in the time-domain. In the frequency-domain, this feature implies that the spectrum is proportional to  $|\lambda|^{-2d}$  as the frequency  $\lambda$  approaches zero. In the time-domain, the autocorrelations decay hyperbolically, instead of geometrically as in ARMA processes ( $\rho_k \sim C|k|^{1-2d}$  as  $k \rightarrow \infty$ , where  $k$  is the lag and  $C$  is a constant). In both cases,  $d$  is the long memory parameter and the above relationships characterize long memory and stationarity if  $d \in (0, 0.5)$ . Good reviews of long memory literature are found in Beran (1994) and Baillie (1996). Hosking (1981) and Granger and Joyeux (1980) proposed at the same time the fractional integration, which has no physical but only mathematical sense. It is represented by a noninteger power of  $(1 - B)$ , where  $B$  is the backward-shift operator such that  $BX_t = X_{t-1}$ , and can generate long memory while still keeping the process stationary. Generalized (seasonal) long memory can be generated by a noninteger power of the filter  $(1 - 2\gamma B + B^2)$  and the periodicity is implicit in the choice of the parameter  $\gamma$ . Gray, Zhang and Woodward (1989) call such processes as Gegenbauer process, because they admit Gegenbauer polynomials in the  $MA(\infty)$  representation. They explore the properties of Gegenbauer and related processes, which present the generalized long memory feature. The noninteger power of a finite polynomial filter is equivalent to an infinite polynomial filter, which can be obtained by a Binomial expansion.

The time series modeling includes a stochastic trend, seasonal dummies to model the weekly pattern and the influence of holydays. After filtering these features from the log-transformed data, there remains a clear seasonal (annual) pattern. Analyzing the autocorrelogram, a damped sinusoid is observed, consistent with a general long memory (Gegenbauer) process. The forecasting performance 24 to 168 hours ahead for the years 1999 and 2000 (each year separately) is compared with that of a benchmark, namely a SARIMA model, and the results are highly favorable to our modeling. For one day (24 hours) ahead, the model yields mean absolute percentage error (MAPE) going from 2.2% (21<sup>th</sup> hour, 1999) to 4.4% (2<sup>nd</sup> hour, 2000), and for seven days (168 hours) ahead it goes from 3.1% (20<sup>th</sup> hour, 1999) to 8.4% (3<sup>rd</sup> hour, 2000). Given the characteristics observed in the data and the results obtained in

forecasting, we conclude for the presence of generalized long memory in the data studied.

The plan of the paper is as follows. The next Section briefly explains ordinary and generalized long memory, both used in the modeling process. Section 3 describes the data and the model proposed to fit the load demand, while Section 4 shows the forecasting results of this modeling. Section 5 offers some concluding remarks.

## 2 – (Generalized) Long Memory

### 2.1 – Ordinary Long Memory

Stationary long memory processes are defined by the behavior of the spectral density function near the frequency zero, as follows. Let  $f(\lambda)$  be the spectral density function of the stationary process  $X_t$ . If there exists a positive function  $c_f(\lambda)$ ,  $\lambda \in (-\pi, \pi]$ , which varies slowly as  $\lambda$  tends to zero, such that  $d \in (0, 0.5)$  and

$$f(\lambda) \sim c_f(\lambda)|\lambda|^{-2d} \text{ as } \lambda \rightarrow 0, \quad (1)$$

then  $X_t$  is a stationary process with long memory with (long-)memory parameter  $d$ . Alternatively, let  $\rho(k)$  be the  $k$ -th order autocorrelation of the series  $X_t$ . If there exists a real number  $d \in (0, 0.5)$  and a positive function  $c_\rho(k)$  slowly varying as  $k$  tends to infinity, such that:

$$\rho(k) \sim c_\rho(k)k^{2d-1} \text{ as } k \rightarrow \infty \quad (2)$$

then  $X_t$  is said to have long memory or long range dependence.

$X_t$  is said to follow an ARFIMA( $p, d, q$ ) model if  $\Phi(B)(1-B)^d X_t = \Theta(B)\varepsilon_t$ , where  $\varepsilon_t$  is a mean-zero, constant variance white noise process,  $B$  is the backward shift operator such that  $BX_t = X_{t-1}$ ,  $d$  is not restricted to integer values as in the ARIMA specification, and  $\Phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p$  and  $\Theta(B) = 1 + \theta_1 B + \dots + \theta_q B^q$  are the autoregressive and moving-average polynomials, respectively. ARFIMA processes are stationary and display long memory if the roots of  $\Phi(B)$  are outside the unit circle and  $d \in (0, 0.5)$ . If  $d < 0$  the process is short memory and said to be “antipersistent” (Mandelbrot, 1977, p.232), and Equation (1) holds so that the spectrum has a zero at the zero frequency. Note that in this case the process does not fit into the definition of long memory, since the parameter  $d$  is outside the range imposed by this definition. If  $d = 0$  the ARFIMA process reduces itself to an ARMA. If the roots of  $\Phi(B)$  are outside the unit circle and  $d < 0.5$ , the process is stationary and if the roots of  $\Theta(B)$  are outside the

unity circle and  $d > -0.5$ , the process is invertible. The autocorrelations of an ARFIMA process follow  $\rho(k) \sim Ck^{2d-1}$  as the lag  $k$  tends to infinity and its spectral function behaves as  $f(\lambda) \sim C|\lambda|^{-2d}$  as  $\lambda$  tends to zero, satisfying thus (1) and (2). A non-integer difference can be expanded into an infinite autoregressive or moving average polynomial using the binomial theorem:

$$(1-B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k, \quad (3)$$

where  $\binom{d}{k} = \frac{\Gamma(d+1)}{\Gamma(k+1)\Gamma(d-k+1)}$  and  $\Gamma(\cdot)$  is the gamma function.

## 2.2 – Generalized Long Memory

Seasonal long memory is usually not defined in the literature, as different spectral behaviors bear analogies with the definition in (1) within a seasonal context. Rather, processes with these analogous properties are defined and their spectral and autocovariance behaviors explored. We in this paper work with Gegenbauer processes as in Gray, Zhang and Woodward (1989) and Chung (1996), but alternative models have been used by Porter-Hudak (1990), Ray (1993) and Arteché (2002), for example. The Gegenbauer processes were suggested by Hosking (1981) and later formalized by Gray, Zhang and Woodward (1989) and are defined as follows. Consider the following process

$$(1 - 2\gamma B + B^2)^d X_t = \varepsilon_t, \quad (4)$$

where  $|\gamma| \leq 1$  and  $\varepsilon_t$  is a white noise. This process is called a Gegenbauer process, because the Wold representation of (4) admits the important class of orthogonal polynomials called Gegenbauer polynomials as the coefficients (Gray et al., 1989). This Wold representation is achieved by using the binomial theorem as in (3), expanding the representation to an  $MA(\infty)$ . If  $|\gamma| < 1$  and  $0 < d < 1/2$ , the autocorrelations of the process defined in (4) can be approximated by

$$\rho(k) = C \cos(k\nu) k^{2d-1} \quad \text{as } k \rightarrow \infty \quad (5)$$

where  $C$  is a constant not depending on  $k$  (but depends on  $d$  and  $\nu$ ) and  $\nu = \cos^{-1}(\gamma)$ . That means that the autocorrelations behave as a damped sinusoid of frequency  $\nu$  and that  $\gamma$  determines the period of the cycle (or seasonality). Moreover, the spectral density function obeys



$$f(\lambda) = \frac{\sigma^2}{2\pi} \{2|\cos \lambda - \cos \nu|\}^{-2d} \quad (6)$$

behaving as  $\lambda \rightarrow \nu$  like

$$f(\lambda) \sim |\lambda^2 - \nu^2|^{-2d} \quad (7)$$

for  $0 \leq \lambda \leq \pi$ . Note that if  $\gamma = 1$  then (4) is an FI(2d), or ARFIMA(0,2d,0), and that is why we can call these processes as having the “generalized” long memory property. Moreover, the (long memory) analogy of Gegenbauer processes with FI processes comes from the fact that the latter have a pole or zero at the zero frequency while the former have a pole or zero at the frequency  $\nu$ , depending on whether  $d$  is respectively positive or negative. Note that the Gegenbauer processes can be generalized into  $k$ -factor Gegenbauer processes as in Gray et al. (1989) and Ferrara and Guégan (2001), by allowing that different Gegenbauer filters  $(1 - 2\gamma_i B + B^2)^{d_i}$ ;  $i = 1, \dots, k$ ; apply to  $X_t$ . We work in this paper with a mix of Gegenbauer and ARFIMA process to model the detrended load after the calendar effect is removed.

