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Exposita Notes

The fuzzy core and the (Π, β)-balanced core*

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Summary. This note provides a new proof of the non-emptiness of the fuzzy core in a pure exchange economy with finitely many agents. The proof is based on the concept of (Π, β)-balanced core for games without side payments due to Bonnisseau and Iehlé (2003).

Keywords and Phrases: Fuzzy core, Payoff-dependent balancedness, Exchange economies.

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1 Introduction

The fuzzy core was introduced by Aubin (1979) as the set of allocations of a pure exchange economy that are robust to blockings by all fuzzy coalitions. To interpret a fuzzy coalition, one thinks of an economy as being comprised of several types of agents, with each type being represented by a continuum of identical individuals. The fuzzy coalition then specifies the number (the mass) of the participants of each type.

The fuzzy core is known to be non-empty under very general conditions. In particular, Florenzano (2003) establishes the non-emptiness of the fuzzy core in an infinite-dimensional production economy (see Proposition 5.2.3, p. 115).

This note provides a new proof of the non-emptiness of the fuzzy core. The proof is based on the concept of the (Π, β)-balanced core for games without side payments, a special case of the so-called core with additional requirements introduced recently in Bonnisseau and Iehlé (2003). A given economy gives rise to a

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non-transferable utility game whose (Π, β)-balanced core corresponds, in a natural way, to the ϵ-fuzzy core of the original economy. The non-emptiness of the fuzzy core can therefore be deduced from the non-emptiness result for the (Π, β)-balanced core in Bonnisseau and Iehlé (2003).

Our non-emptiness result is weaker than that provided by Proposition 5.2.3 in Florenzano (2003). The result in Florenzano (2003) applies to an economy with a non-trivial production sector, while our construction is limited to pure exchange economies. Furthermore, to transform the problem of the non-emptiness of the fuzzy core into that of the non-emptiness of the (Π, β)-balanced core, we have to rely on the existence of the utility functions. This is in contrast to the result in Florenzano (2003), where the consumer preferences may lack completeness or transitivity. On the other hand, we are able to maintain quite general assumptions with respect to the commodity space of an economy. As in Florenzano (2003), the commodity space in this paper is a general (possibly infinite-dimensional) linear Hausdorff topological space.

The rest of the paper is organized as follows. In Section 2 some notation is introduced. In Section 3 the fuzzy core and the ϵ-fuzzy core are defined. In Section 4 some preliminary results are presented, including the non-emptiness result for the (Π, β)-balanced core. In Section 5 the non-emptiness of the ϵ-fuzzy core is demonstrated.

2 Notation

Let n be a positive integer. Then N is the set of integers $\{1, \ldots, n\}$, and $\mathcal{N}$ is the collection of non-empty subsets of the set N. Given an $S \in \mathcal{N}$, the symbol $\mathbb{R}^S$ denotes the space of functions $x : i \in S \mapsto x^i \in \mathbb{R}$. The symbol $\mathbb{R}^{\mathcal{N}}$ denotes the space of functions $\lambda : S \in \mathcal{N} \mapsto \lambda_S \in \mathbb{R}$. Let Δ_N denote the unit simplex in $\mathbb{R}^N$, thus $\Delta_N = \{\pi_N \in \mathbb{R}^N \mid \pi_N^i \geq 0 \text{ for all } i \in N \text{ and } \sum_{i \in N} \pi_N^i = 1\}$. For each $S \in \mathcal{N}$ let Δ_S be a face of Δ_N defined as $\Delta_S = \{\pi_S \in \Delta_N \mid \pi_S^i = 0 \text{ for all } i \in N \backslash S\}$. Given an $\epsilon \in (0, 1/n)$, let $\Delta_S^\epsilon = \{\pi_S \in \Delta_S \mid \pi_S^i \geq \epsilon \text{ for all } i \in S\}$. Let β denote the barycenter of Δ_N. For a subset A of $\mathbb{R}^N$, the symbols $intA$, ∂A, and $\mathrm{co}A$ denote, respectively, the interior, the boundary, and the convex hull of A.

