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# Outside options: Another reason to choose the first-price auction

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## ABSTRACT

In this paper we study equilibrium and experimental bidding behaviour in first-price and second-price auctions with outside options.

We find that bidders do respond to outside options and to variations of common knowledge about competitors' outside options. However, overbidding in first-price auctions is significantly higher with outside options than without. First-price auctions yield more revenue than second-price auctions. This revenue-premium is significantly higher with outside options. In second-price auctions the introduction of outside options has only a small effect.

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## 1. Introduction

During the last decade, auctions have increasingly attracted attention from academia and the wider public. A major part of this increased interest is due to growing popularity of auctions as market institutions for consumer-to-consumer and business-to-consumer transactions, allocating public resources and procurement contracts. Cases in point are worldwide spectrum auctions, online auction platforms such as eBay and Ricardo and virtual business-to-business market places, e.g. Covisint for the automotive industry or Consip's AiR for Italian public procurement offers.

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Typically outside options are available to bidders in addition to the object offered in the particular auction. For example, consider the fierce bidding war of the software giants Oracle and SAP in 2005 to acquire Retek, a developer of specialised software for retailing firms. The “object” in this takeover auction, Retek, offered both firms the opportunity to acquire specialised software and an established customer base. Clearly, both bidders faced the outside option to develop software and build up a customer base independent of the takeover. In fact, the unsuccessful bidder SAP announced in a press release on 22 March 2005 that it precisely plans to follow that outside option after revealing its unwillingness to continue bidding.

Sequential sales of similar products constitute another example for outside options. Bidders in the first sale know that they have the option to bid for the product also in the second sale.

In this paper we augment the standard symmetric independent private value (SIPV) model to allow for public and private outside options. We derive equilibrium bidding functions and implement the auction in the laboratory.

To our knowledge, there is no literature systematically inquiring the effects of outside options in auctions other than [Cherry et al. \(2004\)](#).<sup>1</sup> We show that a model with jointly distributed private values and outside options can be reduced to a standard SIPV model.<sup>2</sup> One special case is analysed by [Holt \(1980\)](#) who assumes that valuations are constant and the same for all bidders. A related case is examined by [Weber \(1983\)](#), [Gale and Hausch \(1994\)](#), and [Reiß and Schöndube \(2007\)](#) who study sequential auction models: A subsequent auction in such a sequential auction process can be interpreted as a specific outside option whose value is endogenously determined. [Brosig and Reiß \(2007\)](#) demonstrate that bidders' behaviour in a prior auction is affected by a subsequent first-price procurement auction. In their complex outside option model, the expected option value depends on beliefs about other bidders' entry behaviour, their bidding behaviour in future auctions and the cognitive abilities to assess the outside option value given beliefs. In contrast, in our paper we isolate the effects of exogenous outside option values from other factors and systematically investigate their impact on allocative efficiency, seller's revenue, and bidders' behaviour in first-price and second-price auctions.

In particular, in our experiments we want to find out the following for the first-price and the second-price auction:

- Do exogenous outside options affect bids at all?
- Do bids in the laboratory deviate from equilibrium bids in the same way as they deviate in standard auctions without outside options?
- How are revenue and efficiency affected if outside options are present?

The plan of the paper is as follows: In Section 2 we introduce outside options into the SIPV auction model and derive equilibrium bidding strategies for the first-price and second-price auctions, Section 3 describes our experimental design, Section 4 provides experimental results and Section 5 concludes.

## 2. The SIPVs auction model with outside options

There are  $n$  risk neutral individuals with single-object-demand. Each individual  $i$  has a valuation  $v_i$  for an object that is for sale in an auction. In addition to the auction offer each individual has access to an outside option that can be substituted for the object offered in the auction. The value that an individual derives from exerting the outside option and not receiving the auctioned object is denoted

<sup>1</sup> [Cherry et al. \(2004\)](#) inquire into the value of laboratory testbed markets regarding field applications. They note that one difference between laboratory and field is the possibility of substitutes in the latter. To test the robustness of the testbed hypothesis, they conduct a second-price auction with the possibility to buy an object identical to that offered in the auction at some price  $p$ . They find that bidding behaviour is affected by the presence of outside options.

<sup>2</sup> In particular, our theoretical analysis will include the case of correlated values and outside options. See Section 2.3.

by  $w_i$ .<sup>3</sup> We assume here that receiving the auctioned object eliminates the value of the outside option entirely. Individuals may exert their outside options before, during or after the auction.

We distinguish between public and private outside options. In the case of public outside options, each individual derives the same benefit from the outside option. This is common knowledge. In contrast, private outside options are individual-specific and private information.

We briefly report equilibrium bidding functions for first-price and second-price auction in the SIPV model in Section 2.1. In Section 2.2 we introduce public outside options. In Section 2.3 we extend the SIPV model to allow for private outside options.

### 2.1. Bidding without outside options

Consider first the case of an object which has individual valuation  $v_i$  for each bidder  $i$ . This valuation is private information and independently and identically distributed according to a cumulative distribution function  $F(v_i)$  where  $v_i \in [\underline{v}, \bar{v}]$ . Without outside options, the symmetric risk neutral Bayes–Nash equilibrium bidding functions for the first- and second-price auctions are well known (cf. Riley and Samuelson, 1981; Vickrey, 1961): For the first-price auction we have  $b^{fp}(v) = v - \int_{\underline{v}}^v F^{n-1}(x) dx / F^{n-1}(v)$  and for the second-price auction  $b^{sp}(v) = v$ .

### 2.2. Public outside options

As in Section 2.1 individual  $i$  has a valuation  $v_i$  for the auctioned object. Now, however, individual  $i$  can also exert the public outside option and obtain a value  $w$  which is, in the case of a public outside option, the same across all individuals. We assume  $w \leq \underline{v}$ . This ensures that every individual voluntarily participates in a standard auction.

We will use the payoff equivalence theorem to derive the equilibrium bidding function.<sup>4</sup> From the bidder's perspective the auction with public outside options can be interpreted as a standard auction where bidders who fail to win the object receive a payment of  $w$ , thus, the application of the payoff equivalence theorem is possible.

Following Riley and Samuelson (1981), let  $\Pi(x, v)$  be the expected payoff that a representative bidder with valuation  $v$  receives if mimicking valuation  $x$ , provided that all competitors adhere to the common equilibrium bidding strategy  $b^{fp-pb}()$ . The superscript indicates the auction format (fp for first-price auction) and the type of outside option (pb for public outside options). From payment equivalence without auction reserve price (cf. Riley and Samuelson, 1981, Eqs. (7) and (8)), we obtain immediately the expected equilibrium payment of the representative bidder with valuation  $v$ . This payment depends on the expected equilibrium payoff to a bidder with the lowest valuation  $\underline{v}$ :

$$P(v) = vF^{n-1}(v) - \int_{\underline{v}}^v F^{n-1}(x) dx - \Pi(\underline{v}, \underline{v}). \quad (1)$$

For a bidder with the lowest valuation to be indifferent between participating and not participating in the auction, the bidder must receive at least the outside option  $w$  in the auction. This yields the condition  $\Pi(\underline{v}, \underline{v}) = w$ . For the first-price design with the modification that each unsuccessful bidder receives  $w$  the expected equilibrium payment is given by

$$P(v) = F^{n-1}(v)b^{fp-pb}(v) - [1 - F^{n-1}(v)]w. \quad (2)$$

Combining both expressions for expected equilibrium payment, (1) and (2), and solving for  $b^{fp-pb}(v)$  leads to the intuitive result that the equilibrium bid under the first-price design with public outside

<sup>3</sup> Valuations of transaction alternatives are net of transaction costs. If, for instance, an alternative object is offered at a posted price, then  $w_i$  represents the value of the outside option net of its price. If there are many alternatives, then  $w_i$  corresponds to the best alternative net of prices.

<sup>4</sup> Myerson (1981) and Riley and Samuelson (1981) derived the payoff equivalence theorem in auction theory. According to the theorem the expected revenue of an auction seller and the expected payoffs of risk neutral bidders in Bayes–Nash equilibrium are invariant within a large class of auction formats encompassing first-price and second-price auction.

option precisely matches that without public outside options reduced by the outside option's value:

$$b^{\text{fp-pb}}(v, w) = b^{\text{fp}}(v) - w, \tag{3}$$

where  $b^{\text{fp}}(v)$  is the equilibrium bidding function in the first-price auction without outside options as defined in Section 2.1.

The weakly dominant bidding strategy<sup>5</sup> for the second-price auction is

$$b^{\text{sp-pb}}(v, w) = v - w. \tag{4}$$

Since  $b^{\text{fp-pb}}$  and  $b^{\text{sp-pb}}$  are monotonic in  $v$  the allocation in equilibrium is always efficient in the public outside options case.