### 3 – Data and model

#### 3.1 - Data

The data comprise hourly electricity load demands from the area covered by the utility in study, ranging from the first day of 1990 to the last of 2000. The data set is then separated into 24 subsets, each containing the load for a specific hour of the day (“sectional” data). Each subset is treated as a single series, all of them modeled following a same overall specification, although estimated independently of the others (univariate modeling). We use an estimation window of four years, believing it sufficient for a good estimation. Some experiments with longer and shorter estimation windows yielded results not much different from the ones obtained here. As the focus is in multiples of 24 hours ahead forecasts, the influence of lags up to 23 may be unconsidered without affecting predictability. Furthermore, using these “sectional” data avoids modeling complicated intra-day patterns in the hourly load, commonly called load profile, and enables each hour to have a distinct weekly pattern. This last feature is desirable, since it is expected that the day of the week will affect more the middle hours, when the commerce may or not be open, compared to the first and last hours of the day, when most people are expected to sleep. Hippert, Pedreira and Souza (2001, p. 49), in

their review of load forecasting papers, report that difficulties in modeling the load profile are common to (almost) all of them.

The hour-by-hour approach has been also tried by Ramanathan et al. (1997), who win a load forecast competition, but their modeling follows a diverse approach than ours. Unlike them, we use neither deterministic components to model the load nor external variables such as those related to temperature. This is a point to draw attention to, as some temperature measures (maximums, averages and others) could improve substantially the prediction if used, particularly in the summer period when the air conditioning appliances constitute great part of the load. The forecasting errors are in general higher in this period and we do not use this kind of data because it is unavailable to us. However, the temperature effect on load is commonly nonlinear as attested by a number of papers (see Hippert, Pedreira and Souza, 2001, p. 50), and including it in our linear model would probably add a nonlinear relationship to it.

### **3.2 – Model**

A wide variety of models and methods have been tried out to forecast energy demand, and a great deal of effort is dedicated to artificial intelligence techniques, in particular to neural network modeling<sup>1</sup>. Against this tidal wave of neural applications in load forecasting, we prefer to adopt statistical linear methods, as they seem to explain the data to a reasonable level, in addition to give an insight into what is being modeled. The forecasting performance makes us believe the choice is correct for the data in study. A contemporaneous work of Soares and Medeiros (2003) analyses the same data set, adopting instead a deterministic components approach, modeling as stochastic only the residuals left after fitting these deterministic components. Their results are fairly comparable to ours (slightly better), but the deterministic components approach should not adapt well to the case where a structural break is present in the data, like during the 2001 Brazilian energetic crisis, when the electricity consumption was reduced by about 20%. On the other hand, our methodology is shown to yield good forecast results in this period (details in Souza and Soares, 2003).

The model for a specific hour is presented below, omitting subscripts for the hour as the specification is the same. Let the load be represented by  $X_t$  and  $Y_t = \log(X_t)$ . Then,

---

<sup>1</sup> Hippert, Pedreira and Souza (2001) provide a review of works that used varied techniques in predicting load demand, paying especial attention to evaluate the use of neural networks.

$$Y_t = L_t + WD_t + \alpha H_t + Z_t \quad (8)$$

where  $L_t$  is a stochastic level (following some trend, possibly driven by macroeconomic and demographic factors),  $WD_t$  is the effect of the day of the week,  $H_t$  is a dummy variable to account for the effect of holidays (magnified by the parameter  $\alpha$ ) and  $Z_t$  is a stochastic process following:

$$(1 - 2\gamma B + B^2)^{d_1} (1 - B)^{d_2} (1 - \phi_t B) Z_t = \varepsilon_t, \quad (9)$$

where  $\varepsilon_t$  is a white noise. The first multiplicative term on the left hand side of (9) takes the annual (long memory) seasonality into account, where  $\gamma$  is such that the period is 365 days and  $d_1$  is the degree of integration. The second refers to the pure long memory, where  $d_2$  is the degree of integration. The third refers to an autoregressive term, with seven different and static values of  $\phi_t$ , one for each day of the week (rather than  $\phi_t$  being a stochastic parameter).

No assumptions were made up on the trend driving the stochastic level, but it is reasonable to think it is related to macroeconomic and demographic conditions.  $L_t$  is estimated simply averaging the series from half a year before to half a year after the date  $t$ , so that the annual seasonality does not interfere with the level estimation. It is forecasted by extrapolating a linear trend estimated by local regression on the estimated  $L_t$ . The calendar effect, in turn, is modeled by  $WD_t$  and  $\alpha H_t$ , where the former is constituted by dummy seasonals,  $H_t$  is a dummy variable which takes value 1 for holidays, 0.5 for “half-holidays” and 0 otherwise, and  $\alpha$  is its associated parameter. The days modeled as “half holidays” are part-time holidays, holidays only in a subset of the area in study or even for a subset of the human activities in the whole region, in such a way that the holiday effect is smooth. For two examples of “half-holidays”, one can cite the Ash Wednesday after Carnival, where the trading time begins in the afternoon; and San Sebastian’s day, feast in honor of the patron saint of one of the cities within the region in study (holiday only in this city).  $H_t$  could allow values in a finer scale (e.g., measuring out the “holiday intensity” of different holiday types), but there is a trade-off between the fit improvement and the measurement error in so doing. As it is, the huge forecast errors present when no account of the holiday effect is taken disappear.  $WD_t$  and  $\alpha$  are estimated using dummy variables by OLS techniques. Seasonal and ordinary long memory are estimated using the Whittle (1951) approach, details of which<sup>2</sup> are

---

<sup>2</sup> Such as consistency and asymptotic normality.

found in Fox and Taqqu (1986) and in Ferrara and Guégan (1999) respectively for ordinary and generalized long memory. Chung (1996, p. 245) notes that when the parameter  $\gamma$  is known, as in the present case, the estimation of seasonal long memory is virtually identical to that of ordinary long memory. However, there are varied long memory estimators in the literature and we chose this one for its comparatively high efficiency. The weekly autoregressive terms are estimated with the remaining residuals, using for it OLS techniques.

Figure 1 shows the autocorrelation function (ACF) of the series from December 26, 1990 to December 25, 1998 (eight years), for selected hours, after the trend and the calendar effects (weekdays and holidays) are removed. The form of the ACF, which resembles a damped sinusoid as in (5), with annual period, highly justifies using Gegenbauer processes to model  $Z_t$ . The exception is hour 20, in which either there is a sinusoidal component of semiannual period or there are two sinusoidal components, one with annual period and the other with semiannual period. This behavior does not occur with any other hour, being specific of hour 20. However, our modeling does not use a  $k$ -factor ( $k$  would equal to 2 in this case) Gegenbauer process as proposed by Gray et al. (1989) and studied by Ferrara and Guégan (2000, 2001) to model hour 20, obtaining good results though. In fact, hour 20 obtains the lowest errors, the exception being one and sometimes two steps ahead, where it is beaten by hour 21. Figure 2 shows the periodogram (raw and smoothed by a Parzen lag window with degree of smoothness 0.9, respectively represented by dots and a solid line) for the same series in log-log scale. Note the resemblance with the behavior described by (7). The eighth Fourier frequency, corresponding to an annual period as we use eight years of data, shows a sharp peak in the raw periodogram, whereas its vicinity shows a smooth decrease (in log-log scale) going farther to either side. The subfigure corresponding to hour 20 in Figure 2 is consistent with the ACF shown in Figure 1, since there are peaks at the eighth and the sixteenth Fourier frequencies, meaning that there may be one (semiannual) or two (annual and semiannual) Gegenbauer factors.

We chose working with logarithms as the weekly seasonality and the holiday effect can be modeled additively while they are multiplicative in the original series  $X_t$ . These effects are believed to be multiplicative while relating to consuming habits, varying proportionally when the number of consumers expands. However, we experimented applying the same modeling as in (8)-(9) to  $X_t$ , instead of  $Y_t$ , yielding

very similar results. The choice of the logarithm transformation is thus only to be coherent to our beliefs and not result-oriented.

#### 4 – Results

We run a forecasting exercise with the data specified in the previous section, comparing the results of model (8)-(9) to a benchmark. The benchmark used is a SARIMA, like the works of Darbellay and Slama (2000) and Soares and Medeiros (2003), for example. It is modeled and estimated using EViews<sup>TM</sup> software package. The “seasonal” period considered for the SARIMA is one week, since there must be in general small variation between one week and the next within the context of annual seasonality. In fact the SARIMA performs quite well in the early hours, when the load is smoother. A different SARIMA is estimated for each hour of the day, as is done with model (8)-(9).

