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ABSTRACT

This paper considers nonparametric identification and estimation of a generalized Roy model that includes a non-pecuniary component of utility associated with each choice alternative. Previous work has found that, without parametric restrictions or the availability of covariates, all of the useful content of a cross-sectional dataset is absorbed in a restrictive specification of Roy sorting behavior that imposes independence on wage draws. While this is true, we demonstrate that it is also possible to identify (under relatively innocuous assumptions and without the use of covariates) a common non-pecuniary component of utility associated with each choice alternative. We develop nonparametric estimators corresponding to two alternative assumptions under which we prove identification, derive asymptotic properties, and illustrate small sample properties with a series of Monte Carlo experiments. We demonstrate the usefulness of one of these estimators with an empirical application. Micro data from the 2000 Census are used to calculate the returns to a college education. If high-school and college graduates face different costs of migration, this would be reflected in different degrees of Roy-sorting-induced bias in their observed wage distributions. Correcting for this bias, the observed returns to a college degree are cut in half.

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1. Introduction

In the original application of his model, Roy (1951) showed that the self-selection of individuals into occupations generally implies that observed wages (conditional on occupation choice) differ markedly from the underlying distribution of wages in the population. The Roy model has subsequently been applied to a wide class of problems in economics as its structure fits any setting in which individuals choose among a set of alternatives to maximize an outcome associated with that choice. Given its wide applicability, an important line of recent research has analyzed identification in the Roy model. Beginning with Heckman and Honore (1990), this literature has produced a series of results that clarify the conditions under which the underlying population distribution of wages can (or cannot) be identified in observational data.

In this paper, we study the nonparametric identification and estimation of a generalized Roy model that includes a non-pecuniary component of utility associated with each alternative. An important limitation of the pure Roy model is that it assumes that individuals maximize only economic returns (e.g., wages). Yet non-pecuniary aspects of decisions are important in many economic applications. In the choice of occupation, for example, non-pecuniary components of utility would include the amenity value or injury risk associated with different jobs.[1] As with the pure Roy model, this generalized version is also applicable to settings in which the outcome of interest is not economic returns. In studying the choice of health behaviors or medical treatments, for example, the relevant outcome might be the survival rate, while the "non-pecuniary" component of utility might capture the enjoyment associated with a behavior (such as smoking) or disutility of side-effects associated with various treatments.[2] In this way, the generalized model developed here can be applied to a wide class of problems in economics.

[1] In modeling the choice of labor market/residence, the non-pecuniary component of utility would capture variation in amenities and cost-of-living across cities.

[2] Likewise, in the study of school choice, the relevant outcome might be achievement scores, while other factors affecting the choice of school (e.g., availability of special education programs) might be included as part of a separate component of utility.

The starting point for our analysis is the well-known result for the pure Roy model in Heckman and Honore (1990) – that any cross-sectional dataset (consisting of the observed distribution of wages in each sector and the probability that each sector is chosen) can be rationalized by an underlying population wage distribution in which wages are distributed independently across sectors. Thus, the correlation of wage offers across sectors is unidentified in a single cross-section.

A common interpretation of this important result is that, without parametric restrictions or access to covariates, all of the useful content of cross-sectional data is absorbed in a restricted specification of the Roy model (i.e., one that imposes independence). While this interpretation is correct if the data is generated in a pure Roy model, one can in fact glean additional information from a single cross-sectional dataset when non-pecuniary considerations matter. In the analysis that follows, we provide two distinct sets of conditions under which the non-pecuniary values associated with each choice alternative are non-parametrically identified. As in Heckman and Honore (1990), the identification of the full population wage distribution requires additional identifying assumptions; the key insight of our paper is that the non-pecuniary value of alternatives in a generalized Roy model can be identified even in a single cross-section.

Throughout the paper, we consider the nonparametric identification of the model in a relatively demanding setting in which (i) the set of available choices is large and (ii) covariates are not available to the researcher.[3] The objects to be identified are the population wage distributions and a common non-pecuniary utility associated with each choice. In developing a first set of conditions for identification, we impose only the relatively innocuous requirement that

[3] The related problem when covariates are available has been studied extensively in the literature, especially in the binomial choice problem. [See, for example, Heckman and Honore (1989) and Heckman (1990)] As there is a close link between Roy models and competing risk models, several many of the papers in the survey in Powell(1994) are also related to the models we explore here. Some more recent related work includes Honore et al.(2002), Lee(2006), Honore and Lleras-Muney (2007), and Khan and Tamer (2007). While identification strategies that rely on the use of covariates can be extended to the multinomial choice setting, the requisite demands on the data are enormous, requiring, for example, the availability of distinct combinations of covariates that compel individuals to select each choice with certainty.

the distribution of pecuniary returns has a finite lower bound. Given this assumption, we demonstrate that the difference in the minimum order statistic for any two alternatives exactly identifies the difference in the non-pecuniary value of those choices. Intuitively, this follows directly from the observation that no individual will choose a less-preferred choice (on the basis of non-pecuniary considerations) unless the wage offered there exceeds this threshold. Thus, the minimum wage observed in the less-preferred sector should be exactly the minimum wage observed in the more-preferred sector plus the difference in non-pecuniary components.[4] Having identified the non-pecuniary component of utility, we show that it is then straightforward to (i) back-out underlying unconditional population wage distributions using transformed versions of the observed conditional wages distributions for each sector and the Kaplan-Meier (1958) procedure, if one assumes independence, or (ii) apply Petersen (1976) bounds to the transformed data to bound the unconditional population wage distribution.

While this estimator works very well in controlled data environments, relying exclusively on differences in minimum order statistics to identify the non-pecuniary component of utility raises concerns about measurement error. As a result, we consider a second set of formal identifying assumptions. Our second identification proof is based on two key assumptions. First, we assume independence.[5] Second, we assume that information is available for (at least) two subsets of the population that differ in their non-pecuniary valuation of the set of choice alternatives. In the application that we present below, we consider the choice of regional labor market; in that context, moving costs (broadly defined) naturally imply that birth region affects the non-pecuniary value one ascribes to a particular destination. We then exploit the fact that

[4] Note that, within the pure Roy model, the minimum order statistics would be identical for all choices and the full empirical content of the data would in fact be absorbed by a specification including independent population wage distributions, as suggested by Heckman and Honore (1990).

[5] Again, following the existing literature, the independence assumption can be relaxed in more generous data environments (e.g., when data is available for more than a single cross-section or when covariates are available). See, for example, Khan and Tamer (2007) who achieve identification results under strong support conditions in a semi-parametric Roy model. Honore and Llera-Muney (2007) establish set identification when the independence assumption is relaxed.

wage offers are likely to be similar for individuals with similar characteristics from neighboring regions while the non-pecuniary value of residing in these regions will vary significantly with an individual's birthplace. We refer to this second assumption as "commonality", i.e., that a common wage distribution characterizes wage offers for all individuals regardless of birthplace. Given this assumption, we prove that both the non-pecuniary components of utility for each population subset and the overall population wage distributions are identified.

In this case, some intuition for why the model is identified by the commonality assumption can again be gained by referring back to Heckman and Honore (1990). Without non-pecuniary components of utility, the observed conditional wage distributions and choice probabilities map uniquely to a set of independent population wage distributions. With at least two subsets of the population that differ in their non-pecuniary valuations of alternatives, however, the resulting unconditional wage distributions that would reconcile the two subsets of the data would differ. What our identification proof ensures is that the identical unconditional wage distributions for each subset can only be reconciled at the true values of the non-pecuniary components of utility for each population subset.

Estimation of this model follows directly from the identification proof. As we show below, it is possible to write a system of equations based on the observed conditional wage distributions that must equal zero identically at the true values of the non-pecuniary parameters for each population subset. These equations serve as natural moments for a minimum distance estimator.

These results add to a sparse literature that has studied the nonparametric identification of a generalized Roy model with many alternatives and non-pecuniary components of utility. Dahl (2001) proposes a multinomial version of the estimator developed in the binomial context by Ahn and Powell (1994). His extension relies on the key assumption that a non-parametric selection correction term can be based on the first-best choice probability. This assumption is not, however, based on a model of utility maximizing behavior. Other work has examined spatial sorting behavior based on wages and non-pecuniary benefits. Falaris (1987) and Davies, Greenwood, and Li (2001) study the determinants of migration decisions in Venezuela and the US, respectively. Falaris applies Lee's (1983) generalized polychotomous choice model to control for non-random selection bias in conditional wage distributions, while Davies, Greenwood, and Li essentially ignore it. The entire literature on wage-hedonics, beginning with Roback (1982), has similarly ignored this problem. In those papers, wage and housing price gradients across cities are used to back-out the value of urban amenities. Wage distributions conditional on non-random selection into cities are typically used to calculate the first of these gradients, leading to biased estimates.

We conclude this paper by applying our estimator to US Census data to study the effect of spatial sorting on returns to a college education, addressing the same question as Dahl (2001). College graduates are more likely to migrate than are high-school graduates, meaning that the bias in their conditional wage distributions induced by Roy sorting will be greater. Controlling for this bias for both high-school and college graduates, we find that the estimated returns to a college education at the median fall from 42% to only 18%.