### 2.3. Private outside options

The valuation of individual  $i$  is, again,  $v_i$ . The value from exerting the outside option is  $w_i$ . Valuation pairs  $(v, w) \in [\underline{v}, \bar{v}] \times [\underline{w}, \bar{w}]$  are independently distributed across individuals according to the probability density function  $f(v, w)$  and are their private information. The joint distribution  $f(v, w)$  allows for correlations between valuation and outside option. Again we assume that no outside option  $w$  is larger than the lowest valuation  $\underline{v}$ , i.e.  $\bar{w} \leq \underline{v}$ . This ensures that each individual will participate in the auction.

#### 2.3.1. Equilibrium bidding: First-price auction

In order to derive the equilibrium bidding strategy in the first-price auction, we represent the bidding model such that we can employ the payoff equivalence theorem. Its application to the case of private outside options is not trivial since interpreting the outside option as an auctioneer's payment to unsuccessful bidders implies bidder-specific payments. This would violate the theorem's assumption of anonymity which requires allocation and payment rules to depend only on submitted bids and not on other individual characteristics. We, therefore, reduce the two-dimensional bidder's type  $(v, w)$ , to a single-dimensional type  $x := v - w$ .

Consider the utility maximisation problem of the representative risk neutral individual  $i$  that submits bid  $b_i$  in some auction satisfying payoff equivalence (cf. Riley and Samuelson, 1981). Let individual  $i$  face outside option  $w_i$  and denote the bidder's expected payment according to the auction rules by  $P(\cdot)$  where  $P(\cdot)$  depends on all submitted bids:

$$\max_{b_i} \Pi = \Pr(b_i \text{ wins}) \cdot v_i - P(b_1, \dots, b_n) + [1 - \Pr(b_i \text{ wins})] \cdot w_i.$$

This program can be rearranged to

$$\max_{b_i} \Pi = \Pr(b_i \text{ wins}) \cdot (v_i - w_i) - P(b_1, \dots, b_n) + w_i. \tag{5}$$

Since the outside option  $w_i$  is known to the individual and a constant we can drop  $w_i$ . Furthermore, we replace  $v_i - w_i$  by  $x_i$ .

$$\max_{b_i} \tilde{\Pi} = \Pr(b_i \text{ wins}) \cdot x_i - P(b_1, \dots, b_n) \tag{6}$$

We interpret  $x \in [\underline{v} - \bar{w}, \bar{v} - \underline{w}]$  as an individual's net valuation of the object. One way to view the transformation of the original maximisation problem into (6) is to suppose that the representative individual exerts the outside option  $w_i$  before bidding in the auction, and, in case the auction is won (since we have single-object-demand) repays the value of the outside option.

We have now a standard bidding problem with net valuation  $x$ . Next, we identify the probability density function of net valuations. Note that infinitely many valuation pairs  $(v, w)$  lead to the same net valuation  $x$ . The probability density of a given net valuation  $x$  is obtained by summing up all

<sup>5</sup> The proof is identical to that for the equilibrium bidding strategy under the second-price auction with private outside options, see Section 2.3.

densities over all pairs  $(v, w)$  with the same net valuation  $x$ ,

$$f_X(x) = \int_{\max\{w, v-x\}}^{\min\{w, v-x\}} f(x+w, w) dw, \tag{7}$$

where  $f_X(x)$  is the probability density function of net valuations and  $F_X(x)$  the associated cumulative distribution function. Similar to the case of public outside options (see Eq. (1)) we obtain the expected equilibrium payment  $P(x)$  using the payoff equivalence theorem:

$$P(x) = xF_X^{n-1}(x) - \int_x^x F_X^{n-1}(y) dy - \tilde{\Pi}(x, x). \tag{8}$$

For the marginal bidder with the lowest net valuation  $x$  to be indifferent between participating and not participating in the auction, the bidder must receive the outside option in the auction. Since, for any net valuation  $x$  we have  $\Pi = \tilde{\Pi} + w_i$  by the above transformation it follows  $\tilde{\Pi}(x, x) = 0$ . Hence, for the first-price auction the expected equilibrium payment is given by

$$P(x) = F_X^{n-1}(x) b^{\text{fp-pr}}(x), \tag{9}$$

where the superscript fp-pr indicates the case of a first-price auction (fp) with private outside options (pr). Combining both expressions for expected equilibrium payment, (8) and (9), solving for  $b^{\text{fp-pr}}(\cdot)$ , and substituting for net valuations leads to the equilibrium bidding function:

$$b^{\text{fp-pr}}(v, w) = v - w - \frac{\int_{v-w}^{v-w} F_X^{n-1}(y) dy}{F_X^{n-1}(v-w)}, \tag{10}$$

which is strictly increasing in  $v$  and strictly decreasing in  $w$  since  $\partial b/\partial x > 0$  and  $\partial x/\partial v = -\partial x/\partial w = 1$ .

As a special case, let us now derive the equilibrium bidding function for the parametrisation we use in the experiment. Suppose  $n = 2$  and let valuation-outside option pairs  $(v, w)$  be uniformly distributed over the domain  $[50, 100] \times [0, 50]$ . The joint probability density function of  $(v, w)$  is  $f(v, w) = 1/2500$ . To find the probability density function of net valuations  $f_X(x)$  we use Eq. (7) and obtain

$$f_X(x) = \begin{cases} \frac{x}{2500} & \text{if } x \in [0, 50], \\ \frac{100-x}{2500} & \text{if } x \in [50, 100]. \end{cases} \tag{11}$$

The associated cumulative distribution function  $F_X(x)$  is

$$F_X(x) = \begin{cases} \frac{x^2}{5000} & \text{if } x \in [0, 50], \\ \frac{200x - x^2 - 5000}{5000} & \text{if } x \in [50, 100]. \end{cases} \tag{12}$$

As a result, the symmetric risk neutral Bayes-Nash equilibrium bidding function is given by

$$b^{\text{fp-pr}}(x) = \begin{cases} \frac{2}{3}x & \text{if } x \in [0, 50], \\ \frac{300x^2 - 2x^3 - 250000}{600x - 3x^2 - 15000} & \text{if } x \in [50, 100]. \end{cases} \tag{13}$$

The bidding function  $b^{\text{fp-pr}}(x)$  is continuous and differentiable at  $x = 50$ .  $b^{\text{fp-pr}}(v, w)$  can be obtained by substituting  $x = v - w$ .

### 2.3.2. Equilibrium bidding: Second-price auction

For the second-price auction without outside options it can be shown with a standard argument that bidding the own valuation in the auction is a weakly dominant strategy for each bidder. Following a similar argument we also find that bidding the own net valuation  $x_i = v_i - w_i$  is a weakly dominant strategy in the second-price auction with outside options.

### 2.3.3. Efficiency with private outside options

Since bidding functions are monotonic in  $x$  the object is in equilibrium allocated to the bidder with the highest net valuation. One can easily see that this leads to an efficient allocation.

## 3. Experimental design and procedures

To test the theoretical implications of public and private outside options in the SIPV model, we use five treatments in a between-subjects design. We vary the type of outside options (none/public/private) and the auction design (first price/second price). In Appendix A we summarise the treatment parameters for each of the 19 experimental sessions. Three hundred and forty subjects participated in the experiment. Sessions lasted for about 75 min. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007).

### 3.1. Treatment types

We distinguish between three types of treatments: A, B, and C. In the baseline treatment, A, we ran standard auction games without outside options and independent private values with two bidders. Valuations were randomly drawn from a uniform distribution with support [50, 100]. In the baseline treatment A we use either a first-price auction (treatment A1) or a second-price auction (treatment A2). There were neither minimum bids nor entry fees. Bids and valuations were denominated in experimental currency units (ECU). In each experimental session twelve auction rounds were played. We used a strangers-matching design such that no bidder was matched with the same opponent in two consecutive rounds.

### 3.2. Strategy method

In standard auction experiments bidders first learn their payoff-relevant valuation before they submit a bid, e.g., Cox et al. (1982), Kagel and Levin (1993) or Ockenfels and Selten (2005). In our experiment we use the strategy method to elicit continuous bidding functions. Fig. 1 shows a typical input screen. We ask participants to submit bids for six hypothetical valuations 50, 60, 70, 80, 90 and 100. Bids for intermediate valuations are interpolated linearly. The bidding function is displayed as a graph at all times. Bidders can adjust their entered bids and, thus, their bid functions until they are satisfied. Bids are entered via keyboard, they have to be non-negative, not larger than 200, and have at most two decimal places. Only after all bids are submitted, values were drawn.

