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**THE EFFECT OF INFLATION ON GROWTH  
INVESTMENTS : A NOTE**

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# THE EFFECT OF INFLATION ON GROWTH INVESTMENTS: A NOTE

## **I. Introduction**

One of the most traditional issues in the theory of capital, is the choice of the optimal duration of growth investments. This problem, also known as capital deepening (cf. Baumol, 1972 and Hirshleifer, 1970) and which can be traced back to the 19<sup>th</sup> century work of the English mathematician W.S. Jevons, is typical of the so called point-input, point-output process. Illustrated, in an idealized way, by the activities of wine aging and tree growing, it consists of determining the length of time that an investor should keep an asset whose value increases with time.

Although a standard topic in text-books presentation of capital theory (besides the above mentioned works of Baumol and Hirshleifer, see, for instance: Allen, 1938; Bierman and Smidt, 1993; Henderson and Quandt, 1971; Simon and Blume, 1994 and Varian, 1990), the effects of inflation and taxation are usually not included in the analysis. Brenner and Venezia (1983) appear to be the first authors to take into account the joint effect of taxes and inflation on the optimal duration. However, despite the importance of their analysis, the very fact of considering the joint effect of taxes and inflation rendered some of the results somewhat confusing.

A previous note focused attention on the isolated effect of taxation on the optimal duration in a non-inflationary environment (de Faro, 1996). Complementing the analysis, the present note is aimed at the investigation of the effects of inflation by itself. Specifically, considering the no-reinvestment case only, our objective is to include in the analysis both a numerical illustration of the peculiar results of BV as well as some indexation procedures that have been used in some high inflation economies.

## **II. Basic Model**

Adapting the notation in BV, let

$F(T)$  - net present value of the investment cash-flow, as measured in monetary units referred to the date of the investment;

$f(T)$  - pre tax net receipt, as measured in monetary units referred to the date of the investment, derived from termination at time  $T$ ;

$h(T)$  - pre tax net receipt, as measured in nominal terms, derived from termination at time  $T$ ;

$R$  - real rate of interest (as measured in instantaneous terms and assumed to be positive);

$C$  - cost of investment, as measured in monetary units referred to the date of the investment;

$T$  - time to termination of investment;

$\theta$  - constant rate of inflation (as measured in instantaneous terms and assumed to be positive);

$\phi$  - tax rate.

Assuming that the asset can be depreciated, for tax purposes, in a lump sum at the time of termination, with no indexation of the investment cost, and that the pre tax net receipt is fully indexed (i.e.  $h(T) = e^{qT} f(T)$ ), the cash flow associated with the investment, as expressed in current prices (i.e. nominal terms), is as depicted in Figure I

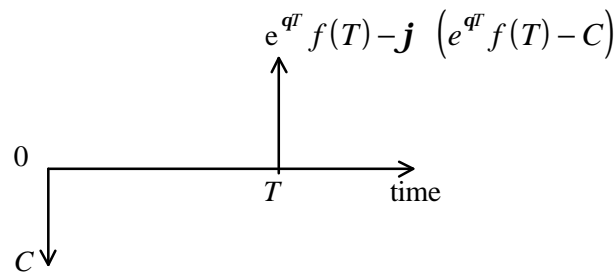


Figure I

### Cash-Flow for the Basic Model

The above cash-flow corresponds to what will be called the basic model, and is the one that was addressed by BV.

Working with constant prices, as referred to the date of the investment, it follows that the function whose maximization determines the optimal duration is:<sup>1</sup>

$$F(T) = -C + e^{-(R+q)T} \left\{ (1-j) e^{qT} f(T) + j C \right\} \quad (1)$$

<sup>1</sup> Observe that, if we denote by  $i=R+\theta$  the corresponding nominal rate of interest, if  $\phi=0$  and if we work directly with  $h(T)$ , equation (1) corresponds to the classical text-book presentation of the problem (c.f. Hirshleifer, 1970, Simon and Blume, 1994 and Varian, 1990).