The remainder of the paper is organized as follows. Section 2 introduces the generalized Roy model, proves identification for the case in which wage distributions are assumed to have a finite lower support, and develops a corresponding estimator. Section 3 proves identification under the alternative assumptions of independence and commonality, and develops a corresponding estimator. Section 4 outlines the asymptotic properties of our estimators, and section 5 shows how each estimator performs in finite samples and under less-than-ideal data circumstances. Section 6 uses the unbounded support estimator to recover an unbiased estimate of the returns to a college education. Section 7 concludes with a discussion of possible extensions to this research.

2. Identification and Estimation – Finite Lower Support

We begin our analysis by describing the generalized Roy model and data environment that we study. We then prove identification under two separate sets of assumptions. The first case is characterized by the assumption that the distribution of the endogenously determined payoffs (e.g., wages in a classic Roy model) has a finite lower support. In this case, our estimation strategy is transparent and easily applied. Our second set of assumptions, described in Section 3, is applicable in situations where a finite lower support cannot be assumed, or where the minimum order statistic provides a noisy measure of the lower bound. In both cases, we first prove identification with a simple model describing the sorting of individuals from a single origin location into one of two destinations (k = 1, 2). We indicate the wage earned by individual i, should he choose to settle in locations #1 and #2 as $\omega_{1,i}$ and $\omega_{2,i}$, respectively. In contrast to the classic Roy model, where sorting is simply across employment sectors and driven entirely by pecuniary compensation, we model sorting in a geographic context where the individual's location decision depends in part on his wage draw in each location, but also on non-wage determinants of utility specific to a particular location, which we label as "tastes".[6] Utility from choosing to settle in location k is given by the sum of wages ($\omega_{k,i}$) and tastes (τ_k):

(1) $$U_{k,i} = \omega_{k,i} + \tau_k$$

[6] Tastes would certainly include natural amenities and local public goods associated with the destination location. In addition, they may include "migration costs"; i.e., costs specific to someone moving from a particular origin to a particular destination. In a narrow sense, these costs would be comprised of re-location expenditures. In broader terms, these costs would likely involve the psychological costs of living far from one's birth location. 2000 Census data indicate that a majority of US household heads live in the narrowly defined region in which they were born. [Bayer, Keohane, and Timmins (2007)]

Without loss of generality, we normalize $\tau_1 = 0$.[7] The goal of our exercise is to recover estimates of τ_2, $f_1(\omega_1)$, and $f_2(\omega_2)$ (i.e., the taste parameter associated with location #2 and the unconditional wage distributions in each location). The difficulty arises from the fact that we only see (i) wage distributions conditional upon optimal sorting behavior, and (ii) an indicator of which location an individual chooses.

2.1 Identification of Tastes Based on a Finite Lower Support

Our first approach uses only the conditional wage distributions and an indicator of location choice to recover τ_2, $f_1(\omega_1)$, and $f_2(\omega_2)$ according to the following argument based on minimum order statistics. For an individual i, we only observe $\omega_{2,i}$ if:

$$\text{(2)} \qquad \omega_{2,i} + \tau_2 \geq \omega_{1,i}$$

and we only observe $\omega_{1,i}$ if:

$$\text{(3)} \qquad \omega_{2,i} + \tau_2 < \omega_{1,i}$$

Denote the smallest wage (i.e., the minimum order statistic) that we observe from someone choosing to settle in location #1 or #2 by $\underline{w}_1$ and $\underline{w}_2$, respectively. Assuming that $f_1(\omega_1)$ and $f_2(\omega_2)$ have finite lower points of supports (denoted by ω_1^* and ω_2^*, respectively), we know that the smallest value of ω_1 that we could ever see given that individuals maximize utility:

[7] As in all random-utility frameworks, utility is only identified up to an additive constant. This requires some sort of a normalization, which we use to eliminate one of the τ's from the two-destination example. In the more general N x N case, we estimate (N-1) τ's for each of the N origins.

$$(4)\qquad \begin{array}{lll} \underline{w}_1 = \omega_1^* & if & \omega_1^* > \omega_2^* + \tau_2 \\ \underline{w}_1 = \omega_2^* + \tau_2 & if & \omega_1^* \le \omega_2^* + \tau_2 \end{array}$$

Similarly, the smallest value of ω_2 that we could ever see would be:

$$(5)\qquad \begin{array}{lll} \underline{w}_2 = \omega_2^* & if & \omega_1^* \le \omega_2^* + \tau_2 \\ \underline{w}_2 = \omega_1^* - \tau_2 & if & \omega_1^* > \omega_2^* + \tau_2 \end{array}$$

In order to make sense of (4) and (5), define the following two cases:

$$(6)\qquad \begin{array}{ll} A: & \omega_1^* > \omega_2^* + \tau_2 \\ B: & \omega_1^* \le \omega_2^* + \tau_2 \end{array}$$

We are not able to tell whether case A or B prevails in the data without recovering an estimate of τ_2. Conveniently, we are able to recover an estimate of τ_2 in either case. In particular:

$$(7)\qquad \tau_2 = \underline{w}_1 - \underline{w}_2$$

Equation (7) therefore describes our first estimator of τ_2 in the simplest 1 x 2 case. The same logic extends easily to any number of potential destinations (i.e., $\tau_k = \underline{w}_1 - \underline{w}_k,\ k = 1, 2, ..., K$). Begin by defining the following indicator variables:

$$(8)\qquad d_{1,i} = I\left[\omega_{1,i} > \max\left(\omega_{2,i} + \tau_2, \omega_{3,i} + \tau_3, ..., \omega_{K,i} + \tau_K\right)\right]$$

$$(9)\qquad d_{j,i} = I\left[\omega_{j,i} + \tau_j > \max\left(\omega_{1,i}, ..., \omega_{j-1,i} + \tau_{j-1}, \omega_{j+1,i} + \tau_{j+1}, ..., \omega_{K,i} + \tau_K\right)\right]$$

where we continue to normalize $\tau_1 = 0$. The observed wage of individual *i* is defined by:

$$(10) \qquad w_i = \sum_{k=1}^{K} \omega_{k,i} d_{k,i}$$

We next define the following indicator variables, which refer to the finite lower bounds in each location:

$$(11) \qquad \delta_1 \;=\; I\left[\omega_1^* > \max(\omega_2^* + \tau_2,\, \omega_3^* + \tau_3, \ldots,\; \omega_K^* + \tau_K)\right]$$

$$(12) \qquad \delta_k \;=\; I\left[\omega_k^* + \tau_k > \max(\omega_1^*, \ldots, \omega_{k-1}^* + \tau_{k-1}, \omega_{k+1}^* + \tau_{k+1}, \ldots,\; \omega_K^* + \tau_K)\right]$$

We then proceed by evaluating the minimum order statistic for an individual choosing to settle in location #1:

$$(13) \qquad \underline{w}_1 \;=\; \min(w_i \mid d_{1,i} = 1) \;=\; \omega_1^* \delta_1 + \sum_{k=2}^{K} (\omega_k^* + \tau_k)\, \delta_k$$

and in each location $j > 1$:

$$(14) \qquad \underline{w}_j \;=\; \min(w_i \mid d_{1,i} = j) \;=\; \omega_j^* \delta_j + \sum_{\substack{k=1 \\ k \neq j}}^{K} (\omega_k^* + \tau_k - \tau_j)\, \delta_k$$

By simple inspection, one can see that $\tau_k = \underline{w}_1 - \underline{w}_k$, $\forall\; k = 1, 2, \ldots, K$.

2.2 *Identification of $f_1(\omega_1)$ and $f_2(\omega_2)$ with Kaplan-Meier*

Having recovered an estimate of τ_2, it is a simple matter to recover $f_1(\omega_1)$ and $f_2(\omega_2)$ by employing a variation of the Kaplan-Meier (1958) procedure typically used in competing-risks models under independence assumptions.[8] The Kaplan-Meier can be interpreted as a nonparametric maximum likelihood estimator of a censored distribution, and has been proven be asymptotically normally distributed- see, e.g. Gill(1980).

Our variation will be to apply the Kaplan Meier procedure to draws from $\omega_1+\tau_1$, where τ_1 can be estimated using the proposed procedure. In particular, we estimate $f_1(\omega_1)$ by first creating a new data vector which corresponds to only those values of utilities (i.e., $\omega_1 + \tau_1$) that are "uncensored" for destination #1 (i.e., observed for individuals who optimally chose destination #1). Note that, because we were able to recover tastes with equation (7), we can treat utility (i.e., the sum of wages and tastes) as observed for the remainder of the exercise – our only goal is to recover its unconditional distribution, from which we can recover the unconditional distribution of wages. This vector of utilities will be of smaller dimension than the vector of all utilities, which includes draws for individuals who chose destination #1 or destination #2.

To implement the Kaplan Meier procedure, we can simply use standard software packages such as Stata. The resulting value of *S* is the Kaplan-Meier estimate of the c.d.f. of U_2 at x. In the final step, we simply deduct our estimate of τ_2 from utility U_2 at each point in the support of its distribution. The resulting distribution is a non-parametric representation of $f_2(\omega_2)$. We then repeat this process in order to recover $f_1(\omega_1)$, recalling that τ_1 had been normalized to zero.