Related implementations of the strategy method have been used in a few other experiments. Selten and Buchta (1999) introduced the strategy method for auction experiments. There bidders could specify a piecewise linear bidding functions with up to 10 000 segments either using a graphical input mode or via keyboard (valuations ranged between 0.00 and 100.00). The number of possible segments in this setup was apparently too high: 46% of observed bidding functions were non-monotonic—the authors observe that bidders drew landscapes rather than bidding functions (p. 81). Pezanis-Christou and Sadrieh (2004) used a highly simplified version of the Selten and Buchta (1999) implementation. Bidders could specify only two segments. In their study, approximately 15% of bidding functions are non-monotonic in their asymmetric auction treatments and approximately 5% in their symmetric auction treatments.<sup>6</sup> Güth et al. (2002, 2003) also use the strategy method, though, with a much smaller set of only 11 possible valuations. For each of these eleven values, bidders had to enter a corresponding value. Intermediate values, which were possible in Selten and Buchta (1999), Pezanis-Christou and Sadrieh (2004), and in our experiment, were not possible in the experiment of Güth et al.

Our experiment constitutes, thus, a compromise between the potentially highly complex bidding function of Selten and Buchta (1999) and the simpler designs of Pezanis-Christou and Sadrieh (2004)

<sup>6</sup> The percentages are inferred from bar charts in Pezanis-Christou and Sadrieh (2004, Figs. 3 and 5).

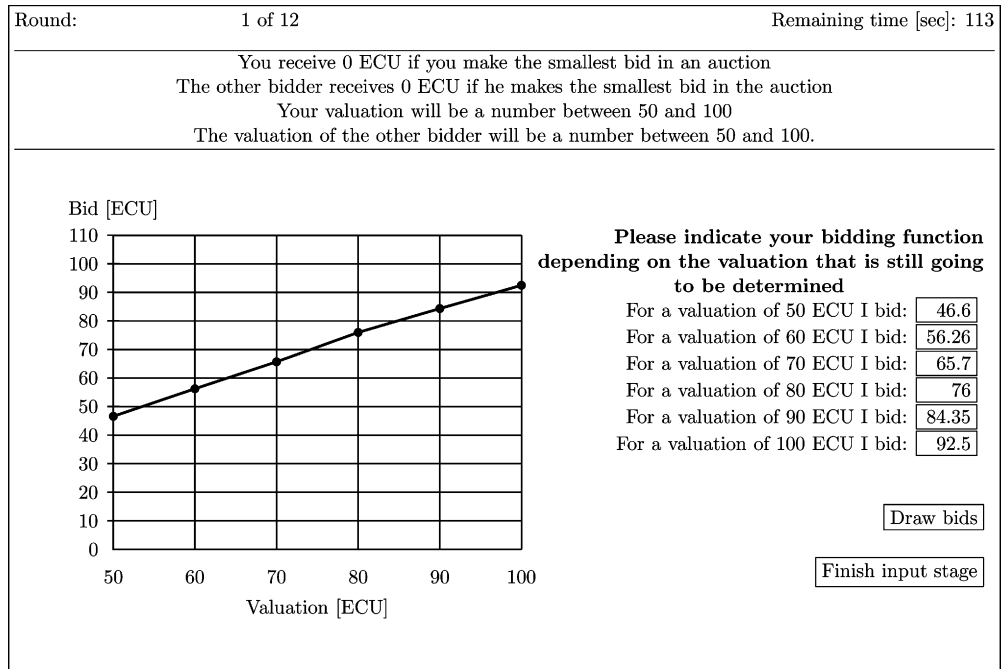


Fig. 1. A typical input screen in the experiment (treatment A1, translated into English).

and GÜth et al. (2002, 2003). Although our design is more complex than the latter, we obtain sensible bidding functions with our interface: Only about 10.15% of all bids are not monotonic in our experiment.

### 3.3. Multiple feedback and risk neutral equilibrium predictions

Another distinctive feature of our design is that pairs of matched bidders participated in five unrelated auctions after specification of their bidding schedules instead of one single auction. For each of these five auctions and for each bidder a valuation was drawn independently from a uniform distribution with support [50, 100]. In each of the five auctions, bids were determined according to the specified bidding functions. One reason to do this is to decrease the random component in income determination and to increase the strategic one. Kirchkamp et al. (2006) show that playing several auctions simultaneously makes bidding functions in the experiment appear substantially more risk neutral. Using risk neutral equilibria as reference points becomes, thus, more plausible. To better understand whether multiple auctions also reduce risk averse behaviour in our experiment we follow Isaac and James (2000) and infer for each subject the parameter of constant relative risk aversion  $r$  from observed bids in our baseline treatment (A1). Risk neutral bidders are characterised by  $r = 1$  and infinitely risk averse bidders have an  $r = 0$ . Isaac and James (2000) report for their experiments where players play only single auctions an average of  $r = 0.503$ . In our experiment the sample average is with 0.821 substantially larger and closer to risk neutrality.

Another reason to play several auctions in our experiment is to make it more plausible that participants have to submit an entire bidding function. For all five auctions, bidders were informed about their valuations, their submitted bid, whether they won the object, the price of each object, and their own income, see Fig. 2. They were also informed about their income for each round which was the sum of the income in the five auctions. No information about competitor's valuations and

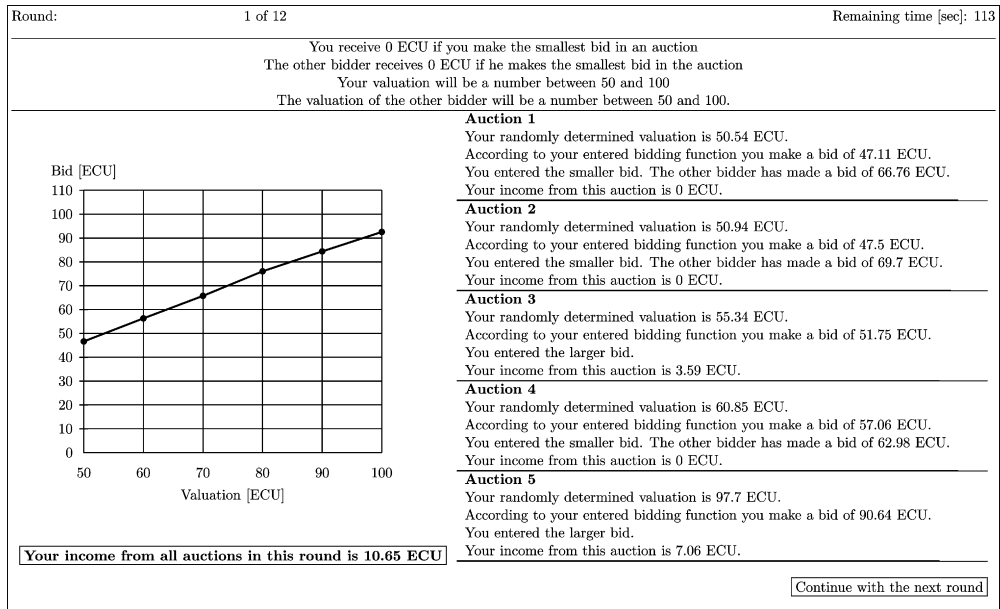


Fig. 2. A typical feedback screen in the experiment (treatment A1, translated into English).

incomes was revealed. The competitor's bid was revealed only if it determined the auction price of the object.<sup>7</sup>

### 3.4. Outside options

Treatments B and C introduce outside options. The outside option is an exogenously given income for the bidder who did not win the auction. The values of these outside options were drawn from a uniform distribution with support  $[0, 50]$  and were held constant for four consecutive auction rounds. Outside options were distributed independently of valuations to rule out incorrect statistical inferences as a possible confounding factor since subjects in auction experiments do not always correctly account for all statistical information available to them (cf. Brosig and Reiß, 2007). The value of the outside option was announced to each individual bidder before they specified their bidding functions. Treatments B and C differed in the amount of information bidders had about their competitors' outside options. In treatment B the outside option was public information and the same for both bidders. In the C treatments outside options were drawn independently for each bidder from a uniform distribution with support  $[0, 50]$ . In this treatment bidders knew their own outside option but not the outside option of their opponent. We use first-price auctions in treatments B1 and C1 and a second-price auction in treatment C2. Table 1 summarises the number of independent observations and the number of participants by treatment.

### 3.5. Matching groups

We get one independent observation per matching group. In 17 of 19 experimental sessions, subjects were randomly divided into two matching groups consisting of either eight or 10 subjects

<sup>7</sup> We have to make the transaction price public in the second-price auction since there the loser's bid determines the winner's price. To be consistent, and in line with typical real-world markets, we make the transaction price also public in the first-price auction.



