Sequential Auctions
and
Alternating Price Competition

Kasper Leufkens

This book was typeset by the author using LaTeX.

Published by Universitaire Pers Maastricht
ISBN: 987 90 5278 768 8
Printed in The Netherlands by Datawyse Maastricht

Sequential Auctions and Alternating Price Competition

PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Universiteit Maastricht,
op gezag van de Rector Magnificus,
Prof. mr. G.P.M.F. Mols,
volgens het besluit van het College van Decanen,
in het openbaar te verdedigen op
vrijdag 31 oktober 2008 om 16.00 uur

door

Kasper Franciscus Hendricus Leufkens

Promotor:

Prof. dr. P.J.J. Herings

Copromotor:

Dr. R.J.A.P. Peeters

Beoordelingscommissie:

Prof. dr. R.J. Müller (voorzitter)

Prof. dr. T.J.S. Offerman (Universiteit van Amsterdam)

Dr. J.P. Reiss

Dit onderzoek werd gefinancieerd door de Maastricht Research School of Economics of Technology and Organizations (METEOR).

Acknowledgements

Four years ago, Ronald Peeters inquired whether I would be interested in a Ph.D. position. I quickly replied that I needed some time to think about it. The truth is that I had no idea what he was talking about. After finding out that being a Ph.D. candidate meant doing research, studying, teaching, and, ultimately, getting a doctorate, I was very pleased when the position was awarded to me.

This dissertation is the result of the research I carried out as a Ph.D. candidate at the Economics Department of Maastricht University. The four years in which I worked on this dissertation went by fast, although this was not caused by things going smoothly all the time. There were some setbacks and doing research can be frustrating at times. However, overall I had a very pleasant stay in academia and I am pleased to have completed this challenge.

The past four years were particularly pleasant thanks to the support of many people. My first words of gratitude go to my supervisor Ronald Peeters. Without his help I would never have started nor finished this book. What I really appreciated about Ronald was that he was always available for advice. I am grateful for the many things he taught me and for the confidence he had in me and my work.

I want to thank my promotor Jean-Jacques Herings for carefully reading all my research output and the comments and suggestions that always followed. I want to thank Dries Vermeulen and Marc Vorsatz for their cooperation on some of the research that is contained in this dissertation. I am grateful to Martin Strobel for introducing me to the world of behavioral and experimental economics and his invaluable advice on conducting statistical analyses. Furthermore, I want to thank the assessment committee consisting of Rudolf Müller, Theo Offerman, and Philipp Reiss for the useful comments they gave after reading the entire manuscript. More general, I want to thank all my colleagues at the Department of Economics of Maastricht University.

I want to thank Jeroen van den Berg, Rik Frehen, Jan (der Bube) Piplack, and Stephan Smeekes, who were not only colleagues but are also great friends. We often joked that if the others had not been there, each of us would have finished his dissertation in half the time. Had that been the case, I am sure it would have felt as a much longer time period than was the case now. I want to thank

Robert Flinzner and Martijn Souren for being my paranymphen. We studied together in Maastricht and it is great that we are still good friends.

On a more personal level, I would like to thank my family and friends for their contributions to my truly enjoyable social life. I am very grateful to my parents, Frans and Rien, and my sister Laura for their support and endurance over all those years. Finally, I want to thank Vera for all the love and support. The expression 'my better half' describes her better than any Dutch translation can.

Kasper Leufkens
Maastricht, August 27, 2008

Contents

Chapter 1

Introduction

Most economic interactions have a recurrent nature. Households continuously decide on consumption and savings, firms frequently adjust their strategic variables, and governments repeatedly decide on market regulations and budgets. In such dynamic settings, decisions are not only based on immediate rewards but also on their influence on future interactions. The dynamic structure of the interactions allows players to respond to previous actions of other players and to influence future decisions by them. The decision making can be complicated further by the presence of uncertainty over future market conditions.

Typically, when a competitive situation is modeled as a one-shot interaction, players have no opportunity to reward cooperative behavior and to punish aggressive competitive behavior. In a dynamic setting, players might act mutually nonaggressive in order to ensure higher profits to all of them. On the other hand, if a large initial market share can lead to long lasting reputational advantages, players will compete much more vigorously in the early time periods than they would in a one-shot interaction. In order to understand mutual aggressive or nonaggressive behavior by players, dynamic models have to be considered.

Insights in dynamic situations of strategic conflict can be acquired by applying non-cooperative game theory. The conflict situation is modeled as a game and this game is subsequently solved to find its equilibria. A game-theoretic equilibrium prescribes self-enforcing behavior of the players in the sense that none of the players has an incentive to change his behavior unilaterally. Equilibria provide information about the strategies of players and consequently market outcomes.

A central concept in economics is the functioning of markets. The field of economics that studies the strategic behavior of firms, the structure of markets and their interactions is named industrial organization. In this dissertation research in the field of industrial organization is presented. Dynamic settings of economic interaction in markets are studied by the application of non-cooperative game theory. The insights gained in market behavior and outcomes are of utmost importance when developing regulatory instruments, as these often affect the underlying incentive structure.

This dissertation consists of two parts and the parts differ in the market setting that is considered. The first part of this dissertation considers a sequential auction setting with one seller and multiple buyers who compete (bid) for winning the objects and experience positive synergies for winning multiple objects. That is, buyers can benefit from earlier wins in later auctions. The effect of the synergy on bidding behavior and payoff, auction revenue, and efficiency is studied. The second part of this dissertation considers a dynamic duopoly setting in which firms adapt their prices in alternating time periods. In this part, among others, the effect of exogenous demand shocks on pricing behavior is studied. Both parts of this dissertation consist of analytical, numerical, and experimental analyses. In the subsequent two sections the different parts are introduced in more detail.

1.1 Sequential auctions with synergies

Auctions have been used for the sale of various objects since ancient times. Recently, governments all over the world started using auctions for allocating spectrum licenses to companies that wish to provide wireless communication services. The uncoordinated sequence of UMTS (third generation mobile telecommunication) auctions by Western European governments between 2000 and 2001 drew a lot of attention, not only from academics and practitioners but also the public at large. When analyzing the results of one of these auctions, it should not be neglected that the auction was only a subgame of one large game, in which the objectives of players may have been to create a pan-European network (Van Damme, 2002). Cramton (2002) indeed argues that one reason for the enormous revenue in the spectrum auction by the United Kingdom is that it was the first in the sequence of European UMTS auctions. Winners in the United Kingdom auction were well positioned for subsequent auctions and hence bidders could view it as a foot-in-the-door to Europe.

In multi-unit procurement auctions winning multiple contracts can lead to cost advantages due to synergies. These synergies can be material, for instance owning specialized equipment, or intangible, such as expertise. A notable characteristic of procurement is its recurrent nature. Construction contracts, military procurement, and public service contracts are all examples of this recurrence. In these settings, auctions take place sequentially and have some time periods in between. A consequence of the presence of synergies in such settings, is that bidders' valuations become stochastically dependent across auctions.

The literature on sequential auctions with synergies focuses completely on price trends and revenues. Branco (1997) is the first to explain the empirical observation of declining prices in sequential auctions of homogeneous objects by the presence of positive synergies. Jeitschko and Wolfstetter (2002) extend this result to a sequential auction of stochastically equivalent objects. Next to a declining trend, the presence of positive synergies also leads to higher revenues for the auctioneer, which appears positive for, for instance, government procurement. That price trends are not necessarily declining but can also be increasing is shown by Tang

Sørensen (2006) for a setting with heterogeneous objects and discrete valuations.

In the first part of this dissertation the focus is on a sequential auction of two objects with positive synergies. Bidders then face an exposure problem as they can end up winning contracts that are too expensive if complementary contracts are not won. Furthermore, the presence of synergies induces future asymmetries between bidders which makes the bidding environment more complex. Therefore, we not only consider price trends and revenues but also the consequences of the presence of synergies for bidders.

In Chapter 2 it is shown that bidders not only dissipate the complete expected rent of having synergies but also forgo a part of their intrinsic share. Namely, the bidding in the first auction is so competitive that bidders have lower total expected payoffs with than without synergies. Positive synergies form a paradox in this setting in the sense that bidders actually suffer from the presence of positive synergies, instead of having benefit from them. This effect has not been observed for auction settings before. Furthermore, in equilibrium serious losses, and hence, bankruptcy problems can occur. The negative economic consequences caused by these bankruptcy problems are of importance for auction design when synergies are present.

The analysis in Chapter 2 is restricted to second-price sealed-bid auctions, which are auctions in which the winner pays the second-highest bid. The results of that chapter also hold for first-price sealed-bid auctions, which are auctions in which the winner pays his bid, if the equilibrium bidding strategies fulfill certain conditions. These mild conditions are discussed at the end of Chapter 2, but it cannot be verified whether these are generally fulfilled. Namely, a closed-form solution for the equilibrium bidding strategies in an asymmetric first-price auction with more than two bidders is not available for the setting considered in this part of the dissertation. In the subsequent two chapters, the performance of the first- and second-price sealed-bid auctions are compared.

Chapter 3 analytically compares the performance of first- and second-price sealed-bid auctions for the setting in case of two bidders. It is shown that both efficiency and revenues are higher in the second-price format. However, in this auction format bidders are also more likely to incur losses which can lead to bankruptcies. Considering the welfare consequences these bankruptcies might have, the first-price auction can well be preferred, in particular for auctions where potential synergies are high.

The use of laboratory experimental methods in economics has been growing rapidly. In addition to analytical research, laboratory experiments can be useful as a ‘testbed’ for possible auction procedures prior to the final selection of auction rules. The U.S. Federal Communications Commission (FCC) has made use of experiments on many occasions in order to get some guidance on which auction rules to implement for spectrum auctions. Many of these experiments explored situations in which theory gave little guidance (see Plott (1997), and Ledyard et al. (1997)).

In Chapter 4 of this dissertation, an experimental analysis of the sequential auction setting with synergies is presented. The performance of the first- and second-price auction format in case of four bidders is compared for a baseline treatment without synergies and for two treatments with positive synergies of different sizes. By comparing the experimental data both within and between the two pricing formats, insights are acquired in how actual bidding behavior is influenced by the presence of an exposure problem and asymmetries between bidders. The responsiveness of the bidding, and the probability of making losses in particular, to the presence and the size of the synergy are important for public policy. Subjects clearly respond to the incentives the presence of synergies provide and the aggressiveness of bidding in the first auction increases in the synergy factor. For small synergies, the first-price format performs better in terms of efficiency, revenue, and the probability on losses. For a large synergy factor, the performance of the two pricing formats becomes more similar, although the first-price auction never performs worse than the second-price auction on any item.

1.2 Alternating price competition

Static duopoly settings were first analyzed by Cournot (1838) for quantity competition and by Bertrand (1883) for price competition. In both of these models it is assumed that firms simultaneously decide on their strategic variable. If firms compete against each other in either way for an infinite number of periods, any equilibrium with payoffs above the competitive level can be supported according to the Folk Theorem (Friedman, 1971). In contrast, when firms compete for a finite number of periods the subgame-perfect equilibrium consists of the competitive static equilibrium being played in every period.

The literature on alternating move models started with the pioneering contribution of Cyert and DeGroot (1970). They note that the reaction functions that are derived by Cournot (1838) are based on a model in which the decision periods for the two firms alternate. That is, each decision prevails for two periods, the period in which it is made and the period during which the rival makes its decision. Furthermore, the reaction functions are based on the assumption that the rival will not change his decision in response to a change by the firm. This assumption is proved false in each period, but the firms continue to use reaction functions that are based on this false assumption. Cyert and DeGroot (1970) overcome this by using an alternating move structure. A different motivation for implementing an alternating move structure is that it captures the idea of short-run price commitments (Maskin and Tirole, 1987).

Cyert and DeGroot (1970) analyze a duopoly with a long but finite time horizon in which firms select quantities. They show that the backwards induction strategies differ from a simple repetition of the competitive equilibrium strategies of the static model. Although they only consider a finite horizon, they do discuss long-run prices for a lengthy horizon. Maskin and Tirole (1987) consider the same setting with an infinite time horizon and show, using a contraction mapping argument, that the finite horizon equilibrium strategies of Cyert and DeGroot (1970) converge to their infinite horizon counterparts as the time horizon lengthens.

Maskin and Tirole (1988) consider a comparable model with homogeneous products where competition is in prices. It is found that an increase in the discount factor makes it more worthwhile for a firm to sacrifice current clientele by raising its price today in the expectation of future profit when the other firm follows suit. Furthermore, they show that two Markov perfect equilibria coexist: a focal price equilibrium and an equilibrium consisting of Edgeworth cycles. In the focal price equilibrium both firms split the market at the monopoly price. In an Edgeworth cycle, firms successively undercut the price of the other until the war becomes too expensive and one firm increases its price. Next, the other firm responds with a match or a slight undercut, after which the process of undercutting resumes.

In Chapter 5 an experimental analysis of the model by Maskin and Tirole (1988) is conducted in order to see whether the focal price equilibrium or the equilibrium consisting of Edgeworth cycles emerges. The focal price equilibrium is observed in a strict majority of the observations and hardly any support is found for the equilibrium consisting of Edgeworth cycles. Furthermore, the setting is analyzed for a long but finite time horizon. The subgame-perfect equilibrium consists of Edgeworth cycles and these cycles are different from those of the Markov perfect equilibrium. Strikingly, the backwards induction strategies do not converge to the Markov perfect equilibrium for the infinite time horizon when the horizon lengthens. Experimentally, a focal price is still observed in the majority of the observations. Nevertheless, price cycles are observed far more often than for the infinite time horizon.

The techniques used in the analysis of duopolies that compete in prices depend heavily on whether the goods sold are homogenous or heterogenous. Eaton and Engers (1990) consider a similar model as Maskin and Tirole (1988) but then in a differentiated product market. They consider a linear city where half of all consumers are located at one of the endpoints and the other half at the other endpoint. They find that two kind of equilibria exist: a disciplined one that is enforced by threats to undercut and that arises when the products are close substitutes, and a spontaneous one in which such threats are not needed and that arises when the products are more differentiated.

Baye and Ueng (1999) consider an alternating move price setting environment with differentiated products and linear demand functions. Using the techniques of Maskin and Tirole (1987) they find closed-form solutions for the Markov perfect equilibrium prices. It is found that in equilibrium prices are strictly higher than the one-shot Nash equilibrium prices, but lower than fully collusive prices.

Eckert (2004) introduces uncertainty in the model of Maskin and Tirole (1988) by making the marginal cost stochastic. It is shown that, provided marginal costs do not fluctuate excessively, equilibria in which firms match the current monopoly price do not exist when the probability is low that the marginal cost remain in their current state. However, focal price equilibria in which firms always match their rival along the equilibrium path do exist. A similar result is found for fluctuating demand and constant marginal costs.

The basic model of horizontal differentiation was introduced by Hotelling (1929). For an infinite time horizon, the Hotelling model with linear transportation costs is discussed by Rath (1998). However, after the firms have chosen location they repeatedly set prices simultaneously. Firms have to resort to 'carrot-and-stick' strategies in order to sustain the cooperative outcome with prices above the base level from the static model.

Chapter 6 considers an infinite alternating move Hotelling model in which consumers are uniformly distributed over the market. The unique linear stationary subgame-perfect equilibrium is determined analytically by applying the techniques of Maskin and Tirole (1987). The equilibrium is found to be dynamically stable as the dynamic reactions converge to a steady state. Moreover, the equilibrium is found numerically. The steady state price turns out to increase in the discount factor and thus firms become less aggressive the more patient they are. The model is then extended by introducing exogenous demand shocks. The introduction of uncertainty into the model has fundamental consequences as the analytical solution can not be found anymore via the conventional analysis. However, the equilibrium can still be found numerically. It is found that in equilibrium, pricing behavior is more competitive when the demand is high and that, in the long-run, prices are higher in case of low demand than in case of high demand. Thus, the observed prices move countercyclically.

In Chapter 7 an alternating move Hotelling setting with exogenous demand shocks is experimentally analyzed. Market demand can either be low or high and the probability of a change from one state to the other is constant. Three treatments are tested which differ in the size of the market demand in the high-state and the transition probability. Both within and between treatments, a comparison is made between the prices and profits in case of low and high demand. No significant differences between average prices and profits in mature behavior are found. The reason for this is that subjects collude in the majority of the observations.

Part I

Sequential auctions with synergies

Chapter 2

The paradox of positive synergies

Winning multiple contracts in multi-unit procurement auctions can lead to cost advantages due to synergies. In this chapter[1] we analyze the effects of the presence of such synergies on auction outcomes. We find that the presence of positive synergies on the bidders' side reduces the bidders' ex ante expected payoffs. Thus, instead of benefiting from the presence of synergies, bidders suffer from it. Furthermore, it is found that serious losses and hence bankruptcy problems can occur. In particular the negative economic consequences caused by these bankruptcy problems are of major importance for auction design when synergies are present.

2.1 Introduction

A distinguishing characteristic of procurement auctions is their recurrent nature. These auctions take place sequentially with time periods between them. Construction contracts, military procurement and the uncoordinated sequence of European spectrum auctions during 2000 and 2001 are examples of such sequential settings. Furthermore, large-scale projects frequently need to be divided into small pieces or subprojects which are then procured sequentially. This can be due to the fact that the project as a whole is too complex to auction at once. Also, there may be too few firms with sufficient resources to complete the project as a whole or essential facilities cannot be shut down simultaneously (Yildirim, 2004).

In multi-unit procurement auctions winning multiple contracts can lead to cost advantages due to synergies. These synergies can be material, for instance owning specialized equipment, or intangible, such as expertise. A consequence for the settings above is that bidders' valuations are stochastically dependent across auctions and this changes the auctions fundamentally.

[1] This chapter is based on Leufkens, Peeters, and Vermeulen (2006).

Hendricks and Porter's (1988) study of drainage lease auctions was the first empirical study to show that interdependencies among the values of objects affect the outcome of a sequential auction. Ausubel et al. (1997) show that synergies associated with winning multiple adjacent licenses in the United States spectrum auctions affected bidding strategies. Rusco and Walls (1999) find that in timber auctions spatial correlation of bids induces more aggressive bidding. De Silva (2005) finds the same for road construction auctions in Oklahoma. De Silva et al. (2005) show that in sequential construction auctions by the Oklahoma Department of Transportation previous winners are more likely to win in later auctions. Finally, Cramton (2002) argues that one reason for the enormous revenue in the spectrum auction by the United Kingdom is that it was the first in the sequence of UMTS auctions throughout Europe (see also Van Damme (2002)). Winners in the UK auction were well positioned for subsequent auctions and hence bidders could view it as a foot-in-the-door to Europe.

The theoretical literature on auctions is voluminous. Much interest in multi-unit auctions has been generated by the spectrum auctions that were conducted all over the world. The literature on sequential auctions with synergies focuses completely on revenues and price trends. Branco (1997) was the first to attribute the empirical observation of declining prices in auctions of homogeneous objects to the presence of positive synergies. Jeitschko and Wolfstetter (2002) extend this result to a sequential auction of stochastically equivalent objects. Next to a declining trend, the presence of positive synergies also leads to higher revenues for the auctioneer, which appears positive for, for instance, government procurement.

In this chapter, we show that for a sequential auction with synergies it is not only important to consider efficiency and prices, but also the consequences for bidders. We analyze a sequential auction of two objects, which are stochastically equivalent in the sense that both valuations are independent draws from the same distribution, and synergies lead to an increase in the valuation for the second object. It is found that bidders not only dissipate the complete expected rent of having synergies but also forgo a part of their intrinsic share. The bidding in the first auction is so competitive that bidders have lower total expected payoffs with than without synergies. Thus, bidders are worse off when positive synergies are present than when this is not the case. This is an effect that has not been observed in previous studies. We also find that in equilibrium serious losses, and hence, bankruptcy problems can occur. We study how the probability on losses is related to the valuation distribution, the synergy factor, and the number of bidders. The negative economic consequences caused by these bankruptcy problems are of importance for auction design when synergies are present.

A closed-form solution for the equilibrium bidding strategies in an asymmetric first-price sealed-bid auction is not always available (see Plum (1992)). Therefore, we restrict the analysis to the second-price sealed-bid auctions. At the end of this chapter, we discuss which condition must hold for the equilibrium bidding strategy in an asymmetric first-price sealed-bid auction for our results to apply also there.

The remainder of this chapter is organized as follows. In Section 2.2 the model is described. The equilibrium bidding strategies and its properties are given in

Section 2.3. In Section 2.4, we analyze the economic consequences. In particular, we discuss the bidders' ex ante expected payoffs, the probability that bidders make losses, prices and price trends, and efficiency. In Section 2.5, we illustrate our findings by means of three examples. The chapter ends with a discussion in Section 2.6.

2.2 The model

We consider a private value auction with $n \geq 2$ risk neutral bidders. Two stochastically equivalent objects are auctioned sequentially using the second-price sealed-bid format.[2] Bidders' valuations are distributed according to the differentiable, cumulative distribution function $F(v)$ with associated density function $f(v)$ on the interval $[0, \infty)$. In particular, $F(0) = 0$, F is non-decreasing, and $\lim_{v \to \infty} F(v) = 1$. We assume that the expected valuation is finite, thus $\mathbb{E}(v) = \int_0^\infty v f(v)\, \mathrm{d}v < \infty$. Valuations are individually uncorrelated, drawn independently from the same identical distribution at the start of each auction, and private information.

Although no bidder knows his valuation for the second object during the first auction, it is common knowledge that winning the first auction increases this valuation by a factor $s > 1$. This synergy factor only applies to the valuation for the second object, v_2. Winning the first auction then increases the second auction valuation from v_2 to sv_2, but does not have any effect on the first auction valuation v_1.

After each auction, the bidders are informed whether or not they won the auction. Whether the identity of the winner and the winning bid are made public or not, is irrelevant in this setting. The first auction informs every bidder whether he or one of his opponents benefits from synergies in the second auction. However, it does not convey any information on the actual valuations of bidders in the second auction. We rule out the possibility of a resale of the first object after the second auction.

As mentioned before, Branco (1997) was the first to attribute declining prices to the presence of positive synergies. He transformed the model of Krishna and Rosenthal (1996) into a sequential auction of two identical objects. Winning both objects increases the payoff for the bidder with a positive constant amount.

Menezes and Monteiro (2004) criticize this way of modeling synergies by noting that for valuations close to zero the marginal synergy is infinite. The synergy between two highly valued objects is the same as between two worthless objects. Therefore they define the valuation for both objects as a function of the value of one. If this function is larger or smaller than two times the value of one object, there are respectively positive or negative synergies.

Jeitschko and Wolfstetter (2002) depart from these models by analyzing a sequential auction of two nonidentical objects. In their model, two bidders are active in both auctions and a bidder's valuation in the second auction is uniformly

[2] Although most applications are in procurement settings, we follow the convention and analyze 'highest bid wins' auctions for expositional ease and without loss of generality.

distributed between zero and one without synergy, and between zero and two with synergy. The focus in this chapter is fundamentally different from theirs. Furthermore, our approach to a sequential second-price sealed-bid auction with synergies is more general.

Our approach to modeling positive synergies is comparable to the way Black and De Meza (1992) model negative synergies. The benefits from synergies are only attributed to the second project, considering the order in which projects are executed. For instance, expertise is created during the first project, and this only gives benefits during the second.

We ensure there is a relationship between the benefits due to synergies and the intrinsic value of the second object. The actual gain due to synergies depends on the combination of the synergy factor and the drawn valuation for the second object. One expects to complete a project at only a fraction of the costs if one has expertise, but the possible cost reduction then depends on the normal costs of the second project.

Our model resembles Engelbrecht-Wiggans' (1994) in having bidders' valuations drawn independently across objects and only be known at the start of each auction. This is applicable since there are time periods in between auctions. Then, the exact valuation for the second object is not known at the time the first auction takes place.

We end this section by introducing some notation. We write b_{ki} and v_{ki} for respectively the bid and valuation in auction $k = 1, 2$ of bidder $i = 1, \ldots, n$. In the second auction the winner of the first auction is denoted by w and bidder ℓ refers to one of the $n-1$ bidders that did not win the first auction. Because of symmetry, we only have three different expected payoffs in our setting. By $\bar{\pi}_{1i}$ we denote the expected instantaneous payoff for bidder i in the first auction, prior to the realization of the valuations for this auction. By $\bar{\pi}_{2w}$ ($\bar{\pi}_{2\ell}$) we denote the expected instantaneous second auction payoff of the winner (a loser) of the first auction, prior to the realization of the valuations for the second auction. For comparisons we sometimes refer to the expected instantaneous payoff of a symmetric one-shot auction, which is denoted by $\bar{\pi}$. In our setting, the expected price of auction k, $\bar{p}_k$, is also prior to the realization of the valuations and thus precisely the seller's expected revenue of that auction. The seller's expected total revenue from the auction sequence is denoted by $\bar{R}$ and the expected total payoff for bidder i is denoted by $\bar{\mu}_i$, both are prior to the realization of valuations. Finally, probabilities are denoted by $\mathbb{P}$ and expectations by $\mathbb{E}$.

2.3 The equilibrium bidding strategies

In this section, we give the equilibrium bidding strategies and discuss the consequences of the presence of synergies for bidding behavior.

The equilibrium bidding in the second auction is fairly straightforward. Each bidder bids his value, where the winner of the first auction takes the synergy factor into account to determine his value. Therefore, the actual value of winning the first auction is not only the value of the first object but also the difference in the expected instantaneous payoff of the second auction between winning and losing the first auction. This difference in the expected instantaneous second auction payoff is denoted by Δ, and will be referred to as the option value. We then obtain the following equilibrium in weakly dominant strategies.[3]

Proposition 2.1 *Consider a sequential second-price sealed-bid auction with independent, individually uncorrelated valuations, n risk-neutral bidders, two objects and synergy factor s. In this auction the bidding strategies given by*

$$b^*_{1i} = v_{1i} + \Delta \quad \textit{with} \quad \Delta = \bar{\pi}_{2w} - \bar{\pi}_{2\ell}$$

and

$$b^*_{2i} = \begin{cases} v_{2i} & \textit{if the first auction is lost;} \\ sv_{2i} & \textit{if the first auction is won.} \end{cases}$$

constitute a symmetric linear Bayesian Nash equilibrium in weakly dominant strategies.[4]

Proof At the beginning of the second auction, all bidders know the outcome of the first auction. The equilibrium bidding strategies for the second auction then follow directly from standard auction theory. The actual value of winning the first auction is then a bidder's valuation of the object plus Δ. □

According to the equilibrium bidding strategies of Proposition 2.1, in the first auction all bidders markup their single-stage bid with the option value Δ. Explicit expressions for $\bar{\pi}_{2w}$, $\bar{\pi}_{2\ell}$, and three useful expressions for Δ are provided in Section 2.7. Lemmas related to some properties of Δ, which will be referred to below, are given in Section 2.8.

We now discuss the impact of the markup by the option value Δ on the first auction bidding.

Theorem 2.2 *For any synergy factor $s > 1$, the option value Δ is strictly larger than zero. Moreover, for a fixed number of bidders, Δ is strictly increasing in s, and $\Delta \to \infty$ as $s \to \infty$.*

Proof Expression 2.3 for Δ, which is given in Section 2.7, reads

$$\Delta = \int_0^\infty f(x) \int_x^{sx} F^{n-2}(v)\,\mathrm{d}v\,\mathrm{d}x.$$

This expression is strictly increasing in s. Moreover, it is clear that $\Delta = 0$ for $s = 1$. Finally, by Lemma 2.12, Δ is divergent in s. □

We obtain the following corollary.

[3]For synergy factor $s = 2$ and uniform distribution of valuations this equilibrium was already shown in Jeitschko and Wolfstetter (2002).

[4]By linear we mean linear in valuation.

Corollary 2.3 *Bidding behavior in the first auction becomes more competitive for increasing values of the synergy factor.*

The effect of an increase in the number of bidders is more subtle. Let $K > 0$ be the smallest number for which $F(K) = 1$.[5]

Theorem 2.4 *Suppose the synergy factor s is fixed. Assume that $K < \infty$. Then*

$$\Delta \to \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K] \text{ as } n \to \infty.$$

When $K = \infty$, then $\Delta \to 0$ as $n \to \infty$.

The proofs can be found in Lemma 2.13 and Lemma 2.14. We obtain the following corollary.

Corollary 2.5 *When $K < \infty$, and the number of bidders increases, competition in the first auction is enhanced by the presence of synergies. When $K = \infty$, bidding behavior in the first auction converges to the bidding behavior in a symmetric one-shot auction when the number of bidders increases.*

Proof For $K < \infty$, the above assertion follows from the proof to Lemma 2.13 together with the observation that, since $sv > K$ for all v in the interval $(K/s, K]$, the expression

$$\lim_{n\to\infty} \Delta = \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K]$$

is strictly positive. The assertion regarding the case $K = \infty$ is an immediate consequence of the proof to Lemma 2.14.[6] □

We end this section by discussing the properties of the bidding strategies in the second auction. From the equilibrium bidding strategies, it follows that all bidders bid the synergy-adjusted value. That is, the first auction losers bid their second auction valuation, and the first auction winner bids his second auction valuation multiplied by the synergy factor. The positive synergy factor induces the first auction winner to upgrade his bid accordingly, and subsequently increase his probability to win. In fact, when $K < \infty$ and the first auction winner's second auction valuation is above $\frac{K}{s}$, his bidding will not leave any opportunity for a first auction loser to win.[7]

2.4 Economic implications

In this section the economic implications of the presence of synergies in the sequential auction setting are discussed. First, the consequences of synergies for the bidders' and the auctioneer's ex ante expected payoffs are analyzed. Then, the probability that bidders make losses is studied. Next, we focus on prices and price trends. Finally, the consequences for efficiency are discussed.

[5] This is a valid definition since F is continuous. We allow for $K = \infty$.

[6] Notice that this does not assert monotonic behavior for $K = \infty$.

[7] Note that in this case any bid by bidder w above $\frac{K}{s}$ is rational. This will not influence any of the results given in the remainder of this chapter.

2.4.1 Ex ante expected payoffs

The difference between the expected instantaneous payoff of the first auction in a situation with synergies when compared to the situation without synergies, is that in the former the winner pays an amount of Δ in addition to the price of the latter situation. The winner has to pay the second highest valuation plus the option value Δ. This means that the synergy has no effect on the probability to win but only on the price. Consequently, the expected first auction instantaneous payoff is lower when synergies are present. Moreover, the expected instantaneous payoff of the first auction is decreasing in the synergy factor, since we know from Theorem 2.2 that Δ is increasing in the synergy factor. This leads to the following theorem on a bidder's total ex ante expected payoff of the two auctions.

Theorem 2.6 *The ex ante expected total payoff for bidders is lower with than without synergies. Moreover, the ex ante expected total payoff is decreasing in the synergy factor and convergent to the expected instantaneous payoff of a symmetric one-shot auction.*

Proof The first auction winner pays, besides the price he would pay if there was only a single auction, the difference in the expected payoff of the second auction between winning and losing the first auction. This means that the ex ante expected total payoff, $\bar{\mu}$, of the auction sequence as a whole equals the expected instantaneous payoff of a single auction without synergies, $\bar{\pi}$, plus the expected payoff in the second auction when the first auction has been lost:

$$\bar{\mu} = \bar{\pi}_{1i} + \tfrac{1}{n}\bar{\pi}_{2w} + \tfrac{n-1}{n}\bar{\pi}_{2\ell} = \bar{\pi} - \tfrac{1}{n}(\bar{\pi}_{2w} - \bar{\pi}_{2\ell}) + \tfrac{1}{n}\bar{\pi}_{2w} + \tfrac{n-1}{n}\bar{\pi}_{2\ell} = \bar{\pi} + \bar{\pi}_{2\ell}.$$

From Section 2.7 we know that

$$\begin{aligned}\bar{\pi}_{2\ell} &= \int_0^\infty vF^{n-2}(v)F(v/s)f(v)\,\mathrm{d}v - \int_0^\infty svF^{n-2}(sv)f(v)(1-F(sv))\,\mathrm{d}v \\ &\qquad -(n-2)\int_0^\infty vF^{n-3}(v)F(v/s)f(v)(1-F(v))\,\mathrm{d}v \\ &\leq \int_0^\infty vF^{n-2}(v)F(v/s)f(v)\,\mathrm{d}v.\end{aligned}$$

Since $\bar{\pi}_{2\ell} \geq 0$, and the latter expression converges to zero as $s \to \infty$ by Lebesgue's Monotone Convergence Theorem, we obtain that $\bar{\pi}_{2\ell} \to 0$ as $s \to \infty$. □

The ex ante expected total payoff is decreasing in the synergy factor. This means that the larger the possible benefit from synergies becomes, the smaller the expected total payoff of the bidders will be. The ex ante expected total payoff of the bidders converges to the expected payoff of a symmetric one-shot auction. Since the ex ante expected total payoff is always larger than the possible expected payoff of a single auction, bidders will participate in both auctions.

It is well known that (part of) a possible rent is dissipated during the competition for that rent. However, in our model not only the possible rent is completely dissipated, bidders are even worse off than in a setting without synergies. When the synergies are large, bidders have half the ex ante expected total payoff they would have if there were no synergies (the sequence would then consist of two one-shot auctions). Given the fact that the total surplus that is divided between the seller and the bidders is larger with than without synergies, Theorem 2.6 is surprising. Instead of benefiting from the presence of synergies bidders suffer from it. Positive synergies form a paradox in this setting in the sense that bidders actually suffer from the presence of positive synergies, instead of having benefit from them.[8]

Next, we determine the effect of the synergy factor on the seller's revenue. The seller's ex ante expected revenue from the auction sequence is the sum of the expected prices of the two auctions: $\bar{R} = \bar{p}_1 + \bar{p}_2$. The expected prices of both the first and second auction are increasing in the synergy factor. For the first auction, we know from Theorem 2.2 that Δ is increasing in the synergy factor and it then follows from Proposition 2.1 that the expected price is increasing in the synergy factor. In the second auction, the $n-1$ losers bid as if there are no synergies and the winner bids sv. Consequently, the expected price in the second auction must also be increasing in the synergy factor. This leads to the following theorem.

Theorem 2.7 *The expected total revenue for the seller is increasing in the synergy factor.*

Theorem 2.6 shows that the increase in the revenue of the seller is not only due to the increased surplus that is divided. The seller also captures a part of the payoffs bidders originally had. The gain from synergies for the seller is more than the value of the synergies itself.

2.4.2 Probability of losses

The uncertainty concerning the benefits from synergies leads to an exposure problem in the sequential auction. Bidders bid above their valuation in the first auction, and consequently it is possible that the instantaneous payoff of the first auction is negative. The winner of the first auction may not win the second auction, or win it but still not recover the loss of the first auction. The total payoff of the sequential auction as a whole is then negative. We compute the probability that this event happens; in particular, in two limit cases: one where the synergy factor is large, and one where the number of bidders is large.

We start with an analysis of the probability that the bidders make a loss when the synergy factor becomes large. Given a synergy factor s, let $P(s)$ denote the ex ante probability in the auction that the winner of the first auction makes a loss when the synergy factor is s. We prove the following statement.

Theorem 2.8 $\liminf_{s\to\infty} P(s) \geq \mathbb{P}[v < \mathbb{E}(v)]$.

Proof Notice that, due to the bidding strategies, the winner of the first auction pays at least Δ. Now, suppose that bidder i won the first auction. When the

[8] By paradox we do not mean a paradox in the purely mathematical sense, but the counter-intuitive fact that positive synergies are in fact a burden instead of a blessing for the bidders.

realization of valuations for bidder i is (v_{1i}, v_{2i}) and $v_{1i} + sv_{2i} < \Delta$, bidder i certainly makes a loss, no matter whether he wins or loses the second auction. Thus,

$$P(s) \geq \mathbb{P}[v_{1i} + sv_{2i} < \Delta \mid i \text{ wins the first auction}].$$

Consequently, it suffices to show that, given $\varepsilon > 0$,

$$\mathbb{P}\left[\tfrac{v_{1i}}{s} + v_{2i} < \tfrac{\Delta}{s} \mid i \text{ wins the first auction}\right] \geq \mathbb{P}[v < \mathbb{E}(v)] - \varepsilon$$

for s sufficiently high. Take $\varepsilon > 0$. Since $\lim_{v\to\infty} F(v) = 1$ we can take $V > 0$ such that $F(V) > 1 - \varepsilon$. Then, $\mathbb{P}[v_{1i} < V] \geq 1 - \varepsilon$. Take $\bar{s}$ such that, for all $s > \bar{s}$, $\frac{V}{s} < \varepsilon$ and $\Delta - \frac{V}{s} \geq \mathbb{E}(v) - \varepsilon$. Then for $s > \bar{s}$,

$$\begin{aligned}
&\mathbb{P}\left[\tfrac{v_{1i}}{s} + v_{2i} < \tfrac{\Delta}{s} \mid i \text{ wins}\right] \\
&\quad = \mathbb{P}[v_{1i} \geq V] \cdot \mathbb{P}\left[\tfrac{v_{1i}}{s} + v_{2i} < \tfrac{\Delta}{s} \mid i \text{ wins, } v_{1i} \geq V\right] \\
&\qquad + \mathbb{P}[v_{1i} < V] \cdot \mathbb{P}\left[\tfrac{v_{1i}}{s} + v_{2i} < \tfrac{\Delta}{s} \mid i \text{ wins, } v_{1i} < V\right] \\
&\quad \geq (1-\varepsilon) \cdot \mathbb{P}\left[\tfrac{v_{1i}}{s} + v_{2i} < \tfrac{\Delta}{s} \mid i \text{ wins, } v_{1i} < V\right] \\
&\quad \geq (1-\varepsilon) \cdot \mathbb{P}\left[\tfrac{V}{s} + v_{2i} < \tfrac{\Delta}{s} \mid i \text{ wins, } v_{1i} < V\right] \\
&\quad = (1-\varepsilon) \cdot \mathbb{P}\left[v_{2i} < \tfrac{\Delta - V}{s} \mid i \text{ wins, } v_{1i} < V\right] \\
&\quad = (1-\varepsilon) \cdot \mathbb{P}\left[v < \tfrac{\Delta - V}{s}\right] \\
&\quad \geq (1-\varepsilon) \cdot \mathbb{P}\left[v < \mathbb{E}(v) - \varepsilon\right].
\end{aligned}$$

The proof is complete once we observe that the probability $\mathbb{P}[v < \mathbb{E}(v) - \varepsilon]$ converges to $\mathbb{P}[v < \mathbb{E}(v)]$ as $\varepsilon \to 0$. □

Theorem 2.8 roughly states that, for large synergy factors s, the probability that the winner of the first auction makes a loss is at least $\mathbb{P}[v < \mathbb{E}(v)]$. Thus, the loss effect is particularly severe for distributions where a bidder has a high probability of a relatively low valuation, and a rather small probability to have an extremely high valuation.

Next, we analyze the probability that the bidders make a loss when the number of bidders becomes large. Given the number of bidders n, let $P(n)$ denote the ex ante probability in the auction that the winner of the first auction makes a loss. The asymptotic behavior of the probability $P(n)$ is different for $K < \infty$ and $K = \infty$. We treat both cases separately.

