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Leadership in Public Good Provision: a Timing Game Perspective

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Abstract

We address in this paper the issue of leadership when two governments provide public goods to their constituencies with cross border externalities as both public goods are valued by consumers in both countries. We study a timing game between two different countries: before providing public goods, the two policymakers non-cooperatively decide their preferred sequence of moves. We establish conditions under which a first- or second-mover advantage emerges for each country, highlighting the role of spillovers and the strategic complementarity or substitutability of public goods. As a result we are able to prove that there is no leader when, for both countries, public goods are substitutable. When public goods are complements for both countries, both countries may emerge as the leader in the game. Hence a coordination issue arises. We use the notion of risk-dominance to select the leading government. Lastly, in the mixed case, the government for whom public goods are substitutable becomes the leader.

1 Introduction

The issue of leadership is much studied in industrial economics, in particular in relation with duopoly theory, but much less in public economics. Yet this issue cannot be under-estimated in this domain of economics. There are many examples of interdependent decisions made by independent public authorities. To name a few examples, let us mention the case of cross-border externalities, federations, military alliances, environmental issues, transnational public goods. These public authorities rule clearly differentiated jurisdictions. It is common to oppose core vs periphery jurisdictions, large vs small governments, or differences in capacities (in military alliances for example). These differences often translate in strategic asymmetries, as one jurisdiction assumes a leading role with respect to others, setting the agenda, deciding and constraining other jurisdictions, considered and acting as followers.

The present paper aims at formally addressing the issue of leadership in public economics, focusing on the problem of providing public goods in the presence of cross-jurisdiction spillovers.[1] We aim at understanding who is leader (follower) in providing public goods and why. We also want to characterize the consequences of leadership relative to its absence for each jurisdiction involved in this setting. In order to answer this question, we adopt a game-theoretic approach and define leadership as the action of moving first.[2]

As our approach is parallel to the one adopted in industrial economics to study leadership in duopolies, a brief summary of this research is relevant here.[3] First, taking as given the order of moves in a duopoly, in other words the presence of a leader, industrial economists were interested in assessing the respective advantages of each firm: that is, in comparing the first-mover and second-mover advantages, defined as

[1] Of course, there are many more issues in public economics which can be related to the problem of leadership. We return to this point in the conclusion.

[2] See Kempf and Taugourdeau (2005) for a first exploration of Stackelberg games over fiscal decisions in a two-country model.

[3] Vives (1999) provides a survey of this literature.

the payoffs for this firm of playing either first or second. Once it was recognized that the two roles of leader and follower lead to different advantages, the next step was to attempt to endogenize the moves, that is consider the sequence of moves as the equilibrium result of non-cooperating strategies played by the two firms given their own relative characteristics and their prospective advantage as a leader or as a follower. d'Aspremont and Gerard-Varet (1980), Gal-Or (1985) and Dowrick (1986) proposed an initial analysis, which has been extended by Hamilton and Slutsky (1990), Pal (1996), Amir and Grilo (1999), van Damme and Hurkens (1999) and more recently by van Damme and Hurkens (2004) or Amir and Stepanova (2006). The determination of simultaneity versus sequentiality of moves, as well as the assignment of roles of the players in the latter case, is then completely endogenous.

In this paper, we shall follow the same logic. First, using a simple yet fairly general two-jurisdiction model of public good provision and externalities (linked to the public goods) across jurisdictions, we shall characterize the first-mover and second-mover advantages. Then we shall set-up a timing game, or equivalently the two-period action commitment game proposed by Hamilton and Slutsky (1990). In the first stage, the policymaker of each jurisdiction states which role (leading or following) it prefers. Once the solution of this stage is obtained, that is when the two roles are attributed to the two jurisdictions, given their statements, the resulting game is played in the second stage of the timing game.[4] It may happen that two equilibria emerge as the outcome of the first stage: that is, two sequences of moves are solutions of the two-stage game. In this case, in order to break-up this multiplicity, we resort to the concepts of Pareto-dominance and risk-dominance offered by Harsanyi and Selten (1988). Applying this concept to two different specifications of our general model, we show how it allows a simple and straightforward explanation

[4]Another presentation of this game has been proposed by van Damme and Hurkens (1999): each player has to move in one of two periods; choices are simultaneous, but if one player choose to move early while the other moves late, the latter behaves as a Stackelberg follower, the former as a leader.

of which jurisdiction ends up as the leader.

We prove that both the magnitude of the moves' advantages and the existence and identity of a leader depend on the characteristics of public goods in the utility function. When for both countries the public goods are substitutes, both jurisdictions experience a first-mover advantage and the solution of the timing game is the Nash simultaneous game. When for both countries the public goods are complements, at least one jurisdiction benefits from a second-mover advantage when public goods are complements and the two sequential "basic games" are solutions of the timing game. Finally, in the mixed case, the country for which the public goods are substitutes benefits from a first-mover advantage and is the leader. Then we prove that the use of the risk-dominance criterion always allows us to identify the leader and provides an explanation of this leadership in each of the two specifications of the utility function we consider. Overall, the issue of leadership is solved for all cases.

Our work refers to the study of inter-jurisdictional spillovers. Several recent contributions can be questioned through our results. The literature on centralization and international unions only considers simultaneous games where countries choose to cooperate or not (see for instance, Lockwood (2002), Besley and Coate (2003) or Alesina, Angeloni, and Etro (2005)). It can be deduced from our analysis that the usual benchmark used to appreciate political centralization, i.e. the simultaneous Nash equilibrium, may not be relevant when public goods are complements. The works on global public goods (Kaul, Grunberg, and Stern (1999) or Barrett (2007)) are also related to our analysis since we establish a taxonomy of international spillovers and their consequences in term of their global provision and the "natural" (since endogenous) emergence of a leader. We emphasize that the free rider issue is not as strong as predicted in the literature, when public goods are complements. Indeed, in this situation, a sequential situation emerges as a Subgame Perfect Equilibrium (SPE), which involves a higher provision of public goods to this at the simultaneous Nash equilibrium.[5]

Finally, our analysis might be fruitful in political science where the concepts of hegemony and leadership, often confounded, remain a hot topic (see Keohane (1984) or Pahre (1999)). By considering the leadership as endogenous, we are able to formalize a clear distinction between hegemony and leadership: hegemony would be a structural variable (an *ad hoc* assumption in the utility functions), while leadership would characterize the solution of a timing game.

The next section sets up the two-jurisdiction model we use and studies the three non-cooperating games, with either synchronous or sequential moves, that can be played, and derives the first- and second-mover advantages for a very general specification of the utility function. The third section tackles the selection of a leader by means of a timing game, relying when necessary on the concept of risk-dominance. The last section concludes.

2 Public good provision non-cooperating games.

We consider an economy consisting of two jurisdictions or countries (A and B). There is no mobility accross countries. Their populations are normalized to 1. Each country i ($\in \{A, B\}$) provides a local public good, in quantity g_i, which generates some externalities for the other country, namely country j ($\neq i$). There is perfect information. Inhabitants in country i are assumed to have preferences that can be

[5] Our analysis may also be linked to the work of Varian (1994), who considers a sequential game of private contribution. This author highlights that the ability to commit to a contribution reinforces the free-rider problem and he concludes that the amount of public good sequentially provided is never larger than the amount simultaneously provided. We extend his analysis in two ways: first we consider the leader as endogenous; second, since Varian (1994) focuses exclusively on the contribution game proposed by Bergstrom, Blume, and Varian (1986), he considers only public goods as substitutes, we study here all the possible configurations. Whereas the conclusions of Varian (1994) remain valid for substitutes and an endogenous leadership, they do not hold anymore as soon as complement public goods considered.

described by the following utility function:

$$U_i(g_i, g_j) = y - g_i + \Psi^i(g_i, g_j).$$

The function $\Psi^i(g_i, g_j)$ represents the public goods provision function, entering linearly into the utility function of the representative agent in country j. We make the following assumptions:

$$\forall i = A, B, \qquad \Psi_1^i(g_i, g_j) \equiv \frac{\partial \Psi^i(g_i, g_j)}{\partial g_i} > 0, \quad \Psi_2^i(g_i, g_j) \gtrless 0,$$
$$\Psi_{11}^i(g_i, g_j) < 0, \quad \Psi_{12}^i(g_i, g_j) \gtrless 0.$$

The derivative $\Psi_{12}^i(g_i, g_j)$ plays a crucial role in this paper. When for any (g_i, g_j), $\Psi_{12}^i(g_i, g_j)$ is positive, we shall say that the two public goods are complements: an increase in the provision of g_j increases the marginal utility of good g_i. When it is negative, they are substitutes. When it is nil, the three studied games yield the same equilibrium payoffs. This last assumption is often implicit in the literature on inter-jurisdictional spillovers and it appears as an important limit to these analysis (see for instance Lockwood (2002), Besley and Coate (2003) or Dur and Roelfsema (2005)).[6]

Despite the use of a quasi-linear utility function which is the workhorse of this literature (see Batina and Ihori (2005)), our formalization remains relatively general with respect to the quoted works. It allows us to encompass several works concerning transnational or global public goods provisions, international organizations or fiscal federalism where the jurisdiction correspond to subnational governments. Moreover, the quasi-linear form involves a strict equivalence between the function $\Psi_1^i(g_i, g_j)$ and the marginal rate of substitution (MRS) between private and public consumptions in country i.

[6] Lockwood (2002) specifies $\Psi^i(g_i, g_j) = g_i + g_j$; Besley and Coate (2003), $\Psi^i(g_i, g_j) = (1-\alpha)\log g_i + \alpha \log g_j$ with $\alpha \in [0,1]$; Dur and Roelfsema (2005), $\Psi^i(g_i, g_j) = b(g_i) + \alpha b(g_j)$.

We consider three possible situations for the determination of non-cooperative national policies, defining three "basic games": the simultaneous Nash game (G^N) and the two Stackelberg games (G_i^S where country i is leader and country j follower).