The period used to estimate the models begins four years before December 25, 1998, and ends on this date (in sample). That is, the model is estimated once using a four-year estimation window, as it seems to be enough. The estimated parameters for each hour are presented in Table 1. The degrees of fractional integration, both seasonal and ordinary, vary throughout the hours, though  $d1$  is always positive (long memory) and  $d2$  always negative (short memory). The (seasonal) long memory is stronger for the hours in the beginning and in the end of the day, coinciding with the hours where the load is smoother. The calendar effect, modeled by  $WD_t$  and  $\alpha H_t$ , is also not the same for all the hours, as it is apparent that business days affect more the load in the middle hours (trading hours) than in the first or last ones. The use of one autoregressive parameter for each day is fully justified as one sees in Table 1 the difference in estimated values there can be (for instance,  $\phi_t$  estimated for the hour 19 varies from  $-0.125$  to  $0.472$ ; respectively  $\phi_{we}$  and  $\phi_{tu}$ ). Note that  $\phi_{tu}$  corresponds to the effect of the residual (of the remaining model) of Monday on that of Tuesday, and so on. We experimented varying the estimation window from three to eight years but the results were practically the same. The out-of-sample, i.e., the period used to test the forecasting accuracy of the models ranges from the first day of 1999 to the last of 2000, split in two sets of one year. Note that the model is not re-estimated during this period. The reason why the out-of-sample is split in two is that the trend is estimated at once (from 1998) for 1999 and 2000 by a linear extrapolation. By visual inspection, there is an apparent break in the trend, as if this was piecewise linear, seeming to have occurred in 2000, so that results

for this year are likely to be worse than those for 1999. Indeed they are, as one can inspect by comparing Tables 2.1 and 3.1. The reason for this apparent break may relate to macroeconomic factors, being beyond the scope of this paper. In a practical application, it would be wise to frequently re-estimate the model, but in this illustration, not re-estimating the model allows one to observe that the model does not suffer from misspecification (remember the trend is forecasted as linear, but not modeled so).

The favorite measure of forecasting accuracy in the load forecasting literature (c.f. Darbellay and Slama, 2000, Park et al., 1991, and Peng et al., 1992) is the Mean Absolute Percentage Error (MAPE), which is defined below, as it measures the proportionality between error and load.

$$MAPE = \frac{1}{n} \sum_{i=1}^n \frac{|\hat{X}_{T+i} - X_{T+i}|}{X_{T+i}}, \quad (10)$$

where  $X_t$  is the load at time  $t$ ,  $\hat{X}_t$  its estimate,  $T$  is the end of the in-sample and  $n$  is the size of the out-of-sample. Other measures could be used, as some works suggest that these measures should penalize large errors and others suggest that the measure should be easy to understand and related to the needs of the decision makers. For an example of the former, Armstrong and Collopy (1992) suggest the Root Mean Square Error (RMSE), while for an example of the latter, Bakirtzis et al. (1996) use Mean Absolute Errors (MAE). However, both measures lack comparability among different data sets, and, maybe because of that, the MAPE still remains as the standard measure. Some authors (c.f., Park et al., 1991, and Peng et al., 1992) achieve MAPEs as low as 2% when predicting the total daily load, but results of different methods cannot be compared on different data sets as some data are noisier than others. If different data sets are used, the same method(s) must be used, and the comparison be made among data sets and not methods. As to the present data, Table 2 shows the MAPE one to seven days ahead for the year of 1999 both for the model proposed in this paper and the SARIMA benchmark. The best method for each hour and number of steps ahead is shown in bold. The proposed model outperforms the benchmark for all hours at one step ahead. The benchmark is better mainly for the first five hours and more than one step ahead. The middle hours see a huge difference in the forecast ability, with our model performing well, attaining MAPEs of around 3% for one step ahead going to 4% for seven steps ahead. In contrast, the benchmark attains two-digit MAPEs for steps ahead higher than one. The peak hours (19-21) see the best MAPEs, in part because the load is

the highest in the day (but the proportional predictability is more accurate for both methods). Table 3 presents the same as Table 2, but for the year 2000. The results are slightly worse, mainly because the linear trend is not re-estimated but seems to suffer a break in 2000, as explained before. Even so, the results are good and are qualitatively equal to the year of 1999, the difference being that the SARIMA fares better also in the sixth hour for steps ahead higher than one. The worst MAPEs from the model (8)-(9), namely those for the first hours seven steps ahead, rise from below 7% in 1999 to above 8% in 2000. As to one step ahead results, the best MAPE rises from 2,23% in 1999 to 2,63% in 2000 (hour 21) and the worst from 3,76% (hour 1) to 4,43% (hour 2). The hour 20 is the best predictable hour for seven steps ahead using the MAPE as the criterion, yielding MAPEs of 3,12% in 1999 and 3,93% in 2000. It is important to note that when we speak of  $h$  steps ahead, we consider the sectional data and hence refer to days. As the primary data are hourly, one must interpret as  $24h$  steps ahead, so that (1, 2, ..., 7) daily steps ahead actually correspond to (7, 14, ..., 168) hourly steps ahead. In practice, it would be interesting to use the model proposed here and the benchmark, each one for the hours and time horizons in which each one fares better, or even in a combined way. The combination is out of the scope of this paper.

Figure 3 illustrates with a typical week the one step ahead forecasting performance of the proposed model and the benchmark. This week goes from May 9, 1999 to May 15, 1999. One can notice that the forecasts from the proposed model fit more closely to the observed loads than those from the benchmark do, reflecting the smaller error obtained by (8)-(9) as presented in Table 2. However, both fits are reasonably good, and the benchmark is not a bad predictor for this horizon (24 hours ahead). Figure 4 illustrates the seven steps ahead forecasting performance of the proposed model and the benchmark, using the same week as Figure 3. The fits are looser than those for one step ahead, as expected, but the proposed model forecasts still track the realized loads. The benchmark forecasts, on the other hand, miss out the real loads too often for this horizon (168 hours ahead), corroborating the superiority of the proposed model.

## **5 – Final remarks**

This paper proposes a stochastic model for the hourly electricity load demand from the area covered by a specific Brazilian utility. This model applies to sectional data, that is, the load for each hour of the day is treated separately as a series. This model can be

applied to other utilities presenting similar seasonal patterns, such as many in Brazil. The model explains the seasonality by generalized (seasonal) long memory using Gegenbauer processes, having in addition a stochastic level (driven by some trend), and a calendar effects component (consisting of dummy variables for the days of the week and for holidays). The Gegenbauer processes fit in well the form of the autocorrelations observed after the estimated level and calendar effects are removed.

A forecasting exercise against a SARIMA model (the benchmark) is highly favorable to our modeling. This exercise included the entire years of 1999 and 2000, forecasting one to seven days ahead (24, 48, ..., 168 hours ahead), using models estimated up to the end of 1998. We conclude for the presence of seasonal long memory in the data in study and suggest that it may be present in the load demand of other utilities with similar seasonal behavior as well.

### **Acknowledgements**

The authors would like to thank the Brazilian Center for Electricity Research (CEPEL) for the data; Dominique Guégan for providing some bibliographical work; and to Marcelo Medeiros and Reinaldo Souza for invaluable comments on a previous version of this work. The second author would also like to thank FAPERJ for the financial support.

### **References**

- Armstrong, J.S. and Collopy, F. (1992), "Error measures for generalizing about forecast methods: empirical comparisons", *International Journal of Forecasting* 8, 69-80.
- Arteche, J. (2002), "Semiparametric robust tests on seasonal or cyclical long memory time series", *Journal of Time Series Analysis* 23, 251-285.
- Baillie, R.T. (1996), "Long memory processes and fractional integration in econometrics", *Journal of Econometrics* 73, 5-59.
- Bakirtzis, A.G., Petridis, V., Kiartzis, S.J., Alexiadis, M.C. and Maissis, A.H. (1996), "A neural network short term load forecasting model for the Greek power system", *IEEE Transactions on Power Systems* 11, 858-863.
- Beran, J. (1994). *Statistics for Long Memory Processes* (Chapman & Hall, London).
- Bunn, D.W. and Farmer, E.D., Eds. (1985). *Comparative Models for Electrical Load Forecasting* (John Wiley & Sons, Belfast).



