Sorting on Skills and Preferences: Tinbergen Meets Sattinger*

Arnaud Dupuy†

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Abstract

This paper proposes an assignment model where sorting occurs on attributes including both skills (Sattinger (1979)) and preferences (Tinbergen (1956)). The key feature of this model is that in equilibrium, the wage function admits both jobs' and workers' attributes as arguments. Even under positive assortative matching, the correlation between the contribution of workers' attributes to wages and that of jobs' attributes can vary from -1 to 1 depending on the parameters of the model, i.e. preference, technology and

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†ROA, IZA. Corresponding address: ROA, Maastricht University PO Box 616, NL-6200 MD, The Netherlands. Email: a.dupuy@roa.unimaas.nl.

the distribution of both sets of attributes. This paper also presents conditions under which nonadditive marginal utility and production function are nonparametrically identified using observations from a single hedonic market and proposes a nonparametric estimator. Finally, possible extensions of the model to include the output market are also proposed.

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1 Introduction

Recent emerging empirical literature (e.g. Borghans et al. (2008)) has shown the importance of personality traits in economics and in particular for earnings (Bowles et al. (2001) and Mueller and Plug (2006)). This literature shows that earnings are related to personality traits like risk aversion or conscientiousness. One possible explanation for these wage differentials would be that personality traits are linked to preferences for certain jobs' attributes so that the correlation between personality and earnings reflects compensating wage differentials for jobs disamenities. Yet, another explanation would be that personality traits are linked to skills that enhance productivity on the job and hence lead to higher wages. For instance, the documented positive effect of conscientiousness on earnings could come about because conscientiousness enhances workers' productivity or because in equilibrium, more conscientious workers are mapped onto jobs whose attributes are associated with negative intrinsic utility (tax controller) and hence require a wage compensation.

The model presented in this paper is the first treatment of personality traits in assignment models. This assignment model is concerned with the process by which heterogenous workers that are characterized by a vector of attributes t, including both skills and preferences, are assigned to heterogenous jobs characterized by a vector of attributes z , including both required skills and disamenities. This paper shows that this type of assignment models yields an equilibrium[1] that is defined by a mapping of workers' attributes t onto jobs' attributes z, a function say $t(z)$ or $z(t)$, together with a wage function $w(z,t)$ that depends on both workers' attributes and jobs' attributes. This wage function is shown to be additive separable in z and t, $w(z,t) = w^z(z) + w^t(t)$.

The fact that the equilibrium wage function admits both workers' and jobs' attributes as arguments has important implications for empirical applications and in particular for two noteworthy segments. First, the model offers a natural explanation for the presence of firms' and workers' fixed-effects in earnings regressions using matched-employer-employee data and their empirically observed low or even negative correlation, see Abowd et al. (1999). Since sorting occurs on both skills and preferences, equilibrium wages are function of both workers' attributes and jobs' attributes. However, even when sorting exhibits positive assortative matching, the model does not imply that wage differentials due to workers' attributes correlate positively with wage differentials associated to jobs' attributes. Both the sign and magnitude of the correlation between workers' and jobs' fixed-effects will

[1] The existence, uniqueness, purity and efficiency of this type of models have been treated elsewhere, e.g. Gretsky et al. (1992; 1999) deal with the so call *endowment* economy where z is endowed to the firm and Ekeland (2005) and more recently Chiappori et al. (2007) deal with the generalization to hedonic *production* economy where z is produced by firms with endowed attributes say y. Chiappori et al. (2007) have shown that hedonic models are equivalent to matching models and both belong to the general class of optimal transportation problems (Monge-Kantorovich). Under the assumptions made in this paper, in particular, about the shape of the profit and utility functions and given absolute continuous distribution of characteristics on both sides, an equilibrium exists, is unique and pure, see for instance Theorem 1, p. 3 in Ekeland (2005).

depend on the preference parameters, the technology parameters and the distribution of workers' and jobs' attributes. This result is illustrated numerically for the quadratic-normal economy.

Second, the model has implications for the estimation of preference (technology respectively) parameters in hedonic models. Recently, Ekeland et al. (2002 and 2004) and Heckman et al. (2005) have shown conditions under which nonparametric identification of additive and nonadditive marginal utility models of the Tinbergen class, where sorting occurs on preferences only, is possible in a single hedonic market. These conditions build on results from Matzkin (2003) on nonparametric estimation of nonadditive random functions and depend crucially on the assumption that $w^z(z)$ is known (or estimated from data on wages and z). In the Tinbergen class of models assumed in Ekeland et al. (2004) and Heckman et al. (2005), wages depend only on z and identification of $w^z(z)$ follows naturally. However, in the unified economy, where workers' attributes include both preferences and skills, wages are given by the function $w(z,t) = w^z(z) + w^t(t)$ where the functions $w^z(z)$ and $w^t(t)$ are unknown.[2] Without further assumptions, $w^z(z)$ is not identified nonparametrically since for any value of t, the value of

[2]If one is willing to guess some parametric shapes for w^z and w^t, however, parametric identification is possible since, as shown by Ekeland et al. (2004), a generic feature of the model is that $w^z(z(t))$ is not colinear to $w^t(t)$. Of course, one would want w^z and w^t to be as flexible as possible and these functions could be parameterized using Cubic B-splines with k knots of the form $w^s(z) = \sum_{j=-1}^{k} a_{sj} B_j(s'b_s)$ for $s = z, t$ and where a_s and b_s are vectors of parameters to be estimated and B_j is the jth B-spline of degree 3.

z is uniquely determined by the mapping function $z = z(t)$. This paper shows conditions under which $w^z(z)$ is identified nonparametrically and proposes a nonparametric estimator. First, it is shown that the mapping function $z(t)$ is identified nonparametrically using results from Matzkin (2003). This is a generalization to the unified economy of Heckman et al.'s (2005) result obtained for the Tinbergen class of models. Following the identification of $z(t)$, a method to identify $w^z(z)$ nonparametrically is proposed. This method relies on imposing shape restrictions on the utility and production functions. In general, the method assumes i) that the production function is additive separable in (z, t_{-i}) and t_i, with $t =< t_{-i}, t_i >$, where the contribution of t_i is a known differentiable function and ii) t_i is a scalar attribute influencing marginal utility. Assumption ii) insures that, fixing t_{-i}, variations in t_i generate variations in $w^z(.)$ through the mapping function while assumption i) guarantees that, at fixed t_{-i}, variations in t_i generate variations in $w^t(t)$ of know magnitude. With these two assumptions, w^z is identified from data in a single hedonic market. A special case of this method is met when attribute t_i is a strict preference that influences marginal utility but not productivity, i.e. the output contribution of t_i is known to be constant. In this case, t_i is an exclusion restriction as variations in t_i generate variations in $z(t)$ and hence $w^z(z)$ holding $w^t(t)$ constant.

The remaining structure of the paper is as follows. The next section reviews the related literature. Section 3 presents the unified model for the hedonic *endowment* economy. Section 4 presents a closed form solution for the wage function in the quadratic-normal setup. Section 5 discusses the implication of the model for firms' and workers' fixed-effects in earnings regressions using matched-employer-employee data as well as for the identification of preference parameters in a single hedonic market. Section 6 discusses first an extension of the model to the hedonic *production* economy, and then a generalization of the model including the product market. Section 7 summarizes and concludes.

2 Related literature

The model presented in this paper nests existing assignment models in the literature. This literature is divided into two distinct classes of models depending on the nature of the process governing assignment. One class of models, led by Tinbergen (1956), focuses on the assignment of workers to jobs based on preferences. Within this class of models, jobs' attributes z are seen as disamenity and workers derive intrinsic disutility from z. Although jobs with different attributes are unequally productive, output at a job with attribute z does not depend on workers' attributes t. Hence productivity is merely determined by jobs' attributes and all workers are equally productive at all jobs. In this class of models, workers select jobs' attributes to maximize their utility. The pricing function $w(z, t)$ does not depend on workers' attributes but merely on jobs' attributes, i.e. $w(z,t) = w^z(z)$, and is therefore interpreted as a compensating wage differential. As an example, the preference class of models indicates that risk loving workers will tend to become firemen as they command lower compensations for the risks taken on the job, but yet, assumes that risk loving workers would just make as good firemen as any other (risk averse) worker. While this class of models explains wage formation due to risk compensation, the model fails to explain wage formation due to productivity differentials across workers.

In contrast, the second class of models, led by Sattinger (1979), focuses on the assignment of workers to jobs based on skills. jobs' attributes are seen as productive capacities and workers derive no intrinsic (dis-)utility from z. Both workers' and jobs' attributes matter for productivity. Workers with certain attributes are more productive at certain jobs than others. In this class of models, workers select jobs' attributes to maximize their wage and the wage function $w(z,t)$ does not depend on jobs' attributes but merely on workers' attributes, i.e. $w(z,t) = w^t(t)$. For instance, the skills class of models indicates that conscientious workers will tend to become tax controllers as conscientiousness is an important factor of productivity on the job, but yet, assumes that conscientious workers derive the same disutility from being a tax controller as less conscientious workers. While this class of models explains wage formation due to differential productivity across workers, these models fail to explain wage formation due to preference compensation.