3 Fuzzy core

We consider a pure exchange economy $\mathcal{E}$ in which the set of agents is N, and the commodity space is $\mathbf{C}$. Each agent $i \in N$ is characterized by a consumption set $X^i \subseteq \mathbf{C}$, initial endowment of commodities $e^i \in \mathbf{C}$, and a utility function $u^i : X^i \to \mathbb{R}$. Given $S \in \mathcal{N}$ we write X_S to denote the product $\underset{i \in S}{\times} X^i$. We employ the following assumptions:

(A1) The commodity space $\mathbf{C}$ is a linear (over the field of real numbers) Hausdorff topological space.

(A2) For each $i \in N$, the set X^i is a closed convex set containing e^i, and the set $F(\mathcal{E}) = \left\{x \in X_N \mid \sum_{i \in N} x^i = \sum_{i \in N} e^i\right\}$ of feasible commodity allocations is a compact set.

(A3) For each $i \in N$ the utility function u^i is a continuous and quasi-concave function.

Definition 1 An allocation $x \in F(\mathcal{E})$ is an element of the fuzzy core $C^f(\mathcal{E})$ of the economy $\mathcal{E}$ if there exist no $\pi \in \Delta_N$ and no $\chi \in X_N$ such that $\sum_{i \in N} \pi^i \chi^i = \sum_{i \in N} \pi^i e^i$ and $u^i(\chi^i) > u^i(x^i)$ for all $i \in N$ with $\pi^i > 0$.

It is instrumental to consider the following modification of the fuzzy core, called the ϵ-fuzzy core. In the ϵ-fuzzy core it is required that for any fuzzy coalition π with support S the mass π^i of the participants of each type $i \in S$ be at least $\epsilon > 0$.

Definition 2 An allocation $x \in F(\mathcal{E})$ is an element of the ϵ-fuzzy core $C^f_\epsilon(\mathcal{E})$ of the economy $\mathcal{E}$ if there exist no $S \subseteq N$, no $\pi \in \Delta^\epsilon_S$ and no $\chi \in X_S$ such that $\sum_{i \in S} \pi^i \chi^i = \sum_{i \in S} \pi^i e^i$ and $u^i(\chi^i) > u^i(x^i)$ for all $i \in S$.

Theorem 1 *For each $\epsilon \in (0, 1/n)$, the ϵ-fuzzy core of the economy $\mathcal{E}$ is non-empty.*

The proof of Theorem 1 is given in Section 5. Our main result is the following.

Theorem 2 *The fuzzy core of the economy $\mathcal{E}$ is non-empty.*

Proof. To prove Theorem 2 we consider fuzzy core as a limit of the ϵ-fuzzy core as ϵ approaches zero. Observe that $C^f_\epsilon(\mathcal{E}) \subseteq C^f_{\epsilon'}(\mathcal{E})$ whenever $0 < \epsilon \leq \epsilon'$. Moreover, the fuzzy core of the economy $\mathcal{E}$ can be written as

$$C^f(\mathcal{E}) = \bigcap_{\epsilon \in (0,1/n)} C^f_\epsilon(\mathcal{E}).$$

Since the non-emptiness of the ϵ-fuzzy core follows from Theorem 1, it suffices to demonstrate that $C^f_\epsilon(\mathcal{E})$ is a compact set. We show that it is a closed subset of $F(\mathcal{E})$.

Let x be a point of $F(\mathcal{E})$ not in $C^f_\epsilon(\mathcal{E})$. Then there exist $S \subseteq N$, $\pi \in \Delta^\epsilon_S$ and $\chi \in X_S$ as in Definition 2. For a fixed χ, the inequalities $u^i(\chi^i) > u^i(x^i)$, $i \in S$, define an open neighborhood of x in $F(\mathcal{E})$ contained entirely in the complement of $C^f_\epsilon(\mathcal{E})$. Thus, the complement of $C^f_\epsilon(\mathcal{E})$ in $F(\mathcal{E})$ is an open set, as desired. □

4 (Π, β)-balanced core

An n-person game with non-transferable utility (hereafter referred to simply as a *game*) is a family of sets $V = \langle V(S) \rangle_{S \in \mathcal{N}}$ satisfying the following assumptions. For all $S \in \mathcal{N}$

(G1) $V(S)$ is a non-empty closed subset of $\mathbb{R}^N$.
(G2) $[v \in V(S),\ \bar{v} \in \mathbb{R}^N,\ \bar{v}^i \leq v^i$ for all $i \in S]$ implies $[\bar{v} \in V(S)]$.
(G3) There is a number M such that $v^i \leq M$ for all $i \in S$ and $v \in V(S)$.