**Table 1**  
Number of independent observations and participants

Type of outside options	First-price auction	Second-price auction
None	A1: 8 (86 participants)	A2: 6 (58 participants)
Public	B1: 6 (52 participants)	–
Private	C1: 8 (72 participants)	C2: 8 (72 participants)

**Table 2**  
Equilibrium bidding predictions

Treatment	Risk neutral Bayes–Nash equilibrium prediction	Bid range
A1	$b^{A1}(v) = 25 + \frac{v}{2}$	$b^{A1} \in [50, 75]$
B1	$b^{B1}(v, w) = 25 + \frac{v}{2} - w$	$b^{B1} \in [0, 75]$
C1	$b^{C1}(x) = \begin{cases} \frac{2}{3}x & \text{if } x \in [0, 50] \\ \frac{100x^2 - 2/3x^3 - 250\,000/3}{200x - x^2 - 5000} & \text{if } x \in [50, 100] \end{cases}$ <p>where <math>x \equiv v - w</math></p>	$b^{C1} \in [0, 50]$
A2	$b^{A2}(v) = v$	$b^{A2} \in [50, 100]$
C2	$b^{C2}(v, w) = x$ , where $x \equiv v - w$	$b^{C2} \in [0, 100]$

depending on the number of subjects in the experimental session.<sup>8</sup> In each of the two remaining sessions, there was a single matching group with 14 subjects. The number of subjects varied slightly over matching groups since in some sessions some participants did not show up. This, however, only affected the size of the matching group (which was never smaller than eight). Auctions were always played by two bidders.

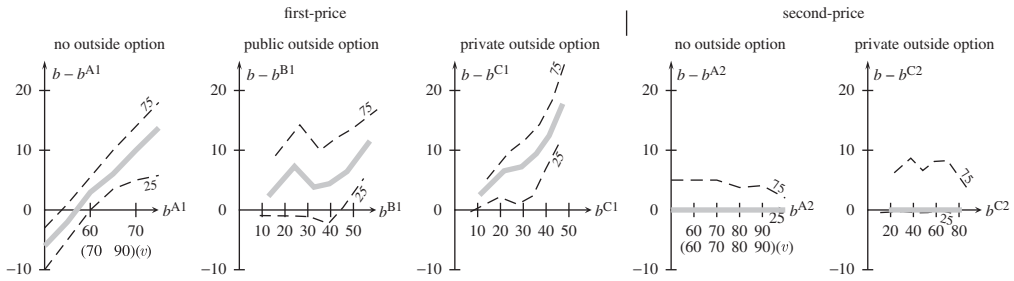
### 3.6. Procedures

At the beginning of each experimental session, participants read written instructions, then they took a brief treatment-specific computerised quiz to ensure their familiarity with the instructions and the experiment. Afterwards they played 12 rounds of the actual experiment. At the end of the last auction round, participants completed a brief computerised questionnaire and received their earned income in cash. Since in treatments A1 and A2 participants earned no income from outside options, they received an additional payment of 3 Euro in these treatments.

### 3.7. Equilibrium predictions

Table 2 summarises the risk neutral Bayes–Nash equilibrium bidding strategies for the different treatments. For all treatments, except for C1, the equilibrium bidding functions are linear in  $v$ . Hence, in these treatments bidders can submit equilibrium bidding functions even if they have to use a stepwise linear bidding function. In treatment C1 the equilibrium bidding function is nonlinear and can only be approximated with a stepwise linear function. However, one can show that the difference between an unrestricted equilibrium bid and a stepwise approximation is very small (far less than 1% of the bid), thus, the equilibrium still constitutes a useful benchmark.

<sup>8</sup> See Appendix A for the number of subjects in each experimental session.



The vertical axis shows for each of our five different treatments the median (thick line) as well as 25% and 75% quantiles (dashed lines) of the amount of overbidding, i.e. differences between actual bids and equilibrium bids (see table 2).

The horizontal axis shows (for the different combinations of valuations  $v$  and outside options  $w$ ) the values of the equilibrium bids in this treatment. In the treatments A1 and A2 this is a simple linear function of the valuation. For these treatments valuations  $v$  are given in parentheses below the equilibrium bids. Quantiles are based on six bands with the same number of observations in each band.

**Fig. 3.** Overbidding: Medians and quantiles. Quantiles are based on six bands with the same number of observations in each band.

**4. Experimental results**

A summary of overbidding behaviour is shown in Fig. 3. The vertical axis shows for each of our five different treatments the medians and the 25% and 75% quantiles of differences between actual bids and equilibrium bids (see Table 2). The horizontal axis shows (for the different combinations of valuations  $v$  and outside options  $w$ ) the values of the equilibrium bids in this treatment. In the treatments A1 and A2 this is a simple linear function of the valuation. For these treatments valuations  $v$  are given in parentheses below the equilibrium bids.

We make two observations. One is standard: There is a substantial amount of overbidding in the first-price auction (A1) and a small amount of overbidding in the second-price auction (A2). In that context we should note that the distribution of bids in the second-price auction is rather asymmetric. Median bids and 25% quantile bids are very close to equilibrium. Only the right tail of the distribution, the 75% quantile bids, show overbidding. The second observation is more interesting: The amount of overbidding is larger in the treatments with outside options.

*4.1. Treatment B1: Public outside options in the first-price auction*

To gain a better understanding of bidding behaviour we regress bids  $b$  on valuations  $v$  and outside options  $w$  following Eq. (14) for treatments A1 and B1. Results are shown in Table 3.

$$b = \beta_v v + \beta_w w + \beta_0 + u. \tag{14}$$

Since observations within a matching group of our experiments might be dependent we compare a standard robust model with two mixed effect models. Both mixed effect models allow for two sources of error, one associated with the specific matching group, the other with the individual observation. The first mixed effect model assumes that only the intercept is subject to a random effect, the second mixed effect model assumes that all coefficients are subject to a random effect. The third column shows  $p$ -values for a Wilcoxon test against the equilibrium values.<sup>9</sup> Since in

<sup>9</sup> For the mixed effect model we use R's lme procedure with REML estimation. For the Wilcoxon test we estimate a linear model for each independent matching group. We then use a standard one-sided Wilcoxon test to compare the vector of these coefficients with the equilibrium value.

**Table 3**  
Estimation of Eq. (14) for the A1 and B1 treatment

	$\beta^{eq}$	Wilcox.	Standard robust model			Mixed effect model I			Mixed effect model II				
		$P$	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$P_{\beta \neq \beta^{eq}}$	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$\hat{\sigma}_{grp}^{rand.}$	$P_{\beta \neq \beta^{eq}}$	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$\hat{\sigma}_{grp}^{rand.}$	$P_{\beta \neq \beta^{eq}}$
<b>A1</b>													
const.	25.0	0.0039	-0.1155	0.525	0.0000	0.0348	0.836	1.895	0.0000	0.3630	2.008	5.497	0.0000
$v$	0.5	0.0039	0.8710	0.007	0.0000	0.8710	0.006		0.0000	0.8666	0.023	0.063	0.0000
$\hat{\sigma}_{res.}^{rand.}$								8.714				8.662	
			$R^2 = 0.7374$			AIC = 44425.2562				AIC = 44372.4140			
<b>B1</b>													
const.	25.0	0.0156	1.8716	0.941	0.0000	3.1076	1.238	1.500	0.0000	4.0864	1.528	2.539	0.0000
$v$	0.5	0.0156	0.7965	0.013	0.0000	0.7965	0.012		0.0000	0.7962	0.013	0.013	0.0000
$w$	-1.0	0.2188	-0.7668	0.016	0.0000	-0.8124	0.020		0.0000	-0.8717	0.058	0.126	0.0265
$\hat{\sigma}_{res.}^{rand.}$								12.766				12.743	
			$R^2 = 0.6281$			AIC = 29727.8468				AIC = 29733.6608			

The table reports estimated fixed effects  $\hat{\beta}_{eff}^{fix.}$  with their estimated standard deviation  $\hat{\sigma}_{\beta}^{fix.}$ . For the mixed models the table also includes the estimated standard deviation of the random effects  $\hat{\sigma}_{grp}^{rand.}$  and the estimated residual standard deviation  $\hat{\sigma}_{res.}^{rand.}$ .  $P$ -values are for tests against the equilibrium values of the coefficients. As a measure of fit the standard robust effect model includes  $R^2$  and the mixed effect model Akaike's Information Criterion AIC. The third column shows  $p$ -values for a Wilcoxon test against the equilibrium values  $\beta^{eq}$ . (see footnote 9).

treatment A1  $w = 0$  we do not estimate  $\beta_w$  in this treatment. The  $p$ -values for comparisons with the equilibrium values are also shown in Table 3.

We find that the estimated coefficient of the public outside option value  $w$  is in absolute terms significantly smaller than the equilibrium prediction  $\beta_w = -1$ . Also the coefficient of the valuation  $v$  is, as found in several other studies, significantly larger in the experiment than the equilibrium prediction  $\beta_v = 0.5$ .

The fact that bidders do not fully exploit their outside option is interesting in the light of the debate on the declining price anomaly, i.e. the observation that prices for identical products which are sold sequentially often follow a decreasing pattern.<sup>10</sup> If this exploitation failure arises with endogenous outside options, too, then one might expect a series of falling prices as observed in the field since bids in the early auctions would be too high.

