On Non-Contemporaneous Short-Run Comovements*

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Abstract

This paper proposes the notion of *Polynomial Serial Correlation Common Features* as a measure of non-contemporaneous cyclical comovements in multiple time series. Statistical inference within this modeling is easily performed by reduced rank regression. We show the implications of the PSCCF in terms of the Beveridge-Nelson decomposition and we illustrate their relevance for empirical analyses.

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1 Motivation

In the empirical economic literature, business cycle regularities are often presented as comovements of the deviations from a "trend" in different macroeconomic time series. Beyond this general intuitive definition, no consensus emerges about how to measure the trend component and how to analyze these short-run comovements. Engle and Kozicki (1993) and Vahid and Engle (1993) proposed the Serial Correlation Common Feature (SCCF) as a measure of cyclical comovements in a similar spirit as cointegration is used for detecting long-run relationships.[1] In this context, common fluctuations describe a system with a reduced number of propagation mechanisms for the transmission of the information contained in the past.[2]

However, one observes that due to technical reasons (e.g. seasonal adjustment) as well as economic reasons (e.g. adjustment costs and habit formation), the hypothesis underlying SCCF, namely that a linear combination of stationary variables is independent from the past, is sometimes too strong. Consequently, different variants of the SCCF have been suggested in order to allow for adjustment delays. Examples are the notions of Codependent Cycles (CC) by Vahid and Engle (1997) and Weak Form Common Features by Hecq, Palm and Urbain (2000).

In this note we propose an alternative indicator of non-contemporaneous cyclical comovements labelled the Polynomial Serial Correlation Common Feature (PSCCF). To illustrate this concept, let us consider the theoretical model of consumer behavior that motivates the CC approach in Vahid and Engle (1997). In the case where the representative agent has to choose between two types of consumption goods, durables D_t and non-durables C_t, the standard permanent income hypothesis leads to the following implications for consumption components:

$$\Delta D_t = \gamma\varepsilon_t + \lambda\varepsilon_{t-1}, \tag{1}$$

$$\Delta C_t = \varepsilon_t, \tag{2}$$

where $\Delta = (1-L)$, L is the lag operator, ε_t is an innovation with respect to the information set $I_t \equiv \{D_\tau, C_\tau, Yd_\tau; \tau < t\}$, Yd_t indicates disposable income, and γ and λ are coefficients implied by preferences parameters.

Vahid and Engle (1997) suggest to test for these implications by looking for linear combinations of ΔD_t and ΔC_t which are independent from I_{t-1}. However, the statistical inference becomes more involved than in the standard SCCF framework since VMA terms are entailed. Instead, by lagging and multiplying by λ both sides of (2) and substracting this new equation from (1), we get

$$\Delta D_t - \lambda\Delta C_{t-1} = \gamma\varepsilon_t, \tag{3}$$

[1] Notice that term "cycle" has in this framework a different meaning from spectral analysis, see Cubadda (1999b).

[2] See Brillinger (1969) for this type of physical interpretation.

and hence there exists a linear combination of ΔD_t and ΔC_{t-1} which is a white noise. In particular, $(1, -\lambda L, 0)'$ is a (restricted) PSCCF vector for the trivariate process $(\Delta D_t, \Delta C_t, \Delta Yd_t)'$.

In the sequel we formalize the notion of PSCCF and we show that statistical inference can be performed by reduced rank regression. Then we analyze the implications of the PSCCF for the multivariate Beveridge and Nelson (1981, BN henceforth) trend-cycle decomposition. Finally, we estimate the model we have just sketched on post-war US data.

2 PSCCF: Definition and Statistical Inference

Consider the VAR(p) model for a n-vector of I(1) time series $\{y_t, t = 1, \dots, T\}$,

$$\Phi(L)y_t = \varepsilon_t, \tag{4}$$

for fixed values of $y_{-p+1}, ..., y_0$ and where $\Phi(L) \equiv I_n - \sum_{i=1}^{p} \Phi_i L^i$, and i.i.d. $N_n(0, \Omega)$ errors ε_t. For notational convenience, deterministic terms are omitted at this level of presentation.

