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# INFORMATION DISCLOSURE AND UNRAVELING IN MATCHING MARKETS

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**ABSTRACT**

This paper explores information disclosure in matching markets, e.g., the informativeness of transcripts given out by universities. We show that the same, "benchmark," amount of information is disclosed in essentially all equilibria. We then demonstrate that if universities disclose the benchmark amount of information, students and employers will not find it profitable to contract early; if they disclose more, unraveling will occur.

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*When recruiters call me up and ask for the three best people, I tell them, "No! I will give you the names of the six best."*

Professor Robert J. Gordon, Director of Graduate Placement,
Northwestern University, Department of Economics

*Harvard wants high schools to give class rank, but high schools do not want to.*

Senior Harvard official

# 1 Introduction

Labor market institutions often suppress some information about job candidates. For example, students at the Stanford Graduate School of Business (GSB) are graded on a curve, resulting in transcripts that very accurately reflect students' performance. These transcripts, however, are not revealed to potential employers:

> *The GSB has no policy on grade disclosure; your grades belong to you and it is your right to use them as you wish. Stanford's nondisclosure norm among MBA students, however, has existed for nearly 40 years.*[1]

Harvard and Chicago Business Schools have similar norms. High profile examples from other areas include Yale Law School, where first semester grades are Credit/No Credit. Stanford Medical School conceals from residency programs a part of the student's record. MIT official undergraduate transcripts available to graduate schools and potential employers also suppress available information: they contain only full letter grades, while internal transcripts distinguish between such grades as B+ and B-. Nearly 40 percent of high schools do not disclose class rank to colleges, even though some of them maintain it internally and report it when "absolutely necessary."[2] In fact, more than 90 percent of private non-parochial schools do not disclose class rank (National Association for College Admission Counseling, 2005).

Concealing information need not require deliberate actions on the part of schools: if revealing full information is not in the interests of a school, it can add noise to transcripts by tolerating (or encouraging) grading policies that make grades less informative. Unless the dean clearly communicates expectations about grading standards, professors are likely to have different ideas regarding the appropriate grade for the average performance. Lack of consistent grading standards adds noise to transcripts, because luck of the draw determines the grading standard adopted by an instructor. A school can reduce this sort of noise by reporting an average grade in each class alongside the grade received by a student or by mandating the use of a forced curve in large classes. Inflated grades could also reduce informativeness of transcripts, perhaps unintentionally. For instance, after years of grade inflation, close to 50% of grades in undergraduate classes at Harvard College are A and A-, often erasing the differences between the good and the great. Figure I suggests that the

[1] https://www.gsb.stanford.edu/mba/academics/learning_methods.html, accessed March 22, 2007.
[2] "Schools Avoid Class Ranking, Vexing Colleges," *New York Times,* March 5, 2006.

informativeness of grades at Harvard, as measured by their entropy, has declined in recent years as the percentage of As and A-s has risen.[3]

All of the practices described above are similar from the employer's perspective. Refusal to reveal part of the student record, inconsistent grading among instructors, or coarse transcripts are all "noise" that reduces the ability of potential employers to correctly judge the ability of students. The above examples suggest that at least some schools are either indifferent regarding how much information to reveal or prefer to conceal some information about the ability of their students: otherwise, they would try to implement policies that minimize the amount of noise in their transcripts.

There is an alternative channel through which information can be suppressed. Each semester before graduation a student's transcript becomes longer and hence more informative. Even if schools make transcripts as informative as possible, students and employers may choose to contract significantly before graduation, thus leading to incomplete information disclosure. We say that unraveling occurs when the timing of contracting reduces the amount of available information in a dynamic setting. Early Action and Early Decision admission programs at many selective colleges (Avery, Fairbanks, and Zeckhauser, 2003) are examples of unraveling. These programs allow high school students to submit their applications in the Fall of their senior year, and admission decisions are made before Fall semester grades are available (in contrast to the regular admission process, which takes Fall semester grades into account). The market for law clerks (Avery, Jolls, Posner, and Roth, 2001) is another dramatic example of uraveling. Avery et al. (2001) report that interviews for clerkship positions are held at the beginning of the second year of three-year law school programs, when only a third of the students' grades are available. Clearly, a lot of information is withheld. Roth and Xing (1994) describe several other markets in which the timing of transactions has unraveled.

This paper shows that there is a remarkably close connection between the equilibrium (or "benchmark") amount of information revealed by schools in the static environment and the incentive for students and employers to unravel in the dynamic environment. If schools disclose the benchmark amount of information, students and employers will not find it profitable to contract early; if they disclose more, unraveling will occur.

## 2 Information Disclosure in a Static Environment

We begin our analysis in the static model. Schools evaluate students and give them transcripts. Subsequently, these transcripts are used by outsiders (e.g., employers, professional schools, clerkship positions) in their hiring decisions. We assume that the ability of each student and the distribution of students among schools are given exogenously.[4] We also assume that wages offered by employers

[3]Some of Harvard's policies actually encourage grade inflation. An instructor who gives an F or a D is asked to write a note explaining the reasons for the poor performance of a student. In contrast, instructors who give many As are not asked to explains their reasons.

[4]The effects of allowing agents in a matching market to invest in their "quality" are analyzed in Cole, Mailath, and Postlewaite (1992, 2001), Peters and Siow (2002), Hopkins (2005), and Peters (2005).

are inflexible, and so the supply of placement slots of a given desirability is exogenously fixed.[5]

The ability of students is perfectly observed by schools but not by outsiders. Each school decides how much information to reveal in its transcripts in order to maximize the average desirability of placement of its alumni. Outsiders use transcripts to infer the expected ability of students and rank them solely according to their expected ability. The desirability of each position is common knowledge, and students rank positions based on desirability. Thus, all students have the same preferences and so do all recruiters.

The key feature of our model is that by introducing noise in students' transcripts, a school can change the distribution of desirabilities of positions to which its students are matched in the job market. Consider, for instance, the competition for admission to medical schools. Introducing noise into transcripts may enable a college to increase placement into moderately desirable medical schools at the cost of reducing the number of students placed at top medical schools. The aggregate distribution of positions in the job market does not depend on the transcripts given out by schools, and so the total desirability of placements is constant. However, as we will see in the next section, in a broad range of situations noise is a necessary feature of transcripts given out in equilibrium.[6]

Consider a population of students. The ability of each student is a real number $a$ in the interval $[a_L, a_H]$. Each student attends one of $I$ schools. The distribution $\lambda_i(\cdot)$ of ability levels at each school $i$ is continuous, exogenous, and commonly known. Without loss of generality, we assume that schools observe the true abilities of their students.[7] Each school decides how much of this information to reveal, i.e., how precise to make its transcripts. A school can make transcripts completely informative, revealing the ability level of each student, or it can make them completely uninformative, or anything in between.

Formally, a school chooses a *transcript structure*, which is a mapping from the abilities of students into *expected abilities* $\hat{a} \in [a_L, a_H]$. This mapping may be stochastic, i.e., for each ability $a$ there can be a probability distribution over the set of expected abilities $\hat{a}$ that a student of ability $a$ can get. However, this mapping has to be statistically correct, in the following sense: the average ability of students "labeled" with expected ability $\hat{a}$ in school $i$ has to equal $\hat{a}$.

**Definition 1** *A transcript structure is a function $F(\cdot|\cdot)$, where $F(\hat{a}|a)$ is a probability distribution with which a student of ability $a \in [a_L, a_H]$ is mapped to expected ability $\hat{a}$, such that the average ability of students labeled with expected ability $\hat{a}$ is equal to $\hat{a}$.*[8]

[5]One can show that our results remain valid when the wages of some (or all) firms are flexible. Moreover, if the wages of all firms are flexible, then under full information revelation the wage schedule will be convex in ability (see Sattinger, 1993, for a survey of the literature on assignment models with flexible wages), and therefore, as we explain in the discussion following Theorem 1, full information revelation by all schools will be an equilibrium outcome.

[6]Several recent papers study strategic disclosure of information in a variety of environments (Matthews and Postlewaite, 1985; Okuno-Fujiwara, Postlewaite, and Suzumura, 1990; Lizzeri, 1999; Chakraborty and Harbaugh, 2005). The distinguishing features of our setup are the general equilibrium approach and the competitive nature of the market.

[7]Suppose nobody observes the true ability, but each school observes a signal regarding the true ability of each of its students. Based on this signal a school can form expectation about a student's ability. All results in the paper continue to hold if instead of "true ability" we use "expected ability based on information available to schools."

[8]This definition is very similar to the definition of "information structure" in Bergemann and Pesendorfer (2001).

Essentially, the definition says that schools give out grades and transcripts to students using some commonly known grading scheme, and then employers can back out each student's expected ability based on his or her transcript, the grading scheme, and the distribution of student abilities in the school. We assume that schools can commit to their transcript structure. This is not a critical assumption.[9] What *is* critical is that employers know the distribution of transcripts given out by a school, as well as the distribution of student abilities there. Employers know the distribution of transcripts if they receive applications from many candidates from a given school. Likewise, the distribution of student abilities in large schools is known to recruiters fairly well, at least if it does not change drastically year-to-year. We rule out the possibility that a school can "fool" employers into thinking that it has better students than it actually does by giving out too many good grades (as in Chan, Li, and Suen, 2005), and focus solely on information compression.

After schools announce transcript structures and announce expected abilities of their students, students and positions are matched. On one side of the market there is a population of students. On the other side there is a set of positions. The desirability of each position, $q \in [q_L, q_H]$, is common knowledge. The distribution $\mu(\cdot)$ of position desirabilities is continuous, exogenous, commonly known, and has positive density on $[q_L, q_H]$. The mass of positions is equal to the mass of students.[10] Students rank positions by desirability, and employers rank students by expected ability.[11] The resulting rankings induce a unique (up to permutations of equally desirable positions) assortative stable matching between students and positions.

Each school selects a transcript structure to maximize the total desirability of positions obtained by its students. Each school is small relative to the labor market and is a "price taker"—its actions have no effect on the placement of students of a given expected ability.[12]

The following series of examples illustrates the model. In these examples, we discuss equilibrium information disclosure—the concept we formally define in the following section.

**Example 1.** Student abilities at each school are distributed uniformly on $[0, 100]$ and position desirabilities are also distributed uniformly on $[0, 100]$. If all schools fully reveal student abilities (i.e., set $\hat{a} \equiv a$), the resulting mapping $Q$ from expected abilities to position desirabilities is linear

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That paper, however, considers information disclosure in a very different environment—a single-seller, single-object auction, whereas we consider a matching market.

[9]If schools could not commit to their transcript structures, equilibrium information disclosure that we explore in the following section would still remain an equilibrium outcome of the resulting cheap-talk game (see Crawford and Sobel, 1982, for a formal analysis of cheap-talk games). Of course, the cheap-talk game has many other equilibria.