#### 4.2. Treatment C1: Private outside options in first-price auctions

In treatment C1 the equilibrium bidding function is not linear. We, thus, estimate the following equation:

$$b = \beta_v v + \beta_w w + \beta_{\Delta} \Delta + \beta_0 + u, \tag{15}$$

where  $\Delta$  captures the nonlinearity and is defined as

$$\Delta \equiv b^{C1}(v - w) - \frac{2}{3}(v - w). \tag{16}$$

We should note that  $\Delta$  is zero for the linear part of the bidding function (where  $(v - w) \in [0, 50]$ ). In that range the equilibrium bidding function is just  $\frac{2}{3}(v - w)$  (see Eq. (13)). For the non-linear part of the bidding function (where  $(v - w) \in [50, 100]$ ) the equilibrium bid is just the sum of  $\frac{2}{3}(v - w)$  and  $\Delta$ .

<sup>10</sup> See Ashenfelter and Graddy (2003) and the references therein. For the theoretical reference solution, see Weber (1983); for an experimental study that reproduces this phenomenon in the laboratory, see Keser and Olson (1996).

**Table 4**  
Estimation of Eq. (15) for the C1 treatment

C1	$\beta^{eq.}$	Wilcox.	Standard robust model			Mixed effect model I			Mixed effect model II				
		<i>P</i>	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	<i>P</i> <sub><math>\beta \neq \beta^{equil.}</math></sub>	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$\hat{\sigma}_{grp}^{rand.}$	<i>P</i> <sub><math>\beta \neq \beta^{eq.}</math></sub>	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$\hat{\sigma}_{grp}^{rand.}$	<i>P</i> <sub><math>\beta \neq \beta^{eq.}</math></sub>
const.	0.0000	0.2734	-1.6588	0.833	0.0466	-1.2863	1.002	1.908	0.1992	-2.0381	2.919	7.990	0.4850
<i>v</i>	0.6667	0.0078	0.8056	0.013	0.0000	0.8059	0.011		0.0000	0.8122	0.035	0.093	0.0000
<i>w</i>	-0.6667	0.0742	-0.7202	0.012	0.0000	-0.7337	0.013		0.0000	-0.7286	0.030	0.077	0.0402
$\Delta$	1.0000	0.0039	0.2214	0.076	0.0000	0.2240	0.071		0.0000	0.1851	0.099	0.192	0.0000
$\hat{\sigma}_{res}^{rand.}$								10.434				10.256	
			$R^2 = 0.7141$			AIC = 39075.3903				AIC = 38944.2364			

The table reports estimated fixed effects  $\hat{\beta}_{eff}^{fix.}$  with their estimated standard deviation  $\hat{\sigma}_{\beta}^{fix.}$ . For the mixed models the table also includes the estimated standard deviation of the random effects  $\hat{\sigma}_{grp}^{rand.}$  and the estimated residual standard deviation  $\hat{\sigma}_{res}^{rand.}$ . *P*-values are for tests against the equilibrium values of the coefficients. As a measure of fit the standard robust effect model includes  $R^2$  and the mixed effect model Akaike's Information Criterion AIC. The third column shows *p*-values for a Wilcoxon test against the equilibrium values  $\beta^{eq.}$  (see footnote 9).

Hence, when we estimate Eq. (15) we should expect that in equilibrium  $\beta_v = \frac{2}{3}$ ,  $\beta_w = -\frac{2}{3}$ ,  $\beta_{\Delta} = 1$ . However, a bidder who follows the linear part of the equilibrium bidding function  $\frac{2}{3}(v - w)$  over the entire range of possible valuations and outside options will still have a  $\beta_v = \frac{2}{3}$ ,  $\beta_w = -\frac{2}{3}$  but a  $\beta_{\Delta} = 0$ . Table 4 summarises the regression results. We see that estimates for  $\beta_v$  and  $\beta_w$  are significantly larger in absolute terms than they should be in equilibrium. The coefficient  $\beta_{\Delta}$  has the correct sign, i.e. bidders do pick up the nonlinearity. However,  $\beta_{\Delta}$  is significantly smaller than its equilibrium value.  $\beta_v$  is larger in absolute terms than  $\beta_w$ , though not significantly so ( $F_{1,7} = 2.82$ ,  $p = 0.1370$ ).

The regression results show that in the first-price auction with private outside options bidders react slightly too sensitively to their own valuation *v*. This is consistent with the case of no outside options and leads to some overbidding. Bidders also reduce their bids according to their outside options almost as they should in equilibrium. In addition to the so far "standard" overbidding, bidders also forget to shade their bids when *v* is large and *w* is small. The nonlinearity in the equilibrium bidding function which lowers equilibrium bids is only to a small degree reflected in the experimental bidding function.

4.3. Treatment A2 and C2: Second-price auctions

For the second-price auction we again estimate Eq. (14). Results are reported in Table 5. The  $\beta_v$  coefficients in both treatments are not significantly different from each other ( $F_{1,13} = 2.28$ ,  $p = 0.1549$ ).<sup>11</sup> Also, coefficients are close to equilibrium values, though in the C2 treatment  $\beta_v$  is (in absolute terms) significantly smaller than  $\beta_w$  ( $F_{1,7} = 7.29$ ,  $p = 0.0307$ ).

Most importantly, the coefficient for the outside option value  $\beta_w$  is estimated, depending on the model, to be somewhere between 0.96 and 0.97, thus, very close to the equilibrium value  $\beta_w = 1$  and not significantly different. It appears that bidders on average fully exploit their outside option in the second-price auction, although they failed to do so in first-price auctions. Recall that in treatment B1 bids decrease only by 0.77 to 0.87 (again, the estimation depends on the model) per unit of outside option value instead of 1.00.

<sup>11</sup> For this, as well as for all other test statistics given in the text, we use Rogers' (1993) procedure that takes into account that observations might be correlated within matching groups but not across matching groups. Standard errors with this procedure are usually more conservative than those of either the standard robust model or the mixed effect models.

**Table 5**  
Estimation of Eq. (14) for the A2 and C2 treatment

	$\beta^{eq}$	Wilcox.	Standard robust model			Mixed effect model I			Mixed effect model II				
			$P$	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$P_{\beta \neq \beta^{equi.}}$	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$\hat{\sigma}_{grp}^{rand.}$	$P_{\beta \neq \beta^{eq}}$	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$\hat{\sigma}_{grp}^{rand.}$
<b>A2</b>													
const.	0	0.0156	4.7325	0.690	0.0000	4.7839	0.848	1.327	0.0000	4.7241	1.257	2.630	0.0002
v	1	0.0313	0.9704	0.009	0.0010	0.9704	0.008		0.0005	0.9712	0.015	0.031	0.0587
$\hat{\sigma}_{res.}^{rand.}$								9.362				9.350	
			$R^2 = 0.7551$			AIC = 30561.1333				AIC = 30559.8348			
<b>C2</b>													
const.	0	0.0078	8.8431	1.176	0.0000	8.7566	1.602	3.248	0.0000	8.7111	2.294	5.681	0.0001
v	1	0.0195	0.9233	0.014	0.0000	0.9233	0.014		0.0000	0.9230	0.027	0.065	0.0038
w	-1	0.2734	-0.9708	0.016	0.0627	-0.9711	0.016		0.0786	-0.9590	0.069	0.189	0.5513
$\hat{\sigma}_{res.}^{rand.}$								16.656				16.443	
			$R^2 = 0.6056$			AIC = 43918.1978				AIC = 43823.3168			

The table reports estimated fixed effects  $\hat{\beta}_{eff}^{fix.}$  with their estimated standard deviation  $\hat{\sigma}_{\beta}^{fix.}$ . For the mixed models the table also includes the estimated standard deviation of the random effects  $\hat{\sigma}_{grp}^{rand.}$  and the estimated residual standard deviation  $\hat{\sigma}_{res.}^{rand.}$ .  $P$ -values are for tests against the equilibrium values of the coefficients. As a measure of fit the standard robust effect model includes  $R^2$  and the mixed effect model Akaike's Information Criterion AIC. The third column shows  $p$ -values for a Wilcoxon test against the equilibrium values  $\beta^{eq}$ . (see footnote 9).

4.4. Non-parametric test results

To further test whether outside options are taken into account, i.e. whether bids are larger with outside options than without, we compare average bids from the A1 and the B1 treatment. For each of the six valuations and 12 rounds we compare the mean bids for each independent observation in the A1 and B1 treatment with a Mann-Whitney  $U$  test. If bidders would disregard outside options the test should find only a small number of significant differences of mean bids among the two treatments. However, bids with public outside options are significantly larger in all 72 cases ( $p < 0.015$ , two-tailed). We also compare the A1 and C1 treatment using the same procedure. Again, in all 72 comparisons average bids are significantly larger with outside options than without ( $p < 0.0008$ , two-tailed). Also in a comparison of the A2 and C2 treatment in all 72 comparisons average bids are significantly larger ( $p < 0.003$ , two-tailed).