We further assume that the process y_t is cointegrated of order (1,1), namely that 1°) $rank(\Phi(1)) = r$, $0 < r < n$, so that $\Phi(1)$ can be expressed as $\Phi(1) = -\alpha\beta'$, with α and β both $(n \times r)$ matrices of full column rank r and that 2°) the matrix $\alpha'_\perp \dot{\Phi}(1)\beta_\perp$ has rank equal to $(n-r)$ where $\dot{\Phi}(1)$ denotes the first derivative of $\Phi(z)$ at $z = 1$. The columns of β span the space of cointegrating vectors, and the elements of α are the corresponding adjustment coefficients. In order to rewrite the system in a VECM form we use the identity $\Phi(L) \equiv \Phi(1)L + \Phi^*(L)\Delta$ where $\Phi^*(L) = I_n - \sum_{i=1}^{p-1} \Phi_i^* L^i$, and $\Phi_i^* = -\sum_{j=i+1}^{p} \Phi_j$ for $i = 1, \dots, p-1$. And finally we obtain

$$\Phi(L) = \Delta - \underline{A}^*(L)L, \tag{5}$$

with the notations

$$\underline{A}^*(L) = \alpha\beta' + \underline{\Phi}^*(L)\Delta, \qquad \text{and} \qquad \underline{\Phi}^*(L) = \Phi^*(L) - I_n. \tag{6}$$

Alternatively, we can focus on the Wold representation of the stationary process Δy_t,

$$\Delta y_t = C(L)\varepsilon_t, \tag{7}$$

with $\sum_{j=1}^{\infty} j\,|C_j| < \infty$, $C_0 = I_n$ and the associate polynomial factorization $C(L) = C(1) + \Delta C^*(L)$ where $C_i^* = -\sum_{i+1}^{\infty} C_j$ for $i \geq 0$. Also note that by combining (4) and (7) we obtain the well-known equation,

$$C(L)\Phi(L)y_t = \Delta y_t. \tag{8}$$

Definition 1 ***Polynomial Serial Correlation Common Features:*** *The Series Δy_t have s PSCCF of order one iff there exists a $n \times s$ polynomial matrix $\delta(L) = \delta_0 + \delta_1 L$ such that the matrix δ_0 is full column rank and $\delta(L)'C(L) = \delta_0'$.*

Notice that the above definition can be easily extended to polynomial matrices of higher orders. In the Appendix, we provide the main results for the more general case.

Some implications of the PSCCF are worth considering. Premultiplying by $\delta(L)'$ both sides of equation (8) and using Definition 1, we obtain

$$\delta_0'\Phi(L)y_t = \delta(L)'\Delta y_t. \tag{9}$$

Hence, combining (5) and (9) we get

$$-\delta_0'\underline{A^*}(L)y_t = \delta_1'\Delta y_t. \tag{10}$$

Note that the converse implication holds as well, i.e. equation (10) implies that the series Δy_t have the PSCCF. In view of equations (6) and (10), we can thus formulate the following proposition:

Proposition 2 *The Series Δy_t have the PSCCF iff the following relations on the VECM coefficient matrices hold:*

$$\begin{array}{ll} Assumption\ 1. & \delta_0'\alpha = 0 \\ Assumption\ 2. & \delta_0'\Phi_i^* = \begin{cases} -\delta_1' & if\ i = 1 \\ 0 & if\ i > 1 \end{cases} \end{array}$$

Proposition 2 suggests a simple strategy for statistical inference. Indeed, since under PSCCF the matrix δ_0' lies in the left null space of all the VECM coefficient matrices but Φ_1^*, a ML approach requires to solve one of the following equivalent canonical correlation programs,

$$CanCor\left\{\Delta y_t, \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} \mid f_t, \Delta y_{t-1}\right\}, \quad \text{or} \quad CanCor\left\{\begin{pmatrix} \Delta y_t \\ \Delta y_{t-1} \end{pmatrix}, \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} \mid f_t\right\}, \tag{11}$$

where $CanCor(Y, X \mid Z)$ denotes the partial canonical correlations between the elements of Y and X conditional on Z, $\hat{\beta}$ is a superconsistent estimate of the cointegrating vectors, and f_t is a vector of fixed elements such a constant, a linear trend, and seasonal dummies. Notice that the only difference of the second program is the presence of n additional unit eigenvalues while the remaining n ones are identical. But the advantage of the second expression is that the eigenvectors associated with the s smallest eigenvalues give estimates for both δ_0 and δ_1 matrices.