Repeating here, for completeness, the presentation of BV, we have:

$$\frac{dF(T)}{dT} = \{(1-\mathbf{j})(f'(T) - Rf(T))e^{qT} - (R + \mathbf{q})\mathbf{j}C\}e^{-(R+\mathbf{q})T} \quad (2)$$

Thus

$$\frac{dF(T)}{dT} = 0 \Rightarrow (1-\mathbf{j})(f'(T) - Rf(T))e^{qT} - (R + \mathbf{q})\mathbf{j}C = 0 \quad (3)$$

On the other hand

$$\begin{aligned} \frac{d^2 F(T)}{dT^2} &= \{(1-\mathbf{j})(f''(T) - Rf'(T))e^{qT} + \mathbf{q}(1-\mathbf{j})(f'(T) - Rf(T))e^{qT}\}e^{-(R+\mathbf{q})T} \\ &\quad - (R + \mathbf{q})\frac{dF(T)}{dT} \end{aligned} \quad (4)$$

Therefore, denoting by  $T^*$  the solution of (3), we have:

$$\left. \frac{d^2 F(T)}{dT^2} \right|_{T^*} = (1-\mathbf{j})\{f''(T^*) - Rf'(T^*) + \mathbf{q}(f'(T^*) - Rf(T^*))\}e^{-RT^*} \quad (5)$$

Thus, as  $1-\mathbf{q} > 0$  and  $e^{-RT^*} > 0$ , it follows that  $T^*$  will be the optimal duration if

$$f''(T^*) - Rf'(T^*) + \mathbf{q}(f'(T^*) - Rf(T^*)) < 0 \quad (6)$$

To investigate the effect of changes in the inflation rate  $\theta$  on the optimal duration  $T^*$ , differentiate (3) holding  $\mathbf{q}$  and  $R$  as constants. It follows that:

$$\frac{dT}{dq} = \frac{jC - T(1-j)(f'(T) - Rf(T))e^{qT}}{(1-j)\{f''(T) - Rf'(T) + q(f'(T) - Rf(T))\}e^{qT}} \quad (7)$$

Taking into account (6), we know that, at  $T^*$ , the denominator of (7) is negative. Accordingly, the sign of  $dT^*/d\theta$  is the opposite of the sign of the numerator of (7), which depends on  $T^*$ .

Rather than pursuing the examination of the sign of the numerator of (7), a cumbersome task indeed, BV turned their attention to the investigation of the sign of  $\frac{\partial^2 F(T)}{\partial T \partial q}$ , at  $T^*$ , as it is the same as the sign of  $dT/d\theta$ , also at  $T^*$ .<sup>1</sup>

From (2), it follows that

$$\begin{aligned} \frac{\partial^2 F(T)}{\partial T \partial q} &= \{T(1-j)(f'(T) - Rf(T))e^{qT} - jC\}e^{-(R+q)T} \\ &\quad - T\{(1-j)(f'(T) - Rf(T))e^{qT} - (R+q)jC\}e^{-(R+q)T} \end{aligned}$$

Thus, taking into account (3), we have:

$$\left. \frac{\partial^2 F(T)}{\partial T \partial q} \right|_{T^*} = \{T^*(1-j)(f'(T^*) - Rf(T^*))e^{qT^*} - jC\}e^{-(R+q)T^*}$$

or, given that  $(1-j)(f'(T^*) - Rf(T^*))e^{-qT^*} = (R+q)jC$

$$\begin{aligned} \left. \frac{\partial^2 F(T)}{\partial T \partial q} \right|_{T^*} &= \{T^*(R+q)jC - jC\}e^{-(R+q)T^*} \\ &= jC\{T^*(R+q) - 1\}e^{-(R+q)T^*} \end{aligned} \quad (8)$$

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<sup>1</sup> This result follows from the so-called envelope theorem (cf. Varian, 1992).

Therefore, the sign of  $dT/d\theta$  at  $T^*$  is the same as the sign of  $T^* (R+\theta) - 1$ . Thus, as  $R+\theta$  was assumed to be positive, it follows that:

$$\frac{dT^*}{dq} \begin{matrix} > \\ < \end{matrix} 0 \quad \text{if} \quad T^* \begin{matrix} > \\ < \end{matrix} \frac{1}{R+q} \quad (9)$$

or

$$\frac{dT^*}{dq} \begin{matrix} > \\ < \end{matrix} 0 \quad \text{if} \quad q \begin{matrix} > \\ < \end{matrix} \frac{1}{T^*} - R \quad (10)$$

Quoting BV, we can say that “for investments with long duration increased inflation increases duration and the opposite holds for short duration. Alternatively, as expressed by (10), at high levels of inflation, increased inflation calls for longer duration and the opposite holds at low levels of inflation. In particular, a change from no inflation to some low level of inflation should decrease optimal duration”.

With the sole purpose to give at least some evidence of the magnitude of the numerical results that may be obtained, let us consider the case where, following an adaptation of an example that was presented in Hirshleifer (1970, pg, 87), we have :

$$f(T) = 90 \log(1 + T) + 120$$

If  $C = 100$ ,  $\phi = 20\%$ ,  $R = 10\%$  and if we have no inflation ( $\theta = 0$ ), it is easily verified that the optimal duration is  $T^* \cong 2,493902$ . On the other hand, if we move to the situation where we start to have inflation, at the very moderate rate  $\theta = 1\%$ , the optimal duration will be slightly reduced to  $T^* \cong 2,475614$ .