The model presented in this paper nests both Tinbergen's and Sattinger's models.[3] Under the assumption that all jobs' attributes lead to intrinsic disutility and workers' attributes do not affect productivity the model collapses to Tinbergen's model. Under the assumption that all workers' attributes contribute to productivity and no job attributes lead to intrinsic disutility the model collapses to Sattinger's differential rents model.

There exist only few examples of closed form solutions for the hedonic price as a function of attributes, i.e. $w(z,t)$. The first was proposed by Tinbergen (1956). Assuming that i) workers derive intrinsic disutility from all their attributes, ii) price enters log linearly in the utility function, iii) intrinsic (dis-)utility is quadratic in jobs' attributes z, iv) the supply of products is exogenous (the hedonic *endowment* economy model) and v) both workers and jobs' attributes are normally distributed, Tinbergen showed that the *log* of the equilibrium price is a function quadratic in attributes z.[4] Assuming i), ii'), iii) and v) and relaxing iv) by allowing firms to produce job attributes and introducing production costs (the hedonic *production* economy model), Epple (1984) provided a closed form solution for the hedonic

[3]Sattinger (1977) developed a compensating wage differential model where workers differ in terms of productivity and jobs in terms of the satisfaction workers receive from working at it, both unidimensional. Workers and jobs attributes are encompassed in the definition of job satisfaction and cannot be distinguished from each other. Moreover, all jobs have similar productivity. There is no complementarity between workers skills and jobs requirements.

[4]In fact, replacing ii) by ii') price enters linearly in the utility function, the *level* of the equilibrium price would be quadratic in z.

price function that is quadratic in z when production costs are also quadratic in z. Sattinger (1979 and 1980) provided closed for solutions when jobs and workers are differentiated along a single attribute (skills demanded and supplied) assuming that workers derive no intrinsic disutility from the level of skills demanded by their job, i.e. maximize their wage. This skills attribute affects productivity but does not provide intrinsic utility. The pricing function of interest in this model is the wage as a function of workers' skills. Since skills provide no intrinsic disutility and merely affect production, sorting in this model occurs on productive attributes rather than preference attributes as in Tinbergen and Epple. Sattinger's (1980) closed form solutions for the wage function are obtained when the distribution of jobs and workers are Pareto, production is multiplicative in attributes (i.e. Cobb-Douglas) and utility depends on wages only. This last assumption is characteristic of the differential rents model that precludes compensating wage differential for intrinsic disutility derived from the type of jobs.

A closed form solution for the unified model is proposed in section 4 of this paper. This solution is derived when workers' and jobs' attributes are normally distributed and intrinsic disutility is quadratic in jobs' attributes and productivity is quadratic in workers' attributes.

Finally, this paper relates to the general literature on hedonic models and not only on that segment focussing on the labor market. For instance, Epple's extension of Tinbergen's *endowment* economy to a *production* economy was originally written in a consumer/producer context, not a worker/firm context. In the consumer/producer model, the restriction that consumers' attributes do not affect the production of goods does not at first sight seem to be too strong. However, the generalization proposed in section 6 of this paper is also relevant in that case. Think for instance of an economy where firms are endowed with a vector of attributes y. In this economy, to produce good z, firms need to hire a fixed number of workers, one and only one worker for simplicity. Suppose further that the attributes of that worker, say t', matter in the production process so that the costs (profits) of producing good z depend on t'. Firms need now to optimize not only on z but also on t'.

3 The unified hedonic *endowment* economy model

3.1 Setup

Consider a static labor market where workers match one-to-one with firms. Let each firm be endowed with a single machine. The supply of machines is therefore assumed exogenous to the model,[5] and the assumption that workers and firms

[5] The assumption that firms are endowed with a machine z can be released by supposing that firms are endowed with a vector of attributes y (investments capacity, managers' attributes etc.) and "produce" their machine z. The distribution of machines is then endogenous to the model. This case corresponds to the hedonic *production* economy and is dealt with section 6 of this paper. The main results of the paper remain unchanged but the mechanic of the model

match one-to-one therefore means that to produce output each machine must be operated by one and only one worker. Let a machine be characterized by a vector of attributes denoted by $z \in \mathbb{R}^{n_z}$. To fix ideas, machines attributes could be the level of physical strength involved in operating the machine, the level of intellectual complexity involved, the level of noise generated by the machine, the degree of risks taken while operating the machine, etc. Let $f_z(z)$ and $F_z(z)$ be the PDF and CDF of z respectively and let F_z be absolutely continuous with respect to Lebesgue measure.

Similarly, suppose that workers are endowed with a vector of attributes $t \in \mathbb{R}^{n_t}$. These attributes could refer to cognitive ability such as physical strength, intellectual ability but also personality traits such as conscientiousness, risk aversion etc.. Let the distribution of t be exogenous and let $f_t(t)$ and $F_t(t)$ be its PDF and CDF respectively and let F_t be absolutely continuous with respect to Lebesgue measure.[6]

In contrast to Tinbergen (1956), Epple (1984), Ekeland et al. (2002 and 2004) and Heckman et al. (2005), the model does not require workers' attributes to be non productive. Let the output of each machine depend on its own attributes

simplifies significantly by assuming machines are endowed.

[6]It should be noted here that the mass of workers is assumed to be equal to the mass of firms. The model could be accommodated to allow for different masses and would inevitably lead to unemployed workers or vacancies in equilibrium depending on whether the mass of workers exceeds that of firms. Although assignment models offer an interesting structure to analyze which agents are kept out of the market by the equilibrium pricing, the primary aim of this paper is to analyze wage formation when workers' attributes are both skills and personality traits. The assumption of equal mass does not seem to be restrictive with respect to this aim.

but also on the attributes of the worker operating this machine. Let $p(z,t;E)$ be a continuous function indicating the units of output produced by the pair (z,t) where E are technology parameters common to all firms. An attribute i is not a productive attribute if and only if $\frac{\partial p(z,t;E)}{\partial t_i} = 0$ for all z and t. Note that some attributes may be productive at some jobs but not at others. While skills of different types will clearly affect productivity, some preferences may also affect productivity, for instance, a risk averse person might also tend to operate a machine slower, conscientious workers may take better care of their machine, etc...

Let $w(z,t)$ be the wage of a worker with attributes t when assigned to a machine with attributes z and let $r(z,t)$ be the rents of a firm owning machine with attributes z when employing a worker with attributes t. Note that, by definition, product is exhausted so that $p(z,t;E) - w(z,t) = r(z,t)$.

In contrast to Sattinger (1979), the model does not require that jobs' attributes do not affect intrinsic disutility. Assume that utility $u \equiv u(c,t,z;A)$ is a continuous function where A are preference parameters common to all workers. Utility u depends on consumption c, equal to $w(z,t)$ by assuming no unearned income, and the job satisfaction derived from the attributes of the machine workers' are assigned to. More specifically, let $j(z,t;A)$ be a continuous function capturing job dissatisfaction. The function j could take the specific form proposed by Tinbergen (1956), $j(z,t;A) = \frac{1}{2}(z-t)' A (z-t)$ where A is a positive definite matrix of parameters. We therefore have $u(c,t,z,A) = w(z,t) - j(z,t;A)$. A job attribute i does not provide intrinsic utility if and only if $\frac{\partial j(z,t;A)}{\partial z_i} = 0$ for all z and t.

3.2 Equilibrium

Definition 1 *An equilibrium is a wage function $w(z,t)$ and a mapping function $t(z;A,E,w_z,w_t)$ so that i) firms' supply of machines with attributes z equals workers' demand for machines with attributes z everywhere on the support of z, ii) workers maximize utility and iii) firms maximize rents.*[7]

Utility maximizing workers seek for a machine with attributes z so that:

$$
\begin{aligned}
\frac{\partial u(w(z,t),j(z,t;A))}{\partial z} &\equiv \frac{\partial w(z,t)}{\partial z} - \frac{\partial j(z,t;A)}{\partial z} = 0 \\
&\Leftrightarrow \\
\frac{\partial w(z,t)}{\partial z} &= \frac{\partial j(z,t;A)}{\partial z} \qquad (1)
\end{aligned}
$$

Let $z(t;A,w_z)$ denote the implicit function that solves Equation 1 for z given parameters A and a function w_z where $w_z \equiv \frac{\partial w}{\partial z}$. This function indicates the

[7] *Existence, uniqueness, purity and efficiency of hedonic models have been studied elsewhere: Gretsky et al. (1992;1999), Ekeland (2005), and Chiappori et al. (2007). Under the standing assumptions formulated in the setup above, an equilibrium allocation exists, is unique, and efficient. It will also be pure if the generalized Spence-Mirrlees condition is satisfied. An equilibrium wage function exists, and is unique if every workers and every firms participate (full employment and no vacancies).*

optimal machine a worker with attributes t chooses given preference parameters A and the shape of the wage function and in particular the wage differential at z.