[10]This is not a restrictive assumption, because unemployment can be viewed as a position of the lowest desirability, and because if the mass of positions is greater than the total mass of students, the same subset of positions gets assigned a student under any information disclosure.

[11]As long as the output of a worker is a function of his ability, we can find a rescaling of ability such that a particular firm is indifferent between having a worker of ability $a_0$ for sure and a worker of uncertain ability with expectation $a_0$. However, we do have to assume that this rescaling is the same for all firms.

[12]This can be reconciled with a finite number of schools by using the standard general equilibrium approach—assume that there are $I$ school types and an infinite number of schools of each type. Technically, different schools of the same type could select different transcript structures. However, in any equilibrium where that could happen, the average transcript structure of schools of a given type would also be an optimal transcript structure for a school of that type to use, and so there exists another equilibrium with the same aggregate properties, in which all schools of the same type behave identically.

($Q(\hat{a}) = \hat{a}$) and no school can benefit by deviating. Thus, fully informative transcripts form an equilibrium.

**Example 2.** Now suppose that at a half of all schools student abilities are distributed uniformly on $[0, 100]$, while the other half has a more able population: student abilities are distributed uniformly on $[50, 100]$. There is a mass .5 of students at each type of school. There is also a mass 1 of positions, distributed uniformly on $[0, 100]$, as before. If all schools fully reveal student abilities, the resulting mapping from expected abilities to desirabilities has two linear pieces:

$$Q(\hat{a}) = \begin{cases} \frac{\hat{a}}{2}, & \text{for } \hat{a} \leq 50 \\ \frac{3\hat{a}}{2} - 50, & \text{for } \hat{a} \geq 50. \end{cases}$$

For instance, a student with expected ability 50 is in the 25th percentile of the student population, and hence gets a job of the 25th desirability percentile. Figure II illustrates this desirability mapping $Q$. Note that again, no school can benefit by deviating and suppressing some information: if a "better than average" school mixes some students of different abilities together, it gets exactly the same payoff as without mixing, while if an "average" school mixes students with abilities above 50 and below 50 together, it gets a strictly lower payoff than without mixing.

**Example 3.** Finally, suppose that there is an "oversupply" of less able students: at a half of all schools student abilities are distributed uniformly on $[0, 100]$, while the other half has a *less* able population: student abilities are distributed uniformly on $[0, 50]$. As before, there is a mass .5 of students at each type of school and a mass 1 of positions, distributed uniformly on $[0, 100]$. Suppose each school reveals student abilities truthfully. Then the resulting mapping (Figure III) is

$$Q(\hat{a}) = \begin{cases} \frac{3\hat{a}}{2}, & \text{for } \hat{a} \leq 50 \\ \frac{\hat{a}}{2} + 50, & \text{for } \hat{a} \geq 50. \end{cases}$$

Now, consider a school that contains students of all true abilities from 0 to 100. The average position desirability obtained by its students is $\frac{1}{2}(\frac{75}{2} + \frac{175}{2}) = 62.5$. Suppose now that the school adopts a "no grade disclosure" policy—every student gets the same empty transcript, looks the same to employers, and therefore has the expected ability 50. Then the average position desirability obtained by the school's students increases to 75! Therefore, full revelation is not an equilibrium in this example. What is?

Suppose each "worse than average" school reveals information truthfully, while each "average" school mixes students in such a way that the distribution of expected abilities there is the one plotted in Figure IV(b): one third of expected abilities are distributed uniformly on $[0, 50]$ and the remaining two thirds are distributed uniformly on $[50, 75]$.[13] Then the *aggregate* distribution of expected abilities in the population is uniform on $[0, 75]$ and the corresponding desirability mapping

[13]Note that this distribution second-order stochastically dominates the distribution of true abilities at the "average" school, and therefore there exists a mixing of students generating such distribution of expected abilities.

$Q'$ (plotted as the dotted line in Figure III) is linear:

$$Q'(\hat{a}) = \begin{cases} \frac{4\hat{a}}{3}, & \text{for } \hat{a} \leq 75 \\ 100, & \text{for } \hat{a} \geq 75. \end{cases}$$

This amount of information disclosure is an equilibrium.

# 3 Equilibrium Information Disclosure

In our setup, the behavior of students and positions is straightforward—they get matched to the agents of the highest quality available to them on the other side of the market (in the next section we give them some flexibility by allowing early contracting). Thus, we focus on the actions of schools and the resulting disclosure of information.

Let $\omega = (F_1, F_2, \ldots, F_I)$ be a profile of transcript structures and let $G$ be the aggregate distribution of expected abilities generated by $\omega$.

**Definition 2** *We say that function $Q(\cdot)$ on $[a_L, a_H]$ is a desirability mapping corresponding to profile $\omega$ if, given that schools give out grades in accordance with $\omega$, the expected desirability of a position matched with a student labeled with expected ability $\hat{a}$ is equal to $Q(\hat{a})$.*

Thus, $Q(a_L) = q_L$, $Q(a_H) = q_H$, and $Q(\hat{a}) = \mu^{-1}(G(\hat{a}))$ if $G$ is continuous at $\hat{a}$. If $G$ is discontinuous at $\hat{a}$, let $\underline{q} = \mu^{-1}(\lim_{a \to \hat{a}-} G(a))$ and let $\overline{q} = \mu^{-1}(\lim_{a \to \hat{a}+} G(a))$. Then $Q(\hat{a}) = \frac{\int_{\underline{q}}^{\overline{q}} q d\mu(q)}{\mu(\overline{q}) - \mu(\underline{q})}$.

**Definition 3** *We say that $\omega$ is an equilibrium profile of transcript structures if each school maximizes the average desirability of placements of its students given the desirability mapping $Q(\cdot)$ corresponding to $\omega$. I.e., if $G_i(\cdot)$ is the distribution of expected abilities of students in school $i$ under the transcript structure $F_i$, and $G_i'(\cdot)$ is the distribution of expected abilities under some alternative transcript structure $F_i'$, then $\int_{a_L}^{a_H} Q(\hat{a}) dG_i(\hat{a}) \geq \int_{a_L}^{a_H} Q(\hat{a}) dG_i'(\hat{a})$.*

Before we can state the main result of this section, we need an additional, somewhat technical definition.

**Definition 4** *Let $\hat{a}_L$ be the lowest and $\hat{a}_H$ the highest expected ability levels produced in an equilibrium. Then we say that the equilibrium is connected if for any point $\hat{a} \in (\hat{a}_L, \hat{a}_H)$ there exists a school that produces students of all expected abilities in some $\epsilon$-neighborhood of $\hat{a}$.*

Connectedness is a mild restriction. Indeed, if at least one school gives out some transcripts with the worst and the best possible expected abilities, and everything in between, this restriction is satisfied. Hence, if we observe a school (e.g., UC Berkeley) that places students in the entire spectrum of positions, then connectedness holds. We discuss this restriction in more detail in Appendix C and provide some sufficient conditions for the existence of a connected equilibrium.

Finally, we assume that if all abilities are revealed truthfully, the corresponding desirability mapping $Q_T(\cdot)$ does not switch from convexity to concavity infinitely often, i.e., there exists a finite increasing sequence of ability levels $a_i$, starting at the lowest and ending at the highest true ability, such that $Q_T(\cdot)$ is convex or concave on each interval $[a_i, a_{i+1}]$.

We are now ready to state and prove the main result of this section. It says that in all connected equilibria desirability mappings (and therefore the aggregate distributions of expected abilities) are the same. In fact, they do not even depend on how students are assigned to schools—only the aggregate distribution of student abilities and the distribution of position desirabilities matter.[14]

**Theorem 1** *Suppose there is a connected equilibrium with desirability mapping $Q_1(\cdot)$. Suppose students are reshuffled among schools so that the aggregate distribution of student abilities remains the same, but the distributions of abilities within schools possibly change, and suppose there is a new connected equilibrium with desirability mapping $Q_2(\cdot)$. Then for any $\hat{a}$, $Q_1(\hat{a}) = Q_2(\hat{a})$, i.e., the desirability mappings coincide. Equivalently, the aggregate distribution of expected abilities in any connected equilibrium is uniquely determined by the distribution of position desirabilities and the aggregate distribution of true abilities, and does not depend on how these abilities are divided among schools.*

The proof of Theorem 1 proceeds in several steps. First, we show that in any equilibrium, desirability mapping $Q$ is an invertible, monotonically increasing, continuous function, i.e., no positive mass of students receives the same expected ability. Next, we show that in any connected equilibrium, the desirability mapping must be convex—otherwise, as in Example 3, at least one school will be able to improve its payoff by mixing some students together. On the other hand, if a school does mix students on some interval, the desirability schedule there cannot be strictly convex (and therefore has to be linear): otherwise, the school would be better off by not mixing the students. Also, we show that the lowest expected ability produced in equilibrium has to be the same as the lowest true ability. The final, and most involved part of the proof, shows that there can only exist one desirability mapping satisfying the above properties for a given pair of distributions of desirabilities and true abilities. This part relies on the assumption that $Q_T$ does not switch from convexity to concavity infinitely often on $[a_L, a_H]$ and proceeds by induction on the number of its inflection points. Along the way, the proof shows how to construct the unique equilibrium desirability mapping and describes what happens in various special cases: For instance, if $Q_T$ is convex, then $Q \equiv Q_T$. If $Q_T$ is concave, then $Q$ is linear on $[a_L, \hat{a}_H]$ for some $\hat{a}_H$ and no students have expected abilities above $\hat{a}_H$ in equilibrium. If $Q_T$ is S-shaped, with inflection point $\hat{a}_i$, then in equilibrium there will be "information compression at the top"—up to some level $\hat{a}_* \leq \hat{a}_i$, abilities will be revealed truthfully and so $Q$ and $Q_T$ will coincide; above $\hat{a}_*$, students of different abilities will be mixed together and $Q$ will be linear. Appendix A makes all these statements formal and gives the full proof of Theorem 1.

[14] Of course, if there existed only one school, and all students went there, this school would be indifferent between all possible amounts of information disclosure. This situation, however, would violate our assumption of price-taking behavior by the schools.

Hence, the same amount of information is disclosed in all connected equilibria. We will call this the *benchmark amount of information*: the amount that is disclosed in equilibrium when schools can release as little or as much information about their students as they want. Before proceeding further, we give a definition that makes the words "amount of information disclosure" precise. Note that if schools introduce *more* noise in their grades, the resulting distribution of expected abilities gets *compressed* and its variance *decreases*. This leads to a natural partial ordering on the set of profiles of transcript structures.

**Definition 5** *Profile of transcript structures $\omega$ is more informative than profile of transcript structures $\omega'$ if distribution $G$ of expected abilities generated by $\omega$ is second-order stochastically dominated by distribution $G'$ of expected abilities generated by $\omega'$.*

This partial ordering has two extreme elements: the completely uninformative profile, which has zero variance, and the profile revealing all student abilities, which has the highest possible variance. Also, it is clear that a more informative profile has a higher variance than a less informative one, since the former is a mean-preserving spread of the latter.