One can test for the null hypothesis that there exist at least s PSCCF vectors through the LR statistic (see *inter alia* Anderson, 1984, Velu *et al.,* 1986)

$$\zeta_{PSCCF} = -T\sum_{i=1}^{s} \ln(1-\hat{\lambda}_i), \qquad s = 1\ldots n, \tag{12}$$

where $0 \leq \ldots \hat{\lambda}_i \ldots < 1$, are the (ordered) smallest squared canonical correlation coming from the solution of (11), with s, the potential number of zero eigenvalues. The test statistic (12), which is appropriate to test for PSCCF, follows asymptotically a $\chi^2_{(v)}$ distribution under the null where $\nu = s \times (n(p-1-1)+r) - s \times (n-s)$. In the empirical section we also use a small sample correction proposed in Hecq (2000) similar to the one often used for cointegration analysis, namely one applies to (12) the scaling factor[3] $(T - n(p-2) - r)/T$.

Alternatively, an efficient estimation of elements of $\delta(L)$, including the standard errors, is obtained, for given s, by FIML in a model with s pseudo structural equations where the RHS variables is given only by the Δy_{t-1} and additional $(n-s)$ unrestricted equations such that

$$\begin{pmatrix} I_s & \delta^{*\prime}_{0,s\times n-s} \\ 0_{(n-s)\times s} & I_{n-s} \end{pmatrix} \Delta y_t = \begin{pmatrix} 0_{s\times r} & \delta'_{1,s\times n} & 0_{s\times n} & \ldots & 0_{s\times n} \\ \tilde{\alpha} & \tilde{\Phi}^*_1 & \tilde{\Phi}^*_2 & \ldots & \tilde{\Phi}^*_{p-1} \end{pmatrix} \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} + \upsilon_t, \tag{13}$$

where the contemporaneous cofeature matrix $\delta'_0 = (I_s, \delta^{*\prime}_{0,s\times n-s})$ has been normalized and just-identified with an identity matrix on the first s elements and where $\tilde{\alpha}$ and $\tilde{\Phi}^*_i$ stands for the remaining $n-s$ coefficient matrices in the VECM. In (13) we have assumed that the first block of δ_0 is full rank. Otherwise, a rotation of the co-feature matrix is necessary. Interestingly, the PSCCF modeling given by equations (13) may be used to test for additional restrictions coming from the economic theory such as those in (3).

Remark 3 *Notice that the constraints given in Proposition 2 are always true in a cointegrated VAR(2) with $\delta_0 = \alpha_\perp$. This is simply an extension of the property put forward by Vahid and Engle (1993) that the SCCF vector is trivially $\alpha_\perp$ in a cointegrated VAR(1). Consequently, VAR(p) with $p > 2$ are worth considering in empirical analyses.*

[3] In a Monte Carlo investigation (not reported here to save space but available upon request), it emerges that the test statistic (12) has good finite sample properties. However, size distortions increase with the number of variables n and the lag length p as expected according to the results by Abadir *et al.* (1999). Testing becomes more difficult when the number s of cofeature vectors is large, and the cointegration rank and vectors are estimated. In most cases, the small sample correction works quite well.

3 PSCCF and Non-Contemporaneous Cycles

In this section we focus on the properties of PSCCF in terms of the multivariate BN representation of the series y_t. In particular, we can decompose y_t in a trend and a cyclical component as follows

$$y_t = \tau_t + \xi_t, \tag{14}$$

where $\Delta\tau_t = C(1)\varepsilon_t$ and $\xi_t = C^*(L)\varepsilon_t$ (see e.g. Vahid and Engle, 1993). Since it holds $C(1) = -\beta_\perp \left(\alpha'_\perp \dot{\Phi}(1)\beta_\perp\right)^{-1} \alpha'_\perp$ (see e.g. Johansen, 1995), we know that under cointegration the series y_t have $(n-r)$ common stochastic trends.