- Chung, C.-F. (1996), "Estimating a generalized long memory process", *Journal of Econometrics* 73, 237-259.
- Darbellay, G.A. and Slama, M. (2000), "Forecasting the short-term demand for electricity. Do neural networks stand a better chance?", *International Journal of Forecasting* 16, 71-83.
- Ferrara, L. and Guégan, D. (1999), "Estimation and applications of Gegenbauer processes", working paper.
- Ferrara, L. and Guégan, D. (2000), "Forecasting financial time series with generalized long memory processes: theory and applications", in C. Dunis (ed.), *Advances in Quantitative Asset Management*, Kluwer Academic Press, 319-342.
- Ferrara, L. and Guégan, D. (2001), "Forecasting with k-factor Gegenbauer processes: theory and applications", *Journal of Forecasting* 20, 581-601.
- Fox, R. and Taqqu, M.S. (1986), "Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series", *Annals of Statistics* 14, 517-532.
- Granger, C. W. G. and R. Joyeux (1980), "An introduction to long memory time series models and fractional differencing", *Journal of Time Series Analysis* 1, 15-29.
- Gray, H.L., Zhang, N.-F. and Woodward, W.A. (1989), "On generalized fractional processes", *Journal of Time Series Analysis* 10, 233-257.
- Hosking, J. (1981), "Fractional differencing", *Biometrika* 68, 1, 165-176.
- Hippert, H.S., Pedreira, C.E. and Souza R.C. (2001), "Neural networks for short-term load forecasting: a review and evaluation", *IEEE Transactions on Power Systems* 16, 44-55.
- Mandelbrot, B.B. (1977). *Fractals: Form, Chance and Dimension* (Freeman, San Francisco).
- Park, D.C., El-Sharkawi, M.A., Marks II, R.J., Atlas, L.E. and Damborg, M.J. (1991), "Electric load forecasting using an artificial neural network", *IEEE Transactions on Power Systems* 6 (2), 442-449.
- Peng, T.M., Hubele, N.F. and Karady, G.G. (1992), "Advancement in the application of neural networks for short-term load forecasting", *IEEE Transactions on Power Systems* 7 (1), 250-256.
- Porter-Hudak, S. (1990), "An application of the seasonal fractional differenced model to the monetary aggregates", *Journal of the American Statistical Association* 85, 338-344.

- Ramanathan, R., Engle, R., Granger, C.W.J., Vahid-Araghi, F. and Brace, C. (1997), "Short-run forecasts of electricity loads and peaks", *International Journal of Forecasting* 13, 161-174.
- Ray, B.K. (1993), "Long-range forecasting of IBM product revenues using a seasonal fractionally differenced ARMA model", *International Journal of Forecasting* 9, 255-269.
- Soares, L.J. and Medeiros, M.C. (2003), "Short-term load forecasting: a two-step modelling", *working paper*.
- Souza, L.R. and Soares, L.J. (2003), "Forecasting electricity load demand: analysis of the 2001 rationing period in Brazil", *working paper*.
- Whittle, P. (1951). Hypothesis testing in time series analysis (Hafner, New York).

**Table 1:** Parameter estimates of model (8)-(9) for each hour.

hour	d1	d2	WD <sub>su</sub>	WD <sub>mo</sub>	WD <sub>tu</sub>	WD <sub>we</sub>	WD <sub>th</sub>	WD <sub>fr</sub>	WD <sub>sa</sub>	$\alpha$	$\phi_{su}$	$\phi_{mo}$	$\phi_{tu}$	$\phi_{we}$	$\phi_{th}$	$\phi_{fr}$	$\phi_{sa}$
1	0.361	-0.370	-0.013	-0.047	0.001	0.006	0.010	0.013	0.031	0.006	0.407	0.695	0.495	0.552	0.289	0.574	0.541
2	0.374	-0.395	-0.013	-0.044	-0.001	0.007	0.010	0.013	0.030	0.015	0.434	0.687	0.513	0.560	0.301	0.558	0.550
3	0.385	-0.407	-0.019	-0.041	0.001	0.010	0.011	0.013	0.025	0.016	0.473	0.665	0.485	0.525	0.335	0.607	0.606
4	0.394	-0.411	-0.026	-0.039	0.003	0.012	0.012	0.015	0.022	0.010	0.450	0.646	0.480	0.582	0.316	0.624	0.639
5	0.398	-0.408	-0.036	-0.034	0.006	0.015	0.015	0.018	0.017	-0.004	0.471	0.617	0.485	0.559	0.331	0.679	0.604
6	0.396	-0.410	-0.063	-0.020	0.015	0.024	0.022	0.023	-0.002	-0.042	0.493	0.537	0.439	0.538	0.338	0.713	0.673
7	0.350	-0.340	-0.121	0.006	0.035	0.042	0.038	0.039	-0.039	-0.126	0.537	0.291	0.400	0.406	0.363	0.700	0.751
8	0.315	-0.280	-0.175	0.028	0.048	0.057	0.052	0.055	-0.066	-0.194	0.613	0.295	0.305	0.340	0.244	0.714	0.620
9	0.280	-0.192	-0.220	0.046	0.059	0.068	0.065	0.068	-0.084	-0.252	0.494	0.309	0.286	0.304	0.096	0.789	0.401
10	0.251	-0.112	-0.246	0.057	0.065	0.072	0.070	0.073	-0.091	-0.285	0.367	0.163	0.278	0.345	-0.002	0.739	0.171
11	0.230	-0.072	-0.257	0.064	0.069	0.076	0.073	0.074	-0.099	-0.300	0.298	0.158	0.389	0.378	0.040	0.527	-0.109
12	0.238	-0.126	-0.257	0.066	0.070	0.078	0.074	0.073	-0.105	-0.306	0.398	0.151	0.379	0.383	0.109	0.631	0.088
13	0.239	-0.140	-0.249	0.066	0.069	0.076	0.072	0.073	-0.105	-0.297	0.439	0.183	0.379	0.354	0.125	0.599	0.089
14	0.244	-0.160	-0.255	0.068	0.072	0.079	0.075	0.075	-0.115	-0.303	0.473	0.338	0.468	0.414	0.116	0.543	0.113
15	0.248	-0.177	-0.261	0.072	0.077	0.085	0.078	0.076	-0.126	-0.311	0.481	0.409	0.511	0.289	0.126	0.547	0.147
16	0.253	-0.201	-0.263	0.073	0.078	0.086	0.080	0.074	-0.128	-0.308	0.508	0.462	0.600	0.212	0.156	0.597	0.208
17	0.219	-0.128	-0.253	0.072	0.075	0.086	0.079	0.066	-0.126	-0.297	0.321	0.336	0.648	-0.029	0.127	0.537	0.128
18	0.198	-0.149	-0.192	0.053	0.054	0.066	0.061	0.048	-0.090	-0.257	0.441	0.165	0.567	-0.024	0.114	0.523	0.203
19	0.271	-0.062	-0.131	0.032	0.039	0.044	0.041	0.030	-0.055	-0.207	0.409	0.086	0.472	-0.125	-0.040	0.296	0.112
20	0.292	-0.121	-0.096	0.021	0.026	0.032	0.028	0.018	-0.029	-0.156	0.526	0.238	0.449	0.127	0.068	0.374	0.203
21	0.279	-0.273	-0.081	0.019	0.023	0.029	0.025	0.015	-0.030	-0.114	0.755	0.457	0.695	0.329	0.439	0.508	0.312
22	0.311	-0.355	-0.072	0.017	0.025	0.030	0.025	0.014	-0.039	-0.097	0.909	0.462	0.608	0.360	0.484	0.523	0.422
23	0.330	-0.346	-0.058	0.010	0.020	0.025	0.023	0.017	-0.038	-0.063	0.776	0.481	0.639	0.350	0.468	0.493	0.461
24	0.343	-0.349	-0.050	0.006	0.010	0.018	0.017	0.024	-0.024	-0.040	0.772	0.477	0.618	0.336	0.509	0.523	0.400

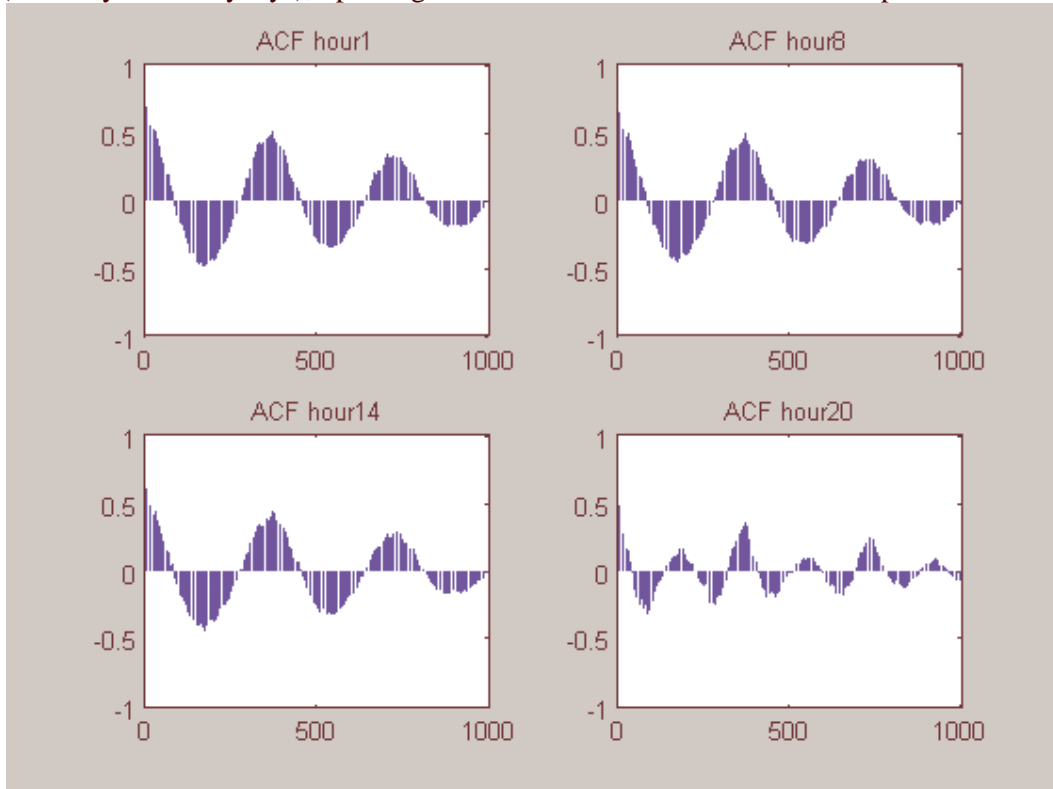
**Table 2:** MAPE for the entire year of 1999 (the best model in bold).