The last result of this section is another corollary of Theorem 1. It says that if truthful revelation of abilities is an equilibrium, then there are no other connected equilibria.

**Corollary 1** *Suppose there are two connected equilibria in a market, and one of them is fully informative. Then the other one also has to be fully informative, and therefore the same.*

**Proof.** By Theorem 1, the desirability mappings of these two equilibria have to be the same. Therefore, the distributions of expected abilities generated in these equilibria also have to be identical (since they are uniquely determined by the mapping and the distribution of position desirability). But the fully informative equilibrium is strictly more informative than any other one, and so the second equilibrium also has to be fully informative, and therefore the same. ■

We conclude this section with brief comments on efficiency implications of information suppression. Our assumptions are insufficient to make unambiguous inferences about efficiency. Indeed, if higher student ability and higher position desirability are complements, then positive assortative matching is efficient, and therefore noisy transcripts will lead to a less efficient allocation of talent than fully informative ones. If, however, they are substitutes, then negative assortative matching is efficient, and suppressing information will in fact lead to a more efficient allocation. Thus, efficiency implications of information suppression may be different for different markets.

Another dimension potentially important for evaluating efficiency is investment in human capital. In our model, a student's ability is exogenously fixed. If learning entails costly effort, noisy transcripts reduce the effort of at least some students (Becker, 1982). However, the efficiency loss may be small, because the loss in human capital is partially compensated by saved effort. Moreover, if signaling high ability is merely a ticket to high-rent jobs, then noisy transcripts may in fact be welfare-improving.

## 4 Unraveling

Sections 2 and 3 analyze information disclosure in a static framework. In this section, we take the actions of schools as given, but add a time dimension to the model: students and positions can decide when to sign employment contracts. We show that there is a strong connection between the static concept of "benchmark information disclosure" and an inherently dynamic phenomenon that frequently occurs in matching markets—"unraveling," or "early contracting," i.e., contracting between students and positions before full information about the former is available. Examples of early contracting include Early Action and Early Decision admission programs at many selective colleges, which allow students to apply before Fall semester grades of their Senior year at high school are available (Avery, Fairbanks, and Zeckhauser, 2003), the market for federal judicial law clerks, where judges interview candidates two years prior to the beginning of the clerkships (Avery, Jolls, Posner, and Roth, 2001), and many others (Roth and Xing, 1994).

A frequently stated reason for early contracting is insurance: a student may prefer to contract early with a mediocre firm to avoid the possibility of being matched with a really bad firm in case of a negative shock in the future (Li and Rosen, 1998; Li and Suen, 2000; Suen, 2000).[15] We consider this explanation in light of our model and establish a close, albeit not obvious, connection between information disclosure and unraveling. To compute the benchmark amount of information disclosure, one only needs to know the distribution of ability in the population and the distribution of job desirability. It is easy to check that in situations where unraveling occurs due to insurance reasons, waiting till all information is revealed will lead to the disclosure of more than the benchmark amount of information. This is not a coincidence. Theorems 2 and 3 show that if the benchmark amount of information is revealed, no unraveling occurs. Consequently, if schools can control the amount of information disclosed to potential employers, the insurance reason for unraveling disappears. The intuition is simple: in equilibrium, due to the convexity of desirability mapping $Q(\cdot)$, the expected position desirability that a student will get tomorrow is higher than the position desirability that he could get today.[16]

It may seem surprising that there is no unraveling under the benchmark amount of information disclosure—after all, imagine all positions have similar desirability except for a few that are terrible, e.g., unemployment. Then one might think that students would be eager to sign contracts earlier to avoid this outcome. However, as the following example shows, this does not happen. What happens instead is that the benchmark distribution of transcripts "mimics" the distribution of desirability—a small group of students gets very bad transcripts, and the rest get compressed transcripts with little information beyond being much better than the bad transcript.

**Example 4.** Suppose mass .8 of position desirabilities is distributed uniformly on $[80, 100]$

[15] Of course, this is not the only possible reason. Unraveling can also occur because of the use of an unstable matching mechanism, exercise of market power, small numbers of participants in a matching market, and other strategic reasons (Roth and Xing, 1994; Avery, Fairbanks, and Zeckhauser, 2003).

[16] Our arguments rely on the assumption that not only ordinal, but also cardinal preferences of schools and students over positions coincide. Otherwise, unraveling may occur even if the benchmark amount of information is disclosed.

("good jobs") and mass .2 of position desirabilities is distributed uniformly on $[0, 80]$ ("bad jobs"). Suppose also that student abilities in each school are distributed uniformly on $[0, 100]$, and the total mass of students is 1 (Figure V).

First, note that it is not an equilibrium for all schools to lump all students into one category. If they do, then a school can profitably deviate by separating a small fraction of the worst students into a new category. Second, providing fully informative signals is not an equilibrium either—the resulting desirability mapping is concave, and so schools can benefit from mixing students (Figure VI). So, what is the benchmark amount of information disclosure in this market?

It turns out that the desirability mapping corresponding to the benchmark amount of information disclosure in this example is linear on the relevant range:

$$Q'(\hat{a}) = \begin{cases} \frac{8}{5}\hat{a}, & \text{for } \hat{a} \leq 62.5 \\ 100, & \text{for } \hat{a} \geq 62.5 \end{cases}$$

and the corresponding distribution of expected abilities, $G$, mimics the distribution of position desirabilities:

$$G(\hat{a}) = \begin{cases} \frac{1}{250}\hat{a}, & \text{for } \hat{a} \leq 50 \\ \frac{1}{5} + \frac{16}{250}(\hat{a} - 50), & \text{for } 50 \leq \hat{a} \leq 62.5. \end{cases}$$

Of course, distribution $G$ has the same mean as the distribution of "true" abilities (uniform on $[0, 100]$) and second-order stochastically dominates it, so there exist transcript structures $F_i$ that give rise to distribution $G$ of expected abilities. Figure VII illustrates the resulting distribution of expected ability and the corresponding desirability mapping.

Notice that in this equilibrium there is no unraveling (or, more precisely, no incentive to unravel), since students become effectively risk-neutral. Consider a student whose first-year transcript indicates an expected ability level corresponding to a particular job desirability. This student can secure a job corresponding to his current expected ability or he can wait for second-year grades. In the absence of private information about ability the expected change in ability implied by the transcript must be zero. It is easy to see that the expected change in position desirability cannot be negative as a result of arrival of new information.

In the remainder of the section, we first present a simple two-period model where no information is available in period 1, which is very similar to the model of Suen (2000).[17] This similarity brings into focus the fact that the schools' ability to control information undermines the insurance reason for unraveling. We then show that the result becomes much stronger if information arrives gradually: if more than the benchmark amount of information is disclosed, unraveling will occur.

[17]We should note that Suen's model is more complicated—it involves wages. However, the main intuition that unraveling is caused by workers' demand for insurance can be applied to our model just as well, as we will show at the end of this section when we demonstrate unraveling in environments where schools cannot fully control information.

## 4.1 Two-period Model, Benchmark Amount of Information

Suppose students stay in school for 2 periods. In period 1 no information about them is known. Therefore, for all students in school $i$ expected ability in period 1 is the same, $\hat{a}_i$. A student has no private information about his ability.[18] Suppose employers and students can sign binding contracts in either year of study based on the information available at that period.

**Theorem 2** *If in period 2 schools reveal the benchmark amount of information, then no position can increase the expected ability of its match by making an early offer.*

**Proof.** Take a student from school $i$ in period 1. His expected ability in period 1 is $\hat{a}_i$. If he waits until period 2, more information about his ability will be revealed; his expected ability will become, say, $\hat{a}$; and he will get a position of desirability $Q(\hat{a})$. By the law of iterated expectations, $E_i[\hat{a}] = \hat{a}_i$. Desirability mapping $Q(\cdot)$ is convex, and therefore $E_i[Q(\hat{a})] \geq Q(\hat{a}_i)$. Thus, a student will only accept an early offer from a position that is at least as desirable as $Q(\hat{a}_i)$. But positions of desirability $Q(\hat{a}_i)$ and higher get a student of expected ability at least $\hat{a}_i$ if they wait until period 2, and so cannot benefit form moving early. ■

## 4.2 Gradual Information Arrival

We now set up a continuous-time model of gradual information arrival, and show a close connection between unraveling and information disclosure.

Students are in school from time $\tau = 0$ until time $\tau = \overline{\tau}$. At time 0 no information about a student is known except for the school $i$ that he attends. While the student stays in school, new information arrives continuously and is added to his transcript (we assume that information about students cannot disappear). Namely, at each time $\tau$ a potential employer can compute the student's expected ability $\hat{a}_\tau$ based on the school and the current transcript. Since employers use Bayes' rule to form beliefs about a student's expected ability, the drift term must be zero and the process is a martingale. We assume that $\hat{a}_\tau$ for students in school $i$ follows a diffusion process

$$d\hat{a}_\tau = \sigma_i(\cdot)dz, \tag{1}$$

where diffusion parameter $\sigma_i(\cdot)$ is a bounded continuous function of $\tau$ and $\hat{a}_\tau$, such that the process does not leave the interval $[a_L, a_H]$ and for all $\hat{a} \in (a_L, a_H)$, $\sigma_i(\hat{a}, \overline{\tau}) > 0$. We also assume that for some $\tau < \overline{\tau}$ function $\hat{Q}_i(\hat{a}_{\tau'}, \tau') = E[Q(\hat{a}_{\overline{\tau}})|\hat{a}_{\tau'}, \tau', i]$ is twice continuously differentiable for all $\tau' \in [\tau, \overline{\tau}]$.[19] Whenever expected ability follows such diffusion process we will say that *information*

[18] Even if students did have private information, unraveling would still not occur. In fact, the result would become even stronger. In the absence of private information, unraveling is a matter of indifference for both students and positions. If students do have private information, adverse selection works against unraveling, because the lowest ability students have higher payoff from unraveling than observationally equivalent students of higher ability. Essentially, only the lowest ability students are eager to unravel, and unraveling cannot occur under equilibrium information disclosure except for a set of measure zero.

[19] More formally, $\hat{Q}_i(\hat{a}_{\tau'}, \tau')$ is twice continuously differentiable on the set of points $\{(a, \tau')|\tau' \in [\tau, \overline{\tau}]$, $a$ is in the domain of $\hat{Q}_i(\cdot, \tau')\}$.

*arrives gradually.* Also, we call the amount of information disclosed by schools at the end, i.e., at time $\overline{\tau}$, the *actual amount of information disclosure.*

Each position's desirability is constant and commonly known, and any student-position pair can enter into a binding match at any time. Unraveling occurs if at some time $\tau < \overline{\tau}$ there is a pair, student $S$ and position $P$, that finds it profitable to sign such a contract.[20]

We now claim that it is an equilibrium for students and firms to sign contracts at time $\overline{\tau}$ without contracting early if the actual amount of information released by schools (i.e., the amount of information disclosed at time $\overline{\tau}$) coincides with the benchmark amount of information. If more than the benchmark amount of information is disclosed, some students and employers will find it profitable to sign contracts earlier.