Let us now see if the PSCCF has any implication for the cyclical components ξ_t. If we combine the equation

$$\delta(L)'C(L) = \delta'_0 + \sum_{i\geq 1}(\delta'_1 C_{i-1} + \delta'_0 C_i)L^i, \tag{15}$$

with the definition of the PSCCF we obtain

$$\delta'_1 C_i = \delta'_0 C_{i+1}, \text{ for } i \geq 0. \tag{16}$$

By construction of $C^*(L)$ and in view of equation (16) we see that

$$\delta(L)'C^*(L) = \delta'_0 C^*_0 - \sum_{j\geq 1}\sum_{i>j}(\delta'_1 C_{i-1} + \delta'_0 C_i)L^j = \delta'_0(I_n - C(1)) = \delta'_1 C(1), \tag{17}$$

where the last equality follows from the relation $\delta(1)'C(1) = \delta'_0$. Equation (17) indicates that $\delta(L)'\xi_t$ is an innovation. We summarize this result in the following proposition:

Proposition 4 *If the series Δy_t have the PSCCF then the BN cycles of y_t have the same PSCCF.*

Interestingly enough, the converse implication of Proposition 4 generally fails. Indeed, from the relation $C_{i+1} = C^*_{i+1} - C^*_i$, for $i \geq 0$, it can be easily shown that if there exists a $n \times s$ polynomial matrix $\delta^*(L) = \delta^*_0 + \delta^*_1 L$ such that the matrix δ^*_0 is full column rank and $\delta^*(L)'C^*(L) = \delta^{*\prime}_0 C^*_0$ then it holds that

$$\delta^*(L)'C(L) = \delta^{*\prime}_0 + (\delta^{*\prime}_1 C(1) - \delta^{*\prime}_0)L, \tag{18}$$

Hence, the polynomial linear combination $\delta^*(L)'\Delta y_t$ will be a VMA(1) process.

Notice that the presence of PSCCF have similar implications on the BN cycles as the notion of CC.[4] However, there are two main differences between these approaches. First, the

[4]Indeed, Vahid and Engle (1997) proved that the same linear combination that renders the series Δy_t a

CC relation involves only contemporaneous elements of the vector series and hence it is not informative on the lead-lag structure of the cyclical components. Secondly, the presence of CC is associated to linear combinations of the series Δy_t with a VMA structure thus rendering optimal statistical inference more involved.

4 Durable and Non-durable Expenditures

We finally illustrate the use of a PSCCF approach for modeling the relationship between durable and non-durable consumption expenditures. We use US data on real per capita durable consumption expenditures (D_t), non-durable plus services consumption expenditures (C_t), and disposal personal income (Yd_t).[5] In order to avoid the observations occurring during the Korean War, during the period of price control (extreme inflation variability) and Treasury-Fed accord, our analysis runs over 1954:Q1-2000:Q1. The first eight observations are used as pre-sample values for relationships containing lags. We determine the lag structure both using information criteria and LR test statistics from 1 to 8, that means that $T = 177$. In the light of the debate in the literature about transforming the variables in logarithms or not, we consider both linear and log-linear versions. In both cases, the model which best summarizes the covariation of the data is a VAR(6) with an unrestricted constant.

Using Johansen trace test we reject at the 5% significance level the null hypothesis of zero cointegrating vectors: $H_{0,r=0} = 45.87$ for the log-linear version and $H_{0,r=0} = 31.26$ for the linear one. Moreover the hypothesis of more than one cointegrating vector is rejected in bot cases: $H_{0,r\leq 1} = 4.24$ for the log version and $H_{0,r\leq 1} = 8.82$ otherwise. Fixing the first cointegrating vector to its estimated value, Table 1 reports PSCCF test statistics. It emerges that we cannot reject the null hypothesis of one PSCCF vector for both versions. Note that the hypothesis that there exists a SCCF structure is rejected with a *p-values* of 0.009 and 0.016 for respectively the log-linear and the linear models.

INSERT TABLES 1 & 2

Normalizing the PSCCF vectors by the coefficient of durable expenditures we have the following estimates for $\Delta y_t = (\Delta D_t, \Delta C_t, \Delta Y d_t)'$ or $(\Delta d_t, \Delta c_t, \Delta y d_t)'$,

$$\text{linear version} \;:\; \delta_0 = (1, \underset{(0.123)}{-0.225}, \underset{(0.067)}{-0.155})' \quad \text{and} \quad \delta_1 = (\underset{(0.086)}{-0.291}, \underset{(0.085)}{0.125}, \underset{(0.032)}{0.035})'$$

$$\text{log-linear version} \;:\; \delta_0 = (1, \underset{(2.491)}{2.550}, \underset{(1.950)}{-6.715})' \quad \text{and} \quad \delta_1 = (\underset{(0.141)}{-0.323}, \underset{(1.482)}{-1.623}, \underset{(0.695)}{1.646})'$$

VMA(1) process is a SCCF relationship for the BN cycles of y_t.