Note that a portion of the unconditional distribution for one of these two locations will necessarily be censored. Suppose we are in case A, where ω_1^* is large relative to $\omega_2^* + \tau_2$. We are therefore able to observe the complete distribution $f_1(\omega_1)$, beginning with $\underline{w}_1 = \omega_1^*$. We are,

[8] As we mentioned previously, an alternative approach in this stage would be to relax independence and apply the Petersen (1976) bounds to the transformed data to bound the unconditional distributions.

however, unable to observe $f_2(\omega_2)$ to the left of $\underline{w}_2 = \omega_1^* - \tau_2 > \omega_2^*$. While we are unable to determine the shape of the distribution $f_2(\omega_2)$ between ω_2^* and $\underline{w}_2$ in the above case, we are able to bound from above the value of ω_2^* (i.e., the lower point of support for the censored distribution). In particular, knowing that $\omega_1^* = \underline{w}_1$, we know that $\omega_2^* < \underline{w}_1 - \tau_2$. We are unable to determine more about the shape of the distribution $f_2(\omega_2)$ between ω_2^* and $\underline{w}_2$ without resorting to parametric assumptions.

3. Identification and Estimation – Unbounded Support

While clean and transparent, there are two practical problems with the technique outlined in Section 2. First, the payoff variable in question may not naturally have a finite lower support (e.g., theory might dictate using the natural log of wages in the utility function). Second, the minimum order statistic can be a very noisy statistic.[9] Unless one has tremendous confidence in the estimate of the minimum order statistic, that noise will be translated directly through to the estimates of the taste parameters and, subsequently, on to the Kaplan-Meier estimates.

As an alternative, we propose in this section an estimator that employs data from the full distribution of conditional wages. Importantly, this approach is valid for an unbounded support.[10] With that flexibility, however, comes the need for an additional identification assumption. In particular, we begin by showing that, without an additional assumption, τ_2, $f_1(\omega_1)$, and $f_2(\omega_2)$ are not identified. This negative proof, however, reveals just how easily identification can be achieved by exploiting the assumption of "commonality" described in Section 3.2.

[9] For example, the bottom 2-3% of wage observations in the US Census data used for our empirical application in Section 7 are implausibly low (i.e., less than 50¢ per hour).
[10] In practice, this means that poorly measured data in the lower tail of the wage distribution will not have a significant impact on the estimation algorithm, whereas it can have severe effects on the minimum order statistic approach.

3.1 Non-Identification in the 1 x 2 Case

We begin with a simple model of individuals sorting over two locations, indexed by 1 and 2. We assume for simplicity that the individuals are from location 1, and we therefore normalize their taste for staying there to zero ($\tau_1 = 0$). Our interest is in recovering estimates of τ_2, $f_1(\omega_1)$, and $f_2(\omega_2)$.

We define a variable d_i, which functions as an indicator that individual i remained in his origin location:

(15) $$d_i = I[\omega_{1,i} > \omega_{2,i} + \tau_2]$$

Using this indicator, we can write down an expression for individual i's observed wage:

(16) $$w_i = d_i \omega_{1,i} + (1 - d_i)\omega_{2,i}$$

i.e., the individual receives his draw from location #1 if it was utility maximizing to stay there. Next, define the following joint probability distributions, both of which are easily observed in the data:

(17) $$\Psi_1(t) = P(d_i = 1, w_i \le t) \qquad \Psi_2(t) = P(d_i = 0, w_i \le t)$$

We will also work with the derivatives of these expressions, which we denote by:

(18) $$\psi_1(t) = \frac{\partial}{\partial t} P(d_i = 1, w_i \le t) \qquad \psi_2(t) = \frac{\partial}{\partial t} P(d_i = 0, w_i \le t)$$

Assuming independent wage draws, we can re-write $\Psi_1(t)$ as:

$$\Psi_1(t) = P(d_i = 1, w_i \le t)$$

(19) $$= P(\omega_{1,i} > \omega_{2,i} + \tau_2, \omega_{1,i} \le t) \quad = \quad P(\omega_{1,i} - \tau_2 > \omega_{2,i}, \omega_{1,i} \le t)$$

$$= \int_{-\infty}^{t} f_1(\omega_1) d\omega_1 \int_{-\infty}^{\omega_1 - \tau_2} f_2(\omega_2)\, d\omega_2 \quad = \quad \int_{-\infty}^{t} f_1(\omega_1)\, F_2(\omega_1 - \tau_2)\, d\omega_1$$

This means that we can define $\psi_1(t)$ as follows:

(20) $$\psi_1(t) = \frac{\partial}{\partial t} \int_{-\infty}^{t} f_1(\omega_1)\, F_2(\omega_1 - \tau_2)\, d\omega_1 = f_1(t)\, F_2(t - \tau_2)$$

An analogous argument defines $\psi_2(t)$:

(21) $$\psi_2(t) = \frac{\partial}{\partial t} \int_{-\infty}^{t} f_2(\omega_2)\, F_1(\omega_2 + \tau_2)\, d\omega_2 = f_2(t)\, F_1(t + \tau_2)$$

Going back to the final integral in equation (19) and carrying out integration-by-parts yields:

(22) $$\Psi_1(t) = \int_{-\infty}^{t} f_1(\omega_1) F_2(\omega_1 - \tau_2) d\omega_1 \quad = \quad F_1(t) F_2(t - \tau_2) - \int_{-\infty}^{t} F_1(s) f_2(s - \tau_2) ds$$

Performing a change of variables $u = s - \tau_2$, equation (22) becomes:

(23) $$\Psi_1(t) = F_1(t)F_2(t-\tau_2) - \int_{-\infty}^{t-\tau_2} F_1(u+\tau_2) f_2(u)du$$

Next, we use the expressions for $\psi_1(t)$ and $\psi_2(t)$ defined in (20) and (21) to re-write equation (23) as follows:

(24) $$\Psi_1(t) = \frac{F_1(t)\psi_1(t)}{f_1(t)} - \int_{-\infty}^{t-\tau_2} \psi_2(u)du$$

Noting that the second integral in (24) is simply $\Psi_2(t-\tau_2)$, we can solve for the distribution of ω_1 as a function of τ_2:

(25) $$\lambda_1(t) = \frac{f_1(t)}{F_1(t)} = \frac{\psi_1(t)}{\Psi_1(t) + \Psi_2(t-\tau_2)}$$

where $\lambda_1(t)$ is a function of the unconditional wage distribution in location #1. (25) is a single equation in two unknowns ($\lambda_1(t)$ and τ_2) for a particular value of t, and it is therefore not surprising that we cannot identify both of these values without making an additional assumption. One solution would involve making a parametric assumption about $F_1(t)$. For example, assuming $F_1(t) \sim N(\mu_1, \sigma_1^2)$ would reduce the equation to three parameters. The number of parameters would not increase, however, as one considered the expression evaluated at different values of t. By forcing the equation to hold for many values of t, we would have more equations than unknowns and could identify the model's parameters.

In the following section, we show how the assumption of commonality can be used to non-parametrically recover $\lambda_1(t)$ and τ_2 .

3.2 Identification via Commonality in the 2 x 2 Case

Consider now the case of individuals born into one of two locations (again indexed by 1 and 2), who decide where to reside based on the maximization of utility. This introduces the need for additional notation – we use a superscript to indicate origin location and a subscript to indicate destination location.

The dummy variable indicating that an individual originating in location #1 chooses to stay in that location is given by:

(26) $$d_i^1 = I[\omega_{1,i}^1 > \omega_{2,i}^1 + \tau_2^1]$$

while the indicator that an individual originating in location #2 chooses not to migrate is given by:

(27) $$d_i^2 = I[\omega_{2,i}^2 > \omega_{1,i}^2 + \tau_1^2]$$

As before, we normalize the taste parameter for those choosing not to migrate to zero (i.e., $\tau_1^1 = \tau_2^2 = 0$). With these indicators, we can now write the expression for the observed wage of an individual *i* who originates in location #1:

(28) $$w_i^1 = d_i^1 \omega_{1,i}^1 + (1 - d_i^1)\omega_{2,i}^1$$

Based on these definitions for *d* and *w*, we define the following expressions analogously to the previous sub-section:

$$\text{(29)} \quad \begin{aligned} &\Psi_1^1(t) = P(d_i^1 = 1, w_i^1 \le t) \qquad \Psi_2^1(t) = P(d_i^1 = 0, w_i^1 \le t) \\ &\Psi_1^2(t) = P(d_i^2 = 0, w_i^2 \le t) \qquad \Psi_2^2(t) = P(d_i^2 = 1, w_i^2 \le t) \end{aligned}$$

Continuing in a manner similar to the previous sub-section, we can use equation (29) to derive the following four expressions:

$$\text{(30)} \quad \lambda_1^1(t) = \frac{f_1^1(t)}{F_1^1(t)} = \frac{\psi_1^1(t)}{\Psi_1^1(t) + \Psi_2^1(t - \tau_2^1)}$$

$$\text{(31)} \quad \lambda_2^1(t) = \frac{f_2^1(t)}{F_2^1(t)} = \frac{\psi_2^1(t)}{\Psi_2^1(t) + \Psi_1^1(t + \tau_2^1)}$$

$$\text{(32)} \quad \lambda_1^2(t) = \frac{f_1^2(t)}{F_1^2(t)} = \frac{\psi_1^2(t)}{\Psi_1^2(t) + \Psi_2^2(t + \tau_1^2)}$$

$$\text{(33)} \quad \lambda_2^2(t) = \frac{f_2^2(t)}{F_2^2(t)} = \frac{\psi_2^2(t)}{\Psi_2^2(t) + \Psi_1^2(t - \tau_1^2)}$$