The second order condition for utility maximization reads as:

$$
\begin{aligned}
\frac{\partial^2 u(w(z,t), j(z,t;A))}{\partial z^2} &< 0 \\
&\Leftrightarrow \\
\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t;A)}{\partial z^2} &< 0
\end{aligned}
$$

Rents maximizing firms will look for a worker with attributes t so that:

$$
\begin{aligned}
\frac{\partial r(z,t)}{\partial t} &= \frac{\partial p(z,t;E)}{\partial t} - \frac{\partial w(z,t)}{\partial t} = 0 \\
&\Leftrightarrow \\
\frac{\partial w(z,t)}{\partial t} &= \frac{\partial p(z,t;E)}{\partial t} \qquad (2)
\end{aligned}
$$

The second order condition for rents maximization reads as:

$$
\frac{\partial^2 p(z,t;E)}{\partial t^2} - \frac{\partial^2 w(t)}{\partial t^2} < 0
$$

Let $t(z; E, w_t)$ denote the implicit function that solves Equation 2 for t given parameters E and a function w_t where $w_t \equiv \frac{\partial w}{\partial t}$. This function indicates the optimal choice of worker for a firm with machine z given productivity parameters E and the shape of the wage function and in particular the differential at t.

If the equilibrium is pure, the two mapping functions $z(t; A, w_z)$ and $t(z; E, w_t)$ are invertible. We therefore have the restriction $t^{-1}(t; E, w_t) = z(t; A, w_z)$ and, for notational clarity, re-write the implicit function as $t(z; A, E, w_z, w_t)$ without loss of generality. Note that for the equilibrium mapping to be pure, i.e. $t(z^a; A, E, w_z, w_t) = t(z^b; A, E, w_z, w_t) \Longrightarrow z^a = z^b$, the generalized Spence-Mirrlees condition must be satisfied, see Ekeland (2005). Write the total surplus of a pair (z, t) as $s(z, t; E, A) \equiv p(z, t; E) - w(z, t) + u(t, z; A) = p(z, t; E) - j(z, t; A)$, the generalized Spence-Mirrlees condition reads as:

$$\begin{aligned}
\frac{\partial s(z, t^a; E, A)}{\partial z} &= \frac{\partial s(z, t^b; E, A)}{\partial z} \Longrightarrow t^b = t^a \\
\frac{\partial s(z^a, t; E, A)}{\partial t} &= \frac{\partial s(z^b, t; E, A)}{\partial t} \Longrightarrow z^b = z^a
\end{aligned}$$

For an equilibrium allocation to be reached, the supply of machines with attributes z should be equal to workers' demand for machines with attributes z for all z. This means that:

$$f_z(z)dz = f_t(t(z; A, E, w_z, w_t)) \left| \frac{\partial t(z; A, E, w_z, w_t)}{\partial z} \right| dz$$

Equilibrium will be reached by choosing the right shape for the function w and in particular the right differentials at t and z. Workers and firms will participate if their wage and rents are larger than their reservation levels. To close the model, the usual assumption (see Ekeland et al. (2002 and 2004) and Sattinger (1979) among others) is to fix a reservation value for the utility, say $\underline{u}$ and rent $\underline{r}$ so that u and r must be larger than their respective thresholds. Ekeland (2005) has shown that the wage function has a unique solution on the support of z. The equilibrium in this economy is therefore characterized by a wage function $w(z, t)$ and a mapping of workers' attributes onto jobs' attributes $t(z; A, E, w_z, w_t)$ so that i) supply equals demand everywhere on the support of z , ii) workers maximize utility and iii) firms maximize rents.

3.3 Proposition

Proposition 2 *In the unified model spelled out above, the equilibrium wage function $w(z, t)$ is additive separable in z and t.*

Proof. To see this, first note that even though the first order conditions tie the partial derivatives of w to the slopes of production $p(.,.)$ and job dissatisfaction $j(.,.)$, there are no restrictions on the second order cross-partial derivative $\frac{\partial^2 w(z,t)}{\partial z \partial t}$. At first sight, all functions $w(z,t)$ satisfying $\frac{\partial w(z,t)}{\partial z} = \frac{\partial j(z,t;A)}{\partial z}$ and $\frac{\partial w(z,t)}{\partial t} = \frac{\partial p(z,t;E)}{\partial t}$ would do, including the additive separable function $\frac{\partial^2 w(z,t)}{\partial z \partial t} = 0$. However, the first order conditions need only be satisfied on the allocation path $t(z)$ or $z(t)$. Replacing t by $t(z)$ into $\frac{\partial w(z,t)}{\partial z} = \frac{\partial j(z,t;A)}{\partial z}$ and integrating over z one obtains:

$$w(z,t(z)) = \int \frac{\partial j(z,t(z);A)}{\partial z} dz + c_t + c$$

where $c_t + c$ is a constant of integration. The use of splitting the constant into c_t and c will become clear below.

Similarly, substituting $z(t)$ for z into $\frac{\partial w(z,t)}{\partial t} = \frac{\partial p(z,t;E)}{\partial t}$ and integrating over t one obtains:

$$w(z(t),t) = \int \frac{\partial p(z(t),t;E)}{\partial t} dt + c_z + c$$

where $c_z + c$ is a constant of integration.

Equilibrium wages $w(z,t)$ must be so that $w(z,t(z)) = w(z(t),t)$ and hence:

$$\begin{aligned} w(z,t) &= w^z(z) + w^t(t) + c \\ &\quad with \\ w^t(t) &= c_t = \int \frac{\partial p(z(t),t;E)}{\partial t} dt \\ w^z(z) &= c_z = \int \frac{\partial j(z,t(z);A)}{\partial z} dz \end{aligned}$$

■

Proposition 1 plays a central role in the nonparametric identification of the model, presented in section 5 of this paper, and in particular for the identification of the mapping function $z(t)$.

3.4 Special cases

The following special cases are noteworthy for their importance in the nonparametric identification of the model. Suppose that not all jobs' attributes provide intrinsic disutility. Suppose for instance that z is partitioned as $z = \begin{pmatrix} z_p \\ z_s \end{pmatrix}$, where z_p is a vector of those jobs' attributes that derive intrinsic disutility, i.e. $\frac{\partial j(z,t;A)}{\partial z_p} > 0$, and z_s its complement containing attributes that derive no intrinsic disutility, $\frac{\partial j(z,t;A)}{\partial z_s} = 0$ for all z and t. From the first order condition to utility maximization, we have:

$$\frac{\partial w(z,t)}{\partial z} = \frac{\partial j(z,t;A)}{\partial z} = \begin{pmatrix} \frac{\partial j(z,t;A)}{\partial z_p} \\ 0 \end{pmatrix}$$

This has two important implications. First, the implicit function $z(t; A, w_z)$ will only have solutions for jobs' attributes z_p, not for z_s. In other words, the first order condition to utility maximization only solves the assignment problem for z_p as a function of all t. All mappings of z_s on t would lead to the same pricing equilibrium. However, if all workers' attributes t are productive, the first order condition to profits maximization in equation 2, yields a mapping of all t on *all* z. Its inverse will provide a unique solution for z_s as a function of t.

Second, the wage differential for jobs' attributes z_s, corresponding to attributes from which no intrinsic disutility is derived, will be 0 and therefore, z_s are not arguments of w^z:

$$\frac{\partial w(z,t)}{\partial z_s} = w^z_{z_s}(z) = \frac{\partial j(z,t;A)}{\partial z_s}|_{t=t(z)} = 0$$

Hence, holding z_p fixed, variations in z_s provide variations in all $t = t(z)$ and hence in wages through w^t while w^z remains constant. Attributes z_s provide exclusion restrictions that enable nonparametric identification of w^t and w^z in a single hedonic market.

Similarly, suppose now that workers' attributes t_p are not productive, i.e. $\frac{\partial p(z,t;E)}{\partial t_p} = 0$ for all t and z, whereas attributes t_s are, $\frac{\partial p(z,t;E)}{\partial t_s} > 0$. From the first order condition to profits maximization, we have:

$$\frac{\partial w(z,t)}{\partial t} = \frac{\partial p(z,t;E)}{\partial t} = \begin{pmatrix} 0 \\ \frac{\partial p(z,t;E)}{\partial t_s} \end{pmatrix}$$

Again, the implicit function $t(z; E, w_t)$ will only have solutions for workers' attributes t_s, not for t_p. In other words, the first order condition to profits maximization only solves the assignment problem for t_s as a function of all z. If all jobs' attributes z are disamenities, however, the first order condition to utility maximization in equation 1, yields a mapping of all z on *all* t. The wage differential for workers' attributes t_p will be 0 and therefore, t_p are not arguments of w^t:

$$\frac{\partial w(z,t)}{\partial t_p} = w^t_{t_p}(t) = \frac{\partial p(z,t;E)}{\partial t_p}|_{z=z(t)} = 0$$

Hence, holding t_s fixed, variations in t_p provide variations in all $z = z(t)$ and hence in wages through w^z while w^t remains constant. Attributes t_p provide exclusion restrictions that enable nonparametric identification of w^t and w^z in a single hedonic market.