Theorem 2.9 *Suppose that $K < \infty$. Then,*

$$\liminf_{n\to\infty} P(n) \geq \mathbb{P}\Big[sv < \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K]\Big].$$

Proof Assume without loss of generality that bidder i won the first auction. The probability that bidder i makes a loss is larger than or equal to the probability that $v_{1i} + sv_{2i} < p_1 + \Delta$, where p_1 denotes the price in the first auction. Now, take $\varepsilon > 0$ and $\delta > 0$. Notice that,

$$\mathbb{P}[v_{1i} + sv_{2i} < p_1 + \Delta \mid i \text{ wins}] \geq \mathbb{P}[v_{1i} < p_1 + \delta \mid i \text{ wins}] \cdot \mathbb{P}[sv_{2i} \leq \Delta - \delta].$$

Now notice that, keeping δ fixed,

$$\mathbb{P}[v_{1i} < p_1 + \delta \mid i \text{ wins}] \to 1 \text{ as } n \to \infty$$

Thus, it suffices to show that, for δ sufficiently small,

$$\mathbb{P}[sv \leq \Delta - \delta] \geq \mathbb{P}\Big[sv < \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K]\Big] - \varepsilon \text{ as } n \to \infty$$

By Lemma 2.13 we know that, for fixed δ,

$$\mathbb{P}[sv \leq \Delta - \delta] \to \mathbb{P}\Big[sv \leq \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K] - \delta\Big] \text{ as } n \to \infty$$

Now, we can take a δ sufficiently small so that

$$\begin{aligned} &\mathbb{P}\Big[sv \leq \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K] - \delta\Big] \\ &\qquad \geq \mathbb{P}\Big[sv < \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K]\Big] - \tfrac{1}{2}\varepsilon \end{aligned}$$

and, given this δ, we can take a N such that for all $n > N$ we have

$$\mathbb{P}[sv \leq \Delta - \delta] \geq \mathbb{P}\Big[sv \leq \mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K] - \delta\Big] - \tfrac{1}{2}\varepsilon.$$

Combined, these two choices give the result. □

For $s > 1$ the expression $\mathbb{P}[sv \geq K] \cdot \mathbb{E}[sv - K \mid sv \geq K]$ is strictly positive, and therefore Theorem 2.9 implies that for a large number of bidders the probability that a bidder makes a loss is strictly positive as well. Thus, the probability of losses is also present in this setting with a large number of bidders, although the effect is not as drastic as in the case we discussed earlier where the synergy factor is large.

Theorem 2.10 *Suppose that $K = \infty$. Then $P(n) \to 0$ as $n \to \infty$.*

Proof Notice that $P(n)$ is smaller than or equal to the expression

$$1 - \sum_{i=1}^{n} \frac{\mathbb{P}[sv_{2i} \geq \Delta]}{n} = 1 - \mathbb{P}[sv \geq \Delta].$$

However, by Lemma 2.14, $\mathbb{P}[sv \geq \Delta] \to 1$ as $n \to \infty$. □

Apparently, when $K = \infty$ the probability of losses vanishes when the number of bidders becomes sufficiently large. The intuition is that, for a fixed synergy factor s, even when a bidder wins the first auction the probability of winning the second auction as well becomes very small when the number of bidders increases. This is due to the fact that, when $K = \infty$, arbitrarily high valuations are possible. This lowers the value of Δ, and hence increases the probability that $sv > \Delta$.

In this subsection, we analyzed the probability that bidders make losses in relation to the valuation distribution, the synergy factor and the number of bidders. In any case, the ex ante expected total payoff of the sequential auction is, of course, nonnegative for bidders. Therefore, the losses that bidders can make in equilibrium are not a major concern when these bidders enter many similar settings. However, some projects may be much larger than others and typically such projects are not auctioned regularly. The spectrum auctions, special military procurement, and large building projects are all examples of this. Then, losses made on one project are difficult to be recovered and bankruptcy problems are likely to occur.

2.4.3 Prices

We now turn to price trends. There is ample empirical evidence of declining price trends in sequential auctions, which is known as the declining price anomaly or afternoon effect. For instance, declining prices are observed in wine auctions (Ashenfelter, 1989), real estate auctions (Ashenfelter and Genesove, 1992), and impressionist and modern paintings auctions (Beggs and Graddy, 1997). Branco (1997) was the first to attribute the declining price anomaly to the presence of positive synergies, a theoretical finding that was extended to heterogenous objects by Jeitschko and Wolfstetter (2002). For heterogeneous objects with discrete valuations Tang Sørensen (2006), on the other hand, shows that prices can both be increasing or decreasing.

In our setting, all bidders increase their first auction bids with the aim of obtaining an advantage for the second auction. The only change in the bids of the second auction is the increased bid of the participant of type w. Therefore, it can be that the expected price is higher in the first auction than in the second auction. The expected price for the first object equals the expected second highest valuation plus Δ. The expected price in the second auction is the sum of the expected payments made by each of the $n-1$ bidders of type ℓ and the single bidder of type w. The expected prices in both auctions are then

$$\bar{p}_1 = n(n-1)\int_0^\infty (v+\Delta)F^{n-2}(v)f(v)(1-F(v))\,\mathrm{d}v$$

and

$$\begin{aligned}\bar{p}_2 \;=\; &(n-1)\int_0^\infty vF^{n-2}(v)f(v)(1-F(v/s))\,\mathrm{d}v\\ &+(n-1)\Big\{\int_0^\infty svF^{n-2}(sv)f(v)(1-F(sv))\,\mathrm{d}v\\ &\quad+(n-2)\int_0^\infty vF^{n-3}(v)F(v/s)f(v)(1-F(v))\,\mathrm{d}v\Big\}.\end{aligned}$$

Declining prices will be observed if

$$\bar{p}_1 > \bar{p}_2 \quad\Longleftrightarrow\quad \int_0^\infty vf(v)G(v)\,\mathrm{d}v > 0$$

where

$$\begin{aligned}G(v) \;=\; &(n-1)[sF^{n-1}(sv)-nF^{n-1}(v)+(n-1)F^{n-2}(v)F(v/s)]\\ &-(n-2)[sF^{n-2}(sv)-(n-1)F^{n-2}(v)+(n-2)F^{n-3}(v)F(v/s)].\end{aligned}$$

Rewriting the above expression for $G(v)$ yields

$$\bar{p}_1 > \bar{p}_2 \iff \int_0^\infty f(v) \int_v^{sv} \left[(n-1)F^{n-1}(x) - (n-2)F^{n-2}(x)\right] \mathrm{d}x \, \mathrm{d}v > 0.$$

In Section 2.5, we show that in case of uniformly distributed valuations, prices are declining for any number of bidders and any synergy factor. Moreover, for any distribution function declining prices are found if the synergy factor is sufficiently large.

Theorem 2.11 *For any $F(v)$ and $n \geq 2$ there exists some $\bar{s}$ such that for any $s > \bar{s}$ a declining price trend will be observed.*

Proof From Lemma 2.12 it follows that $\bar{p}_1$ diverges as $s \to \infty$. However, for any synergy factor s, $\bar{p}_2$ is smaller than or equal to the expected value of the highest valuation in the single auction without synergies, that is, the expected value of the random variable $\max_i\{v_i\}$. To see this, observe that, for any realization $(v_{21}, \ldots, v_{2n})$ in the second auction, in case the winner of the first auction wins the second auction, we have $p_2 \leq \max_i\{v_{2i}\}$ and also in case another bidder, say j, wins the second auction we have $p_2 \leq \max_i\{v_{2i}\}$, because $p_2 \leq v_{2j}$. Finally, recall that in our model $\mathbb{E}(v) < \infty$. Now, the above statement follows from the observation that $\mathbb{E}(\max_i\{v_i\}) \leq n\mathbb{E}(v)$. $\square$

Nevertheless, within our model declining prices are in general not guaranteed. In Section 2.5 an example is given where prices are increasing.

2.4.4 Efficiency

In the first auction the bidder with the highest valuation wins. The winner of the second auction is the bidder with the highest valuation for the second object, taking positive synergies into account. The sequential auction is therefore efficient ad interim.

Ex post it might have been better if a different bidder had won the first auction and there are two types of inefficiencies that may occur. First, two different bidders win one object whereas it would have been better that a single bidder (not the first auction winner) had won both objects. Second, a bidder wins both objects whereas it would have been better had a different bidder won both auctions.

Under the first type of ex post inefficiency, bidder i has the highest valuation in the first auction and thus wins that auction. However, his synergy-adjusted second auction valuation is not sufficient to win the second auction and therefore it might have been better had a different bidder won the first auction. This bidder could be the winner of the second auction or a bidder that originally does not win any auction. This type of ex post inefficiency occurs if there exist bidders i and k $(i \neq k)$, such that $v_{1i} > v_{1j}$ for all $j \neq i$, $v_{2k} > sv_{2i}$ and $v_{2k} > v_{2j}$ for all $j \neq i, k$, but there exists a bidder $j \neq i$ such that $v_{1j} + sv_{2j} > v_{1i} + v_{2k}$.

The second type of ex post inefficiency can occur if the second auction valuation of bidder i is lower than the valuation of at least one other bidder j, but bidder

i still wins the second auction due to the synergies. For instance, a bidder's first auction valuation could be marginally below that of bidder i and his synergy-adjusted second auction valuation could more than offset this. This type of ex post inefficiency occurs if there exists a bidder i, such that $v_{1i} > v_{1j}$ for all $j \neq i$ and $sv_{2i} > v_{2j}$ for all $j \neq i$, but there exists a bidder $j \neq i$ such that $v_{1j} + sv_{2j} > v_{1i} + sv_{2i}$.

Without synergies the sequential auction is efficient ex post. Due to the synergies the valuations are stochastically dependent across auctions and only then the lack of information can give rise to an ad interim efficient but ex post inefficient allocation. When the synergies become large, the probability of a non-optimal ex post allocation converges to $\frac{n-1}{n}$. In the limit the allocation is only optimal if the bidder that wins the first auction also draws the highest valuation in the second auction.

2.5 Examples

In this section we illustrate our findings by means of three examples. The first example considers a (trivial) case where there are no losses possible, even though the synergy induces increased competition in the auction for the first object. Next, we present the typical example of valuations being uniformly drawn from the unit interval. Finally, we present an example with an increasing price trend.

2.5.1 No losses

First, consider the trivial case with each bidder having a value of v in each auction, such that the winner of the first object will win the second object with certainty. The excess surplus that a win of the first object generates in the auction for the second object is then $\Delta = (s-1)v$. In the first auction all bidders upgrade their truthful bid of v with this option value effect, whereas in the second auction they just bid their value. This guarantees a decreasing price trend ($\bar{p}_1 = sv$ and $\bar{p}_2 = v$). Moreover, all bidders end up with an overall payoff of 0. This means that there is no bankruptcy problem and the seller roams off precisely the excess surplus (no more and no less) that is generated by the synergy. All these properties, both qualitatively and quantitatively, are independent of the number of bidders, n.

The absence of bankruptcy problems in this example is not driven by the fact that the winner of the first object, wins the second object with certainty. In case the values are drawn from the interval $[\underline{v}, \overline{v}]$ with $\underline{v} < \overline{v} < s\underline{v}$, the latter property is satisfied whereas bankruptcies may occur. It is precisely the lack of uncertainty about current and future valuations of all bidders, and hence the complete information structure, that guarantees absence of bankruptcies.

2.5.2 Uniform distribution

Let for each bidder the valuation be a random draw from the interval $[0, 1]$ according to a uniform distribution. The symmetric linear equilibrium gives then rise to the following specification:

$$b^*_{1i} = v_{1i} + (\tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-2}{n}\, \tfrac{1}{s})$$

$$b^*_{2i} = \begin{cases} v_{2i} & \text{if auction 1 is lost} \\ sv_{2i} & \text{if auction 1 is won} \end{cases}$$

$$\bar{\pi}_{1i} = \tfrac{1}{n(n+1)} - \tfrac{1}{n}\, (\tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-2}{n}\, \tfrac{1}{s})$$

$$\bar{\pi}_{2i} = \begin{cases} \tfrac{1}{n(n+1)}\, \tfrac{1}{s} & \text{if auction 1 is lost} \\ \tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-1}{n+1}\, \tfrac{1}{s} & \text{if auction 1 is won} \end{cases}$$

$$\bar{\mu}_i = \tfrac{1}{n(n+1)} + \tfrac{1}{n(n+1)}\, \tfrac{1}{s}$$

$$\bar{p}_1 = \tfrac{n-1}{n+1} + (\tfrac{1}{2}\, s - \tfrac{n-1}{n} + \tfrac{1}{2}\, \tfrac{n-2}{n}\, \tfrac{1}{s})$$

$$\bar{p}_2 = \tfrac{n-1}{n} - \tfrac{n-1}{n(n+1)}\, \tfrac{1}{s}$$

$$\bar{R} = \tfrac{1}{2}\, s + \tfrac{n-1}{n+1} + \tfrac{1}{2}\, \tfrac{n-3}{n+1}\, \tfrac{1}{s}.$$

It can easily be verified that the effect of the synergy factor, s, on the payoff in the second auction, $\bar{\pi}_{2i}$, is positive in case the first auction is won, but negative if the first auction is lost. Hence, the option value and the bids in the first auction are increasing in s. This implies that the synergy enhances competition in the first auction and the expected payoff in the first auction, $\bar{\pi}_{1i}$, is consequently decreasing in s. Moreover, we see that the overall payoff, $\bar{\mu}_i$, is decreasing in s, indicating that (in expectation) bidders suffer from the synergy. Both auctions' prices, $\bar{p}_1$ and $\bar{p}_2$, and consequently the auction revenue, $\bar{R}$, are increasing in s. Finally, the first auction's price, $\bar{p}_1$, is always larger than the second auction's price, $\bar{p}_2$, such that for valuations uniformly distributed over the unit interval, prices are declining.

The payoff in the second auction, $\bar{\pi}_{2i}$, is decreasing in the number of bidders, n, regardless of the outcome of the first auction. The option value, though, is strictly decreasing in n. Consequently, the bidding in the first auction becomes less aggressive the more bidders are present. The effect of n on the payoff in the first auction, $\bar{\pi}_{1i}$, is however ambiguous. For instance, for $s = 1.7$ this payoff is decreasing when the number of bidders increases from 3 to 4, but increasing when the number of bidders increases from 5 to 6. Despite this ambiguity, the overall payoff, $\bar{\mu}_i$, is decreasing in n. Where the expected price in the second auction, $\bar{p}_2$, is clearly increasing in n, this is ambiguous for the price in the first auction, $\bar{p}_1$. The derivative of $\bar{p}_1$ with respect to n is given by $\frac{\mathrm{d}\bar{p}_1}{\mathrm{d}n} = \frac{2}{(n+1)^2} - (1 - \frac{1}{s})\frac{1}{n^2}$ and can be negative as well as positive. For instance, for $s = 9$ and $n = 2$ the derivative is equal to zero, such that for any lower (larger) s the derivative is positive (negative). Nevertheless, the expected revenue, $\bar{R}$, is unambiguously increasing in n. This implies that the increase of $\bar{p}_2$ dominates an eventual decrease of $\bar{p}_1$.

2.5.3 Increasing prices

Let for each bidder the value be $\overline{v}$ with probability θ and $\underline{v}$ with probability $1-\theta$ with $\overline{v} > \underline{v}$. Moreover, let the synergy factor be such that $\overline{v} > s\,\underline{v}$, such that winning the first item does not automatically lead to winning both objects.

The expected prices for the first and second object are given by

$$\bar{p}_1 = (1-\theta)^n \underline{v} + n\theta(1-\theta)^{n-1}\underline{v} + [1-(1-\theta)^n - n\theta(1-\theta)^{n-1}]\overline{v} + \Delta$$

and

$$\begin{aligned}\bar{p}_2 \;=\; & (1-\theta)^n \underline{v} + n\theta(1-\theta)^{n-1}[\tfrac{1}{n}\underline{v} + \tfrac{n-1}{n}s\underline{v}] \\ & + [1-(1-\theta)^n - n\theta(1-\theta)^{n-1}]\overline{v},\end{aligned}$$

where Δ indicates the expected benefit from having the synergy and is given by

$$\begin{aligned}\Delta \;=\; & \theta\,(1-\theta)^{n-1}[(s\overline{v}-\underline{v}) - (s\underline{v}-\underline{v})] + \theta[1-(1-\theta)^{n-1}](s\overline{v}-\overline{v}) \\ & + (1-\theta)^n(s\underline{v}-\underline{v}) \\ =\; & \theta\,(s-1)\,\overline{v} + (1-\theta)^{n-1}\,(s-1)\,\underline{v}.\end{aligned}$$

In expectation there is an increasing price trend if $\bar{p}_2 > \bar{p}_1$. This is the case if

$$s\underline{v} < \overline{v} < (1-\theta)^{n-1}[(n-1) - \tfrac{1}{\theta}]\underline{v}.$$

A configuration for which both these inequalities are satisfied is: $\underline{v} = 0.5$, $\overline{v} = 0.6$, $\theta = 0.1$, $n = 21$, and $s = 1.1$.

2.6 Discussion

In this chapter, it was shown that even though positive synergies on the bidders' side appear beneficial for them, this is not necessarily the case. The equilibrium bidding strategies result in such a fierce competition that bidders lose all the benefits from synergies, and on top of that part of their intrinsic expected payoff. The larger the possible benefits due to synergies are, the smaller the expected payoff of the bidders will be, which is paradoxical.

The sequential auctions are efficient ad interim. However, due to the uncertainty concerning the valuation for the second object during the first auction and the stochastic dependence between the valuations for both auctions, inefficiencies can occur ex post.

Only the seller benefits from positive synergies. He captures all the gains from synergies and on top of that part of the share bidders would have had without synergies. In our setting, it would be profitable for the auctioneer to announce future auctions well in advance. However, a transparent procurement policy can be a two-edged sword. The winner of the first auction can make a loss and consequently go bankrupt. Especially for large governmental procurement projects this can be a severe problem.[9]

[9] See, for instance, the problems caused when MobilCom returned its UMTS-license in Germany.

The probability of making a loss is particularly severe for valuation distributions where a bidder has a high probability of a relatively low valuation, and a rather small probability on having an extremely high valuation. Even when the number of bidders is large, the probability on a loss remains strictly positive in case the expected valuation is finite.

Finally, we find that positive synergies always lead to declining prices in case of uniformly distributed valuations. Furthermore, there always exists a critical synergy factor above which prices are declining.

A closed-form solution for the equilibrium bidding strategies in an asymmetric first-price sealed-bid auctions is not generally available. Therefore, we have to restrict our formal analysis to the second-price sealed-bid format. Nevertheless, if for an asymmetric first-price sealed-bid auction there exists a positive Δ, that is $\bar{\pi}_{2w} > \bar{\pi} > \bar{\pi}_{2\ell}$, our main results also hold there. Then, the equilibrium bidding strategy in the first auction will be the bidding strategy in an auction without synergies plus Δ. The bidding in the first auction is more competitive with than without synergies and for Δ large enough bidders bid above their value and possibly incur losses. It is known that these conditions hold for an asymmetric first-price sealed-bid auction with two bidders and uniformly distributed valuations (see Chapter 3). The paradox then also applies: the ex ante expected payoff for bidders is given by $\bar{\pi} + \bar{\pi}_{2\ell}$ which is lower than the payoff without synergies.

2.7 Appendix A: Formulas

In this section we will provide explicit expressions for the expected payoffs. Furthermore, we will give three useful expressions for Δ.

In the first auction all bidders are symmetric. Without synergies ($s = 1$) the expected instantaneous payoff of each auction is given by

$$\bar{\pi} = \int_0^\infty vF^{n-1}(v)f(v)\,\mathrm{d}v - (n-1)\int_0^\infty vF^{n-2}(v)f(v)(1-F(v))\,\mathrm{d}v.$$

When synergies are present, so $s > 1$, the expected instantaneous payoff for a bidder i is then given by

$$\bar{\pi}_1 = \int_0^\infty vF^{n-1}(v)f(v)\,\mathrm{d}v - (n-1)\int_0^\infty (v+\Delta)F^{n-2}(v)f(v)(1-F(v))\,\mathrm{d}v.$$

The bidder of type w wins the second auction if his synergy-adjusted bid is higher than that of all the other bidders and the price he has to pay is determined by the highest bid among the $n-1$ bidders of type ℓ. Thus the expected instantaneous second auction payoff for the winner of the first auction is given by

$$\bar{\pi}_{2w} = \int_0^\infty svF^{n-1}(sv)f(v)\,\mathrm{d}v - (n-1)\int_0^\infty vF^{n-2}(v)f(v)(1-F(v/s))\,\mathrm{d}v.$$

A bidder of type ℓ only wins if his bid is above that of all other losers and the bid of bidder w. There are two possibilities to consider for the expected instantaneous second auction price a bidder of type ℓ has to pay; one of the remaining $n-2$ bidders of type ℓ has the second highest bid (third term) or bidder w has the second highest bid (second term). Thus the expected instantaneous second auction payoff for a loser is given by

$$\begin{aligned}\bar{\pi}_{2\ell} \;=\; & \int_0^\infty vF^{n-2}(v)F(v/s)f(v)\,\mathrm{d}v - \int_0^\infty svF^{n-2}(sv)f(v)(1-F(sv))\,\mathrm{d}v \\ & - (n-2)\int_0^\infty vF^{n-3}(v)F(v/s)f(v)(1-F(v))\,\mathrm{d}v.\end{aligned}$$

A bidder's ex ante expected total payoff of the auction sequence is given by

$$\bar{\mu}_i = \bar{\pi}_{1i} + \tfrac{1}{n}\,\bar{\pi}_{2w} + \tfrac{n-1}{n}\,\bar{\pi}_{2\ell}.$$

This follows from the fact that all bidders are symmetric ex ante and hence bidder i wins the first auction with probability $\frac{1}{n}$.

Since $\Delta = \bar{\pi}_{2w} - \bar{\pi}_{2\ell}$, we can substitute the above formulas for $\bar{\pi}_{2w}$ and $\bar{\pi}_{2\ell}$. The result can be rewritten to

$$\begin{aligned}\Delta \;=\; & s\int_0^\infty vF^{n-2}(sv)f(v)\,\mathrm{d}v + (n-2)\int_0^\infty vf(v)F^{n-3}(v)F(v/s)\,\mathrm{d}v \\ & - (n-1)\int_0^\infty vf(v)F^{n-2}(v)\,\mathrm{d}v\end{aligned} \tag{2.1}$$

or alternatively

$$\begin{aligned}\Delta \;=\; & s\int_0^\infty vF^{n-2}(sv)f(v)\,\mathrm{d}v \\ & - (n-2)\int_0^\infty vf(v)F^{n-3}(v)(F(v)-F(v/s))\,\mathrm{d}v \\ & - \int_0^\infty vf(v)F^{n-2}(v)\,\mathrm{d}v.\end{aligned} \tag{2.2}$$

We focus on the second term of this latter expression for Δ. This term can be rewritten as follows.

$$\begin{aligned}& (n-2)\int_0^\infty vf(v)F^{n-3}(v)(F(v)-F(v/s))\,\mathrm{d}v \\ & \quad = (n-2)\int_0^\infty vf(v)F^{n-3}(v)\int_{v/s}^{v} f(x)\,\mathrm{d}x\,\mathrm{d}v \\ & \quad = (n-2)\int_0^\infty \int_{v/s}^{v} vf(v)F^{n-3}(v)f(x)\,\mathrm{d}x\,\mathrm{d}v \\ & \quad = (n-2)\int_0^\infty \int_{x}^{sx} vf(v)F^{n-3}(v)f(x)\,\mathrm{d}v\,\mathrm{d}x \\ & \quad = (n-2)\int_0^\infty f(x)\int_{x}^{sx} vf(v)F^{n-3}(v)\,\mathrm{d}v\,\mathrm{d}x \\ & \quad = \int_0^\infty f(x)\int_{x}^{sx} v\mathrm{d}F^{n-2}(v)\,\mathrm{d}x\end{aligned}$$

$$
\begin{aligned}
&= \int_0^\infty f(x)\left[sxF^{n-2}(sx) - xF^{n-2}(x) - \int_x^{sx} F^{n-2}(v)\mathrm{d}v\right]\mathrm{d}x \\
&= \int_0^\infty f(x)sxF^{n-2}(sx)\,\mathrm{d}x - \int_0^\infty f(x)xF^{n-2}(x)\,\mathrm{d}x \\
&\qquad - \int_0^\infty f(x)\int_x^{sx} F^{n-2}(v)\,\mathrm{d}v\,\mathrm{d}x.
\end{aligned}
$$

Plugging this back into the expression for Δ and canceling equal terms yields

$$
\Delta = \int_0^\infty f(x)\int_x^{sx} F^{n-2}(v)\,\mathrm{d}v\,\mathrm{d}x. \tag{2.3}
$$

2.8 Appendix B: Analysis of Δ

In this section we state and prove some technical facts regarding the option value Δ.

Lemma 2.12 $\lim_{s\to\infty}\frac{\Delta}{s} = \mathbb{E}(v)$.

Proof We separately analyze the behavior of the three terms of Expression 2.1 for Δ. Regarding the first integral, define the function $G\colon [0,\infty)\to[0,\infty)$ by

$$
G(v) = \begin{cases} 0 & \text{if } v = 0 \\ vf(v) & \text{else.} \end{cases}
$$

Since $F(0) = 0$, and for any $v > 0$ the value sv becomes large when s becomes large, it can be seen that $vF^{n-2}(sv)f(v)$ is non-decreasing in s and converges pointwise to G as $s\to\infty$. Thus, by Lebesgue's Theorem of Monotone Convergence, we know that

$$
\int_0^\infty vF^{n-2}(sv)f(v)\,\mathrm{d}v \to \int_0^\infty G(v)\,\mathrm{d}v = \mathbb{E}(v) \text{ as } s\to\infty.
$$

Regarding the second integral, recall that F is continuous, non-decreasing, and $F(0) = 0$. Thus, $f(v)F^{n-3}(v)F(v/s)\downarrow 0$ pointwise as $s\to\infty$. Hence,

$$
\int_0^\infty vf(v)F^{n-3}(v)F(v/s)\,\mathrm{d}v \to 0 \text{ as } s\to\infty.
$$

Finally observe that the third integral does not depend on s. The result now follows. □

The asymptotic behavior of Δ when the number of bidders becomes large is more complicated. First we consider the case where $K < \infty$.

Lemma 2.13 *Assume that $K < \infty$. Then*

$$
\lim_{n\to\infty}\Delta = \mathbb{P}[sv \geq K]\cdot\mathbb{E}[sv - K \mid sv \geq K].
$$

Proof Since $K < \infty$, Expression 2.1 for Δ becomes

$$\begin{aligned}\Delta \;=\; & s\int_0^K vF^{n-2}(sv)f(v)\,\mathrm{d}v + (n-2)\int_0^K vf(v)F^{n-3}(v)F(v/s)\,\mathrm{d}v \\ & - (n-1)\int_0^K vf(v)F^{n-2}(v)\,\mathrm{d}v.\end{aligned}$$

First notice that, concerning the second term,

$$\begin{aligned}&(n-2)\int_0^K vf(v)F^{n-3}(v)F(v/s)\,\mathrm{d}v \\ &\quad= (n-2)\int_0^K vf(v)F^{n-3}(v)\int_0^{v/s} f(x)\,\mathrm{d}x\,\mathrm{d}v \\ &\quad= (n-2)\int_0^{K/s}\int_{sx}^K vf(v)F^{n-3}(v)\,\mathrm{d}v\; f(x)\,\mathrm{d}x \\ &\quad= \int_0^{K/s}\int_{sx}^K v\,\mathrm{d}F^{n-2}(v)\; f(x)\,\mathrm{d}x.\end{aligned}$$

Using integration by parts for the inner integral, this expression can be rewritten to

$$KF(K/s) - s\int_0^{K/s} xF^{n-2}(sx)f(x)\,\mathrm{d}x - \int_0^{K/s}\int_{sx}^K F^{n-2}(v)\,\mathrm{d}vf(x)\,\mathrm{d}x.$$

Furthermore, the third term of this expression for Δ can be rewritten as

$$(n-1)\int_0^K vf(v)F^{n-2}(v)\,\mathrm{d}v \;=\; \int_0^K v\,\mathrm{d}F^{n-1}(v) \;=\; K - \int_0^K F^{n-1}(v)\,\mathrm{d}v.$$

Plugging all this into the formula for Δ we obtain

$$\begin{aligned}\Delta \;=\; & s\int_{K/s}^K vF^{n-2}(sv)f(v)\,\mathrm{d}v - K(1-F(K/s)) \\ & - \int_0^{K/s}\int_{sx}^K F^{n-2}(v)\,\mathrm{d}vf(x)\,\mathrm{d}x \\ & - \int_0^K F^{n-1}(v)\,\mathrm{d}v.\end{aligned}$$

Since by Lebesgue's Monotone Convergence Theorem the latter two integrals converge to zero, and since $F^{n-2}(sv) = 1$ for $K/s \le v \le K$, we see that

$$\Delta \to s\int_{K/s}^K vf(v)\,\mathrm{d}v - K(1-F(K/s)) \text{ as } n \to \infty.$$

Finally notice that

$$s\int_{K/s}^K vf(v)\,\mathrm{d}v - K(1-F(K/s)) = \mathbb{P}[sv \ge K]\cdot\mathbb{E}[sv - K \mid sv \ge K].$$

This concludes the proof. □

The intuition for this expression for Δ is the following. When the number of bidders is large, the additional value of obtaining the second object is approximately s times the expected value, given that the value is larger or equal to K/s, this being the minimum value for which winning is guaranteed when you bid s times your valuation, and the others bid their valuation. The price you pay in that case when the number of bidders is large is approximately K. The difference between these two gets multiplied by $1 - F(K/s)$, which is the probability of having a value higher than K/s, which is for a large number of bidders approximately the probability of winning the second auction, knowing that you won the first auction.

Lemma 2.14 *Assume that* $K = \infty$*. Then* $\lim_{n\to\infty} \Delta = 0$*.*

Proof Expression 2.3 for Δ yields

$$\begin{aligned}\Delta &= s\int_0^\infty vF^{n-2}(sv)f(v)\,\mathrm{d}v \\ &\qquad - (n-2)\int_0^\infty vf(v)F^{n-3}(v)(F(v)-F(v/s))\,\mathrm{d}v \\ &\qquad\quad - \int_0^\infty vf(v)F^{n-2}(v)\,\mathrm{d}v \\ &\leq s\int_0^\infty vF^{n-2}(sv)f(v)\,\mathrm{d}v.\end{aligned}$$

Since $K = \infty$ we know that $F(v) < 1$ for all v. Thus, by Lebesgue's Monotone Convergence Theorem and Theorem 2.2, Δ converges to zero as $n \to \infty$. □

The explanation for Lemma 2.14 is that when the probability of winning the second auction converges to zero, the possibility to benefit from a synergy vanishes. Consequently, the option value converges to zero.

Chapter 3

A reason to prefer first-price auctions

Theoretically, second-price sealed-bid auctions are often preferred to first-price sealed-bid auctions in private value settings. In this chapter[1] a comparison is made between these two auction formats for a sequential auction of two objects with synergies. We find that the second-price auction performs better in terms of efficiency and revenue. However, the first price auction performs much better on a so far neglected dimension. Namely, the likelihood that the winner of the first object goes bankrupt is larger when using the second-price format.

3.1 Introduction

In private value settings second-price sealed-bid auctions are generally preferred to first-price sealed-bid auctions due to the desirable properties they have. First of all, the dominant bidding strategy is truth-revealing and therefore does not require any information on the situation or intention of competitors. Second, in equilibrium the auction is guaranteed to be efficient. Besides, the revenue-equivalence theorem (Myerson, 1981) shows that the expected revenues from both auction formats are identical under certain assumptions.

Still, second-price auctions are hardly observed in practice and several theoretical explanations have been given for this. Rothkopf et al. (1990) argue that bidders might be reluctant to reveal their true valuation, since this can have negative implications for future auctions. Both in one-shot and repeated settings, collusion among bidders is less stable in first-price auctions than in second-price auctions (see for instance Robinson (1985), Fehl and Güth (1987), and Skrzypacz and Hopenhayn (2004)). Finally, when bidders are risk-averse or when their valuations are asymmetrically distributed, the revenue-equivalence theorem does not hold anymore. Holt (1980) shows that when bidders are risk-averse the first-price

[1] This chapter is based on Leufkens and Peeters (2007).

auction generates higher revenues since bidders then shade their bids less. When the valuation distribution of one type of bidders stochastically dominates that of the other type of bidders, Maskin and Riley (2000) show that a first-price auction generates more revenue than a second-price auction. However in such a situation the first-price auction is not guaranteed to be efficient anymore.

This chapter discusses asymmetries between the valuation distributions of two bidders, but here the asymmetry is not exogenously given. We analyze a sequential auction of two stochastically equivalent objects in which positive synergies are present and compare the first- and second-price format. We find that both efficiency and revenue are higher for the second-price auction format. However, bidders are more likely to receive losses under this auction format. These losses can ultimately lead to bankruptcy, in particular for auctions where potential synergies are high, and therefore induce disastrous welfare consequences for society.

This chapter is organized as follows. In Section 3.2 the model is described and the equilibrium bidding strategies are derived for the first- and second-price auction. In Section 3.3 the two auction formats are compared. The chapter ends with a discussion in Section 3.4.

3.2 The model

Two objects, indexed by $k = 1, 2$, are auctioned sequentially among two risk-neutral bidders, indexed by $i = 1, 2$, who participate in both auctions. Valuations are private information and independently drawn across bidders and objects from the interval $[0, 1]$ according to the uniform distribution. The valuation for the second object is not known during the auction of the first object. After each auction bidders are informed whether or not they won the object. We incorporate the presence of positive synergies by multiplying the second auction valuation with a factor $s > 1$ if the first auction is won. Although the valuation for the second object is not known during the first auction, bidders know that winning the first object increases this valuation from v_2^i to $s\,v_2^i$ and thus increases their expected payoff of the second auction.

As mentioned in Chapter 2, a closed-form solution for the equilibrium strategies in an asymmetric first-price auction is not generally available. For a first-price auction with uniformly distributed valuations and the present type of asymmetry, a solution is only known in case of two bidders (see Plum (1992)). Consequently, the analysis in this chapter has to be restricted to a setting with two bidders.

The present setting represents a recurring auction like the annual auctioning of contracts for public services. The exact details of future contracts are not specified yet and therefore contracts are considered as a priori identical. The benefits from synergies are only attributed to the second object as the result of the order in which projects are executed. For instance, expertise is created during the first project and this gives benefits for the second, or specialized equipment is needed which then does not need to be acquired for a possible second project. Our setting can also find its application in other situations such as spectrum auctions where bidders benefit from a win in one area in the creation of a domain consisting of multiple contiguous areas.

Similar auction settings have been studied in Jeitschko and Wolfstetter (2002) and Tang Sørensen (2006). In both articles the focus is only on revenues and price trends whereas in this chapter the focus is on the wider (social) consequences the presence of positive synergies has.

Proposition 3.1 *For the first-price sealed-bid auction format, the bidding strategies given by*

$$b_1^i(v_1^i) = \tfrac{1}{2}v_1^i + \Delta^1$$

$$\textit{with} \quad \Delta^1 = \tfrac{s^2}{s^2-1}\left\{\tfrac{1}{2}s - 1 - \tfrac{1}{s} + \tfrac{1}{2}\tfrac{1}{s^2} + \tfrac{1}{2}\tfrac{\operatorname{arcsinh}(\sqrt{s^2-1})+\arcsin(\frac{1}{s}\sqrt{s^2-1})}{\sqrt{s^2-1}}\right\},$$

and

$$b_2^i(v_2^i) = \begin{cases} \dfrac{v_2^i}{1+\sqrt{1-(1-\frac{1}{s^2})(v_2^i)^2}} & \textit{if auction 1 is lost;} \\ \dfrac{s\,v_2^i}{1+\sqrt{1+(1-\frac{1}{s^2})(s\,v_2^i)^2}} & \textit{if auction 1 is won} \end{cases}$$

constitute a unique symmetric Bayesian Nash equilibrium.

Proof See Section 3.5. □

The factor Δ^1 in the proposition is the option value of winning the first auction. This option value is the difference in expected instantaneous payoffs for the second auction, before knowing the second auction valuation, between winning and losing. In line with intuition, the option value is positive and increasing in the synergy factor.

Proposition 3.2 *For the second-price sealed-bid auction format, the bidding strategies given by*

$$b_1^i(v_1^i) = v_1^i + \Delta^2 \quad \textit{with} \quad \Delta^2 = \tfrac{1}{2}s - \tfrac{1}{2}$$

and

$$b_2^i(v_2^i) = \begin{cases} v_2^i & \textit{if auction 1 is lost;} \\ s\,v_2^i & \textit{if auction 1 is won} \end{cases}$$

constitute a unique symmetric linear Bayesian Nash equilibrium in weakly dominant strategies.

Proof See Proposition 2.1. □

Again the factor Δ^2 represents the option value effect of winning the first auction and equals the difference in expected instantaneous payoffs for the second auction between winning and losing the first auction before the second auction valuation is known. Also here the option value is positive and increasing in the synergy factor.

3.3 Comparing the first- and second-price format

The option value may cause bidders to overbid their valuation in the first auction. In the first-price format bidders overbid if the option value exceeds half the valuation for the first object. In the second-price format bidders always overbid. As a consequence of the overbidding, the winner of the first auction can make an instantaneous loss in the first auction.

Losses that are made in the first auction can, but are not guaranteed to, be recovered in the second auction. In case the first auction winner is not able to recover its loss in the second auction, bankruptcy results in our setting. Especially for large procurement contracts a loss that is not recovered could well lead to bankruptcy of the firm. Figure 3.1 displays the likelihood that bankruptcy occurs in both the first-price and the second-price sealed-bid sequential auction with positive synergies for different values of the synergy factor. We see that for most values of s bankruptcy is more likely to occur in the second-price format than in the first-price format.

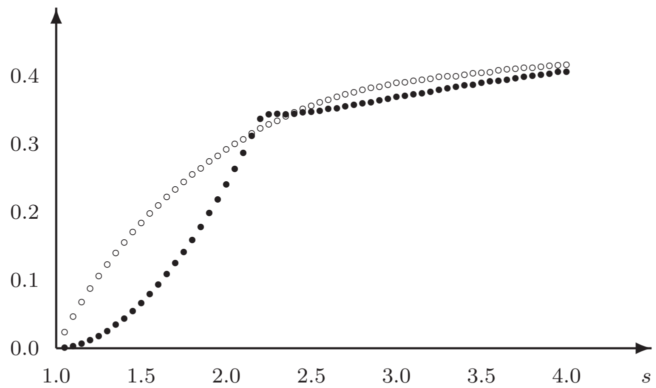

Figure 3.1: The probability of bankruptcy for the first-price ($\bullet$) and the second-price ($\circ$) format.