**Theorem 3** *Suppose that information about ability of students arrives gradually (see equation (1)). If at time $\overline{\tau}$ transcripts contain the benchmark amount of information, then it is an equilibrium for all students and positions to wait until time $\overline{\tau}$ to sign contracts. If at time $\overline{\tau}$ transcripts contain more than the benchmark amount of information, then some agents are strictly better off not waiting till time $\overline{\tau}$ to sign contracts.*

The proof of the first statement of Theorem 3 follows the same intuition as the proof of Theorem 2: If the benchmark amount of information is disclosed at time $\overline{\tau}$, desirability mapping $Q(\cdot)$ is convex, making students effectively risk-neutral or risk-seeking, and thus giving them an incentive to wait for additional information.

The proof of the second statement involves several steps. When the actual amount of information disclosed at time $\overline{\tau}$ is between the benchmark and the full amounts, we show that at the points of strict convexity of the benchmark desirability mapping all three desirability mappings coincide, and students of abilities below and above such points are not mixed together. Thus, any additional information revelation in the actual vs. the benchmark amounts has to take place in an interval where the benchmark desirability mapping is linear. But then at some expected ability level $\hat{a}_*$ in this interval the actual desirability mapping $Q(\cdot)$ will be locally concave. This implies that at some time $\tau$ sufficiently close to $\overline{\tau}$, a student of expected ability $\hat{a}_*$ will strictly prefer immediately signing a contract with a position of desirability $Q(\hat{a}_*)$ to waiting until time $\overline{\tau}$. See Appendix B for the detailed proof.

# 5 Conclusion

Information suppression by schools is a widespread phenomenon, taking many forms from non-disclosure policies to coarse transcripts and inconsistent grading standards. We show that such behavior may be necessary in equilibrium: for many distributions of student abilities and job desirabilities, if all schools revealed full information about their students, then some of them could

[20] It is profitable for the pair to sign such a contract if by waiting till time $\overline{\tau}$, $P$ would get a student of expected ability no higher than the expected ability of $S$ given the information available at time $\tau$; $S$, in expectation, would get a position of desirability no higher than that of $P$; and at least one of these two inequalities is strict.

benefit by giving similar transcripts to students of different abilities, thus increasing the placement into moderately desirable positions and reducing the placement into very desirable and very undesirable ones. We also show that an essentially unique amount of information is disclosed in all equilibria. We call this the benchmark amount of information disclosure.

Schools are not the only actors in this market who can suppress information. By signing contracts early, students and employers can forgo the information about the students' performance in the last few semesters. We show that these two seemingly distinct ways of suppressing information are in fact closely related. If schools disclose the benchmark amount of information, students and employers will not find it profitable to contract early. If they disclose more, unraveling will occur.

The intuition behind this connection is in fact very natural. Under the benchmark amount of information disclosure, the mapping from expected student abilities inferred from their transcripts to job desirabilities must be convex: otherwise, a school could "mix" some students together and increase its payoff. Hence, if the benchmark amount of information is disclosed at graduation, a student is effectively risk-neutral or even risk-seeking: in expectation, additional information does not hurt him. If, however, schools disclose more than the benchmark amount of information, some parts of the ability–desirability mapping become concave, and students in the relevant ability range become effectively risk-averse, thus trying to avoid the arrival of future information by contracting early.

Educational institutions are often criticized for not revealing full and accurate information about their students, either by means of non-disclosure policies[21] or as a result of grade inflation, which can compress grades so that they lose some of their informativeness.[22] Our results show that information suppression may be inevitable. Even if schools reveal full information about their students, some of that information will be suppressed via a different channel—unraveling will occur. Unraveling in various markets and its consequences are documented in Roth and Xing (1994), Avery et al. (2001), and Niederle and Roth (2003). Avery et al. give many colorful quotes from judges and law school students who experience the effects of unraveling in the market for federal judicial law clerks, such as "The unseemly haste to hire law clerks is a disgrace to the federal bench" and "Some judges scrapped decorum and even bare civility."[23] Anyone who claims that more information needs to be disclosed has to keep in mind the "unseemly haste" that may follow.

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# A Proof of Theorem 1

We first show that in equilibrium there is a one-to-one mapping from expected ability to position desirability, i.e., the distribution of *expected* abilities, as well as the corresponding desirability mapping $Q$ are continuous in equilibrium. This implies that $Q$ is an invertible function.

**Lemma 1** *In equilibrium, any two students of the same expected ability $\hat{a}$ obtain equally desirable positions.*

**Proof.** Suppose in equilibrium students of expected ability $\hat{a}$ get jobs of desirabilities from $q_1$ to $q_2$, $q_1 < q_2$, i.e., there is a positive mass of students of expected ability $\hat{a}$. Let $\hat{q}$ be the average desirability that students of expected ability $\hat{a}$ get. $q_1 < \hat{q} < q_2$. Since there is a positive mass of students of expected ability $\hat{a}$, there must be at least one school producing a positive mass of such students. This school has to include some students of lower and some students of higher ability in this mass. Thus, it can select a small subset from the mass (say, $\epsilon$-share of the mass) such that its expected ability is $\hat{a} - \delta$, where $\delta$ is also small. Then the remaining mass has expected ability higher than $\hat{a}$, and therefore all students there get positions of desirability $q_2$ or higher. For sufficiently small $\epsilon$ and $\delta$, the net change in average desirability is positive, i.e., the school was able to improve upon its equilibrium transcript structure—contradiction. ■

Desirability mapping $Q(\cdot)$ is monotonically increasing. This, however, does not necessarily mean that a student of a higher true ability will get matched to a better position than a student of a lower true ability: if a school gives out transcripts that are not fully informative, the lower ability student may receive a better transcript than the higher ability student and thus get a better position.

We will say that an equilibrium is *fully informative* at a particular value of position desirability $q$ if there is an ability level that is necessary and sufficient for receiving a position of this quality. More precisely, equilibrium is *fully informative* at desirability $q$ and ability $a$ if $Q(a) = q$, no students with true ability below $a$ get matched with jobs of desirability above $q$, and no students with true ability above $a$ get matched with jobs of desirability below $q$. It is straightforward to show that an equilibrium is fully informative (i.e., schools do not suppress any information) if and only if it is fully informative at every position desirability.

Now, suppose a school produces students of expected abilities $b$ and $c$. This could only be optimal for the school if by mixing students of these abilities it could not raise its payoff, i.e., if $\alpha Q(b) + (1-\alpha)Q(c) \geq Q(\alpha b + (1-\alpha)c)$ for any $\alpha \in [0,1]$. Since this reasoning can be applied to every pair of points, and in a connected equilibrium there is a school producing students in a neighborhood of any point, $Q(\hat{a})$ has to be convex.

Next, if a school does mix students of true abilities $b$ and $c$, by convexity of desirability mapping $Q(\hat{a})$ this could only be optimal if the desirability mapping is linear on the interval $[b, c]$. Consequently, if $Q(\hat{a})$ is strictly convex at a certain expected ability level $a$, it is fully informative at $Q(a)$: students with ability above $a$ get positions better than $Q(a)$ and students with ability below $a$ get positions worse than $Q(a)$. Therefore, in that case $Q(a) = Q_T(a)$ (recall that $Q_T(a)$ is the desirability mapping that would arise if all schools revealed all abilities truthfully).

The next lemma shows that the lowest expected ability produced by schools is equal to the lowest true ability. This is similar to the "lowest type not signalling" in a separating equilibrium of a signalling game.

**Lemma 2** *In a connected equilibrium, let $\hat{a}_L$ be the lowest expected ability level, and $a_L$ be the lowest true ability level. Then $\hat{a}_L = a_L$.*

**Proof.** It is clear that $\hat{a}_L \geq a_L$, since it is impossible to produce students of expected ability lower than the lowest true ability.

Suppose $\hat{a}_L > a_L$. Take a school that has students of true ability $a_L$ (i.e., a positive mass of students of abilities $(a_L, a_L + \epsilon)$ for any positive $\epsilon$). Since the school does not produce any students of ability below $\hat{a}_L$, it has to "bundle" students in the interval $(a_L, a_L + \epsilon)$ with higher ability students $(0 < \epsilon < \hat{a}_L - a_L)$. But then, since $Q(\hat{a})$ is increasing and convex, the school would increase the average desirability of placements of its students by "unbundling" these low ability students—contradiction. ■

We are now ready to prove Theorem 1. The proof proceeds by induction on the number of intervals on which the convexity or concavity of $Q_T(a)$ does not change (and, along the way, shows how to construct the equilibrium desirability mapping). For convenience, we will call such intervals "convexity intervals". Recall that by assumption, $Q_T(a)$ does not switch from convexity to concavity infinitely often, and hence has a finite number of convexity intervals.[24]

**Step 1.** Suppose $Q_T(a)$ has only one convexity interval.

**Step 1. Case "Convex".** Suppose $Q_T(a)$ is convex on $[a_L, a_H]$. Then truthful revelation is an equilibrium profile of transcript structures. Suppose there is another equilibrium profile of transcript structures $\omega$, involving some mixing of students, with desirability mapping $Q(\hat{a})$ on $[a_L, \hat{a}_H]$, where $\hat{a}_H \leq a_H$. Take any point $x_1$ on $(a_L, \hat{a}_H)$ such that $Q_T(x_1) \neq Q(x_1)$. Equilibrium $\omega$ is not fully informative at $x_1$, and is therefore linear on some interval containing $x_1$. Take the largest such interval $[a_1, a_2]$. Equilibrium $\omega$ has to be fully informative at $a_1$, and therefore $Q_T(a_1) = Q(a_1)$.

With $a_2$, there are two possibilities.

If $a_2 < \hat{a}_H$ or $a_2 = \hat{a}_H = a_H$, then $\omega$ also has to be fully informative at $a_2$, with $Q_T(a_2) = Q(a_2)$. But then $Q_T$ is convex, $Q$ is linear on $[a_1, a_2]$, $Q_T(a_1) = Q(a_1)$, $Q_T(a_2) = Q(a_2)$, and $Q_T(x_1) \neq Q(x_1)$ (with $a_1 < x_1 < a_2$), which implies that $Q_T(x_1) < Q(x_1)$, which in turn implies that for all $x \in (a_1, a_2)$, $Q_T(x) < Q(x)$. This, in turn, implies that *every* firm of desirability $q$ strictly between $q_1 = Q_T(a_1) = Q(a_1)$ and $q_2 = Q_T(a_2) = Q(a_2)$ is matched to a better (in expectation) student under truthful revelation than under equilibrium with mixing $\omega$, which, finally, implies that the *total* ability of students matched to those positions in equilibrium $\omega$ is strictly higher than the total

[24]We do not discuss in detail intervals on which $Q_T$ is linear, and effectively assume its interval-wise strict concavity or convexity. Considering the intervals on which the mapping is linear is not hard conceptually, but would make the proof more cumbersome.

ability of students matched to them under truthful revelation, i.e.,

$$\int_{q_1}^{q_2} \left( a_1 + \frac{q - q_1}{q_2 - q_1}(a_2 - a_1) \right) d\mu(q) < \int_{q_1}^{q_2} Q_T^{-1}(q) d\mu(q) = \int_{a_1}^{a_2} a dG(a).$$

But this is impossible, because by construction desirability mapping $Q$ is strictly convex at both $a_1$ and $a_2$, and so the set of students matched with positions in the range $[q_1, q_2]$ in equilibrium $\omega$ is the same as the set of students matched with those positions under truthful revelation, and so all of the integrals above have to be equal.