Supposing now that  $C = 56$ , and maintaining the values of  $\phi$  and  $R$ , consider the situation where we have rampant inflation, with  $\theta = 50\%$ . In this state of affairs we will have  $T^* \cong 2,503765$ .



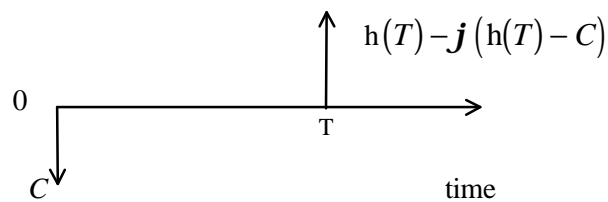
On the other hand, if inflation increases even further, say  $\theta = 60\%$ , we will also have an increase (though small) in the optimal duration, as  $T^* \cong 2,529576$ .

### **III. Variations of the Basic Model**

The persistent presence of high inflation in some countries, has caused the implementation of indexation procedures. In particular, in the Brazilian case, the so called monetary correction scheme,<sup>1</sup> which was made official in 1964 and is in use even nowadays, makes indexation a crucial factor for tax purposes. Accordingly, based primarily in the Brazilian experience, we are going to consider the following variations of the basic model.

#### **a) absence of indexation**

As the first variation of the basic model, one which is more appropriate for low inflation economies, let us consider the case where neither the pre tax net receipt nor the investment, the latter for tax purposes, are indexed to the inflation rate. In this situation, the cash flow associated with the investment, as expressed in current prices, is as depicted in Figure II.



**Figure II**

#### **Cash-Flow in the Absence of Indexation**

Thus, at constant prices, as referred to the date of the investment, we have:

$$F(T) = -C + e^{-(R+q)T} \{(1-j)h(T) + jC\} \quad (11)$$

or, working with the nominal interest rate  $i = R + \theta$

$$F(T) = -C + e^{-iT} \{(1-j)h(T) + jC\} \quad (11')$$

That is, if the classical Fisher equation relating  $i$ ,  $R$  and  $\theta$  holds, and if we do not have indexation, the optimal duration is a function of the nominal interest rate  $i$ . In other words, with  $h(T)$  playing the role of  $f(T)$ , we have, formally, exactly the case treated in equation (2) in BV. The difference is that, rather than analyzing the effects of changes in  $\varphi$  on the optimal duration  $T^*$ , we are interested now in investigating the effects of changes in the inflation rate  $\theta$ .

Proceeding with the maximization of  $F(T)$ , we have:

$$\frac{dF(T)}{dT} = \{(1-j)(h'(T) - ih(T)) - ijC\}e^{-iT^*} \quad (12)$$

Thus

$$\frac{dF(T)}{dT} = 0 \Rightarrow i = \frac{(1-j)h'(T)}{(1-j)h(T) + jC} \quad (13)$$

That is, the optimal duration  $T^*$  has to satisfy the extended version of the classical Jevon's formula, as given by (13).

With regard to the second order condition, we have:

$$\left. \frac{d^2 F(T)}{dT^2} \right]_{T^*} = \{(1-j)(h''(T^*) - ih'(T^*))\}e^{-iT^*} \quad (14)$$

Thus, if the nominal pre tax net receipt  $h(T)$  is a well behaved function of  $T$ , i.e. increasing and concave, the considered solution  $T^*$  is indeed optimal.

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<sup>1</sup> For assessments of the monetary correction scheme see, besides the early work of Fishlow (1974), Barbosa (1993) and the very comprehensive surveys of Simonsen (1983 and 1995).

On the other hand, to investigate the effects of changes in the inflation rate  $\theta$  on the optimal duration  $T^*$ , let us start considering the effects of changes in the nominal interest rate  $i$ . To do this, differentiating (13), holding the tax rate  $\phi$  constant, we have:

$$\frac{dT}{di} = \frac{(1-j)h(T) + jC}{(1-j)(h''(T) - ih'(T))} \quad (15)$$

Therefore, given the assumptions on the behavior of  $h(T)$ , it follows that increases in the nominal rate of interest  $i$  shortens the optimal duration  $T^*$ . Making use of the chain rule of differentiation, and taking into account that  $di/d\theta > 0$ , it follows then that:

$$\frac{dT}{d\theta} = \left( \frac{dT}{di} \right) \left( \frac{di}{d\theta} \right) < 0 \quad (16)$$

Thus, if there is no indexation at all, increases in the rate of inflation shortens the optimal duration.