The core of the game V is a subset of $\mathbb{R}^N$ given by

$$C(V) = V(N) \Big\backslash \bigcup_{S \in \mathcal{N}} int V(S).$$

Given $v \in \mathbb{R}^N$, let $\mathcal{S}(v) = \{S \in \mathcal{N} \mid v \in \partial V(S)\}$.

The (Π, β)-balanced core is a special case of the so-called core with additional requirements for games with non-transferable utility introduced in Bonnisseau and Iehlé (2003). It generalizes the idea of the socially stable core for socially structured games in Herings, Laan, and Talman (2003).

Definition 3 Let V be a game. For each $S \in \mathcal{N}$ let $\Pi_S : \partial V(S) \to \Delta_S$ be a correspondence with non-empty convex values and with a closed graph. Then the (Π, β)-balanced core of the game V is the set of points v in the core of V such that $\beta \in \operatorname{co}\{\Pi_S(v) \mid S \in \mathcal{S}(v)\}$.

Thus the (Π, β)-balanced core is a selection from the core. If the correspondence Π_N maps each utility tuple v into the one-point set $\{\beta\}$, then the (Π, β)-balanced core of V coincides with the core of V.

The (Π, β)-balancedness condition for games without side payments belongs to the class of payoff-dependent balancedness conditions studied in Bonnisseau and Iehlé (2003). A related reference is Predtetchinski and Herings (2004), where payoff-dependent balancedness is shown to be a necessary and sufficient condition for the non-emptiness of the core.

Definition 4 Let V be a game. For each $S \in \mathcal{N}$ let $\Pi_S : \partial V(S) \to \Delta_S$ be a correspondence with non-empty convex values and with a closed graph. The game V is said to be (Π, β)-balanced provided that the following condition is satisfied: if $v \in \mathbb{R}^N$ and $\beta \in \operatorname{co}\{\Pi_S(v) \mid S \in \mathcal{S}(v)\}$, then $v \in V(N)$.

For the proof of Theorem 3 the reader is referred to Bonnisseau and Iehlé (2003).

Theorem 3 *Let V be a game. For each $S \in \mathcal{N}$ let $\Pi_S : \partial V(S) \to \Delta_S$ be a correspondence with non-empty convex values and with a closed graph. Suppose that the game V is (Π, β)-balanced. Then the (Π, β)-balanced core of the game V is non-empty.*

5 The non-emptiness of the ϵ-fuzzy core

In this section we prove Theorem 1. Given a number $\epsilon \in (0, 1/n)$ and an economy $\mathcal{E}$, we construct a non-transferable utility game V^ϵ and the correspondences Π_S as follows: In the game V^ϵ, the utility tuple v is considered feasible for a coalition S if it is feasible for some fuzzy coalition π_S with support S in the original economy $\mathcal{E}$. Given a vector of utilities v in $\partial V^\epsilon(S)$, we let $\Pi_S(v)$ to consist of those fuzzy coalitions π_S that can achieve the utility of v. We then show that the game V^ϵ is (Π, β)-balanced and that its (Π, β)-balanced core corresponds, under the utility functions, to the ϵ-fuzzy core of the economy $\mathcal{E}$.

Given $S \in \mathcal{N}$, let

$$Z^\epsilon(S) = \left\{ (x_S, \pi_S) \in X_S \times \Delta_S^\epsilon \mid \sum_{i \in S} \pi_S^i x_S^i = \sum_{i \in S} \pi_S^i e^i \right\} \tag{1}$$

$$V^\epsilon(S) = \left\{ v \in \mathbb{R}^N \;\middle|\; \begin{array}{c} \text{there exists } (x_S, \pi_S) \in Z^\epsilon(S) \text{ such that} \\ u^i(x_S^i) \geq v^i \text{ for all } i \in S \end{array} \right\}. \tag{2}$$

Proposition 1 *For each* $\epsilon \in (0, 1/n)$, $Z^\epsilon(S)$ *is a non-empty and compact set.*