If $a_2 = \hat{a}_H < a_H$, then $Q_T(\hat{a}_H) < Q_T(a_H) = Q(\hat{a}_H)$, and by convexity of $Q_T$ and linearity of $Q$ on $[a_1, \hat{a}_H]$, for all $x \in (a_1, \hat{a}_H)$, $Q_T(x) < Q(x)$, and therefore for all $q \in (Q_T(a_1), Q_T(a_H))$, $Q_T^{-1}(q) > Q^{-1}(q)$, which is again impossible because equilibrium $\omega$ is fully informative at $a_1$ and the set of students matched to positions above $Q_T(a_1)$ is the same under $\omega$ and under truthful revelation (in both cases, it is the set of students with abilities above $a_1$).

**Step 1. Case "Concave".** Suppose $Q_T(a)$ is concave on $[a_L, a_H]$. In equilibrium, the desirability mapping $Q$ has to be linear on the entire interval $[a_L, \hat{a}_H]$ for some $\hat{a}_H \leq a_H$. Indeed, suppose there is a point, $\hat{a}$, at which $Q$ is not linear. Then it has to be strictly convex (and equilibrium fully informative) at $\hat{a}$. By an argument analogous to that of Case "Convex", this is impossible.

Moreover, there exists only one $\hat{a}_H$ that can arise in equilibrium—it is the unique one that guarantees that the total ability of students assigned to all schools is equal to the total ability of students in the population, i.e., the unique $\hat{a}_H$ such that

$$\int_{q_L}^{q_H} \left( a_L + \frac{q - q_L}{q_H - q_L}(\hat{a}_H - a_L) \right) d\mu(q) = \int_{a_L}^{a_H} a dG(a).$$

**Step 2.** We are now ready to prove the inductive step. Suppose the theorem is true for all $n < k$ and suppose there are $k > 1$ convexity intervals in $Q_T$. Take the first one, i.e., the one that begins at $a_L$ and ends at some value $b_1$. It is now more convenient to consider the two cases in the reverse order.

**Step 2. Case "Concave".** Suppose $Q_T$ is concave on $[a_L, b_1]$. By an argument analogous to the one above, equilibrium desirability mapping $Q$ has to be linear on interval $[a_L, c_1]$ for some $c_1 > b_1$. Let us find this point $c_1$ and show that it is uniquely determined. Consider the graph of of $Q_T$ on a two-dimensional plane, and take the infinite ray that starts at the point $(a_L, q_L)$ and has a slope of zero. Start rotating this ray around its origin, increasing its slope. Once the ray begins to intersect with the graph of $Q_T$ at points $(a_i, q_i)$ other than the origin, for each of these points (and there is always a finite number of them, at most two per convexity interval) keep checking whether they could potentially be the $c_1$ we are looking for. Specifically, check whether the total ability of all students of ability below $a_i$ is equal to the hypothetical total ability of students assigned to positions of quality below $q_i$ under the linear desirability mapping implied by the ray, i.e., whether

$$\int_{a_L}^{a_i} adG(a) = \int_{q_L}^{q_i} \left( a_L + \frac{q - q_L}{q_i - q_L}(a_i - a_L) \right) d\mu(q).$$

As soon as such a point exists, stop, and consider this point $(a^*, q^*)$. If, by coincidence, there are several such points on the ray, consider the one with the largest coordinates; this is Subcase "Partially Linear" below. If no such point exists for any slope less than or equal to $\frac{q_H - q_L}{a_H - a_L}$, let $q^* = q^H$, take the unique point $(a^*, q^H)$ such that the total ability of students assigned to positions $[q_L, q_H]$ implied by desirability mapping $q_L + \frac{q_H - q_L}{a_* - a_L}(\hat{a} - a_L)$ is equal to the total ability of all students in the population, i.e., $\int_{q_L}^{q_H} \left( a_L + \frac{q - q_L}{q_H - q_L}(a^* - a_L) \right) d\mu(q) = \int_{a_L}^{a_H} adG(a)$; this is Subcase "Fully Linear" below.

We now claim that in any connected equilibrium, $c_1 = a^*$ and the desirability mapping on $[a_L, a^*]$ is a straight line between $(a_L, q_L)$ and $(a^*, q^*)$.

**Step 2. Case "Concave". Subcase "Partially Linear".** Suppose there exists an equilibrium, $\omega$, for which $c_1 \neq a^*$. Consider students assigned to positions $[q_L, q^*]$ under $\omega$ and under truthful revelation. Under truthful revelation, matching is based on true ability, and so these positions get the worst possible students. Hence, the total ability of these students has to be at most as high as the total ability of students assigned to these positions under $\omega$. Now, consider desirability mapping $Q$ corresponding to $\omega$. By construction, the slope of $Q$ at $a_L$ is at least as high as $\frac{q^* - q_L}{a^* - a_L}$ and $Q(a^*) > q^*$, which implies that for all $q \in [q_L, q^*]$, $Q^{-1}(q) \leq a_L + \frac{q - q_L}{q_i - q_L}(a_i - a_L)$, and for a positive mass of positions $q$ from this interval, $Q^{-1}(q) < a_L + \frac{q - q_L}{q_i - q_L}(a_i - a_L)$. But this leads us to a contradiction, because then the total ability of students assigned to positions $[q_L, q^*]$ under $\omega$, $\int_{q_L}^{q_i} Q^{-1}(q) d\mu(q)$, is strictly less than $\int_{q_L}^{q^*} \left( a_L + \frac{q - q_L}{q^* - q_L}(a^* - a_L) \right) d\mu(q)$, which by construction is equal to $\int_{a_L}^{a_i} adG(a)$, i.e., the total ability of students assigned to these positions under truthful revelation.

To complete the inductive step for this case, it is now sufficient to note that if $a^* = a_H$, we are done; otherwise, the equilibrium desirability mapping for expected ability levels above $a^*$ is uniquely determined as the equilibrium desirability mapping of the original economy excluding the students of ability below $a^*$ and positions of desirability below $q^*$; in this truncated economy, the number of convexity intervals is less than $k$, satisfying the assumptions of the inductive step.

**Step 2. Case "Concave". Subcase "Fully Linear".** This substep follows from the same ideas as Subcase "Partially Linear" and Case "Concave" of Step 1, and is therefore omitted.

**Step 2. Case "Convex".** Suppose $Q_T$ is convex on $[a_L, b_1]$. Our method for finding the unique equilibrium desirability mapping $Q$ is based on the following observation. Suppose $Q$ and $Q_T$ do not coincide on $[a_L, b_1]$. Then there exists $a \in [a_L, b_1)$ such that

1. $Q(x) = Q_T(x)$ for all $x \in [a_L, a]$,

2. $Q(x)$ is linear on $[a, b_1]$,

3. the slope of $Q(x)$ on $[a, b_1]$ is less than or equal to the right derivative of $Q_T(x)$ at $a$, and

4. if $a > a_L$, the slope of $Q(x)$ on $[a, b_1]$ is greater than or equal to the left derivative of $Q_T(x)$ at $a$.

Indeed, suppose for some $x \in (a_L, b_1)$, $Q(x) \neq Q_T(x)$. Then we know that $Q(x)$ has to be linear on some interval around $x$. Take the largest such interval $[a, b]$. The equilibrium is fully informative at $a$; it is also fully informative at $a_L$. If $a = a_L$, statement (1) above follows trivially; otherwise it follows from the convexity of $Q_T(x)$ on $[a_L, a]$ by the argument analogous to Step 1, Case "Convex". By a similar argument, $b$ has to be strictly greater than $b_1$, giving us (2). Statement (4) follows immediately from the convexity of equilibrium desirability mapping $Q$ at any point, including, of course, $a$. To prove (3), suppose the slope of $Q(x)$ on $[a, b_1]$ is strictly greater than the right derivative of $Q_T$ at $a$. Then the total ability of students assigned to positions in some small interval $[Q(a), Q(a) + \epsilon]$ in this equilibrium is strictly lower than the total ability of students assigned to these positions under full information revelation, which is impossible.

Let $\bar{r}$ be the left derivative of $Q_T$ at $b_1$. The observation above implies that any equilibrium has to be either fully informative on $[a_L, b_1]$ or be fully informative up to some ability level $a \geq a_L$ and linear with some slope $r < \bar{r}$ on $[a, b_1]$. Crucially, it also implies that for any slope $r < \bar{r}$, there exists exactly one point $a(r)$ on $[a_L, b_1]$ where an equilibrium switches from being fully informative to being linear with slope $r$; this point $a(r)$ is simply the point at which a line with slope $r$ is tangent to the graph of function $Q$.

We now proceed in essentially the same way as in Step 2, Case "Concave", starting with a ray from $(a_L, q_L)$ and a slope of zero and gradually increasing the slope, looking first for a "Partially Linear" subcase and then, after the ray crosses the point $(a_H, q_H)$, looking for the "Fully Linear" subcase. There are, however, two differences. First, as we increase the slope, we also gradually move the origin of the ray along the graph of mapping $Q_T$, keeping the ray tangent to it. Second, the slope may reach $\bar{r}$ before encountering either "Partially Linear" or "Fully Linear" subcase. If that happens, we know that the equilibrium has to be fully informative on $[a_L, b_1]$, and the rest of the desirability mapping is uniquely pinned down as the equilibrium desirability mapping of the original economy excluding the students of ability below $b_1$ and positions of desirability below $Q_T(b_1)$; in this truncated economy, the number of convexity intervals is equal to $k - 1$, satisfying the assumptions of the inductive step.