4.5. Outside options increase overbidding in first-price auctions

To assess the impact of outside options on overbidding we estimate the following equation for the first-price treatments:

$$b = \beta_A b_A^{eq} + \beta_B b_B^{eq} + \beta_C \frac{2}{3}(v - w) + \beta_A \Delta + u. \tag{17}$$

$b_A^{eq}$  is the equilibrium bid in the first-price auction without outside options and zero otherwise.  $b_B^{eq}$  is the equilibrium bid in the first-price auction with public outside options and zero otherwise. The term  $2/3(v - w)$  is the linear part of the equilibrium bid in the first-price auction with private outside options and zero in the case without outside options.  $\Delta$  is the nonlinear part of the bidding function with private outside options as defined in Eq. (16) and zero in the case without outside options. The equation does not contain a constant to make sure that over- or underbidding is only reflected in one parameter. As a result we cannot estimate our first mixed effect model where only the constant is

**Table 6**  
First price overbidding depending on the outside option: Estimating Eq. (17)

A1, B1, C1	$\beta^{eq}$	Wilcox.	Standard robust model			Mixed effect model			
		$P$	$\hat{\beta}$	$\hat{\sigma}_{\beta}^{fix}$	$P_{\beta \neq \beta^{equil}}$	$\hat{\beta}_{eff}^{fix}$	$\hat{\sigma}_{\beta}^{fix}$	$\hat{\sigma}_{grp}^{rand.}$	$P_{\beta \neq \beta^{eq}}$
$\beta_A$	1	0.0039	1.0561	0.002	0.0000	1.1115	0.010	0.391	0.0000
$\beta_B$	1	0.0156	1.1291	0.007	0.0000	1.1526	0.014	0.446	0.0000
$\beta_C$	1	0.0039	1.2314	0.006	0.0000	1.2111	0.017	0.549	0.0000
$\Delta$	1	0.0078	0.4675	0.062	0.0000	0.2533	0.074		0.0000
$\hat{\sigma}_{res.}^{rand.}$								10.645	
			$R^2 = 0.9559$			AIC = 114664.3254			

The table reports estimated fixed effects  $\hat{\beta}_{eff}^{fix}$  with their estimated standard deviation  $\hat{\sigma}_{\beta}^{fix}$ . For the mixed models the table also includes the estimated standard deviation of the random effects  $\hat{\sigma}_{grp}^{rand.}$  and the estimated residual standard deviation  $\hat{\sigma}_{res.}^{rand.}$ .  $P$ -values are for tests against the equilibrium values of the coefficients. As a measure of fit the standard robust effect model includes  $R^2$  and the mixed effect model Akaike's Information Criterion AIC. The third column shows  $p$ -values for a Wilcoxon test against the equilibrium values  $\beta^{eq}$ . (see footnote 9).

allowed to vary. Instead we only present the second mixed effect model where  $\beta_A, \beta_B, \beta_C$  are allowed to vary over groups. Results are given in Table 6.

As expected, we see overbidding in the standard case without outside options. The coefficient  $\beta_A$  is significantly larger than one. But once public outside options are introduced we observe more overbidding: The coefficient  $\beta_B$  is significantly larger than  $\beta_A$ .<sup>12</sup> Introducing private outside options leads to even more overbidding than with public outside options: The coefficient  $\beta_C$  is significantly larger than  $\beta_B$ .<sup>13</sup> Not only do bidders in the case with private outside options bid more than without outside options, furthermore they fail to correct for the concavity of the bidding function: The coefficient  $\beta_{\Delta}$  is significantly smaller than one.<sup>14</sup> Since  $\Delta$  is always negative a  $\beta_{\Delta}$  which is smaller than the equilibrium value means again more overbidding. It is possible that the difference in common knowledge which stems from different types of outside options affects bidding behaviour.<sup>15</sup>

When we repeat this exercise for second-price auctions we find that overbidding is less affected by outside options.<sup>16</sup> Table 7 shows estimates of Eq. (18) for the second-price treatments.

$$b = \beta_A b_A^{eq} + \beta_C b_C^{eq} + u. \tag{18}$$

Overbidding in second-price auctions has already been observed by Kagel et al. (1987). Also in our experiment we see a small but significant amount of overbidding in the second-price auction. Introducing outside options in the second-price auction does; however, not further increase overbidding.<sup>17</sup>

To support the observation that outside options affect overbidding in the first-price auction but not in the second-price auction we run the following non-parametric test: For each of the six

<sup>12</sup>  $F_{1,21} = 4.82, p = 0.0396$ .

<sup>13</sup>  $F_{1,21} = 6.72, p = 0.0170$ .

<sup>14</sup>  $F_{1,21} = 12.76, p = 0.0018$ .

<sup>15</sup> Güth and Ivanova-Stenzel (2003) report that the manipulation of common knowledge in asymmetric private value auctions (competitor's valuation distribution is known/unknown) "changes behavior only slightly and hardly ever in significant ways." (p. 198f.) However, it is also possible that the difference in bidding behaviour is not due to differences in common knowledge, but driven by the symmetry/asymmetry of outside options. Since in our treatments B and C, symmetry and common knowledge of outside options are different, we cannot conclusively assess the effects of common knowledge about outside options. We are grateful to a referee for pointing this out to us.

<sup>16</sup> For second-price auctions, we define overbidding as bidding above net valuation. This definition embodies the standard notion of overbidding without outside options since then, with an outside option value of zero, the object value coincides with the net valuation. Due to the fact that, for second-price auctions, the equilibrium bid equals the net valuation, we can equivalently identify overbids in second-price auctions as bids exceeding equilibrium bids.

<sup>17</sup> The coefficient  $\beta_B$  is not significantly different from  $\beta_A$  ( $F_{1,13} = 1.22, p = 0.2892$ ).

**Table 7**  
Second-price overbidding depending on the outside option: Estimating Eq. (18)

A2, C2	$\beta^{eq}$	Wilcox.	Standard robust model			Mixed effect model			
		$P$	$\hat{\beta}$	$\hat{\sigma}_{\beta}^{fix.}$	$P_{\beta \neq \beta^{equil.}}$	$\hat{\beta}_{eff}^{fix.}$	$\hat{\sigma}_{\beta}^{fix.}$	$\hat{\sigma}_{grp}^{rand.}$	$P_{\beta \neq \beta^{eq}}$
$\beta_A$	1	0.0156	1.0303	0.002	0.0000	1.0234	0.008	0.088	0.0023
$\beta_C$	1	0.0273	1.0538	0.004	0.0000	1.0294	0.017	0.113	0.0828
$\hat{\sigma}_{res.}^{rand.}$								13.796	
			$R^2 = 0.9583$			$AIC = 75802.2927$			

The table reports estimated fixed effects  $\hat{\beta}_{eff}^{fix.}$  with their estimated standard deviation  $\hat{\sigma}_{\beta}^{fix.}$ . For the mixed models the table also includes the estimated standard deviation of the random effects  $\hat{\sigma}_{grp}^{rand.}$  and the estimated residual standard deviation  $\hat{\sigma}_{res.}^{rand.}$ .  $P$ -values are for tests against the equilibrium values of the coefficients. As a measure of fit the standard robust effect model includes  $R^2$  and the mixed effect model Akaike's Information Criterion AIC. The third column shows  $p$ -values for a Wilcoxon test against the equilibrium values  $\beta^{eq}$  (see footnote 9).

**Table 8**  
Average excess revenue

Treatment			Excess revenue	Robust $\hat{\sigma}$	$t$	$P >  t $
A2	Second-price	No outside option	1.519887	0.3717887	4.09	0.000
C2	Second-price	Private outside option	1.31426	0.9255003	1.42	0.164
A1	First-price	No outside option	7.185551	0.5245006	13.70	0.000
B1	First-price	Public outside option	10.82654	0.6744602	16.05	0.000
C1	First-price	Private outside option	12.36949	0.4828499	25.62	0.000

The table shows average excess revenue (the difference between expected revenue in the lab and the expected revenue with equilibrium bids).

valuations and 12 rounds we compare average overbidding for each independent observation in A1 and the C1 treatment. If bidders would disregard outside options the test should find only a small number of significant differences in the average amount of overbidding among the two treatments. However, the amount of overbidding is significantly larger in the outside options treatment in 63 out of 72 comparisons ( $p < 0.1$ , two-tailed). When we repeat this exercise with the second-price auctions A2 and C2, average overbidding is significantly larger with outside options in only 6 out of 72 comparisons ( $p < 0.1$ , two-tailed).