Proof. For each $\epsilon \in (0, 1/n)$, the set Δ_S^ϵ is non-empty, and $Z^\epsilon(S)$ contains a non-empty subset $\{e_S\} \times \Delta_S^\epsilon$, where $e_S^i = e^i$ for all $i \in S$. Define $F_S(\mathcal{E})$ to be the set of feasible commodity allocations of the coalition S:

$$F_S(\mathcal{E}) = \left\{ x_S \in X_S \middle| \sum_{i \in S} x_S^i = \sum_{i \in S} e^i \right\}.$$

First we argue that $F_S(\mathcal{E})$ is a compact set. The set $F_N(\mathcal{E})$ is compact by assumption (**A2**). For a proper subset S of N the set $F_S(\mathcal{E})$ can be considered a subset of $F_N(\mathcal{E})$ given by

$$F_N(\mathcal{E}) \bigcap \left\{ x_N \in X_N \mid x_N^i = e^i \text{ for all } i \in N \backslash S \right\}.$$

By assumption (**A2**), **C** is a Hausdorff space, so a one-point set $\{e^i\}$ is closed in X^i for each i. It follows that $F_S(\mathcal{E})$ is closed in $F_N(\mathcal{E})$; therefore, it is a compact set.

To see that $Z^\epsilon(S)$ is a compact set, let $|S|$ be the cardinality of S; let $\mathbf{C}^{|S|}$ be the $|S|$-fold product of **C**, and let φ be a continuous function from the space $\mathbf{C}^{|S|} \times \Delta_S^\epsilon$ to itself that carries a point (x_S, π_S) to a point $(\widetilde{x}_S, \pi_S)$, where

$$\widetilde{x}_S^i = \frac{1}{\pi_S^i} x_S^i + \left(1 - \frac{1}{\pi_S^i}\right) e^i$$

for each $i \in S$. Then

$$Z^\epsilon(S) = (X_S \times \Delta_S^\epsilon) \bigcap \varphi(F_S(\mathcal{E}) \times \Delta_S^\epsilon).$$

The product of the compact sets $F_S(\mathcal{E})$ and Δ_S^ϵ is a compact set, and so is its image under φ. Thus $Z^\epsilon(S)$ is a closed subset of a compact set; therefore, it is a compact set. □

Proposition 2 *For each* $\epsilon \in (0, 1/n)$, *the family of sets* $V^\epsilon = \langle V^\epsilon(S) \rangle_{S \in \mathcal{N}}$ *satisfies conditions* (**G1**)-(**G3**).

Proof. Condition (**G2**) is satisfied by definition of $V^\epsilon(S)$. To verify conditions (**G1**) and (**G3**), let us write the set $V^\epsilon(S)$ as

$$V^\epsilon(S) = \left\{ v \in \mathbb{R}^N \middle| \begin{array}{c} \text{there exists } \dot{v} \in \dot{V}^\epsilon(S) \text{ such that} \\ \dot{v}^i \geq v^i \text{ for all } i \in S \end{array} \right\},$$

where $\dot{V}^\epsilon(S)$ is defined by

$$\dot{V}^\epsilon(S) = \left\{ v \in \mathbb{R}^S \middle| \begin{array}{c} \text{there exists } (x_S, \pi_S) \in Z^\epsilon(S) \text{ such that} \\ u^i(x_S^i) = v^i \text{ for all } i \in S \end{array} \right\}.$$

The set $\dot{V}^\epsilon(S)$ is the image of the set $Z^\epsilon(S)$ under the composition of the maps

$$X_S \times \Delta_S^\epsilon \longrightarrow X_S \longrightarrow \mathbb{R}^S$$

where the first map is the natural projection, and the second map is a Cartesian product of the functions u^i for $i \in S$. It follows that $\dot{V}^\epsilon(S)$ is a non-empty compact subset of $\mathbb{R}^S$; in particular, it is bounded from above. From this one can easily see that the set $V^\epsilon(S)$ satisfies conditions (**G1**) and (**G3**). □

We remark that for $\epsilon = 0$ the family of sets V^ϵ may fail to meet conditions (**G1**) and (**G3**), since $Z^0(S)$ is not, generally, a compact set. It is to avoid this difficulty that the ϵ-fuzzy cores and the corresponding games V^ϵ were introduced.