hour	model (8)-(9)							SARIMA						
	steps			ahead				steps			ahead			
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	<b>3.76%</b>	5.14%	5.82%	6.08%	6.30%	6.49%	6.71%	3.93%	<b>4.93%</b>	<b>5.63%</b>	<b>5.84%</b>	<b>5.83%</b>	<b>5.63%</b>	<b>4.72%</b>
2	<b>3.68%</b>	5.09%	5.84%	6.12%	6.33%	6.59%	6.81%	3.85%	<b>4.79%</b>	<b>5.37%</b>	<b>5.51%</b>	<b>5.46%</b>	<b>5.40%</b>	<b>4.67%</b>
3	<b>3.54%</b>	4.97%	5.75%	6.00%	6.18%	6.47%	6.70%	3.67%	<b>4.55%</b>	<b>5.21%</b>	<b>5.36%</b>	<b>5.26%</b>	<b>5.14%</b>	<b>4.40%</b>
4	<b>3.35%</b>	4.67%	5.54%	5.73%	5.92%	6.18%	6.40%	3.46%	<b>4.39%</b>	<b>5.07%</b>	<b>5.24%</b>	<b>5.20%</b>	<b>5.02%</b>	<b>4.17%</b>
5	<b>3.21%</b>	4.39%	5.20%	5.39%	5.56%	5.83%	6.04%	3.37%	<b>4.16%</b>	<b>4.95%</b>	<b>5.18%</b>	<b>5.16%</b>	<b>4.90%</b>	<b>4.01%</b>
6	<b>2.91%</b>	<b>4.09%</b>	<b>4.70%</b>	<b>4.93%</b>	<b>5.07%</b>	<b>5.35%</b>	5.54%	3.26%	4.31%	5.29%	5.90%	5.95%	5.43%	<b>4.30%</b>
7	<b>2.85%</b>	<b>3.69%</b>	<b>4.16%</b>	<b>4.30%</b>	<b>4.43%</b>	<b>4.63%</b>	<b>4.72%</b>	3.62%	6.21%	8.24%	9.08%	9.16%	8.42%	6.27%
8	<b>2.74%</b>	<b>3.37%</b>	<b>3.75%</b>	<b>3.88%</b>	<b>4.04%</b>	<b>4.22%</b>	<b>4.31%</b>	4.09%	8.43%	11.04%	11.80%	11.99%	11.38%	8.38%
9	<b>2.75%</b>	<b>3.45%</b>	<b>3.76%</b>	<b>3.86%</b>	<b>3.98%</b>	<b>4.12%</b>	<b>4.21%</b>	4.63%	10.33%	13.18%	13.76%	13.98%	13.62%	10.05%
10	<b>2.82%</b>	<b>3.46%</b>	<b>3.78%</b>	<b>3.86%</b>	<b>3.93%</b>	<b>4.00%</b>	<b>4.07%</b>	4.91%	11.38%	14.42%	14.89%	15.07%	14.84%	11.05%
11	<b>2.92%</b>	<b>3.59%</b>	<b>3.86%</b>	<b>3.97%</b>	<b>4.01%</b>	<b>4.04%</b>	<b>4.05%</b>	4.95%	11.81%	15.19%	15.48%	15.67%	15.57%	11.40%
12	<b>2.89%</b>	<b>3.61%</b>	<b>3.88%</b>	<b>3.96%</b>	<b>3.97%</b>	<b>3.98%</b>	<b>4.00%</b>	5.02%	12.06%	15.61%	15.81%	15.93%	15.96%	11.58%
13	<b>2.99%</b>	<b>3.79%</b>	<b>4.07%</b>	<b>4.17%</b>	<b>4.19%</b>	<b>4.19%</b>	<b>4.24%</b>	5.04%	11.69%	15.32%	15.64%	15.72%	15.50%	11.19%
14	<b>3.15%</b>	<b>4.06%</b>	<b>4.40%</b>	<b>4.51%</b>	<b>4.49%</b>	<b>4.49%</b>	<b>4.52%</b>	5.10%	12.01%	15.91%	16.14%	16.20%	16.04%	11.45%
15	<b>3.22%</b>	<b>4.16%</b>	<b>4.52%</b>	<b>4.63%</b>	<b>4.67%</b>	<b>4.67%</b>	<b>4.73%</b>	5.16%	12.45%	16.78%	17.08%	17.16%	16.89%	11.79%
16	<b>3.21%</b>	<b>4.11%</b>	<b>4.48%</b>	<b>4.60%</b>	<b>4.70%</b>	<b>4.74%</b>	<b>4.77%</b>	5.11%	12.40%	16.92%	17.33%	17.34%	16.93%	11.70%
17	<b>3.26%</b>	<b>3.84%</b>	<b>4.17%</b>	<b>4.27%</b>	<b>4.35%</b>	<b>4.40%</b>	<b>4.43%</b>	4.82%	11.57%	16.16%	16.57%	16.64%	16.11%	10.97%
18	<b>2.95%</b>	<b>3.49%</b>	<b>3.71%</b>	<b>3.79%</b>	<b>3.81%</b>	<b>3.85%</b>	<b>3.90%</b>	4.27%	8.97%	11.95%	12.46%	12.56%	12.19%	8.65%
19	<b>2.82%</b>	<b>3.38%</b>	<b>3.57%</b>	<b>3.73%</b>	<b>3.87%</b>	<b>3.94%</b>	<b>3.99%</b>	3.54%	6.04%	7.55%	8.02%	8.01%	7.89%	6.12%
20	<b>2.35%</b>	<b>2.87%</b>	<b>2.93%</b>	<b>3.04%</b>	<b>3.05%</b>	<b>3.07%</b>	<b>3.12%</b>	3.07%	4.90%	5.63%	5.98%	5.94%	5.77%	4.88%
21	<b>2.23%</b>	<b>2.83%</b>	<b>3.10%</b>	<b>3.21%</b>	<b>3.27%</b>	<b>3.35%</b>	<b>3.47%</b>	2.85%	4.47%	5.44%	5.84%	5.82%	5.44%	4.42%
22	<b>2.56%</b>	<b>3.39%</b>	<b>3.75%</b>	<b>3.88%</b>	<b>3.93%</b>	<b>4.02%</b>	<b>4.13%</b>	3.08%	4.85%	6.21%	6.73%	6.70%	6.09%	4.76%
23	<b>3.04%</b>	<b>3.99%</b>	<b>4.45%</b>	<b>4.66%</b>	<b>4.72%</b>	<b>4.87%</b>	5.01%	3.35%	4.83%	6.18%	6.67%	6.55%	6.07%	<b>4.63%</b>
24	<b>3.54%</b>	<b>4.75%</b>	<b>5.33%</b>	<b>5.60%</b>	<b>5.72%</b>	<b>5.92%</b>	6.10%	3.76%	4.97%	6.00%	6.38%	6.34%	5.93%	<b>4.76%</b>