The presence of synergies causes the second auction to be asymmetric. Asymmetric first-price sealed-bid auctions are known to possess inefficiencies, even when bidders are risk-neutral. The second-price counterpart never possesses such inefficiencies. As a consequence, in addition to possible ex post inefficiencies due to absence of hindsight in future valuations, even ex ante inefficiencies can be observed for the first-price format. To be more specific, the inefficiency that can occur is that the loser of the first auction can win the second auction although that the synergy-adjusted value of the first auction winner is higher. Figure 3.2 displays the likelihood by which inefficiencies occur in the two auction formats. We see that the probability on an inefficient outcome is bounded from above by 8.6% for the first-price auction format.

From the two figures we can conclude that for only a small region of the synergy factor the second-price auction performs better on both efficiency and bankruptcy.

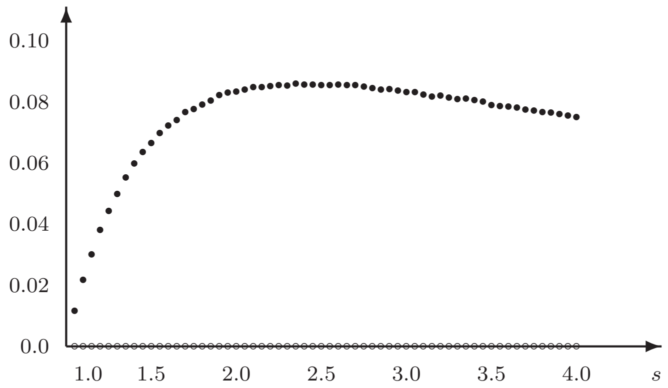

Figure 3.2: The probability of inefficiency for the first-price ($\bullet$) and the second-price ($\circ$) format.

For all remaining values of the synergy factor the first-price auction performs better regarding bankruptcy but is outperformed by the second-price auction regarding efficiency.

The smaller probability on bankruptcy in a first-price auction is a consequence of the inefficiency in the second auction. As already mentioned, the only kind of ex ante inefficiency that can occur is that the loser of the first auction wins even though the (synergy adjusted) valuation of the first auction's winner is higher. This inefficiency has a negative impact on the option value effect of winning the first auction. In a second-price auction this inefficiency does not exist and consequently the option value in the second-price format is larger than for the first-price format (see Güth et al. (2005)). Hence in the first-price auction the bidding competition will be less fierce in the first auction and consequently decreases the probability on bankruptcy.

As predicted by Maskin and Riley (2000), the expected revenue from the second auction is higher for the first-price format. The expected revenue from the first auction is higher for the second-price format owing to the larger option value. The total revenue from both auctions is larger for the second-price format but can be shown to be never more than 4% above that from the first-price format.

3.4 Discussion

In sequential auctions with synergies, the second-price sealed-bid auction format guarantees efficiency in contrast to the first-price sealed-bid auction format. Also, the total revenue resulting from the second-price auction is higher. Still, the first-price format can be preferred since the probability that the winner of the first object goes bankruptcy is smaller for most synergy levels. Our findings support the common use of the first-price auction format when synergies are present, most notably for procurement settings.

3.5 Appendix

The expected instantaneous payoff in auction 1 for bidder i prior to the realization of the valuations for this auction is denoted by $\bar{\pi}_{1i}$. The expected instantaneous payoff in auction 2 for bidder i, $\bar{\pi}_{2i}$, is prior to the realization of the valuations for this auction but given the outcome of the first auction. The winner and loser of the first auction are referred to as respectively bidder w and ℓ in the second auction.

Proof to Proposition 3.1
1. Second auction bid functions. The bid functions for this asymmetric auction follow directly from Plum (1992).
2. The factor Δ^1. Note that both second auction bid functions are strictly increasing functions on $[0,1]$ with minimum value 0 and maximum value $\frac{s}{s+1}$. The first auction winner's bid coincides with the first auction loser's bid if and only if

$$b_w^*(v_w) = b_\ell^*(v_\ell) \qquad \Longleftrightarrow \qquad v_w = \frac{1}{s}\frac{v_\ell}{\sqrt{1-(1-\frac{1}{s^2})v_\ell^2}} \quad \text{or} \quad v_\ell = \frac{s\,v_w}{\sqrt{1+(1-\frac{1}{s^2})(s\,v_w)^2}}$$

Therefore, in equilibrium, the expected instantaneous payoffs for the second auction after having learned the second auction valuation are

$$\bar{\pi}_w^*(v_w) = (s\,v_w - b_w^*(v_w)) \cdot \Pr\{b_w^*(v_w) \geq b_\ell^*(v_\ell)\} = \frac{(s\,v_w)^2}{1+\sqrt{1+(1-\frac{1}{s^2})(s\,v_w)^2}}$$

for the first auction winner, and

$$\bar{\pi}_\ell^*(v_\ell) = (v_\ell - b_\ell^*(v_\ell)) \cdot \Pr\{b_\ell^*(v_\ell) \geq b_w^*(v_w)\} = \frac{v_\ell^2}{1+\sqrt{1-(1-\frac{1}{s^2})v_\ell^2}}\frac{1}{s}$$

for the first auction loser. Hence, the expected instantaneous payoffs for the second auction before knowing the second auction valuation are

$$\begin{aligned}
\bar{\pi}_w^* &= \int_0^1 \bar{\pi}_w^*(v_w)\,\mathrm{d}v_w = \int_0^1 \frac{(s\,v_w)^2}{1+\sqrt{1+(1-\frac{1}{s^2})(s\,v_w)^2}}\,\mathrm{d}v_w \\
&\overset{3}{=} \int_0^1 \frac{(s\,v_w)^2\left[1-\sqrt{1+(1-\frac{1}{s^2})(s\,v_w)^2}\right]}{1-(1+(1-\frac{1}{s^2})(s\,v_w)^2)}\,\mathrm{d}v_w \\
&= \int_0^1 \frac{\sqrt{1+(1-\frac{1}{s^2})(s\,v_w)^2}-1}{(1-\frac{1}{s^2})}\,\mathrm{d}v_w = \frac{s^2}{s^2-1}\left\{\int_0^1 \sqrt{1+(s^2-1)v_w^2}\,\mathrm{d}v_w - 1\right\} \\
&\overset{6}{=} \frac{s^2}{s^2-1}\left\{\left[\frac{1}{2}v_w\left(\sqrt{1+(s^2-1)v_w^2} + \frac{\operatorname{arcsinh}(\sqrt{s^2-1})}{\sqrt{s^2-1}}\right)\right]_0^1 - 1\right\} \\
&= \frac{s^2}{s^2-1}\left\{\frac{1}{2}s + \frac{1}{2}\frac{\operatorname{arcsinh}(\sqrt{s^2-1})}{\sqrt{s^2-1}} - 1\right\}
\end{aligned}$$

for the winner,[2] and

$$
\begin{aligned}
\bar{\pi}_\ell^* &= \int_0^1 \bar{\pi}_\ell^*(v_\ell)\,\mathrm{d}v_\ell = \int_0^1 \frac{v_\ell^2}{1+\sqrt{1-(1-\frac{1}{s^2})v_\ell^2}}\frac{1}{s}\,\mathrm{d}v_\ell \\
&\overset{3}{=} \int_0^1 \frac{v_\ell^2\left[1-\sqrt{1-(1-\frac{1}{s^2})v_\ell^2}\right]}{1-(1-(1-\frac{1}{s^2})v_\ell^2)}\frac{1}{s}\,\mathrm{d}v_\ell \\
&= \int_0^1 \frac{1-\sqrt{1-(1-\frac{1}{s^2})v_\ell^2}}{(1-\frac{1}{s^2})}\frac{1}{s}\,\mathrm{d}v_\ell = \frac{s}{s^2-1}\left\{1-\int_0^1 \sqrt{1-(1-\frac{1}{s^2})v_\ell^2}\,\mathrm{d}v_\ell\right\} \\
&\overset{6}{=} \frac{s}{s^2-1}\left\{1-\left[\frac{1}{2}v_\ell\left(\sqrt{1-(1-\frac{1}{s^2})v_\ell^2}+\frac{\arcsin(\sqrt{1-\frac{1}{s^2}})}{\sqrt{1-\frac{1}{s^2}}}\right)\right]_0^1\right\} \\
&= \frac{1}{s^2-1}\left\{s-\frac{1}{2}-\frac{1}{2}\frac{s^2\arcsin(\frac{1}{s}\sqrt{s^2-1})}{\sqrt{s^2-1}}\right\}
\end{aligned}
$$

for the loser.[3]

The option value effect of winning the first auction Δ^1 is therefore equal to the difference in expected instantaneous payoffs for the second auction before knowing the second auction valuation between winning and losing. Hence,

$$\Delta^1 = \bar{\pi}_w^* - \bar{\pi}_\ell^* = \frac{s^2}{s^2-1}\left\{\frac{1}{2}s - 1 - \frac{1}{s} + \frac{1}{2}\frac{1}{s^2} + \frac{1}{2}\frac{\operatorname{arcsinh}(\sqrt{s^2-1})+\arcsin(\frac{1}{s}\sqrt{s^2-1})}{\sqrt{s^2-1}}\right\}$$

3. First auction bid functions. The effective first auction valuation has two components: (1) the value of the first object, and (2) the option value of winning. This means that the effective valuations are uniformly distributed over the interval $[\Delta^1, \Delta^1+1]$. The bid functions stated in the theorem follow directly from here.

[2] Here, equality 3 is obtained by multiplying the nominator and denominator by $1-\sqrt{1+(1-\frac{1}{s^2})(s\,v_w)^2}$ and equality 6 is obtained by realizing that $\frac{1}{2}x(\sqrt{1+ax^2}+\frac{\operatorname{arcsinh}(\sqrt{a})}{\sqrt{a}})$ is the antiderivative of $\sqrt{1+ax^2}$.

[3] Here, equality 3 is obtained by multiplying the nominator and denominator by $1-\sqrt{1-(1-\frac{1}{s^2})v_\ell^2}$ and equality 6 is obtained by realizing that $\frac{1}{2}x(\sqrt{1-ax^2}+\frac{\arcsin(\sqrt{a})}{\sqrt{a}})$ is the antiderivative of $\sqrt{1-ax^2}$.

Chapter 4

An experimental comparison of first- and second-price auctions

The presence of synergies in recurrent procurement auctions leads to asymmetries among bidders and an exposure problem for bidders. This chapter[1] considers sequential first- and second-price auctions with synergies in a setting with four bidders. Such a setting cannot be solved analytically for the first-price auction format. In a series of experiments, the performance of the two pricing formats is compared for three different sizes of the synergy. We find that for small synergies, the first-price auction performs better in terms of efficiency, revenue, and the probability on losses. However, once the synergy factor becomes very large the performance of the two pricing formats becomes more similar. We also find that even though the potential total surplus that can be divided between buyers and seller increases in the synergy factor, subjects' earnings do not significantly change in the synergy factor. Finally, we observe that the two pricing formats give rise to different price trends within the auction sequence.

4.1 Introduction

Research on multi-object auctions has mainly focussed on objects that are auctioned within a short period of time. The auctions take place either simultaneously or in a sequence immediately after each other. However, a notable characteristic of procurement is its recurrent nature. Construction contracts, military procurement, and service contracts are all examples of this recurrence. In these settings, auctions take place sequentially but with time periods in between. A consequence of the presence of synergies in such settings, is that bidders' valuations are stochas-

[1]This chapter is based on Leufkens, Peeters, and Vorsatz (2007).

tically dependent across auctions. Bidders then face an exposure problem as they can end up winning contracts that are too expensive if complementary contracts are not won. Furthermore, the presence of synergies induces future asymmetries between bidders which makes the bidding environment more complex.

In Chapter 2 the consequences for the bidders due to the presence of positive synergies in sequential second-price auctions were analyzed. It was shown that bidders' expected total payoffs from the auction sequence decreases in the synergy factor and that there is a serious probability of making losses. In Chapter 3 a comparison was made between sequential first- and second-price auctions with synergies and two bidders. It was found that although expected revenue and efficiency are higher in second-price auctions, first-price auctions may be preferred based on a less severe exposure problem. However, theoretical research on sequential auctions with synergies is seriously restricted by the absence of an explicit closed-form expression for the equilibrium bidding functions in an asymmetric first-price auction with more than two bidders. A comparison between sequential first- and second-price auctions with synergies has to be restricted to a setting with two bidders. For more than two bidders, it is not clear how the two pricing formats will perform.

In this chapter, we experimentally analyze a sequential auction with synergies with four bidders. Based on the aforementioned sequentiality of procurement auctions, we consider a sequential auction of two stochastically equivalent objects where the valuation of the second object is uncertain during the first auction. The winner of the first auction receives an upgrade for his valuation in the second auction. We compare sequential first- and second-price auctions for a baseline treatment without synergies and for two treatments with positive synergies of different sizes.

The setting analyzed in this chapter cannot be solved analytically for a first-price auction. By comparing the experimental data both within and between the two pricing formats, we gain insights in how bidding behavior is influenced by the presence of an exposure problem and asymmetries between bidders. The responsiveness of the bidding, and the probability of making losses in particular, to the presence and the size of the synergy are important for public policy.

Experimental research on multi-object auctions was spurred by the spectrum auctions, for which guidance on the auction rules was needed. Still, few in depth experimental analyses on the exposure problem in sequential auctions have been conducted. Février et al. (2007) analyze sequential auctions of two identical objects with a buyer's option, which means that the winner of the first object has the option to buy the second object at the winning price. They consider a setting in which the valuation for the second object increases if the first is won, and compare the revenue and usage of the buyer's option in the four main auction institutions. They note that subjects bid too conservatively compared to theory which can be attributed to the presence of the exposure problem.

In our setting, the upgrade for the valuation of the winner of the first auction makes the second auction asymmetric. Güth et al. (2005) experimentally analyze asymmetric first- and second-price auctions with two bidders in which the role of strong and weak bidder are assigned exogenously and remain fixed. They find that in first-price auctions a weak bidder bids more aggressively than a strong bidder. Furthermore, they test bidders' preference for first- versus second-price auctions and find that a strong bidder is willing to pay more than a weak bidder to dictate the auction rule.

Grimm et al. (2006) consider an asymmetric setting comparable to Güth et al. (2005). They randomly select one of the two bidders prior to an auction and give him an investment opportunity to become the strong bidder. When the investment opportunity is given, the auction valuation is not known yet. They find that bidders invest more often prior to second-price auctions than to first-price auctions. However, by not having a competition for the possibility and the cost at which one can become the strong bidder, their setting differs fundamentally from ours.

We find that the aggressiveness of bidding in the first auction increases in the synergy factor. For small synergies, the first-price auction performs better in terms of efficiency, revenue, and the probability on losses. However, once the synergy factor becomes very large the performance of the two different pricing formats becomes more similar, although the first-price auction never performs worse than the second-price auction on all three aspects. Although we observe differences in bid shading between strong and weak bidders in asymmetric first-price auctions, this difference is too small to cause significant inefficiencies. The average first auction payoff decreases in the synergy factor and is negative for large synergies. Furthermore, we observe that even though the potential total surplus that can be divided between buyers and seller increases in the synergy factor, subjects' earnings within a pricing rule do not significantly change in the synergy factor. Finally, we find that the two pricing formats give rise to different price trends within the auction sequence.

This chapter is organized as follows. In Section 4.2, we discuss the setting and theoretical insights. In Section 4.3, the experimental design and the laboratory procedures are described. The analysis of the data is presented in Section 4.4. The chapter ends with a discussion in Section 4.5.

4.2 The model

We consider a private value auction of two stochastically equivalent objects. The objects are auctioned sequentially under identical auction rules, and the same four bidders participate in both auctions. Valuations are individually uncorrelated and drawn independently according to a uniform distribution between 0 and 100. Furthermore, the valuation for the second object is not yet known during the first auction.

If the first auction is won, the drawn second auction valuation will be multiplied with a synergy factor $s \geq 1$. That is, if bidder i wins the first object, his valuation in the second auction (weakly) increases from v_{2i} to $s\,v_{2i}$. Since the second auction valuation is not yet known during the first auction, winning the first auction leads to an increase in the expected valuation for the second object. After each auction, bidders are instantly informed whether or not they won the object, the price at which the object was sold, and their payoff, which is the difference between the valuation and the price in case the auction is won and zero otherwise.

Before we discuss the theoretical insights, we need to introduce some notation. We write b_{ki} and v_{ki} for respectively the bid and valuation in auction k of bidder i. With $\bar{\pi}_{ki}$ we denote the expected instantaneous payoff of auction k for bidder i prior to the realization of the valuations for this auction and, for $k = 2$, given the history of the previous auction. The expected price of auction k, $\bar{p}_k$, is also prior to the realization of the valuations and thus the seller's expected revenue of that auction. In the second auction, bidder w and bidder ℓ are a bidder i that, respectively, won or lost the first auction.

The symmetric equilibrium for a sequential second-price auction with four risk-neutral bidders is given by:

$$b^*_{1i} = v_{1i} + (50\,s - 75 + 25\,\tfrac{1}{s})$$

$$b^*_{2i} = \begin{cases} v_{2i} & \text{if auction 1 is lost} \\ s\,v_{2i} & \text{if auction 1 is won} \end{cases}$$

$$\bar{\pi}_{1i} = 5 - \tfrac{1}{4}\,(50\,s - 75 + 25\,\tfrac{1}{s})$$

$$\bar{\pi}_{2i} = \begin{cases} 5\,\tfrac{1}{s} & \text{if auction 1 is lost} \\ 50\,s - 75 + 30\,\tfrac{1}{s} & \text{if auction 1 is won} \end{cases}$$

$$\bar{p}_1 = 60 + (50\,s - 75 + 25\,\tfrac{1}{s})$$

$$\bar{p}_2 = 75 - 15\,\tfrac{1}{s}$$

The intuition for the equilibrium bidding strategies is straightforward. The second auction is a one-shot asymmetric second-price auction, in which truthful bidding is (weakly) dominant. When positive synergies are present, the expected instantaneous payoff of the second auction is larger if the first auction is won than if the first auction is lost. The actual value of winning the first auction is then not only the value of the first object, but also the difference in the expected instantaneous payoff of the second auction between winning and losing the first auction. For the remainder of this chapter we refer to this difference in the expected instantaneous second auction payoff as the option value. It then immediately follows that bidding the valuation plus the option value forms a (weakly) dominant strategy in the first auction. Notice that the option value, and hence the first auction bidding, increases in the synergy factor.

For a first-price auction, the present setting cannot be solved analytically for its equilibrium when there are more than two bidders. This is due to the absence of a closed-form expression for the equilibrium bidding functions in the asymmetric auction of the second object. Consequently, we provide a discussion on the qualitative impact the presence of positive synergies is expected to have on the bidding in this auction format.

Winning the first auction leads to an increase in the expected valuation for the second object. Therefore, suppose that the expected instantaneous second auction payoff of the winner of the first auction increases in the synergy factor and decreases for the loser of the first auction.[2] The option value is then positive and increases in the synergy factor. In the first auction, bidders not only bid for the object but also for the option value. This shifts the supports of the distribution up by the option value, and, consequently, in equilibrium all bidders add the option value to the bid they would make in a one-shot auction (assuming risk-neutrality).

Based on the equilibrium bidding strategies for the second-price auction and the suppositions for the first-price auction, we conclude that the bidding behavior in the first auction increases in the synergy factor in both pricing formats. In a second-price auction, bidders bid above their valuation as soon as $s > 1$, which can lead to an instantaneous loss when the first object is won. In a first-price auction, bidders bid above their valuation if the synergy factor is sufficiently large to offset the bid-shading. Then, winning the auction always results in an instantaneous loss. The valuation of the second object is uncertain during the first auction. Bidders face an exposure problem, since a (possible) instantaneous loss in the first auction might not be recovered during the second auction.

In the first auction, a bidder bids the same as in a one-shot auction plus the option value. The expected probability of winning the first auction is one fourth. Therefore, the expected instantaneous first auction payoff equals the payoff of a one-shot auction, minus one fourth times the option value. It then follows that the ex-ante expected total payoff of the auction sequence as a whole, equals the expected payoff of a one-shot auction plus the expected instantaneous payoff in the second auction when the first auction has been lost:

$$\bar{\mu}_i = \bar{\pi}_{1i} + \tfrac{1}{4}\bar{\pi}_{2w} + \tfrac{3}{4}\bar{\pi}_{2\ell} = \bar{\pi} - \tfrac{1}{4}(\bar{\pi}_{2w} - \bar{\pi}_{2\ell}) + \tfrac{1}{4}\bar{\pi}_{2w} + \tfrac{3}{4}\bar{\pi}_{2\ell} = \bar{\pi} + \bar{\pi}_{2\ell}.$$

The ex-ante expected total payoff is thus decreasing in the synergy factor for both pricing formats. This means that the larger the possible benefit from synergies becomes, the smaller the expected total payoff of the bidders will be.

It is well known that (part of) a possible rent is dissipated during the competition for that rent. However, in this setting not only the possible rent is completely dissipated, bidders are even worse off than in a setting without synergies. When the synergies are large, bidders have half the ex-ante expected total payoff they would have if there were no synergies. Given the fact that the total surplus that can be divided between the seller and the bidders is larger with than without synergies, this is especially surprising. Instead of benefiting from the presence of synergies bidders suffer from it.

[2] For an asymmetric first-price auction, this does not necessarily hold for certain special cases with common supports but differences in the cumulative distribution functions of the bidders' valuations (Lebrun, 1998). This is not the case in the present setting, and Table 4.9 shows that the supposition does hold for our experiment.

4.3 Experimental design and procedures

In order to analyze the impact positive synergies have on a sequential sealed-bid auction of two objects we implemented a 3×2 between-subjects design, which is depicted in Table 4.1. The dimensions relate to the size of the synergy factor and the pricing rule of the auctions. For both pricing-rules we conducted a baseline treatment without synergies and two treatments with two different positive synergy factors. Subjects were randomly assigned to a group of four bidders. Although it was common knowledge that group composition did not change during the experiment, collusion is not a concern for this setting with four bidders. All groups played fifty rounds and a round consisted of two sequential auctions. In total, there are ten independent observations per treatment.

	First-price	Second-price
No synergy	FP1.0	SP1.0
Synergy factor 1.5	FP1.5	SP1.5
Synergy factor 2.0	FP2.0	SP2.0

Table 4.1: The experimental treatments.

In the first auction of each round, the valuation of a bidder was given by 50 plus an independently drawn and individually uncorrelated integer between 0 and 100. The constant of 50 is introduced in order to allow for underbidding at all valuations.[3] In each auction, the random component of the valuation and the total valuation were listed below each other on the screen. In the baseline treatments, the procedure in the second auction was identical to that in the first. Hence, subjects played two one-shot auctions per round.

In the treatments with positive synergies, the second auction valuations of the losers of the first auction were again determined by 50 plus a randomly drawn integer between 0 and 100. For the winner of the first auction, the second auction valuation was determined by multiplying the randomly drawn integer with the synergy factor and then adding 50. In the second auction, the winner of the first auction observed both the drawn random component, the upgraded random component and the valuation. Thus, in the first auction of a round, all bidders' valuations lay between 50 and 150. In the second auction, the valuations of the losers lay between 50 and 150. The valuation of the winner lay between 50 and 200 in treatments with synergy factor 1.5 and between 50 and 250 in treatments with synergy factor 2.0. Via control questions we tested whether subjects understood this composition of their valuation and that their auction payoff would be based on the valuation.

Subjects did not know their second auction valuation during the first auction. In the treatments with synergies, subjects were informed during the first auction of a round that the random component of the second auction valuation would be

[3] See Kirchkamp and Reiss (2004) for a discussion on the importance of allowing bidding below valuation at all valuations in first-price auctions.

upgraded by the synergy factor, s, if this auction was won. After an auction, each subject received an overview of the auction outcome; the valuation, the bid submitted, whether or not the auction was won, the price at which the object was sold and the payoff from the auction. Subjects never (directly) observed the valuations and bids of others. At the end of each round, subjects also received an update on their total payoff.

To maximize the comparability of the treatments, series of valuations were independently drawn for forty bidders and then used in all treatments with the same group compositions. Bidders' valuations were expressed in Experimental Currency Units (ECU). Subjects could submit any bid between 0 and 999 ECU. Bids did not have to be integers. In case of tied highest bids, the winner was randomly selected from these bidders. The experiments were conducted in the experimental laboratory of the Faculty of Economics and Business Administration at Maastricht University in March 2007. In total, 240 undergraduate students participated, and sessions lasted approximately 90 minutes. We conducted two sessions of twenty subjects per treatment which resulted in ten independent observations per treatment. The groups were formed randomly and group members were anonymous to each other. Subjects earned ECU during the experiment that were converted into Euros at a known exchange rate at the end of the experiment. Since we only had theoretical predictions for the second-price treatments and the revenue ranking between first- and second-price auctions was ambiguous, we used the same exchange rates in both pricing formats. For the baseline treatments, synergy factor 1.5, and synergy factor 2.0, the exchange rate for 1 ECU were respectively 2.2, 2.4, and 2.4 Eurocents. The average payoff was €11.34 including an initial endowment of €5.-. None of the subjects received a negative payoff.

The experiment was announced via email and subjects could register online using their matriculation number. This ensured that students could participate only once. When students arrived at the laboratory, they had to draw a card from a deck that determined at which computer terminal they were placed. In case more than twenty students showed up for a session, we included blank cards in the deck. Students that drew a blank card could not participate and were paid €3.- as compensation.

All interactions took place via computers that were connected to a network and the computer terminals were placed in such a way that subjects could neither see the screens of others nor make eye contact with them. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007). Before the start of a session, subjects read the instructions and were allowed to privately ask questions that were then privately answered. After reading the instructions, subjects had to answer control questions, which tested their understanding of the instructions.[4] One of the experimenters checked the answers, and the experiment only started after all subjects answered each question correctly. Payment took place privately, and subjects had to leave the laboratory immediately after receiving their payment.

[4]See Section 4.6 for the instructions and control questions.

4.4 Results

In this section, we discuss the results for efficiency, revenue, and bidders' payoffs. Thereby, we concentrate on the results in the last 25 rounds.[5] The game played is rather complex, and we are interested in mature behavior only. The results are robust to adding or dropping a couple of rounds.

4.4.1 Bid functions

The presence of positive synergies should lead to higher bidding in the first auction of a round. To see whether this is indeed the case, the first auction bid functions are estimated using a generalized least squares random-effect model with valuation as independent variable and a constant. Each group is treated as an independent unit in order to correct for dependency within groups. The estimation results per treatment are reported in Table 4.2.[6] The estimated bid function, $\hat{\beta}_0 + \hat{\beta}_1 v$, can been seen in Figure 4.1.

	FP1.0	FP1.5	FP2.0
$\hat{\beta}_0$	5.85 (0.55)	8.07 (1.24)	7.80 (1.78)
$\hat{\beta}_1$	0.88 (0.01)	0.90 (0.01)	0.94 (0.01)

	SP1.0	SP1.5	SP2.0
$\hat{\beta}_0$	2.05 (1.07)	7.51 (1.63)	14.04 (3.17)
$\hat{\beta}_1$	1.02 (0.01)	1.04 (0.02)	1.06 (0.02)

Table 4.2: Estimated parameters of the bid functions with standard errors in brackets.

The estimation results in Table 4.2 show that the effect of the synergy factor on the bid function in the first-price auctions is not completely as predicted. The constant is lower in FP2.0 than in FP1.5, and the coefficient of the valuation increases in the synergy factor. Within the second-price treatments, the constant increases in the synergy factor. Still, this increase is well below the theoretical prediction in case of risk neutrality. Furthermore, we observe that the coefficient of the valuation is always above one and increases slightly in the synergy factor.

From Figure 4.1 we can conclude that, for both price rules, the bidding increases in the synergy factor. In the first-price treatments, bids are above valuation in FP2.0 over almost the complete domain, and in FP1.5 for low valuations. In the second-price treatments, bids are always above valuation.

[5] This choice was made upfront and is based on Leufkens, Peeters, and Vorsatz (2006).

[6] Bids that were more than 1.5 times the interquartile range above the third quartile, were omitted from the analysis. For the first-price treatments we also omitted a small number of bids of (almost) zero. The pattern of the estimated bid functions in Figure 4.1 does not change if we include all observations.

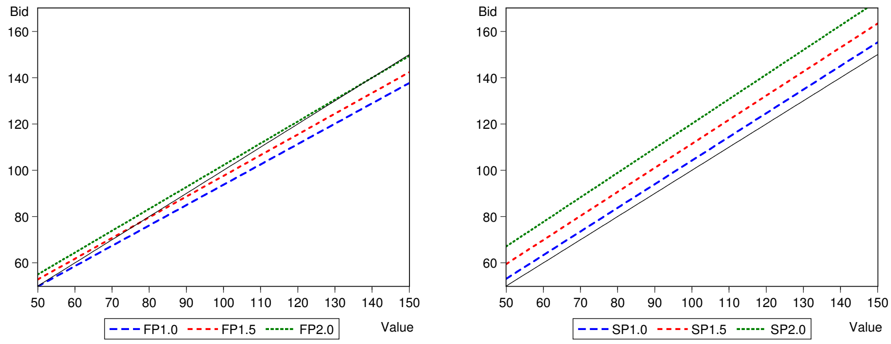

Figure 4.1: The estimated bid functions. Bids equal to valuation are shown by the thin solid line in the figure.

4.4.2 Efficiency

The presence of positive synergies should not affect the efficiency of the first auction of a round. Within a treatment, all bidders should add the same positive constant to the equilibrium bid of a one-shot auction, and, hence, the bidder with the highest valuation still wins. The second auction of a round is an asymmetric auction with four bidders. Bidding truthfully is still a weakly dominant strategy in a second-price auction. However, it has been shown for two bidders that an asymmetric first-price auction is not guaranteed to be efficient (Plum, 1992). The reason is that the bidder that draws from the more favorable valuation distribution shades his bid more at a given valuation than the other bidder.

We measure efficiency by $(v_{win} - 50)/(v_{max} - 50)$, where v_{win} denotes the valuation of the winner of the auction and v_{max} corresponds to the highest valuation within the group. We subtract the constant for normalization.[7] We also report the relative occurrence of an efficient auction.

	FP1.0	SP1.0	FP1.5	SP1.5	FP2.0	SP2.0
Occurrence	89.60	89.60	86.40	82.00	83.60	77.20
Efficiency	99.10	99.07	98.65	96.65	98.02	94.67
	(0.971)		(0.280)		(0.015)	

Table 4.3: The efficiency and occurrence of an efficient auction in percentages for auction 1. In brackets are the exact two-sided p-values for Mann-Whitney tests on the average efficiency per group.

In the upper part of Table 4.3, it can be seen that the results are (almost) the same for the baseline treatments of both pricing-rules. However, for a given positive synergy factor, the first-price auction appears to perform better than the second-price auction. The last row of Table 4.3 contains the significance levels of a two-sided Mann-Whitney test based on average efficiency per independent observation.

[7] See Ledyard et al. (1997) for a discussion on efficiency measurement in auctions.

The difference is only significant between FP2.0 and SP2.0. Mann-Whitney tests based on the relative occurrence of an efficient auction lead to the similar results.

It can also be seen in Table 4.3 that, within both the first- and second-price treatments, the efficiency decreases the larger the synergies are. For the first-price treatments, we find by means of two-sided Mann-Whitney tests, that the efficiency in FP1.0 is significantly higher than in FP2.0 ($p = 0.024$), but not than in FP1.5 ($p = 0.190$). Similarly, for the second-price treatments, the efficiency in SP1.0 is significantly higher than in both SP1.5 ($p = 0.043$) and SP2.0 ($p = 0.000$). Comparisons between the efficiency for the two different positive synergy factors within a pricing rule does not lead to any significant result. Thus, for both pricing-rules the presence of positive synergies leads to a lower efficiency in the first auction, which conflicts with theory.

	FP1.0	SP1.0	FP1.5	SP1.5	FP2.0	SP2.0
Occurrence	88.80	88.80	90.80	89.20	90.00	88.40
Efficiency	98.99	99.24	99.32	98.93	98.45	97.44
	(0.739)		(0.342)		(0.247)	

Table 4.4: The efficiency and occurrence of an efficient auction in percentages for auction 2. In brackets are the exact two-sided p-values for Mann-Whitney tests on the average efficiency in a group.

In Table 4.4, the efficiency measures are presented for the second auction in a round. In the upper part of the table, it can be seen that the efficiency and occurrence of an efficient auction are again (almost) the same for the baseline treatments of both pricing-rules. In contrast to expectation, the efficiency in the first-price auction is not lower than that in the second-price auction for a given positive synergy factor. The last row of the table contains the significance levels of a two-sided Mann-Whitney test based on the average efficiency per independent observation. We do not find any significant differences. The results for the second auction in a round are surprising since Güth et al. (2005) found that for two bidders, the efficiency in an asymmetric first-price auction is lower than in a second-price auction.

We cannot compare the efficiency within the first-price (or second-price) treatments with the efficiency measure defined above. The reason is that given there is an inefficiency, the expected efficiency is lower the larger the synergy factor. Consequently, we conducted the two-sided Mann-Whitney tests on the relative occurrence of an efficient auction per independent observation.

Based on the theory by Plum (1992) for two bidders, we would expect the efficiency in FP1.0 to be significantly higher than in FP1.5 and FP2.0. However, Table 4.4 shows that the relative occurrence of an efficient auction is higher when synergies are present. We cannot conclude that the efficiency in the second auction of FP1.0 is significantly different from FP1.5 ($p = 0.509$) or FP2.0 ($p = 0.896$).

Within the second-price treatments, there should not be any difference in the efficiency, since truthful bidding is still a weakly dominant strategy. Therefore, we conduct Mann-Whitney tests based on the relative occurrence of an efficient auction per independent observation. From this we cannot conclude that the efficiency in the second auction of SP1.0 significantly differs from SP1.5 ($p = 0.914$) or SP2.0 ($p = 0.977$).

The theoretical cause of inefficiencies in an asymmetric first-price auction is the difference in the bid shading for the two types of bidders. Güth et al. (2005) indeed observe this in their experiment, and, by applying the same analysis, we will now investigate whether this property also holds in our experiment with four bidders. In Table 4.5, we compare the median degree of bid shading between the two types of bidders for valuations that lay within the stated intervals.[8] The degree of bid shading is defined as $(v - b(v))/v$.

	FP1.5		FP2.0	
	Losers	Winners	Losers	Winners
$v \in [\ 50,\ 75]$	0.028	0.028	0.020	0.034
$v \in (\ 75, 100]$	0.034	0.026	0.024	0.046
$v \in (100, 125]$	0.035	0.059	0.039	0.052
$v \in (125, 150]$	0.069	0.082	0.042	0.076

Table 4.5: Median degree of bid shading in the second auction.

In general, we observe that, for both types of bidders, the median degree of bid shading increases in the valuation. In FP1.5, the winner shades more than the losers when the valuations are in the two upper intervals. In FP2.0, the first auction winner always shades his second auction bid more than the losers. The finding that the efficiency does not decrease for an asymmetric first-price auction is therefore not caused by the absence of a distinction in bid shading by the two types of bidders. Compared to Güth et al. (2005), we find much lower shading which can be explained by the fact that we have four bidders of which three are of the unfavorable type. The differences in shading of winners and losers are small and, therefore, as well the probability of an inefficient auction caused by this difference.

In this subsection we discussed the efficiency results. We observed that in the first auction of a round, the average efficiency within a pricing rule decreases in the synergy factor. When positive synergies are present, the efficiency is lower in the second-price auctions than in the first-price auctions, although this difference is only significant in a comparison between FP2.0 and SP2.0. For the second auction of a round, we do not observe any significant decrease in efficiency within either of the two pricing rules. Surprisingly, the efficiency in the asymmetric first-price auction is not lower than that in a second-price auction for the same positive synergy factor. A possible explanation for this is that the difference in bid-shading between a first auction winner and loser is rather small.

[8]We take the median instead of the average because subjects with low valuations occasionally submitted a bid of zero. Removing these bids and then taking the average leads to the same insights as Table 4.5.

4.4.3 Revenue and price trends

The seller's revenue consists of the prices received in each auction of a round. In Table 4.6, the average prices in the first and second auction are presented and compared for a given synergy factor. In the last two rows of the table, the same is done for the revenue. The exact two-sided Mann-Whitney statistics are based on average prices per group.

	FP1.0	SP1.0	FP1.5	SP1.5	FP2.0	SP2.0
Price 1	120.38	115.20	124.77	123.30	131.90	132.41
	(0.035)		(0.631)		(1.000)	
Price 2	119.16	113.32	128.84	119.24	137.75	127.74
	(0.015)		(0.000)		(0.002)	
Revenue	239.53	228.51	253.61	242.54	269.66	260.15
	(0.003)		(0.000)		(0.105)	

Table 4.6: The average price of each auction in a round and the revenue for all treatments. In brackets are the exact two-sided p-values for Mann-Whitney tests on the averages per group.

Within both the first- and second-price treatments, average prices significantly increase in the synergy factor.[9] The underlying forces at work are similar for both pricing formats. Namely, in the first auction, the option value effect increases in the synergy factor. In the second auction, the valuation of one of the four bidders increases (in expectation).

For both the first and second auction, a comparison between average prices in FP1.0 and SP1.0 shows that prices are significantly higher in the first-price auctions. Although it opposes to the revenue-equivalence theorem, this is a common experimental finding.

When positive synergies are present, the prices in the first auction of a round do not differ significantly, for a given synergy factor, between the first- and second-price treatments. The increase in the average price for the different synergy factors is larger in the second-price treatments. This suggests that bidders value the option value more in a second-price auction than in a first-price auction.

The prices in the second auction of a round are significantly higher in the first-price auction for a given synergy factor. Maskin and Riley (2000) showed that with this kind of asymmetry, indeed the expected revenue from a first-price auction is higher than from a second-price auction. It appears then that this finding also holds experimentally for four bidders.

Average revenue in FP2.0 is not significantly higher than in SP2.0. We already observed that average prices do not differ in the first auction and that average prices in FP2.0 are higher in the second auction. Still, the overall effect on revenue

[9] One-sided Mann-Whitney tests show that all differences within pricing rules are significant.
Price 1: FP1.0 $<_{.018}$ FP1.5 $<_{.001}$ FP2.0 and SP1.0 $<_{.003}$ SP1.5 $<_{.014}$ SP2.0
Price 2: FP1.0 $<_{.000}$ FP1.5 $<_{.000}$ FP2.0 and SP1.0 $<_{.003}$ SP1.5 $<_{.000}$ SP2.0

is indeterminate. When comparing first auction prices in FP1.5 and SP1.5 we also do not find a significant difference, but total revenue in FP1.5 is significantly higher.

There has been a lot of interest in price trends in sequential auctions. Weber (1983) shows that if bidders demand a single unit, the prices in a sequential auction of identical objects are a martingale; that is, in expectation prices drift neither up nor down. However, there is ample empirical evidence of declining price trends in sequential auctions, which is known as the declining price anomaly or afternoon effect. For instance, declining prices are observed in wine auctions (Ashenfelter, 1989, and McAfee and Vincent, 1993), real estate auctions (Ashenfelter and Genesove, 1992), and impressionist and modern paintings auctions (Beggs and Graddy, 1997). Declining prices have also been observed in experimental settings (Burns, 1985, Keser and Olson, 1996, and Neugebauer and Pezanis-Christou, 2007).