4.6. Outside options boost revenue-dominance of first-price auction

Based on the payoff equivalence theorem which also applies to the case of outside options the expected revenue of a first-price auction should equal the expected revenue of a second-price auction. Since the experimental study of Cox et al. (1982) it is well known that, at least in the absence of outside options, the first-price auction generates larger revenues than the second-price auction. Table 8 shows for all treatments the difference between the expected revenue given the bidding functions in the lab and the expected revenue in equilibrium.<sup>18</sup> Also in our experiment first-price auctions obtain a higher revenue than second-price auctions. The difference is significant among the

<sup>18</sup> We determine the expected revenue by evaluating the bidding function for each participant and each period 100 times for a random valuation and matching the bidder as in the experiment.

**Table 9**  
Relative frequency of efficient allocations

Treatment		Relative frequency of efficient allocations	$\hat{\sigma}$	95% conf. int.		Mean efficiency rate	$\hat{\sigma}$	95% conf. int.	
A2	Second-price	0.876	0.018	0.840	0.911	0.990	0.003	0.983	0.997
C2	Second-price	0.825	0.021	0.784	0.867	0.978	0.004	0.969	0.987
A1	First-price	0.856	0.007	0.841	0.870	0.988	0.001	0.985	0.991
B1	First-price	0.807	0.011	0.784	0.829	0.982	0.002	0.976	0.988
C1	First-price	0.836	0.015	0.805	0.867	0.982	0.003	0.976	0.989

no outside option treatments A1 and A2<sup>19</sup> and also significant among the private outside option treatments C1 and C2<sup>20</sup>. More importantly, the difference in revenue between the first-price and the second-price auction increases when outside options are introduced.<sup>21</sup>

#### 4.7. Efficiency

In our auction we call an allocation efficient if the object is obtained by the bidder with the highest net valuation. Since in all treatments bidding functions are monotonic and the same for all bidders in the net valuation we always have an efficient allocation in equilibrium. Before we did our experiment we suspected that with outside options the situation is more complex, hence, we would find more inefficient allocations. This, however, does not seem to be the case. The left part of Table 9 shows relative frequencies of efficient allocations for the different treatments. We see that differences in efficiency are small and also not significant.

To confirm this finding we also report mean efficiency rates in the right part of the table, i.e. the ratio of realised total surplus and maximum total surplus. With outside options, realised surplus is given by the sum of the winner's surplus and the surplus generated by the outside option available to the unsuccessful bidder. Again, we see that losses in efficiency due to outside options are very small.

## 5. Conclusion

We have introduced a bidding model that allows for public and private outside option and we have experimentally tested its properties. A key feature of our experimental design is that we observe entire bidding functions. We find that, in line with the theoretical prediction, higher-valued outside options lead to less aggressive bidding (i.e. lower bids).

In contrast to the theoretical revenue equivalence of first-price and second-price auctions, our laboratory analysis shows that the first-price auction generates more revenue than the second-price auction. Importantly, outside options significantly magnify the revenue-premium of the first-price auction since overbidding in first-price auctions is more prominent with outside options than without. There is no such effect for second-price auctions.

Choosing first-price auctions is, hence, more attractive especially when bidders have outside options.

Our finding that bidders do not take full advantage of their outside options offers another explanation for the declining price anomaly. Future auctions in an auction sequence can be viewed as endogenous outside options. The failure to fully account for outside options, which we observe in our

<sup>19</sup>  $F_{1,35} = 77.66$ ,  $p = 0.0000$ .

<sup>20</sup>  $F_{1,35} = 112.16$ ,  $p = 0.0000$ .

<sup>21</sup> In the no outside option treatment the revenue-premium of the first-price auction is  $7.19 - 1.52 = 5.67$  ECU while the introduction of private outside options almost doubles the premium to  $12.37 - 1.31 = 11.06$  ECU. The difference among the revenue premia is significant,  $F_{1,35} = 19.33$ ,  $p = 0.0001$ .



experiment, leads in sequential sales to bids which are higher than theoretically predicted. This, in turn, results in a sequence of falling prices.

Taken together, our analysis suggests that outside options crucially influence bidding behaviour in a way that is qualitatively predicted by theory. Furthermore we note that the particular nature of outside options matters.

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## Appendix A. List of experimental sessions

Seventeen sessions were conducted at the experimental laboratory at the SFB 504 at the University of Mannheim in 2003 and 2004; two sessions were conducted at the MaXLab at the University of Magdeburg in April 2005.

Date	Treatment	Outside option	Auction	ECU/Euro	Participants
20031211-18:23	A1	None	First price	25	14
20031212-10:45	A1	None	First price	25	14
20040519-15:53	A1	None	First price	35	18
20050414-08:55	A1	None	First price	35	20
20050414-13:17	A1	None	First price	35	20
20031212-14:23	B1	Public	First price	120	18
20031212-15:53	B1	Public	First price	120	16
20031212-17:33	B1	Public	First price	120	18
20031211-10:19	C1	Private	First price	150	16
20031211-12:31	C1	Private	First price	150	18
20031211-14:25	C1	Private	First price	150	20
20031211-16:17	C1	Private	First price	150	18
20041130-17:41	A2	None	Second price	35	20
20041201-14:09	A2	None	Second price	35	20
20041201-15:57	A2	None	Second price	35	18
20041129-10:27	C2	Private	Second price	150	20
20041129-15:29	C2	Private	Second price	175	16
20041130-10:43	C2	Private	Second price	175	18
20041130-15:59	C2	Private	Second price	175	18

## Appendix B. Conducting the experiment and instructions

Participants were recruited by email and could register for the experiment on the internet. At the beginning of the experiment participants drew balls from an urn to determine their allocation to seats. Being seated participants then obtained written instructions in German. These instructions vary slightly depending on the treatment. In the following we give a translation of the instructions.

After answering control questions on the screen participants entered the treatment described in the instructions. After completing the treatment they answered a short questionnaire on the screen and were then paid in cash. The experiment was conducted with the help of z-Tree (Fischbacher, 2007).

### B.1. General information

You are participating in a scientific experiment that is sponsored by the Deutsche Forschungsgemeinschaft (German Research Foundation). If you read the following instructions carefully then you can—depending on your decision—gain a considerable amount of money. It is, hence, very important that you read the instructions carefully.

The instructions that you have received are only for your private information. *During the experiment no communication is permitted.* Whenever you have questions, please raise your hand. We will then answer your question at your seat. Not following this rule leads to the exclusion from the experiment and all payments.

During the experiment we are not talking about Euro, but about ECU. Your entire income will first be determined in ECU. The total amount of ECU that you have obtained during the experiment will be converted into Euro at the end and paid to you in *cash*. The conversion rate will be shown on your screen at the beginning of the experiment.

### B.2. Information regarding the experiment

Today you are participating in an experiment on auctions. The experiment is divided into separate rounds. We will conduct 12 *rounds*. In the following we explain what happens in each round.

In each round you bid for an object that is being auctioned. Together with you another participant is also bidding for the same object. Hence, in each round, there are *two bidders*. In each round you will be allocated randomly to another participant for the auction. Your co-bidder in the auction changes in every round. {The following sentence was only included in the instructions for A1, B1, and C1: The bidder with the highest bid obtains the object. If bids are the same the object will be allocated randomly.}

{The following paragraph was only included in the instructions for A2 and C2:

In each round you submit your maximum bid for an object that is auctioned. The maximum bid is the largest amount that you want to pay for the object. Your co-bidder submits his maximum bid for the object at the same time as you do. In the auction, the price for the object will be increased in steps of 0.01 ECU. As soon as the price matches one of both maximum bids (either yours or that of your co-bidder), the corresponding player stops bidding in the auction. The bidder who remains alone in the auction obtains the object. *Thus, the bidder with the higher maximum bid obtains the object. The price is equal to the lower maximum bid.* In case you or your co-bidder stop bidding at the same time, the bidder who obtains the object is randomly determined.}

For the auctioned object you have a valuation in ECU. This valuation lies between 50 and 100 ECU and is randomly determined in each round. *From this range you will obtain in each round new and random valuations for the object.* The other bidder in the auction also has a valuation for the object. The valuation that the other bidder attributes to the object is determined by the same rules as your valuation and changes in each round, too. All possible valuations of the other bidder are also in the interval from 50 to 100 from which also your valuations are drawn. All valuations between 50 and 100 are equally likely. Your valuations and those of the other player are determined independently. You will be told your valuation in each round. You will not know the valuation of the other bidder.

{The following three paragraphs were only included in the instructions for treatments B1, C1, and C2:

The auction income is calculated as follows:

- {B1 and C1: The bidder with the higher bid obtains the valuation he had for the object in this auction added to his account minus his bid for the object.} {C2: The bidder who remains alone in the auction obtains the valuation he had for the object in this auction added to his account minus the price of the object. The price is given by the smaller one of both maximum bids, i.e. the price at which one of the bidders stops bidding.}
- {B1: The bidder with the smaller bid obtains a payment that both bidders learn} {C1: The bidder with the smaller bid obtains a randomly determined payment that he learns} {C2: The bidder that

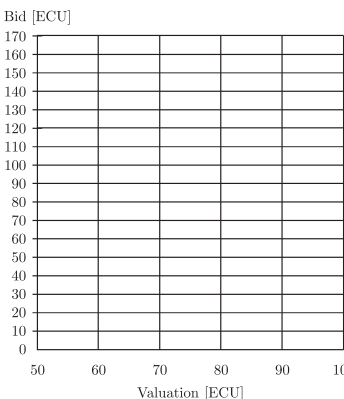
first stops bidding in the auction obtains the randomly determined payment that he learns} before. The determination of this payment is explained further below.