[5]The data come from historical national income and product accounts series and are available from the BEA website at www.bea.doc.gov (also on the homepage www.personeel.unimaas.nl/a.hecq). They reflect the latest annual revision of the NIPA's, released on July 28, 2000 and reported in the August 2000 Survey of Current Business. More precisely the variable we consider are from NIPA *Table 8.7. Selected Per Capita Product and Income Series in Current and Chained Dollars.*

where the FIML asymptotic standard errors are in brackets. Table 2 reports additional exclusion restrictions implied by the theoretical model. It emerges that only for the linear model exclusion restrictions of the growth of disposal income are not rejected.

The PSCCF approach is very interesting especially when additional exclusion restrictions are tested on the common feature space. In this application, we cannot reject the presence of one PSCCF relationship between disposal income, durable and non-durable expenditures. However the exclusion restrictions put forward by the theoretical model are rejected. Several reasons can be given such as the misclassification of some durable goods as non-durables, the use of seasonally adjusted series (Cubadda, 1999a) or some kind of misspecification of the empirical model. Notice that a similar kind of exclusion restrictions may be used in business cycle analyses for the extraction of a composite leading indicator (CLI). For instance, suppose that the vector series $\Delta y_t = (\Delta gdp_t, \Delta x_t')'$ has the PSCCF and there exist a polynomial matrix $\delta(L)$ with $\delta_0' = (1, 0')'$ and $\delta_1' = (0, \delta_1^{*\prime})'$ where $\delta_1^{*\prime} \neq 0$. In this case, a CLI with desirable properties is given by the combination $\delta_1^{*\prime}\Delta x_t$, see Cubadda and Hecq (2001) for further details.

5 Appendix

This Appendix generalizes the results for a Polynomial Serial Correlation Common Feature of order m.

Definition 5 ***PSCCF(m):*** *The Series Δy_t have s polynomial serial correlation common features of order m, henceforth PSCCF(m), iff there exists a $n \times s$ polynomial matrix $\delta(L) = \sum_0^m \delta_i L^i$ such that the matrix δ_0 is full column rank and $\delta(L)'C(L) = \delta_0'$.*

$$-\delta_0' \underline{A}^*(L) y_{t-1} = \sum_{i=1}^{m} \delta_i' \Delta y_{t-i}. \tag{A1}$$

Proposition 2 can be easily generalized as follows,

Proposition 6 *The Series Δy_t have the PSCCF(m) iff the following relations on the VECM coefficient matrices hold:*

$$\begin{array}{lrcl} \textit{Assumption 1.} & \delta_0'\alpha & = & 0 \\ \textit{Assumption 2.} & \delta_0'\Phi_i^* & = & \left\{ \begin{array}{ll} -\delta_i' & if\ i = 1, ..., m \\ 0 & if\ i > m \end{array} \right. \end{array}$$

Let us now see the implications of the PSCCF(m) for the BN cyclical components ξ_t. If we premultiply by $\delta(L)'$ both sides of the equation $C(L) = C(1) + \Delta C^*(L)$ and in view of Definition 5 we get

$$\delta_0' = \delta(L)'C(1) + \Delta\delta(L)'C^*(L). \tag{A2}$$

Combining equation (A2) with the equation $\delta(1)'C(1) = \delta_0'$ we see that

$$\Delta\delta(L)'C^*(L) = \sum_{i=1}^{m}(1 - L^i)\delta_i' C(1). \tag{A3}$$

Finally, since $(1 - L^i) = \Delta \sum_0^{i-1} L^j$, we can rewrite equation (A3) as follows

$$\delta(L)'C^*(L) = \sum_{i=1}^{m}(\sum_{j=0}^{i-1} L^j)\delta_i' C(1). \tag{A4}$$

Equation (A4) allows us to formulate the following proposition:

Proposition 7 *When the series Δy_t have the PSCCF(m) then $\delta(L)'\xi_t$ is a VMA($m-1$) process.*

However, the converse implication of Proposition 7 generally fails. Indeed, it is easy to see that if there exists a $n \times s$ polynomial matrix $\delta^*(L)$ of order m such that the matrix δ_0^* is full column rank and $\delta^*(L)'C^*(L)$ is a polynomial matrix of order $(m-1)$ then $\delta^*(L)'\Delta y_t$ will be a VMA(m) process.