Jeitschko and Wolfstetter (2002) conclude that, in case of two bidders, the expected price in the second auction is below that in the first auction for both first- and second-price auctions. From the average prices in Table 4.6, it can be seen that we indeed observe a higher average price in the first auction in SP1.5 and SP2.0. However, for both FP1.5 and FP2.0 the average price is lower in the first auction than in the second auction.

	FP1.0	SP1.0	FP1.5	SP1.5	FP2.0	SP2.0
Declining	1	1	0	3	0	2
No difference	9	9	7	7	6	8
Increasing	0	0	3	0	4	0

Table 4.7: Price trends over the auctions within a round.

Within each independent observation, we analyze the price trend by comparing the prices in the first and second auction of a round using a one-sided Wilcoxon signed-rank test. Table 4.7 shows for each treatment the number of independent observations in which the prices significantly decline or increase within a round, at a significance level of five percent. The pricing format appears to influence the observed price trend when positive synergies are present. For both baseline treatments, we do not observe any difference between the price trends. However, once positive synergies are present, there are more observations with significantly declining prices in SP1.5 and SP2.0 and with significantly increasing prices in FP1.5 and FP2.0. Performing a sign-test or increasing the significance level to ten percent results in the same insight.

In this subsection, we discussed revenues and price trends. When positive synergies are present, the prices in the first auction of a round do not differ significantly between the first- and second-price treatments for given positive synergies. In contrast, the prices in the second auction of a round are always significantly higher in the first-price treatments. The overall revenue is also always higher in the first-price treatments, although this is not significant at five percent significance level for synergy factor 2.0. Finally, it appears that the presence of positive synergies gives rise to opposing price trends for both pricing-formats.

4.4.4 Payoffs

The average auction payoffs for a bidder are shown in Table 4.8. Within both the first- and second-price treatments, the first auction payoff decreases in the synergy factor whereas the second auction payoff increases in the synergy factor.[10] In both FP2.0 and SP2.0, the average first auction payoff is even negative. In the bottom part of the table, we compare the average round payoffs between the two pricing formats for a given synergy factor.

	FP1.0	SP1.0	FP1.5	SP1.5	FP2.0	SP2.0
Auction 1	2.33	3.63	1.17	1.14	-0.74	-1.39
	(0.003)		(0.579)		(0.393)	
Auction 2	2.58	4.08	3.73	6.58	5.96	8.69
	(0.004)		(0.002)		(0.000)	
Round	4.91	7.71	4.90	7.73	5.22	7.30
	(0.001)		(0.015)		(0.089)	

Table 4.8: The average payoff in ECU per auction for each treatment. The exact two-sided p-values of Mann-Whitney tests on average payoffs per group are given between brackets.

The test results in the table are in line with what was previously observed for prices. For both the first and second auction, a comparison between average payoffs in FP1.0 and SP1.0 shows that payoffs are significantly lower in the first-price auctions. When positive synergies are present, the payoffs in the first auction of a round do not differ significantly for a given synergy factor. The payoffs in the second auction are significantly lower in the first-price auction for a given synergy factor. Finally, note that the difference in average round payoff is smaller between FP2.0 and SP2.0 than between FP1.0 and SP1.0.

Although the possible surplus that can be divided between bidders and seller increases in the synergy factor, we showed in Section 4.2 that the expected round payoff decreases in the synergy factor. Within both the first- and second-price treatments, the realized average round payoff remains approximately the same when the synergy factor increases. For the second-price treatments, the average round payoff is even lowest in SP2.0. Comparing the payoffs between treatments with the same pricing rule using a one-sided Mann-Whitney test based on average round payoffs per group never leads to the rejection of equality.[11] Thus, subjects

[10] One-sided Mann-Whitney tests show that all differences within pricing rules are significant.
Payoff 1: FP1.0 $<_{.007}$ FP1.5 $<_{.001}$ FP2.0 and SP1.0 $<_{.000}$ SP1.5 $<_{.014}$ SP2.0
Payoff 2: FP1.0 $>_{.004}$ FP1.5 $>_{.000}$ FP2.0 and SP1.0 $>_{.001}$ SP1.5 $>_{.004}$ SP2.0

[11] FP1.0 $\sim_{.434}$ FP1.5, FP1.0 $\sim_{.485}$ FP2.0, FP1.5 $\sim_{.485}$ FP2.0, SP1.0 $\sim_{.485}$ SP1.5, SP1.0 $\sim_{.427}$ SP2.0, SP1.5 $\sim_{.427}$ SP2.0

do not bid aggressive enough to suffer from the presence of positive synergies. Still, the seller always reaps the increase in surplus that is due to the positive synergies.

In the first auction of a round, subjects do not only bid for the object but also to be the strong bidder in the second auction. In Table 4.9, we show the average second auction payoff for a bidder, depending on whether he won or lost the first auction of a round. The average option value of winning the first auction is then the difference between the payoffs of both types.

	FP1.0	SP1.0	FP1.5	SP1.5	FP2.0	SP2.0
Winner	2.70	4.47	10.62	19.51	21.95	31.00
Loser	2.54	3.94	1.43	2.27	0.64	1.25
Option value	0.16	0.53	9.20	17.24	21.31	29.75
	(0.986)		(0.003)		(0.000)	

Table 4.9: The average second auction payoff for the winner and a loser of the first auction in a round. The exact two-sided p-values of Mann-Whitney tests on average payoffs per group are given between brackets.

Given the synergy factor, the average second auction payoffs are lowest for both types in the first-price treatments. When positive synergies are present, the payoff of the winner increases in the synergy factor, whereas that of the loser decreases. Consequently, the option value increases in the synergy factor. Based on a two-sided Wilcoxon signed-rank test on the average option value per independent observation, we cannot reject that the option value equals zero in both FP1.0 ($p = 0.770$) and SP1.0 ($p = 1.000$). As can be seen in Table 4.9, the option value is always significantly larger in the second-price treatment for a given positive synergy factor.

The larger the option value, the higher bidders should bid for it. For the first auction of a round, the larger option value then explains the more rapid decrease of payoffs in the second-price treatments. Without positive synergies, the option value is zero and we observe significantly higher payoffs in the second-price treatments. When positive synergies are present, the option value is higher in the second-price treatment, and, as a consequence, the payoff difference between the two pricing formats disappears. The same reasoning applies for the first auction prices.

In this auction setting, bidders face an exposure problem as they can end up winning one object that is too expensive if the second object is not also won. We already observed that average first auction payoffs decrease in the synergy factor and even become negative. In order to analyze the consequences of the exposure problem, we count the number of times a bidder receives a negative payoff from a round. We only count round losses that are caused by a loss made in the first auction of a round. This measure excludes round losses that are caused by the second auction in a round, since such losses are not stemming from the presence of positive synergies.[12] In Table 4.10, the average and relative number of round

[12] Our findings are robust to counting all negative round payoffs.

losses are listed per treatment. The one-sided Mann-Whitney statistics are based on the number of round losses per independent observation.

	FP1.0	SP1.0	FP1.5	SP1.5	FP2.0	SP2.0
Average	0.30	3.40	2.30	5.90	6.10	7.50
Percentage	1.20	13.60	9.20	23.60	24.40	30.00
	(0.000)		(0.006)		(0.171)	

Table 4.10: The average and relative number of round losses for each treatment. The p-values of Mann-Whitney tests are given between brackets

The average number of round losses is always larger in the second-price treatments. In SP1.0 and FP1.0, losses can only be caused by irrational bidding behavior. As is often observed in experiments, bidding above valuation occurs in the second-price auction but not in the first-price auction.

When positive synergies are present, the option value makes the value of winning the first auction higher than just the bidder's valuation for the object. For a second-price auction this immediately implies that it is optimal to bid above one's valuation for the object. In a standard first-price auction, bidders shade their bids, and therefore, the presence of the option value does not immediately imply bidding above one's valuation. In Chapter 3 it was shown that, for two bidders, the number of round losses tends to be lower in first-price auctions, but that the difference almost vanishes when the synergy factor becomes large. When comparing FP1.5 with SP1.5 we observe a significant negative difference that is larger than between the baseline treatments. For synergy factor 2.0, there is no significant difference between the number of round losses. It is interesting to note that given that a loss was made, the average loss was 14.43 in FP2.0 and 21.84 in SP2.0.

In this subsection we discussed the payoffs of subjects. We clearly observe differences in the average second auction payoff for the winner and the loser of the first auction. The option value is always significantly larger in the second-price treatments. The presence of a positive option value leads to more aggressive bidding in the first auction of a round. Consequently, the average first auction payoff decreases in the synergy factor and is even negative for large synergies. Still, the average earnings per subject over a round remain approximately constant within a pricing rule for the different sizes of the synergy factor. The occurrence of losses increases in the synergy factor, and with the exception of synergy factor 2.0, is always significantly smaller in the first-price auction.

4.5 Discussion

Winning multiple contracts in multi-unit auctions can lead to cost advantages due to synergies. In recurrent procurement auctions, the presence of synergies leads to an exposure problem and asymmetries among bidders. We consider sequential first- and second-price auctions in which winning the first auction leads to an increase in the valuation of the second object. In a series of experiments, we compare the performance of the two auction formats for three different sizes of the synergy.

We find that subjects indeed respond to the incentives provided by the presence of positive synergies. The average second auction payoff of a first auction winner is much larger than that of a first auction loser when positive synergies are present. The first auction is then not only about winning the first object, but also about increasing the expected second auction payoff. Consequently, the presence of a positive option value leads to more aggressive bidding in the first auction. The average first auction payoff decreases in the synergy factor and is even negative for large synergies.

Bidders face an exposure problem, since an instantaneous loss in the first auction might not be recovered during the second auction. For subjects, the faced exposure problem differs between the two pricing formats. In a second-price auction, bidders bid above their valuation as soon as the option value is positive, which can, but does not necessarily, lead to an instantaneous loss when the first object is won. In a first-price auction, bidders bid above their valuation if the option value is sufficiently large to offset the bid-shading. Then, winning the auction always results in an instantaneous loss. Furthermore, for a given positive synergy factor, the option value is always significantly larger in the second-price treatments. Both factors explain why the first auction bidding increases more in the synergy factor for the second-price treatments. However, the average earning of a subject remains approximately constant within a pricing rule, which contrasts the theoretical prediction. The occurrence of losses increases in the synergy factor, and with the exception of synergy factor 2.0, is always significantly smaller in the first-price auction.

When positive synergies are present, the prices in the first auction do not differ significantly between the first- and second-price treatments for given positive synergies. In contrast, the prices in the second auction are always significantly higher in the first-price treatments. The overall revenue is also always higher in the first-price treatments, although this is not significant at five percent significance level for synergy factor 2.0. Finally, it appears that the presence of positive synergies gives rise to opposing price trends for both pricing-formats. Within the first-price treatments, we observe an increase in the number of observations with an increasing price trend, whereas within the second-price treatments we observe an increase in the number of observations with a declining price trend.

We observe that in the first auction, the average efficiency within a pricing rule decreases in the synergy factor. When positive synergies are present, the efficiency is lower in the second-price auctions than in the first-price auctions, although this difference is only significant for a comparison with synergy factor 2.0. For the second auction, we do not observe any significant decrease in efficiency within either of the two pricing rules. Surprisingly, the efficiency in the asymmetric first-price auction is not lower than that in the second-price auction for the same positive synergy factor. A possible explanation for this is that the difference in bid-shading of a first auction winner and loser is rather small.

For small synergies, the first-price auction performs better in terms of efficiency, revenue and the probability on losses. Once the synergy factor becomes very large, the performance of the two different pricing formats becomes more similar, although the first-price auction never performs worse than the second-price auction on all three aspects. Our results provide support for the common use of first-price rather than second-price auctions when the exposure problem is present, in particular for recurrent procurement settings.

4.6 Appendix

The instructions of treatment FP1.5 are provided here. Trivial modifications were made for the instructions of other treatments.

Dear participant,

welcome to this experiment. This experiment will last about 1.5 hours and you will be compensated according to your performance. In order to ensure that the experiment takes place in an optimal setting, we would like to ask you to follow the general rules during the whole experiment:

- Read these instructions carefully! It is important that you understand the rules of this experiment. These instructions are identical for all students that participate together with you. If something is not explained well, please raise your hand. Do not ask the question out loud, but wait until one of the experimenters approaches you to answer the question privately.
- Switch off your mobile phone!
- Do not communicate with your fellow students! Even though the experiment may get exiting at times, it is very important that you remain silent through the proceedings.
- Focus on your *own* computer screen and not on other participants!
- There is paper and a pen on your table, which you can use during the experiment.
- After the experiment, please remain seated until you are paid off.
- If you do not obey the rules, the data becomes useless for us. Therefore we will have to exclude you from this experiment and you will not receive any compensation.

Your decisions and earnings in this experiment will remain anonymous.

General set-up In this experiment all of you are buyers of a fictitious object that is auctioned off. You can earn ECU (Experimental Currency Units) which will be exchanged into Euros at the end of the experiment. The exchange rate will be given in the instructions below.
If you win an auction you do not receive the object but you receive an amount of ECU equal to your value of the object v. In return you have to pay the price p resulting from the auction. Thus you receive a payoff of $v - p$ ECU. The rules of the auction will be discussed below.
Before the experiment starts, you will be randomly divided into groups of 4 potential buyers. You will not know the identity of the 3 potential buyers you are matched with. The groups remain unchanged throughout the whole experiment.

This experiment consists of 50 rounds. Each round consists of 2 auctions which are held after each other. Now we will explain the procedures in each round.

Auction rules In each auction you and the 3 other potential buyers in your group will be bidding for a single object that is auctioned off. In each auction the following happens. First you observe your value of the object. Then all of you are asked to submit a bid. Your bid can be any nonnegative number below 1000.
Within each group, the winner is the subject who submitted the highest bid. The price the winner has to pay equals his bid. The payoff to the winner will be the difference between his value v and the price he has to pay p. The payoff to the winner is thus given by the difference between his value and his bid. All other subjects get a payoff of 0. In case the highest bid was submitted by more than one subject, the computer will randomly select a winner among those subjects.

Auction 1 Each round consists of two auctions and the proceedings in the first auction of each round are as follows.
First the value of the object will be determined for each subject. This value will be $v_1 = 50+x_1$ where x_1 is a randomly determined integer between 0 and 100 with each number being equally likely. Notice that x_1 is determined independently for each subject. Consequently the value of the object, v_1, will be a number between 50 and 150 for each bidder.
After observing your value, all of you are asked to submit a bid. Thus you will submit this bid after observing your value of the object but without observing the values or bids of the other subjects. To submit a bid you can fill in a number in the box and click on *submit*. Notice that your bid is not restricted to the interval [50,150] but can be any nonnegative number below 1000.
The bidding procedure in the first auction is illustrated in Figure 1 below. Note that the numbers are omitted in the illustration.

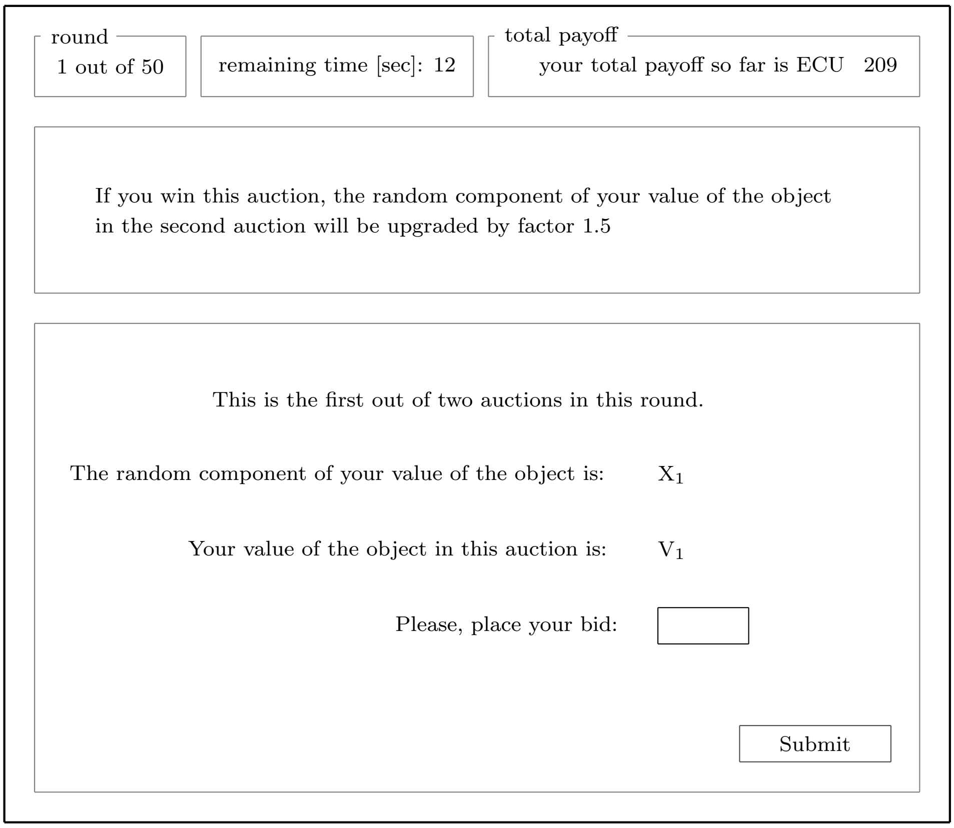

Figure 1: illustration of bidding in auction 1. On your screen you can see how your value of the object was constructed. The random component of your value is x_1 and consequently your value of the object in the first auction is v_1. To make a bid you can enter a number in the corresponding box and click on submit. The screen also shows that this is the first out of 50 rounds and that your total payoff so far is 209.

The meaning of the sentence '*If you win this auction, the random component of your value of the object in the second auction will be upgraded by factor 1.5*' will be clarified below.

After all of you submitted a bid, the winner and the price will be determined according to the rules mentioned above. The feedback on the result of the auction that will then appear on your screen is illustrated in Figure 2. Click on *continue* when you are ready for the second auction of this round.

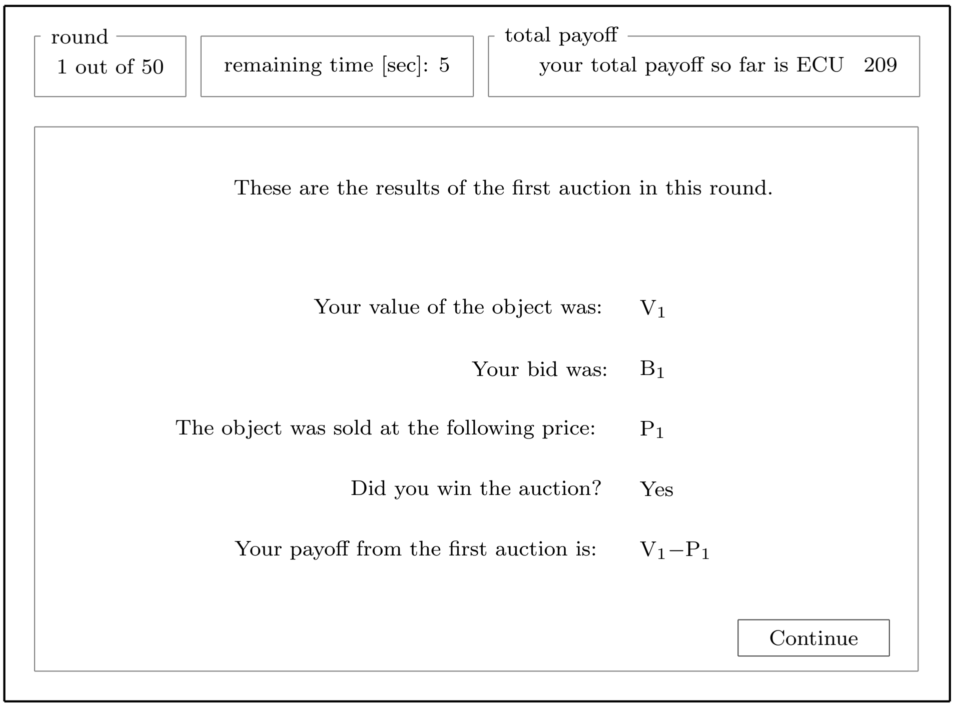

Figure 2: illustration of the results of auction 1. Your value was v_1 and your bid was b_1. You won the auction and the price you have to pay is p_1 (which hence equals b_1). Consequently your payoff from the first auction in this round is the difference between your value and the price, namely $v_1 - p_1$.

Auction 2 The proceedings in the second auction of each round are as follows. A new value of the object will be determined for each subject. In case the first auction was not won, the value will be $v_2 = 50 + x_2$ where, x_2 is a randomly determined integer between between 0 and 100 with each number being equally likely. In case the first auction was won, the value will be $v_2 = 50 + 1.5 \cdot x_2$ where, again, x_2 is a randomly determined integer between between 0 and 100 with each number being equally likely. Thus, for the subject that won the first auction the random component of the value of the object is multiplied by 1.5. Notice that x_2 is determined independently for each subject.
Consequently, the values of the 3 subjects that did not win the first auction lie between 50 and 150 and the value of the subject that won the first auction lies between 50 and 200. Again notice that your bid is not restricted to the interval [50,150] or [50,200] but can be any nonnegative number below 1000.
The bidding procedure in the second auction is illustrated in Figure 3 below.

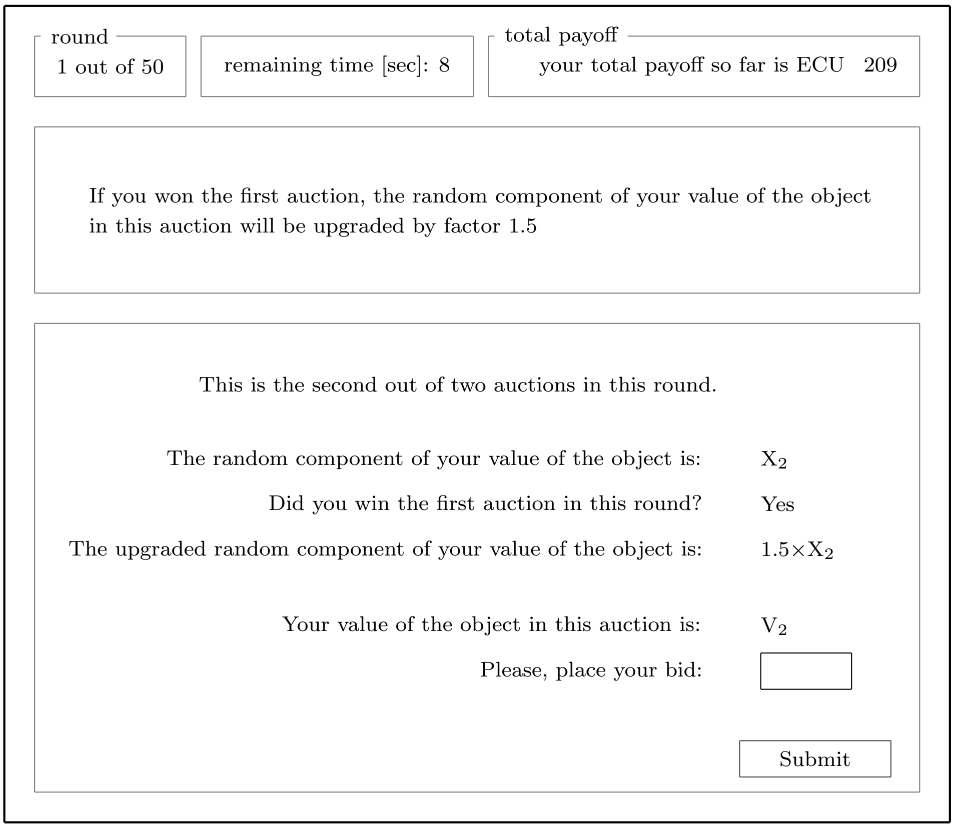

Figure 3: illustration of bidding in auction 2. On your screen you can see how your value of the object was constructed. Since you won the first auction, the random component of your value is multiplied by 1.5. Your value of the object after this upgrade is v_2. To make a bid you can enter a number in the corresponding box and click on submit.

In case the first auction was not won, the number that appears on your screen behind '*The upgraded random component of the object in this auction is:*' is just a repetition of your random component of the value, since no upgrade takes place. After all of you submitted a bid, the winner and the price will be determined according to the rules mentioned above. The feedback on the result of the auction and the round that will then appear on your screen is illustrated in Figure 4.

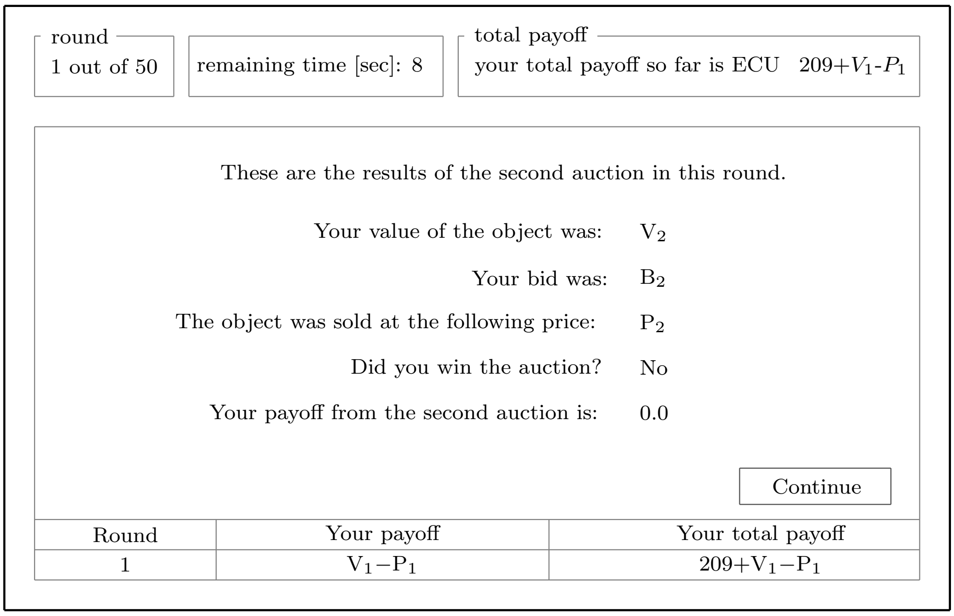

Figure 4: illustration of the results of auction 2. Your value of the object object was v_2. Your bid was b_2 and you did not win the auction. Consequently your payoff from the second auction in this round is zero. Your payoff from the round is the sum of the payoffs of both auctions and thus in this case $v_1 - p_1$. This was the first round and therefore the total payoff in the upper right corner changes from 209 into $209 + (v_1 - p_1)$.

This ends the round and the first auction of a new round starts after all of you clicked *Continue*. In total you will participate in 50 rounds of 2 auctions each. After the last round of the experiment, we would like to ask you to complete a short questionnaire that will appear on your screen. Payments will be made by the experimenters afterwards.

ECU are transformed into Euros according to the following conversion rate: 1 ECU = 0.024 Euro. You will get an initial endowment of 5 Euro (209 ECU).
Just like a profit is automatically added to your total payoff at the end of a round, a loss will be automatically deducted. If at the end of the experiment your total payoff is negative we will ask you to pay this amount of money to us. This situation is very unlikely to occur and under your control.

Before we start with the experiment we would like you to answer the questionnaire on the next page. One of the experimenters will go around and check the answers and discuss any problems.

Control questions

Please answer the following questions. When you are finished, please raise your hand. One of the experimenters will come to you and check whether everything is correct.

1.) How many subjects are bidding in an auction (including yourself)?

__

2.) Suppose that in the first auction the random component of your value is 82. What is your value in the first auction?

__

3.) Suppose that you have a value of 86 for the object in the first auction. What can you conclude about the values of the three other potential buyers?

__ Their value is for sure 86.

__ Their values might be 86 and might be different from that but all lie between 50 and 150 for sure.

__ Their values lie between 50 and 150 and are for sure different from 86.

4.) Suppose the four participants A, B, C, D submitted the following bids: A submitted 101, B submitted 93, C submitted 74, and D submitted 137. Who wins the auction?

__ A __ B __ C __ D

5.) What price does this subject have to pay?

__ 101 __ 93 __ 74 __ 137

6.) Suppose the random component of your value is 45 in the first auction. What can you conclude about the random component of your value in the second auction before upgrading?

__ The random component will be 45 for sure.

__ The random component might be 45 and might be different from that but lies between 0 and 100 for sure.

__ The random component lies between 0 and 100 and is different from 45 for sure.

7a.) Suppose you did not win the first auction and the drawn random component of your value in the second auction is 80. What is the upgraded random component of your value?

—

7b.) and what is your value in the second auction?

—

8a.) Suppose you won the first auction and the drawn random component of your value in the second auction is 80. What is the upgraded random component of your value?

—

8b.) and what is your value in the second auction?

—

9.) Suppose you buy the object for a price of 100. Your value of the object is 121. What is your payoff from this auction?

—

10.) Suppose you buy the object for a price of 120. Your value of the object is 101. What is your payoff from this auction?

—

Part II

Alternating price competition

Chapter 5

An experimental analysis of focal prices and price cycles

In this chapter[1] we experimentally analyze the alternating price setting game of Maskin and Tirole (1988). When the time horizon is infinite, a focal price equilibrium and an equilibrium consisting of Edgeworth cycles coexist. We test which of these two equilibria emerges in an experiment. It is found that in 20 out of 27 observations the focal price equilibrium emerges. Price cycles are observed in only one observation. Furthermore, we analyze the game in case of a long but finite horizon and find that the corresponding subgame-perfect equilibrium consists of Edgeworth cycles. Experimentally, we still observe a focal price in the majority of the observations. Nevertheless, price cycles are observed far more often than for the infinite horizon setting.

5.1 Introduction

In oligopolistic markets, prices often fluctuate even though demand and supply conditions are stable. Frequently, the rocket-feather pricing pattern is observed, where a series of small price decrements is followed by a sudden substantial increment, after which prices start declining again, possibly after a period of stable prices. Empirical studies have found such a pattern for local gasoline markets in the United States (Castanias and Johnson, 1993, and Doyle et al., 2007), Canada (Eckert, 2003, and Noel, 2007), and Australia (Wang, 2005). Similarly, a price pattern consisting of periods of stability followed by gradual undercutting of competitors' prices have been observed for the commercial airline industry (Ross, 1997, and Busse, 2002).

The traditional price setting model for an oligopolistic market was introduced by Bertrand (1883). For homogeneous objects this model predicts that, in equilibrium, prices equal marginal cost. Consequently, prices are stable and firms do

[1] This chapter is based on Leufkens and Peeters (2008b).

not make any profit. Edgeworth (1925) showed that the static price equilibrium of Bertrand does not arise when firms face capacity constraints and set prices repeatedly, but prices would cycle. In this so-called Edgeworth cycle, firms successively undercut each others' prices until the 'war' becomes too expensive and one firm increases its price. Next, the other firms respond with a match or a slight undercut, after which the process of undercutting resumes. This result, however, is induced by the competitors' mutual irrational expectation that opponents maintain their prices from the previous period.

In opposition, Chamberlain (1933) claims that a small number of firms will not start undercutting but rather charge the monopoly price. The firms will realize their interdependence and that competitors will retaliate a price cut by also cutting their prices. Therefore, the result of a price cut will be a decrease in its own profits. Hall and Hitch (1939) and Sweezy (1939) formalize this conjecture by showing that a focal price equilibrium can be sustained for a kinked demand curve.

Maskin and Tirole (1988) show that the equilibria envisioned by Edgeworth and Chamberlain can coexist for an infinite time horizon. They show that when firms face short-run price commitments, which is modeled via alternating price setting, both a focal price equilibrium and an equilibrium consisting of Edgeworth cycles emerge as a Markov perfect equilibrium. In the focal price equilibrium, prices are stable at the monopoly price. In the Edgeworth cycles equilibrium, prices decline to marginal cost from which one of the firms raises it again after which the undercutting starts again.

A different line of theoretical explanations for periods of stable prices and periods of successive undercutting concerns tacit collusion. Price wars can erupt due to uncertainty related to the market conditions. Green and Porter (1984) consider a model in which firms face uncertainty about current demand such that price cutting detection is hindered. Optimal punishment is not to resort to the Bertrand equilibrium forever but involves a finite number of periods after which a collusive price is again adopted. Furthermore, Rotemberg and Saloner (1986) show that collusive prices move countercyclically in case market demand is stochastic.

Evidence of price cycles in experiments is rather limited. Cason et al. (2005) find that for some variations of the Edgeworth hypothesis, the data of repetitive posted price offers displays a cycle in a setting with six sellers. Furthermore, Kruse et al. (1994) observe price cycles when capacity is restricted and there are four players. Guillén (2004) finds price cycles for an experiment involving simultaneous price and quantity setting. However, in the same setting, price cycles are not observed in case of two or three sellers (Brandts and Guillén, 2007).

In this chapter we investigate the potential to observe price cycles in the alternating price setting of Maskin and Tirole (1988) with two players by means of a laboratory experiment. We consider the infinite and the finite time horizon versions of this model. For the infinite time horizon there exists a Markov perfect equilibrium with stable prices besides a Markov perfect equilibrium that displays price cycles. For the finite time horizon, the subgame-perfect equilibrium induces prices that cycle. Strikingly, the backwards induction strategies do not converge to the Markov perfect equilibrium for the infinite time horizon when the horizon lengthens. In our experiment, we find that in an infinite horizon setting, the focal price equilibrium emerges in 20 out of 27 observations and price cycles in only one observation. For the finitely repeated setting, we also observe the focal price outcome in the majority of the observations, even though it is not an equilibrium. However, we observe clear price cycles in three and price wars in two out of the fifteen observations.

The remainder of this chapter is organized as follows. In Section 5.2, we present the basic model and the Markov perfect equilibria for the infinite time horizon of the model and the subgame-perfect Nash equilibrium in case the time horizon version is finite. In Section 5.3, the experimental design and procedures are described. The analysis of the data is presented in Section 5.4. The chapter ends with some concluding thoughts in Section 5.5.

5.2 The setting

The setting that we consider is precisely the illustrating example of the exogenous-timing duopoly model of Maskin and Tirole (1988). The two firms compete in a homogenous product market with prices being the strategic variable. Firms interact dynamically in discrete time and can adapt their prices alternately. So, in the periods in which a firm cannot adapt its price, the price remains equal to the price set in the previous period. Consequently, firms set prices for two periods. This alternating move structure captures the idea of short-run price commitments. Without loss of generality, we assume that firm A can adapt its price in odd periods and firm B in even periods.

The prices the firms can charge are the seven integers between (and including) 0 and 6. Given the firms prices at a certain period, p^A and p^B, the market price for that period equals $p^* = \min\{p^A; p^B\}$. The market demand for that period is given by $D(p^*) = 6 - p^*$. We assume that the firms do not incur any costs for production. Consequently, the market profit equals $\Pi(p^*) = p^* \cdot D(p^*)$. Table 5.1 summarizes the possible market prices and the resulting market demand and market profit. The products are homogeneous and therefore only the firm with the lowest price

Market price	0	1	2	3	4	5	6
Market demand	6	5	4	3	2	1	0
Market profit	0	5	8	9	8	5	0

Table 5.1: The output and profit for the seller with the lowest price.

sells output. This means that if a firm has the unique lowest price, this firm receives the full market profit, and the profit of the other firm equals zero. In case both firms charge the same price, the market output and the market profit are split equally among them.

Next, we consider this setting with an infinite and a finite time horizon.

5.2.1 Infinite horizon

Suppose that the time horizon is infinite and future profits are discounted by a factor δ. It is assumed that this discount factor is sufficiently close to, and strictly smaller than one. Firms maximize the present value of the infinite stream of profits. Maskin and Tirole (1988) solve this setting for strategies that only depend on the payoff relevant state, which is in this case the price set by the other firm in the previous period. Consequently, strategies are dynamic reaction functions that give a response price for each price set by the other firm. There are two Markov perfect equilibria (MPE) that coexist: a focal price equilibrium and an equilibrium that consists of Edgeworth cycles.[2]

In the focal price equilibrium, the firms always set prices equal to 3 which is then also the market price. Hence, the market demand and profit is continuously split equally and each firm makes a profit of 4.5 in all periods. The symmetric equilibrium strategy and the prices set along the equilibrium path are illustrated in Figure 5.1. As soon as one of the two firms sets a price of 3, prices remain there

Price	Response price	
6	3	
5	3	
4	3	
3	3	
2	1	
1	1	with probability $\beta(\delta)$
	3	with probability $1-\beta(\delta)$
0	3	
$\beta(\delta) \equiv (5+\delta)/(5\delta+9\delta^2)$		

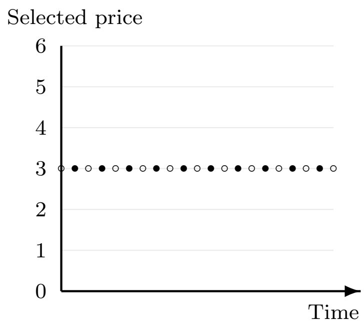

Figure 5.1: Focal price equilibrium. Left panel: symmetric MPE strategy. Right panel: selected price by the respective firm in the respective period (● = firm A; ○ = firm B).

ever after. From any price above 3, the best response brings the price immediately down to 3. From prices below 3, prices are not instantly changed to 3, but do end up there with probability one. Bringing the price up from 1 to 3 is costly and therefore each firm wants the other firm to do so. The probability $\beta(\delta)$ is determined in such a way that the opponent firm is indifferent between raising and not raising the price.

The other Markov perfect equilibrium induces a cyclical pricing pattern on the equilibrium path. The symmetric equilibrium strategy and the prices set along the equilibrium path are illustrated in Figure 5.2. The firms undercut each other's price successively until the price equals zero and neither firm makes a positive profit. At that price, each firm has an incentive to raise its price. Furthermore, each firm prefers the other firm to raise its price first, so that it can start undercutting in the subsequent period. The probability $\alpha(\delta)$ is determined in such a

[2]For proofs see Maskin and Tirole (1988). They refer to the focal price equilibrium as a kinked demand curve. For our experimental context it is more intuitive to interpret it as the focal price equilibrium.

Price	Response price	
6	4	
5	4	
4	3	
3	2	
2	1	
1	0	
0	0	with probability $\alpha(\delta)$
	5	with probability $1-\alpha(\delta)$
$\alpha(\delta) \equiv (3\delta^2-1)(1+\delta^2+\delta^4)/(8+7\delta^2+2\delta^4+3\delta^6)$		

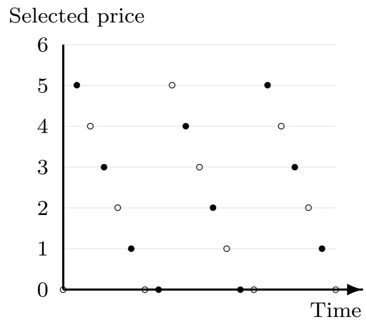

Figure 5.2: Edgeworth cycle equilibrium. Left panel: symmetric MPE strategy. Right panel: price set by the respective firm in the respective period (• = firm A; ◦ = firm B).

way that the opponent firm is indifferent between raising and not raising the price. After one firm has raised the price, the undercutting starts again. The price cycles consist of two phases: an undercutting phase and a coordination phase concerning which firm is going to raise the price when the price equals zero. In equilibrium, all prices except 6 are observed.