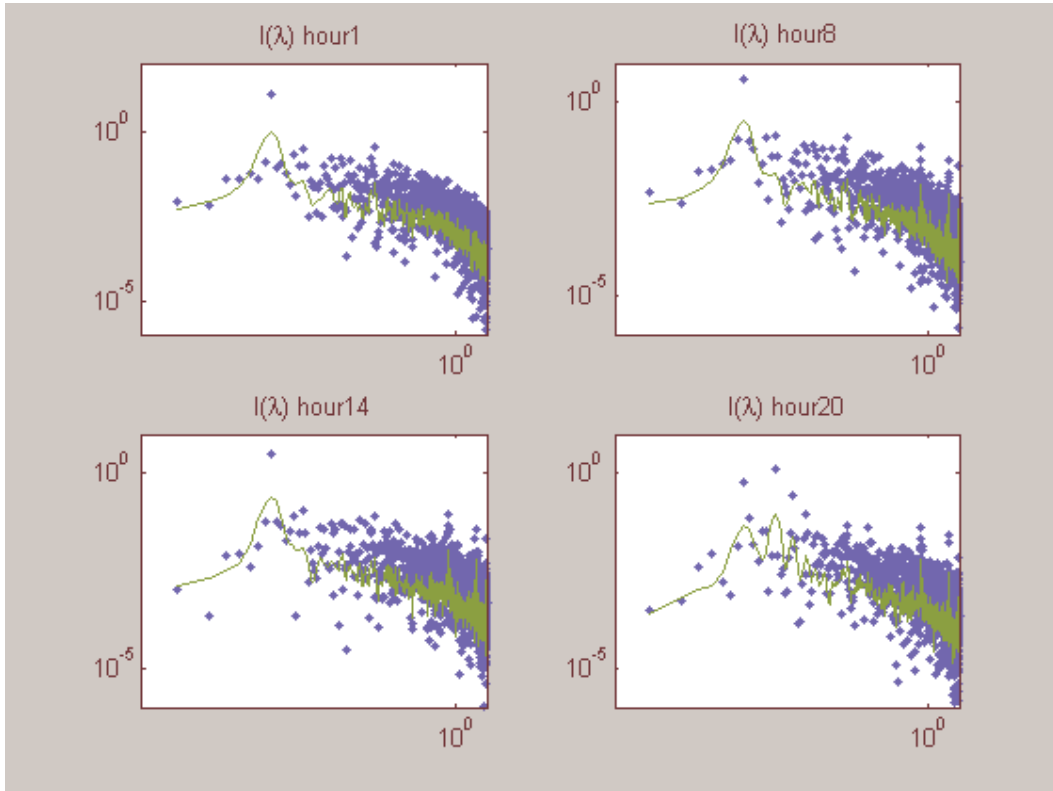
**Table 3:** MAPE for the entire year of 2000 (the best model in bold).

hour	model (8)-(9)							SARIMA						
	1	2	steps 3	ahead 4	5	6	7	1	2	steps 3	ahead 4	5	6	7
1	<b>4.37%</b>	6.42%	7.38%	7.92%	8.18%	8.20%	8.16%	4.53%	<b>5.77%</b>	<b>6.55%</b>	<b>6.71%</b>	<b>6.73%</b>	<b>6.65%</b>	<b>5.70%</b>
2	<b>4.43%</b>	6.54%	7.62%	8.15%	8.43%	8.45%	8.42%	4.53%	<b>5.73%</b>	<b>6.42%</b>	<b>6.55%</b>	<b>6.54%</b>	<b>6.50%</b>	<b>5.65%</b>
3	<b>4.37%</b>	6.45%	7.61%	8.15%	8.46%	8.44%	8.45%	4.44%	<b>5.58%</b>	<b>6.27%</b>	<b>6.36%</b>	<b>6.35%</b>	<b>6.33%</b>	<b>5.48%</b>
4	<b>4.23%</b>	6.24%	7.40%	7.93%	8.26%	8.28%	8.26%	4.35%	<b>5.30%</b>	<b>6.05%</b>	<b>6.15%</b>	<b>6.15%</b>	<b>6.06%</b>	<b>5.24%</b>
5	<b>4.03%</b>	5.99%	7.12%	7.69%	8.01%	8.06%	8.09%	4.24%	<b>5.07%</b>	<b>5.82%</b>	<b>5.92%</b>	<b>5.97%</b>	<b>5.87%</b>	<b>5.11%</b>
6	<b>3.81%</b>	5.60%	6.58%	7.05%	7.32%	7.40%	7.46%	4.14%	<b>5.10%</b>	<b>6.21%</b>	<b>6.48%</b>	<b>6.46%</b>	<b>6.16%</b>	<b>5.14%</b>
7	<b>3.60%</b>	<b>5.08%</b>	<b>5.88%</b>	<b>6.21%</b>	<b>6.43%</b>	<b>6.53%</b>	6.59%	4.47%	6.93%	8.68%	9.12%	9.16%	8.72%	<b>6.57%</b>
8	<b>3.22%</b>	<b>4.61%</b>	<b>5.26%</b>	<b>5.54%</b>	<b>5.73%</b>	<b>5.81%</b>	<b>5.88%</b>	4.70%	9.08%	11.41%	12.08%	12.13%	11.58%	8.61%
9	<b>3.03%</b>	<b>4.28%</b>	<b>4.75%</b>	<b>5.03%</b>	<b>5.22%</b>	<b>5.30%</b>	<b>5.35%</b>	4.92%	10.69%	13.48%	14.28%	14.33%	13.86%	10.25%
10	<b>2.97%</b>	<b>4.06%</b>	<b>4.49%</b>	<b>4.67%</b>	<b>4.80%</b>	<b>4.86%</b>	<b>4.92%</b>	5.24%	12.01%	15.02%	15.80%	15.85%	15.52%	11.48%
11	<b>3.00%</b>	<b>3.97%</b>	<b>4.39%</b>	<b>4.56%</b>	<b>4.69%</b>	<b>4.76%</b>	<b>4.81%</b>	5.39%	12.69%	16.11%	16.81%	16.79%	16.53%	11.99%
12	<b>3.05%</b>	<b>3.99%</b>	<b>4.41%</b>	<b>4.56%</b>	<b>4.68%</b>	<b>4.73%</b>	<b>4.75%</b>	5.60%	12.78%	16.47%	17.14%	17.12%	16.84%	12.10%
13	<b>3.00%</b>	<b>4.00%</b>	<b>4.44%</b>	<b>4.59%</b>	<b>4.71%</b>	<b>4.76%</b>	<b>4.78%</b>	5.40%	12.56%	16.27%	16.94%	16.93%	16.49%	11.71%
14	<b>3.17%</b>	<b>4.29%</b>	<b>4.72%</b>	<b>4.93%</b>	<b>5.05%</b>	<b>5.10%</b>	<b>5.10%</b>	5.55%	12.85%	16.80%	17.65%	17.60%	17.09%	12.11%
15	<b>3.38%</b>	<b>4.51%</b>	<b>4.98%</b>	<b>5.19%</b>	<b>5.31%</b>	<b>5.35%</b>	<b>5.36%</b>	5.74%	13.29%	17.70%	18.40%	18.45%	17.89%	12.41%
16	<b>3.43%</b>	<b>4.55%</b>	<b>4.98%</b>	<b>5.23%</b>	<b>5.33%</b>	<b>5.41%</b>	<b>5.43%</b>	5.77%	13.17%	17.77%	18.47%	18.57%	17.90%	12.29%
17	<b>3.60%</b>	<b>4.43%</b>	<b>4.77%</b>	<b>4.89%</b>	<b>4.98%</b>	<b>5.04%</b>	<b>5.02%</b>	5.39%	12.19%	16.65%	17.26%	17.39%	16.63%	11.25%
18	<b>3.44%</b>	<b>4.19%</b>	<b>4.43%</b>	<b>4.52%</b>	<b>4.58%</b>	<b>4.58%</b>	<b>4.59%</b>	4.79%	9.24%	12.18%	12.66%	12.68%	12.09%	8.67%
19	<b>3.21%</b>	<b>3.77%</b>	<b>4.07%</b>	<b>4.30%</b>	<b>4.44%</b>	<b>4.50%</b>	<b>4.53%</b>	4.03%	6.43%	7.81%	8.13%	8.14%	7.80%	6.18%
20	<b>2.71%</b>	<b>3.30%</b>	<b>3.60%</b>	<b>3.74%</b>	<b>3.87%</b>	<b>3.93%</b>	<b>3.93%</b>	3.47%	5.15%	5.74%	5.95%	6.05%	5.80%	4.99%
21	<b>2.63%</b>	<b>3.45%</b>	<b>3.85%</b>	<b>4.11%</b>	<b>4.16%</b>	<b>4.23%</b>	<b>4.22%</b>	3.17%	4.89%	5.67%	5.84%	5.95%	5.69%	4.56%
22	<b>2.98%</b>	<b>4.12%</b>	<b>4.62%</b>	<b>4.94%</b>	<b>5.11%</b>	<b>5.17%</b>	5.16%	3.36%	5.30%	6.61%	6.92%	6.90%	6.56%	<b>4.92%</b>
23	<b>3.58%</b>	<b>5.15%</b>	<b>5.82%</b>	<b>6.22%</b>	<b>6.41%</b>	<b>6.43%</b>	6.41%	3.99%	5.64%	6.95%	7.28%	7.24%	6.79%	<b>5.30%</b>
24	<b>4.14%</b>	5.96%	6.81%	7.34%	7.56%	7.60%	7.56%	4.50%	<b>5.79%</b>	<b>6.78%</b>	<b>7.09%</b>	<b>6.98%</b>	<b>6.70%</b>	<b>5.59%</b>