2.1 Characterizing the simultaneous game (G^N).

At the simultaneous non-cooperative equilibrium, each country chooses its own policy taking as given the provision of the other public good, that without taking in account the externalities its decision creates on the other country. We denote by $\left(g_A^N, g_B^N\right)$ the Nash equilibrium of this game. This pair must verify the following set of definitions:

$$\begin{cases} g_A^N \equiv \arg\max\limits_{g_A \geqslant 0} U_A\left(g_A, g_B\right), & g_B \text{ given,} \\ g_B^N \equiv \arg\max\limits_{g_B \geqslant 0} U_B\left(g_B, g_A\right), & g_A \text{ given.} \end{cases}$$

The non-cooperative equilibrium levels of public good provisions are implicitly given by:

$$\forall i \in \{A, B\}, \quad -1 + \Psi_1^i\left(g_i^N, g_j^N\right) = 0 \tag{1}$$

Following Vives (1999), a sufficient condition for the best-replies to be contractions is: $\frac{\partial^2 U_i(g_i,g_j)}{\partial g_i^2} + \left|\frac{\partial^2 U_i(g_i,g_j)}{\partial g_i \partial g_j}\right| < 0$, which yields in our model

$$\Psi_{11}^i\left(g_i, g_j\right) + \left|\Psi_{12}^i\left(g_i, g_j\right)\right| < 0, \tag{2}$$

This insures the existence and the unicity of the Nash Equilibrium. In the following, we always assume that condition (2) is satisfied.

2.2 Characterizing the Stackelberg game (G_i^S).

Under this scenario, we assume that one of the two jurisdictions denoted by i is the first player to set its provision of g_i, and then jurisdiction j (the follower F) chooses its own level g_j. In other words, jurisdiction i behaves as a Stackelberg leader (L).

Applying backward induction, we first consider the maximization program of the follower which is given by:

$$g_j^F(g_i) \equiv \arg\max_{g_j} U_j(g_j, g_i), \quad g_i \text{ given.}$$

The FOC, which is equivalent to (1) for country j, yields:

$$-1 + \Psi_1^j\left(g_j^F(g_i), g_i\right) = 0 \tag{3}$$

Applying the envelop theorem, we remark that:

$$\frac{dg_j}{dg_i} = -\frac{\Psi_{12}^j(g_j, g_i)}{\Psi_{11}^j(g_j, g_i)} \lesseqgtr 0 \Leftrightarrow \Psi_{12}^j(g_j, g_i) \lesseqgtr 0$$

The leader solves the following program:

$$g_i^L \equiv \arg\max_{g_i} U_i\left(g_i, g_j^F(g_i)\right)$$

which implies the following FOC:

$$-1 + \Psi_1^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) + \frac{dg_j^F(g_i)}{dg_i}\Psi_2^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) = 0$$

or equivalently,

$$-1 + \Psi_1^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) - \frac{\Psi_{12}^j\left(g_j^F\left(g_i^L\right), g_i^L\right)}{\Psi_{11}^j\left(g_j^F\left(g_i^L\right), g_i^L\right)}\Psi_2^i\left(g_i^L, g_j^F\left(g_i^L\right)\right) = 0 \tag{4}$$

The SOC is assumed to be satisfied.

2.3 Comparison of the levels of public good provisions.

Our analysis offers a taxonomy of inter-jurisdictional interactions. Indeed, we consider six cases depending on the sign of the spillovers and the presence of complementarity or substitutability among national public goods.[7] The literature focuses essentially on one kind of interactions, when either complementarity or substitutability is considered.[8] Indeed, many authors implicitly assume a standard technology of agregation of jurisdiction's contribution, the weighted summation, which may be relied to the canonical model of Bergstrom, Blume, and Varian (1986). This hypothesis involves substitutability among national public goods. Thus, the standard formalization of interjurisdictional interactions corresponds to the case where $\Psi_2^{i,j}(.) > 0$ and $\Psi_{12}^{i,j}(.) < 0$. Under these assumptions, Bloch and Zenginobuz (2007) renewes the analysis of the issue of local public good provision with spillovers and highlights their effects. Ellingsen (1998), Redoano and Scharf (2004), and Alesina, Angeloni, and Etro (2005) consider the issues of international agreements or centralization under similar assumptions.[9]

Hirshleifer (1983)[10] proposed two other agregation technologies: the "weakest-link" and the "best-shot". These functions have been used in the context of global public goods (see Kaul, Grunberg, and Stern (1999) or Barrett (2007)). These two cases generate several difficulties since the underlying functions are not differentiable. Cornes (1993) proposed a class of functions which encompasses the different preceeding cases. Cornes and Hartley (2007) renewes the analysis of Bergstrom, Blume, and Varian (1986) by considering a CES composition function. The case of complements and positive spillovers ($\Psi_2^{i,j}(.) > 0$ and $\Psi_{12}^{i,j}(.) > 0$) might then correspond to the analysis of defence expenditures between two allied countries (see

[7] We do not assume that the signs of the spillovers differ for the two countries.

[8] Several papers assume functional forms such that $\Psi_{12}^{i,j}(.) = 0$.

[9] These authors use a more restrictive function than our, since they assume: $\Psi^i(g_i, g_j) = v(g_1 + \beta g_2)$, with $0 < \beta < 1$ and $v''(.) < 0$.

[10] See Hirshleifer (1985) for a correction.

Ihori (2000)) and some works on the global public goods, higlighting a better-shot agregation technology (see Barrett (2007)). Finally, to our knowledge, the mixed cases have not been considered in the literature. Different public goods can logically be linked to different cases, depending on their complementarity or substitutability and the sign of their induced spillovers. For example, national programs on airfield infrastructure building can be considered as complementary and generating positive externalities. On the other hand, national plans for biodiversity conservation can be substitutes and related to positive externalities.

The comparison of the levels of public goods requires to consider six cases depending on the sign of $\Psi_2(.)$ and $\Psi_{12}(.)$ (when it is assumed to be non-null).[11] In the appendix, we establish the following PROPOSITION:

Proposition 1 *The public good provision levels solutions of the Nash and Stackelberg games are such that:*
(i) If $\Psi_2^i(.) > 0$ *and* $\Psi_{12}^i(.) > 0$ *(positive spillovers and complements):*

$$\begin{cases} g_i^L > g_i^F > g_i^N \\ g_j^L > g_j^F > g_j^N \end{cases} \quad or \quad \begin{cases} g_i^L > g_i^F > g_i^N \\ g_j^F > g_j^L > g_j^N \end{cases}$$

(ii) If $\Psi_2^i(.) < 0$ *and* $\Psi_{12}^i(.) > 0$ *(negative spillovers and complements):*

$$\begin{cases} g_i^N > g_i^F > g_i^L \\ g_j^N > g_j^F > g_j^L \end{cases} \quad or \quad \begin{cases} g_i^N > g_i^L > g_i^F \\ g_j^N > g_j^F > g_j^L \end{cases}$$

(iii) If $\Psi_2^i(.) > 0$ *and* $\Psi_{12}^i(.) < 0$ *(positive spillovers and substitutes):*

$$\begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^F > g_j^N > g_j^L \end{cases}$$

(iv) If $\Psi_2^i(.) < 0$ *and* $\Psi_{12}^i(.) < 0$ *(negative spillovers and substitutes):*

$$\begin{cases} g_i^L > g_i^N > g_i^F \\ g_j^L > g_j^N > g_j^F \end{cases} .$$

(v) If $\Psi_2^{i,j}(.) > 0$ *and* $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$ *(positive spillovers for both countries,*

[11] The equilibria of the different games are identical when $\Psi_{12}^i(.) = 0$, that is in the absence of any interaction between the two countries.

complements for country i and substitutes for country j):

$$\begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^L > g_j^F > g_j^N \end{cases} \quad or \quad \begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^F > g_j^L > g_j^N \end{cases}$$

(vi) If $\Psi_2^{i,j}(.) < 0$ and $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$ (negative spillovers for both countries, complements for country i and substitutes for country j):

$$\begin{cases} g_i^L > g_i^N > g_i^F \\ g_j^N > g_j^L > g_j^F \end{cases} \quad or \quad \begin{cases} g_i^L > g_i^N > g_i^F \\ g_j^N > g_j^F > g_j^L \end{cases}$$

Proof. See Appendix A.1. ■

Consider the first case ($\Psi_2^i(.) > 0$ and $\Psi_{12}^i(.) > 0$). When the leader, say A, increases its level of provision relative to the Nash equilibrium value, it induces its follower, B, to increase its own provision level because of the complementarity property between the two public goods. In turn, this increases the leader's payoff because of the positive externality assumption. Hence we get $g_A^L > g_A^N$ and $g_B^F > g_B^N$. However it may happen that $g_A^F > g_A^L$. This comes from the differences in the $\Psi^i(g_i, g_j)$ functions. It may happen that the externalities and interaction effects are much stronger from B to A, than from A to B. Then g_A^L is very close to g_A^N and g_B^L is very far from g_B^N as well as g_A^F from g_A^N. This explains the two possible rankings obtained for (i). The other cases may be explained by means of similar reasonings, that is by the interplay between the externality effect and the interaction effect.

When the two countries are symmetric, that is when $\Psi^i(g_i, g_j) = \Psi(g_i, g_j)$, $\forall i$, the mixed cases disappear. It is immediate to derive from the previous proposition the comparison between the public good provisions under this assumption:

Corollary 1 *When $\Psi^i(g_i, g_j) = \Psi(g_i, g_j)$, $\forall i$, the public good provision levels solutions of the Nash and Stackelberg games are such that:*

(i) If $\Psi_2(.) > 0$ and $\Psi_{12}(.) > 0$ (positive spillovers and complements):

$$g^L > g^F > g^N,$$

(ii) If $\Psi_2(.) < 0$ and $\Psi_{12}(.) > 0$ (negative spillovers and complements):

$$g^N > g^F > g^L,$$

(iii) If $\Psi_2(.) > 0$ *and* $\Psi_{12}(.) < 0$ *(positive spillovers and substitutes):*

$$g^F > g^N > g^L,$$

(iv) If $\Psi_2(.) < 0$ *and* $\Psi_{12}(.) < 0$ *(negative spillovers and substitutes):*

$$g^L > g^N > g^F.$$

Proof. See APPENDIX A.1. ■

This proposition can be explained following the same reasoning as for the previous proposition. We notice that the coexistence of two rankings in cases (i) and (ii) disappears with the asymmetries between the two $\Psi^i(g_i, g_j)$ functions.