4 Quadratic-normal example

4.1 The model

Let $n_z = n_t = n$. Suppose that, as in Tinbergen (1956) job dissatisfaction is defined as $j(z, t; A) = \frac{1}{2}(z-t)' A (z-t)$, where A is a positive definite matrix of preference parameters. Suppose further that productivity is given by $p(z, t; E) = b_0 + b'z + c't + \frac{1}{2}z'Bz + \frac{1}{2}t'Ct + t'Dz$ with $E = \{b_0, b, c, B, C, D\}$ and where b_0 is a positive constant, b and c are vectors filled with positive constants or zeros and B, C and D are matrices of parameters. The parameters contained in b and B indicate how productive a machine with attributes z is, independently of the attributes of the worker operating this machine, i.e. $\frac{\partial p(z,t;E)}{\partial z} = b + Bz$ The parameters contained in c and C indicate the extent to which workers' attributes affect productivity, independently of the attributes of the machine, i.e. $\frac{\partial p(z,t;E)}{\partial t} = c + Ct$. The parameters contained in D indicate the extent to which the attributes of machines complement or substitute workers' attributes, i.e. $\frac{\partial p(z,t;E)}{\partial z \partial t} = D$.

Note that since t and z take on negative values with positive probability, i) the productivity of machines decreases with workers' attributes for some machines with negative attributes, $\frac{\partial p}{\partial t} = c + Ct + Dz < 0$ for some z, and ii) the productivity of workers decreases with machines attributes for some workers with negative attributes, $\frac{\partial p}{\partial z} = b + Bz + Dt < 0$ for some t. However, the share of machines and the share of workers for which i) and ii) hold can be made arbitrarily small by varying the parameters of the distribution of z and t.

The generalized Spence-Mirrlees condition will be satisfied as long as $|D + A| \neq 0$. As long as $|D + A| \neq 0$, equilibrium is pure for any distributions F_t and F_z so that the mapping function $t(z)$ is invertible with inverse $z(t) \equiv t^{-1}(t)$. The mapping function $t(z)$ is linear when the distributions of t and z are normal.

The first order conditions read now as:[8]

$$\begin{aligned}
\frac{\partial w(z,t)}{\partial z} &= A(z-t) \\
\frac{\partial w(z,t)}{\partial t} &= c + Ct + Dz
\end{aligned}$$

Note that the wage differential for jobs' attributes is (positively) related to workers preference parameters A while the wage differential for workers' attributes

[8] The second order conditions are trivial and given by:

$$\begin{aligned}
\frac{\partial^2 w(z,t)}{\partial z^2} - A &< 0 \\
C - \frac{\partial^2 w(z,t)}{\partial t^2} &< 0
\end{aligned}$$

is related to (a subset of) E.

It is now easy to see that the first order conditions yield linear mapping of jobs' attributes on workers' attributes if and only if $\frac{\partial w(z,t)}{\partial t}$ is linear in t and $\frac{\partial w(z,t)}{\partial z}$ is linear in z. This, in turns, implies that the equilibrium wage function is quadratic and reads as:

$$w(z,t) = \delta_0 + \delta' t + \lambda' z + \frac{1}{2} t' \Delta t + \frac{1}{2} z' \Lambda z \tag{3}$$

where $w^z(z) = \lambda' z + \frac{1}{2} z' \Lambda z$ and $w^t(t) = \delta' t + \frac{1}{2} t' \Delta t$.

Using equation 3 in equations 1 and 2 respectively and rearranging yields:

$$\lambda + (\Lambda - A)\, z = -At \tag{4}$$

$$\delta - c + (\Delta - C)\, t = Dz \tag{5}$$

These are linear functions and the reduced form solution will be of the form $t = \pi_0 + \Pi_1 z$ or $z = -\Pi_1^{-1}\pi_0 + \Pi_1^{-1} t$. Plugging $t = \pi_0 + \Pi_1 z$ into 4 yields $\lambda = -A\pi_0$ and $\Lambda = A(I - \Pi_1)$. Plugging $z = -\Pi_1^{-1}\pi_0 + \Pi_1^{-1} t$ into Equation 5 yields $\delta = c - D\Pi_1^{-1}\pi_0$ and $\Delta = C + D\Pi_1^{-1}$.

As noted earlier by Tinbergen (1956) and Epple (1984), when attributes on both sides of the labor market are normally distributed, i.e. $z- > N(\mu_z, \Sigma_z)$ and $t- > N(\mu_t, \Sigma_t)$, linear mapping functions of the form $t = \pi_0 + \Pi_1 z$ equilibrate supply and demand. Indeed, the equilibrium condition $f_t(t)dt_1 dt_2 ... dt_N = f_z(z)dz_1 dz_2 ... dz_N$ given normally distributed attributes, is equivalent to equating the means, i.e. $\mu_t = \pi_0 + \Pi_1 \mu_z$ and equating the variances, i.e. $\Sigma_t = \Pi_1' \Sigma_z \Pi_1$.

To find the solution for π_0 and Π_1, first note that $\Sigma_t = \Pi_1' \Sigma_z \Pi_1 = (-\Pi_1)' \Sigma_z (-\Pi_1)$. There are therefore two solutions to this equilibrium condition, one with positive assortative matching $\Pi_1 > 0$ and one with negative assortative matching $\Pi_1 < 0$. These two solutions however will give rise to different total surplus. The one that maximizes total surplus will prevail. If workers' and jobs' attributes are globally complements (substitutes) in surplus, i.e. $D + A > 0$, then total surplus will be maximized by mapping higher t with higher (lower) z, i.e. $\Pi_1 > 0$ (< 0). The solution for π_0 and Π_1 is:[9]

[9] Note that the power p, $p \in R$, $p \neq 0$, of a square matrix A of size $n \times n$ is obtained as $A^p X = X diag(\lambda)$ where X is matrix of size $n \times n$ formed of the n eigenvectors of A and λ is the vector containing the corresponding eigenvalues. If in addition A is symmetric, then X is orthogonal so that $X'X = XX' = I$ and, post-multiplying both sides by X', the result simplifies to $A^p = X diag(\lambda)^p X'$. The matrix A^p will be real if and only if all eigenvalues λ are real and strictly positive that is if and only if A is positive definite. Since Σ_t and Σ_z are symmetric, the above result applies to $\Sigma_t^{-1/2}$ and $\Sigma_z^{1/2}$. So, $\Sigma_t^{-1/2}$ and $\Sigma_z^{1/2}$ will be real if and only if all eigenvalues of Σ_t and Σ_z respectively are real and positive. (See Bosch (1987))

$$
\begin{aligned}
\pi_0 = \mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z \\
\Pi_1 = \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}
\end{aligned}
\quad \text{if } D + A > 0
$$

$$
\begin{aligned}
\pi_0 = \mu_t + \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z \\
\Pi_1 = -\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}
\end{aligned}
\quad \text{if } D + A < 0
$$

To see this, note first that $\Pi_1 \left(\Pi_1^{-1}\right)' = \Pi_1^{-1}\Pi_1' = I$ where I is the identity matrix. Post-multiply both sides of the equation $\Sigma_t = \Pi_1'\Sigma_z\Pi_1$ by $\left(\Pi_1^{-1}\right)' = \Sigma_t^{-1/2}\Sigma_z^{1/2}$. This yields $\Sigma_t\Sigma_t^{-1/2}\Sigma_z^{1/2} = \Pi_1'\Sigma_z$. Pre-multiply both sides of this equation by Π_1^{-1}. This yields the identity $\Sigma_z^{1/2}\Sigma_t^{-1/2}\Sigma_t\Sigma_t^{-1/2}\Sigma_z^{1/2} = \Sigma_z$.

If $D + A > 0$, the equilibrium wage function has for parameters:

$$
\begin{aligned}
\lambda &= A\left(\mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) && (6)\\
\Lambda &= A\left(\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1} - I\right) && (7)\\
\delta &= c - D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)\left(\mu_t - \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) && (8)\\
\Delta &= C + D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right) && (9)
\end{aligned}
$$

If $D + A < 0$, the equilibrium wage function has for parameters:

$$\lambda = A\left(\mu_t + \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) \tag{10}$$

$$\Lambda = -A\left(\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1} + I\right) \tag{11}$$

$$\delta = c + D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)\left(\mu_t + \left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right)^{-1}\mu_z\right) \tag{12}$$

$$\Delta = C - D\left(\Sigma_z^{1/2}\Sigma_t^{-1/2}\right) \tag{13}$$

The constant δ_0 is not identified. To close the model, the usual assumption (see Ekeland et al. (2002 and 2004), Heckman et al. (2005) and Sattinger (1979) among others) is to fix a reservation value for the utility, say $\underline{u}$ and rent $\underline{r}$ so that u and r must be larger than their respective thresholds.

To summarize, we have shown that the equilibrium parameters of the wage function λ, Λ, δ and Δ are retrieved from the distribution parameters of workers' and jobs' attributes using the mapping function $t(z) = \pi_0 + \Pi_1 z$ where Π_1 is identified by the variances and covariances of workers' and jobs' attributes only and π_0 is identified from the means of workers and jobs' attributes and from Π_1. The second order coefficients of the wage function Λ and Δ depend only on the matrix of variances and covariances of workers' and jobs' attributes. The first order coefficients λ and δ depend on the matrix variances and covariances and the means of workers' and jobs' attributes.