By itself, the expansion of the 1 x 2 case to the 2 x 2 case does nothing to help with identification. It does, however, allow us to introduce an additional assumption – commonality. Under the assumption of commonality, $\lambda_1^1(t) = \lambda_1^2(t)$ and $\lambda_2^1(t) = \lambda_2^2(t)\ \forall\ t$. Under this assumption, we can re-write equations (30)-(33) as the following two equations:

$$\text{(34)} \quad \lambda_1^1(t) = \frac{\psi_1^1(t)}{\Psi_1^1(t) + \Psi_2^1(t - \tau_2^1)} = \frac{\psi_1^2(t)}{\Psi_1^2(t) + \Psi_2^2(t + \tau_1^2)} = \lambda_1^2(t)$$

$$\text{(35)} \quad \lambda_2^1(t) = \frac{\psi_2^1(t)}{\Psi_2^1(t) + \Psi_1^1(t + \tau_2^1)} = \frac{\psi_2^2(t)}{\Psi_2^2(t) + \Psi_1^2(t - \tau_1^2)} = \lambda_2^2(t)$$

Estimation proceeds by forming minimum distance criterion functions based on equations (34) and (35):

$$(36)\qquad \lambda_1^1(t;\tau_2^1)-\lambda_1^2(t;\tau_1^2)=0$$

$$(37)\qquad \lambda_2^1(t;\tau_2^1)-\lambda_2^2(t;\tau_1^2)=0$$

and then relying on the properties of M-estimators to recover τ_2^1 and τ_1^2. [Davidson and MacKinnon (1993)] We then use these taste parameters along with a Kaplan-Meier procedure to recover estimates of $f_1(\omega_1)$ and $f_2(\omega_2)$ as described in Section 2.2.

We now provide sufficient conditions for identification and estimation of the taste parameters in the 2 x 2 setting with commonality. We begin by rearranging the expressions (34) and (35):

$$(38)\qquad \Psi_2^1(t-\tau_2^1)\psi_1^2(t)-\psi_1^1(t)\Psi_2^2(t+\tau_1^2)=\Psi_1^2(t)\psi_1^1(t)-\Psi_1^1(t)\psi_1^2(t)=H(t)$$

$$(39)\qquad \Psi_1^1(t+\tau_2^1)\psi_2^2(t)-\psi_2^1(t)\Psi_1^2(t-\tau_1^2)=\Psi_2^2(t)\psi_2^1(t)-\Psi_2^1(t)\psi_2^2(t)=J(t)$$

Note that the right-hand-side of each of these expressions is an observable function of the data for a particular value of *t*. Our identification result begins with the following lemma:

Lemma 1: *At the true parameter values (* τ_2^1*, τ_1^2* *), we have*

$$(40)\qquad \left(\Psi_2^1(t-\tau_2^1*)\psi_1^2(t)-\psi_1^1(t)\Psi_2^2(t+\tau_1^2*)-H(t)\right)^2+\left(\Psi_1^1(t+\tau_2^1*)\psi_2^2(t)-\psi_2^1(t)\Psi_1^2(t-\tau_1^2*)-J(t)\right)^2=0$$

$\forall\, t\in\Re$ *in the intersection of the supports of* $\psi_1^1(t)$, $\psi_2^1(t)$, $\psi_1^2(t)$, *and* $\psi_2^2(t)$.

This is simply a re-statement of our minimum distance criterion function described above. We will now show that, for each set of values of the taste parameters different from (τ_2^1*, τ_1^2*), denoted by ($\tilde{\tau}_2^1, \tilde{\tau}_1^2$), we must have:

$$(41)\qquad \left(\Psi_2^1(t-\tilde{\tau}_2^1)\psi_1^2(t)-\psi_1^1(t)\Psi_2^2(t+\tilde{\tau}_1^2)-H(t)\right)^2+\left(\Psi_1^1(t+\tilde{\tau}_2^1)\psi_2^2(t)-\psi_2^1(t)\Psi_1^2(t-\tilde{\tau}_1^2)-J(t)\right)^2>0$$

for some $t\in\Re$ in the intersection of the supports of $\psi_1^1(t)$, $\psi_2^1(t)$, $\psi_1^2(t)$, and $\psi_2^2(t)$. To prove this result, first note that if $\tilde{\tau}_2^1=\tau_2^1*$, then only $\tilde{\tau}_1^2=\tau_1^2*$ will make equation (40) hold, by the monotonicity of the conditional c.d.f.'s that make-up that expression. By a similar argument, if $\tilde{\tau}_1^2=\tau_1^2*$, then $\tilde{\tau}_2^1=\tau_2^1*$ in order for equation (40) to hold. Therefore, we need only consider the case in which $\tilde{\tau}_2^1\neq\tau_2^1*$ and $\tilde{\tau}_1^2\neq\tau_1^2*$. I.e., is it possible that an imposter pair ($\tilde{\tau}_2^1, \tilde{\tau}_1^2$) could satisfy equation (40)?

Consider the following condition which we argue will be sufficient to rule out this possibility:

(42) $$\psi_2^1(t-\tau_2^1*)\psi_1^2(t)\psi_2^1(t)\psi_1^2(t-\tau_1^2*) \neq \psi_1^1(t+\tau_2^1*)\psi_2^2(t)\psi_1^1(t)\psi_2^2(t+\tau_1^2*)$$

for some $t \in \Re$ in the intersection of the supports of $\psi_1^1(t)$, $\psi_2^1(t)$, $\psi_1^2(t)$, and $\psi_2^2(t)$. This condition has a simple interpretation – i.e., that the Jacobian matrix associated with equations (38) and (39) is non-singular. There are situations in which this condition will not hold; for example, when the two conditional wage distributions are identical and $\tau_2^1 = -\tau_1^2$.[11] We consider this to be a pathological case.

To establish the sufficiency of the above condition for identification, consider a local linearization of equations (38) and (39) around the true values of τ_2^1 and τ_1^2 and evaluated at *t*. For any pair of perturbations, Δ_2^1 and Δ_1^2, we require the net effect on the left-hand-side of each equation to be zero (since *H(t)* and *J(t)* are functions of only *t*).

(43) $$\psi_2^1(t-\tau_2^1*)\psi_1^2(t)\Delta_2^1 + \psi_1^1(t)\psi_2^2(t+\tau_1^2*)\Delta_1^2 = 0$$

(44) $$\psi_1^1(t+\tau_2^1*)\psi_2^2(t)\Delta_2^1 - \psi_2^1(t)\psi_1^2(t-\tau_1^2*)\Delta_1^2 = 0$$

If condition (42) holds, then the only solution to these expressions is given by $\Delta_2^1 = \Delta_1^2 = 0$, implying that no imposter values of $(\tilde{\tau}_2^1, \tilde{\tau}_1^2)$ could satisfy the system. Of course, we do not know the value(s) of *t* where equation (42) holds, requiring that we evaluate our minimum distance estimator at all available values of *t*. In doing so, we are restricted to only using values of $t \in \Re$ in the intersection of the supports of $\psi_1^1(t)$, $\psi_2^1(t)$, $\psi_1^2(t)$, and $\psi_2^2(t)$. Without any overlap, this identification strategy is not applicable.

[11] This would be the case if we took a single location and arbitrarily divided it into two locations with the exact same wage distributions and amenities. This condition therefore places a practical constraint on the level of geographic precision at which we can apply our estimator – i.e., at the level at which we can observe different spatial wage distributions.

4. Asymptotics

Having described two identification strategies for both the taste parameters and unconditional wage distributions, we now outline the arguments that will be used in developing the asymptotic properties of our proposed estimators. We begin with a discussion of our minimum order statistic estimator. In practice, we simply replace population extreme quantiles in the identification argument with sample minimum order statistics. Asymptotic properties of minimum or maximum order statistics have been studied in recent work by Porter and Hirano (2003). Chernozhukov and Hong (2004) obtain similar results. As a preliminary step, we establish the rate of convergence of the estimator. The result is based on the following regularity conditions:

A1 The K+1 vectors of observed wage and choice indicators (w_i, $d_{k,i}$) are i.i.d. across individuals.
A2 The unconditional wage distributions for alternatives k = 1, 2, …, K are continuously distributed with positive density on $[\ell_k, \infty)$.
A3 $\min_{k=1,2,\ldots K} \ell_k > -\infty$
A4 $\min_{k=1,2,\ldots K} P(d_{k,i} = 1) > 0$

Theorem 0.1 *Under Assumptions A1-A4, we have*

(45) $$\hat{\tau}_k - \tau_k = O_p(n^{-1})$$

A proof that our estimator attains this rate of convergence under Assumption A2 follows from arguments similar to those used in van der Vaart (1998), Section 21.4.

Turning attention to the second stage estimator of the unconditional wage distributions, we proposed applying Kaplan-Meier to yield a consistent estimator of the distribution of $\omega_{k,i}+\tau_k$. We note the first stage estimator, which was shown to be "super-consistent", will have no effect on the limiting distribution of the second stage estimator. The next theorem establishes the limiting distribution of this estimator.