At the beginning of each round, you are informed about the payment {B1: That is obtained by the bidder who does} {C1, C2: That you obtain if you do} not receive the object in that round. The value of this payment will be randomly determined and remains constant for four rounds. Thus, you are assigned a new randomly determined value for the payment after four rounds. {B1: This payment is identical for you and your co-bidder. For the value of this payment, all values in the range of 0 and 50 ECU are equally likely.} {C1, C2: This randomly determined payment can be any value in the range of 0 and 50 ECU with equal probability. Also the other bidder is assigned such a payment. That will be determined according to the same rules as your payment. You are not informed about the payment of the other bidder. Your payment and the payment of the other bidder are independent of one another.}}

### B.2.1. Experimental procedure

The experimental procedure is the same in each round and will be described in the following. Each round in the experiment has two stages.

*First stage:* In the first stage of the experiment you see the following screen<sup>22</sup>:

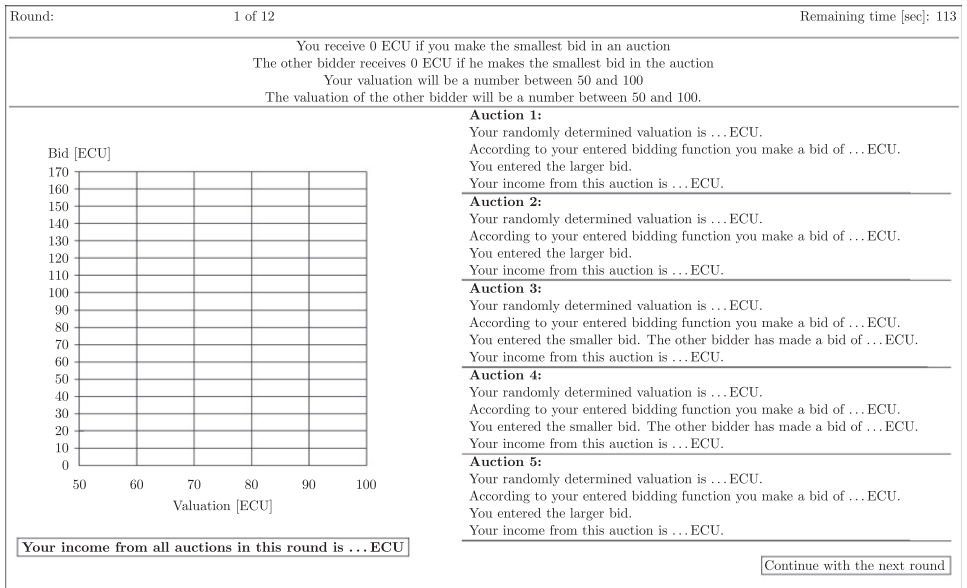
Round:	1 of 12	Remaining time [sec]: 113
<p>You receive 0 ECU if you make the smallest bid in an auction          The other bidder receives 0 ECU if he makes the smallest bid in the auction          Your valuation will be a number between 50 and 100          The valuation of the other bidder will be a number between 50 and 100.</p>		
<p>Bid [ECU]</p> 	<p>Please indicate your bidding function depending on the valuation that is still going to be determined</p> <p>For a valuation of 50 ECU I bid: <input type="text"/></p> <p>For a valuation of 60 ECU I bid: <input type="text"/></p> <p>For a valuation of 70 ECU I bid: <input type="text"/></p> <p>For a valuation of 80 ECU I bid: <input type="text"/></p> <p>For a valuation of 90 ECU I bid: <input type="text"/></p> <p>For a valuation of 100 ECU I bid: <input type="text"/></p> <p style="text-align: right;"><input type="button" value="Draw bids"/></p> <p style="text-align: right;"><input type="button" value="Finish input stage"/></p>	

At that stage you do not know your own valuation for the object in this round. {B1: The payment that the bidder with the smaller bid obtains} {C1, C2: The payment that you obtain if you do not receive the object} {B1, C1, C2: Is displayed on the screen.} On the right side of the screen you are asked to enter a {A1, B1, C1: bid} {A2, C2: Maximum bid} for six hypothetical valuations that you might have for the object. These six hypothetical valuations are 50, 60, 70, 80, 90, and 100 ECU. Your input into this table will be shown in the graph on the left side of the screen when you click on {A1, B1, C1: “draw bids”} {A2, C2: “draw maximum bids”}. In the graph the hypothetical valuation is shown on the horizontal axis, the {A1, B1, C1: Bids} {A2, C2: Maximum bids} are shown on the vertical axis. Your input in the table is shown as six points in the diagram. Neighbouring points are connected with a line automatically. These lines determine your {A1, B1, C1 : bid} {A2, C2 : maximum bid} for all valuations between the six points for which you have made an input. For the other bidder the screen in the

<sup>22</sup> This figure does not show the bidding function in the graph and the specific bids that would be shown during the experiment. Figures are slightly treatment dependent.

first stage looks the same and there are as well  $\{A1, B1, C1 : \text{bids}\}$   $\{A2, C2 : \text{maximum bids}\}$  for six hypothetical valuations. The other bidder cannot see your input.

*Second stage:* The actual auction takes place in the second stage of each round. In each round we will play not only a single auction but *five auctions*. This is done as follows: *Five times a random valuation is determined* that you have for the object. Similarly for the other bidder five random valuations are determined. You see the following screen<sup>23</sup>:



For each of your five valuations the computer determines your  $\{A1, B1, C1 : \text{bid}\}$   $\{A2, C2 : \text{maximum bid}\}$  according to the graph from stage 1. If a valuation is precisely at 50, 60, 70, 80, 90, or 100 the computer takes the  $\{A1, B1, C1 : \text{bid}\}$   $\{A2, C2 : \text{maximum bid}\}$  that you entered for this valuation. If a valuation is between these points your  $\{A1, B1, C1 : \text{bid}\}$   $\{A2, C2 : \text{maximum bid}\}$  is determined according to the joining line. In the same way the  $\{A1, B1, C1 : \text{bids}\}$   $\{A2, C2 : \text{maximum bids}\}$  of the other bidder are determined for his five valuations.  $\{A1, B1, C1 : \text{Your bid is compared with the one of the other bidder. The bidder with the higher bid has obtained the object.}\}$   $\{A2, C2 : \text{In each of the five auctions the price at which one bidder stops bidding will be determined from your maximum bid and the maximum bid of your co-bidder. The price is equal to the smaller one of both maximum bids. The bidder who remains alone in the auction obtains the object.}\}$

*Your income from the auction:* For each of the five auctions the following holds:

- $\{A1, B1, C1 : \text{The bidder with the higher bid gets the valuation he had for the object in this auction added to his account minus his bid for the object.}\}$   $\{A2, C2 : \text{The bidder who remains alone in the auction gets the valuation he had for the object in this auction added to his account minus the price of the object. The price is given by the smaller one of both maximum bids, ie. the price at which one of the bidders stops bidding.}\}$
- $\{A1 : \text{The bidder with the smaller bid obtains no income from this auction.}\}$   $\{B1 \text{ and } C1 : \text{The bidder with the smaller bid obtains the randomly determined payment that [B1 : Is used in this round.] [C1 : He is informed about.]\}$   $\{A2, C2 : \text{That bidder that first stops bidding in the auction obtains}\}$

<sup>23</sup> This figure does not show the bidding function in the graph and the specific bids, gains and losses that would be shown during the experiment. Figures are slightly treatment dependent.

{A2: no income from this auction.} {C2: The randomly determined payment that he is informed about.}

Your total income in a round is {A1, A2: The sum of the ECU income from those auctions in this round where} {A1: You have made the higher bid.} {A2: You were the only remaining bidder in the auction.}

The following box was only included in the instructions for treatments B1, C1, and C2:}

For each of the five auctions where you {B1, C1: submitted the higher bid;}  
 {C2: were the only remaining bidder;}  
**Your valuation in this auction minus {B1, C1: your bid} {C2: the price}**  
 +  
 For each of the five auctions where you {B1, C1: submitted the smaller bid;}  
 {C2: stopped first bidding in the auction;}  
**{B1: The} {C1, C2: Your} randomly determined payment used in this round.**

This ends one round of the experiment and you see in the next round again the input screen from stage 1.

At the end of the experiment your total ECU income from all rounds will be converted into Euro and paid to you in cash.

Please raise your hand if you have questions.

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