2.4 First-mover and second-mover advantages.

Given these rankings, we can compute the first- and second-mover advantages. These advantages has been extensively used and discussed in the duopoly theory.[12] Here they will allow us to understand the stakes linked to the possible existence of leadership in public good provision. Following Amir and Stepanova (2006) we define the notions of "first-" and "second-mover advantage" as follows:

Definition 1 *Country i has a first-mover advantage (a second-mover advantage) if its equilibrium payoff in the Stackelberg game in which it leads, denoted by G_i^S, is higher (lower) than in the Stackelberg game in which it follows (G_j^S).*

Formally, country i benefits from a first-mover advantage when

$$U_i\left(g_i^L, g_j^F\right) > U_i\left(g_i^F, g_j^L\right)$$

and from a second-mover advantage when

$$U_i\left(g_i^F, g_j^L\right) > U_i\left(g_i^L, g_j^F\right).$$

Given this definition we offer the following PROPOSITION:

[12] See Bagwell and Wolinsky (2002).

Proposition 2 *(i) If public goods are complements in both countries* $(\Psi_{12}^{i}(.) > 0)$, *at least one country has a second-mover advantage;*
(ii) If public goods are substitutes in both countries $(\Psi_{12}^{i}(.) < 0)$, *each country has a first-mover advantage;*
(iii) If $\Psi_{12}^{i}(.) > 0 > \Psi_{12}^{j}(.)$, *country* j, *the country for which the public goods are substitutes, has a first-mover advantage. Country* i *has a first-mover advantage (a second-mover advantage) when externalities from the foreign public good are positive (negative).*

Proof. see APPENDIX A.2. ■

Let us first focus on the case when the public goods are complements for both countries (though affecting them differently) and spillovers are positive. At least one player prefers to be a second-mover as then, it optimally benefits from the higher provision decided by the first-mover, which generate positive externalities. To be second-mover allows this country to reduce its own provision of public good, in other terms it free-rides the leader. However this free-riding is less than in the simultaneous Nash game where spillovers are positive. The other cases can be explained by similar reasonings. In the case of substitutable public goods and positive spillovers, which is the case studied by Varian (1994), our result is consistent with his.

3 Selecting a leader through a timing game.

We have just seen that the existence and identity of a leader matters because of first- or second-mover advantages. This puts to the fore the issue of determining whether a leader emerges, and if yes, its identity. In order to address this issue, we want to endogenously define the order of moves, and not take it as given as in the previous section by resorting to a timing game, following the seminal study of Hamilton and Slutsky (1990).

A timing game is a sequential game in the first stage of which players non-cooperatively choose their preferred order of moves. Once the order of moves has been defined, players act accordingly in the second stage, that is non-cooperatively choose their level of public good provision, applying the order of moves selected in the first stage. In other words, a timing game is an extended game $\widetilde{G}$ which encompasses the preceding games. Following Hamilton and Slutsky (1990) and Amir and Stepanova (2006) we restrict our attention to the SPE of $\widetilde{G}$.

More precisely, $\widetilde{G}$ is defined as follows: at the first or *preplay* stage to the "basic game", players simultaneously and independently of each other decide whether they prefer to move early or late in the "basic game". In the same way as Hamilton and Slutsky (1990), we assume that if country i chooses leadership (strategy *Leads*), it commits itself to setting its national policy as leader and if it chooses to be a follower (*Follows*), it commits itself to following the other country's decision. Once the timing choice of each player is announced, the order of moves is defined according to the following rules: if countries choose complementary roles, there preferred moves are enforced and one of the two possible Stackelberg games will emerge. If both choose lead or follow, as their decisions are inconsistent, it is decided that the simultaneous non-cooperative game will be enforced. That is, the second stage corresponds to the realization of the selected "basic game": it is one of the three games studied in the previous section.

Notice that, in choosing their role (leader or follower), the two players also choose which kind of behavior they prefer. Therefore this game has the following normal form:[13]

Country B

Country A		*Leads*	*Follows*
	Leads	U_A^N, U_B^N	U_A^L, U_B^F
	Follows	U_A^F, U_B^L	U_A^N, U_B^N

[13] We remark that the literature on endogenous timing remains divided to qualify the situation where both players choose to lead. Indeed, Dowrick (1986) and more recently van Damme and Hurkens (1999) consider a Stackelberg warfare, where both countries apply their action as a leader. In contrast, Hamilton and Slutsky (1990) or Amir and Stepanova (2006) apprehend this situation as the static Nash game. Hamilton and Slutsky (1990) (p. 42) emphasize that Stackelberg warfare can occur only through error, since the underlying strategy of one player is not consistent with the other player's strategy.

where $U_i^N = U_i\left(g_i^N, g_i^N\right)$, $U_i^F = U_i\left(g_i^F, g_i^L\right)$ and $U_i^L = U_i\left(g_i^L, g_i^F\right)$.

3.1 Solutions to the leadership problem.

The solution to this reduced form game is equivalent to characterizing the solution to the leadership problem. There is no leader when both government choose the same action; a leader emerges when they choose complementary roles. The result of the timing game can be related to the nature of the interactions between the two countries. We obtain the following PROPOSITION:

Proposition 3 *(i) If public goods are complements, the subgame perfect equilibria are the two Stackelberg situations whatever is the sign of spillovers;*
(ii) If public goods are substitutes, the subgame perfect equilibrium is the simultaneous moves situation whatever is the sign of spillovers;
(iii) If public goods are complements for country i and substitutes for country j, the subgame perfect equilibrium is the Stackelberg situation where country j leads and country i follows whatever is the sign of spillovers.

Proof. See APPENDIX A.3. ■

The following table summarizes the results stated in Propositions (2) and (3):[14]

$\Psi_2^{A,B}(.) \gtrless 0$	$\Psi_{12}^{B}(.) > 0$	$\Psi_{12}^{B}(.) < 0$
$\Psi_{12}^{A}(.) > 0$	Second-mover advantage for A or B SPE: (L, F) or (F, L).	First-mover advantage for B SPE: (F, L).
$\Psi_{12}^{A}(.) < 0$	First-mover advantage for A SPE: (L, F).	First-mover advantage for A and B SPE: (L, L) or (F, F).

We remark that the nature of the spillovers (positive or negative) does not affect the presence of a first- or second-mover advantage, nor the kind of the SPE. Given this proposition, it is straightforward to see that there is no leader in the case of substitutable public goods as the solution to the timing game is the Nash game, and that the leader in the mixed case is the government for whom the public goods are substitutable. In the former case, this outcome comes from the fact that both countries would like to exploit a first-mover advantage and suffer as a second-mover compared to the Nash solution. Consequently *Leads* is a dominant strategy for each of them and no leader emerges from the timing game.

In the latter case (the mixed case), the presence of a leader comes from the fact that the government which benefits from complementary public goods prefers being a follower to the Nash case, whereas the government which benefits from substitutable public goods always prefers to lead. There is then a unique SPE.

[14] We denote by (L, F) the SPE where country A leads and B follows, and by (F, L) the reverse.

On the other hand, when the public goods are complements, there are two possible Stackelberg equilibria solutions to the timing game. This comes from the fact that in any possible case with complement public goods, both the first- and second-movers are better off than under a Nash equilibrium. This raises a coordination issue: how to solve the tie? In the next section, we provide a solution for the selection of a leader by means of the risk-dominance criterion.

3.2 Selecting a leader in the complementarity case.

To solve the coordination issue when the public goods are complements, more formally, when $\Psi_{12}^{i}(.) > 0$, two criteria can be used for the selection of the leader in this case: the Pareto-dominance and the risk-dominance. Given PROPOSITION 2, it is clear that no equilibrium Pareto-dominates the other one when both firms have a second-mover advantage. Hence, following Harsanyi and Selten (1988), we have to turn to the risk-dominance criterion. It amounts to a minimization of the risk of a coordination failure due to strategic uncertainty.[15]

Harsanyi and Selten (1988) define the concept of risk-dominance as follows:

Definition 2 *An equilibrium risk-dominates another equilibrium when the former is less risky than the latter, that is the risk-dominant equilibrium is the one for which the product of the deviation losses is the largest.*

Risk-dominance allows a simple characterization for 2×2 games with two Nash equilibria: the equilibrium *(A Leads, B Follows)* risk-dominates the equilibrium *(A Follows, B Leads)* if the former is associated with the larger product of deviation losses. More formally, we have: *(A Leads, B Follows)* risk-dominates *(A Follows, B*

[15] This uncertainty comes from the fact that a player is always unsure of the other player's move because of the multiplicity of solutions. Consider then a mixed-strategy equilibrium, where p and q are the probabilities corresponding to the choice of *Leads* by countries A and B, respectively. A pure-strategy equilibrium risk-dominates the other one if it has a larger basin of attraction in the (p, q) space.

Leads) if and only if:

$$\Pi \equiv \left(U_A^L - U_A^N\right)\left(U_B^F - U_B^N\right) - \left(U_A^F - U_A^N\right)\left(U_B^L - U_B^N\right) > 0. \tag{5}$$

In our framework, Pareto-dominance always involves risk-dominance, the inverse is not true. There is here no trade-off between risk and payoff-dominance. When $g_i^L > g_i^F > g_i^N$ for both countries, Pareto-dominance is not relevant. We then have to consider only the notion of risk-dominance to solve the coordination issue. But, as stressed by Amir and Stepanova (2006), a resolution of the problem does not seem possible without resorting to an explicit specification of the payoff functions. These authors use a linear demand function as in van Damme and Hurkens (1999).

We will consider in the following sub-sections two cases in the presence of asymmetries between countries, relying on two specifications of the $\Psi^i(.)$ functions.[16] These cases are characterized by complementary public goods, but they exhibit different assumptions on externalities.