4.2 Relations to Tinbergen (1956) and Sattinger (1979)

Tinbergen (1956), Epple (1984) and Ekeland et al. (2002 and 2004) consider the case where workers' attributes do not contribute to production, i.e. $\frac{\partial p(z,t;E)}{\partial t} = 0$ for all t and z, so that the first order condition to rents maximization in equation 2 indicate no wage differentials across workers' attributes. In the quadratic-normal example, this condition is met when $D = C = \boxed{0}$ where $\boxed{0}$ is a matrix filled with zeros, $c' = \underline{0}$, so that equation 5 yields $\Delta = C = \boxed{0}$ and $\delta = c = \underline{0}$. Since $D = \boxed{0}$, the generalized Spence-Mirrlees condition for pure equilibrium is now satisfied for $|D + A| \neq 0$, the equilibrium will be pure in Tinbergen's model only if $|A| \neq 0$.

The model proposed above admits Sattinger's differential rents model as a special case. This is the case when t and z are unidimensional and z carries no intrinsic disutility so that $\frac{\partial j(z,t;A)}{\partial z} = 0$ for all z and t or $A = \boxed{0}$ in the quadratic-normal example. From equation 2 (equation 4 respectively in the quadratic-normal example) we then have $\Lambda = (A =)\boxed{0}$ and $\lambda = \underline{0}$ so that the wage function depends merely on t. As soon as t is loaded with intrinsic disutility, $A \neq \boxed{0}$, the slope of the rents function increases and the increase is more pronounced for higher z. Again, since $A = \boxed{0}$, the generalized Spence-Mirrlees condition is satisfied if $|D + A| \neq 0$.

5 Implications for empirical applications

5.1 Literature on matched employer-employee data

The model naturally generates equilibrium wage functions with both workers' and jobs' (firms') attributes as arguments. Moreover, the model presented above does not restrict the correlation between both contributions to wages. The correlation could be anything between -1 and 1 depending on the production parameters E, the preference parameters A and the distribution of attributes F_t and F_z.

This links the model to the empirical literature estimating wage functions using matched employer-employee data, e.g. Abowd et al. (1999). This literature typically finds that both workers' and firms' fixed-effects contribute significantly to wage formation and the correlation between workers' and firms' fixed-effects is low or even negative. While Shimer's (2005) unidimensional assignment model with coordinative frictions could generate low or even negative correlation if frictions are large enough, the unified model presented above shows that a frictionless economy could also be characterized by a low or negative correlation between the contribution of workers' attributes to wages and that of their job/firm even when sorting exhibits positive assortative matching. This is possible as long as sorting occurs on both skills and preferences.

To illustrate this result, consider the unidimensional quadratic-normal model, i.e. $n_z = n_t = 1$. For the sake of the argument, think of persons with high t as risk loving persons and jobs with high z as risky jobs. Let $A = D = 3$, $c = C = 0$ and let $t- > N(\mu_t = 4, \Sigma_t = 3)$ and $z- > N(\mu_z, \Sigma_z)$ where $\mu_z \in [1, 7]$ and $\Sigma_z \in [1, 5]$. In the simulations presented below, each economy will be associated with different combinations of μ_z and Σ_z. However, note that since $A + D = 3 > 0$ in all economies, $\Pi_1 = \frac{\sqrt{3}}{\sqrt{\Sigma_z}} > 0$ so that in all economies simulated, assignment satisfies positive assortative matching so that larger values of t are assigned to larger values of z in equilibrium.

For a given economy, i.e. for given values of μ_z and Σ_z, I simulate a matched employer-employee data set. In this data set, there are 400 firms of identity $j =< 1, ..., 400 >$ and 400 workers of identity $i =< 1, ...400 >$ observed in five successive years. It is assumed that in each period a firm employs one and only one worker and a worker works in one and only one firm. Each firm j draws an attribute z_j from the distribution F_z and each worker i draws an attribute t_i from F_t. In each year of the panel, assignment of workers to firms occurs according to the unified model and the equilibrium wage is the solution provided above in the quadratic-normal example. The worker with the highest t gets assigned to the firm with the highest z and etc. since $\Pi_1 = \frac{\sqrt{3}}{\sqrt{\Sigma_z}} > 0$. To identify a firm fixed-effect, we must observe at least two different workers working at that firm over the time span. Similarly, to identify a worker fixed-effect we must observe that worker at least in two different firms over the time span. To create this type of mobility in the data while preserving the underlying equilibrium mapping $t(z) = \pi_0 + \Pi_1 z$ and wage function $w(z,t) = \delta_0 + \delta t + \lambda z + \frac{1}{2}\Delta t^2 + \frac{1}{2}\Lambda z^2$ in each time period, that is holding A, D , c, C, μ_z, μ_t, Σ_z and Σ_t constant over time, assume that each period, 5% of the workers, randomly chosen, observe an increase in their attribute of 0.1 standard deviation, i.e. from t to $t+$ $0.1 \times \sqrt{\Sigma_t} = t + 0.1 \times \sqrt{3}$, and 5% of the workers, randomly chosen, observe a decrease in their attribute of 0.1 standard deviation, i.e. from t to $t - 0.1 \times \sqrt{3}$. This has the effect of shuffling the rank of workers and hence the identity of the workers working in the various firms while keeping the distribution of t constant over time and therefore the hedonic assignment occurring in each year. The correlation between firms' and workers' fixed-effects is therefore only determined by the correlation between $\delta t_i + \frac{1}{2}\Delta t_i^2$ and $\lambda z_j + \frac{1}{2}\Lambda z_j^2$ for each pair $< i, j >$ along the equilibrium mapping $t(z) = \pi_0 + \Pi_1 z$ and not due to changes in the equilibrium wage function.

The econometrician only observes pairs $< i, j >$ and the associated wage w_{ij} in each period. The econometrician cannot estimate the function $w(.,.)$.[10] However, since there is mobility of workers across firms, the econometrician can estimate fixed-effects for workers and firms by running a linear regression of the form $w = Pa + Fb + e$ where P is a matrix of dummy variables capturing the person effects and F is a matrix of dummy variables capturing the firm effects. The estimate a is an

[10] Note that, as a well-known general comment on the quadratic-normal economy, since the mapping is linear, even if we would observe t and z, we would not be able to estimate the wage function due to multicolinearity.

estimate of a worker's fixed-effect given by $\overline{\delta t_i + \frac{1}{2}\Delta t_i^2}$, equal to $\delta t_i + \frac{1}{2}\Delta t_i^2$ for those workers with fixed t over the 5-year window or to the mean of $\delta t_i + \frac{1}{2}\Delta t_i^2$ for those with varying attributes, and the estimate of b is an estimate of $\lambda z_j + \frac{1}{2}\Lambda z_j^2$. For each firm we can calculate the mean of workers' fixed-effects and then calculate the correlation between firms' fixed-effects and firms' mean of workers' fixed-effects.

Figure 1 plots the correlation between firms' fixed-effects and (mean at the firm of) workers' fixed-effects as μ_z increases from 1 to 8. Each curve on this figure represents a different value of the variance of job attributes $\Sigma_z =< 1, 3, 5 >$. The figure indicates that the correlation varies between -0.6 and 0.4 in these examples. Even though the true underlying model generating the panel data sets is a frictionless assignment model with positive assortative matching, i.e. $\Pi_1 = \frac{\sqrt{3}}{\sqrt{\Sigma_z}} > 0$, the correlation between firms' and workers' fixed-effects is not close to 1 and even is negative for a fairly large range of the parameter set.

5.2 Identification and estimation in a single hedonic market

5.2.1 Identification

Since the seminal work by Rosen (1974), the traditional approach to estimate preference parameters, i.e. parameters A of the function $j(z, t; A)$,[11] has consisted

[11] All techniques below apply also to the estimation of productivity parameters by symmetry.

of two steps. In the first step, using market data on wages and jobs' attributes, one estimates the wage function to get $\widehat{w}(z)$ applying the functional form that fits best the data. In the second step, one uses the first order condition in Equation 1 together with the marginal wage derived from the first step, i.e. $\frac{\partial \widehat{w}(z)}{\partial z}$, to recover preference estimates $j(z,t;A)$. Early literature by Brown and Rosen (1982), Epple (1987), Bartik (1987) and Kahn and Lang (1988) has argued that $j(z,t;A)$ cannot be identified in a single hedonic market unless an arbitrary nonlinear marginal utility is assumed. Recently, Ekeland et al. (2002 and 2004) have shown that nonlinearity is a generic feature of the hedonic model not an arbitrary choice and Ekeland et al. (2002 and 2004) and Heckman et al. (2005) have provided conditions under which nonparametric identification of additive and nonadditive hedonic models of the Tinbergen class is possible in a single hedonic market.