Theorem 0.2 *Under Assumptions A1-A4, our second stage estimator of the unconditional wage distribution has the following linear representation. Let $\pi(t) = P(\omega_{k,i} \leq t)$ and define the set $\Omega = \{t : \pi(t) < 1\}$. Then for any $t \in \Omega$,*

(46) $$\sqrt{n}(\hat{F}_{\omega_{k,i}+\tau_k}(\cdot) - F_{\omega_{k,i}+\tau_k}(\cdot)) \Rightarrow F_{\omega_{k,i}+\tau_k}(\cdot)W(\varphi(\cdot))$$

where W is Brownian motion, and $\varphi(t) = \int_t^\infty \pi^{-1}(s)ds$.

A proof of the above theorem can be found by using the same arguments as in Fleming and Harrington (1991). We omit the details here.

We now turn our attention to the asymptotic properties of the unbounded support estimator. To illustrate the basic arguments involved, we will focus on the two-region setting. Our estimator of the taste parameter vector, $\hat{\tau} = (\hat{\tau}_2^1, \hat{\tau}_1^2)$, is obtained by minimizing the minimum distance objective function:

(47) $$\hat{\tau} = \arg\min_{\tau} \frac{1}{n}\sum_{i=1}^{n} Q(\tau, t_i)$$

The asymptotic properties of our unbounded support estimator are based on the following assumptions:

B1 The K+1 vectors of observed wage and choice indicators (w_i, $d_{k,i}^j$) are i.i.d. across individuals.

B2 The true vector τ_0 lies in the interior of a compact parameter space.

B3 The functions $\psi_m^l(\cdot)$, l, m = 1, 2 are assumed to be uniformly bounded and twice continuously differentiable, with uniformly bounded first and second derivatives.

B4 The kernel function $K(\cdot)$ used to approximate $\psi_m^l(\cdot)$ has bounded support, integrates to one, and has mean zero.

B5 The bandwidth h associated with kernel function $K(\cdot)$ satisfies $\sqrt{n}h^2 \to 0$ and $nh \to \infty$.

Theorem 0.3 *Under Assumptions B1-B4,*

(48) $$\hat{\tau} \xrightarrow{p} \tau_0$$

The proof of the above result can be shown by establishing the four sufficient conditions in Theorem 2.1 in Newey and McFadden (1994), which can be characterized as compactness, identification, uniform convergence, and continuity.

Furthermore, by Newey and McFadden (1994) Theorem 8.11, we can establish the parametric rate of convergence as well as the asymptotic normality of our estimator. The parametric rate is attainable despite the nonparametric rate of convergence achieved by some components because the parameter of interest (τ_0) is a smooth functional of the nonparametric components. Our next theorem is based on the following assumptions:

B6 The functions $\psi_m^l \quad l, m = 1, 2$ are assumed to be uniformly bounded and p times continuously differentiable, with uniformly bounded p^{th} order derivatives.

B7 The kernel function K integrates to one, has mean zero, and is of p^{th} order.

B8 The bandwidth h associated with the kernel function satisfies $\sqrt{n}h^p \to 0$ and $nh \to \infty$.

The following theorem establishes the root-*n* consistency and asymptotic normality of our estimator. Its proof is omitted as it follows from the same arguments used in proving Theorem 8.11 in Newey and McFadden (1994).

Theorem 0.4 *Under Assumptions B1, B2, B4-B8,*

(49) $$\sqrt{n}(\tau - \tau_0) \Rightarrow N(0, \Omega_x)$$

Where $\Omega_x > 0$.

Remark 1 *The exact form of* Ω_x *is complicated, as it involves higher order derivates of the functions* ψ_m^l. *Consequently, for inference on* τ_0 *in our application, we employ sampling methods to avoid using further nonparametric methods.*

Remark 2 *Note that, in this case, the first stage estimator converges at the parametric rate, and consequently will affect the limiting distribution of the second stage estimator. While the precise effect on the limiting distribution can be derived using arguments similar to those used by Newey and McFadden (1994) Section 8, we omit the details here.*

5. Monte Carlo Results

In this section, we use Monte Carlo experiments to describe the properties of both estimators in small samples and with less-than-ideal data. We consider a simple setting with just three locations that serve as both origins and destinations, and we model the sorting decisions of individuals who care about both pecuniary returns (i.e., wages) and non-pecuniary factors (i.e., migration costs and amenities) in deciding where to live. In each experiment, we consider some number of identical individuals (N) originating in each location, and we use their simulated behavior to recover the matrix of taste parameters:

$$(50)\qquad \begin{bmatrix} \tau_1^1 & \tau_2^1 & \tau_3^1 \\ \tau_1^2 & \tau_2^2 & \tau_3^2 \\ \tau_1^3 & \tau_2^3 & \tau_3^3 \end{bmatrix} = \begin{bmatrix} 0 & -0.5 & -0.2 \\ -0.4 & 0 & -0.6 \\ -0.3 & -0.1 & 0 \end{bmatrix}$$

For the sake of simplicity in exposition, we focus our attention on the performance of the estimators in recovering these taste parameters. Unconditional wage distributions in each location could be recovered by applying the Kaplan-Meier technique described in section 2.2 for each set of Monte Carlo estimates.

We begin by looking at the minimum order statistic estimator. The results of nine Monte Carlo experiments are described in Table 1. The first three experiments use the baseline framework in which wages are random variables determined by the following (j denotes origin location):

$$(51)\qquad \begin{aligned} \omega_1^j &= \sqrt{x^2 + 2.25} \\ \omega_2^j &= \sqrt{x^2 + 1.75} \\ \omega_3^j &= \sqrt{x^2 + 2.75} \end{aligned} \qquad j = 1, 2, 3 \qquad x \sim N(0, \tfrac{1}{2})$$

Columns describe the various taste parameter estimates, while rows summarize the mean, standard deviation, and mean squared error of 500 Monte Carlo simulations for each experiment.

With an increasing number of individuals in each origin location, the minimum order statistic becomes a better measure of the true lower bound on wages in a particular location, and our estimates of the taste parameters improve accordingly. This is evident in the declining MSE as N increases from 1,000 to 10,000 to 50,000 for each parameter. Even with as few as 1,000 observations, however, taste parameter estimates based on the minimum order statistic are quite precise.

The fourth and fifth experiments in Table 1 relax the assumption of a finite lower bound for the unconditional wage distribution. In particular,

$$
(52) \qquad \begin{aligned} & f(\omega_1^j) \sim N(2.25, 0.5) \\ & f(\omega_3^j) \sim N(1.75, 0.5) \qquad j = 1, 2, 3 \\ & f(\omega_3^j) \sim N(2.75, 0.5) \end{aligned}
$$

The impact of this model mis-specification is evident in an increase in the MSE by a factor of 100 to 10,000, depending upon the parameter. Conditional upon this mis-specification, however, MSE's still fall as N increases from 10,000 to 50,000.

The sixth and seventh experiments address an important concern with our minimum order statistic estimator – measurement error. Because the estimator relies on a single value of wages for each origin and destination combination, it could become severely biased if that value were mis-measured. In these experiments, we return to the same wage distributions used in the first three experiments (i.e., assuming a finite lower bound), but we add to each wage an i.i.d. normally distributed random variable with zero mean and variance equal to 0.25. This has a significant impact on the precision of our estimates, raising the MSE's associated with our taste parameters by nearly as much as the absence of a finite lower bound. In contrast to that model mis-specification, however, this is primarily the result of an increase in the bias of our estimator, as opposed to its standard deviation.

In the eighth and ninth experiments, we demonstrate a desirable feature of the minimum order statistic estimator – the fact that it is robust to arbitrary forms of correlation in wage draws. Using the same wage distributions as in our baseline specifications, we assume a correlation of 0.25 between wage draws in all locations. As is evident from the table, taste parameter estimates are virtually identical to the baseline case.

Table 2 describes the results of nine Monte Carlo experiments that similarly illustrate the properties of our unbounded support estimator. Using the same matrix of taste parameters, we assume in our baseline experiment that wages are drawn from the same distributions as described in equation (52). The first three experiments demonstrate the effect of increasing the number of individuals originating from each location (N) from 1,000 to 50,000. MSE's of all taste parameter estimates fall with an increase in the sample size. In general, however, results are not as precise as under the (properly specified) minimum order statistic estimator (conditional upon N).

In the next two experiments, we show the implications of violating our key identifying assumption – commonality. In particular, we allow non-migrants to receive a higher wage on average than individuals migrating into their birth location (i.e., a "home advantage" in the labor market).

$$\begin{array}{llllll} & f(\omega_1^j) \sim N(2.25, 0.5) & \mathit{if} & j=2,3 & \mathit{otherwise} & f(\omega_1^1) \sim N(2.5, 0.5) \\ (53) & f(\omega_3^j) \sim N(1.75, 0.5) & \mathit{if} & j=1,3 & \mathit{otherwise} & f(\omega_2^2) \sim N(2, 0.5) \\ & f(\omega_3^j) \sim N(2.75, 0.5) & \mathit{if} & j=1,2 & \mathit{otherwise} & f(\omega_3^3) \sim N(3, 0.5) \end{array}$$

We assume, moreover, that the researcher properly identifies this home advantage and uses only moments formed between pairs of migrant groups (e.g., migrants from locations #2 and #3 living in location #1) in forming our minimum distance objective function. Not surprisingly, with this limited set of moments the model does not perform as well as in the baseline specification. It does, however, do a reasonable job of estimating all parameters (even with only 10,000 observations per origin location). When N is set equal to 50,000, the estimates become quite precise, indicating that our estimation strategy is indeed valid under situations of "limited commonality".