3.3 A Cobb-Douglas specification.

We consider a Cobb-Douglas specification of the function $\Psi^i(.)$, that is, we assume:[17]

$$\begin{array}{c} \Psi^i(g_i, g_j) = g_i^{\alpha_i} g_j^{\gamma_i}, \quad \text{with} \\ i, j \in \{A, B\}, \quad j \neq i. \end{array} \quad \left\{ \begin{array}{c} (\alpha_i, \alpha_j, \gamma_i, \gamma_j) \in \,]0,1[^4 \\ \alpha_i + \gamma_i < 1 \\ \alpha_j + \gamma_j < 1 \end{array} \right. \tag{6}$$

[16] Obviously this criterion is not relevant in the symmetric case. In the symmetric case, both possible equilibria lead to the same value of Π. Therefore, the risk-dominance criterion does not apply. A solution could then be to select an equilibrium randomly. But notice that the two equilibria are not equivalent from the point of view of a particular government. Depending on which equilibrium is chosen, one country loses and the other gains with respect to the discarded equilibrium.

[17] The condition (2) of existence and uniqueness of a simultaneous Nash equilibrium involves:

$$\forall g_A, g_B, \quad \frac{\gamma_B}{1-\alpha_B} < \frac{g_A}{g_B} < \frac{1-\alpha_A}{\gamma_A}$$

The externality effect is always positive for both countries. We obtain the following values for the levels of public good provision:

	Nash Equilibrium G^N	Country A leads G^S_A	Country B leads G^S_B
Country 1	$g^N_A = \alpha_A^{\frac{1-\alpha_B}{\delta}} \alpha_B^{\frac{\gamma_A}{\delta}}$	$g^L_A = \alpha_B^{\frac{\gamma_A}{\delta}} \Omega_A^{\frac{1-\alpha_B}{\delta}}$	$g^F_A = \alpha_A^{\frac{1-\alpha_B}{\delta}} \Omega_B^{\frac{\gamma_A}{\delta}}$
Country 2	$g^N_B = \alpha_A^{\frac{\gamma_B}{\delta}} \alpha_B^{\frac{1-\alpha_A}{\delta}}$	$g^F_B = \alpha_B^{\frac{1-\alpha_A}{\delta}} \Omega_A^{\frac{\gamma_B}{\delta}}$	$g^L_B = \alpha_A^{\frac{\gamma_B}{\delta}} \Omega_B^{\frac{1-\alpha_A}{\delta}}$

where $\delta = (1-\alpha_A)(1-\alpha_B) - \gamma_A\gamma_B > 0$ since $\alpha_i + \gamma_i < 1$, $\Omega_A = \alpha_A + \frac{\gamma_A\gamma_B}{1-\alpha_B} < 1$ and $\Omega_B = \alpha_B + \frac{\gamma_A\gamma_B}{1-\alpha_A} < 1$. Since $\Omega_i > \alpha_i$, we have $g^N_i < g^L_i$ and $g^N_i < g^F_i$ without ambiguity. However, the comparison between g^F_i and g^L_i remains ambiguous. We obtain the three possible rankings established in Proposition 1.[18]

The product of deviation losses is given by:

$$\begin{aligned}\Pi(.) \;=\; & (1-\alpha_A)(1-\alpha_B)\,\alpha_A^{-1+\frac{1-\alpha_B+\gamma_B}{\delta}}\,\alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}}. \\ & \left\{\left(\left(\frac{\Omega_A}{\alpha_A}\right)^{\frac{\gamma_B}{\delta}}-1\right)\left[\frac{1-\Omega_A}{1-\alpha_A}\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}-1\right]\right. \\ & \left.-\left(\left(\frac{\Omega_B}{\alpha_B}\right)^{\frac{\gamma_A}{\delta}}-1\right)\left[\frac{1-\Omega_A}{1-\alpha_A}\left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A}{\delta}}-1\right]\right\}\end{aligned}$$

(See Appendix A.4). Hence we can establish the following

Proposition 4 *Assuming $\alpha_A = \alpha_B$, the equilibrium (*A Leads, B Follows*) risk-dominates the other equilibrium (*A Follows, B Leads*) when $\gamma_A < \gamma_B$, and the equilibrium (*A Follows, B Leads*) risk-dominates the other equilibrium (*A Leads, B Follows*) when $\gamma_A > \gamma_B$.*

[18] Note that $g^F_A > g^L_A$ and $g^F_B > g^L_B$ is impossible conforming to Lemma 1. Indeed, we have:

$$\left\{\begin{array}{l} g^L_A < g^F_A \\ g^L_B < g^F_B \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} 1+\frac{\gamma_A\gamma_B}{\alpha_A(1-\alpha_B)} < \left(1+\frac{\gamma_A\gamma_B}{(1-\alpha_A)\alpha_B}\right)^{\frac{\gamma_A}{1-\alpha_B}} \\ 1+\frac{\gamma_A\gamma_B}{(1-\alpha_A)\alpha_B} < \left(1+\frac{\gamma_A\gamma_B}{\alpha_A(1-\alpha_B)}\right)^{\frac{\gamma_B}{1-\alpha_A}} \end{array}\right.$$

$$\Rightarrow \quad 1+\frac{\gamma_A\gamma_B}{\alpha_A(1-\alpha_B)} < \left(1+\frac{\gamma_A\gamma_B}{\alpha_A(1-\alpha_B)}\right)^{\frac{\gamma_A\gamma_B}{(1-\alpha_B)(1-\alpha_A)}}$$

which is impossible since $\frac{\gamma_A\gamma_B}{(1-\alpha_A)(1-\alpha_B)} < 1$.

Proof. For $\alpha_A = \alpha_B = \alpha$, we have $\Omega_A = \Omega_B = \Omega$ and then

$$\Pi(.) = (1-\alpha)^2 \alpha^{-2+\frac{2-2\alpha+\gamma_A+\gamma_B}{\delta}} \left[\frac{1-\Omega}{1-\alpha}\left(\frac{\Omega}{\alpha}\right)^{-1+\frac{1-\alpha}{\delta}} - 1\right]\left[\left(\frac{\Omega}{\alpha}\right)^{\frac{\gamma_B}{\delta}} - \left(\frac{\Omega}{\alpha}\right)^{\frac{\gamma_A}{\delta}}\right]$$

Thus we have $\gamma_B > \gamma_A$ involves $\Pi(.) > 0$.□ ■

Notice that $\alpha + \gamma_i < 1$ involves $g_i^L > g_i^F$ $(i = A, B)$. When $\gamma_A < \gamma_B$, the spillover effect is smaller from B to A than from A to B: B values more the foreign public good than it is the case for A. Hence, B has more interest in *Follows* than country A. Both countries lose in case of simultaneous moves. Hence, since both players know that B is more interested in being the follower than A and that a consistent choice of moves delivers for both players positive advantages compared with the outcome of disagreements over moves, the equilibrium is that A chooses *Leads* and B *Follows*. B is able to push A to assume leadership by selecting *Follows* and thus can extract the second-mover advantage. The reverse explanation holds when $\gamma_A > \gamma_B$.

In the Appendix A.4, we explore other combinations of the parameters, allowing the four coefficients α_i and γ_i to vary and we give a set of sufficient conditions so that the timing game ends up with one leader.

More precisely, if

$$\begin{cases} \frac{\gamma_A\gamma_B(1-\alpha_B-\gamma_A\gamma_B)}{(1-\alpha_B)[\gamma_A\gamma_B+(1-\alpha_B)\log 2]} < \alpha_A < \frac{\gamma_A\gamma_B}{1-\alpha_B} \\ 0 < \alpha_A < \alpha_A^* < \alpha_B < 1 \\ 0 < \gamma_A < \gamma_B < 1 \\ \alpha_A + \gamma_A < 1 \\ \alpha_B + \gamma_B < 1 \end{cases} \tag{7}$$

then,

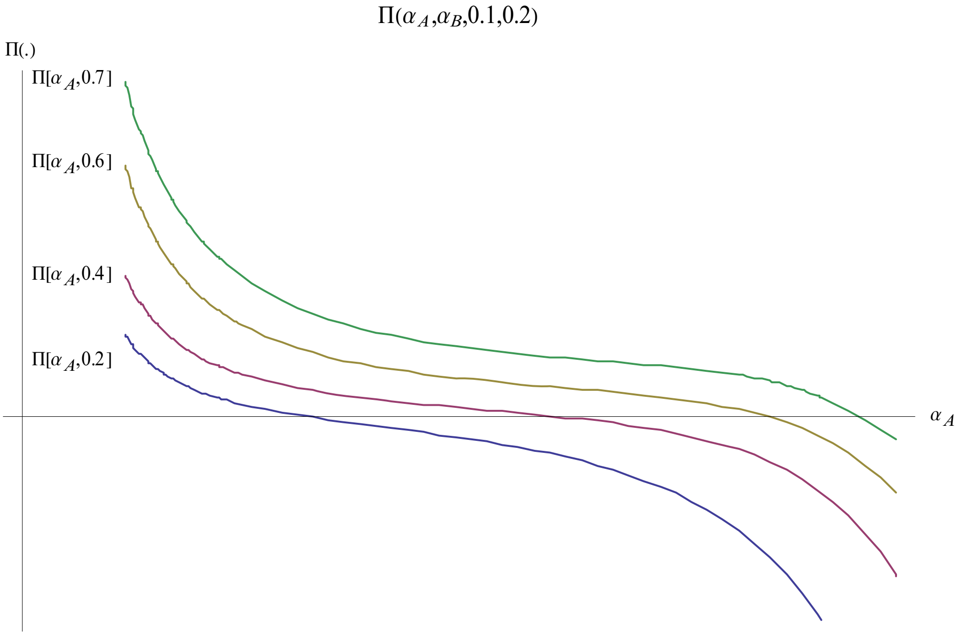
$$\Pi(\alpha_A, \alpha_B, \gamma_A, \gamma_B) > 0$$

When the set of conditions given above is satisfied, using the risk-dominance criterion, the outcome of the timing game is that A leads and B follows.