## B Proof of Theorem 3

Suppose the benchmark amount of information is disclosed and there is no unraveling. We then show that no student has an incentive to deviate, i.e., to sign a contract earlier than $\overline{\tau}$. Consider an arbitrary school $i$. Let the interval of expected abilities of students at school $i$ at time $\overline{\tau}$ be $[a_i, b_i]$. By the law of iterated expectations, no student at school $i$ can have expected ability outside of this interval at any time $\tau \leq \overline{\tau}$. Take any time $\tau < \overline{\tau}$ and any student from school $i$ who has expected ability $\hat{a}_\tau$ inside the interval at time $\tau$. If he signs now, the best position he can get is of desirability $Q(\hat{a}_\tau)$. If he waits until time $\overline{\tau}$, the expected desirability of position he gets is $E[Q(\hat{a}_{\overline{\tau}})|\hat{a}_\tau, \tau, i]$. By assumption, at time $\overline{\tau}$ the school produces a positive density of students on an interval, and transcript structures form an equilibrium—thus, $Q(\hat{a}_{\overline{\tau}})$ is convex on the interval. $E[\hat{a}_{\overline{\tau}}] = \hat{a}_\tau$, and so $E[Q(\hat{a}_{\overline{\tau}})|\hat{a}_\tau, \tau, i] \geq Q(\hat{a}_\tau)$, and the student does not have an incentive to deviate—this is the same logic as in the proof of Theorem 2.

Now suppose more than the benchmark amount of information is disclosed at time $\overline{\tau}$. We first show that the corresponding desirability mapping is not convex. Let $F$ be the distribution of expected abilities under benchmark information disclosure, $G$ the distribution of expected abilities actually disclosed at time $\overline{\tau}$, and $H$ the distribution of true abilities. We know that $H$ is more informative than $G$, which in turn is more informative than $F$. Suppose at some expected ability level $a$, the desirability mapping corresponding to $F$ is strictly convex. Then, as we have previously explained, in the static equilibrium schools do not mix students with abilities below $a$ and above $a$, and so under both $F$ and $H$, a student with reported expected ability $a$ gets matched with a job of the same desirability $d$. Moreover, the average (and the total) ability of students matched with positions of desirabilities less than $d$ is the same under $F$ and $H$.

Consider two arbitrary distributions of expected abilities, $\beta$ and $\gamma$, desirability level $\delta$, and expected ability level $\alpha$ corresponding to $\delta$ if expected abilities are distributed according to $\beta$. Note that if $\gamma$ is less informative than $\beta$, then the average (or total) ability of students matched with positions of desirability less than $\delta$ under $\gamma$ is a least as large as under $\beta$. Moreover, the two are equal only if distribution $\beta$ restricted to $[a_L, \alpha]$ is a mean-preserving spread of distribution $\gamma$ restricted to the same interval, i.e., under $\gamma$ (relative to $\beta$), students of expected ability levels below $\alpha$ do not get mixed with students of expected ability levels above $\alpha$.

But then it has to be the case that under distribution $G$, which in terms of informativeness is between distributions $F$ and $H$, students of ability below $a$ do not get mixed with students of ability above $a$. Therefore, any piece of additional disclosure of information under $G$ vs. $F$ has to take the form of a mean-preserving spread of the distribution of expected abilities in a region where the desirability mapping under $F$ is linear. It is easy to see that any amount of additional information in such a region leads to a desirability mapping that is not convex. Hence, there exists some point $\hat{a}_*$ inside that region such that $Q''(\hat{a}_*) < 0$.

Since $\hat{a}_*$ is inside the region of reported abilities, there exists some $\tau_1 < \overline{\tau}$ and school $i$ such that a positive mass of expected abilities is produced by school $i$ in a small $\epsilon$-neighborhood of $\hat{a}_*$ for any $\tau \in [\tau_1, \overline{\tau}]$. By assumption, $\hat{Q}_i(\hat{a}_\tau, \tau) = E[Q(\hat{a}_{\overline{\tau}})|\hat{a}_\tau, \tau, i]$ is twice continuously differentiable; also, $\hat{Q}_i(\hat{a}, \overline{\tau}) = Q(\hat{a})$. Therefore, there exists $\tau_2 < \overline{\tau}$, $\tau_2 \geq \tau_1$ such that $\frac{\partial^2 \hat{Q}_i(\hat{a}_*, \tau)}{\partial \hat{a}^2} < 0$ for all $\tau \in [\tau_2, \overline{\tau}]$. Finally, there exists $\tau_3 < \overline{\tau}$, $\tau_3 \geq \tau_2$ such that diffusion parameter $\sigma_i(\hat{a}_*, \tau)$ is strictly positive for all $\tau \in [\tau_3, \overline{\tau}]$.

By construction, $\hat{Q}_i$ is a martingale, and therefore $E[d\hat{Q}_i(\hat{a}, \tau)] = 0$. By Ito's lemma,

$$0 = E[d\hat{Q}_i(\hat{a}, \tau)] = \frac{1}{2}\sigma_i^2 \frac{\partial^2 \hat{Q}_i(\hat{a}, \tau)}{\partial \hat{a}^2} + \frac{\partial \hat{Q}_i(\hat{a}, \tau)}{\partial \tau}.$$

For $\tau \in [\tau_3, \overline{\tau}]$, $\frac{1}{2}\sigma_i^2 \frac{\partial^2 \hat{Q}_i(\hat{a}_*,\tau)}{\partial \hat{a}^2} < 0$, and so $\frac{\partial \hat{Q}_i(\hat{a}_*,\tau)}{\partial \tau} > 0$. But this implies that $Q(\hat{a}_*) = \hat{Q}_i(\hat{a}_*, \overline{\tau}) > \hat{Q}_i(\hat{a}_*, \tau_3)$, and so at time $\tau_3$ a student of expected ability $\hat{a}_*$ strictly prefers unraveling and immediately matching with a position of desirability $Q(\hat{a}_*)$ to waiting until time $\overline{\tau}$ and getting, in expectation, $\hat{Q}_i(\hat{a}_*, \tau_3)$, while the employer is indifferent.

# C The Connectedness Restriction

In this appendix, we discuss the connectedness restriction. First, we give a sufficient condition for the existence of a connected equilibrium: all schools are identical. Next, we show that in some markets, a connected equilibrium may not exist, even if for every *true* ability level $a \in (a_L, a_H)$ there exists a school that has students of all *true* abilities in some $\epsilon$-neighborhood of $a$.

**Theorem C.1** *If all schools have identical distributions of student abilities, there exists a symmetric equilibrium in pure strategies. This equilibrium is connected.*

**Proof.** We first prove the existence of a symmetric equilibrium in pure strategies. Let $S$ be the set of a school's strategies. Let $B(s)$ be the best responce correspondence—the set of best responces for a school given that all other schools play $s$. We need to show that correspondence $B(\cdot)$ has a fixed point.

Define $S_n$ as the set of all strategies that generate a finite number of expected abilities $\{\hat{a}_i^n\}$, $i \in \{1, 2, \ldots, 2^n - 1\}$, such that expected ability $\hat{a}_{2^{n-1}}^n$ corresponds to the average true ability in the population, $\hat{a}_{2^{n-1}+2^{n-2}}^n$ corresponds to the expected ability of a better-than-average student, $\hat{a}_{2^{n-1}-2^{n-2}}^n$ corresponds to the expected ability of a worse-than-average student, and so on (there can be a zero mass of students with a particular expected ability). $S_n$ is not empty for $n \geq 1$ because it contains the strategy that assigns the same expected ability to all students.

Note that $S_n$ is just the set of distributions on the set of the above $2^n - 1$ points that second-order stochastically dominate the underlying distribution of student abilities.[25] $S_n$ is convex (if each of two distributions dominates $F$, their affine combinations do too, and they are also concentrated on the set of $2^n - 1$ points), compact, and the payoff function is continuous on $S_n$ (Each element in $S_n$ is just a vector of $2^n - 1$ positive numbers adding up to 1, and so we can use the induced metric from $R^{2^n-1}$). Consider now the best response correspondence $B_n(\cdot)$, which for every strategy $s \in S_n$ returns the (nonempty) set of best responses to $s$ from the set $S_n$. Note that due to the continuity of payoffs, $B_n(\cdot)$ is upper hemicontinuous. Note also that for any $s$, the set $B_n(s)$ is convex, due to the linearity of payoffs (if strategies $s_1$ and $s_2$ are in $B_n(s)$ and thus give identical payoffs to the school, for any $\alpha \in [0, 1]$, strategy $(\alpha s_1 + (1 - \alpha)s_2)$ is also in $S_n$ and gives the same payoff to the school, and therefore also belongs to $B_n(s)$). Thus, by Kakutani's Fixed Point Theorem there exists $s_n^*$ such that $s_n^* \in B_n(s_n^*)$.

[25] Clearly, any distribution in $S_n$ second-order stochastically dominates the underlying distribution of true student abilities. On the other hand, any distribution $s$ that second-order stochastically dominates the underlying distribution of true student abilities can be obtained from that distribution by mixing some students together, because the underlying distribution of true student abilities is a mean-preserving spread of $s$.

Take the sequence $\{s_n^*\}$ for $n \to \infty$. Since all distributions $s_n^*$ have supports on subsets of a bounded interval $[a_L, a_H]$, this sequence has a weakly converging subsequence. Let $s$ be the limit of this subsequence. Note that the payoff function of a school is continuous both in its own strategy and in the strategy of other players. Therefore, $s$ is a best response to itself, and thus corresponds to a symmetric equilibrium.

Let us show that this equilibrium is connected. Suppose it is not. This implies that there is an interval $(a, b)$ such that no school produces students of ability in this interval, but each (since the equilibrium is symmetric) produces positive masses of students on both sides of the interval, i.e., for any open interval containing $a$ or $b$. But then the school can increase its payoff by mixing some students of ability slightly below $a$ and some students of ability slightly above $b$ so that the expected ability in this mix equals $a$. ■

The symmetry condition is sufficient for the existence of a connected equilibrium, but it is not necessary, as Examples 2, 3, and 4 illustrate. However, it is not clear what a more general sufficient condition on the primitives of the model could be: as we show in the following example, a connected equilibrium may not exist, even if for any *true* ability level $a \in (a_L, a_H)$ there exists a school that has students of all *true* abilities in some $\epsilon$-neighborhood of $a$.

**Example C.1** There is mass .8 of students in "bad" schools, with abilities distributed uniformly on $[0, 50]$ and mass .2 of students in "good" schools, with abilities distributed uniformly on $[0, 150]$. There is also mass 1 of positions, distributed uniformly on $[0, 100]$.

Suppose this market has a connected equilibrium. Under truthful information revelation the desirability mapping would be concave. Hence, in the connected equilibrium, the desirability mapping would be linear, and so the observed distribution of expected abilities would be uniform on $[0, \hat{a}_H]$. The average true ability in this market is $.8 \cdot 25 + .2 \cdot 75 = 35$, and the average expected ability has to be the same. Therefore, $\hat{a}_H = 70$. But no matter what mixing strategy it uses, a "good" school will produce a positive mass of students with expected ability 75 or higher—contradiction.