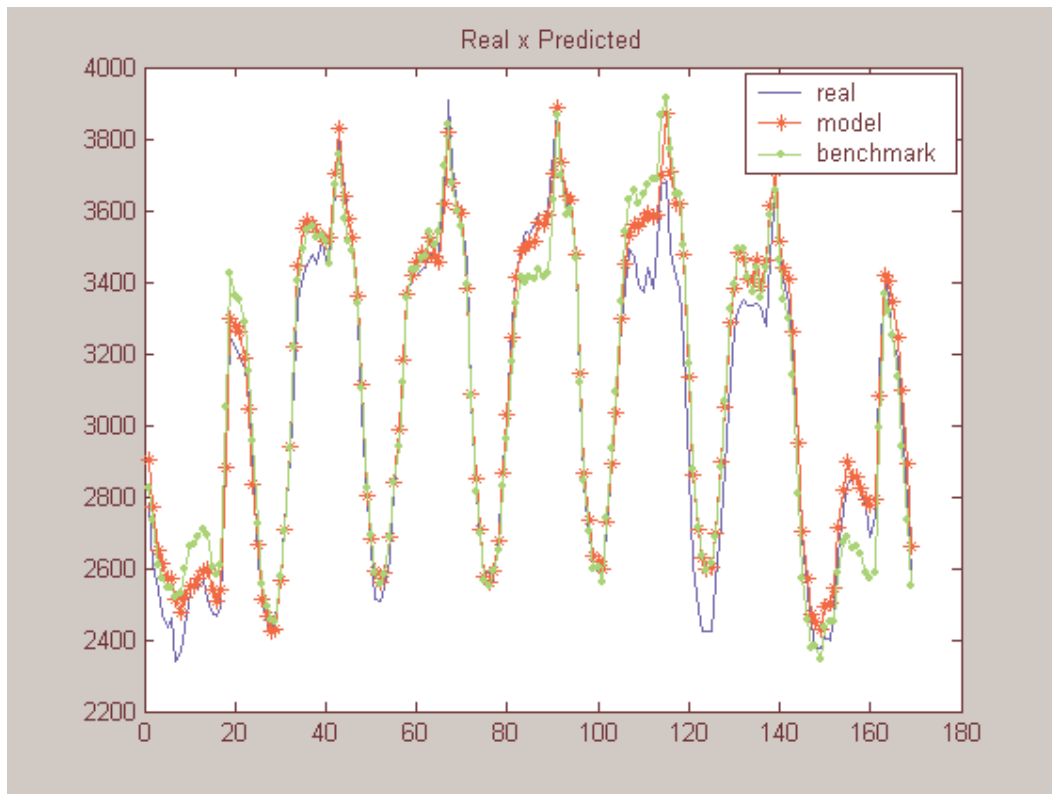
**Figure 1:** Autocorrelation function of the series from January 1, 1990 to December 31, 1998, for hours 1, 8, 14 and 20, after removal of the trend and the calendar effects (weekdays and holydays), up to lag 1000. Note the resemblance to a damped sinusoid.



**Figure 2:** Raw and smoothed periodogram of the series from January 1, 1990 to December 31, 1998, for hours 1, 8, 14 and 20, after removal of the trend and the calendar effects.

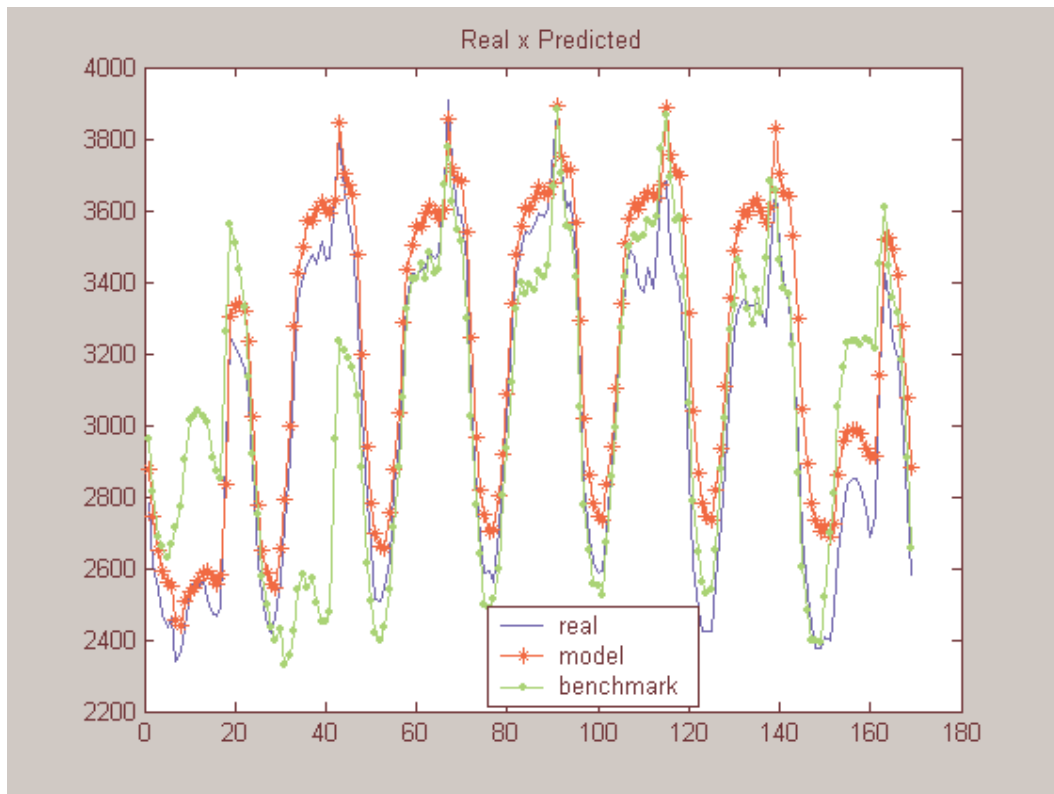


**Figure 3:** Real versus predicted load (MWh), for both the model proposed here and the SARIMA benchmark, one step ahead, from May 9, 1999 to May 15, 1999 (one week, from Sunday to Saturday).





**Figure 4:** Real versus predicted load (MWh), for both the model proposed here and the SARIMA benchmark, seven steps ahead, from May 9, 1999 to May 15, 1999 (one week, from Sunday to Saturday).



# ENSAIOS ECONÔMICOS DA EPGE

441. TESTING PRODUCTION FUNCTIONS USED IN EMPIRICAL GROWTH STUDIES – Pedro Cavalcanti Ferreira; João Victor Issler; Samuel de Abreu Pessôa – Março de 2002 - 28 págs.
442. THREE PRINCIPLES OF COMPETITIVE NONLINEAR PRICING – Frank H. Page Jr; Paulo K. Monteiro – Abril de 2002 – 44 págs.
443. O MECANISMO MONETÁRIO DE TRANSMISSÃO NA ECONOMIA BRASILEIRA PÓS-PLANO REAL – Marcelo Fernandes; Juan Toro – Abril de 2002 – 33 págs.
444. A ROBUST POVERTY PROFILE FOR BRAZIL USING MULTIPLE DATA SOURCES – Francisco H. G. Ferreira; Peter Lanjouw; Marcelo Neri – Abril de 2002 – 38 págs.
445. THE MISSING LINK: USING THE NBER RECESSION INDICATOR TO CONSTRUCT COINCIDENT AND LEADING INDICES OF ECONOMIC ACTIVITY – João Victor Issler; Farshid Vahid – Maio de 2002 – 31 págs.
446. SPECULATIVE ATTACKS ON DEBTS, DOLLARIZATION AND OPTIMUM CURRENCY AREAS – Aloisio Araujo; Marcia Leon – Maio de 2002 – 74 págs.
447. VINTAGE CAPITAL, DISTORTIONS AND DEVELOPMENT – Samuel de Abreu Pessoa; Rafael Rob – Maio de 2002 – 43 págs.
448. GOVERNMENT ACTIONS TO SUPPORT COFFEE PRODUCERS – AN INVESTIGATION OF POSSIBLE MEASURES FROM THE EUROPEAN UNION – Germán Calfat; Renato G. Flôres Jr. – Junho de 2002 – 30 págs.
449. A MODEL OF CAPITAL ACCUMULATION AND RENT-SEEKING – Paulo Barelli; Samuel de Abreu Pessôa – Junho de 2002 – 50 págs.
450. THE MISSING LINK: USING THE NBER RECESSION INDICATOR TO CONSTRUCT COINCIDENT AND LEADING INDICES OF ECONOMIC ACTIVITY – João Victor Issler; Farshid Vahid – Junho de 2002 – 29 págs.
451. LENDER LIABILITY IN THE CONSUMER CREDIT MARKET – Elisabetta Iossa; Giuliana Palumbo – Agosto de 2002 – 20 págs.
452. DECISION RULES AND INFORMATION PROVISION: MONITORING VERSUS MANIPULATION - Elisabetta Iossa; Giuliana Palumbo – Agosto de 2002 – 37 págs.
453. ON CERTAIN GEOMETRIC ASPECTS OF PORTFOLIO OPTIMISATION WITH HIGHER MOMENTS – Gustavo M. de Athayde; Renato G. Flôres Jr. – Setembro de 2002 – 21 págs.
454. MENSURANDO A PRODUÇÃO CIENTÍFICA INTERNACIONAL EM ECONOMIA DE PESQUISADORES E DEPARTAMENTOS BRASILEIROS - João Victor Issler; Tatiana Caldas de Lima Aché Pillar – Setembro de 2002 – 39 págs.
455. FOREIGN DIRECT INVESTMENT SPILLOVERS: ADDITIONAL LESSONS FROM A COUNTRY STUDY - Renato G. Flores Jr; Maria Paula Fontoura; Rogério Guerra Santos – Setembro de 2002 – 30 págs.