5.2.2 Finite horizon

In this subsection, we consider a similar setting but then with a finite time horizon without discounting. For a fixed horizon, the resulting game can be solved by application of the backwards induction procedure. Obviously, the best response in the last period is to undercut the opponent if possible. Considering the profit structure of the game, this still does not imply that for long horizons prices constant at 1 are observed. It appears that when the horizon lengthens, the backwards induction outcome converges to a pricing pattern that contains recurrent cycles (as long as the final stage is not too near). The best responses within a cycle not only depend on the current price of the opponent, but also on the number of periods that have elapsed since the start of the cycle. The non-stationary pricing behavior within a cycle is illustrated in Figure 5.3.

The seven boxes represent periods 1 to 7 of a price cycle. In each box, an arrow shows the best response for each current price of the opponent. The price that will be set along the equilibrium path is shown in boldface in each box. In box 1, the best response is to set the price equal to 1, regardless of the opponent's current price. Note that the actual period in time at which this behavior is observed is not specified, we take it as the first of the cycle since the resulting price is 1 in any case. Next, in box 2, the price of 1 is responded with one of the prices 4, 5 and 6. By overshooting the current market price, the opponent foregoes any immediate profit in exchange for future profits. In box 3, no matter which of the three prices were chosen in box 2, the response is to undercut this price by setting the price equal to 3. Thereby the maximum immediate profit of 9 is gained. Consequently, in the previous box the opponent is indeed indifferent between setting a price of 4, 5, or 6. Next, in box 4, the opponent undercuts this price by setting its price equal

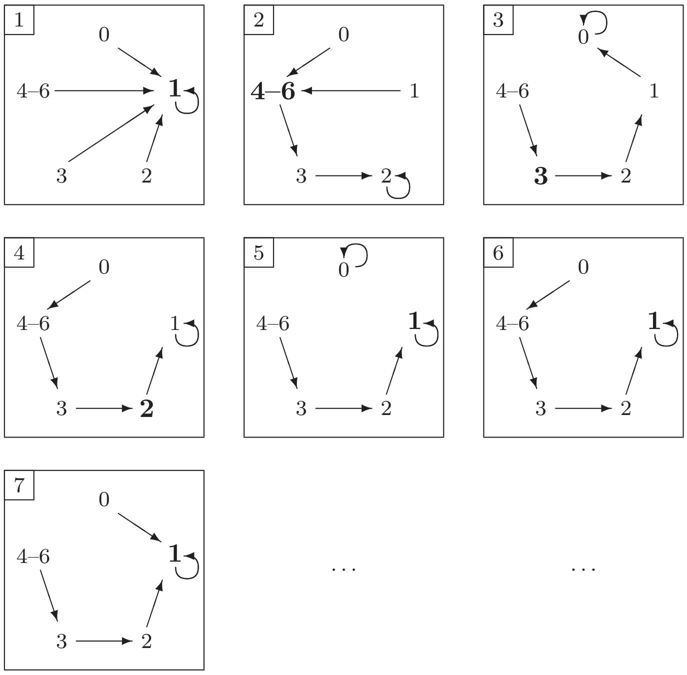

Figure 5.3: Best responses for large finite horizons.

to 2, and this price is again undercut by a price of 1 in box 5. Then, the price remains at 1 in box 6, 7 and 1 in order to be brought up in box 2 and subsequently to start the gradual undercutting.

Although the figure only presents seven boxes, the cycle has a length of fourteen periods. Namely, if it is the one firm that played according to the action displayed in box 1, after having reached box 7, it is the other firm that continues with the action that is depicted in box 1. Nevertheless, the dynamics of the prices being selected over periods follows a cyclical pattern with cycle length of seven. A graphical illustration of the cycle is given in Figure 5.4. The figure does not contain prices observed in initial periods or the endgame effects that are observed for any finite horizon.

5.3 Experimental design and procedures

In our experiment, we study subjects' behavior in the alternating price setting model for the infinite and the finite (but lengthy) time horizon. Our experimental design, hence, consists of two treatments that are equal apart from the way the number of periods is determined. In the first treatment we implemented a random continuation rule, whereas in the other treatment we have a fixed number of periods.

In the treatment with random ending, for each period there was a probability of two percent that the experiment ended after that period. Of course, it requires

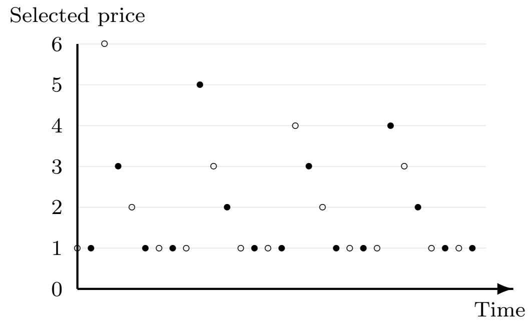

Figure 5.4: Cyclical pattern of selected prices (• = firm A; ◦ = firm B).

a sufficiently long horizon in order to be able to find mature behavior. In order to increase the chance on having a session with a sufficiently long horizon, we scheduled two sessions for this treatment. Within a session, all subjects faced the same horizon. In the end, the experiment consisted of 67 periods in the first session, and of 40 periods in the second session. [3] In the treatment with fixed ending, the experiment ended after 80 periods. In both treatments, subjects were perfectly informed about the determination of the number of periods. The treatments are summarized in Table 5.2.

	Random ending		Fixed ending
	Session 1	Session 2	
Periods	67	40	80
Observations	15	12	15

Table 5.2: The experimental treatments.

At the beginning of the session, subjects were randomly matched and it was common knowledge that the matching did not change throughout the experiment. Before the first period started, for each matched pair of subjects, it had to be decided which of the two subjects could adapt its price in the first (and hence each odd) period and what price was responded to in this first period. Therefore, the experiment started with a pre-stage phase in which both subjects simultaneously had to select an initial price. Next, it was randomly decided which of the two subjects could adapt the price in the first period. In the first period, this subject responded to the price that the other had set in the pre-stage phase. The initial price selected by the subject that could adapt its price in the first period was never revealed.

Every period, the subjects that were able to adapt their prices could observe the current period price of their opponent. Prices were selected by marking one of the seven possibilities. At the end of each period, subjects received an overview

[3]See Selten et al. (1997) and Dal Bó (2005) for discussions on approximating infinitely repeated games in experiments.

of the results of that period, which consisted of both prices, own profit, and own total profit so far. In periods where subjects could not adapt their price, they only observed the result screen of that period.

The experiment was conducted in the behavioral and experimental laboratory (BeeLab) of the Faculty of Economics and Business Administration at Maastricht University in November 2007. The laboratory has a capacity of 32 students and we allowed precisely 32 students to register for each of the three sessions. The experiment was announced via email and subjects could register online using their matriculation number, which ensured that students could participate only once. When students arrived at the laboratory, they had to draw a card from a deck that determined at which computer terminal they were placed. In case an odd number of students showed up for a session, we included a blank card in the deck. Students that drew the blank card could not participate and were paid €3.- as compensation. In total, 84 undergraduate students participated in the experiment. Students not showing up or canceling on short notice led to the dispersion in the number of independent observations.

All interactions took place via computers that were connected to a network and the computer terminals were placed in such a way that subjects could neither see the screens of others nor make eye contact with them. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007). Before the start of a session, subjects read the instructions and were allowed to privately ask questions that were then privately answered. After reading the instructions, subjects had to answer control questions, which tested their understanding of the instructions.[4] One of the experimenters checked the answers, and the experiment only started after all subjects answered each question correctly. During the experiment, subjects earned ECU that were converted into Euros at a known exchange rate at the end of the experiment. We used the same exchange rates in both treatments and 20 ECU was exchanged for 1 Euro. The average payoff was €17.78 including a show-up fee of €5.-. Sessions lasted, depending on the treatment, between 45 and 70 minutes. Payment took place privately, and subjects had to leave the laboratory immediately after payment.

5.4 Results

In this section we analyze the experimental price setting behavior. All prices set over time are displayed in Section 5.6 for all observations in this experiment. For both treatments, it is found that the major share of the groups coordinated on a price of 3 (the focal point equilibrium for the infinite time horizon). Groups where the price settled down at 2 form a small minority. Finally, there is a large minority of groups where prices did not converge and displayed some kind of cyclical behavior. For the groups in the latter category, we pursue a deeper investigation into the underlying pricing dynamics. In doing so, we neglect premature decision making in both treatments and endgame effects in the fixed ending treatment.

[4] See Section 5.7 for the instructions and control questions.

Consequently, we always omit the prices set in the first fourteen periods and, if applicable, the last six periods.[5]

5.4.1 Random ending

For this treatment we have in total 27 independent observations spread over two sessions. In the first session, 15 pairs of subjects played for 67 periods. The remaining 12 pairs of subjects played in the second session which consisted of 40 periods. If subjects' behavior would be consistent with common belief in sequential rationality and Markovian behavior, we should observe one of the two equilibria of Subsection 5.2.1. That is, prices are constantly 3, or would follow the rocket–feather pattern of prices gradually falling to 0, where at some point the price rockets to 5. All decisions made throughout these experimental sessions are presented in Subsection 5.6.1.

Apart from five groups, denoted by R1.6, R2.5, R2.6, R2.7 and R2.11, subjects managed to coordinate on a common price. Groups R1.15 and R2.1 settled down at a price of 2. The remaining 20 groups settled down at a price of 3. So, a majority of the groups played according to the focal price equilibrium. Four of these 20 groups, R1.9, R1.10, R1.13, and R2.9, needed more than 15 periods for getting to this equilibrium. The other 16 groups managed to get to this equilibrium rather quickly. Next, we study the dynamics of pricing behavior for those five groups that did not settle down at a single common price in more detail.

One property of a price cycle is that a range of prices is observed. In addition, subjects should gradually undercut each other's price until the prices reach a certain bottom level. Undercutting behavior above the bottom price can be identified by the conditional probability that $p_{t+1} = p_t - 1$. When prices have reached the bottom, they are likely to stay there for some periods due to the coordination problem for bringing prices up. Consequently, having the mode at the bottom of the price range may indicate some form of cycling behavior.[6]

For the relevant five groups, Table 5.3 displays the conditional switching probabilities for the observations after period 15. For Group R1.6 these probabilities are based on 52 decisions, and for Groups R2.5, R2.6, R2.7 and R2.11 on 25 decisions each. Each entry in the table shows the probability that a current price of p_t is followed by a price of p_{t+1} in the subsequent period. The last row shows the number of observations on which the probabilities are based for each of the possible current prices.

All five observations share the common property that once a price larger or equal to 4 is set, it will immediately be undercut. In Group R2.5 a price of 3 is as

[5] Changing the categories by a couple of periods does not lead to different conclusions.

[6] In empirical studies a negative median price change is sometimes taken as an indication of cycling prices (see Lewis (2006) and Doyle et al. (2007)). In these studies prices are set on a much finer grid and therefore hardly ever the same in subsequent periods. For the present experiment this is not a suitable classification measure since the coordination stage at the bottom of the cycle can take quite long and hence the median price change will be zero even if prices do cycle. Actually, the median price change equals zero in the backwards induction equilibrium of the finite horizon setting which clearly consists of price cycles.

Group R1.6								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6			0	0	0	0	0	0
5			0	0	0	0	0	0
4			0	0	0	0.07	0	1
3			1	0	0.04	0	0.33	4
2			0	1	0.81	0	0.33	26
1			0	0	0.15	0.60	0.17	15
0			0	0	0	0.33	0.17	6

Group R2.5								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6			0	0	0	0		0
5			0	0	0	0		0
4			0	0.14	0	0		1
3			1	0.43	0.17	0.20		7
2			0	0.43	0.67	0.20		12
1			0	0	0.17	0.60		5
0			0	0	0	0		0

Group R2.6								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6			0	0	0	0	0	0
5			0	0	0	0	0	0
4			0	0	0	0.09	0	1
3			1	0	0.11	0.09	0	3
2			0	1	0.78	0	0	9
1			0	0	0.11	0.82	1	11
0			0	0	0	0	0	1

Group R2.7								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6	0		0	0	0	0.11		1
5	0		0	0	0	0		0
4	1		0	0	0	0.44		5
3	0		1	0	0	0.11		5
2	0		0	1	0	0		5
1	0		0	0	1	0.33		9
0	0		0	0	0	0		0

Group R2.11								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6		0		0	0	0		0
5		0		0	0	0.08		1
4		0		0	0	0		0
3		1		0	0	0.08		1
2		0		1	0.60	0.23		10
1		0		0	0.40	0.62		13
0		0		0	0	0		0

Table 5.3: Probability that p_{t+1} is set conditional on p_t after period 14.

likely being undercut as being matched. In the other four out of five observations, also a price of 3 is immediately undercut. Apart from Group R2.7, prices of 2 and 1 are matched with high probability. These groups therefore do not display active cyclical behavior as the price often tends to stagnate at 1 or 2. In these observations prices larger or equal to 3 are not frequently observed. In Group R2.7, a price of 2 is undercut with certainty. One period later, when the price has reached the bottom of the price cycle at price 1, in this group, the price is matched with probability 0.33 and brought up with probability 0.67 in order to continue the process of gradual undercutting.

The only observation, out of the 27 in total, that comprises the conditions of a price cycle is Group R2.7. All prices higher than 1 are followed by a price that is one below it in the subsequent period. After the price has dropped to 1, it either remains there or it is (substantially) increased. Even though prices do clearly cycle, the behavior does not completely match the price cycle equilibrium of Maskin and Tirole (1988). Namely, in this equilibrium, prices are brought up to 5 and coordination takes place at a price of 0. A reason for the coordination to take place at price 1 may be that undercutting to 0 would induce zero profit for at least two periods, whereas matching at 1 results in a profit of at least 2.5.

In the other four observations that do not coordinate at a common price, no real cycling behavior is observed over the mature periods. In Groups R1.6, R2.5, and R2.6 undercutting mainly takes place before period 28. Moreover, in these groups and in Group R2.11 prices fluctuate instead of cycle between 0 and 3 or between 1 and 3.

To summarize, for 20 of the 27 observations, the subjects' behavior is consistent with the focal price equilibrium, possibly after some learning. Cyclical behavior appears to persist in only one observation: Group R2.7. The remaining six observations are difficult to classify as being in line with either the focal price equilibrium or the equilibrium involving price cycles. Among these there are two observations where the price settled at 2. Although this is not an equilibrium, there is some logic behind prices stabilizing at 2. At that price the immediate benefit from undercutting the other firm is much lower that at a price of 3, while the resulting per period profit is only a halve lower. Moreover, it would take (at least) eight periods, to compensate for the immediate loss of inducing a switch to the focal price.

5.4.2 Fixed ending

For this treatment we have 15 independent observations that are gathered in one single session. Within this session, subjects interacted in pairs for 80 periods. The only behavior that is consistent with common belief of rationality results in the cyclical price pattern of Figure 5.4. So, unlike for the treatment with random ending, there is no (subgame-perfect) equilibrium with prices settling down at 3. Nevertheless, in experiments with long time horizons behavior is often observed to be more in line with an infinite than with a finite horizon until shortly before

Group F.2								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6	0		0	0	0	0.14		4
5	0		0	0	0	0		0
4	0		0	0	0	0.07		2
3	1		1	0	0.18	0.04		9
2	0		0	1	0.35	0.04		17
1	0		0	0	0.47	0.71		28
0	0		0	0	0	0		0

Group F.7								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6				0	0	0		0
5				0	0	0		0
4				0	0	0		0
3				0.73	0	0.23		26
2				0.23	0.25	0		8
1				0.04	0.75	0.77		26
0				0	0	0		0

Group F.8								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6	0	0	0	0	0	0.17	0	1
5	0	0	0	0	0.20	0.08	0	3
4	0	0	0	0.03	0	0	0	1
3	1	1	1	0.78	0	0	1	32
2	0	0	0	0.19	0.40	0	0	10
1	0	0	0	0	0.40	0.67	0	12
0	0	0	0	0	0	0.08	0	1

Group F.9								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6	0	0	0	0	0	0.19		5
5	0.60	0	0	0	0	0.08		6
4	0	1	0	0	0	0		5
3	0.40	0	1	0.22	0	0		9
2	0	0	0	0.78	0	0.08		9
1	0	0	0	0	1	0.65		26
0	0	0	0	0	0	0		0

Group F.10								
	p_t							
p_{t+1}	6	5	4	3	2	1	0	#
6	0	0	0	0	0	0.06	0.17	2
5	1	0.13	0	0	0.10	0	0.67	8
4	0	0.88	0	0	0	0	0.17	8
3	0	0	1	0	0	0.06	0	9
2	0	0	0	1	0.10	0	0	10
1	0	0	0	0	0.80	0.53	0	17
0	0	0	0	0	0	0.35	0	6

Table 5.4: Probability that p_{t+1} is set conditional on p_t after period 14 and before period 75.

the end.[7] Hence, in line with the previous subsection, prices constant 3 are not unlikely to be observed. All decisions made throughout this session are presented in Subsection 5.6.2.

In ten out of the 15 groups, subjects coordinated on a common price. One of these ten groups (Group F.13) settled down at a price of 2, the other nine at a price of 3. From the latter nine, seven of the groups managed to coordinate on price within 10 periods. The other two groups needed more than 25 periods. In all observations with stable prices from some point on, an endgame effect with length of at most three periods is observed. To be precise, in two observations there is an endgame effect of length one, in five observations of length two, and in three observations of length three. This signals that subjects seemed to apply the procedure of backwards induction only when the end of the session was very near.

The other five groups (Groups F.2, F.7, F.8, F.9 and F.10) seem to behave more in line with the notion of common belief in sequential rationality. In order to see how close these groups matched equilibrium behavior, we study the dynamics of their pricing behavior in more detail. Table 5.4 displays for each group the conditional switching probabilities for the observations after period 15 and before period 74. Each entry in the table shows the probability that a current price of p_t is followed by a price of p_{t+1} in the subsequent period. The last row shows the number of observations on which the probabilities are based for each of the possible current prices.

In Group F.7 the price never got above 3. In the other observations the price quickly declined towards 3 when it was above it. A price of 3 was undercut with certainty in Groups F.2 and F.10, and in Group F.9 with a probability of 0.78. In Groups F.7 and F.8 a price of 3 was matched with high probability, but this price stability is most prominent in the periods between 15 and 45. In all groups a price of 2 was likely to be followed by a price of 1, although in quite some cases the price of 2 was matched. Once the price was 1, with high probability the price was either matched or (substantially) brought up. In Group F.10 the price frequently further decreased to 0, before being brought up.

The switching probabilities indicate that the price dynamics in Groups F.2, F.9 and F.10 seem to closely follow the subgame-perfect equilibrium prediction. Also the mode price being equal to 1, is in line with the equilibrium prediction. In Groups F.7 and F.8, another kind of cycle is observed. Prices are rather stable at 3, with only a small probability on an undercut. At some point, an undercut takes place and is likely to be followed by another undercut. Next, the price stabilizes at 1, until it is brought up in order to stabilize once more at 3. This pricing pattern has the structure of the classical price war of alternations between periods of gentle and periods of severe price competition.

To summarize, nine of the 15 groups behave according to the focal price equilibrium of the infinite horizon counterpart. Of the remaining five observations, three observations clearly display price cycles. The other two show cyclical behavior that is comparable to that of the classical price war.

[7] See for instance Selten and Stoecker (1986), Engle-Warnick and Slonim (2004), and Normann and Wallace (2006).

5.5 Discussion

We experimentally analyzed the alternating price setting game of Maskin and Tirole (1988). When the time horizon is infinite, two Markov perfect equilibria coexist; one consisting of a focal price and one consisting of Edgeworth cycles. We experimentally find that the focal price equilibrium emerges in 20 out of 27 observations. Only in a single observation subjects' behavior displays price cycles.

We also analyze the alternating move price setting game in case the number of periods is fixed. We find that the subgame-perfect equilibrium consists of prices that cycle. Experimentally, we still observe the focal price in nine out of 15 observations, even though this is not a subgame-perfect equilibrium. Of the remaining five observations, three clearly display price cycles. The other two observations show cyclical behavior that is comparable to that of the classical price war. Consequently, there is less cooperative behavior in the treatment with fixed ending than in the treatment with random ending, although it is impossible to validate it by means of statistical tests.

It has been difficult to find undercutting behavior, and hence price cycles, in the laboratory so far. Especially for settings with few players there had been no experimental observations of them. In our experiments we were able to find cycling prices in a dynamic setting with only two players, although in many instances the players seemed to collude. From this we conclude that the alternating move structure has the potential to enhance undercutting behavior.

Even though it is quite a strong result to find cycling prices for the present setting, future research is needed to see whether prices cycle more often if some of the settings of the alternating move game are changed. For instance, the number of players and hence the number of periods for which prices are committed could be increased. Furthermore, the addition of exogenous demand shocks would make undercutting in periods of high demand more profitable and hence could lead to more cycling behavior. Finally, the structure of the profit table could be altered to make undercutting the focal price more profitable, although the coexistence of the two equilibria needs to be retained.