If the two relative coefficients pondering the national public goods (α_A and α_B) are sufficiently distant ($\alpha_A << \alpha_B$), then the country with the lowest coefficient, A, emerges as the leader from the timing game. As a leader, by increasing its own level compared to the Nash level[19], it will trigger a larger increase in the other country's provision because of the complementarity assumption, which is beneficial to the leader because of the positive externality effect; on the other hand, if B is a leader, given that A as a follower will not tend to act much, i.e. increase its provision level (for opposite reasons), the gain of A as a follower with respect to the Nash solution, is not that large. The opposite reasoning explains why the country with the largest coefficient, B, has the more to lose from not being a follower. Since this is understood by both players, B chooses *Follows* and A *Leads*.

Resorting to simulations, we can also identify the roles assumed by the two countries in the equilibrium of the timing game. The following graphic give some illustrations of the function $\Pi\left(\alpha_A, \alpha_B, \gamma_A, \gamma_B\right)$. When the curb is under the abscissa axe, the risk-dominant equilibrium is the Stackelberg one where country A follows.

[19] cf PROPOSITION 1.



Risk-dominance with $\gamma_A = 0,1 < \gamma_B = 0,2$.

3.4 A log specification.

Let us now assume a log specification of the $\Psi^i(.)$, that is:[20]

$$\Psi^i(g_i, g_j) = \theta_i \log(g_i + \beta g_j), \quad \text{with } -1 < \beta < 1. \tag{8}$$

The parameter β is the degree of spillovers generated by public goods. Depending on the sign of β, the externalities might be positive or negative. We consider here only negative spillovers ($\beta < 0$) since for $\beta > 0$, the public goods are substitutes and no coordination issue appears. Wlog, we assume that $\theta_A > \theta_B = 1$. Country A' s agents value more the public good basket than agents in country 2, and therefore are more adversely affected by the other country's provision (ceteris paribus). An

[20]The condition (2) always holds for $-1 < \beta$ since

$$\Psi^i_{11}(g_i, g_j) + \left|\Psi^i_{12}(g_i, g_j)\right| = -\frac{\theta_i(1+\beta)}{(g_i + \beta g_j)^2} < 0$$

exemple of such public goods is defense spending among rival countries: inhabitants of country A are more sensitive to their national security. The following table gives the levels of public good in the different games.

	Nash Equilibrium G^N	Country A leads G_A^S	Country B leads G_B^S
Country 1	$g_A^N = \frac{\theta_A - \beta}{1-\beta^2}$	$g_A^L = \theta_A - \frac{\beta}{1-\beta^2}$	$g_A^F = \theta_A + \frac{\beta(\beta(\theta_A+\beta)-1)}{1-\beta^2}$
Country 2	$g_B^N = \frac{1-\beta\theta_A}{1-\beta^2}$	$g_B^F = 1 + \frac{\beta\left(\beta-\theta_A(1-\beta)^2\right)}{1-\beta^2}$	$g_B^L = 1 - \frac{\beta}{1-\beta^2}\theta_A$

We deduce the following rankings of the level of the public goods, directly from Proposition 1:

$$\begin{aligned} \forall\beta &\in \left]-1,0\right[, \quad g_A^N > g_A^F > g_A^L \\ \forall\beta &\in \left]-1,-\frac{1}{\theta_A}\right[, \quad g_B^N > g_B^L > g_B^F \\ \forall\beta &\in \left]-\frac{1}{\theta_A},0\right[, \quad g_B^N > g_B^F > g_B^L. \end{aligned}$$

Let us denote by $\Pi(\theta_A, \beta)$ the difference of the products given in (5):

$$\Pi(\theta_A, \beta) = \frac{\beta^3}{\left(1-\beta^2\right)^2}\left(1-\theta_A^2\right)\left(\beta^2 + \left(1-\beta^2\right)\log\left(1-\beta^2\right)\right).$$

We then offer the following

Proposition 5 *When (8) is assumed,*
*(i) the equilibrium (*A Follows, B Leads*) Risk-dominates (*A Leads, B Follows*) for* $\theta_A > 1$.
(ii) this equilibrium is always Pareto-dominant for $\theta_A > 1$.

Proof. See Appendix A.5.□ ■

Playing first is less risky for the country that values the less the basket of public goods. This safer equilibrium in which country A moves second is the neutral focal point and, adopting the risk-dominance concept, the players will coordinate on it.

With the logarithm specification, we are able to establish that this equilibrium is also Pareto-dominant. In other terms, country B always has a first-mover advantage, while country A has a second-mover advantage.

Remark that the complementarity effect is higher for country A, as $\Psi^A_{12}(g,g) = \theta_A \Psi^B_{12}(g,g)$, as well as the negative externality effect (in absolute values) as $\Psi^A_2(g,g) = \theta_A \Psi^B_2(g,g)$. As a follower, by decreasing its own level compared to the Nash level[21], it will trigger a larger decrease in B's provision because of the lower complementarity effect for B: this is beneficial to A because of the negative externality effect ($\left(U^F_A - U^N_A\right)$ large). On the other hand, if A is a leader, given that B as a follower and given the negative externality, the gain of A as a leader with respect to the Nash solution, is not that large ($\left(U^L_A - U^N_A\right)$ small). This explains why the risk-dominance effect favors A as a follower.

4 Conclusion.

The tools applied to study leadership in duopoly theory can be used to address the relevance of leadership in public economics. Doing so in the matter of public good provisions by two interdependent yet non-cooperating jurisdictions generates neat and quite general results, which can easily be explained and may be applied to various situations. Formally, leadership is equated to moving first in a non-cooperative game, that is being a first-mover.

As a first step, a positive comparison between the equilibrium levels obtained in the simultaneous and sequential (non-simultaneous) games allows us to stress the role of externalities and the nature of the interactions between the public goods in the utility functions of the various agents (which are not assumed to be identical across countries): whether they are complements or substitutes.

Then we can make explicit who benefits from a first- or second-mover advantage

[21] using the formulas given above.

in the sequential game. This solely depends on the complementarity or substitutability property of the public goods. As a result, it happens that being a leader is not always beneficial but is so under certain circumstances only.

We are then able to tackle the issue of the existence and if any, the identity, of a leader by means of a timing game. Again the complementarity or substitutability property of the public goods plays a critical role. In particular, when public goods are substitutes for all agents, then there is no leader. On the other hand, when public goods are complements, the timing game generates two equilibria with each country as a leader, respectively. We resort to the risk-dominance criterion to break the tie in two cases with particular specifications of the utility function, exhibiting complementarity. On the whole, the results we obtain are strikingly simple yet general, and easy to understand.

We provide a taxonomy of inter-jurisdictional interactions. Our results may be used to reconsider the literature on political integration or centralization, where the used benchmark is the simultaneous Nash equilibrium. When public goods are complements, it could be argue that the relevant benchmark for assessing the gains from centralization is not the simultaneous Nash equilibrium but a Stackelberg equilibrium as the two countries may be better off with such a non-cooperative equilibrium and may even resort to a timing game to select it.

As we have just said above, the tools that we have been using may be used to study other issues in public economics. An immediate candidate which comes to mind is the issue of tax competition either on production factors or on commodities. One also can think of applying them in the context of international trade as well as political economy problems.

It is also possible to relax the assumption of perfect information for both players and adapt the notions we have just used to handle cases with imperfect information.[22] These issues are left for future research.

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[22] See Mailath (1993) and Daughety and Reinganum (1994).

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A Appendix

A.1 Proof of Proposition 1 (Comparison of the level of public goods).

By definitions of the Stackelberg and the Nash equilibria, we have

$$U_i\left(g_i^L, g_j^F\left(g_i^L\right)\right) \geqslant U_i\left(g_i^N, g_j^N\right) \tag{9}$$

The leader of the Stackelberg game always has a utility level superior or equal to the utility level obtained at the Nash equilibrium.

Moreover, we may establish from the FOCs at the Nash and Stackelberg equilibria that:

$$\Psi_1^i\left(g_i^F, g_j^L\right) = \Psi_1^i\left(g_i^N, g_j^N\right) = 1 \tag{10}$$

We distinguish six cases depending on the signs of $\Psi_2^i(.)$ and $\Psi_{12}^i(.)$ for each country.
If $\Psi_{12}^i(.) > 0$, expression (10) then yields:

$$\begin{array}{c} g_i^F > g_i^N \Leftrightarrow g_j^L > g_j^N \\ g_i^F < g_i^N \Leftrightarrow g_j^L < g_j^N \end{array} \tag{11}$$

If $\Psi_{12}^i(.) < 0$, we have

$$\begin{array}{c} g_i^F > g_i^N \Leftrightarrow g_j^L < g_j^N \\ g_i^F < g_i^N \Leftrightarrow g_j^L > g_j^N \end{array} \tag{12}$$

If $\Psi_2^i(.) > 0$, we have from the definition of the Nash equilibrium:

$$U_i\left(g_i^N, g_j^N\right) = \arg\max_{g_i} U_i\left(g_i, g_j^N\right) \geqslant U_i\left(g_i^L, g_j^N\right)$$

The inequality $g_j^N > g_j^F$ then involves

$$U_i\left(g_i^N, g_j^N\right) \geqslant U_i\left(g_i^L, g_j^N\right) > U_i\left(g_i^L, g_j^F\right),$$

which contradicts the relation (9). We deduce that $\Psi_2^i(.) > 0 \Leftrightarrow g_j^F > g_j^N$.
In a similar way, if $\Psi_2^i(.) < 0$, we deduce from the definition of the Nash equilibrium that the inequality $g_j^N < g_j^F$ contradicts the relation (9), and we establish that $\Psi_2^i(.) < 0 \Leftrightarrow g_j^F < g_j^N$.

Combining our preceding results allows us to present the following table when both countries have the same signs of $\Psi_{12}^i(.)$ and $\Psi_2^i(.)$.

$\forall i \in \{A, B\}$	$\Psi_{12}^i(.) > 0$	$\Psi_{12}^i(.) < 0$
$\Psi_2^i(.) > 0$	$\begin{cases} g_i^N < g_i^F \\ g_i^N < g_i^L \end{cases}$	$\begin{cases} g_i^N < g_i^F \\ g_i^N > g_i^L \end{cases}$
$\Psi_2^i(.) < 0$	$\begin{cases} g_i^N > g_i^F \\ g_i^N > g_i^L \end{cases}$	$\begin{cases} g_i^N > g_i^F \\ g_i^N < g_i^L \end{cases}$

(13)

Now we consider the different cases.