The sixth and seventh experiments describe what happens when another key assumption used in the derivation of the unbounded support estimator – independence – is violated. Recall that, in the derivation of equation (19), we assumed individuals received draws from independent wage distributions. Here, we assume that wage draws exhibit a positive correlation (0.25) across locations. MSE's for all taste parameters rise dramatically, highlighting this as an important shortcoming of our estimation strategy. In current research, we are exploring how correlation might be better handled using panel data. With only cross-sectional data, these results highlight the importance of controlling for as many forms of observable heterogeneity as possible (i.e., wages may be systematically higher for certain groups – the estimation algorithm should be run separately for them). Our final set of experiments describe the effect of measurement error on our unbounded support estimator. As was the case for the minimum support estimator, we simply add to each wage an i.i.d. normal measurement error with mean zero and variance 0.25. In contrast to the minimum order statistic estimator, however, the results of the unbounded support estimator are affected very little.

In summary, Monte Carlo simulations suggest that our minimum order statistic estimator performs extremely well when properly specified. It is, moreover, robust to arbitrary forms of correlation in an individual's wage draws, but it performs very poorly when wages are observed with error or when they are drawn from a distribution without a finite lower bound. These failures motivate our derivation of the unbounded support estimator. When properly specified, experiments show that it also performs well. Moreover, its performance is not adversely affected by measurement error in wages or by limited commonality (if the researcher properly recognizes this in forming the minimum distance objective function). In contrast to the minimum order statistic estimator, however, it performs poorly when wage draws are correlated across locations (motivating our current work with panel data).

6. Empirical Application: Measuring the Returns to College Education

In order to demonstrate the performance of our estimator in an empirical setting, we examine a question similar to that posed by Dahl (2001) – i.e., what are the returns to a college education (relative to graduating from high school) before and after controlling for the non-random spatial sorting of workers across the United States? The results of the basic Roy model (1951) suggest that sorting shifts the means of the (observed) conditional wage distributions up from their (unobserved) unconditional values. Whether spatial sorting increases or reduces the estimated returns to a college education will depend upon whether this shift is proportionally bigger for high school or college educated individuals. If, for example, college educated individuals were more mobile and, hence, more able to migrate in response to favorable idiosyncratic wage draws, we would expect spatial sorting to create an upward bias in the estimated returns to a college education. Whether or not this is the case (and how big is the resulting bias) is an empirical question.

In order to answer that question, we use data extracted from the 2000 US Census 1% microsample, available from the IPUMS (www.ipums.org). Specifically, we consider a sample of 470,918 high school graduates taken from each of nine divisions of the United States used by the Census Bureau, along with a corresponding sample of 429,584 college graduates.[12] We use only data describing male household heads.[13] For each individual, we observe annual income from wages and salary, the individual's region of residence, and the individual's region of birth.[14]

[12] Regional Definitions: (1) *New England* (CT, ME, MA, NH, RI, VT), (2) *Middle Atlantic* (NJ, NY, PA), (3) *East North Central* (IL, IN, MI, OH, WI), (4) *West North Central* (IA, KS, MN, MO,NE, SD, ND), (5) *South Atlantic* (DE, DC, FL, GA, MD, NC, SC, VA, WV), (6) *East South Central* (AL, KY, MS, TN), (7) *West South Central* (AR, LA, OK, TX), (8) *Mountain* (AZ, CO, ID, MT, NV, NM, UT), and (9) *Pacific* (AK, CA, HI, OR, WA).

[13] We use only household heads because we assume they are more likely to have made their own geographic location decision, and we use only individuals less than 35 years of age as they are more likely to have recently migrated. Older individuals may have migrated further in the past in response to different wage or amenity distributions.

[14] We drop any individuals reporting zero annual income, self-employed individuals, individuals not born in the United States, and individuals who worked fewer than 45 weeks in the previous year. The US Census describes both the individual's birth state as well as the PUMA in which he/she was living five years prior. We use birth state to define birth region, which becomes our measure of "origin location", but

Tables 3 and 4 summarize the long-run migration probabilities observed in the data for high school and college graduates, respectively, for each of four summary birth and destination regions. In particular, each row indicates the birth region while each column indicates the region in which the individual is observed in the 2000 Census. Each entry describes the fraction of individuals originating in the row birth region who are found to be living in the column destination region. 80.6% of high-school graduates born in New England are found to be living in New England. The fraction of high school graduate "stayers" is similarly high for other regions.[15] For college graduates, a noticeably lower percentage remains tied to their respective birth regions.

Because Census wage data, which are derived from self-reported income and hours information, are quite noisy in the lower tail (see footnote 5), and because we see individuals from multiple birth places, we opt for our unbounded support estimator. This estimator makes an independence assumption and assumes that individuals from different birth regions will receive wage draws from a common destination wage distribution. Note that, with different data, the extreme quantile estimator (which only assumes that wage distributions have a finite lower bound) might be used instead. Deleire and Timmins (2007) use this estimator, along with CPS wage data, to recover an estimate of the value of a statistical life (VSL) controlling for Roy sorting across occupations.

Tables 5 and 6 report the estimates of the taste parameters for high school and college graduates, respectively. Results are measured in terms of the natural log of hourly wages, standard errors are derived from the results of 750 bootrstrap simulations, and point estimates are bias-corrected. A college graduate from the mid-Atlantic, for example, faces a statistically significant cost of -0.622 per year in moving to the Pacific region. Considering the mean wage

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a similar analysis could be performed using location five years prior as the "origin", leading to a short-run measure of mobility cost.

[15] Note that the fraction of "stayers" would be smaller if we had used a finer geographic division (e.g., states), but would still constitute a clear plurality.

amongst all college graduates ($26.22), this amounts to a compensating variation of $22.62 per hour. All off-diagonal taste parameters are negative and significant, revealing the tendency for individuals of all levels of education to remain in their birth regions.

Next, we use these estimates to recover the unconditional income distributions for each region and education group with the Kaplan-Meier procedure described in Section 2.2. Results are reported in Figures 1 and 2. In every case, the unconditional wage distribution lies below the observed distribution. Importantly, the correction for Roy sorting is generally larger for college graduates, who are more prone to migrate from their birth regions. We record the medians and 75^{th} percentiles of each of these distributions in Table 7. The median log-wage for a high-school graduate from the South Atlantic, for example, falls from 2.63 to 2.55. Defining the returns to a college education at the median to be the difference between the median of the college and high-school graduate log-wage distributions, we report those returns in Table 8. Returns are analogously defined at the 75^{th} percentile. In every region, the returns to a college education fall once we control for Roy sorting. On average, they fall from 42% to 18% at the median, and from 45% to 34% at the 75^{th} percentile. These results suggest that observed wage distributions, which are distorted by Roy sorting, seriously overstate the true returns to a college education, particularly for those in the heart of the wage distribution.

7. Conclusion

This paper considers nonparametric identification and estimation of a generalized multi-sector Roy model which includes a non-pecuniary component of utility associated with each alternative. Two identification results are established – one under a support condition and the other under a commonality/independence assumption. Estimation procedures based on both identification results are proposed, and their asymptotic properties are derived. The latter estimator is used to recover an estimate of the returns to a college education, controlling for different migration rates of high-school and college graduates. The results suggest that an estimate based on conditional distributions may overstate returns by more than a factor of two at the median. An application of our extreme quantile estimator to sorting across occupations, where individuals care about pecuniary returns and other job attributes (including fatality risk) yields similarly stark results. The wage-hedonic estimate of the value of a statistical life rises by a factor of three and becomes statistically significant. (Deleire and Timmins, 2007)

Our work here leaves many import areas for extensions and future research. In particular, it would be useful to explore how the presence of covariates would aide in achieving identification of our generalized Roy model, as has proven to be the case in the standard Roy model (i.e., might they enable us to relax the independence assumption in the unbounded support estimator). Furthermore, it would be useful to derive efficiency bounds for the non-pecuniary parameters to see if more efficient estimators than those proposed here can be constructed.