456. A CONTRACTIVE METHOD FOR COMPUTING THE STATIONARY SOLUTION OF THE EULER EQUATION - Wilfredo L. Maldonado; Humberto Moreira – Setembro de 2002 – 14 págs.
457. TRADE LIBERALIZATION AND THE EVOLUTION OF SKILL EARNINGS DIFFERENTIALS IN BRAZIL - Gustavo Gonzaga; Naércio Menezes Filho; Cristina Terra – Setembro de 2002 – 31 págs.
458. DESEMPENHO DE ESTIMADORES DE VOLATILIDADE NA BOLSA DE VALORES DE SÃO PAULO - Bernardo de Sá Mota; Marcelo Fernandes – Outubro de 2002 – 37 págs.
459. FOREIGN FUNDING TO AN EMERGING MARKET: THE MONETARY PREMIUM THEORY AND THE BRAZILIAN CASE, 1991-1998 - Carlos Hamilton V. Araújo; Renato G. Flores Jr. – Outubro de 2002 – 46 págs.
460. REFORMA PREVIDENCIÁRIA: EM BUSCA DE INCENTIVOS PARA ATRAIR O TRABALHADOR AUTÔNOMO - Samantha Taam Dart; Marcelo Côrtes Neri; Flavio Menezes – Novembro de 2002 – 28 págs.
461. DECENT WORK AND THE INFORMAL SECTOR IN BRAZIL – Marcelo Côrtes Neri – Novembro de 2002 – 115 págs.
462. POLÍTICA DE COTAS E INCLUSÃO TRABALHISTA DAS PESSOAS COM DEFICIÊNCIA - Marcelo Côrtes Neri; Alexandre Pinto de Carvalho; Hessia Guilherme Costilla – Novembro de 2002 – 67 págs.
463. SELETIVIDADE E MEDIDAS DE QUALIDADE DA EDUCAÇÃO BRASILEIRA 1995-2001 - Marcelo Côrtes Neri; Alexandre Pinto de Carvalho – Novembro de 2002 – 331 págs.
464. BRAZILIAN MACROECONOMICS WITH A HUMAN FACE: METROPOLITAN CRISIS, POVERTY AND SOCIAL TARGETS – Marcelo Côrtes Neri – Novembro de 2002 – 61 págs.
465. POBREZA, ATIVOS E SAÚDE NO BRASIL - Marcelo Côrtes Neri; Wagner L. Soares – Dezembro de 2002 – 29 págs.
466. INFLAÇÃO E FLEXIBILIDADE SALARIAL - Marcelo Côrtes Neri; Maurício Pinheiro – Dezembro de 2002 – 16 págs.
467. DISTRIBUTIVE EFFECTS OF BRAZILIAN STRUCTURAL REFORMS - Marcelo Côrtes Neri; José Márcio Camargo – Dezembro de 2002 – 38 págs.
468. O TEMPO DAS CRIANÇAS - Marcelo Côrtes Neri; Daniela Costa – Dezembro de 2002 – 14 págs.
469. EMPLOYMENT AND PRODUCTIVITY IN BRAZIL IN THE NINETIES - José Márcio Camargo; Marcelo Côrtes Neri; Maurício Cortez Reis – Dezembro de 2002 – 32 págs.
470. THE ALIASING EFFECT, THE FEJER KERNEL AND TEMPORALLY AGGREGATED LONG MEMORY PROCESSES - Leonardo R. Souza – Janeiro de 2003 – 32 págs.
471. CUSTO DE CICLO ECONÔMICO NO BRASIL EM UM MODELO COM RESTRIÇÃO A CRÉDITO - Bárbara Vasconcelos Boavista da Cunha; Pedro Cavalcanti Ferreira – Janeiro de 2003 – 21 págs.
472. THE COSTS OF EDUCATION, LONGEVITY AND THE POVERTY OF NATIONS - Pedro Cavalcanti Ferreira; Samuel de Abreu Pessoa – Janeiro de 2003 – 31 págs.

473. A GENERALIZATION OF JUDD'S METHOD OF OUT-STEADY-STATE COMPARISONS IN PERFECT FORESIGHT MODELS - Paulo Barelli; Samuel de Abreu Pessoa – Fevereiro de 2003 – 7 págs.
474. AS LEIS DA FALÊNCIA: UMA ABORDAGEM ECONÔMICA - Aloísio Pessoa de Araújo – Fevereiro de 2003 – 25 págs.
475. THE LONG-RUN ECONOMIC IMPACT OF AIDS - Pedro Cavalcanti G. Ferreira; Samuel de Abreu Pessoa – Fevereiro de 2003 – 30 págs.
476. A MONETARY MECHANISM FOR SHARING CAPITAL: DIAMOND AND DYBVIIG MEET KIYOTAKI AND WRIGHT – Ricardo de O. Cavalcanti – Fevereiro de 2003 – 16 págs.
477. INADA CONDITIONS IMPLY THAT PRODUCTION FUNCTION MUST BE ASYMPTOTICALLY COBB-DOUGLAS - Paulo Barelli; Samuel de Abreu Pessoa – Março de 2003 – 4 págs.
478. TEMPORAL AGGREGATION AND BANDWIDTH SELECTION IN ESTIMATING LONG MEMORY - Leonardo R. Souza - Março de 2003 – 19 págs.
479. A NOTE ON COLE AND STOCKMAN - Paulo Barelli; Samuel de Abreu Pessoa – Abril de 2003 – 8 págs.
480. A HIPÓTESE DAS EXPECTATIVAS NA ESTRUTURA A TERMO DE JUROS NO BRASIL: UMA APLICAÇÃO DE MODELOS DE VALOR PRESENTE - Alexandre Maia Correia Lima; João Victor Issler – Maio de 2003 – 30 págs.
481. ON THE WELFARE COSTS OF BUSINESS CYCLES IN THE 20TH CENTURY - João Victor Issler; Afonso Arinos de Mello Franco; Osmani Teixeira de Carvalho Guillén – Maio de 2003 – 29 págs.
482. RETORNOS ANORMAIS E ESTRATÉGIAS CONTRÁRIAS - Marco Antonio Bonomo; Ivana Dall'Agnol – Junho de 2003 – 27 págs.
483. EVOLUÇÃO DA PRODUTIVIDADE TOTAL DOS FATORES NA ECONOMIA BRASILEIRA: UMA ANÁLISE COMPARATIVA - Victor Gomes; Samuel de Abreu Pessoa; Fernando A . Veloso – Junho de 2003 – 45 págs.
484. MIGRAÇÃO, SELEÇÃO E DIFERENÇAS REGIONAIS DE RENDA NO BRASIL - Enestor da Rosa dos Santos Junior; Naércio Menezes Filho; Pedro Cavalcanti Ferreira – Junho de 2003 – 23 págs.
485. THE RISK PREMIUM ON BRAZILIAN GOVERNMENT DEBT, 1996-2002 - André Soares Loureiro; Fernando de Holanda Barbosa - Junho de 2003 – 16 págs.
486. FORECASTING ELECTRICITY DEMAND USING GENERALIZED LONG MEMORY - Lacir Jorge Soares; Leonardo Rocha Souza – Junho de 2003 – 22 págs.
487. USING IRREGULARLY SPACED RETURNS TO ESTIMATE MULTI-FACTOR MODELS: APPLICATION TO BRAZILIAN EQUITY DATA - Álvaro Veiga; Leonardo Rocha Souza – Junho de 2003 – 26 págs.