1. $\forall i \in \{A, B\}$, $\Psi_2^i(.) > 0$ and $\Psi_{12}^i(.) > 0$: the public goods involve positive externalities

and they are complements in both countries. We have:

$$-\frac{\Psi_{12}^{i}(.)}{\Psi_{11}^{i}(.)}\Psi_{2}^{j}(.) > 0 \tag{14}$$

Combining the FOC for three different games and the sign of (14) yields

$$\forall i \in \{A, B\}, \quad \Psi_{1}^{i}\left(g_{i}^{L}, g_{j}^{F}\right) < \Psi_{1}^{i}\left(g_{i}^{F}, g_{j}^{L}\right) = \Psi_{1}^{i}\left(g_{i}^{N}, g_{j}^{N}\right) \tag{15}$$

From (15), we may establish that

$$g_{i}^{L} < g_{i}^{F} \Longrightarrow g_{j}^{F} < g_{j}^{L} \tag{16}$$

From TABLE (13) and (16), we deduce two following possible rankings:

$$\left\{\begin{array}{l} g_{i}^{L} > g_{i}^{F} > g_{i}^{N} \\ g_{j}^{L} > g_{j}^{F} > g_{j}^{N} \end{array}\right. \quad \text{or} \quad \left\{\begin{array}{l} g_{i}^{L} > g_{i}^{F} > g_{i}^{N} \\ g_{j}^{F} > g_{j}^{L} > g_{j}^{N} \end{array}\right. \tag{17}$$

We note that in the symmetric case only one possible ranking is possible:

$$g^{L} > g^{F} > g^{N}$$

2. $\forall i \in \{A, B\}$, $\Psi_{2}^{i}(.) < 0$ and $\Psi_{12}^{i}(.) > 0$: the public goods involve negative externalities and they are complements in both countries. We have:

$$-\frac{\Psi_{12}^{i}(.)}{\Psi_{11}^{i}(.)}\Psi_{2}^{j}(.) < 0 \tag{18}$$

Combining the FOC for three different games and the sign of (18) yields

$$\forall i \in \{A, B\}, \quad \Psi_{1}^{i}\left(g_{i}^{L}, g_{j}^{F}\right) > \Psi_{1}^{i}\left(g_{i}^{F}, g_{j}^{L}\right) = \Psi_{1}^{i}\left(g_{i}^{N}, g_{j}^{N}\right) \tag{19}$$

From (19), we have

$$g_{i}^{L} > g_{i}^{F} \Longrightarrow g_{j}^{F} > g_{j}^{L} \tag{20}$$

From TABLE (13) and (20), we have two following rankings:

$$\left\{\begin{array}{l} g_{i}^{N} > g_{i}^{F} > g_{i}^{L} \\ g_{j}^{N} > g_{j}^{F} > g_{j}^{L} \end{array}\right. \quad \text{or} \quad \left\{\begin{array}{l} g_{i}^{N} > g_{i}^{L} > g_{i}^{F} \\ g_{j}^{N} > g_{j}^{F} > g_{j}^{L} \end{array}\right. \tag{21}$$

In the symmetric case, we have:

$$g^{N} > g^{F} > g^{L}$$

3. $\forall i \in \{A, B\}$, $\Psi_{2}^{i}(.) > 0$ and $\Psi_{12}^{i}(.) < 0$: the public goods involve positive externalities and they are substitutes in both countries. From TABLE (13), we have $g_{j,i}^{F} > g_{j,i}^{N}$ and then $g_{j,i}^{L} < g_{j,i}^{N}$. We have also expression (18) and deduce the inequality (19). We then obtain: $g_{j}^{F} > g_{j}^{L} \Longrightarrow g_{i}^{L} < g_{i}^{F}$. We obtain the following ranking:

$$\left\{\begin{array}{l} g_{i}^{F} > g_{i}^{N} > g_{i}^{L} \\ g_{j}^{F} > g_{j}^{N} > g_{j}^{L} \end{array}\right. \tag{22}$$

In the symmetric case, we have:

$$g^{F} > g^{N} > g^{L}$$

4. $\forall i \in \{A, B\}, \quad \Psi_2^i(.) < 0$ and $\Psi_{12}^i(.) < 0$: the public goods involve negative externalities and they are substitutes in both countries. From Table (13), we have $g_{j,i}^F < g_{j,i}^N$ and $g_{j,i}^L > g_{j,i}^N$. We obtain expression (14) and thus inequality (15). We then have: $g_j^F < g_j^L \Longrightarrow g_i^L > g_i^F$. We deduce the following rankings

$$\begin{cases} g_i^L > g_i^N > g_i^F \\ g_j^L > g_j^N > g_j^F \end{cases} \tag{23}$$

In the symmetric case, we have:

$$g^L > g^N > g^F$$

5. $\forall i \in \{A, B\}, \quad \Psi_2^i(.) > 0$ and $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$: the public goods involve positive externalities in both countries, but in contrast to the preceding case, they are complements for country i and substitutes in country j.

$$-\frac{\Psi_{12}^i(.)}{\Psi_{11}^i(.)}\Psi_2^j(.) > 0 \quad \text{and} \quad -\frac{\Psi_{12}^j(.)}{\Psi_{11}^j(.)}\Psi_2^i(.) < 0$$

We then have

$$\begin{aligned} \Psi_1^i\left(g_i^L, g_j^F\right) > \Psi_1^i\left(g_i^F, g_j^L\right) = \Psi_1^i\left(g_i^N, g_j^N\right) \\ \Psi_1^j\left(g_j^L, g_i^F\right) < \Psi_1^j\left(g_j^F, g_i^L\right) = \Psi_1^j\left(g_j^N, g_i^N\right) \end{aligned} \tag{24}$$

Considering (11), (12) and the definition of the Nash equilibrium, we know that $g_i^N > g_i^F$ and $g_j^N > g_j^F$ contradict relation (9) for both countries. We then have: $g_i^N < g_i^F$ and $g_j^N < g_j^F$. From expression (24), we deduce that:

$$\begin{aligned} g_i^N < g_i^F \Leftrightarrow g_j^L > g_j^N \\ g_j^N < g_j^F \Leftrightarrow g_i^L < g_i^N \end{aligned}$$

We can deduce that: $g_i^F > g_i^N > g_i^L$ and $\min\left\{g_j^F, g_j^L\right\} > g_i^N$ hold. Moreover, from expression (24) we can establish that

$$g_j^L \lesseqgtr g_j^F \Longrightarrow g_i^L < g_i^F$$

There are two possible rankings:

$$\begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^L > g_j^F > g_j^N \end{cases} \quad \text{or} \quad \begin{cases} g_i^F > g_i^N > g_i^L \\ g_j^F > g_j^L > g_j^N \end{cases} \tag{25}$$

6. $\forall i \in \{A, B\}, \quad \Psi_2^i(.) < 0$ and $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$: the public goods involve negative externalities in both countries, but in contrast to the preceding case, they are complements for country i and substitutes in country j.

$$-\frac{\Psi_{12}^i(.)}{\Psi_{11}^i(.)}\Psi_2^j(.) < 0 \quad \text{and} \quad -\frac{\Psi_{12}^j(.)}{\Psi_{11}^j(.)}\Psi_2^i(.) > 0$$

We then have

$$\begin{aligned} \Psi_1^i\left(g_i^L, g_j^F\right) < \Psi_1^i\left(g_i^F, g_j^L\right) = \Psi_1^i\left(g_i^N, g_j^N\right) \\ \Psi_1^j\left(g_j^L, g_i^F\right) > \Psi_1^j\left(g_j^F, g_i^L\right) = \Psi_1^j\left(g_j^N, g_i^N\right) \end{aligned} \tag{26}$$

Since $\Psi_2^{i,j}(.) < 0$, we have $g_{j,i}^F < g_{j,i}^N$. From (26), we deduce that:

$$\begin{aligned} g_i^F < g_i^N \Leftrightarrow g_j^L > g_j^N \\ g_j^F < g_j^N \Leftrightarrow g_i^L > g_i^N \end{aligned}$$

and

$$g_j^F \lesseqgtr g_j^L \Longrightarrow g_i^L > g_i^F$$

There are two possible rankings:

$$\left\{ \begin{array}{l} g_i^L > g_i^N > g_i^F \\ g_j^N > g_j^L > g_j^F \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} g_i^L > g_i^N > g_i^F \\ g_j^N > g_j^F > g_j^L \end{array} \right. . \tag{27}$$

□

A.2 Proof of Proposition 2 (First-mover and second-mover advantages).

We consider the three cases:

1. For $\forall i \in \{A, B\}$, $\Psi_{12}^i(.) > 0$.

   - If $\Psi_2^i(.) > 0$, the provision levels are given by (17). Using the definition of the first- (second-) mover advantage, we get that, when $g_j^L > g_j^F$,

     $$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^L, g_j^L\right) > U_i\left(g_i^L, g_j^F\right),$$

     where the first inequality results from the definition of the follower's maximization program and the second from the fact that $g_j^L > g_j^F$ and $\Psi_2(.) > 0$. Since $g_i^L > g_i^F$ always holds for at least one of the two countries in either one of the two possible rankings given by (17), we deduce that at least one country has a second-mover advantage.

   - If $\Psi_2(.) < 0$, we have the rankings given by (21). In a similar way as the preceding case, we deduce that when $g_j^F > g_j^L$,

     $$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^L, g_j^L\right) > U_i\left(g_i^L, g_j^F\right),$$

     where the first inequality results from the definition of the follower's maximization program and the second from the fact that $g_j^L < g_j^F$ and $\Psi_2(.) < 0$.