Define the correspondences $\Pi_S : \partial V^\epsilon(S) \to \Delta_S^\epsilon$ as follows. Given $v \in \partial V^\epsilon(S)$ let

$$\Pi_S(v) = \left\{ \pi_S \in \Delta_S^\epsilon \;\middle|\; \begin{array}{c} \text{There exists } x_S \in X_S \text{ such that} \\ \sum_{i\in S} \pi_S^i x_S^i = \sum_{i\in S} \pi_S^i e^i \\ u^i(x_S^i) \geq v^i \text{ for all } i \in S \end{array} \right\}. \tag{3}$$

Proposition 3 *For each $v \in \partial V^\epsilon(S)$ the set $\Pi_S(v)$ is non-empty and convex. The correspondence Π_S has a closed graph.*

Proof. The set $\Pi_S(v)$ is non-empty by definition of $V^\epsilon(S)$. To prove that it is a convex set, let $\pi_S, \pi'_S \in \Pi_S(v)$ and $t \in [0,1]$ be given. We must show that $\widetilde{\pi}_S = t\pi_S + (1-t)\pi'_S$ is an element $\Pi_S(v)$. Let x_S, x'_S be points in X_S such that $\sum_{i\in S} \pi_S^i x_S^i = \sum_{i\in S} \pi_S^i e^i$ and $u^i(x_S^i) \geq v^i$ for all $i \in S$, and similarly for x'_S. Given $i \in S$ let $\widetilde{x}_S^i$ be defined as

$$\widetilde{x}_S^i = \frac{1}{\widetilde{\pi}_S^i}[t\pi_S^i x_S^i + (1-t)\pi_S'^i x_S'^i]$$

By convexity of the consumption sets one has $\widetilde{x}_S^i \in X^i$. By quasi-concavity of the utility function, $u^i(\widetilde{x}_S^i) \geq v^i$ for all $i \in S$. Finally,

$$\begin{aligned} \sum_{i\in S} \widetilde{\pi}_S^i \widetilde{x}_S^i &= t\sum_{i\in S} \pi_S^i x_S^i + (1-t)\sum_{i\in S} \pi_S'^i x_S'^i \\ &= t\sum_{i\in S} \pi_S^i e^i + (1-t)\sum_{i\in S} \pi_S'^i e^i \\ &= \sum_{i\in S} \widetilde{\pi}_S^i e^i. \end{aligned}$$

Thus, $\widetilde{\pi}_S \in \Pi_S(v)$, as desired.

To see that the graph G of the correspondence Π_S is closed, let us write it as the set

$$G = \left\{ (v, \pi_S) \in \partial V^\epsilon(S) \times \Delta_S^\epsilon \;\middle|\; \begin{array}{c} \text{there exists } \dot{v} \in \mathbb{R}^S \text{ such that} \\ (\dot{v}, \pi_S) \in \dot{G} \text{ and} \\ \dot{v}^i \geq v^i \text{ for all } i \in S \end{array} \right\},$$

where $\dot{G}$ is defined by

$$\dot{G} = \left\{ (v, \pi_S) \in \mathbb{R}^S \times \Delta_S^\epsilon \;\middle|\; \begin{array}{c} \text{there exists } x_S \in X_S \text{ such that} \\ (x_S, \pi_S) \in Z^\epsilon(S) \\ u^i(x_S^i) = v^i \text{ for all } i \in S \end{array} \right\}.$$

Then the set $\dot{G}$ is the image of $Z^\epsilon(S)$ under the continuous map $X_S \times \Delta_S^\epsilon \to \mathbb{R}^S \times \Delta_S^\epsilon$, which is a Cartesian product of the utility functions u^i for $i \in S$ and the identity map on Δ_S^ϵ. It follows from Proposition 1 that $\dot{G}$ is a compact set. From this one can easily see that the set G is closed in $\partial V^\epsilon(S) \times \Delta_S^\epsilon$. □

Theorem 4 *Let the game V^ϵ and the correspondences $\Pi_S : \partial V^\epsilon(S) \to \Delta_S^\epsilon$ be defined by Equations 1–3. Let the commodity allocation x be an element of the ϵ-fuzzy core of the economy $\mathcal{E}$. Then the utility tuple v is an element of the (Π, β)-balanced core of the game V^ϵ, where $v^i = u^i(x^i)$ for all $i \in N$. Conversely, let the utility tuple v be an element of the (Π, β)-balanced core of the game V^ϵ. Then there exists a commodity allocation x in the ϵ-fuzzy core of the economy $\mathcal{E}$ such that $v^i \geq u^i(x^i)$ for all $i \in N$.*