These conditions build on results from Matzkin (2003) on nonparametric estimation of additive and nonadditive random functions. All the results from Ekeland et al. and Heckman et al. crucially depend on the assumption that $w_z^z(z)$ is known (or estimated from data on wages and z). In the Tinbergen class of models, where wages depend only on z, identification of $w_z^z(z)$ follows by assumption. However, in the unified economy, wages are given by the unknown function $w(z,t) = w^z(z) + w^t(t)$. The function $w^z(z)$ is not known in general.[12] Moreover, without further assumption, $w^z(z)$ is not identified non parametrically since for

[12]See footnote 2.

any value of t, the value of z is uniquely determined by the mapping function $z = z(t)$.

Since all the identification results presented in Heckman et al. follow once $w^z(z)$ is known, I focus in this paper on the identification of $w^z(z)$ and refer the reader to Heckman et al. for identification results of $j(z, t; A)$. The method proposed to identify $w^z(z)$ relies on shape restrictions on the production function $p(z, t; E)$.

Following Ekeland et al. and Heckman at al., assume that z is a scalar and $t =< t^o, t^u >$ where t^o is a vector of observed attributes and t^u is a scalar unobserved (to the econometrician) attribute. Assume further that t^u is independent of t^o. The identification method requires first an identification of the mapping function $z = z(t^o, t^u)$. Lemma 1 is a generalization of the identification proof provided in Heckman et al. (2005) to the unified model. The additive separability of the wage function guarantees that we end up with the same expression of the mapping function as in Heckman et al. (2005), page 6.

Lemma 3 *The mapping function $z = z(t^o, t^u)$ is identified in the unified hedonic model.*

Proof. See the Appendix. ■

Once the mapping function $z(t^o, t^u)$ is known, we can proceed to the identification of $w^z(z)$. The next theorem shows that imposing some shape restrictions on the production function $p(z, t; E)$ allows us to identify $w^z(z)$.

Theorem 4 *Let there be at least one attribute* t_i^o *so that* $\frac{\partial j(z,t;A)}{\partial z \partial t_i^o} \neq 0$ *and let* $p(z,t;E) = q(z, t_{-i}^o, t^u) + r(t_i^o)$ *where* $r(.)$ *is a known differentiable function. Then for any* (z, t^o, t^u)*, the function* $w^z(z)$ *is identified.*

Proof. Consider the first order condition to utility maximization. We have $\frac{\partial w(z(t),t)}{\partial z} - \frac{\partial j(z(t),t;A)}{\partial z} = 0$. Totally differentiating with respect to t_i^o and rearranging yields:

$$\begin{aligned}\frac{\partial z(t^o,t^u)}{\partial t_i^o} &= \frac{\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t_i^o} - \frac{\partial^2 w(z(t^o,t^u),t;A)}{\partial z \partial t_i^o}}{\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t;A)}{\partial z^2}} \\ &= \frac{\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t_i^o}}{\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t;A)}{\partial z^2}}\end{aligned}$$

The second step holds from the additive separability of $w(z,t)$. This means that $\frac{\partial z(t^o,t^u)}{\partial t_i^o} \neq 0$ if $\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t_i^o} \neq 0$. Hence, varying t_i^o while holding t_{-i}^o and t^u constant will vary z and hence $w^z(z)$. However, since $\frac{\partial p(z,t;E)}{\partial t_i^o} = r'(t_i)$ we have:

$$\begin{aligned}w^t(\bar{t}_{-i}^o, t_i^o, \bar{t}^u) &= \int \frac{\partial p(z(\bar{t}_{-i}^o, t_i^o, \bar{t}^u), \bar{t}_{-i}^o, t_i^o, \bar{t}^u; E)}{\partial t_i^o} dt_i^o \\ &= \int \frac{\partial r(t_i^o)}{\partial t_i^o} dt_i \\ &= r(t_i^o) + const\end{aligned}$$

where *const* is a constant of integration.

Holding t^o_{-i} fixed at $\bar{t}^o_{-i}$ and t^u fixed at $\bar{t}^u$, variations in t^o_i generate variations in z and hence in $w^z(z)$ together with know variations in $w^t(t)(= r(t^o_i))$. We can therefore identify $w^z(z(\bar{t}^o_{-i}, t^o_i, \bar{t}^u))$, up to a constant, as $w(t^o_i) - r(t^o_i)$, where $w(t^o_i)$ is the wage observed in the data for workers with attributes t^o_i, $\bar{t}^o_{-i}$ and $\bar{t}^u$. ■

An important special case is met when $r(t^o_i)$ is a constant. This occurs when t^o_i is a pure preference attribute, i.e. $\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t^o_i} \neq 0$ and $\frac{\partial p(z,t;E)}{\partial t^o_i} = r'(t^o_i) = 0$ for all z and t^o_i. This means that t^o_i plays the role of an exclusion restriction in the wage equation. Attribute t^o_i is an argument of the mapping function $z(.)$ but not of $w^t(.)$. This holds for all values of t^o_{-i} and t^u. Holding $t^o_{-i} = \bar{t}^o_{-i}$ and $t^u = \bar{t}^u$, we can therefore identify $w^z(z(\bar{t}^o_{-i}, t^o_i, \bar{t}^u))$, up to a constant, as $w(t^o_i)$, where $w(t^o_i)$ is the wage observed in the data for workers with attribute t^o_i and $\bar{t}^o_{-i}$ and $\bar{t}^u$.

5.2.2 Estimation

Since the estimation results presented in Heckman et al. (2005) follow once we have estimated $w^z(z)$, this paper focuses on the estimation of $w^z(z)$ and refers the reader to Heckman at al. (2005) for the estimation of $j(z, t; A)$. To present the problem in terms of random functions, let W, T^o, Z be the observable variables of our model and let T^u be the unobservable variable. All variables are of dimension 1 except T^o that has dimension of at least 1. Let our model be $W = w^z(Z) + w^t(T^o, T^u)$ where w^z and w^t are unknown functions continuous in Z and (T^o, T^u) respectively. The functions w^z and w^t are assumed to belong to the set of functions derived from the unified economy outlined above. Let $F_{T^u}(.)$ be the distribution of T^u and let $F_{W,Z,T^o}(.; w', F'_{T^u})$ be the joint distribution of the observable variables when $w = w'$ and $F'_{T^u} = F_{T^u}$. Assume that T^u is independent of T^o. Our data consist of a sample of N draws of W, Z and T^o from a single hedonic market.

To estimate w^z, we need first to estimate the mapping function. From the proof of Lemma 1 we know that $z(t^o, t^u)$ is strictly increasing in its last argument and since by assumption T^u is independent of T^o, we have:

$$\begin{aligned} F_{T^u}(t^u) &= \Pr(T^u < t^u) \\ &= \Pr(T^u < t^u | T^o = t^o) \\ &= \Pr(z(T^o, T^u) < z(t^o, t^u) | T^o = t^o) \\ &= F_{Z|T^o=t^o}(z(t^o, t^u)) \end{aligned} \tag{14}$$

The first equality follows by the definition of F_{T^u}, the second by the independence of T^o and T^u, the third by the monotonicity of $z(t^o, .)$ and the fourth by the definition of $F_{Z|T^o=t^o}$.

Suppose we know F_{T^u}, since $F^{-1}_{Z|,T^o}$ exists from the monotonicity of $z(t^o, .)$, we recover $z(t^o, t^u)$ as:

$$z(t^o, t^u) = F^{-1}_{Z|,T^o}(F_{T^u}(t^u))$$

Suppose instead that we normalize the mapping function so that for some $\bar{t}^o$ and all t^u we have $z(\bar{t}^o, t^u) = t^u$. From Equation 14, we have $F_{T^u}(t^u) = F_{Z|T^o=\bar{t}^o}(t^u)$ and we therefore identify the distribution of T^u from the distribution of Z conditional on $T^o = \bar{t}^o$. The expression of the function z is now given by noting that $F_{Z|T^o=t^o}(z(t^o, t^u)) = F_{T^u}(t^u) = F_{Z|T^o=\bar{t}^o}(z(\bar{t}^o, t^u))$. The first equality follows from the monotonicity of $z(t^o, .)$ in its last argument and the second holds from the previous normalization. This means that we have:

$$z(t^o, t^u) = F^{-1}_{Z|T^o=t^o}\left(F_{Z|T^o=\bar{t}^o}(t^u)\right)$$

Estimates of z are obtained by replacing the true distributions F by their kernel estimators $\widehat{F}$ following the definitions provided in section 3.4 in Matzkin (2003) or pp. 32-33 in Heckman et al. (2005), in the above equalities. Denote $\widehat{z}(t^o, t^u)$ the estimated mapping function. Theorem 1 suggests the following estimator $\widehat{w}^z(z)$ of w^z for any $\bar{t}^o_{-i}, t^o_i, \bar{t}^u$:

$$\widehat{w}^z(\widehat{z}(\bar{t}^o_{-i}, t^o_i, \bar{t}^u)) = w(t^o_i) - r(t^o_i)$$

where $w(t^o_i)$ is the wage data for workers with attributes $t =< \bar{t}^o_{-i}, t^o_i, \bar{t}^u >$ and using the known function r to calculate $r(t^o_i)$.