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Table 1: Monte Carlo Simulations
Minimum Order Statistic Estimator

		τ_2^1	τ_3^1	τ_1^2	τ_3^2	τ_1^3	τ_2^3
		-0.5	-0.2	-0.4	-0.6	-0.3	-0.1
	Baseline (N = 1,000)						
1	Mean	-0.542	-0.203	-0.408	-0.610	-0.316	-0.113
	Std Dev	0.031	0.002	0.006	0.007	0.011	0.009
	MSE	0.003	1.28×10^{-5}	1.03×10^{-4}	1.53×10^{-4}	3.63×10^{-4}	2.50×10^{-4}
	Baseline (N = 10,000)						
2	Mean	-0.510	-0.201	-0.402	-0.602	-0.303	-0.103
	Std Dev	0.007	4.06×10^{-4}	0.001	0.001	0.002	0.002
	MSE	1.40×10^{-4}	5.36×10^{-7}	4.68×10^{-6}	7.26×10^{-6}	1.71×10^{-5}	1.23×10^{-5}
	Baseline (N = 50,000)						
3	Mean	-0.503	-0.200	-0.400	-0.600	-0.301	-0.101
	Std Dev	0.002	1.45×10^{-4}	4.19×10^{-4}	5.00×10^{-4}	7.70×10^{-4}	6.51×10^{-4}
	MSE	1.79×10^{-5}	6.63×10^{-8}	5.54×10^{-7}	8.12×10^{-7}	2.02×10^{-6}	1.33×10^{-6}
	Unbounded Support (N = 10,000)						
4	Mean	-0.595	-0.173	-0.389	-0.564	-0.350	-0.184
	Std Dev	0.202	0.212	0.204	0.198	0.215	0.208
	MSE	0.050	0.046	0.042	0.040	0.048	0.050
	Unbounded Support (N = 50,000)						
5	Mean	-0.569	-0.177	-0.383	-0.572	-0.354	-0.161
	Std Dev	0.193	0.189	0.202	0.187	0.183	0.199
	MSE	0.042	0.036	0.041	0.036	0.036	0.043
	Measurement Error (N = 10,000)						
6	Mean	-0.668	-0.248	-0.502	-0.711	-0.439	-0.228
	Std Dev	0.062	0.048	0.050	0.055	0.057	0.056
	MSE	0.032	0.005	0.013	0.015	0.023	0.020
	Measurement Error (N = 50,000)						
7	Mean	-0.660	-0.248	-0.491	-0.697	-0.430	-0.223
	Std Dev	0.050	0.043	0.044	0.044	0.046	0.047
	MSE	0.028	0.004	0.010	0.011	0.019	0.017
	Correlated Wage Draws (N = 10,000)						
8	Mean	-0.516	-0.201	-0.402	-0.603	-0.305	-0.104
	Std Dev	0.011	3.89×10^{-4}	0.001	0.002	0.003	0.002
	MSE	3.78×10^{-4}	4.61×10^{-7}	5.38×10^{-6}	9.76×10^{-6}	2.96×10^{-5}	1.97×10^{-5}
	Correlated Wage Draws (N = 50,000)						
9	Mean	-0.506	-0.200	-0.401	-0.601	-0.302	-0.101
	Std Dev	0.004	1.40×10^{-4}	4.68×10^{-4}	5.76×10^{-4}	0.001	7.84×10^{-4}
	MSE	4.83×10^{-5}	5.54×10^{-8}	6.87×10^{-7}	1.02×10^{-6}	3.41×10^{-6}	2.12×10^{-6}

Table 2: Monte Carlo Simulations
Unbounded Support Estimator

		τ_2^1	τ_3^1	τ_1^2	τ_3^2	τ_1^3	τ_2^3
		-0.5	-0.2	-0.4	-0.6	-0.3	-0.1
				Baseline (N = 1,000)			
1	Mean	-0.731	-0.230	-0.441	-0.652	-0.404	-0.173
	Std Dev	0.444	0.332	0.422	0.544	0.323	0.156
	MSE	0.250	0.111	0.180	0.298	0.115	0.029
				Baseline (N = 10,000)			
2	Mean	-0.620	-0.199	-0.390	-0.584	-0.373	-0.171
	Std Dev	0.079	0.066	0.081	0.063	0.053	0.057
	MSE	0.021	0.004	0.007	0.004	0.008	0.008
				Baseline (N = 50,000)			
3	Mean	-0.614	-0.197	-0.381	-0.573	-0.375	-0.181
	Std Dev	0.047	0.033	0.043	0.032	0.029	0.042
	MSE	0.015	0.001	0.002	0.002	0.006	0.008
				Home Advantage (N = 10,000)			
4	Mean	-0.576	0.175	-0.103	-0.218	-0.290	-0.303
	Std Dev	0.163	0.415	0.347	0.460	0.133	0.269
	MSE	0.032	0.313	0.208	0.358	0.018	0.114
				Home Advantage (N = 50,000)			
5	Mean	-0.510	-0.101	-0.309	-0.426	-0.354	-0.193
	Std Dev	0.073	0.204	0.156	0.157	0.112	0.128
	MSE	0.005	0.078	0.032	0.055	0.015	0.025
				Correlated Wage Draws (N = 10,000)			
6	Mean	-1.362	-0.441	-0.959	-1.367	-0.814	-0.356
	Std Dev	0.487	0.322	0.625	1.032	0.657	0.176
	MSE	0.980	0.162	0.702	1.652	0.696	0.096
				Correlated Wage Draws (N = 50,000)			
7	Mean	-1.368	-0.419	-0.950	-1.268	-0.706	-0.421
	Std Dev	0.290	0.057	0.155	0.107	0.171	0.135
	MSE	0.838	0.051	0.327	0.458	0.194	0.121
				Measurement Error (N = 10,000)			
8	Mean	-0.643	-0.206	-0.401	-0.601	-0.382	-0.176
	Std Dev	0.077	0.068	0.083	0.066	0.055	0.060
	MSE	0.026	0.005	0.007	0.004	0.010	0.009
				Measurement Error (N = 50,000)			
9	Mean	-0.632	-0.202	-0.395	-0.592	-0.385	-0.187
	Std Dev	0.052	0.035	0.046	0.034	0.029	0.042
	MSE	0.020	0.001	0.002	0.001	0.008	0.009

Table 3: Mobility Matrix, High School Graduates
2000 US Census, 5% IPUMS Random Sample

		Destination Region								
		New England	Mid-Atlantic	E North Central	W North Central	South Atlantic	E South Central	W South Central	Mountain	Pacific
Birth Region	New England	0.806	0.035	0.011	0.005	0.083	0.008	0.012	0.017	0.024
	Mid-Atlantic	0.016	0.809	0.011	0.005	0.100	0.008	0.012	0.019	0.020
	E North Central	0.004	0.013	0.766	0.024	0.072	0.034	0.026	0.031	0.031
	W North Central	0.002	0.006	0.053	0.770	0.028	0.011	0.034	0.053	0.043
	South Atlantic	0.008	0.036	0.016	0.007	0.863	0.029	0.017	0.010	0.015
	E South Central	0.003	0.009	0.065	0.009	0.082	0.776	0.032	0.008	0.015
	W South Central	0.002	0.007	0.022	0.025	0.035	0.022	0.814	0.030	0.043
	Mountain	0.004	0.009	0.017	0.032	0.027	0.010	0.049	0.747	0.105
	Pacific	0.006	0.011	0.021	0.027	0.035	0.012	0.040	0.097	0.750

Table 4: Mobility Matrix, College Graduates
2000 US Census, 5% IPUMS Random Sample

		Destination Region								
		New England	Mid-Atlantic	E North Central	W North Central	South Atlantic	E South Central	W South Central	Mountain	Pacific
Birth Region	New England	0.619	0.073	0.033	0.012	0.129	0.012	0.024	0.029	0.070
	Mid-Atlantic	0.060	0.546	0.053	0.012	0.182	0.014	0.030	0.035	0.070
	E North Central	0.017	0.033	0.600	0.040	0.112	0.028	0.043	0.051	0.077
	W North Central	0.010	0.020	0.091	0.537	0.074	0.018	0.075	0.085	0.090
	South Atlantic	0.020	0.049	0.046	0.014	0.709	0.043	0.043	0.028	0.049
	E South Central	0.009	0.019	0.068	0.017	0.189	0.560	0.076	0.024	0.038
	W South Central	0.008	0.016	0.032	0.028	0.078	0.032	0.696	0.047	0.064
	Mountain	0.012	0.021	0.037	0.040	0.062	0.014	0.074	0.562	0.179
	Pacific	0.014	0.021	0.032	0.022	0.061	0.012	0.042	0.091	0.706

Table 5
Taste Parameter Estimates
High School Graduates

		Destination Region								
		New England	Mid-Atlantic	E North Central	W North Central	South Atlantic	E South Central	W South Central	Mountain	Pacific
Birth Region	New England	0	-0.863 (0.05)	-0.562 (0.05)	-0.899 (0.15)	-0.967 (0.06)	-0.972 (0.10)	-0.865 (0.08)	-0.686 (0.05)	-0.948 (0.08)
	Mid-Atlantic	-0.803 (0.06)	0	-0.877 (0.08)	-1.092 (0.13)	-0.902 (0.05)	-0.988 (0.09)	-0.921 (0.08)	-0.324 (0.03)	-0.976 (0.09)
	E North Central	-1.036 (0.17)	-0.895 (0.18)	0	-0.960 (0.12)	-1.203 (0.08)	-1.055 (0.08)	-1.011 (0.07)	-0.685 (0.06)	-1.211 (0.10)
	W North Central	-0.390 (0.05)	-0.823 (0.07)	-0.631 (0.06)	0	-1.119 (0.11)	-0.830 (0.13)	-0.815 (0.10)	-0.788 (0.05)	-0.899 (0.09)
	South Atlantic	-0.847 (0.08)	-0.742 (0.08)	-0.727 (0.05)	-1.069 (0.09)	0	-0.704 (0.06)	-0.808 (0.06)	-0.801 (0.07)	-0.528 (0.08)
	E South Central	-0.405 (0.04)	-0.585 (0.04)	-0.445 (0.06)	-0.433 (0.03)	-0.874 (0.07)	0	-0.816 (0.06)	-0.995 (0.09)	-0.932 (0.17)
	W South Central	-0.763 (0.08)	-1.231 (0.15)	-0.682 (0.09)	-1.009 (0.09)	-1.108 (0.09)	-0.969 (0.08)	0	-0.699 (0.07)	-1.075 (0.08)
	Mountain	-0.878 (0.15)	-0.555 (0.05)	-0.644 (0.05)	-0.774 (0.10)	-0.952 (0.07)	-0.774 (0.11)	-0.859 (0.06)	0	-0.784 (0.05)
	Pacific	-0.710 (0.18)	-0.804 (0.07)	-0.550 (0.05)	-0.806 (0.06)	-0.775 (0.06)	-0.994 (0.10)	-0.648 (0.05)	-0.305 (0.03)	0