Proof. Let the commodity allocation x be in the ϵ-fuzzy core of the economy $\mathcal{E}$. Let $v \in \mathbb{R}^N$ be given by $v^i = u^i(x^i)$ for all $i \in N$. Then v is in the core of the game V^ϵ, for otherwise x would not be stable against deviations by fuzzy coalitions π_S in Δ_S^ϵ. Furthermore, because x is feasible for the coalition N, the inclusion $\beta \in \Pi_N(v)$ holds, and therefore v is in the (Π, β)-balanced core of V^ϵ.

Conversely, let the vector of utilities v be in the (Π, β)-balanced core of the game V^ϵ. Then for each $S \in \mathcal{S}(v)$, there exists a vector $\pi_S \in \Pi_S(v)$ and a non-negative number λ_S such that

$$\beta = \sum_{S \in \mathcal{S}(v)} \lambda_S \pi_S.$$

This can be rewritten componentwise as

$$1 = \sum_{S \in \mathcal{S}(v)} n \lambda_S \pi_S^i \text{ for all } i \in N.$$

It follows from the definition of the game V^ϵ (Equations 1 and 2) that for every $S \in \mathcal{S}(v)$ there exists a commodity allocation $x_S \in X_S$ such that

$$\sum_{i \in S} \pi_S^i x_S^i = \sum_{i \in S} \pi_S^i e^i$$
$$u^i(x_S^i) \geq v^i \text{ for all } i \in S.$$

Define the commodity allocation $\bar{x}_N \in X_N$ as follows. For each $i \in N$ let

$$\bar{x}_N^i = \sum_{\substack{S \in \mathcal{S}(v) \\ S \ni i}} n \lambda_S \pi_S^i x_S^i.$$

The consumption vector $\bar{x}^i_N$ is a weighted average of the vectors x^i_S over those coalitions $S \in \mathcal{S}(v)$ that contain player i. By convexity of the consumption sets, $\bar{x}^i_N$ is an element of X^i. By quasi-concavity of the utility functions,

$$u^i(\bar{x}^i_N) \geq v^i \text{ for all } i \in N.$$

Moreover,

$$\begin{aligned}\sum_{i \in N}[\bar{x}^i_N - e^i] &= \sum_{i \in N} \sum_{\substack{S \in \mathcal{S}(v) \\ S \ni i}} n\lambda_S \pi^i_S [x^i_S - e^i] \\ &= \sum_{S \in \mathcal{S}(v)} n\lambda_S \sum_{i \in S} \pi^i_S [x^i_S - e^i] = 0.\end{aligned}$$

We see that the allocation $\bar{x}_N$ is an element of the set $F(\mathcal{E})$. Since v is not an interior point of any set $V^\epsilon(S)$, the allocation $\bar{x}_N$ is robust to deviations by fuzzy coalitions $\pi_S \in \Delta^\epsilon_S$ for all $S \in \mathcal{N}$. Thus, $\bar{x}_N$ is in the ϵ-fuzzy core of the economy $\mathcal{E}$. □

Theorem 5 *The game V^ϵ is (Π, β)-balanced.*

We omit the proof of this theorem as it is similar to the proof of Theorem 4. Theorem 5 implies that the (Π, β)-balanced core of the game V^ϵ is non-empty. Our argument is now complete.

References

Aubin, J.-P.: Mathematical methods of game and economic theory. Amsterdam: North-Holland 1979

Bonnisseau, J.-M., Iehlé V.: Payoffs-dependent balancedness and cores. Cahiers de la MSE Série Bleue #2003.45, Université Paris-I (2003)

Florenzano, M.: General equilibrium analysis: existence and optimality properties of equilibria. Boston: Kluwer 2003

Herings, P.J.J., van der Laan, G., Talman, D.: Socially structured games and their applications. METEOR Research Memorandum 03/009, Universiteit Maastricht, pp. 1–24 (2003)

Predtetchinski, A., Herings, P.J.J.: A necessary and sufficient condition for non-emptiness of the core of a non-transferable utility game. Journal of Economic Theory **116**, 84–92 (2004)