6 Extensions

6.1 The unified hedonic *production* economy model

Suppose that instead of being endowed with a machine, firms can produce their own machine. For instance, firms could invest in less noisy machines, safer machines, machines requiring less physical strength to operate, high-tech machines etc. Suppose further that firms are endowed with a vector of attributes y, $y \in \mathbb{R}^{n_y}$. To fix ideas, these attributes could be related to investments capacities but also to the managers' attributes, again, either skills or preferences. Let $f_y(y)$ and $F_y(y)$ be the PDF and CDF of y respectively and let F_y be absolutely continuous with respect to Lebesgue measure.

Let the costs of producing a machine with attributes z for a firm with attributes y be given by the continuous function $c(y, z; G)$ with parameters G. It is still assumed that to produce output each machine needs to be operated by one and only one worker so that workers and firms match one-to-one. The profits of a firm with attributes y producing output with machine z and employing worker t are now given by:

$$r(z,t,y) = p(z,t;E) - w(z,t) - c(y,z;G)$$

The first order condition to utility maximization is unchanged and given by equation 1. We therefore have the mapping function $z(t;A,w_z)$ indicating the optimal machine demanded by a worker with attributes t when wage differential at z is given by w_z. However, firms are now maximizing profits by selecting the optimal combination of worker t and machine z. First order conditions for profit maximization read as:

$$\frac{\partial w(z,t)}{\partial z} = \frac{\partial p(z,t;E)}{\partial z} - \frac{\partial c(y,z;G)}{\partial z} \tag{15}$$

$$\frac{\partial w(z,t)}{\partial t} = \frac{\partial p(z,t;E)}{\partial t} \tag{16}$$

Let $t(z;E,w_t)$ denote the implicit function that solves Equation 16 for t given parameters E and function w_t where $w_t \equiv \frac{\partial w}{\partial t}$. This function indicates the optimal worker t to select for a firm supplying machine with attributes z when the wage differential at t is given by w_t. Let $z(y, t; G, w_z)$ denote the implicit function that solves Equation 15 for z given parameters E and G and function $w_z \equiv \frac{\partial w}{\partial z}$. This function indicates the optimal machine z to supply for a firm with attributes y employing worker with attributes t. Substituting $t(z; E, w_t)$ for t in $z(y, t; G, w_z)$ we obtain an implicit function $z(y; E, G, w_t, w_z)$ indicating the optimal machine $z^* = z(y; E, G, w_t, w_z)$ to supply for a firm with attributes y given productivity parameters E, costs parameters G and wage function w.

Assume further that the total surplus function $s(z, t; E, A, G) \equiv p(z, t; E) - c(y, z; G) - j(z, t; A)$ satisfies the generalized Spence-Mirrlees condition so that equilibrium is pure and the mapping functions $z(y; E, G, w_t, w_z)$ and $t(z; A, w_z)$ are invertible. Define these inverse functions as $y(z; E, G, w_t, w_z)$ and $z(t; A, w_z)$ respectively. Workers' demand for machines with attributes z is then given by $f_z^d(z)dz = f_t(t(z; A, w_z)) \left| \frac{\partial t(z;A,w_z)}{\partial z} \right| dz$ while firms' supply is given by $f_z^s(z)dz = f_y(y(z; E, G, w_t, w_z)) \left| \frac{\partial y(z;E,G,w_t,w_z)}{\partial z} \right| dz$. For an equilibrium to be reached, the supply of machines with attributes z should be equal to workers' demand for machines with attributes z for all z. This means that:

$$f_t(t(z; A, w_t)) \left| \frac{\partial t(z; A, w_z)}{\partial z} \right| dz = f_y(y(z; E, G, w_t, w_z)) \left| \frac{\partial y(z; E, G, w_t, w_z)}{\partial z} \right| dz$$

Equilibrium will be reached by choosing the right shape for the function w and in particular the right differentials at t and z. The equilibrium in this economy is therefore characterized by a wage function $w(z,t)$ and a mapping of workers' attributes onto jobs' attributes $t(z;A,w_z)$ and a mapping function of firms attributes onto jobs' attributes and workers' attributes $y(z;E,G,w_t,w_z)$ so that i) supply equals demand everywhere on the support of z −provided all workers and firms receive more than their reservation levels− , ii) workers maximize utility and iii) firms maximize profits (rents minus costs of producing z).

6.2 Generalization to other markets

This paper relates to the general literature on hedonic models and not only on that segment focussing on the labor market. For instance, Epple's (1984) extension of Tinbergen's endowed economy to a production economy was originally written in a consumer/producer context, not a worker/firm context. The classical consumer/producer setting reads as follow.

There is a market for a good of attributes z and let $p(z)$ be the hedonic equilibrium price. Producers are endowed attributes y, $y \in \mathbb{R}^{n_y}$. To fix ideas, these attributes could be related to investments capacities but also to the managers' attributes, again, either skills or preferences. Let $f_y(y)$ and $F_y(y)$ be the PDF and CDF of y respectively and let F_y be absolutely continuous with respect to

Lebesgue measure. Firms profits are given by $p(z) - c(z, y; G)$. Consumers are endowed with attributes t, $t \in \mathbb{R}^{n_t}$, that reflect their preferences for the product. Let $f_t(t)$ and $F_t(t)$ be the PDF and CDF of t respectively and let F_t be absolutely continuous with respect to Lebesgue measure. Utility is given by $k(z, t; K) - p(z)$ where $k(z, t; K)$ is the indirect utility derived from consumption of z and K are preference parameters common to all consumers. In this classical setting, output is either produced without labor input or, labor input belongs to y and is fixed at the time firms decide what z to produce. Firms choose to produce the z that maximizes their profits and consumers/workers choose to consume the z that maximizes their utility.

An extension of the classical hedonic model related to the unified model outline above would be to allow firms to choose the type of worker to hire simultaneously with their choice of z. Suppose that to produce a unit of good of quality z, firms need to hire one and only one worker, $-$say y is a machine that needs to be operated by fixed quantity of workers, one and only one worker$-$. Suppose that the costs of producing z for a firm y when employing worker with attributes t are $c(z, y, t; G)$. Profits for firm y employing t to produce z are $p(z) - c(z, y, t; G) - w(z, t)$ where $w(z, t)$ is the wage of worker t at job z. Suppose workers t derive indirect disutility $j(z', t; A)$ while working at producing z'. Workers utility is then given by $k(z, t; K) - j(z', t; A) + w(z', t) - p(z)$.

In this economy, firms/producers optimize on z and t' and workers/consumers optimize on z and z'. The first order conditions for profits maximization read as:

$$
\begin{aligned}
p'(z) &= \frac{\partial c(z,y,t;G)}{\partial z} + \frac{\partial w(z,t')}{\partial z} && (17) \\
\frac{\partial c(z,y,t;G)}{\partial t} &= \frac{\partial w(z,t')}{\partial t'} && (18)
\end{aligned}
$$

Let $z(y,t';G,w_z,p')$ be the implicit function that solves Equation 17 for z the optimal good to produce for firm y with worker t' and let $t'(z,y;G,w_t)$ be the implicit function solving Equation 18 for t' the optimal worker to hire for firm y to produce z. Substituting $t'(z,y;G,w_t)$ for t' in $z(y,t';G,w_z,p')$ yields $z(y;G,w_t,w_z,p')$ the optimal quality of good to produce for a firm with attributes y given production technology G, the equilibrium wage function $w(z,t)$ –the slopes– and the equilibrium price function $p(z)$.

The first order conditions for utility maximization read as:

$$
\begin{aligned}
\frac{\partial k(z,t;K)}{\partial z} &= p'(z) && (19) \\
\frac{\partial j(z',t;A)}{\partial z} &= \frac{w(z',t)}{\partial z} && (20)
\end{aligned}
$$

Let $t(z; K, p')$ be the implicit function that solves Equation 19. This function indicates the attributes of workers consuming good of quality z in equilibrium, given product tastes K, and the equilibrium price function. Let $z'(t; A, w_z)$ be the implicit function that solves Equation 20. This function indicates the optimal job to choose for workers with attributes t given job tastes A and the equilibrium wage function $w(z, t)$.

In equilibrium, the worker that consumes good of quality z might not necessarily be the one that produces z, i.e. $t(z; K, p') \neq t'(z, y; G, w_t)$ in general.

7 Conclusion

This paper unifies the two classes of models within the sorting literature. The model nests both Tinbergen's model of sorting on job preferences and Sattinger's model of sorting on productivity. Under the assumption that all jobs' attributes lead to intrinsic disutility but workers' attributes do not affect productivity the model collapses to Tinbergen's model. Workers care about their job satisfaction but are equally productive at all jobs. This means that the equilibrium wage function does not depend on workers' attributes but merely on jobs' attributes. Opposite to this, under the assumption that all workers' attributes contribute to productivity but no jobs' attributes lead to intrinsic disutility the model collapses to Sattinger's differential rents model. Workers do not care about job satisfaction, only about their wage, but workers with different attributes are unequally productivity. This means that the equilibrium wage function does not depend on jobs' attributes but merely on workers' attributes. In the more general case depicted in the unifying model, workers do care about job satisfaction and productivity depends on workers' attributes. As a result, the equilibrium wage function has both workers' and jobs' attributes as arguments. it is shown that this wage function is additive separable and an example of closed form solution is provided when productivity and job satisfaction are quadratic and attributes on both sides are normally distributed.