5.6 Appendix A: Figures

5.6.1 Treatments with random ending

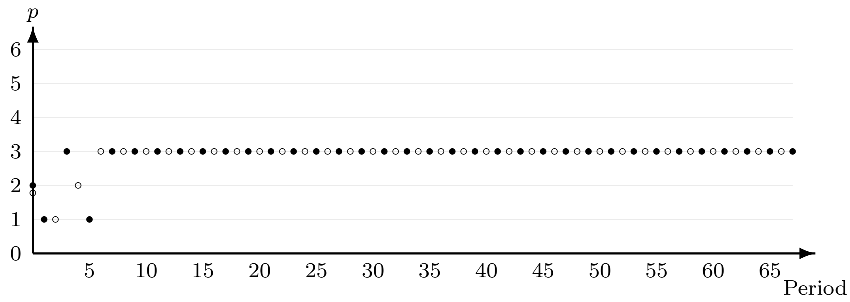

Figure 5.5: Group R1.1

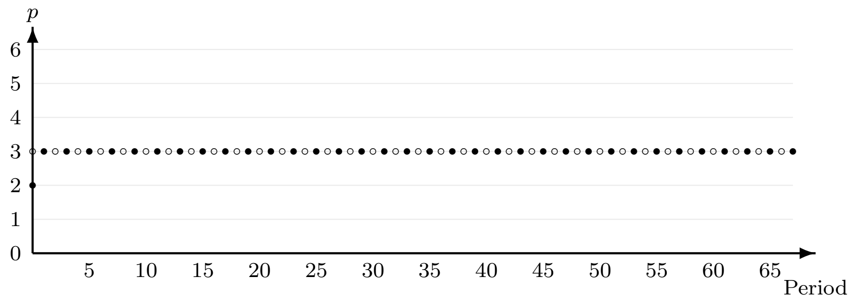

Figure 5.6: Group R1.2

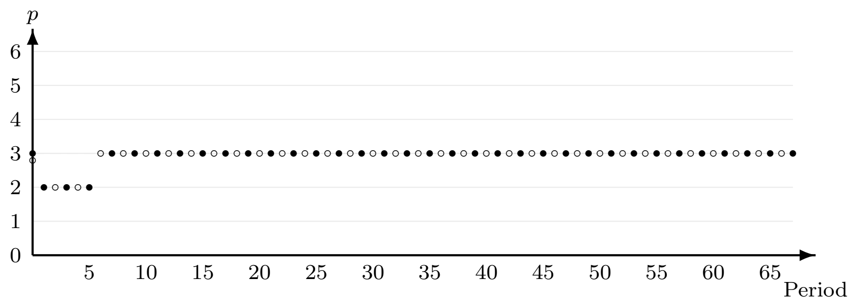

Figure 5.7: Group R1.3

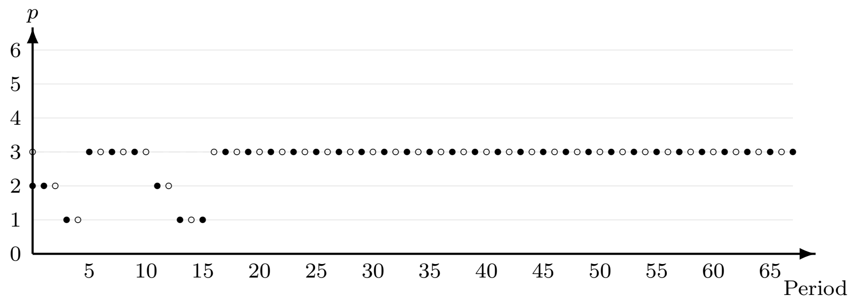

Figure 5.8: Group R1.4

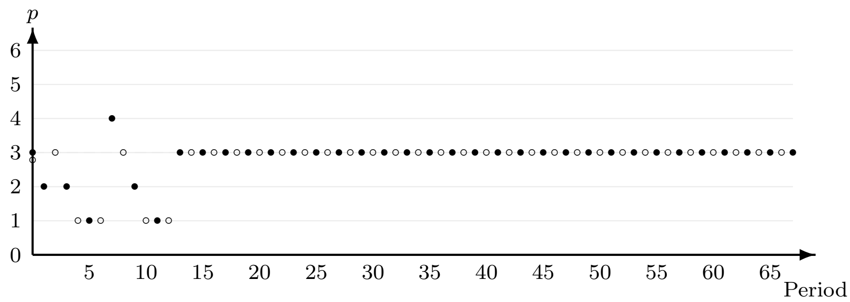

Figure 5.9: Group R1.5

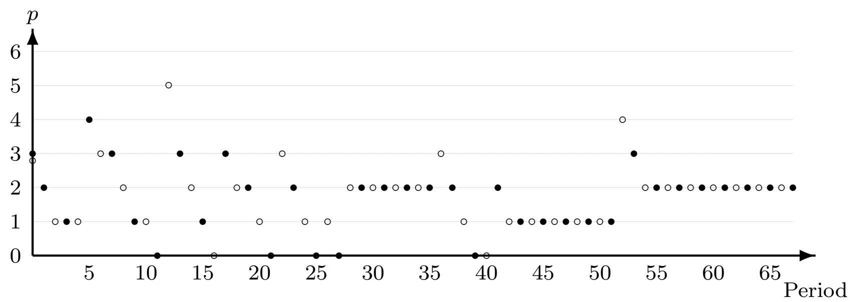

Figure 5.10: Group R1.6

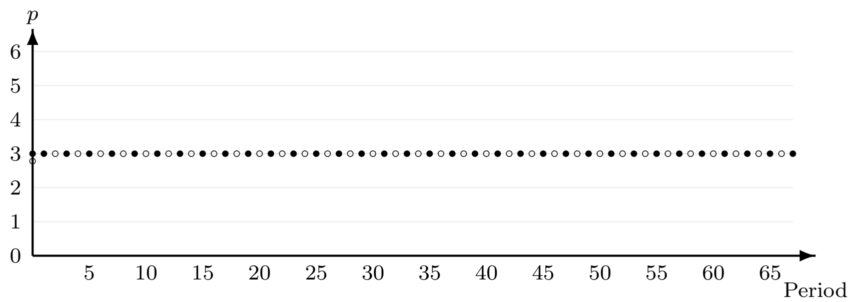

Figure 5.11: Group R1.7

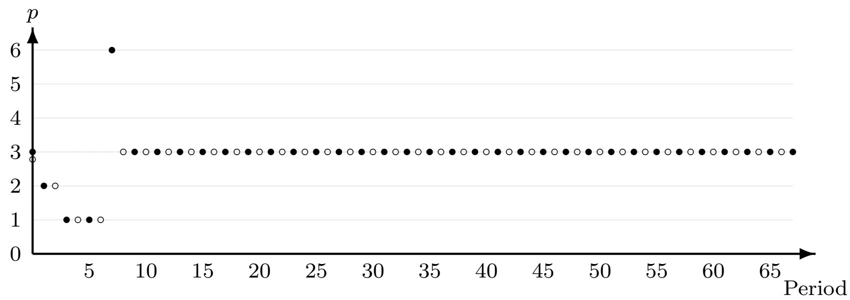

Figure 5.12: Group R1.8

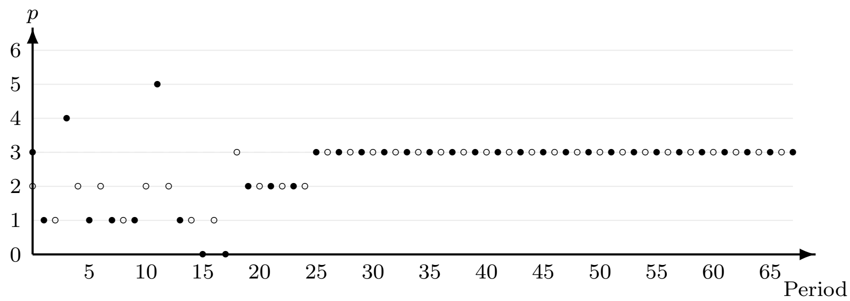

Figure 5.13: Group R1.9

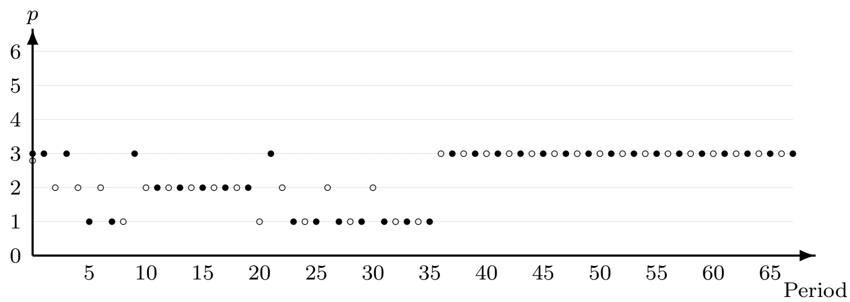

Figure 5.14: Group R1.10

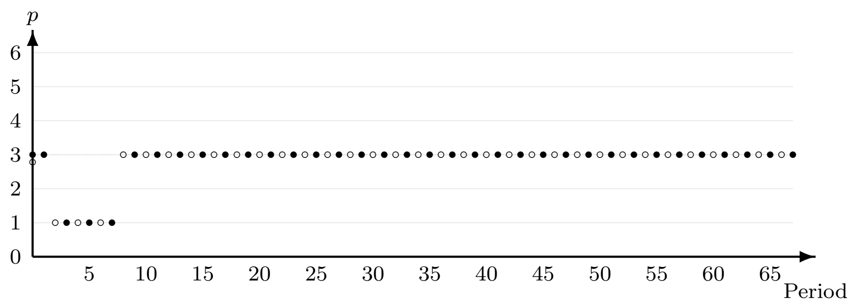

Figure 5.15: Group R1.11

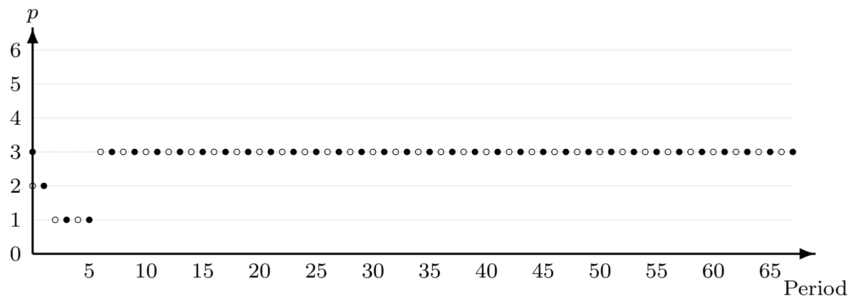

Figure 5.16: Group R1.12

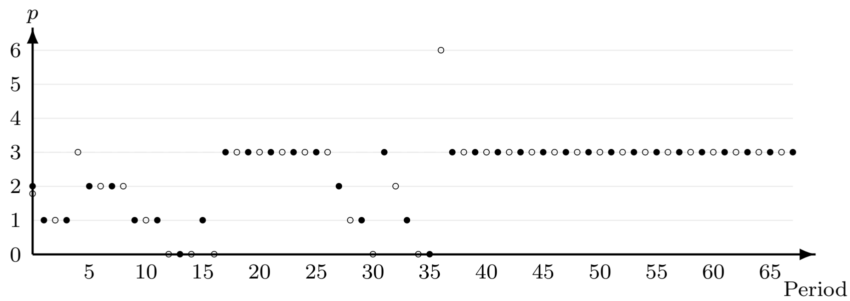

Figure 5.17: Group R1.13

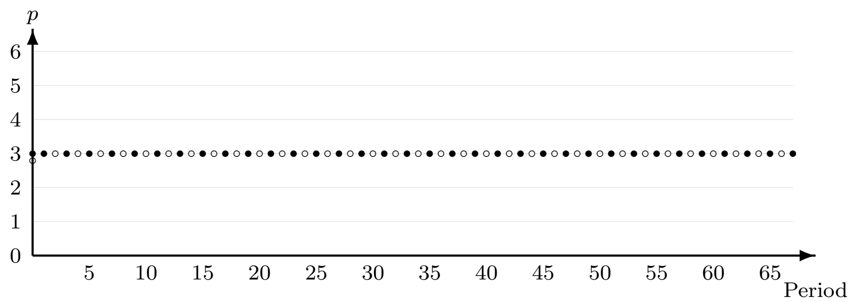

Figure 5.18: Group R1.14

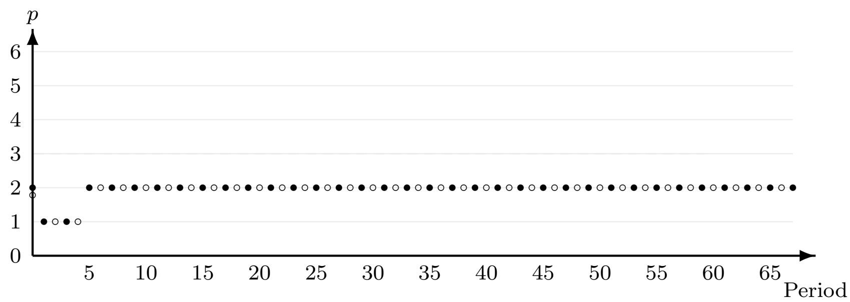

Figure 5.19: Group R1.15

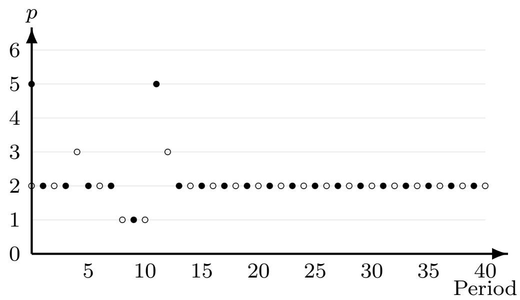

Figure 5.20: Group R2.1

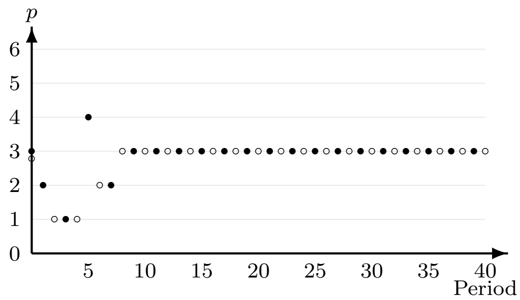

Figure 5.21: Group R2.2

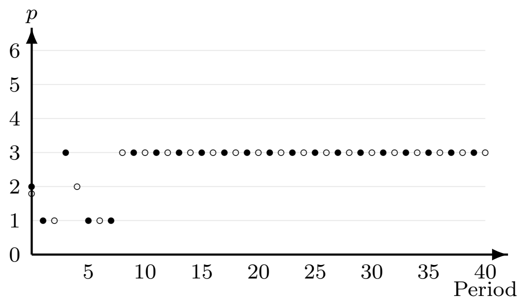

Figure 5.22: Group R2.3

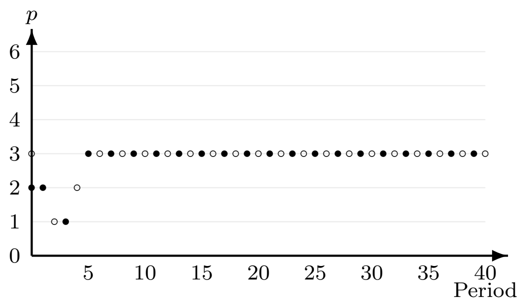

Figure 5.23: Group R2.4

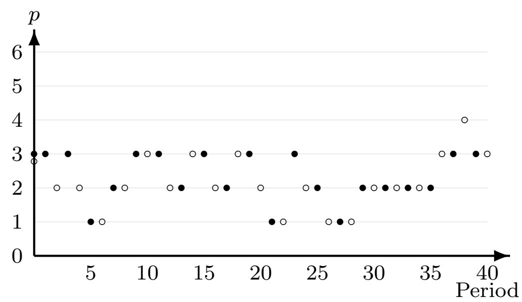

Figure 5.24: Group R2.5

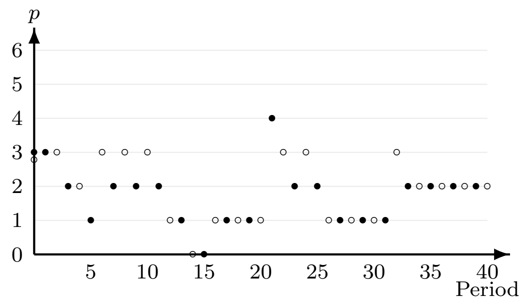

Figure 5.25: Group R2.6

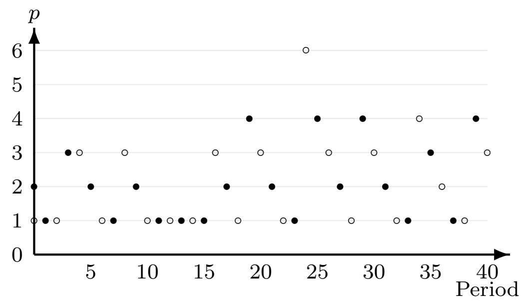

Figure 5.26: Group R2.7

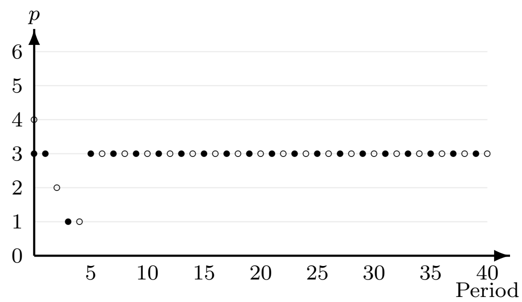

Figure 5.27: Group R2.8

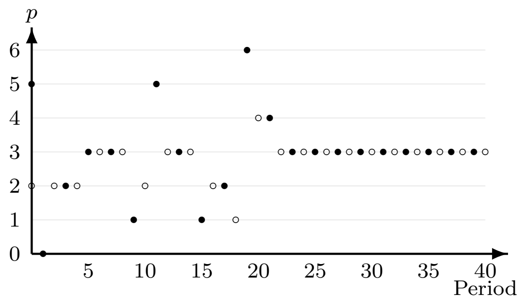

Figure 5.28: Group R2.9

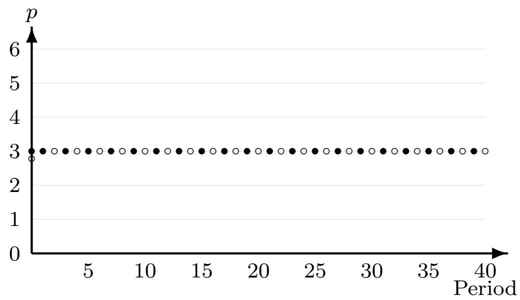

Figure 5.29: Group R2.10

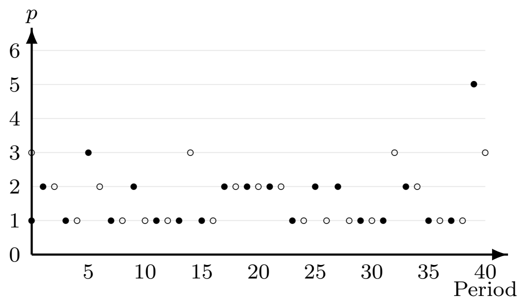

Figure 5.30: Group R2.11

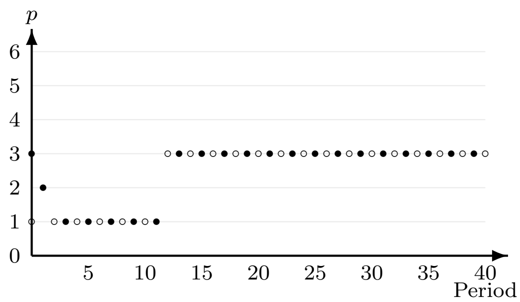

Figure 5.31: Group R2.12

5.6.2 Treatment with fixed ending

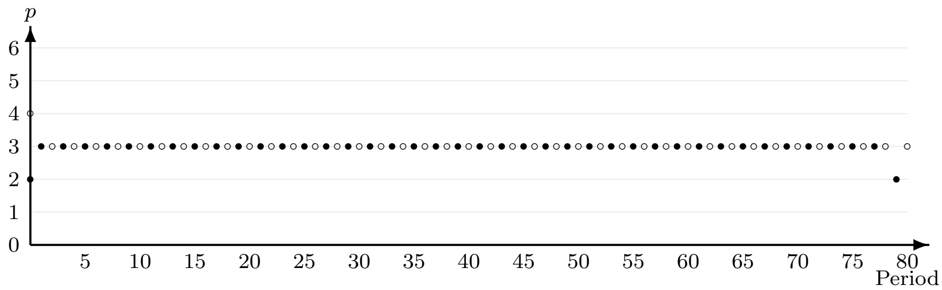

Figure 5.32: Group F.1

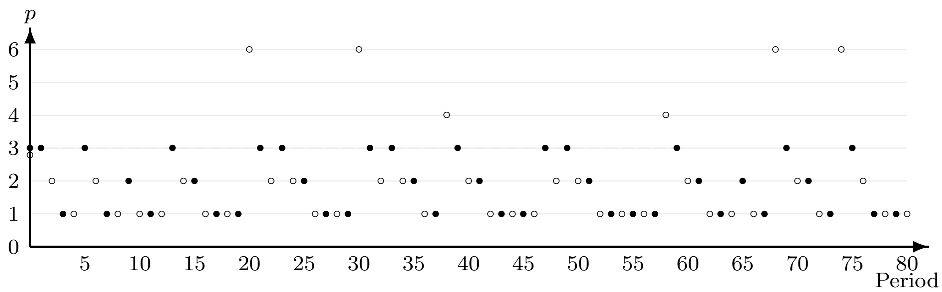

Figure 5.33: Group F.2

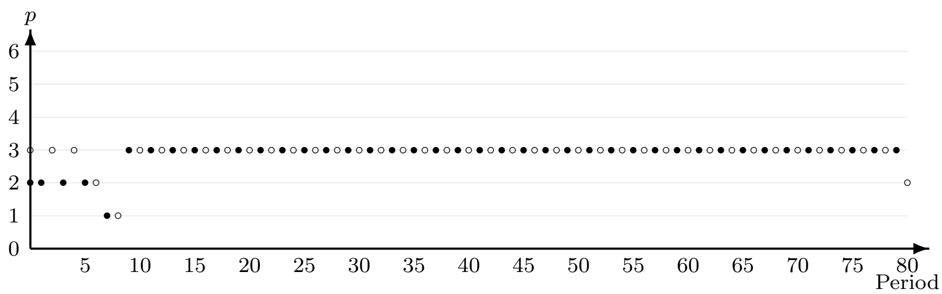

Figure 5.34: Group F.3

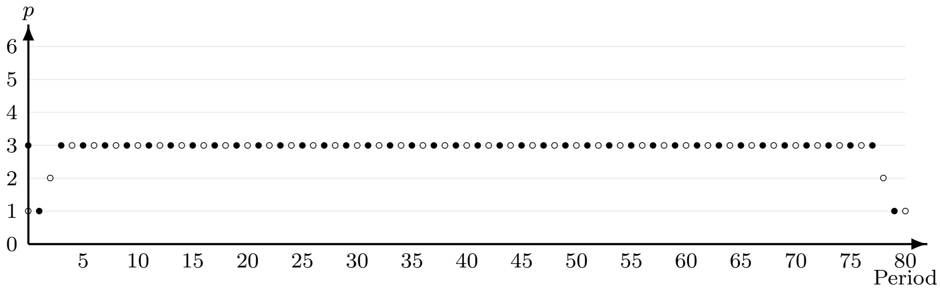

Figure 5.35: Group F.4

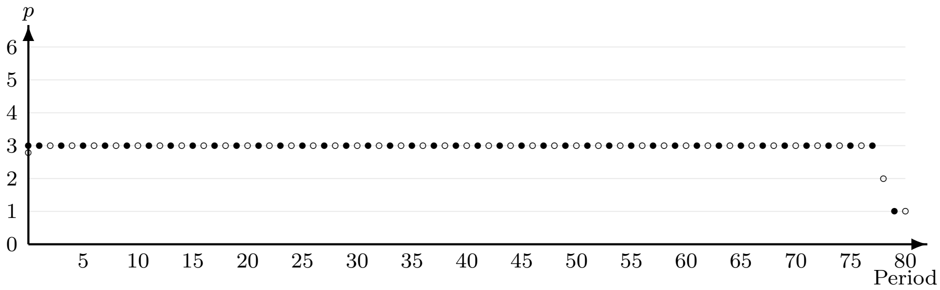

Figure 5.36: Group F.5

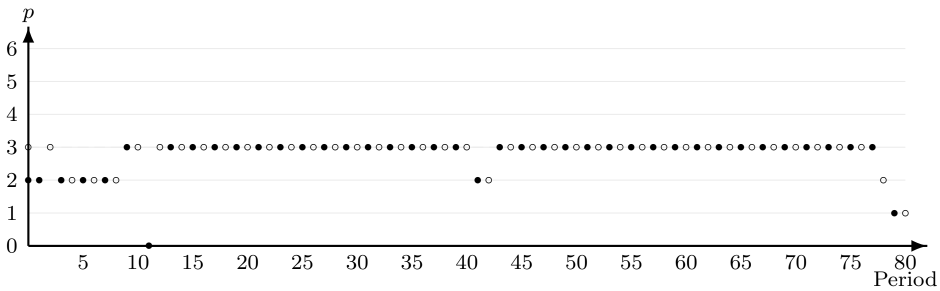

Figure 5.37: Group F.6

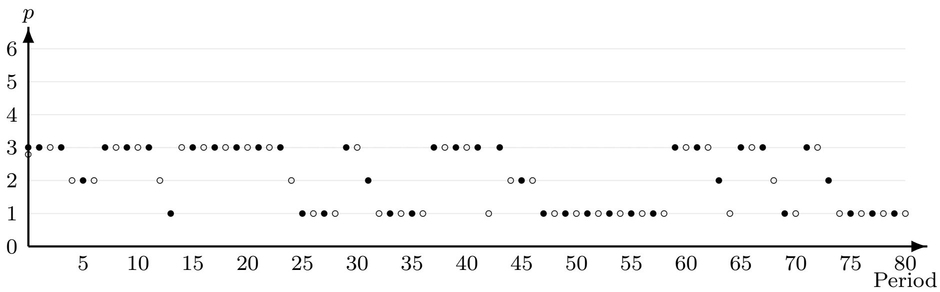

Figure 5.38: Group F.7

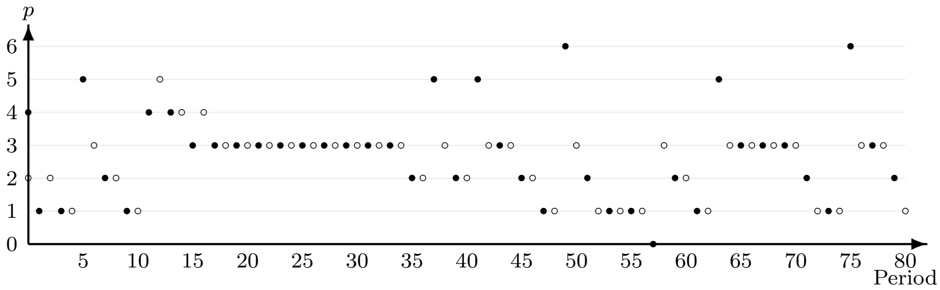

Figure 5.39: Group F.8

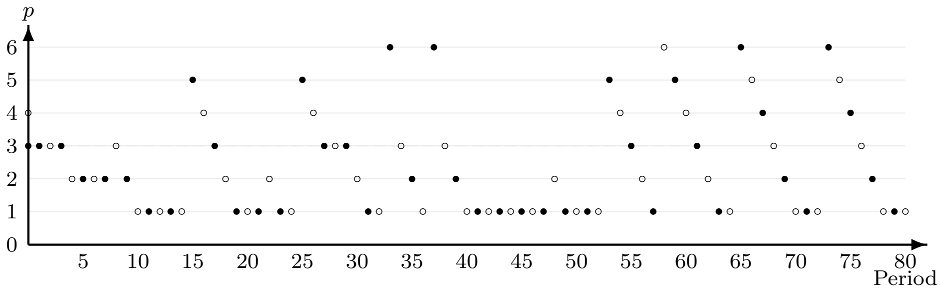

Figure 5.40: Group F.9

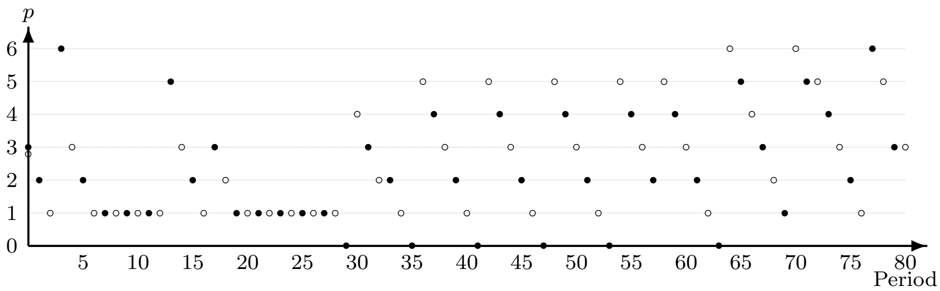

Figure 5.41: Group F.10

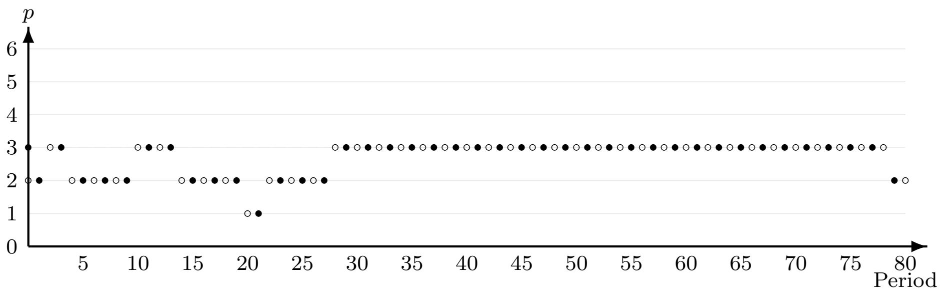

Figure 5.42: Group F.11

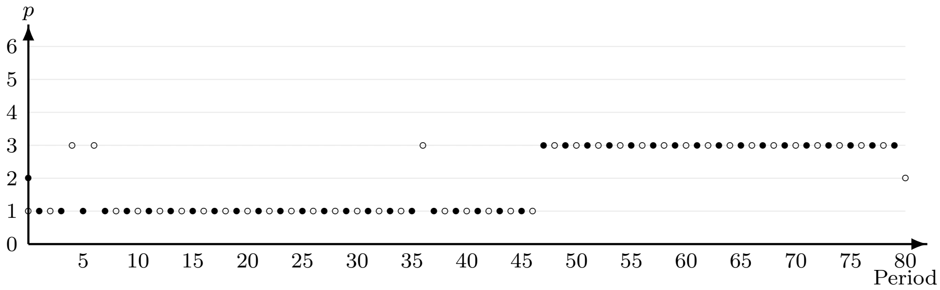

Figure 5.43: Group F.12

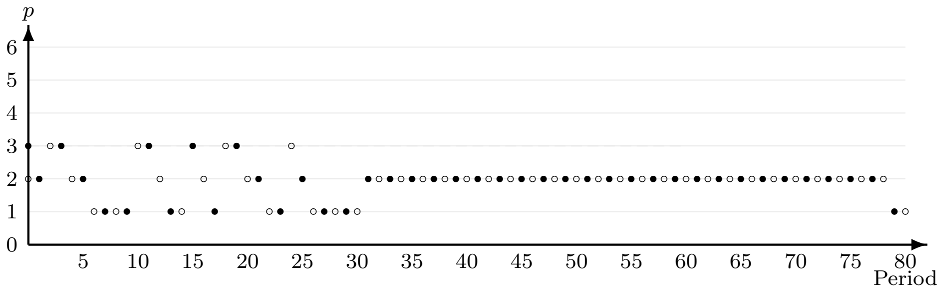

Figure 5.44: Group F.13

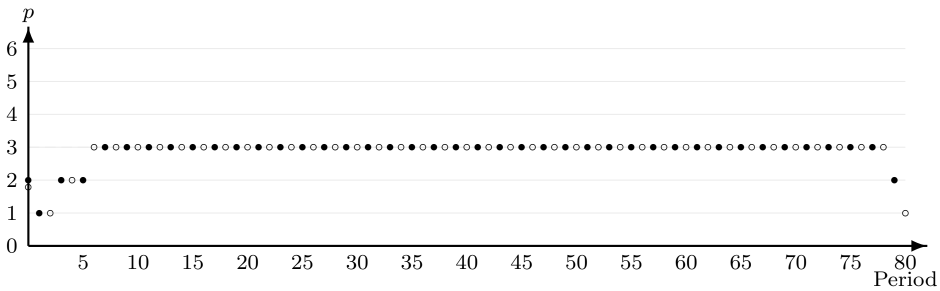

Figure 5.45: Group F.14

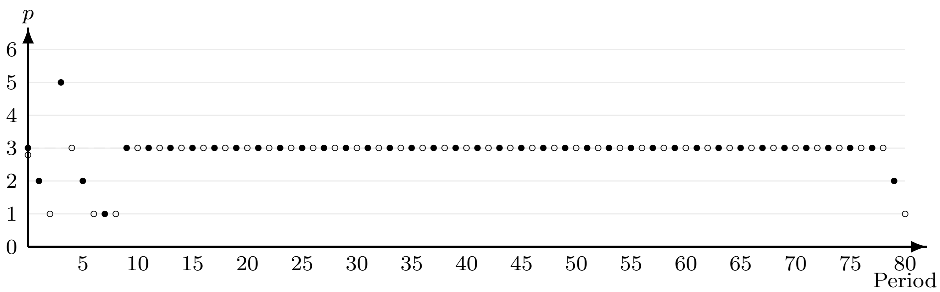

Figure 5.46: Group F.15

5.7 Appendix B: Experimental instructions

The instructions that students received before the experiment started are given here. In case the instruction differed for the two treatments, the text for the treatment with random ending is reported between [].

Dear participant,

welcome to this experiment. You will be compensated according to your performance. In order to ensure that the experiment takes place in an optimal setting, we want to ask you to follow the general rules during the whole experiment:

- Read these instructions carefully! It is important that you understand the rules of this experiment. These instructions are identical for all subjects that participate together with you. If something is not explained well, please raise your hand. Do not ask the question out loud, but wait until one of the experimenters approaches you to answer the question in private.

- Switch off your mobile phone!

- Do not communicate with your fellow students! Even though the experiment may get exiting at times, it is very important that you remain silent through the proceedings.

- Focus on your *own* computer screen and not on other participants!

- There is paper and a pen on your table which you can use during the experiment.

- After the experiment, please remain seated until you are paid off.

- If you do not obey the rules, the data becomes useless for us. Therefore we will have to exclude you from this experiment and you will not receive any compensation.

Your decisions and earnings in this experiment will remain anonymous.

General set-up

In this experiment all of you are sellers of a fictitious commodity. You can earn ECU (Experimental Currency Units) which will be exchanged into Euros at the end of the experiment. The exchange rate will be given in the instructions below.

Before the experiment starts, you will be randomly divided into groups of two sellers. You will not know the identity of the seller you are matched with. The groups remain unchanged throughout the whole experiment.

Procedures

This experiment consists of 80 [multiple] periods. In each period, only one of the two sellers can adapt its price. The price of the seller that cannot adapt its price remains equal to its price in the previous period. The seller that can adapt its price switches after each period. Consequently, one seller can adapt its price only in the *odd* periods, whereas the other seller can adapt its price only in the *even* periods.

Before setting your price for the current and the subsequent period, you observe the price of the other seller for the current period. Remember that the other seller can adapt its price in the next period, after having observed your price. This procedure of alternating price-adaptation continues until the experiment ends.

Possible prices are the integers between (and including) 0 and 6. To decide on a price you can select a price on your screen and then click on *OK* (see the figure below).

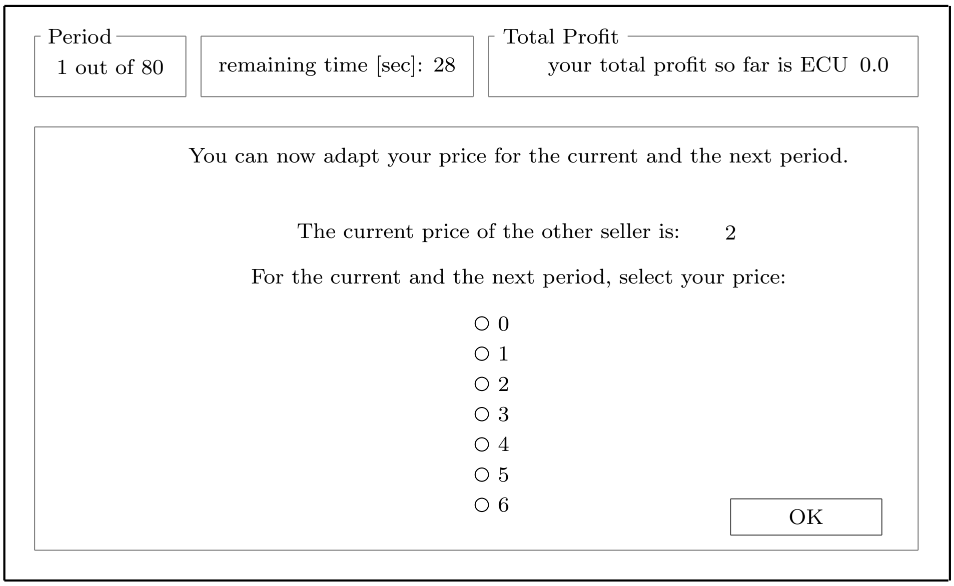

Figure 1: Screenshot of price adaptation screen.

After the seller that could adapt its price has made its decision, the profits for that period are determined. Only the seller that has the lowest price sells output. The amount of output depends on its price. There are no production costs. Hence, the profit for the seller that has the lowest price is equal to the output multiplied by its price. Table 1 shows the output and profit at each possible price for the seller that has the lowest price. The other seller has a profit equal to zero in this period.

Price	0	1	2	3	4	5	6
Output	6	5	4	3	2	1	0
Profit	0	5	8	9	8	5	0

Table 1: The output and profit for the seller with the lowest price.

In case both sellers have the same price, output is split equally. The profit for each seller is then half of the profit reported in Table 1 at that price.

At the end of each period, both sellers receive an overview of the results of that period. You can observe your price, the price of the other seller, your profit, and your total profit so far. In a period in which you cannot adapt your price, you only observe the result screen of that period.

This procedure continues until the end of the experiment.

[Number of periods

The number of periods in this experiment is unknown until the experiment has ended. After each period, the experiment ends with a probability of 2 percent. This means that with a probability of 98 percent the experiment continues with the next period. The decision to continue or not, is made by a computer. Notice that although you are matched with one other seller, the number of periods played will be the same for all subjects in the current session.]

The first period

Before the procedures above start, it has to be decided who of the sellers is able to adapt its price in the first (and all odd) periods and to which price this seller responds. Therefore, the first period contains an initial period in which both sellers are asked to set an initial price. Next the computer randomly decides which of the two sellers can adapt its price in the odd periods. In the first period this seller will respond to the other's initial price.

Closing

After the last period of the experiment, we would like you to complete a short questionnaire that will appear on your screen. Payments will be made by the experimenters afterwards.

ECU are transformed into Euros according to the following conversion rate: 20 ECU = 1 Euro. In addition to your earnings during the course of the experiment, you will also receive a show up fee of 5 Euro.

Before we start with the experiment we want you to answer the questionnaire on the next page. One of the experimenters will go around and check the answers and discuss any problems.

Questionnaire

Please answer the following questions. When you are finished, raise your hand. One of the experimenters will come to you and check whether everything is correct.

1. How many sellers are in your group (including yourself)?

   —

2. Suppose that you can adapt your price in period 4. What does this imply for your price in period 5?

   __ My price in period 5 will be equal to my price in period 4.

   __ My price in period 5 can be any integer between (and including) 0 and 6.

3. Suppose that you can adapt your price in period 4. What do you know about the price of the other seller in period 5?

   __ The other's price in period 5 will be equal to its price in period 4.

   __ The other's price in period 5 can be any integer between (and including) 0 and 6.

4. Suppose that you can adapt your price in period 4. What does this imply for your price in period 6?

   __ My price in period 6 will be equal to my price in period 4.

   __ My price in period 6 will be equal to my price in period 5.

   __ My price in period 6 can be any integer between (and including) 0 and 6.

5. Suppose you can adapt your price in the current period and the price of the other seller is 5.

   (a) What will be your profit in this period when you set a price of 6?

   —

   (b) What will be your profit in this period when you set a price of 5?

   —

   (c) What will be your profit in this period when you set a price of 4?

   —

[6. Suppose you are currently in period 17, what is the probability that the experiment continues with period 18?

   —]

Chapter 6

Price competition with exogenous demand shocks

In this chapter[1] we consider an infinite alternating move Hotelling model in which consumers are uniformly distributed over the market. The analytic solution for this setting is given and the unique linear stationary subgame-perfect equilibrium is computed for different values of the discount factor. The base model is then extended by the introduction of exogenous demand shocks, which makes it impossible to find an analytical solution using the conventional analysis. For this extended model, the bounds between which long-run prices fluctuate are determined via numerical computations for different values of the shock probability. It is found that the presence of exogenous demand shocks leads to countercyclical pricing behavior.

6.1 Introduction

When a competitive situation is modeled in a static setting, firms are found to behave very competitively. For instance, in an oligopoly with homogeneous products the equilibrium prices equal marginal cost and firms do not make any profits (Bertrand, 1883). However, in a dynamic setting firms might act mutually nonaggressive in order to ensure higher profits to all of them (Chamberlain, 1933). Such behavior cannot be observed in a static model due to a lack of opportunities to reward nonaggressive behavior and to punish aggressive behavior.

For a homogeneous product market, Maskin and Tirole (1988) show that in an alternating move setting, pricing behavior becomes less competitive when the discount factor increases. The reason for this is that an increase in the discount factor makes it more worthwhile for a firm to sacrifice current demand by raising its price today in the expectation of future profit when the other firm follows suit. Eaton and Engers (1990) consider a similar setting for a linear city where half of

[1]This chapter is based on Leufkens and Peeters (2008a).

all consumers are located at one of the endpoints and the other half at the other endpoint. For this model of a differentiated product market two kind of equilibria exist: a 'disciplined' one that is enforced by threats to undercut and that arises when the products are close substitutes, and a 'spontaneous' one in which such threats are not needed and that arises when the products are more differentiated.

This chapter presents a model with two firms located at the end points of a linear city. Consumers are uniformly distributed over the market and the firms set prices in alternating periods. We determine the unique linear stationary subgame-perfect equilibrium (SSPE) which is found to be dynamically stable as the dynamic reactions converge to a steady state. The SSPE is also found numerically by applying backwards induction after truncation of the horizon. Namely, Maskin and Tirole (1987) show that the finite horizon equilibrium strategies converge to the unique linear SSPE of the infinite horizon model if the truncation of the horizon lengthens. It is found that the steady state price increases in the discount factor and hence firms become less aggressive the more patient they are.

Then, the model is extended by the introduction of exogenous demand shocks. There are two levels of demand and given the current level there is a probability that the demand changes in the next period. The introduction of uncertainty into the model has fundamental consequences as the analytical solution can not longer be found via the conventional analysis. Therefore, the SSPE is computed numerically by applying backward induction after truncating the horizon. We determine the margin in which long-run prices fluctuate and the dependency of this margin on the shock probability and the discount factor. It is found that in equilibrium, pricing behavior is more competitive when the demand is high and that in the long-run prices are higher in case of low demand. Our results are therefore in line with the often observed countercyclical pricing (see for instance Rotemberger and Saloner (1986)). However, in contrast to most models for which countercyclical prices are found, our results do not depend on (tacit) collusion.

This chapter is organized as follows. In the next section the static model and its solution are presented. Section 6.3 contains the infinite alternating move variant of this model and presents the dynamic reaction functions that constitute a stationary subgame-perfect equilibrium. The reaction functions are computed numerically and the movement of prices in the long-run are discussed. In Section 6.4 the model is extended by including exogenous demand shocks. The chapter ends with a discussion in Section 6.5.

6.2 The model

The basic model of horizontal differentiation was introduced by Hotelling (1929). Let there be two firms, each located at one of the endpoints of a unit interval on which consumers are uniformly distributed. We assume that firm A is located at $x = 0$ and firm B is located at $x = 1$. Furthermore, consumers have unit demands and the basic utility consumers achieve by consumption of the product, β, is sufficiently high and equal for each consumer, which guarantees that all consumers acquire a product. Moreover, consumers are assumed to incur linear transportation costs of τ per unit of distance from the chosen firm. Finally, we assume that the firms' production costs are equal to zero.

The situation described above is a two-stage model, where in the first stage both firms determine prices and in the second stage the consumers make their decisions. Consumers select from which firm to buy on the basis of their location on the unit interval and the firms' prices, and thereby determine the sales of the two firms. The model can be solved by applying backwards induction.

In the second stage, given prices p^A and p^B, the consumer located at x has a utility of

$$U^x = \begin{cases} \beta - p^A - \tau \cdot x & \text{if the consumer buys from firm } A, \\ \beta - p^B - \tau \cdot (1-x) & \text{if the consumer buys from firm } B. \end{cases}$$

The indifferent consumer is then located at

$$\hat{x} = \tfrac{1}{2} + \tfrac{p^B - p^A}{2\tau}.$$

Then, given the prices p^A and p^B, firm A attracts $q^A = \hat{x}$ consumers and firm B the remaining $q^B = 1 - \hat{x}$ consumers.

In the first stage, knowing the reaction of the potential consumers, both firms simultaneously decide on prices. The profits of the firms are given by

$$\pi^A(p^A, p^B) = \tfrac{p^B - p^A + \tau}{2\tau}\, \eta\, p^A \qquad \text{and} \qquad \pi^B(p^A, p^B) = \tfrac{p^A - p^B + \tau}{2\tau}\, \eta\, p^B,$$

where η represents the total mass of consumers and hence the fixed total demand.

Given p^B, firm A's profit is maximized by setting $p^A = \frac{p^B + \tau}{2}$. Given p^A, firm B's profit is maximized by setting $p^B = \frac{p^A + \tau}{2}$. In equilibrium both firms must be optimally responding to each other and therefore both equations have to be satisfied. This leads to the equilibrium prices, quantities and profits given by

$$p^{A*} = p^{B*} = \tau, \quad q^{A*} = q^{B*} = \tfrac{1}{2} \quad \text{and} \quad \pi^{A*} = \pi^{B*} = \tfrac{1}{2}\tau.$$

Although this solution could give a good prediction for market prices when the firms set their prices only once, in reality firms interact and set prices over time, and are able to react on each other's prices.

6.3 Equilibrium strategies and price dynamics

In order to capture the dynamic feature of reality in combination with short-run price commitments, we assume that the firms move alternately over an infinite horizon. The alternating structure ensures that firms react on the price set by the opponent while knowing that this price remains constant in the subsequent period.[2] Assume that firm A adapts its price in odd periods and firm B adapts

[2]Maskin and Tirole (1987 and 1988) consider endogenous timing of the firms' moves and show that in many cases the equilibrium behavior is exactly as in the imposed alternating timing framework.

its price in even periods. After having adapted the price, a firm is committed to this price for two periods. The prices set are perfectly observable for both firms. Consumers purchase the product every period anew and do not incur any costs of switching. Given prices p^A and p^B in a period, the temporal profits are determined according to the Hotelling model of the previous section.

At any period in time, given the opponent's present price, firms maximize the present value of the complete stream of future profits by application of the discount factor $\delta \in (0, 1)$. Consequently, in each period in which a firm can adapt its price, it does not only take into account the current profits but also anticipates on the possible reaction of the opponent in the next period and its own reaction on that and so on. In comparison to Eaton and Engers (1990) our model assumes more heterogeneity among consumers such that the concept of undercutting, which plays an important role in their study, has no longer a sensible meaning.

A strategy for a firm is a specification of its price setting behavior depending on the opponent's current price, time, and the full history of events. A pair of strategies constitutes an equilibrium if no firm is able to improve the present value of the stream of profits by a unilateral (one-shot) deviation. A strategy is called stationary if it is independent of time and history and consequently only depends on the current state. In this model the current state is specified by the price the opponent is committed to. Hence, in a stationary strategy firm A specifies its response-price $p^A(p^B)$ for each possible price p^B. Therefore, stationary strategies can be seen as dynamic reaction functions. Once a pair of stationary strategies $(p^A(\cdot), p^B(\cdot))$ constitutes an equilibrium, this equilibrium is called a stationary subgame-perfect equilibrium (SSPE).[3]

In the remainder of this chapter, the analysis is restricted to the use of stationary strategies.[4] Several technical and pragmatic motivations for this restriction can be found in Maskin and Tirole (2001). An experimental motivation for this restriction can be found in McKelvey and Palfrey (1995).

Theorem 6.1 *For any discount factor δ there exists a unique linear stationary subgame-perfect equilibrium.*

Proof See Theorem 6.7 and its proof in Section 6.6. □

The SSPE is found analytically by applying the techniques used in Maskin and Tirole (1987). The stationary equilibrium can be computed numerically by gradual truncation of the horizon and applying backwards induction to the truncated games. In all maximization problems during the backwards induction procedure, the objective function is quadratic in the decision variable. Then, the finite horizon equilibrium strategies converge uniformly to the SSPE of the infinite horizon

[3]Following Maskin and Tirole (2001) we do not name this equilibrium a Markov perfect equilibrium. The reason is that we impose the payoff relevant state exogenously and therefore cannot guarantee that it is the coarsest partition of histories.

[4]As in Maskin and Tirole (1987), we only consider linear dynamic reaction functions. That is, they are affine functions of the opponent's current price. Consequently, we cannot rule out nonlinear equilibria.

model when the horizon lengthens (see Maskin and Tirole (1987) and Lau (2002)).[5] Moreover, the backwards induction procedure will lead to a linear SSPE. For different values of the discount factor δ, Table 6.1 displays the symmetric unique SSPE strategies of the two firms which are found numerically and appear to be symmetric. It can be verified that these strategies coincide with those from the analytical solution in Section 6.6.

δ	$p^{i*}(p^j)$	δ	$p^{i*}(p^j)$
0.00	$0.500000 \cdot p^j + 0.500000 \cdot \tau$	0.50	$0.384410 \cdot p^j + 0.762062 \cdot \tau$
0.05	$0.487578 \cdot p^j + 0.525226 \cdot \tau$	0.55	$0.374173 \cdot p^j + 0.787992 \cdot \tau$
0.10	$0.475316 \cdot p^j + 0.550867 \cdot \tau$	0.60	$0.364228 \cdot p^j + 0.813566 \cdot \tau$
0.15	$0.463221 \cdot p^j + 0.576861 \cdot \tau$	0.65	$0.354584 \cdot p^j + 0.838725 \cdot \tau$
0.20	$0.451304 \cdot p^j + 0.603136 \cdot \tau$	0.70	$0.345245 \cdot p^j + 0.863419 \cdot \tau$
0.25	$0.439579 \cdot p^j + 0.629612 \cdot \tau$	0.75	$0.336213 \cdot p^j + 0.887605 \cdot \tau$
0.30	$0.428063 \cdot p^j + 0.656206 \cdot \tau$	0.80	$0.327489 \cdot p^j + 0.911251 \cdot \tau$
0.35	$0.416772 \cdot p^j + 0.682833 \cdot \tau$	0.85	$0.319070 \cdot p^j + 0.934329 \cdot \tau$
0.40	$0.405723 \cdot p^j + 0.709406 \cdot \tau$	0.90	$0.310952 \cdot p^j + 0.956821 \cdot \tau$
0.45	$0.394931 \cdot p^j + 0.735842 \cdot \tau$	0.95	$0.303130 \cdot p^j + 0.978714 \cdot \tau$

Table 6.1: Stationary subgame-perfect equilibrium strategies.

When δ equals zero, the dynamic reaction functions coincide with their static counterparts which were given in Section 6.2. If δ converges to 1, these linear dynamic reaction functions converge to $p^{i*}(p^j) = 0.295598\, p^j + \tau$. The decrease in the coefficients in front of p^j for increasing values of δ indicates that the sensitivity to each other's prices is less when the firms are more patient. The increase in the coefficients in front of τ for increasing values of δ indicates that the firms' willingness to coordinate on a higher price increases when the firms are more patient.

In Figure 6.1, impressions of the shape of the equilibrium strategies (linear dynamic reaction functions) are given. Notice that even though the lines in the figure have a similar shape as the best-response curves of the static model, they do not represent best-response curves. Here, the complete lines correspond to equilibrium behavior. Given that firm B plays the stationary strategy $p^{B*}(\cdot)$, the whole line $p^{A*}(\cdot)$ depicts the best response for firm A and $p^{A*}(p^B)$ is just the present stage's price realization when in the previous stage firm B has set the price p^B. So, the only best response drawn for firm A in this figure, is the best response against firm B's strategy $p^{B*}(\cdot)$.[6]

Let the initial state (p_0^A, p_0^B) be given by point 0 in the figure. In the first stage firm A will face the price $p_1^B := p_0^B$ and react with $p_1^A = p^{A*}(p_1^B)$, which leads to the price pair (p_1^A, p_1^B) in point 1. In the next stage firm A is committed to its first stage price: $p_2^A := p_1^A$. Given the state p_2^A, firm B's decision as specified by

[5] The convergence of the strategies (reaction functions) when the horizon lengthens was already pointed out by Cyert and DeGroot (1970), which is the pioneering contribution on alternating move models.

[6] Cyert and DeGroot (1970) point out that in the best-response dynamics of the static model the underlying assumption is that the rival will not change his decision in response to a change by the firm. This assumption is proved false in each period, but firms continue to use reaction functions that are based on this false assumption.

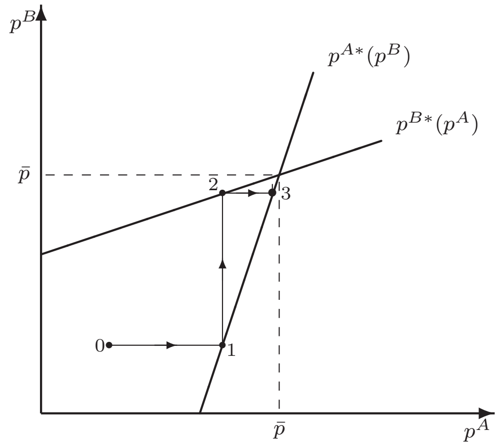

Figure 6.1: SSPE strategies.

its stationary strategy $p^{B*}(\cdot)$ is $p_2^B = p^{B*}(p_2^A)$, which brings the system to point 2. Repeating this procedure, we see that the state dynamics converges to one single point where both firms apply the steady-state price $\bar{p}$. The SSPE is dynamically stable if for any initial pair of prices, the dynamic process induced by the firms pricing behavior converges to the steady-state prices.

Theorem 6.2 *For any discount factor δ, the stationary subgame-perfect equilibrium is dynamically stable. Each firm's steady state price is equal to $\bar{p} = \frac{b}{1-a}\,\tau$, where $b\,\tau$ is the intercept and a the slope of the stationary subgame-perfect equilibrium strategy.*

Proof See Theorem 6.7 and its proof in Section 6.6. □

Regardless of the price-pair (p_0^A, p_0^B) in which the procedure starts, the price dynamics converges to the point $(\bar{p}, \bar{p})$. Hence, in the long-run prices converge to $\bar{p}$. For different values of δ, Table 6.2 lists the values of the corresponding long-run price. The long-run price can be computed to converge to $\bar{p} = 1.419643 \cdot \tau$ when δ converges to 1.

δ	$\bar{p}$	δ	$\bar{p}$
0.00	$1.000000 \cdot \tau$	0.50	$1.237938 \cdot \tau$
0.05	$1.024988 \cdot \tau$	0.55	$1.259121 \cdot \tau$
0.10	$1.049904 \cdot \tau$	0.60	$1.279651 \cdot \tau$
0.15	$1.074672 \cdot \tau$	0.65	$1.299511 \cdot \tau$
0.20	$1.099216 \cdot \tau$	0.70	$1.318690 \cdot \tau$
0.25	$1.123463 \cdot \tau$	0.75	$1.337184 \cdot \tau$
0.30	$1.147340 \cdot \tau$	0.80	$1.354997 \cdot \tau$
0.35	$1.170782 \cdot \tau$	0.85	$1.372136 \cdot \tau$
0.40	$1.193729 \cdot \tau$	0.90	$1.388612 \cdot \tau$
0.45	$1.216129 \cdot \tau$	0.95	$1.404444 \cdot \tau$

Table 6.2: Long-run prices.

Proposition 6.3 *The steady state price $\bar{p}$ is equal to the static Hotelling solution for $\delta = 0$, and grows with the discount factor.*

The positive correlation between the discount factor and the long-run prices reveals that by increasing patience the firms become mutually less aggressive. For any discount factor larger than zero, the long-run prices are higher than in the static model.