2. For $\forall i \in \{A, B\}$, $\Psi_{12}^i(.) > 0$.

   - If $\Psi_2^i(.) > 0$, we have the rankings (22). Since we have $g_{i,j}^N > g_{i,j}^L$, we establish that

     $$U_i\left(g_i^L, g_j^F\right) \geqslant U_i\left(g_i^N, g_j^N\right) > U_i\left(g_i^F, g_j^N\right) > U_i\left(g_i^F, g_j^L\right)$$

     where the first inequality results from (9), the second from the definition of the Nash maximization program, and the third from the fact that $g_j^N > g_j^L$ and $\Psi_2(.) > 0$. Each country then has a first-mover advantage.
   - If $\Psi_2^i(.) < 0$, we have the rankings (23). In a similar way as in the preceding case, we establish that

     $$U_i\left(g_i^L, g_j^F\right) \geqslant U_i\left(g_i^N, g_j^N\right) > U_i\left(g_i^F, g_j^N\right) > U_i\left(g_i^F, g_j^L\right)$$

     where the first inequality results from (9), the second from the definition of the Nash maximization program, and the third from the fact that $g_j^N < g_j^L$ and $\Psi_2(.) < 0$. Each country then has a first-mover advantage.

3. $\Psi_{12}^{i}(.) > 0 > \Psi_{12}^{j}(.)$.

   - If $\Psi_2^i(.) > 0$, $\forall i \in \{A, B\}$, we have the rankings (25). Since $g_i^N > g_i^L$ always holds, we establish that

     $$U_j\left(g_j^L, g_i^F\right) \geqslant U_j\left(g_j^N, g_i^N\right) > U_j\left(g_j^F, g_i^N\right) > U_j\left(g_j^F, g_i^L\right)$$

     Country j then has a first-mover advantage. When $g_j^L > g_j^F$, we have

     $$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^L, g_j^L\right) > U_i\left(g_i^L, g_j^F\right)$$

     Country i has a first- (second-) mover advantage when $g_j^L < (>) g_j^F$.

   - If $\Psi_2^i(.) < 0$, $\forall i \in \{A, B\}$, we have the rankings (27). Since $g_i^L > g_i^N$ always holds, we establish that

     $$U_j\left(g_j^L, g_i^F\right) \geqslant U_j\left(g_j^N, g_i^N\right) > U_j\left(g_j^F, g_i^N\right) > U_j\left(g_j^F, g_i^L\right),$$

     which means that country j has a first-mover advantage. It may happen that country i also benefits from a first-mover advantage, that is when the ranking $g_j^L > g_j^F$ obtains. If $g_j^L < g_j^F$, then it benefits from a second-mover advantage. $\square$

A.3 Proof of Proposition 3 (Subgame Perfect Equilibria).

From (9) we always have: $U_i\left(g_i^L, g_j^F\right) > U_i\left(g_i^N, g_j^N\right)$, $\forall i \in \{A, B\}$. In order to determine the SPE, we only have to compare the utility levels when the country follows and when it plays simultaneously ($U_i\left(g_i^F, g_j^L\right) \lesseqgtr U_i\left(g_i^N, g_j^N\right)$). We consider the six preceding cases:

1. $\forall i \in \{A, B\}, \quad \Psi_{12}^{i}(.) > 0:$

   - If $\Psi_2^i(.) > 0$, we have the rankings (17), and we establish that

     $$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^N, g_j^L\right) > U_i\left(g_i^N, g_j^N\right),$$

     where the first inequality results from the definition of the follower's maximization program and the second from the fact that $g_j^L > g_j^N$ and $\Psi_2(.) > 0$. We deduce that the SPE correspond to the two Stackelberg situations.

   - If $\Psi_2^i(.) < 0$, the rankings (21) yield

     $$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^N, g_j^L\right) > U_i\left(g_i^N, g_j^N\right),$$

     since $g_j^L < g_j^N$ and $\Psi_2^i(.) < 0$. As in the preceding case, the SPEs are the two Stackelberg situations.

2. $\forall i \in \{A, B\}, \quad \Psi_{12}^{i}(.) < 0:$

   - If $\Psi_2^i(.) > 0$, the ranking (22) yields

     $$U_i\left(g_i^N, g_j^N\right) \geqslant U_i\left(g_i^F, g_j^N\right) > U_i\left(g_i^F, g_j^L\right),$$

     where the first inequality results from the definition of the Bertrand-Nash maximization program and the second from the fact that $g_j^N > g_j^L$ and $\Psi_2(.) > 0$. We deduce that the SPE corresponds to the Bertrand-Nash situation (*Leads* is a strictly dominant strategy for both countries).

- If $\Psi_2^i(.) < 0$, the ranking (23) yields

$$U_i\left(g_i^N, g_j^N\right) \geqslant U_i\left(g_i^F, g_j^N\right) > U_i\left(g_i^F, g_j^L\right),$$

since $g_j^N < g_j^L$ and $\Psi_2(.) < 0$. We deduce that the SPE corresponds to the Bertrand-Nash situation.

3. $\Psi_{12}^i(.) > 0 > \Psi_{12}^j(.)$.

- If $\Psi_2^{i,j}(.) > 0$, the rankings (25) yield

$$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^N, g_j^L\right) > U_i\left(g_i^N, g_j^N\right),$$

since $g_j^L > g_j^N$ and $\Psi_2^i(.) > 0$. We have also

$$U_j\left(g_j^N, g_i^N\right) \geqslant U_j\left(g_j^F, g_i^N\right) > U_j\left(g_j^F, g_i^L\right),$$

since $g_i^L < g_i^N$ and $\Psi_2^i(.) > 0$. Country j, the country for which public goods are substitutes, has a strict dominant strategy (*Leads*). Country i always prefers to follow than to play the simultaneous game ($U_i\left(g_i^F, g_j^L\right) > U_i\left(g_i^N, g_j^N\right)$). The SPE corresponds to the situation where country j leads and country i follows.

- If $\Psi_2^{i,j}(.) < 0$, the rankings (27) yield

$$U_i\left(g_i^F, g_j^L\right) \geqslant U_i\left(g_i^N, g_j^L\right) > U_i\left(g_i^N, g_j^N\right),$$

since $g_j^L < g_j^N$ and $\Psi_2^i(.) < 0$. We have also

$$U_j\left(g_j^N, g_i^N\right) \geqslant U_j\left(g_j^F, g_i^N\right) > U_j\left(g_j^F, g_i^L\right),$$

since $g_i^L > g_i^N$ and $\Psi_2^i(.) > 0$. We have the same SPE as in the preceding case.

□

A.4 Proof of Proposition 4.

Using (6), after solving for the various games, we obtain:

$$\begin{aligned}
U_A^N &= w + (1-\alpha_A)\,\alpha_A^{-1+\frac{1-\alpha_B}{\delta}}\,\alpha_B^{\frac{\gamma_B}{\delta}}, \\
U_B^N &= w + \alpha_A^{\frac{\gamma_B}{\delta}}\,(1-\alpha_B)\,\alpha_B^{-1+\frac{1-\alpha_A}{\delta}}, \\
U_A^L &= w + \alpha_A^{\frac{\gamma_A}{\delta}}\,(1-\Omega_A)\,\Omega_A^{-1+\frac{1-\alpha_B}{\delta}}, \\
U_B^F &= w + (1-\alpha_B)\,\alpha_B^{-1+\frac{1-\alpha_A}{\delta}}\,\Omega_A^{\frac{\gamma_B}{\delta}}, \\
U_A^F &= w + (1-\alpha_A)\,\alpha_A^{-1+\frac{1-\alpha_B}{\delta}}\,\Omega_B^{\frac{\gamma_A}{\delta}}, \\
U_B^L &= w + \alpha_A^{\frac{\gamma_B}{\delta}}\,(1-\Omega_B)\,\Omega_B^{-1+\frac{1-\alpha_A}{\delta}}.
\end{aligned}$$

and therefore:

$$\begin{aligned}\left(U_A^L-U_A^N\right)\left(U_B^F-U_B^N\right) &= \left(1-\alpha_B\right)\alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}}\left(\alpha_A^{\frac{\gamma_B}{\delta}}-\Omega_A^{\frac{\gamma_B}{\delta}}\right)\\ &\quad \left[\left(1-\alpha_A\right)\alpha_A^{-1+\frac{1-\alpha_B}{\delta}}-\left(1-\Omega_A\right)\Omega_A^{-1+\frac{1-\alpha_B}{\delta}}\right]\\ \left(U_A^F-U_A^N\right)\left(U_B^L-U_B^N\right) &= \left(1-\alpha_A\right)\alpha_A^{-1+\frac{1-\alpha_B+\gamma_B}{\delta}}\left(\alpha_B^{\frac{\gamma_A}{\delta}}-\Omega_B^{\frac{\gamma_A}{\delta}}\right)\\ &\quad \left[\left(1-\alpha_B\right)\alpha_B^{-1+\frac{1-\alpha_A}{\delta}}-\left(1-\Omega_B\right)\Omega_B^{-1+\frac{1-\alpha_A}{\delta}}\right]\end{aligned}$$

where $\delta=\left(1-\alpha_A\right)\left(1-\alpha_B\right)-\gamma_A\gamma_B$, $\Omega_A=\alpha_A+\frac{\gamma_A\gamma_B}{1-\alpha_B}<1$, $\Omega_B=\alpha_B+\frac{\gamma_A\gamma_B}{1-\alpha_A}<1$.[23]

Applying the definition of Harsanyi and Selten, the equilibrium where country A follows (F,L) risk-dominates the other (L,F) if and only if

$$\Pi\left(\alpha_A,\alpha_B,\gamma_A,\gamma_B\right)\equiv\left(U_A^L-U_A^N\right)\left(U_B^F-U_B^N\right)-\left(U_A^F-U_A^N\right)\left(U_B^L-U_B^N\right)<0$$

Using the preceding expression, it yields

$$\begin{aligned}\Pi\left(\alpha_A,\alpha_B,\gamma_A,\gamma_B\right) &= \left(1-\alpha_B\right)\alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}}\left(\alpha_A^{\frac{\gamma_B}{\delta}}-\Omega_A^{\frac{\gamma_B}{\delta}}\right)\left[\left(1-\alpha_A\right)\alpha_A^{-1+\frac{1-\alpha_B}{\delta}}-\left(1-\Omega_A\right)\Omega_A^{-1+\frac{1-\alpha_B}{\delta}}\right]\\ &\quad -\left(1-\alpha_A\right)\alpha_A^{-1+\frac{1-\alpha_B+\gamma_B}{\delta}}\left(\alpha_B^{\frac{\gamma_A}{\delta}}-\Omega_B^{\frac{\gamma_A}{\delta}}\right)\left[\left(1-\alpha_B\right)\alpha_B^{-1+\frac{1-\alpha_A}{\delta}}-\left(1-\Omega_B\right)\Omega_B^{-1+\frac{1-\alpha_A}{\delta}}\right]\end{aligned}$$