Hence, there is no connected equilibrium in this market. Is there another equilibrium? It turns out, there is, and moreover, the desirability mapping in it is not convex. In this equilibrium, each "bad" school reveals full information about its students, and each "good" school "compresses" the distribution of its students' abilities from the true distribution of $U[0, 150]$ to the distribution of expected ability $U[50, 100]$. The resulting desirability mapping is

$$Q(\hat{a}) = \begin{cases} \frac{8}{5}\hat{a}, & \text{for } \hat{a} \leq 50 \\ 80 + \frac{2}{5}(\hat{a} - 50), & \text{for } \hat{a} \in [50, 100] \end{cases}$$

and it is easy to check that each school behaves optimally.

Finally, we would like to point out that there is a powerful force that is not captured in our model and that would tend to eliminate disconnected equilibria with non-convex desirability mappings as in Example C.1. This force is arbitrage. In our model, we abstract away from how students get assigned to schools; in fact, as long as there exists a connected equilibrium, that doesn't matter.

If, however, schools can compete for students and can facilitate monetary transfers between them (e.g., in the form of a high tuition and heterogeneous financial aid), then a non-convexity in the desirability mapping would allow a school to get students from other schools, mix them together, and get a higher average payoff for them. Thus, any non-convexity in the desirability mapping is an arbitrage opportunity, which cannot persist in equilibrium.

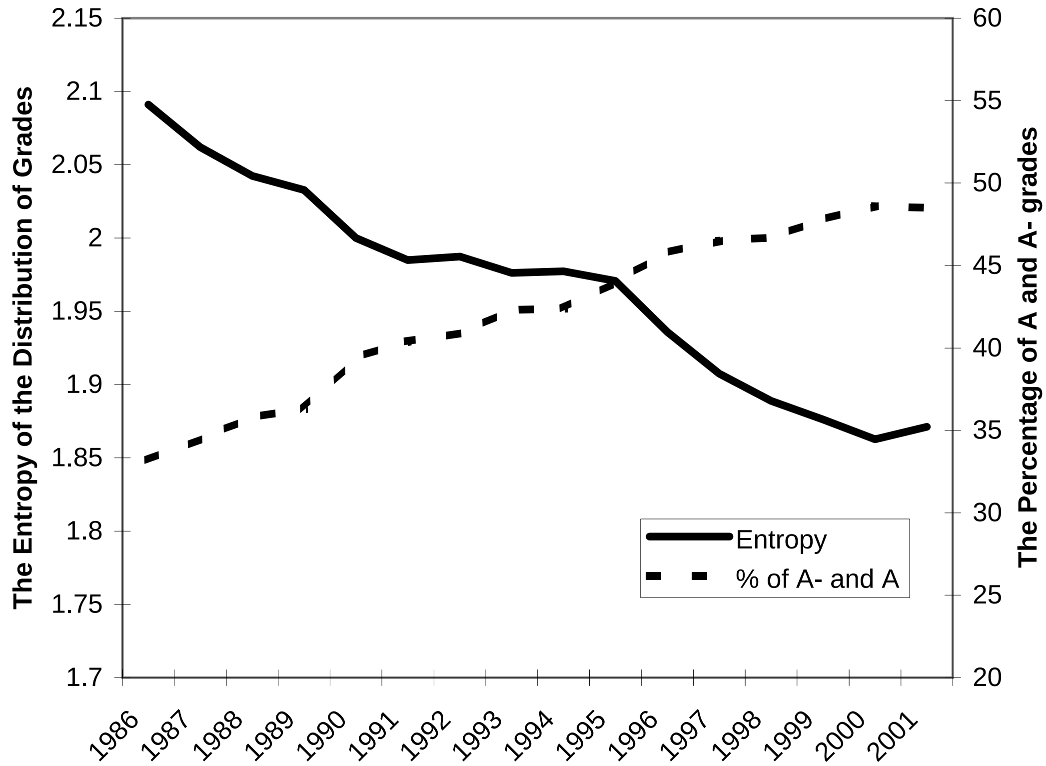



Figure I

Informativeness of Grades at Harvard

We use entropy as a proxy for the informativeness of grades. It is equal to $(-\Sigma_i s_i * \ln(s_i))$, where $i$ ranges from the worst grade to the best and $s_i$ is the share of grade $i$ among all grades. If all students receive the same grade, no information is revealed and entropy is minimized. If a trascript structure is modified in a way that reduces the amount of information (e.g., students who had a C or a D can no longer be distinguished), entropy goes down.

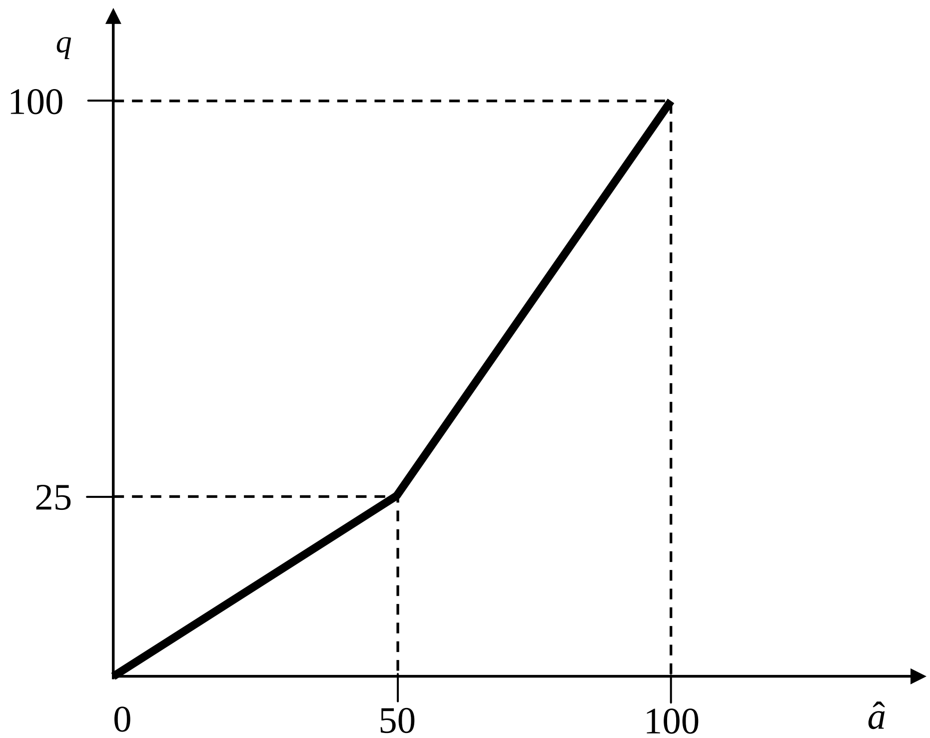
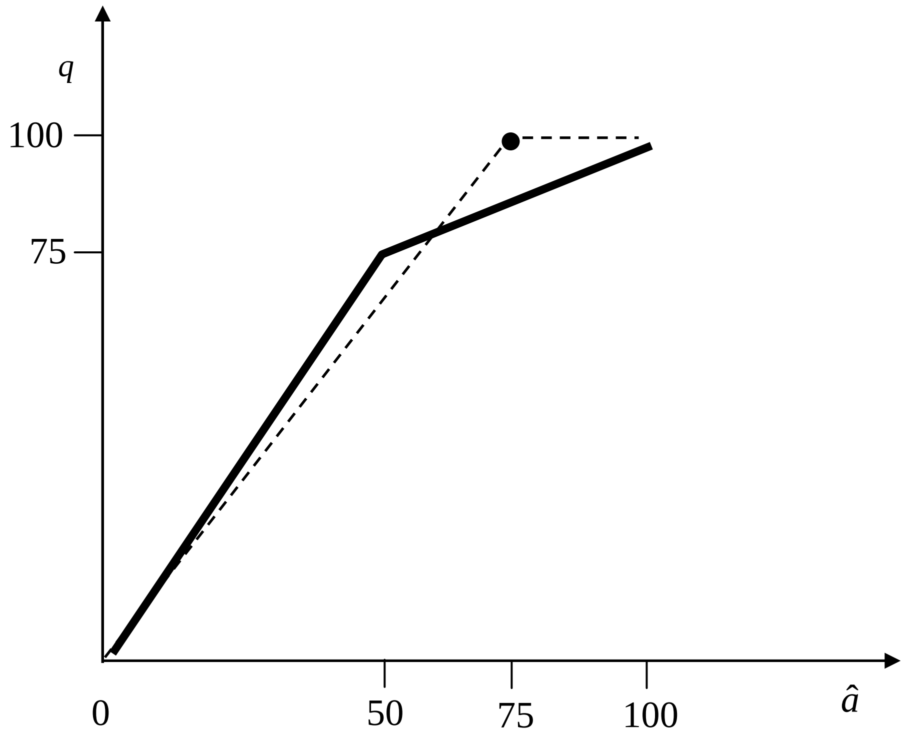



Figure II

Desirability Mapping *Q* in Example 2 under Full Information Revelation

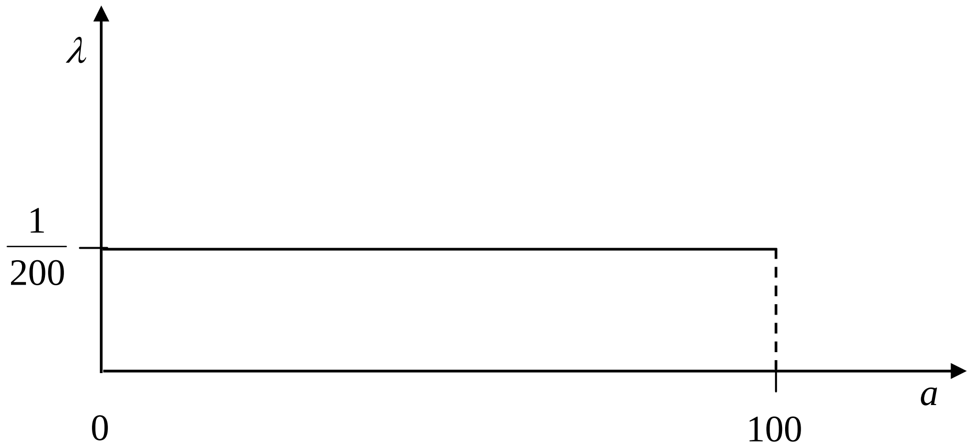



$Q(\hat{a})$ – under truthful information revelation

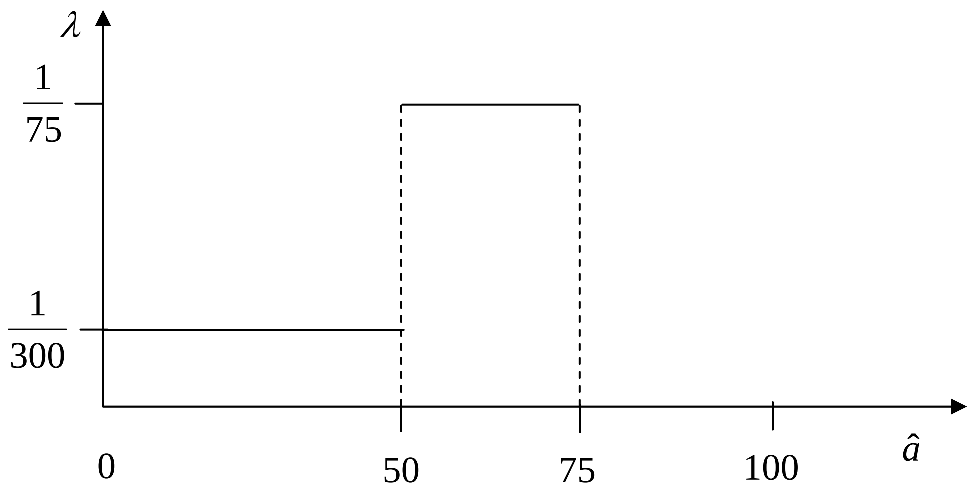
$Q'(\hat{a})$ – under equilibrium information revelation