6.4 Exogenous demand shocks

We extent the alternating move Hotelling model of the previous section by introducing exogenous shocks in the total demand. Let there be two states of demand; a high-demand state (H) and a low-demand state (L). In the low-demand state, the mass of consumers, η_L, is normalized to one and the high-demand state the mass, η_H, equals $h > 1$. The probability of a transition from the one state of demand in a certain period to the other state of demand in the subsequent period is equal to α. Hence, the state of demand remains unchanged from one period to the other with probability $1 - \alpha$. In a stationary strategy for this setting, firm A specifies its response-price $p^A(D, p^B)$ for each state of demand $D \in \{L, H\}$ and each possible price p^B firm B can be committed to.

An example of the present setting is two ice cream vendors that are located at the ends of a beach and both are selling the same brand of ice cream. The seaside visitors, which are the potential customers, are uniformly distributed over the beach. Depending on the prices of the vendors, each visitor decides to which vendor to go, taking into account the transportation cost over the distance from their blanket to the chosen vendor. The weather changes on a daily basis between sunny and cloudy with probability α (hence, tomorrow's weather is related to today's weather) and demand for ice creams is high when it is sunny.

For $\alpha = 0$, $\alpha = 1$ or $h = 1$, there is no uncertainty on the next period's demand and the model is analytically solvable by application of the techniques used in Maskin and Tirole (1987). For all other parameter configurations these techniques are no longer applicable. Namely, in their analysis Maskin and Tirole (1987) use the optimal response to the opponent's optimal response to one's price set (see also Equation (6.2) and Equation (6.3) in Section 6.6). Due to the exogenous demand shocks the optimal response of the opponent to one's price is no longer uniquely defined. Still, the stationary equilibrium can be computed numerically by gradual truncation of the horizon and applying backwards induction to the truncated games. The reason for this is that the demand shocks are exogenous. The alternating move game is still linear-quadratic such that the equilibrium strategies of the finite horizon converge uniformly to the linear SSPE of the infinite horizon when the horizon lengthens.

For the relative demand difference fixed at $h = 1.50$, the discount factor fixed $\delta = 0.95$, and for different values of the exogenous demand shock parameter α, Table 6.3 contains the symmetric linear SSPE strategies (second and third column). The first term of the elements in the second column indicate that in the

α	$p^{i*}(H, p^j)$	$p^{i*}(L, p^j)$	$\tilde{p}^j$
0.00	$0.303130 \cdot p^j + 0.978714 \cdot \tau$	$0.303130 \cdot p^j + 0.978714 \cdot \tau$	$\infty \cdot \tau$
0.10	$0.307865 \cdot p^j + 0.965787 \cdot \tau$	$0.296352 \cdot p^j + 0.997095 \cdot \tau$	$2.719361 \cdot \tau$
0.20	$0.312373 \cdot p^j + 0.954155 \cdot \tau$	$0.290446 \cdot p^j + 1.012004 \cdot \tau$	$2.638254 \cdot \tau$
0.30	$0.316682 \cdot p^j + 0.943610 \cdot \tau$	$0.285230 \cdot p^j + 1.024266 \cdot \tau$	$2.564416 \cdot \tau$
0.40	$0.320817 \cdot p^j + 0.934019 \cdot \tau$	$0.280573 \cdot p^j + 1.034417 \cdot \tau$	$2.494732 \cdot \tau$
0.50	$0.324797 \cdot p^j + 0.925291 \cdot \tau$	$0.276375 \cdot p^j + 1.042831 \cdot \tau$	$2.427409 \cdot \tau$
0.60	$0.328638 \cdot p^j + 0.917369 \cdot \tau$	$0.272559 \cdot p^j + 1.049774 \cdot \tau$	$2.361205 \cdot \tau$
0.70	$0.332357 \cdot p^j + 0.910180 \cdot \tau$	$0.269065 \cdot p^j + 1.055441 \cdot \tau$	$2.295093 \cdot \tau$
0.80	$0.335965 \cdot p^j + 0.903716 \cdot \tau$	$0.265843 \cdot p^j + 1.059976 \cdot \tau$	$2.228402 \cdot \tau$
0.90	$0.339475 \cdot p^j + 0.897945 \cdot \tau$	$0.262856 \cdot p^j + 1.063485 \cdot \tau$	$2.160561 \cdot \tau$
1.00	$0.342898 \cdot p^j + 0.892857 \cdot \tau$	$0.260068 \cdot p^j + 1.066042 \cdot \tau$	$2.090849 \cdot \tau$

Table 6.3: Stationary subgame-perfect equilibrium strategies.

high-demand state the sensitivity to each others prices increases in the likeliness of a transition to the low-demand state. Moreover, the second terms signal a decreasing willingness to coordinate on high prices if the probability on a low demand increases. For the low-demand state exactly the opposite effects are observed.

The optimal response-price in the low-demand state $p^{i*}(L, p^j)$ is larger than the response-price in the high-demand state $p^{i*}(H, p^j)$ if and only if p^j is less than the value $\tilde{p}^j$ that is depicted in the fourth column of Table 6.3. The second and third column of Table 6.4 indicate where prices converge to in the long run if the system would be continuously in the high-demand state and the low-demand state respectively, regardless of initial prices. From this we learn that prices will

α	$\hat{p}(H)$	$\hat{p}(L)$	$\check{p}(H)$	$\check{p}(L)$
0.00	$1.404444 \cdot \tau$	$1.404444 \cdot \tau$	$1.404444 \cdot \tau$	$1.404444 \cdot \tau$
0.10	$1.395374 \cdot \tau$	$1.417037 \cdot \tau$	$1.402043 \cdot \tau$	$1.410617 \cdot \tau$
0.20	$1.387605 \cdot \tau$	$1.426254 \cdot \tau$	$1.399678 \cdot \tau$	$1.415028 \cdot \tau$
0.30	$1.380924 \cdot \tau$	$1.433001 \cdot \tau$	$1.397416 \cdot \tau$	$1.418147 \cdot \tau$
0.40	$1.375210 \cdot \tau$	$1.437835 \cdot \tau$	$1.395301 \cdot \tau$	$1.420264 \cdot \tau$
0.50	$1.370389 \cdot \tau$	$1.441121 \cdot \tau$	$1.393363 \cdot \tau$	$1.421572 \cdot \tau$
0.60	$1.366416 \cdot \tau$	$1.443105 \cdot \tau$	$1.391628 \cdot \tau$	$1.422207 \cdot \tau$
0.70	$1.363273 \cdot \tau$	$1.443960 \cdot \tau$	$1.390090 \cdot \tau$	$1.422250 \cdot \tau$
0.80	$1.360946 \cdot \tau$	$1.443800 \cdot \tau$	$1.388782 \cdot \tau$	$1.421774 \cdot \tau$
0.90	$1.359441 \cdot \tau$	$1.442710 \cdot \tau$	$1.387709 \cdot \tau$	$1.420822 \cdot \tau$
1.00	$1.358780 \cdot \tau$	$1.440730 \cdot \tau$	$1.386880 \cdot \tau$	$1.419417 \cdot \tau$

Table 6.4: Bounds on the prices in the long run.

gradually decrease below $\tilde{p}^j$ once they are above $\tilde{p}^j$ and that once prices are below $\tilde{p}^j$ there is no force that will push the price above this critical value. Hence, in the long run prices will always be below the value $\tilde{p}^j$ and we can state the following proposition.

Proposition 6.4 *In the long run, the response-price in the low-demand state is larger than in the high-demand state.*

The intuition behind these countercyclical price-dynamics is that when the demand is high, competition for the market gets fiercer, resulting in lower prices. This effect is reinforced by a coordination on higher prices in the low-demand state in order to be able to reap higher profits once the high-demand state emerges.

The highest and lowest price that can be observed in the long run are the values $\hat{p}(H)$ and $\hat{p}(L)$ that are indicated in the second and third column of Table 6.4. These values result if the system remains continuously in the same state of demand, either in the high-demand or in the low-demand state. This means that the highest price that can ever be observed in the high-demand state is when responding to $\hat{p}(L)$. Similarly, the lowest price that can be observed in the low-demand state is when responding to $\hat{p}(H)$. These two price-bounds are listed in the fourth and fifth column of Table 6.4 that are indicated by $\check{p}(H) = p^{i*}(H, \hat{p}(L))$ and $\check{p}(L) = p^{i*}(L, \hat{p}(H))$. It can be concluded that in the long run prices will be between $\check{p}(H)$ and $\hat{p}(H)$ in the high-demand state and between $\hat{p}(L)$ and $\check{p}(L)$ in the low-demand state. The price-bounds are graphically illustrated in Figure 6.2.

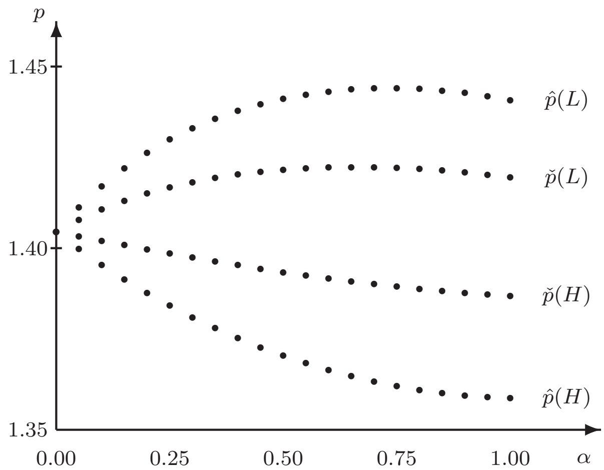

Figure 6.2: Bounds on long run prices.

The figure reveals that the lower bound on the prices in the low-demand state is always above the upper bound on the prices in the high-demand state. Extensive numerical computations demonstrate that this property is found to hold true for all different values of the discount factor δ and the relative demand difference h (with $h > 1$). Supported by these extensive numerical simulations, we formulate the following proposition.

Proposition 6.5 *In the long run, prices in the low-demand state are larger than prices in the high-demand state.*

The figure indicates a downward trend in the price-bounds in the high-demand state for increasing values of the shock probability. The intuition is that when the probability to leave the high-demand state increases, the incentive to reap the immediate profits at stake becomes larger. For the low-demand state the price-bounds increase until the shock probability attains a certain level (in the figure at a point close to $\alpha = 0.75$) and decrease thereafter.[7] The intuition is that when the probability of getting to the high-demand state gets larger, the incentive to give up immediate rewards for the future rewards in the stages to come. But if the probability gets above a certain threshold, the return to the low-demand state in two periods time becomes too probable, resulting in a diminishing incentive for such a nonaggressive behavior. In the end, it would be the opponent firm that benefits from one's leniency.

6.5 Discussion

In this chapter an alternating move Hotelling model is analyzed. It is found that the long-run prices in the alternating move model are higher than for the static model. These long-run prices grow with the discount factor, since the firms become more patient. The presence of exogenous demand shocks leads to countercyclical price behavior for this setting. It can be shown that the introduction of exogenous demand shocks into an infinite horizon alternating move homogenous Cournot model leads to the same insights. However, with respect to quantities the findings are mirrored.

6.6 Appendix

The proofs in this appendix are similar to the proofs of Maskin and Tirole (1987). We are interested in pairs of dynamic reaction functions (R^A, R^B) that form an SSPE. In order to show that a pair of dynamic reaction functions forms an SSPE, it is enough to rule out profitable one-shot deviations (see Herings and Peeters (2004) for an explanation of the one-deviation property). Hence, (R^A, R^B) is an SSPE if and only if there exist value functions $((V^A, W^A), (V^B, W^B))$ such that for any pair of prices (p^A, p^B):

$$V^A(p^B) = \max_p \{\pi^A(p, p^B) + \delta \cdot W^A(p)\}$$
$$R^A(p^B) \in \operatorname{argmax}_p \{\pi^A(p, p^B) + \delta \cdot W^A(p)\}$$
$$W^A(p^A) = \pi^A(p^A, R^B(p^A)) + \delta \cdot V^A(R^B(p^A))$$

and similar expressions for firm B's value functions and dynamic reaction function. Here $V^A(p^B)$ is firm A's present discounted profit if it is about to move, the other firm's price is p^B and the firms use (R^A, R^B) forever; $W^A(p^A)$ is firm A's present discounted profit if firm B is about to move and when firm A is currently committed to price p^A and the firms continue with strategies R^A and R^B stationarily.

Lemma 6.6 *When they exist, the dynamic reaction functions are upward sloping.*

[7]Numerical simulations show that for low discount factors the influence of future periods is too small and the bending point in the price-bound is not observed. For larger values of h the bending point is located at a lower value of α.

Proof Assume, to the contrary, that $p^A > \bar{p}^A$ and $R^B(p^A) < R^B(\bar{p}^A)$. By definition, $R^B(p^A)$ is a best response to p^A, thus

$$\pi^B(p^A, R^B(\bar{p}^A)) + \delta \cdot W^B(R^B(\bar{p}^A)) \le \pi^B(p^A, R^B(p^A)) + \delta \cdot W^B(R^B(p^A)),$$

and $R^B(\bar{p}^A)$ is a best response to $\bar{p}^A$,

$$\pi^B(\bar{p}^A, R^B(p^A)) + \delta \cdot W^B(R^B(p^A)) \le \pi^B(\bar{p}^A, R^B(\bar{p}^A)) + \delta \cdot W^B(R^B(\bar{p}^A)).$$

Combining these two inequalities we find that

$$\pi^B(p^A, R^B(\bar{p}^A)) - \pi^B(p^A, R^B(p^A)) + \pi^B(\bar{p}^A, R^B(p^A)) - \pi^B(\bar{p}^A, R^B(\bar{p}^A)) \le 0,$$

which is equivalent to

$$\int_{\bar{p}^A}^{p^A} \int_{R^B(p^A)}^{R^B(\bar{p}^A)} \tfrac{\partial^2}{\partial x\, \partial y} \pi^B(x, y)\, \mathrm{d}y\, \mathrm{d}x \le 0.$$

But $\frac{\partial^2}{\partial x \partial y}\pi^B(x, y) = \frac{1}{2\tau} > 0$. We have a contradiction. □

The first-order condition for the optimization problem is

$$\tfrac{\partial}{\partial x}\pi^A(R^A(p^B), p^B) + \delta \cdot \tfrac{\mathrm{d}}{\mathrm{d}p} W^A(R^A(p^B)) = 0.$$

Since $p^A = R^A(p^B)$, we have

$$\tfrac{\partial}{\partial x}\pi^A(p^A, (R^A)^{-1}(p^A)) + \delta \cdot \tfrac{\mathrm{d}}{\mathrm{d}p} W^A(p^A) = 0, \tag{6.1}$$

and since $p^B = R^B(p^A)$, we have

$$\tfrac{\partial}{\partial x}\pi^A(R^A(R^B(p^A)), R^B(p^A)) + \delta \cdot \tfrac{\mathrm{d}}{\mathrm{d}p} W^A(R^A(R^B(p^A))) = 0. \tag{6.2}$$

Moreover, from the maximization problem we can formulate the following Bellman equation

$$\begin{aligned} W^A(p^A) \;=\; & \pi^A(p^A, R^B(p^A)) + \delta \cdot \pi^A(R^A(R^B(p^A)), R^B(p^A)) \\ & + \delta^2 \cdot W^A(R^A(R^B(p^A))). \end{aligned}$$

Differentiation of the Bellman equation gives

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}p} W^A(p^A) \;=\; & \tfrac{\partial}{\partial x}\pi^A(p^A, R^B(p^A)) + \tfrac{\partial}{\partial y}\pi^A(p^A, R^B(p^A)) \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^A} R^B(p^A) \\ & + \delta \cdot \tfrac{\partial}{\partial x}\pi^A(R^A(R^B(p^A)), R^B(p^A)) \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^B} R^A(R^B(p^A)) \\ & \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^A} R^B(p^A) \\ & + \delta \cdot \tfrac{\partial}{\partial y}\pi^A(R^A(R^B(p^A)), R^B(p^A)) \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^A} R^B(p^A) \\ & + \delta^2 \cdot \tfrac{\mathrm{d}}{\mathrm{d}p} W^A(R^A(R^B(p^A))) \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^B} R^A(R^B(p^A)) \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^A} R^B(p^A). \end{aligned}$$

Substitution of (6.2) gives

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}p} W^A(p^A) \;=\; & \tfrac{\partial}{\partial x}\pi^A(p^A, R^B(p^A)) + \tfrac{\partial}{\partial y}\pi^A(p^A, R^B(p^A)) \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^A} R^B(p^A) \\ & + \delta \cdot \tfrac{\partial}{\partial y}\pi^A(R^A(R^B(p^A)), R^B(p^A)) \cdot \tfrac{\mathrm{d}}{\mathrm{d}p^A} R^B(p^A) \end{aligned}$$

and subsequent substitution of (6.1) gives

$$\begin{aligned} -\tfrac{1}{\delta}\cdot\tfrac{\partial}{\partial x}\pi^A(p^A,(R^A)^{-1}(p^A)) \;=\; & \tfrac{\partial}{\partial x}\pi^A(p^A,R^B(p^A)) + \tfrac{\partial}{\partial y}\pi^A(p^A,R^B(p^A)) \\ & \cdot\tfrac{\mathrm{d}}{\mathrm{d}p^A}R^B(p^A) \\ & +\delta\cdot\tfrac{\partial}{\partial y}\pi^A(R^A(R^B(p^A)),R^B(p^A)) \\ & \cdot\tfrac{\mathrm{d}}{\mathrm{d}p^A}R^B(p^A). \end{aligned}$$

The latter expression can be simplified to

$$\tfrac{\mathrm{d}}{\mathrm{d}p^A}R^B(p^A) = \frac{-\tfrac{\partial}{\partial x}\pi^A(p^A,(R^A)^{-1}(p^A)) - \delta\cdot\tfrac{\partial}{\partial x}\pi^A(p^A,R^B(p^A))}{\delta\cdot\tfrac{\partial}{\partial y}\pi^A(p^A,R^B(p^A)) + \delta^2\cdot\tfrac{\partial}{\partial y}\pi^A(R^A(R^B(p^A)),R^B(p^A))}. \quad (6.3)$$

Next, we look for linear dynamic reaction functions:

$$R^A(p^B) = \alpha^A + \beta^A p^B \qquad \text{and} \qquad R^B(p^A) = \alpha^B + \beta^B p^A.$$

Moreover, we know that

$$\tfrac{\partial}{\partial p^A}\pi^A = \tfrac{1}{2} + \tfrac{p^B-2p^A}{2\tau} \qquad \text{and} \qquad \tfrac{\partial}{\partial p^B}\pi^A = \tfrac{p^A}{2\tau}.$$

Therefore condition (6.3) boils down to

$$\begin{aligned} \beta^B \;=\; & (-[\tfrac{1}{2} + \tfrac{(p^A-\alpha^A)/\beta^A-2p^A}{2\tau}] - \delta\cdot[\tfrac{1}{2} + \tfrac{\alpha^B+\beta^B p^A-2p^A}{2\tau}]) \\ & /\,(\delta\cdot\tfrac{p^A}{2\tau} + \delta^2\cdot\tfrac{\alpha^A+\beta^A(\alpha^B+\beta^B p^A)}{2\tau}), \end{aligned}$$

or equivalently

$$\begin{aligned} & [\delta^2(\beta^A)^2(\beta^B)^2 + 2\delta\beta^A\beta^B - 2(1+\delta)\beta^A + 1]p^A = \\ & \quad -\delta^2\alpha^B(\beta^A)^2\beta^B - \delta^2\alpha^A\beta^A\beta^B - \delta\alpha^B\beta^A + \alpha^A - (1+\delta)\beta^A\tau. \end{aligned} \quad (6.4)$$

As a constant times p^A is constant, the first constant must be zero, thus

$$\delta^2(\beta^A)^2(\beta^B)^2 + 2\delta\beta^A\beta^B - 2(1+\delta)\beta^A + 1 = 0.$$

By symmetry we also have

$$\delta^2(\beta^A)^2(\beta^B)^2 + 2\delta\beta^A\beta^B - 2(1+\delta)\beta^B + 1 = 0.$$

From these two equations it is clear that $\beta^A = \beta^B = \beta$. Hence, we can drop the superscripts:

$$\delta^2\beta^4 + 2\delta\beta^2 - 2(1+\delta)\beta + 1 = 0.$$

This equation has four solutions of which two of them are real. One of these lies in the interval $(0, \frac{1}{2})$ and the other in the interval $(\frac{1}{\sqrt{\delta}}, \frac{1}{\delta})$. The root in the second interval can be shown to give rise to a dynamically unstable path (see Maskin and Tirole (1987)). The root in the first interval is relevant for the present purpose. This root leads to dynamic reaction functions for which there is a steady-state price and that are therefore dynamically stable.

But, also the right-hand side of (6.4) has to be equal to zero, thus

$$\delta^2\beta^3\alpha^B + \delta^2\beta^2\alpha^A + \delta\beta\alpha^B - \alpha^A = -(1+\delta)\beta\tau,$$

and again by symmetry

$$\delta^2\beta^3\alpha^A + \delta^2\beta^2\alpha^B + \delta\beta\alpha^A - \alpha^B = -(1+\delta)\beta\tau.$$

Combining these two equalities gives

$$(\delta^2\beta^3 - \delta^2\beta^2 + \delta\beta + 1)(\alpha^B - \alpha^A) = 0,$$

from which we see that $\alpha^A = \alpha^B$ and again we can drop the superscripts:

$$\delta^2\beta^3\alpha + \delta^2\beta^2\alpha + \delta\beta\alpha - \alpha = -(1+\delta)\beta\tau.$$

Solving this equation for α gives

$$\alpha = \frac{-(1+\delta)\tau\beta}{\delta^2\beta^3 + \delta^2\beta^2 + \delta\beta - 1}.$$

Theorem 6.7 *For any discount factor δ: (1) there exists a unique linear SSPE, (2) this SSPE is dynamically stable, and (3) each firm's steady state price is equal to $\bar{p} = \frac{\alpha}{1-\beta}$, is equal to the static Hotelling solution for $\delta = 0$, and grows with the discount factor.*

Proof (1) Since the dynamic reaction functions are linear and the profit functions quadratic, the valuation functions are quadratic. From this it is easily found that the objection function is concave. This means that the first-order conditions are not only necessary but also sufficient. All candidate linear SSPEs satisfy the fourth degree polynomial equation that determines β. But, only the dynamics associated with one of the roots is consistent with an SSPE. Thus, the symmetric pair of dynamic reaction functions that are determined by the α and β above form a unique linear SSPE.

(2) Dynamic stability is obtained by the slope of the dynamic reaction curves that have a slope larger than 0 and less than $\frac{1}{2}$ and thus less than 1 in absolute value.

(3) The behavior of the equilibrium dynamic reaction functions and of the steady state are subject in Sections 6.3 and 6.4. □

Chapter 7

An alternating Hotelling experiment

This chapter contains an experimental analysis of an alternating move Hotelling setting with exogenous demand shocks. We compare the prices set in case of low and high demand. The consumer mass in the high-demand state and the transition probability are varied between treatments. We compare the prices and profits in case of low and high demand both within and between treatments. We do not find any significant differences in average prices and profits in mature behavior. The reason is that subjects collude in many observations.

7.1 Introduction

The movement of prices in the presence of exogenous shocks to market demand has received much interest. Prices that fall during demand booms and rise when market demand shrinks have been termed countercyclical. Rotemberg and Saloner (1986) show that if firms tacitly collude, prices move countercyclically in case of stochastic market demand. They apply their findings to empirical observations for the cement industry, the automobile industry between 1954 and 1956 (Bresnahan, 1981), and the railroad cartel which operated in the 1880's on the Chicago-New York route (Porter, 1983). Furthermore, Yoeli (2003) validates the findings of Rotemberg and Saloner (1986) for De Beers diamonds. The theoretical finding of countercyclical pricing also applies when the demand shocks are serially correlated (Kandori, 1991). In contrast, Haltiwanger and Harrington (1991) show that if demand shocks are cyclical, and hence expected, pricing can be procyclical.

Not all explanations for countercyclical pricing rely on the presence of tacit collusion between producers. Bagwell (2004) finds countercyclical pricing for a model where consumers select from which firm to buy without knowing its actual current price due to high search costs. Consumer decisions are based on price reputations, and market demand alternates stochastically between fast growth and slow growth. Another model that does not rely on tacit collusion was presented in Chapter 6. There it was shown that in an alternating move model of horizontal differentiation with two demand states, countercyclical prices are observed in the competitive outcome.

In this chapter we attempt to induce countercyclical pricing in an experiment. We experimentally test an alternating move Hotelling model that is comparable to that of Chapter 6. Market demand can either be low or high and the probability of a change from one state to the other is constant. We consider three treatments which differ in the market demand for the high-state and the transition probability. Between the two demand states, we do not find any significant difference in average prices and profits. The reason for this is that subjects collude in the majority of the observations in all treatments.

This chapter is organized as follows. In Section 7.2 the model used in the experiment is presented. In Section 7.3 we describe the experimental design and the procedures. The results are discussed in Section 7.4. The chapter ends with a discussion in Section 7.5.

7.2 The model

We consider an alternating move Hotelling model with demand shocks which is slightly different from the one presented in Chapter 6. Two firms compete in a heterogenous product market with prices being the strategic variable. Firms can adapt their prices alternately and in the periods in which a firm cannot adapt its price, the price remains equal to the price set in the previous period.

Each of the two firms is located at one of the endpoints of a unit interval and consumers are uniformly distributed over this interval. We assume that firm A is located at $x = 0$ and firm B is located at $x = 1$. The consumers have unit demands and the basic utility consumers achieve by consumption of the product is given by $\beta = 50$. Consumers incur linear transportation costs per unit of distance from the chosen firm, which are given by $\tau = 10$. Finally, we assume that the firms' production costs are equal to zero and that the prices that firms can set are restricted to be between 0 and 50.

Given prices p^A and p^B, a consumer located at x has a utility of

$$U^x = \begin{cases} 0 & \text{if the consumer does not buy,} \\ 50 - p^A - 10x & \text{if the consumer buys from firm } A, \\ 50 - p^B - 10(1-x) & \text{if the consumer buys from firm } B. \end{cases}$$

It then follows that for a given p^A and p^B, firm A's temporal market share, s^A, is given by

$$s^A = \begin{cases} 1 & \text{if } p^A \leq p^B - 10, \\ \frac{1}{2} + \frac{1}{20}(p^B - p^A) & \text{if } p^B - 10 \leq p^A \leq p^B + 10 \text{ and } p^A \leq 90 - p^B, \\ 5 - \frac{1}{10}p^A & \text{if } p^B - 10 \leq p^A \leq p^B + 10 \text{ and } p^A \geq 90 - p^B, \\ 0 & \text{if } p^A \geq p^B + 10. \end{cases}$$

A similar expression can be given for the temporal market share of firm B. In case the price of one firm is at least 10 below that of the other firm, the former captures the whole market. If the price difference is less than 10 and the sum of the prices does not exceed 90, each firm serves halve of the market plus 0.05 times the price difference. In case the sum of the prices exceeds 90, each firm has a market share of 0.5 minus one tenth of its price. In this case the market is not fully served.

There are two states of market demand, namely a high-demand state (H) and a low-demand state (L). In the low-demand state, the mass of consumers, η_L, is normalized to one and in the high-demand state the mass, η_H, equals $h > 1$. The probability of a transition from the one state of demand in a period to the other state of demand in the subsequent period is given by α. Hence, the state of demand remains unchanged from one period to the other with probability $1 - \alpha$.

The temporal profits of the firms are then given by

$$\pi^A = p^A \cdot s^A \cdot \eta \qquad \text{and} \qquad \pi^B = p^B \cdot s^B \cdot \eta,$$

where $\eta \in \{\eta_L, \eta_H\}$ is the temporal total mass of consumers.

7.3 Experimental design and procedures

We want to study subjects' behavior for the alternating price setting model with demand shocks. As was discussed in Chapter 6, differences between prices set in case of low and high demand are influenced by the high-demand state consumer mass and the transition probability. A larger difference between the mass for low and high demand can result in more aggressive pricing in the high-demand state. A larger transition probability α, means that demand is more likely to change for the subsequent period. In order to see how this influences pricing behavior, we implemented a between-subjects design with three treatments, which are summarized in Table 7.1. We have a benchmark treatment (BE), a treatment with increased demand (ID) in the high-demand state, and a treatment with an increased transition probability (IT).

	BE	ID	IT
h	1.5	2.0	1.5
α	0.2	0.2	0.5
periods	80	80	80
observations	14	12	14

Table 7.1: The experimental treatments.

In all three treatments, the experiment consisted of 80 periods which was common knowledge. Although the experiment consisted of a finite number of periods, we still approximate the infinite horizon. Namely, the finite horizon subgame-perfect equilibrium strategies converge to the unique linear stationary subgame-perfect equilibrium of the infinite horizon model if the horizon lengthens (See Chapter 6 and Maskin and Tirole (1987)). The usage of a fixed instead of a random ending reduces the amount of uncertainty that the subjects have to deal with and was therefore preferred for this experiment.

The state of demand in a period was independently drawn for each independent observation. In treatment BE and ID the same realizations were used. For treatment IT this was not possible, but overall the number of periods with low and high demand was similar to the other two treatments.

At the beginning of the session, subjects were randomly matched and it was common knowledge that the matching did not change throughout the experiment. Before the first period started, for each matched pair of subjects, it had to be decided who of the two subjects could adapt the price in the first (and hence each odd) period and what price was responded to in this first period. Therefore, the experiment started with a pre-stage phase in which both subjects simultaneously had to set an initial price. Next, it was randomly decided which of the two subjects could adapt the price in the first period. In this first period, this subject responded to the other's price set in the pre-stage phase.

Every period, the subjects that were able to adapt their prices could observe the current price of their opponent and the state of demand. Prices were restricted to the integers between and including 0 and 50. At the end of each period, subjects received an overview of the results of that period, which consisted of the state of demand, both prices and market shares, own profit, and own total profit so far. In periods where subjects could not adapt their price, they only observed the result screen of that period. The results of all previous periods could be reviewed at the bottom of the result screen.

The experiment was conducted in the behavioral and experimental laboratory (BeeLab) of the Faculty of Economics and Business Administration at Maastricht University in March 2008. The experiment was announced via email and subjects could register online using their matriculation number. The laboratory has a capacity of 32 students and we allowed precisely 32 students to register for each of the three sessions. Students not showing up or canceling on short notice led to some dispersion in the number of independent observations. When students arrived at the laboratory, they had to draw a card from a deck that determined at which computer terminal they were placed. In case an odd number of students showed up for a session, we included a blank card in the deck. Students that drew the blank card could not participate and were paid €3.- as compensation. In total, 80 undergraduate students participated in the experiment.

All interactions took place via computers that were connected to a network. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007). Before the start of a session, subjects read the instructions and were allowed to privately ask questions that were then privately answered. After reading the instructions, subjects had to answer control questions, which tested their understanding of the instructions.[1] One of the experimenters checked the answers, and the experiment only started after all subjects answered all questions correctly. During the experiment, subjects earned ECU that were converted into Euros at a known exchange rate at the end of the experiment. The average payoff

[1]See Section 7.6 for the instructions and control questions.

was €23.22 including a show-up fee of €6.-. Sessions lasted approximately 90 minutes. Payment took place privately, and subjects had to leave the laboratory immediately after payment.

7.4 Results

In this section we discuss the results for the prices set in the different states of demand and the realized profits. We neglect the decisions made in the first 14 periods in order to have settled down behavior. The experiment consisted of a fixed number of periods and endgame effects are then likely to be observed. Therefore, we also neglect the last six periods. Unless specifically stated, the results reported in this section are based on periods 15 to 74.

We compare the prices set in the low and high state of demand, both within and between treatments. For all treatments, the average prices in both demand states are reported in Table 7.2.

	BE		ID		IT	
	p_L	p_H	p_L	p_H	p_L	p_H
Average	37.40	36.86	40.45	39.24	41.17	40.79
	(0.452)		(0.133)		(0.213)	

Table 7.2: The average prices in both demand states for all treatments. In brackets are the exact p-values of one-sided Wilcoxon signed-ranks tests.

In case pricing is countercyclical, the prices set in the low-demand state should be higher than those set in the high-demand state. For all three treatments, it is indeed observed that the average price in case of low demand is above that for high demand. Within each treatment, the prices are compared using one-sided Wilcoxon signed-ranks tests with corrections for ties and which are based on the average prices in each independent observation. From the table it can be concluded that although a price difference is observed, it is not significant.

A possible reason for the lack of a significant difference is the fact that many groups of bidders coordinate on a high price. Both subjects tacitly agreed on a price of 44 or 45 which was kept constant throughout the experiment. In treatments BE and IT this behavior is observed for respectively 8 and 10 out of 14 groups. In treatment ID it is observed for 7 out of 12 groups.

The subjects that managed to coordinate on a high price, often needed quite some periods to do so. When we compare the average prices in both demand states for all periods, a significant difference is found within treatment ID ($p = 0.039$) and IT ($p = 0.012$). Thus by including the first and last couple of periods, the price differences become significant. The reason for this is that the number of periods in which collusion is observed is then seriously reduced. Furthermore, the effect of coordination on high prices can also be mitigated by taking the first quartile instead of the average price over the mature periods. For treatment ID it is again found that the difference between the first quartile in case of low and high demand is significant ($p = 0.013$).

The larger consumer mass for the high-demand state, the more aggressive the pricing might get in that state. Similarly, it could well be that the pricing in the low-demand state responds in the other direction to a change in the difference between the consumer masses. A higher transition probability may lead to more aggressive pricing when demand is high, since it is very likely that demand is low in the subsequent period. Furthermore, even if in the subsequent period demand is still high, the opponent might reasons that it is likely to be low in the subsequent period and hence price aggressively which makes it again better to price aggressively now. Therefore we compare the average prices set in the two demand states between BE and ID, and between BE and IT. In Table 7.2 it can be seen that the average prices in both demand states are lower in BE than in ID and IT. We compare the average prices between the treatments using two-sided Mann-Whitney tests and we do not find any significant differences.[2]

	BE		ID		IT	
	π_L	π_H	π_L	π_H	π_L	π_H
Average	18.70	28.42	19.97	38.02	20.23	30.16
			(0.990)	(0.002)	(0.591)	(0.811)

Table 7.3: The average profits and in brackets the p-values of a comparison with the profit for that demand state in BE.

In Table 7.3 the average temporal profits in the two states of demand are reported. In all treatments, the profits are much higher in case of high demand. For BE and IT, the average profit in the high-demand state is about 1.5 times that in the low-demand state. In ID the average profit for high demand is about double of that in case of low demand.

Displayed in brackets in the table are the two-sided p-values of a comparison with the profit in BE for the same state of demand. Between BE and the other two treatments, the average profit does not significantly differ for low demand. For high demand, the average profit in ID is significantly higher than in BE, whereas in IT this is not the case.

The results for the profits are in line with those observed for the prices. The pricing behavior does not differ much between the states and consequently subjects fully profit from the increased consumer mass in the high-demand state. When looking at the non-collusive groups, particularly in ID it appears that the average profit in case of high demand is above but less than double of the profit for low demand. Still, there are too few observations without collusion to draw unambiguous results on this.

Although we observe collusion in all treatments, it could be that there are differences in the time it takes to arrive at collusive prices. The median period

[2] For p_L: BE $\sim_{.337}$ ID, BE $\sim_{.153}$ IT.
For p_H: BE $\sim_{.475}$ ID, BE $\sim_{.235}$ IT.

number after which prices are constant at a collusive level is 11 for BE, 14 for ID, and 17 for IT. It could well be that endgame effects are initiated earlier in certain treatments and are related to the state of demand. Especially since Rotemberg and Saloner (1986) have shown that cartels are harder to preserve in periods of high demand than in periods of low demand. The median period in which the collusion breaks down is 79 for BE, 77.5 for ID, and 78 for IT. Furthermore, the breakdown cannot clearly be linked to the demand state. Overall we cannot conclude that clear differences are observed between the treatments for the groups that collude.

7.5 Discussion

In this chapter we analyzed an alternating move Hotelling model with demand shocks by means of an experiment. Due to collusion in the majority of the observations, we do not observe significantly lower prices in the high-demand state than in the low-demand state. Furthermore, no significant differences between prices are found for treatments that differ in the size of market demand in the high state and the transition probability between states. Finally, we do not observe apparent differences in the evolution of the collusion between treatments.

We cannot conclude that we observe countercyclical pricing in this experiment. Still, in the observations in which there is no collusion, prices appear to move countercyclical. Especially, for treatment ID this is observed. Further research is needed to get more insights on this. It is crucial to reduce the number of observations in which subjects collude, which is difficult since rematching is not possible in this setting. One line of research that could facilitate this is by having more than two firms competing on a circle instead of a line.

7.6 Appendix

The instructions for the benchmark treatment (h=1.5, α=0.2) are provided here. In addition to these instructions, a table with the profits for all possible price combinations and each state of demand was provided to the subjects. Furthermore, subjects received a one page summary of these instructions. Trivial modifications were made to these instructions for other treatments.

Dear participant,

welcome to this experiment. This experiment will last about approximately two hours and you will be compensated according to your performance. In order to ensure that the experiment takes place in an optimal setting, we want to ask you to follow the general rules during the whole experiment:

- Read these instructions carefully! It is important that you understand the rules of this experiment. These instructions are identical for all subjects that participate together with you. If something is not explained well, please raise your hand. Do not ask the question out loud, but wait until one of the experimenters approaches you to answer the question in private.
- Switch off your mobile phone!
- Do not communicate with your fellow students! Even though the experiment may get exiting at times, it is very important that you remain silent through the proceedings.
- Focus on your *own* computer screen and not on other participants!
- It is prohibited to use the computer for anything else than this experiment! Thus, do not open a webbrowser or any other application.
- There is paper and a pen on your table which you can use during the experiment.
- After the experiment, please remain seated until you are paid off.
- If you do not obey the rules, the data becomes useless for us. Therefore we will have to exclude you from this experiment and you will not receive any compensation.

Your decisions and earnings in this experiment will remain anonymous.

General set-up In this experiment all of you are sellers of a fictitious commodity. You can earn ECU (Experimental Currency Units) which will be exchanged into Euros at the end of the experiment. The exchange rate will be given in the instructions below.

Before the experiment starts, you will be randomly divided into groups of two sellers. You will not know the identity of the seller you are matched with. The groups remain unchanged throughout the whole experiment.

Alternating moves This experiment consists of 80 periods. In each period, only one of the two sellers can adapt its price. The price of the seller that cannot adapt its price remains equal to its price in the previous period. The seller that can adapt its price switches after each period. Consequently, one seller can adapt its price only in the *odd* periods, whereas the other seller can adapt its price only in the *even* periods.

Before setting your price for the current and the subsequent period, you observe the price of the other seller for the current period. Remember that the other seller can adapt its price in the next period, after having observed your price. This procedure of alternating price-adaptation continues until the experiment ends.

Prices Possible prices are the integers between (and including) 0 and 50. To decide on a price you can enter a price in the box on your screen and then click on *OK* (see the figure below). You have 60 seconds to enter a price.