Table 6
Taste Parameter Estimates
College Graduates

		Destination Region								
		New England	Mid-Atlantic	E North Central	W North Central	South Atlantic	E South Central	W South Central	Mountain	Pacific
Birth Region	New England	0	-0.745 (0.02)	-0.578 (0.01)	-1.361 (0.03)	-0.540 (0.01)	-1.068 (0.03)	-0.492 (0.01)	-0.435 (0.01)	-0.448 (0.01)
	Mid-Atlantic	-0.614 (0.01)	0	-0.602 (0.01)	-0.351 (0.01)	-0.347 (0.01)	-1.004 (0.04)	-0.700 (0.02)	-0.294 (0.01)	-0.622 (0.01)
	E North Central	-0.562 (0.02)	-1.087 (0.03)	0	-0.798 (0.03)	-0.515 (0.01)	-0.932 (0.02)	-0.647 (0.02)	-0.679 (0.02)	-0.341 (0.01)
	W North Central	-0.475 (0.01)	-0.672 (0.02)	-0.420 (0.01)	0	-0.908 (0.02)	-1.238 (0.05)	-0.861 (0.02)	-0.573 (0.01)	-0.647 (0.02)
	South Atlantic	-1.097 (0.02)	-0.975 (0.03)	-0.821 (0.02)	-1.303 (0.03)	0	-0.958 (0.02)	-0.890 (0.02)	-0.858 (0.02)	-0.835 (0.02)
	E South Central	-1.129 (0.04)	-1.063 (0.03)	-0.631 (0.01)	-0.450 (0.01)	-0.513 (0.01)	0	-0.646 (0.02)	-0.924 (0.02)	-0.803 (0.02)
	W South Central	-0.781 (0.02)	-1.330 (0.04)	-0.972 (0.03)	-0.632 (0.02)	-0.630 (0.01)	-0.136 (0.00)	0	-0.823 (0.02)	-0.693 (0.02)
	Mountain	-1.106 (0.03)	-1.186 (0.04)	-0.663 (0.01)	-0.840 (0.02)	-0.542 (0.01)	-0.898 (0.02)	-0.666 (0.02)	0	-0.442 (0.01)
	Pacific	-1.253 (0.05)	-1.125 (0.08)	-0.861 (0.02)	-0.809 (0.02)	-0.654 (0.02)	-0.499 (0.01)	-0.761 (0.02)	-0.443 (0.01)	0

Table 7
Log Wages by Education and Region
2000 US Census, 5% IPUMS Random Sample
Raw Data and Corrected for Spatial Selection

	High School				College			
	Median		75^{th} Percentile		Median		75^{th} Percentile	
	Raw Data	Selection Corrected	Raw Data	Selection Corrected	Raw Data	Selection Corrected	Raw Data	Selection Corrected
New England	2.77	2.67	3.03	2.97	3.20	2.87	3.51	3.32
Mid-Atlantic	2.76	2.65	3.04	2.97	3.24	2.76	3.56	3.32
E. North Central	2.74	2.61	3.03	2.96	3.14	2.76	3.44	3.25
W. North Central	2.63	2.50	2.92	2.83	2.98	2.53	3.31	3.05
South Atlantic	2.63	2.55	2.93	2.87	3.09	2.83	3.43	3.3
E. South Central	2.60	2.48	2.91	2.82	3.01	2.59	3.36	3.11
W. South Central	2.60	2.48	2.92	2.84	3.04	2.78	3.39	3.24
Mountain	2.67	2.49	2.96	2.86	3.04	2.63	3.36	3.14
Pacific	2.79	2.63	3.07	2.97	3.22	2.97	3.52	3.4
Average	*2.69*	*2.56*	*2.98*	*2.90*	*3.11*	*2.75*	*3.43*	*3.24*

Table 8
Percentage Returns to College Education

	Median		75th Percentile	
	Raw Data	Selection Corrected	Raw Data	Selection Corrected
New England	0.43	0.20	0.48	0.35
Mid-Atlantic	0.48	0.11	0.52	0.35
E. North Central	0.40	0.15	0.41	0.29
W. North Central	0.35	0.03	0.39	0.22
South Atlantic	0.46	0.28	0.50	0.43
E. South Central	0.41	0.11	0.45	0.29
W. South Central	0.44	0.30	0.47	0.40
Mountain	0.37	0.14	0.40	0.28
Pacific	0.43	0.34	0.45	0.43
Average	*0.42*	*0.18*	*0.45*	*0.34*

Figure 1: High School Graduates
Conditional and Unconditional Log Wage Distributions by Destination Region

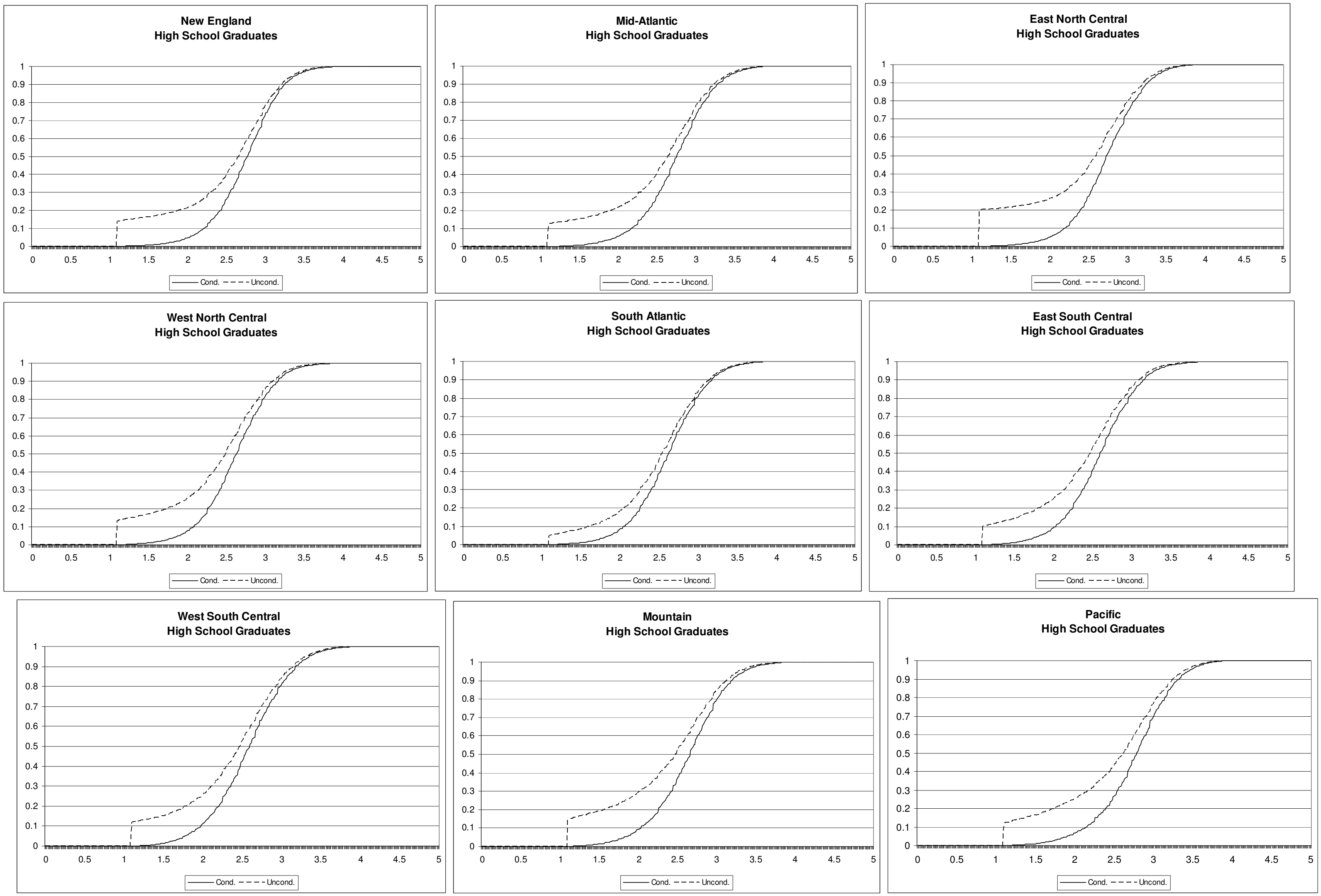

Figure 2: College Graduates
Conditional and Unconditional Log Wage Distributions by Destination Region