### b) full indexation

In the Brazilian case, the usual procedure with regard to taxation, at least up to the implementation of the Real Plan in 1994, was to have the investment value indexed to the inflation. Therefore, if the nominal value of the pre tax net receipt  $h(T)$  is also indexed, i.e.  $h(T) = e^{qT} f(T)$ , we have a situation that will be called of full indexation, and the cash flow associated with the investment, as expressed in current prices, is as depicted in Figure III

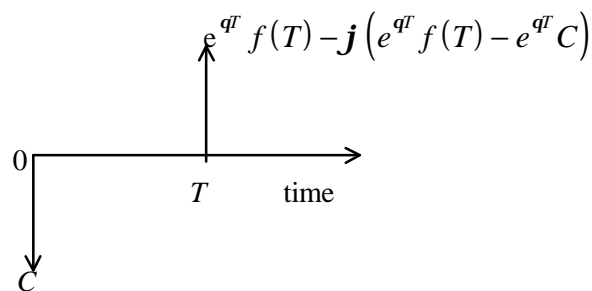


Figure III

### Cash-Flow in the Case of Full Indexation

Thus, as measured in constant prices, as referred to the date of the investment, the present value function to be maximized is:

$$F(T) = -C + e^{-RT} \{(1-j)f(T) + jC\} \quad (17)$$

It is then obvious that the optimal duration does not depend on the inflation rate  $\theta$ .

### c) partial indexation

Concluding the analysis, let us now consider the case where while the investment value is indexed for tax purposes, the nominal value of the pre tax net receipt  $h(T)$  is not. In this situation, assuming that  $h(T) > Ce^{qt}$ , for  $T > 0$ , it follows that the cash flow associated with the investment, as expressed in current prices, is as depicted in Figure IV.<sup>1</sup>

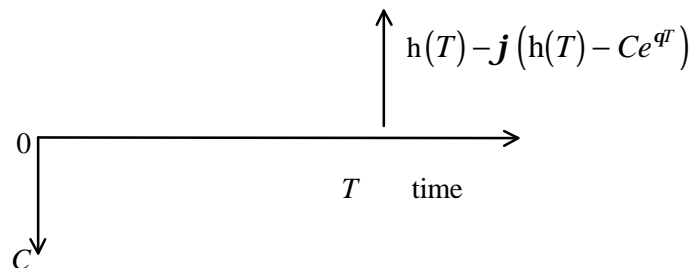


Figure IV

### Cash-Flow in the Case of Partial Indexation

In terms of constant prices, as expressed at the date of the investment, the correspondent present value is:

$$F(T) = -C + e^{-(R+q)T} \{(1-j)h(T) + jCe^{qt}\} \quad (18)$$

<sup>1</sup> Note that, more generally, the value of the tax is given by  $\max \{0; j(h(T) - Ce^{qt})\}$

with

$$\frac{dF(T)}{dT} = \{(1-j)(h'(T) - (R+q)h(T)) - Rj Ce^{qt}\} e^{-(R+q)T} \quad (19)$$

Therefore, the optimal duration  $T^*$  has to satisfy the following equation:

$$(1-j)(h'(T) - (R+q)h(T)) - Rj Ce^{qt} = 0 \quad (20)$$

On the other hand, as

$$\left. \frac{d^2 F(T)}{dT^2} \right]_{T^*} = \{(1-j)(h''(T^*) - (R+q)h'(T^*)) - qRj Ce^{qT^*}\} e^{-(R+q)T^*} \quad (21)$$

$T^*$  will be indeed the optimal duration if

$$(1-j)(h''(T^*) - (R+q)h'(T^*)) - qRj Ce^{qT^*} < 0 \quad (22)$$

Also, differentiating (20), holding constant  $R$  and  $\varphi$ , we have that:

$$\frac{dT^*}{dq} = \frac{(1-j)h(T^*) + T^* Rj Ce^{qT^*}}{(1-j)(h''(T^*) - (R+q)h'(T^*)) - qRj Ce^{qT^*}} \quad (23)$$

Thus, as the numerator of (23) is obviously positive, it follows from (22) that an increase in the rate of inflation decreases the optimal duration.

#### **IV. Conclusion**

Taking into account some indexation procedures that have been used in some high inflation economies, particularly in the Brazilian case, we have extended the basic model presented by BV in order to cover three additional situations. The fundamental conclusion is that while the effect of an increase in the inflation rate is somewhat ambiguous in the basic situation, as it depends on the relative size of the optimal duration itself, the effect is very much predictable on the other three situations. Thus, while the optimal duration is independent of the inflation rate in the case of full indexation, an increase in the rate of inflation always shortens the optimal duration both in the case of partial indexation and in the case of absence of indexation.

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