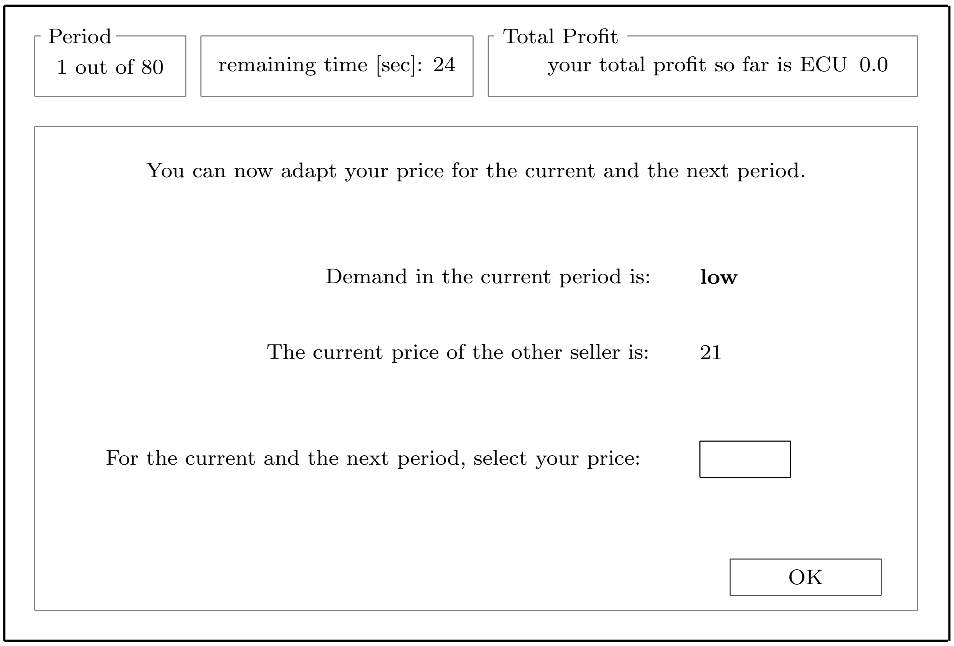

Figure 1: Screenshot of the price adaptation screen.

Market share After the seller that could adapt its price has made its decision, the market shares for that period are determined. Your current market share depends on your current price and the current price of the other seller. We will now give the formula that is used to calculate market shares. We explain its meaning in words below.

When your current price is p and the current price of the other seller is p', then your current market share in percentage is given by

$$s = \begin{cases} 100\,\% & \text{if } p \leq p' - 10 \\ 50 + 5(p' - p)\,\% & \text{if } p' - 10 \leq \ p \ \leq p' + 10 \text{ and } p \leq 90 - p' \\ 10\,(50 - p)\,\% & \text{if } p' - 10 \leq \ p \ \leq p' + 10 \text{ and } p \geq 90 - p' \\ 0\,\% & \text{if } p \geq p' + 10 \end{cases}$$

Now, we explain each row of the formula above.
If in a certain period your price is at least 10 ECU below the other's price (first line in formula), you will serve the complete market in that period. Exactly opposite to this, when your price exceeds the other's price by at least 10 ECU in a certain period (fourth line in formula), all potential customers are served by the other seller in that period.

None of these two events apply once the difference (positive or negative) between the prices is less than 10 ECU. In that case there are two possible scenarios.
In case the sum of the two prices is below 90 ECU (second line in formula), each seller will have a market share of fifty percent plus five times the price difference. Here, the price difference is defined as the price of the other seller minus your price. Notice that the seller with the higher (lower) price will serve less (more) than fifty percent of the market. Furthermore, the larger the price difference, the larger the difference in market share.
In the other case, when the sum of the two prices exceeds 90 ECU (third line in formula), each seller has a market share of 10 times the difference between 50 and its price. Notice that in this scenario the market will not be fully served, since the market shares do not add up to 100 %.

A short summary of the way in which the market share is determined, is provided on the last sheet of these instructions. You are advised to keep this in front of you while making your decisions throughout the experiment.

Market demand In this experiment there are two possible market demands. Market demand can be low (equals 1) or high (equals 1.5). While deciding on a price, you can see whether the demand in the current period is low or high (see the screenshot on the previous page). However, there is a 20 % probability that the demand will change for the next period. To clarify, if current demand is low, there is a 80 % chance that it remains low in the next period and a 20 % chance that it is high in the next period. If current demand is high, there is a 80 % chance it remains high in the next period and a 20 % chance that it changes to low.

Profit Your current period profit is given by:

$$\text{price x } \frac{\text{market share}}{100\,\%} \text{ x market demand.}$$

That is, your profit equals your relative market share (that is your market share in percentage divided by 100) multiplied with your price and that then multiplied with the market demand. Notice that for a given price and market share, profits are a factor 1.5 higher in case of high demand than in case of low demand. Also observe that there are no costs, so you can never make losses. Furthermore, we provided two tables, one for low market demand and one for high market demand, that gives the profits for different possible price combinations.

Overview at end of period At the end of each period, both sellers receive an overview of the results of that period. You can observe, the state of market demand, your price, the price of the other seller, your market share, the other's market share, your profit, and your total profit so far. The result screen also shows the history of all previous periods. In a period in which you cannot adapt your price, you only observe the result screen in that period. After you observed the screen, click on *OK*.

Period
1 out of 80

remaining time [sec]: 17

Total Profit
your total profit so far is ECU 8.40

These are the results of period: 1

Demand in the current period was: low

Your price is: 24
The price of the other seller is: 21

Your market share (%) is: 35
The market share of the other seller (%) is: 65

Your profit is: 8.40

OK

Period	Demand	Price	Price of other seller	Your market share (%)	Market share of other seller (%)	Your profit	Total profit
1	low	24	21	35	65	8.40	8.40

Figure 2: Screenshot of the results screen.

This ends the current period and this procedure continues until the last period.

The first period Before the procedures above start, it has to be decided who of the sellers is able to adapt its price in the first (and all odd) periods and to which price this seller responds in the first period. Therefore, the first period contains an initial period in which both sellers are asked to set an initial price. Next the computer randomly decides which of the two sellers can adapt its price in the odd periods. The state of demand for the first period will already be shown in the initial period. In the first period this seller will respond to the other's initial price.

Closing After the last period of the experiment, we would like you to complete a short questionnaire that will appear on your screen. Payments will be made by the experimenters afterwards.

ECU are transformed into Euros according to the following conversion rate: 100 ECU = 1 Euro. In addition to your earnings throughout the experiment, you will also receive a show up fee of 6 Euro.

Final remarks In order to allow for a smooth running of this session, we would like to point out the following. Use the waiting time in the periods in which you cannot alter your price to consider your future play. Also, when you are in a period in which you can adapt your price, please do so within the 60 seconds. Finally, please pay attention to your screen so that no unnecessary waiting times arise.

Before we start with the experiment we want you to answer the questionnaire on the next page. One of the experimenters will go around and check the answers and discuss any problems.

Questionnaire

Please answer the following questions. When you are finished, raise your hand. One of the experimenters will come to you and check whether everything is correct.

1. How many sellers are in your group (including yourself)?

   —

2. Suppose that you can adapt your price in period 4. What does this imply for your price in period 5?

   __ My price in period 5 will be equal to my price in period 4.

   __ My price in period 5 can be any integer between (and including) 0 and 50.

3. Suppose that you can adapt your price in period 4. What do you know about the price of the other seller in period 5?

   __ The other's price in period 5 will be equal to its price in period 4.

   __ The other's price in period 5 can be any integer between (and including) 0 and 50.

4. Suppose that you can adapt your price in period 4. What does this imply for your price in period 6?

   __ My price in period 6 will be equal to my price in period 4.

   __ My price in period 6 will be equal to my price in period 5.

   __ My price in period 6 can be any integer between (and including) 0 and 50.

5. Suppose current market demand is low. What is the probability that it will be low in the next period?

   —

6. Suppose current market demand is high. What is the probability that it will be high in the next period?

   —

7. Suppose your current price equals 20. Determine your current period profit for the following questions. You can use the profit tables provided to you.

   (a) Demand is low and the price of the other seller is 32.

   —

   (b) Demand is low and the price of the other seller is 8.

   —

   (c) Demand is low and the price of the other seller is 26.

   —

   (d) Demand is high and the price of the other seller is 26.

   —

Profits in case of low demand

price of other seller

your price	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
2	1	1.2	1.4	1.6	1.8	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
4	1.6	2	2.4	2.8	3.2	3.6	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
6	1.8	2.4	3	3.6	4.2	4.8	5.4	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
8	1.6	2.4	3.2	4	4.8	5.6	6.4	7.2	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
10	1	2	3	4	5	6	7	8	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
12	0	1.2	2.4	3.6	4.8	6	7.2	8.4	9.6	10.8	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
14	0	0	1.4	2.8	4.2	5.6	7	8.4	9.8	11.2	12.6	14	14	14	14	14	14	14	14	14	14	14	14	14	14
16	0	0	0	1.6	3.2	4.8	6.4	8	9.6	11.2	12.8	14.4	16	16	16	16	16	16	16	16	16	16	16	16	16
18	0	0	0	0	1.8	3.6	5.4	7.2	9	10.8	12.6	14.4	16.2	18	18	18	18	18	18	18	18	18	18	18	18
20	0	0	0	0	0	2	4	6	8	10	12	14	16	18	20	20	20	20	20	20	20	20	20	20	20
22	0	0	0	0	0	0	2.2	4.4	6.6	8.8	11	13.2	15.4	17.6	19.8	22	22	22	22	22	22	22	22	22	22
24	0	0	0	0	0	0	0	2.4	4.8	7.2	9.6	12	14.4	16.8	19.2	21.6	24	24	24	24	24	24	24	24	24
26	0	0	0	0	0	0	0	0	2.6	5.2	7.8	10.4	13	15.6	18.2	20.8	23.4	26	26	26	26	26	26	26	26
28	0	0	0	0	0	0	0	0	0	2.8	5.6	8.4	11.2	14	16.8	19.6	22.4	25.2	28	28	28	28	28	28	28
30	0	0	0	0	0	0	0	0	0	0	3	6	9	12	15	18	21	24	27	30	30	30	30	30	30
32	0	0	0	0	0	0	0	0	0	0	0	3.2	6.4	9.6	12.8	16	19.2	22.4	25.6	28.8	32	32	32	32	32
34	0	0	0	0	0	0	0	0	0	0	0	0	3.4	6.8	10.2	13.6	17	20.4	23.8	27.2	30.6	34	34	34	34
36	0	0	0	0	0	0	0	0	0	0	0	0	0	3.6	7.2	10.8	14.4	18	21.6	25.2	28.8	32.4	36	36	36
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.8	7.6	11.4	15.2	19	22.8	26.6	30.4	34.2	38	38
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	8	12	16	20	24	28	32	36	40
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.2	8.4	12.6	16.8	21	25.2	29.4	33.6	33.6
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.4	8.8	13.2	17.6	22	26.4	26.4	26.4
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.6	9.2	13.8	18.4	18.4	18.4	18.4
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4.8	9.6	9.6	9.6	9.6	9.6
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

For convenience only even prices are displayed. Notice that you can also select odd prices.

The underlined numbers indicate the maximum current period profit (among those in the table), given the price of the other seller.

Notice that these are the profits of only a single period and that your current price decisions may affect future profits.

References

1. Ashenfelter, O. (1989). How auctions work for wine and art. *Journal of Economic Perspectives*, 3 (3), 23-36.
2. Ashenfelter, O. and D. Genesove (1992). Testing for price anomalies in real-estate auctions. *American Economic Review*, 82 (2), 501-505.
3. Ausubel, L., P. Cramton, R.P. McAfee, and J. McMillan (1997). Synergies in wireless telephony: Evidence from the broadband PCS auctions. *Journal of Economics and Management Strategy*, 6 (3), 497-527.
4. Bagwell, (2004). Countercyclical pricing in customer markets. *Economica*, 71 (284), 519-542.
5. Baye, R.B. and S.F. Ueng (1999). Commitment and price competition in a dynamic differentiated-product duopoly. *Journal of Economics*, 69 (1), 41-52.
6. Beggs, A. and K. Graddy (1997). Declining values and the afternoon effect: Evidence from art auctions, *RAND Journal of Economics*, 28 (3), 544-565.
7. Bertrand, J. (1883). Review of theorie mathematique de la richesse sociale and recherches sur les principes mathematicque de la theoire des richesse. *Journal des Savants*, 67, 499-508.
8. Black, J. and D. De Meza (1992). Systematic price differences between successive auctions are no anomaly. *Journal of Economics and Management Strategy*, 1 (4), 607-628.
9. Branco F. (1997). Sequential auctions with synergies: An example. *Economics Letters*, 54 (2), 159-163.
10. Brandts J. and P. Guillén (2007). Collusion and fights in an experiment with price-setting firms and production in advance. *Journal of Industrial Economics*, 55 (3), 453-473.
11. Bresnahan, T.F. (1981). Competition and collusion in the American automobile industry: The 1955 price war. Working paper, Stanford University, Palo Alto.

12. Burns, P. (1985). Experience and decision-making: A comparison of students and businessmen in a simulated progressive auction. *Research in Experimental Economics III*, 139-157.
13. Busse, M. (2002). Firm Financial Condition and Airline Price Wars. *The RAND Journal of Economics*, 33 (2), 298-318.
14. Cason, T.N., D. Friedman, and F. Wagener (2003). The dynamics of price dispersion, or Edgeworth variations. *Journal of Economic Dynamics and Control*, 29 (4), 801-822.
15. Castanias, R. and H. Johnson (2001). Gas wars: Retail gasoline price fluctuations. *The Review of Economics and Statistics*, 75 (1), 171-174.
16. Chamberlain, E. (1933). *The theory of monopolistic competition.* Harvard university Press, Massachusetts.
17. Cournot, A. (1838). *Reserches sur les principes mathématique de la thérie des richess.*
18. Cramton, P. (2002). Spectrum auctions. In: Cave, M., S. Majumdar, and I. Vogelsang, *Handbook of telecommunications economics.* Amsterdam: Elsevier, pp. 605-639.
19. Cyert, R.M. and M.H. DeGroot (1970). Multiperiod decision models with alternating choice as the solution to the duopoly problem. *Quarterly Journal of Economics*, 84 (3), 410-429.
20. Dal Bó, P. (2005). Cooperation under the shadow of the future: Experimental evidence from infinitely repeated games. *The American Economic Review*, 95 (5), 1591-1604.
21. De Silva, D.G. (2005). Synergies in recurring procurement auctions: An empirical investigation. *Economic Inquiry*, 43 (1), 55-66.
22. De Silva, D.G., T.D. Jeitschko, and G. Kosmopoulou (2005). Stochastic synergies in sequential auctions. *International Journal of Industrial Organization*, 23 (3-4), 183-201.
23. Doyle, J., E. Muehlegger, and K. Samphantharak (2007). Market concentration and Edgeworth cycles in gasoline markets. Working paper.
24. Eaton, J. and M. Engers (1990). Intertemporal price competition. *Econometrica*, 58 (3), 637-659.
25. Eckert, A. (2003). Retail price cycles and the presence of small firms. *International Journal of Industrial Organization*, 21 (2), 151-170.
26. Eckert, A. (2004). An alternating-move price-setting duopoly model with stochastic costs. *International Journal of Industrial Organization*, 22 (7), 997-1015.
27. Edgeworth, F.Y. (1925). The pure theory of monopoly. In: *Papers Relating to Political Economy*, London: MacMillan, pp. 111-142.
28. Engelbrecht-Wiggans, R. (1994). Sequential auctions of stochastically equivalent objects. *Economics Letters*, 44 (1-2), 87-90.

29. Engle-Warnick, J. and R.L. Slonim (2004). The evolution of strategies in a repeated trust game. *Journal of Economic Behavior and Organization*, 55 (4), 553-573.

30. Fehl, U. and W. Güth (1987). Internal and external stability of bidder cartels in auctions and public tenders. *International Journal of Industrial Organization*, 5 (3), 303-313.

31. Février, P., L. Linnemer, and M. Visser (2007). Buy or wait, that is the option: The buyer's option in sequential laboratory auctions. *RAND Journal of Economics*, 38 (1), 98-118.

32. Fischbacher, U. (2007). z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics*, 10 (2), 171-178.

33. Friedman, J. (1971). A non-cooperative equilibrium for supergames. *Review of Economic Studies*, 38 (1), 1-12.

34. Green, E. and R. Porter (1984). Non-cooperative collusion under imperfect price information. *Econometrica*, 52 (1), 87-100.

35. Grimm, V., F. Mengel, L. Viianto, and G. Ponti (2006). Investment incentives in procurement auctions: An experiment. In: Hinloopen, J. and H.T. Normann, *Experimental Economics for Antitrust Policy.* Cambridge: Cambridge University Press, forthcoming.

36. Guillén, P. (2004). Price-quantity competition and Edgeworth cycles. Working paper, Universitat Autónoma de Barcelona.

37. Güth, W., R. Ivanova-Stenzel, and E. Wolfstetter (2005). Bidding behavior in asymmetric auctions: An experimental study. *European Economic Review*, 49 (7), 1891-1913.

38. Hall, R. and C. Hitch (1939). Price theory and business behavior. *Oxford Economic Papers*, 2, 12-45.

39. Haltiwanger, J. and J.E. Harrington Jr. (1991). The impact of cyclical demand movements on collusive behavior. *The RAND Journal of Economics*, 22 (1), 89-106.

40. Hendricks, K. and R. Porter (1988). An empirical study of an auction with asymmetric information. *The American Economic Review*, 78 (5), 865-883.

41. Herings, P.J.J. and R.J.A.P. Peeters (2004). Stationary equilibria in stochastic games: Structure, selection and computation. *Journal of Economic Theory*, 118 (1), 32-60.

42. Holt, C. (1980). Competitive bidding for contracts under alternative auction procedures. *Journal of Political Economy*, 88 (3), 433-445.

43. Hotelling, H. (1929). Stability in competition. *Economic Journal*, 39 (152), 41-57.

44. Jeitschko, T.D. and E. Wolfstetter (2002). Scale economies and the dynamics of recurring auctions. *Economic Inquiry*, 40 (3), 403-414.

45. Kandori, M. (1991). Correlated demand shocks and price wars during booms. *The Review of Economic Studies*, 58 (1), 171-180.
46. Keser, C. and M. Olson (1996). Experimental examination of the declining price anomaly. In: Ginsburgh, V. and P.M. Menger, *Economics of the arts: Selected essays*. Amsterdam: Elsevier, pp. 151-175.
47. Kirchkamp, O. and J.P. Reiss (2004). The overbidding-myth and the underbidding-bias in first-price auctions. Sonderforschungsbereich 504 Publications, 04-32, University of Mannheim.
48. Klemperer, P. (2002). How (not) to run auctions: The European 3G telecom auctions. *European Economic Review*, 46 (4-5), 829-845.
49. Krishna, V. and R. Rosenthal (1996). Simultaneous auctions with synergies. *Games and Economic Behavior*, 17 (1), 1-31.
50. Kruse, J.B., S. Rassenti, S.S. Reynolds, and V.L. Smith (1994). Bertrand-Edgeworth competition in experimental markets. *Econometrica*, 62 (2), 343-371.
51. Lau, S.-H.P. (2002). Solution of multi-player linear-quadratic alternating-move games and its application to the timing pattern of wage adjustment. *Computational Economics*, 19 (3), 341-357.
52. Lebrun, B. (1998). Comparative statics in first-price auctions. *Games and Economic Behavior*, 25 (1), 97-110.
53. Ledyard, J., D. Porter, and A. Rangel (1997). Experiments testing multiobject allocation mechanisms. *Journal of Economics and Management Strategy*, 6 (3), 639-675.
54. Leufkens, K. and R. Peeters (2007). Synergies are a reason to prefer first-price auctions! *Economics Letters*, 97 (1), 64-69.
55. Leufkens, K. and R. Peeters (2008a). Intertemporal price competition with exogenous demand shocks. *Economics Letters*, 99 (2), 301-303.
56. Leufkens, K. and R. Peeters (2008b). Focal prices and price cycles in an alternating price duopoly experiment. METEOR Research Memorandum 08/021, Universiteit Maastricht.
57. Leufkens, K., R. Peeters, and D. Vermeulen (2006). Sequential auctions with synergies: The paradox of positive synergies. METEOR Research Memorandum 06/018, Universiteit Maastricht.
58. Leufkens, K., R. Peeters, and M. Vorsatz (2006). Sequential auctions with synergies: An experimental analysis. METEOR Research Memorandum 06/040, Universiteit Maastricht.
59. Leufkens, K., R. Peeters, and M. Vorsatz (2007). An experimental comparison of sequential first- and second-price auctions with synergies. METEOR Research Memorandum 07/055, Universiteit Maastricht.
60. Lewis, M.S. (2008). Temporary wholesale gasoline price spikes have long-lasting retail effects: The aftermath of hurricane Rita. *Journal of Law and Economics*, forthcoming.

61. Maskin, E. and J. Riley (2000). Asymmetric auctions. *Review of Economic Studies*, 67 (3), 413-438.

62. Maskin, E. and J. Tirole (1987). A theory of dynamic oligopoly III: Cournot competition. *European Economic Review* , 31 (4), 947-968.

63. Maskin, E. and J. Tirole (1988). A theory of dynamic oligopoly II: Price competition, kinked demand curves and Edgeworth cycles *Econometrica*, 56 (3), 571-599.

64. Maskin, E. and J. Tirole (2001). Markov perfect equilibrium I: Observable actions. *Journal of Economic Theory*, 100 (2), 191-219.

65. McAfee, R.P. and D. Vincent (1993). The declining price anomaly. *Journal of Economic Theory*, 60 (1), 191-212.

66. McKelvey, R.D. and T.R. Palfrey (1995). The holdout game: An experimental study of an infinitely repeated game with two-sided incomplete information. In: W. Barnett, H. Moulin, M. Salles, and N. Schofield, *Social Choice, Welfare, and Ethics*, Proceedings of the 8th International Symposium in Economic Theory and Econometrics. Cambridge University Press, Cambridge, pp. 321-349.

67. Menezes, F. and P. Monteiro (2004). Auctions with synergies and asymmetric buyers. *Economics Letters*, 85 (2), 287-294.

68. Myerson, R.B. (1981). Optimal auction design. *Mathematics of Operation Research*, 6 (1), 58-73.

69. Neugebauer, T. and P. Pezanis-Christou (2007). Bidding behavior at sequential first-price auctions with(out) supply uncertainty: A laboratory analysis. *Journal of Economic Behavior and Organization*, 63 (1), 55-72.

70. Noel, M.D. (2007). Edgeworth price cycles, cost-based pricing, and sticky pricing in retail gasoline markets. *Review of Economics and Statistics*, 89 (2), 324-334.

71. Normann, H.T. and B. Wallace (2006). The impact of termination rule on the cooperation in a prisoner's dilemma experiment. Working paper.

72. Plott, C. (1997). Laboratory experimental testbeds: Application to the PCS auction. *Journal of Economics and Management Strategy*, 6 (3), 605-638.

73. Plum, M. (1992). Characterization and computation of Nash-equilibria for auctions with incomplete Information. *International Journal of Game Theory*, 20 (4), 393-418.

74. Porter, R.H. (1983). The joint economic committee, 1880-1886. *Bell Journal of Economics*, 14 (2), 301-314.

75. Rath, K.P. (1998). Stationary and nonstationary strategies in Hotelling's model of spatial competition with repeated pricing decisions. *International Journal of Game Theory*, 27 (4), 525-537.

76. Robinson, M.S. (1985). Collusion and the choice of auction. *RAND Journal of Economics*, 16 (1), 141-145.

77. Ross, L.B. (1997). When will an airline stand its ground? An analysis of fare wars. *International Journal of the Economics of Business*, 4 (2), 109-127.

78. Rotemberg, J. and G. Saloner (1986). A supergame-theoretic model of business cycles and price wars during booms. *The American Economic Review*, 76 (3), 390-407.

79. Rothkopf, M.H., T.J.Teisberg, and E.P. Kahn (1990). Why are Vickrey auctions rare? *Journal of Political Economics*, 98 (1), 94-109.

80. Rusco, F. and D. Walls (1999). Competition in a repeated spatial auction market with an application to timber sales. *Journal of Regional Science*, 39 (3), 449-465.

81. Selten, R., M. Mitzkewitz, and G.R. Uhlich (1997). Duopoly strategies programmed by experienced players. *Econometrica*, 65 (3), 517-555.

82. Selten, R. and R. Stoecker (1986). End behavior in sequences of finite prisoner's dilemma supergames. *Journal of Economic Behavior and Organization*, 7 (1), 47-70.

83. Skrzypacz, A. and H. Hopenhayn (2004). Tacit collusion in repeated auction. *Journal of Economic Theory*, 114 (1), 153-169.

84. Sweezy, P. (1939). Demand under Conditions of Oligopoly. *Journal of Political Economy*, 47 (4), 568-573.

85. Tang Sørensen, S. (2006). Sequential auctions for stochastically equivalent complementary objects. *Economics Letters*, 91 (3), 337-342.

86. Van Damme, E. (2002). The European UMTS-auctions. *European Economic Review*, 46 (4-5), 846-869.

87. Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance*, 16 (1), 8-37.

88. Yildirim, H. (2004). Piecewise procurement of a large-scale project. *International Journal of Industrial Organization*, 22 (8-9), 1349-1375.

89. Yoeli, E. (2003). In December, diamonds are forever: Empirical evidence of countercyclical pricing. Working paper, Stanford University, Palo Alto.

90. Wang Z. (2005). Strategy, timing and oligopoly pricing: Evidence from a repeated game in a timing-controlled gasoline market. Working paper, Northeastern University, Boston.

91. Weber, R.J. (1983). Multiple-objects auctions. In: Engelbrecht-Wiggans, R., M. Shubik, and R.M. Stark, *Auctions, bidding and contracting: Uses and theory.* New York: New York University Press, pp. 165-194.

Nederlandse samenvatting

(Summary in Dutch)

Een centraal thema binnen de economie is het functioneren van markten. Het vakgebied genaamd industriële organisatie bestudeert het strategisch gedrag van bedrijven, de structuur van markten, en de interactie tussen beide. In dit proefschrift worden dynamische interactiesituaties bestudeerd door niet-coöperatieve speltheorie toe te passen. De conflictsituatie wordt gemodelleerd als een spel en voor dit spel worden vervolgens evenwichten gezocht. Een evenwicht is een stabiele situatie waarin geen speler op zichzelf een reden heeft om zijn strategie te wijzigen. Zulke evenwichten geven informatie over de strategische keuzes van de spelers en vervolgens over het marktgedrag en marktresultaat. De inzichten die op deze manier worden verkregen zijn uiterst belangrijk bij het bepalen van marktregulerende instrumenten.

Dit proefschrift bestaat uit twee delen. De delen verschillen in de marktsituatie die bestudeerd wordt. Het eerste deel van dit proefschrift betreft sequentiële veilingen met één verkoper en meerdere bieders. De bieders profiteren van positieve synergieën indien ze meerdere objecten winnen. Het effect van de synergiemogelijkheden op het biedgedrag, verkoopprijs, en efficiëntie wordt geanalyseerd. Het tweede deel van dit proefschrift bestudeert een dynamische duopolie waarin bedrijven hun prijzen alternerend kunnen aanpassen. Onder andere het effect van exogene schokken in de marktvraag wordt bestudeerd in dit deel. Beide delen van dit proefschrift bestaan uit analytische, numerieke, en experimentele analyses. Hieronder worden beide delen uitgebreider samengevat.

Sequentiële veilingen met synergieën

Veilingen worden al sinds de oudheid gebruikt als marktmechanisme. Overheden maken steeds meer gebruik van veilingen, bijvoorbeeld voor de allocatie van frequenties voor draadloze communicatie services. De UMTS veilingen (mobiele communicatie van de derde generatie) van verschillende West-Europese overheden in 2000 and 2001 trokken niet alleen de aandacht van vakmensen en academici, maar ook van een breder publiek. De meest succesvolle UMTS veiling was die van het Verenigd Koninkrijk. Eén van de redenen hiervoor is dat deze veiling de eerste was in de reeks van veilingen door verschillende landen. De meeste bieders in de veiling van het Verenigd Koninkrijk waren van plan om een netwerk te creëren dat heel Europa omvat (Van Damme, 2002). Winnaars in de veiling van het Verenigd Koninkrijk waren daarom goed gepositioneerd voor de latere veilingen van andere landen.

Openbare aanbestedingsprojecten hebben de eigenschap dat ze herhalend van aard zijn. Wegwerkzaamheden, bouwprojecten, frequenties voor draadloze communicatie en militaire uitrustingen worden met regelmaat aanbesteed. Dit gebeurt tegenwoordig meestal door middel van veilingen. Daardoor ontstaan reeksen van veilingen van vergelijkbare projecten met mogelijk lange tijdsperiodes tussen opeenvolgende veilingen. In zulke sequentiële veilingen kan het winnen van meerdere objecten resulteren in kostenvoordelen door synergieën. Deze synergieën kunnen tastbaar zijn, bijvoorbeeld het bezitten van speciale apparatuur, of niet-tastbaar, zoals ervaring. Een gevolg van de aanwezigheid van synergieën in sequentiële veilingen is dan dat de waarde van objecten in latere veilingen afhangen van de uitslag van eerdere veilingen.

Het eerste deel van dit proefschrift bestudeert een sequentiële veiling van twee objecten waar het winnen van beide objecten leidt tot voordelen door positieve synergieën. Bieders hebben dan in de eerste veiling van de reeks het probleem dat het onzeker is of ze kunnen profiteren van positieve synergieën aangezien ze nog geen informatie hebben over hun winkansen in de tweede veiling. Verder zijn de bieders in de tweede veiling asymmetrisch aangezien de bieder die de eerste veiling heeft gewonnen positieve synergiemogelijkheden heeft en de andere bieders niet. Daarom wordt bekeken wat de gevolgen van positieve synergiemogelijkheden in sequentiële veilingen zijn voor zowel de bieders als de verkoper.

In Hoofdstuk 2 wordt aangetoond dat de verwachte uitbetaling van bieders in de veilingreeks lager is met synergieën dan zonder. Door de synergiemogelijkheden wordt het biedgedrag in de eerste veiling zo agressief dat bieders slechter af zijn dan wanneer er geen synergiemogelijkheden zijn. Verder lopen bieders het risico op ernstige verliezen, wat zou kunnen leiden tot een faillissement. De aanwezigheid van positieve synergieën in sequentiële veilingen heeft dus negatieve gevolgen voor bieders.

Hoofdstuk 3 bestudeert twee verschillende veilingmechanismen voor de sequentiële veiling met positieve synergieën. De prestaties van een gesloten veiling waarin de winnaar zijn bod moet betalen (een eerste-prijs veiling) worden vergeleken met die van een gesloten veiling waarin de winnaar het één na hoogste bod moet betalen (een tweede-prijs veiling). Normaal gesproken heeft een tweede-prijs veiling een aantal aangename eigenschappen ten opzichte van een eerste-prijs veiling (Vickrey, 1961). In dit hoofdstuk wordt aangetoond dat in de veilingreeks de efficiëntie en de verkoopopbrengst hoger zijn in het geval van een tweede-prijs veiling. Echter, de kans dat winnaars een verlies maken is ook een stuk groter dan in een eerste-prijs veiling. Gezien de catastrofale gevolgen die zulke verliezen kunnen hebben, is het goed mogelijk dat eerste-prijs veilingen worden verkozen boven tweede-prijs veilingen voor sequentiële veilingen met synergieën.

De analyses in Hoofdstuk 2 en 3 zijn beperkt in het aantal bieders dat actief kan zijn in een veiling. Namelijk, een asymmetrische eerste-prijs veiling kan alleen worden opgelost voor twee bieders. Daarom richt Hoofdstuk 2 zich hoofdzakelijk op tweede-prijs veilingen met (on)eindig veel bieders. De milde voorwaarden waaraan de strategieën in een eerste-prijs veiling zouden moeten voldoen voor generalisatie van de resultaten naar zulke veilingen met meer dan twee bieders, worden ook besproken in dit hoofdstuk. Hoofdstuk 3 vergelijkt beide veilingmechanismen in het geval van twee bieders.

In Hoofdstuk 4 worden eerste- en tweede-prijs veilingen met vier bieders vergeleken door middel van een experiment. Voor beide veilingmechanismen worden experimenten uitgevoerd voor sequentiële veilingen zonder synergieën, met milde synergieën, en met sterke synergieën. Door de resultaten van de verschillende experimenten zowel binnen als tussen de twee veilingmechanismen te vergelijken, worden inzichten verkregen in de effecten van positieve synergieën op het daadwerkelijke biedgedrag.

De proefpersonen reageren duidelijk op de aanwezigheid van synergieën en het biedgedrag in de eerste veiling van de reeks wordt agressiever naarmate de synergie groter wordt. Voor milde synergieën leidt een eerste-prijs veiling tot betere resultaten op het gebied van efficiëntie, veilingopbrengst en de kans dat winnaars verliezen maken. Voor sterke synergieën zijn de resultaten van beide veilingmechanismen ongeveer gelijk, hoewel de eerste-prijs veiling op geen van de voorgenoemde aspecten slechtere resultaten genereert dan de tweede-prijs veiling.

Alternerende prijs competitie

Statische modellen voor markten met twee bedrijven zijn geïntroduceerd door Cournot (1838) voor competitie in hoeveelheden en door Bertrand (1883) voor competitie in prijzen. In beide duopolie modellen wordt verondersteld dat de bedrijven tegelijk en onafhankelijk van elkaar beslissen over hun strategische variabele en dit slechts eenmalig doen. Voor beide vormen van competitie geldt dat wanneer het aantal periodes waarin de bedrijven met elkaar concureren eindig is, het unieke deelspel perfecte Nash-evenwicht bestaat uit een herhaling van de statische oplossing in iedere periode. Echter, indien het aantal periodes oneindig is dan kunnen vele evenwichten met uitbetalingen boven het competitieve niveau gerealiseerd worden via het 'Folk Theorem' (Friedman, 1971).

De modellen van Cournot en Bertrand gaan ervan uit dat de bedrijven hun strategische variabele op hetzelfde moment aanpassen. Het is echter zeer waarschijnlijk dat bedrijven niet op hetzelfde moment hun strategische variabele kunnen aanpassen door rigiditeiten. Daarom gaan alternerende duopolie modellen ervan uit dat bedrijven om en om hun strategische variabele kunnen aanpassen. De strategische variabele van bedrijven blijft dan minstens twee tijdsperiodes gelijk. Door deze structuur krijgen bedrijven de mogelijkheid om daadwerkelijk te reageren op de huidige hoeveelheid of prijs van hun concurrent.

Cyert en DeGroot (1970) waren de eerste die een alternerende duopolie analyseerden en deden dit voor competitie in hoeveelheden. Baanbrekende wetenschappelijke bijdragen werden later geleverd door Maskin en Tirole (1987) voor competitie in hoeveelheden en Maskin en Tirole (1988) voor competitie in prijzen. In dit proefschrift worden alleen alternerende duopolies met competitie in prijzen geanalyseerd.

In Hoofdstuk 5 wordt een alternerende duopolie met homogene goederen bestudeerd. Maskin en Tirole (1988) hebben aangetoond dat er twee evenwichten bestaan in het geval van een oneindige tijdshorizon: een evenwicht bestaande uit de middelpuntsprijs en een evenwicht bestaande uit prijscyclussen. De middelpuntsprijs bevindt zich halverwege het prijsinterval en maximaliseert de som van de winsten van de bedrijven. De prijscyclus van het andere evenwicht bestaat uit periodes waarin de bedrijven elkaars prijzen onderbieden totdat de prijzen zo laag zijn dat er geen winst meer wordt gemaakt. Vervolgens worden de prijzen in één keer fors verhoogd, waarna het onderbieden weer begint. In dit hoofdstuk wordt met behulp van een experiment geanalyseerd welk van deze twee evenwichten daadwerkelijk wordt gespeeld. Het middelpuntsprijs evenwicht wordt in de meeste observaties waargenomen en er zijn bijna geen prijscyclussen. Met behulp van een numerieke analyse wordt aangetoond dat het unieke deelspel perfecte Nash-evenwicht voor een (lange) eindige tijdshorizon bestaat uit prijscyclussen. Echter, experimenteel wordt ook voor deze tijdshorizon vooral de middelpuntsprijs geobserveerd, hoewel dit geen evenwicht is voor deze situatie. Desondanks worden prijscyclussen nu veel vaker waargenomen dan voor de oneindige tijdshorizon.

Voor competitie in hoeveelheden tonen Maskin en Tirole (1987) aan dat de evenwichtsstrategieën van Cyert en DeGroot (1970) voor de eindige tijdshorizon convergeren naar die van de oneindige tijdshorizon naarmate de tijdshorizon langer wordt. Baye en Ueng (1999) bewijzen dat dit ook geldt voor prijscompetitie met gedifferentieerde goederen en een lineaire vraagfunctie. In Hoofdstuk 6 wordt aangetoond dat dit ook geldt voor een alternerend Hotelling model. Vervolgens wordt het Hotelling model uitgebreid door de vraag van consumenten onderhevig te laten zijn aan exogene schokken. Door de schokken in de vraag is het onmogelijk om het model analytisch op te lossen voor de oneindige tijdshorizon. Echter, de convergentie van de evenwichtsstrategieën van de eindige horizon naar die van de oneindige horizon geldt nog steeds. Daarom worden de evenwichtsstrategieën voor dit model bepaald door een numerieke analyse van de eindige tijdshorizon. De resulterende strategieën leiden tot lagere prijzen indien de vraag hoog is dan wanneer de vraag laag is. Hieruit kan geconcludeerd worden dat prijzen zich countercyclisch bewegen in dit model.

In Hoofdstuk 7 wordt een experimentele analyse gedaan van het alternerende Hotelling model met schokken in de vraag uit Hoofdstuk 6. Drie situaties worden onderzocht en de situaties verschillen in de kans op een schok in de vraag en de toe- en afname van de vraag in het geval van een dergelijke schok. Voor de drie situaties worden er geen significante verschillen gevonden tussen de gemiddelde prijzen en winsten voor volwassen gedrag. Dit wordt veroorzaakt doordat er in de meeste observaties een samenzwering optreedt.

Curriculum Vitae

Kasper Franciscus Hendricus Leufkens was born on December 6, 1981 in Heerlen, The Netherlands. He attended high school (VWO) at the St.-Janscollege in Hoensbroek between 1994 and 2000. Subsequently, he studied Economics at Maastricht University and graduated with a Master's degree in Infonomics in 2004.

Kasper Leufkens was a Ph.D. candidate at the Department of Economics at Maastricht University between 2004 and 2008. His promotor is Prof. dr. Jean-Jacques Herings and his copromotor is Dr. Ronald Peeters. The results of the research done during that period are presented in this dissertation.

Kasper presented his work at various international conferences and parts of this dissertation are, or will be, published in international refereed academic journals. Besides his research activities, Kasper coordinated, lectured, and tutored various courses at the Faculty of Economics and Business Administration and at University College Maastricht. Furthermore, he was an academic advisor at University College Maastricht, and co-founder and board member of the Ph.D. Activity Committee of the Faculty of Economics and Business Administration.