The model is flexible enough to allow the correlation between the contribution of workers' attributes to wages and that of jobs' attributes to vary between -1 to 1. This correlation depends on preference parameters, technology parameters and the distribution of workers and jobs' attributes. The model therefore provides an explanation for Abowd et al.'s (1999) puzzling finding of a low or even negative correlation between workers' and firms' fixed-effects in wage regressions using matched employer-employee data that does not require (large) frictions (Shimer (2005)).

The model has implications for the estimation of preference (technology respectively) parameters in hedonic models. Recently, Ekeland et al. (2002 and 2004) and Heckman et al. (2005) have shown conditions under which nonparametric identification of additive and nonadditive marginal utility models of the Tinbergen class is possible in a single hedonic market. These conditions depend crucially on the assumption that $w^z(z)$ is known (or estimated from data on wages and z). While this is true by definition in the Tinbergen class of models studied by Ekeland et al. and Heckman et al., in the unified economy, wages are given by the function $w(z,t) = w^z(z) + w^t(t)$ where the functions $w^z(z)$ and $w^t(t)$ are unknown. Without further assumption, $w^z(z)$ is not identified nonparametrically since for any value of t, the value of z is uniquely determined by the mapping function $z = z(t)$. This paper first shows in Lemma 1 that the mapping function $z(t)$ is identified nonparametrically using results from Matzkin (2003). Lemma 1 generalizes Heckman et al.'s (2005) results to the unified hedonic model. Using the identification result for $z(t)$, this paper shows conditions under which $w^z(z)$ is identified nonparametrically. These conditions impose shape restrictions on the production function $p(z,t)$. In particular, the method assumes that $p(z,t) = q(z, t_{-i}) + r(t_i)$ where $r(.)$ is a known differentiable function and with $t =< t_{-i}, t_i >$ and where t_i is a preference attribute, i.e. so that $\frac{\partial j(z,t;A)}{\partial z \partial t_i} \neq 0$. A special case is met when t_i is a pure preference attribute, that is, t_i affects utility but not productivity, i.e. $\frac{\partial j(z,t;A)}{\partial z \partial t_i} \neq 0$ and $\frac{\partial p(z,t;E)}{\partial t_i} = 0$ for all z and t_i. Attribute t_i is an exclusion restriction in $w^t(.)$ since it does not affect productivity but not in $w^z(.)$ since it matters for job satisfaction.

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Appendix:

Proof Lemma 1. To see this, consider the first order condition to utility maximization, $\frac{\partial w(z(t),t)}{\partial z} - \frac{\partial j(z(t),t;A)}{\partial z} = 0$. Totally differentiating with respect to t^u and rearranging yields:

$$\begin{aligned}\frac{\partial z(t^o,t^u)}{\partial t^u} &= \frac{\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t^u} - \frac{\partial^2 w(z(t^o,t^u),t;A)}{\partial z \partial t^u}}{\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t;A)}{\partial z^2}} \\ &= \frac{\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t^u}}{\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial^2 j(z,t;A)}{\partial z^2}}\end{aligned}$$

The second step holds from the additive separability of $w(z,t)$. Using the second order condition, we have $\frac{\partial z(t^o,t^u)}{\partial t^u} > 0$ if $\frac{\partial^2 j(z(t^o,t^u),t)}{\partial z \partial t^u} < 0$. The mapping function is strictly increasing in t^u. Therefore, $z(t^o,t^u)$ is identified using normalization results from Matzkin (2003) and assuming that t^o and t^u are independently distributed. Normalization is required since there exist monotonic transformations g so that $(g \circ z, F_{t^u} \circ g^{-1})$ and (z, F_{t^u}), where F_{t^u} is the CDF of t^u, generate the same data. However, one can show that $z(t^o,t^u)$ is identified nonparametrically using a normalization (choosing one function g). One could either normalize the distribution of t^u (uniform for instance) or normalize the shape of the function $z(t^o,t^u)$ by imposing $z(\bar{t}^o,t^u) = t^u$ for some $\bar{t}^o$ for instance.

Suppose that we assume a certain distribution on t^u, so that F_{t^u} is known, then $F_{z|t^o=x}(z) = F_{t^u}(y)$ tells us that $z(x,y)$ is the same quantile of the distribution of z given t^o as the quantile that y is of the distribution of t^u. We recover $z(t^o,t^u)$

from $F^{-1}_{z|t^o}(\ F_{t^u}(t^u))$.

Suppose instead that we normalize the mapping function so that for some $\bar{t}^o$ and all t^u we have $z(\bar{t}^o, t^u) = t^u$. We can then show that $F_{t^u}(t^u) = F_{z|t^o=\bar{t}^o}(z(\bar{t}^o, t^u))$. We therefore identify the distribution of t^u from the distribution of z conditional on $t^o = \bar{t}^o$. The expression of the function z is now given by noting that $F_{z|t^o}(z(t^o, t^u)) = F_{t^u}(t^u) = F_{z|t^o=\bar{t}^o}(z(\bar{t}^o, t^u))$. The first equality holds since $z(t^o, t^u)$ is strictly increasing over t^u and the second holds from the previous normalization. This means that $z(t^o, t^u) = F^{-1}_{z|t^o}\left(F_{z|t^o=\bar{t}^o}(t^u)\right)$. ■

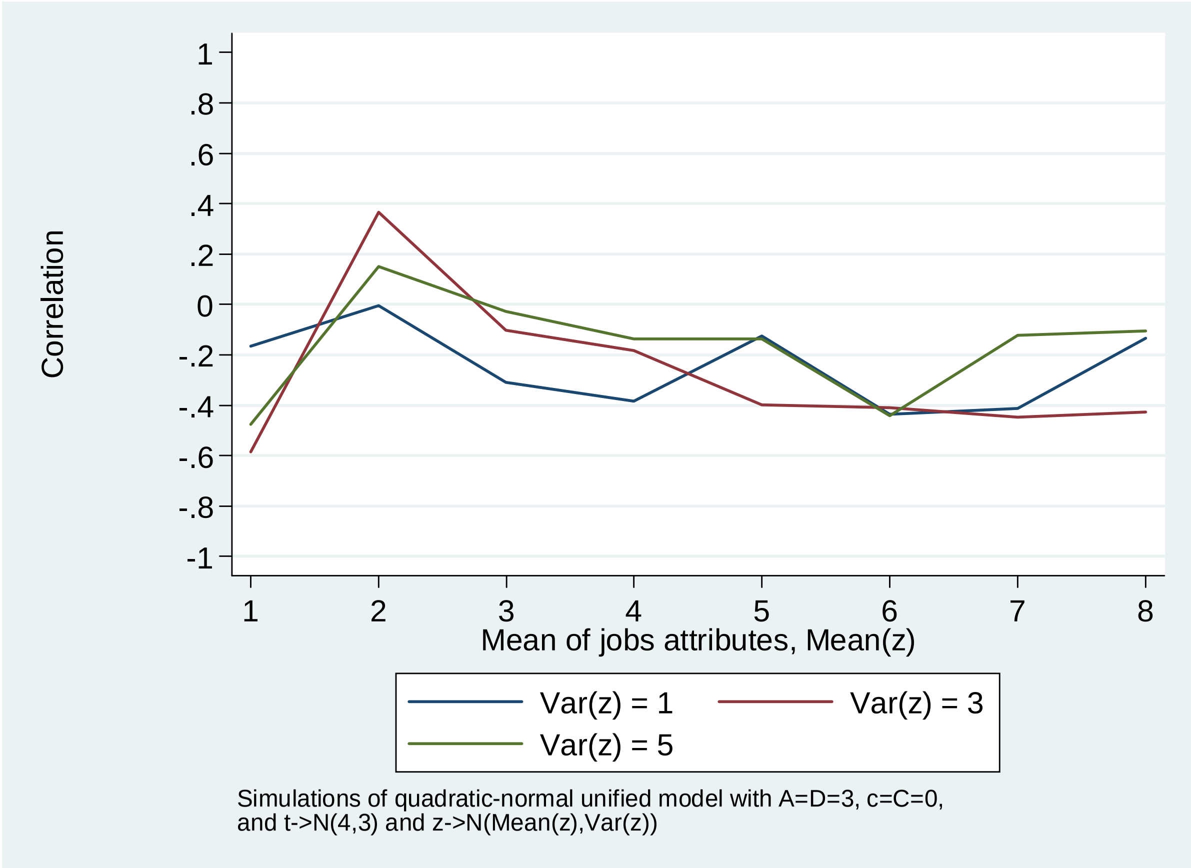

Figure 1: Simulated correlation between firms and workers fixed effects in panel data earnings regressions. Panel data generated using the unified assignment model.