Figure III

Desirability Mappings in Example 3



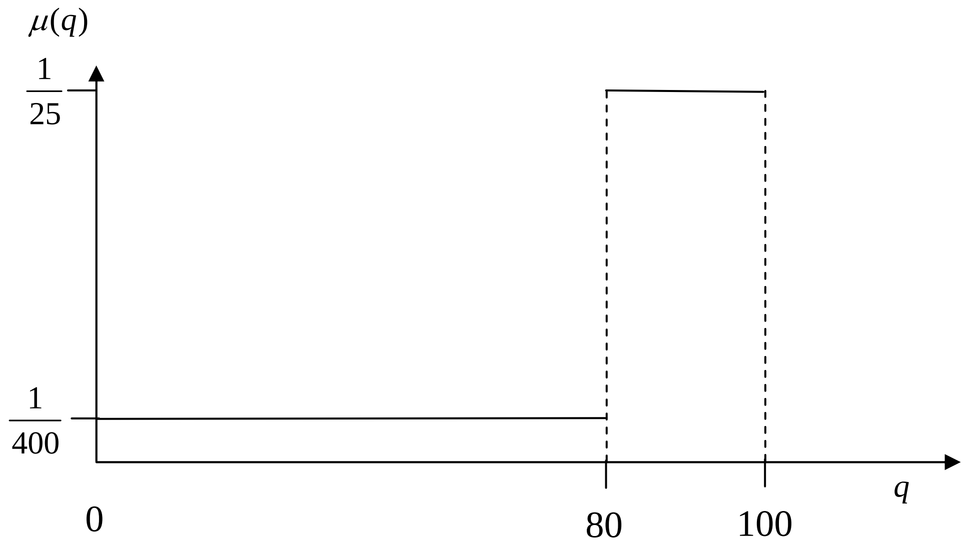

(a) Measure of True Abilities



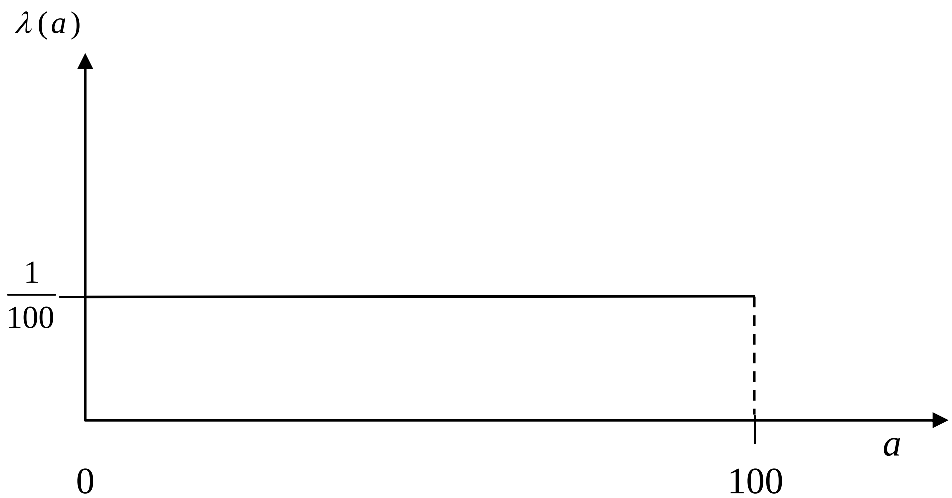

(b) Measure of Expected Abilities

Figure IV

Measures of True and Expected Abilities at an Average School in Example 3




(a) Distribution of Desirabilities




(b) Distribution of True Abilities

Figure V

Distributions of Desirabilities and True Abilities in Example 4

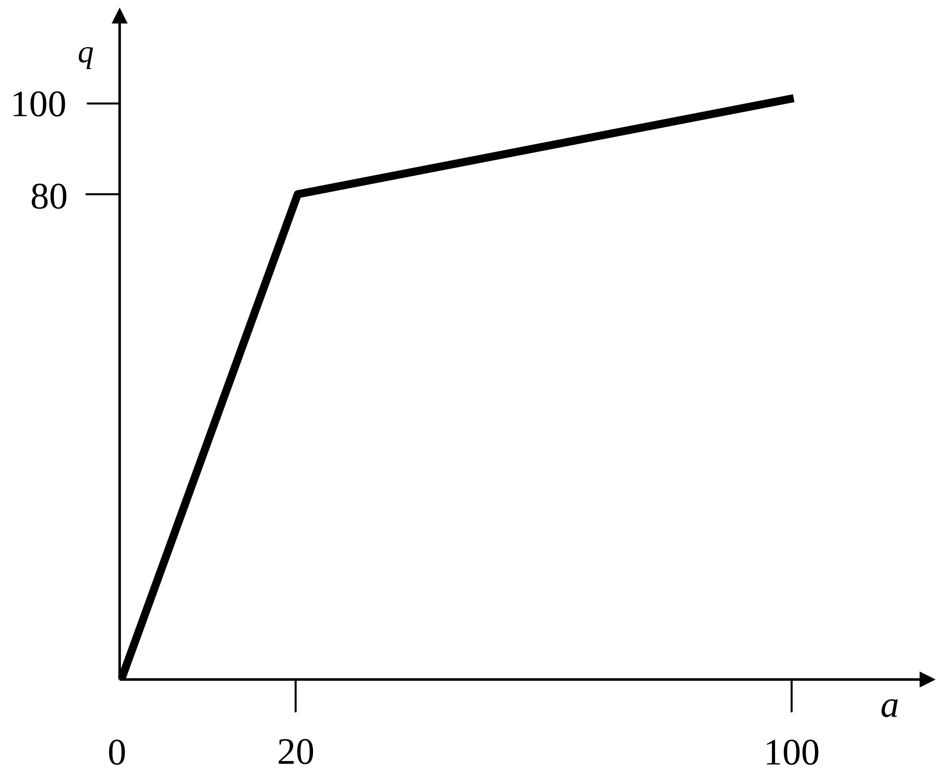



Figure VI

Desirability Mapping *Q* in Example 4 under Full Information Revelation

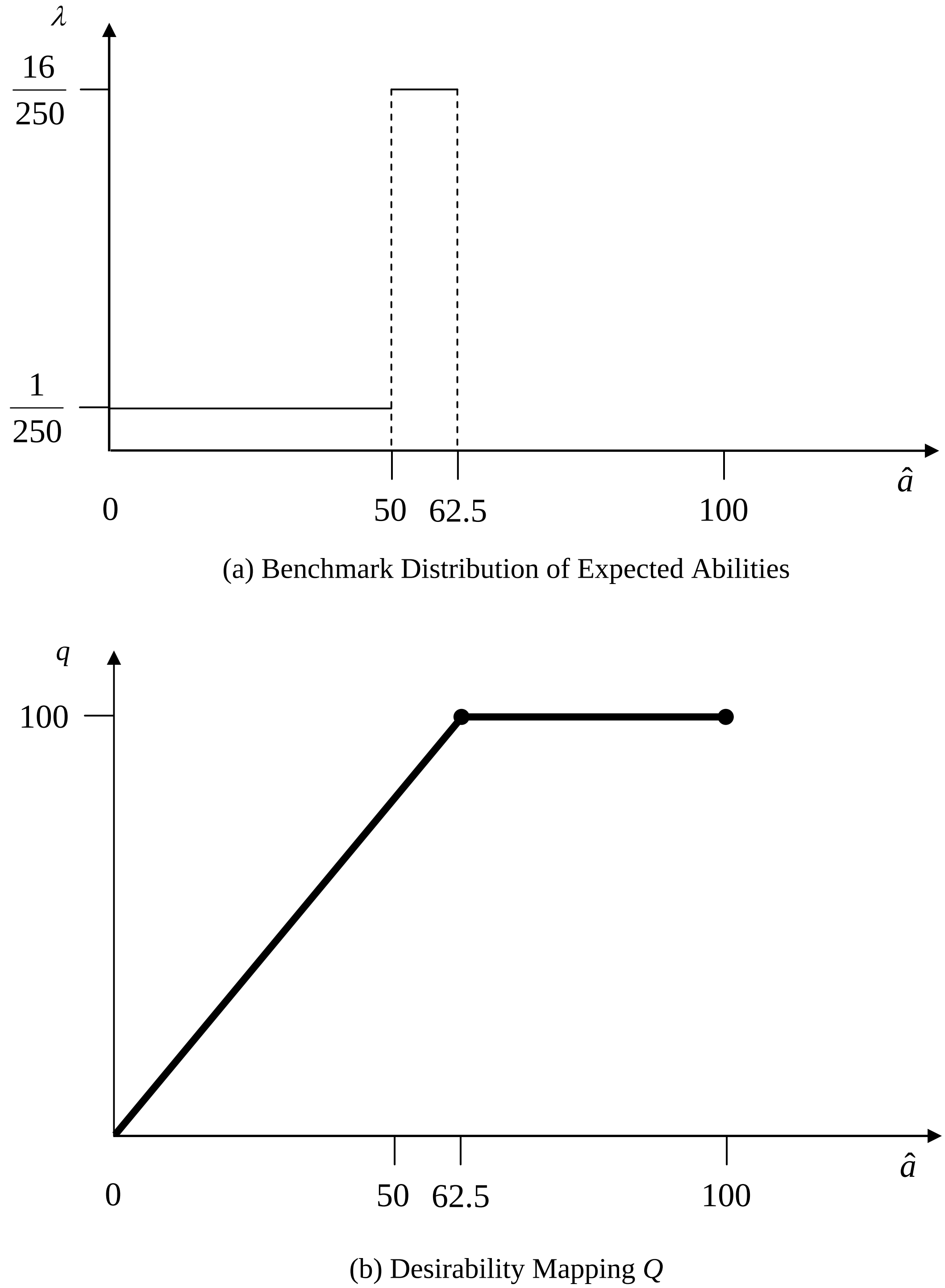



Figure VII

Benchmark Distribution of Expected Abilities and the Corresponding Desirability Mapping $Q$ in Example 4