References

[1] Abadir, K.M., K. Hadri and E. Tzavalis (1999), The Influence of VAR Dimensions on Estimator Biases, *Econometrica*, 67, 163-181.

[2] Anderson, T.W. (1984), *An Introduction to Multivariate Statistical Analysis*, 2nd Ed. (John Wiley & Sons).

[3] Beveridge, S. and C.R. Nelson (1981), A New Approach to Decomposition of Economic Time Series Into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle, *Journal of Monetary Economics*, 7, 151-174.

[4] Brillinger, D.R. (1969), The Canonical Analysis of Stationary Time Series, in *Multivariate Analysis-II*, ed. P.R. Krishnaiah, New York: Academic Press, 331-350.

[5] Cubadda, G. (1999a), Common Cycles in Seasonal Non-Stationary Time Series, *Journal of Applied Econometrics*, 14, 273-291.

[6] Cubadda, G. (1999b), Common Serial Correlation and Common Business Cycles: a Cautious Note, *Empirical Economics*, 24, 529-535.

[7] Cubadda, G. and A. Hecq (2001), The Role of Common Cyclical Features for Composite Leading Indicators Building, *Maastricht University Research Memorandum.*

[8] Engle, R. F. and S. Kozicki (1993), Testing for Common Features (with comments), *Journal of Business and Economic Statistics*, 11, 369-395.

[9] Hecq, A. (2000), Common Cyclical Features in Multiple Time Series and Panel data, Maastricht University Ph.D. Thesis.

[10] Hecq, A., F.C. Palm and J.P. Urbain (2000), Permanent-Transitory Decomposition in VAR Models with Cointegration and Common Cycles, *Oxford Bulletin of Economics and Statistics,* 62, 511–532.

[11] Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models* (Oxford University Press: Oxford)

[12] Vahid, F. and R.F. Engle (1993), Common Trends and Common Cycles, *Journal of Applied Econometrics*, 8, 341-360.

[13] Vahid, F. and R.F. Engle (1997), Codependent Cycles, *Journal of Econometrics*, 80, 199-221.

[14] Velu, R.P., G.C. Reinsel and D.W. Wichern (1986), Reduced Rank Models for Multivariate Time Series, *Biometrika*, 73, 105-118.

	H_0	$\hat{\lambda}$	$Test$	$Test^{small}$	$\chi^2 df$	p-value	p-value^{small}
log-linear	$s \geq 1$	0.087	16.20	15.00	11	.134	.182
	$s \geq 2$	0.186	52.71	48.84	24	<.001	.002
	$s = 3$	0.291	113.67	105.32	39	<.001	<.001
linear	$s \geq 1$	0.065	11.83	10.96	11	.376	.445
	$s \geq 2$	0.184	47.92	44.40	24	.002	.007
	$s = 3$	0.305	112.37	104.12	39	<.001	<.001

Table 1: PSCCF Tests Statistics (1956:Q1-2000:Q1)

	H_0	$Test$	$\chi^2 df$	p-value
linear model	$H_0^1 : \begin{cases} \delta_0 = (1, *, 0)' \\ \delta_1 = (*, *, 0)' \end{cases}$	4.59	2	.101
	$H_0^2 : \begin{cases} \delta_0 = (1, 0, 0)' \\ \delta_1 = (*, *, 0)' \end{cases}$	10.79	3	.013
	$H_0^3 : \begin{cases} \delta_0 = (1, *, 0)' \\ \delta_1 = (0, *, 0)' \end{cases}$	14.05	3	.003
	$H_0^4 : \begin{cases} \delta_0 = (1, 0, 0)' \\ \delta_1 = (0, *, 0)' \end{cases}$	15.34	4	.004
log-linear model	$H_0^1 : \begin{cases} \delta_0 = (1, *, 0)' \\ \delta_1 = (*, *, 0)' \end{cases}$	20.67	2	<.001
	$H_0^4 : \begin{cases} \delta_0 = (1, 0, 0)' \\ \delta_1 = (0, *, 0)' \end{cases}$	29.96	4	<.001

Table 2: PSCCF Exclusion Test Statistics