Notice that

$$\Pi\left(0,\alpha_B,\gamma_A,\gamma_B\right)=\left(1-\alpha_B\right)\alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}}\left(\Omega_A^{\frac{\gamma_B}{\delta}}\right)\left[\left(1-\Omega_A\right)\Omega_A^{-1+\frac{1-\alpha_B}{\delta}}\right]>0$$

For $\alpha_A,\alpha_B\neq0$, since $\frac{1-\Omega_A}{1-\alpha_A}=\frac{1-\Omega_B}{1-\alpha_B}$, expression $\Pi\left(\alpha_A,\alpha_B,\gamma_A,\gamma_B\right)$ becomes

$$\Pi\left(.\right)=\left(1-\alpha_A\right)\left(1-\alpha_B\right)\alpha_A^{-1+\frac{1-\alpha_B+\gamma_B}{\delta}}\alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}}\left\{\begin{array}{c}\left(\left(\frac{\Omega_A}{\alpha_A}\right)^{\frac{\gamma_B}{\delta}}-1\right)\left[\frac{1-\Omega_A}{1-\alpha_A}\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}-1\right]\\ -\left(\left(\frac{\Omega_B}{\alpha_B}\right)^{\frac{\gamma_A}{\delta}}-1\right)\left[\frac{1-\Omega_A}{1-\alpha_A}\left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A}{\delta}}-1\right]\end{array}\right\}$$

For $\gamma_A<\gamma_B$, we have

$$\Pi\left(\alpha,\alpha,\gamma_A,\gamma_B\right)=\left(1-\alpha\right)^2\alpha^{-2+\frac{2-2\alpha+\gamma_A+\gamma_B}{\delta}}\left[\frac{1-\Omega}{1-\alpha}\left(\frac{\Omega}{\alpha}\right)^{-1+\frac{1-\alpha}{\delta}}-1\right]\left\{\left(\frac{\Omega}{\alpha}\right)^{\frac{\gamma_B}{\delta}}-\left(\frac{\Omega}{\alpha}\right)^{\frac{\gamma_A}{\delta}}\right\}>0$$

Assuming $\alpha_A<\alpha_B$ involves the following inequalities

$$\left\{\begin{array}{l}\Omega_A<\Omega_B\\ \frac{\Omega_A}{\alpha_A}>\frac{\Omega_B}{\alpha_B}\end{array}\right.$$

Moreover we consider the situation where $\gamma_A<\gamma_B$, which yields $\left(\frac{\Omega_A}{\alpha_A}\right)^{\frac{\gamma_B}{\delta}}>\left(\frac{\Omega_B}{\alpha_B}\right)^{\frac{\gamma_A}{\delta}}$. We then

[23] >From $\alpha_i+\gamma_i<1$, we have $\frac{\gamma_i}{1-\alpha_i}<1$ and then $\Omega_j=\alpha_j+\frac{\gamma_i\gamma_j}{1-\alpha_i}<1$.

have

$$\Pi(.) > (1-\alpha_A)(1-\alpha_B)\,\alpha_A^{-1+\frac{1-\alpha_B+\gamma_B}{\delta}}\,\alpha_B^{-1+\frac{1-\alpha_A+\gamma_A}{\delta}}\,\frac{1-\Omega_A}{1-\alpha_A}$$
$$\left[\left(\frac{\Omega_B}{\alpha_B}\right)^{\frac{\gamma_A}{\delta}}-1\right]\left[\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}-\left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A}{\delta}}\right]$$

We focus on the sign of the last term. Since we face a transcendental expression, we can only establish a sufficient condition. We note that

$$\frac{\partial}{\partial\alpha_A}\left[\left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A}{\delta}}\right]=\left(\frac{\Omega_B}{\alpha_B}\right)^{-2+\frac{1-\alpha_A}{\delta}}\left[\frac{\gamma_A\gamma_B}{\delta}\frac{\Omega_B}{\alpha_B}\log\left(\frac{\Omega_B}{\alpha_B}\right)+\left(-1+\frac{1-\alpha_A}{\delta}\right)\frac{\partial\left(\frac{\Omega_B}{\alpha_B}\right)}{\partial\alpha_A}\right]>0,$$

since $\frac{\Omega_B}{\alpha_B}>1$ and $\frac{\partial}{\partial\alpha_A}\left(\frac{\Omega_B}{\alpha_B}\right)=\frac{\gamma_A\gamma_B}{(1-\alpha_A)^2\alpha_B}>0$. However, the sign of $\frac{\partial}{\partial\alpha_A}\left[\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}\right]$ is ambiguous since $\frac{\partial}{\partial\alpha_A}\left(\frac{\Omega_A}{\alpha_A}\right)=-\frac{\gamma_A\gamma_B}{\alpha_A^2(1-\alpha_B)}<0$. We have:

$$\frac{\partial}{\partial\alpha_A}\left[\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}\right]=\left(\frac{\Omega_A}{\alpha_A}\right)^{-2+\frac{1-\alpha_B}{\delta}}\left[\frac{(1-\alpha_B)^2}{\delta^2}\frac{\Omega_A}{\alpha_A}\log\left(\frac{\Omega_A}{\alpha_A}\right)+\left(-1+\frac{1-\alpha_B}{\delta}\right)\frac{\partial\left(\frac{\Omega_A}{\alpha_A}\right)}{\partial\alpha_A}\right]$$

We note that

$$\forall\alpha_A>\frac{\gamma_A\gamma_B}{1-\alpha_B},\quad \log\left(\frac{\Omega_A}{\alpha_A}\right)<\log 2, \tag{28}$$

which involves

$$\frac{\partial}{\partial\alpha_A}\left[\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}\right]<\left(\frac{\Omega_A}{\alpha_A}\right)^{-2+\frac{1-\alpha_B}{\delta}}\left[\frac{(1-\alpha_B)^2}{\delta^2}\frac{\Omega_A}{\alpha_A}\log 2+\left(-1+\frac{1-\alpha_B}{\delta}\right)\frac{\partial\left(\frac{\Omega_A}{\alpha_A}\right)}{\partial\alpha_A}\right]$$

Moreover, when α_A respects the following condition

$$\alpha_A<\frac{\gamma_A\gamma_B(1-\alpha_B-\gamma_A\gamma_B)}{(1-\alpha_B)\left[\gamma_A\gamma_B+(1-\alpha_B)\log 2\right]}, \tag{29}$$

we have

$$\frac{(1-\alpha_B)^2}{\delta^2}\frac{\Omega_A}{\alpha_A}\log 2+\left(-1+\frac{1-\alpha_B}{\delta}\right)\frac{\partial\left(\frac{\Omega_A}{\alpha_A}\right)}{\partial\alpha_A}<0\Longrightarrow\frac{\partial}{\partial\alpha_A}\left[\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}\right]<0.$$

Under conditions (28) and (29), we establish that $\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}}$ is decreasing in α_A. Since expression $\left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A}{\delta}}$ is also increasing in α_A, we deduce that:

If it exists $\alpha_A^*\in[0,1]$ such that

$$\left(\frac{\Omega_A}{\alpha_A^*}\right)^{-1+\frac{1-\alpha_B}{\delta}}=\left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A^*}{\delta}},$$

then the value of α_A^* is unique under conditions (28) and (29). We obtain a third condition on α_A:

$$\alpha_A < \alpha_A^*, \tag{30}$$

which involves

$$\left(\frac{\Omega_A}{\alpha_A}\right)^{-1+\frac{1-\alpha_B}{\delta}} > \left(\frac{\Omega_B}{\alpha_B}\right)^{-1+\frac{1-\alpha_A}{\delta}}$$

Finally combining the different preceding conditions and the assumptions on the others parameters: $(\alpha_B, \gamma_A, \gamma_B) \in [0,1]^3$, we obtain the following sufficient set of conditions:

If

$$\begin{cases} \frac{\gamma_A\gamma_B(1-\alpha_B-\gamma_A\gamma_B)}{(1-\alpha_B)[\gamma_A\gamma_B+(1-\alpha_B)\log 2]} < \alpha_A < \frac{\gamma_A\gamma_B}{1-\alpha_B} \\ 0 < \alpha_A < \alpha_A^* < \alpha_B < 1 \\ 0 < \gamma_A < \gamma_B < 1 \\ \alpha_A + \gamma_A < 1 \\ \alpha_B + \gamma_B < 1 \end{cases}$$

then,

$$\Pi(.) > 0$$

Notice that $\frac{\gamma_A\gamma_B(1-\alpha_B-\gamma_A\gamma_B)}{(1-\alpha_B)[\gamma_A\gamma_B+(1-\alpha_B)\log 2]} < \alpha_A < \frac{\gamma_A\gamma_B}{1-\alpha_B}$ involves:

$$\begin{cases} \alpha_A < \frac{1+\log 2}{2} \\ \gamma_B > \gamma_A > \frac{1-\log 2}{2} \\ \alpha_B > 1 - \frac{2\gamma_A\gamma_B}{1-\log 2} \end{cases}$$

□

A.5 Proof of Proposition 5.

- **Risk-dominance:**
  It is obvious that $\Pi(\theta_A, \beta) > 0$, $\forall \beta < 0$ and $\theta_A < 1$ since $\left(\beta^2 + \left(1-\beta^2\right)\log\left(1-\beta^2\right)\right)$ is always negative on $[-1, 0]$.
  For $\theta_A < 1$, (*A Leads, B Follows*) Risk-dominates (*A Follows, B Leads*).
  For $\theta_A > 1$, (*A Follows, B Leads*) Risk-dominates (*A Leads, B Follows*).

- **Pareto-dominance:**
  We have:

$$\begin{aligned} U_A^L - U_A^F &= \frac{\beta^2(\beta+\theta_A)}{1-\beta^2} + \theta_A \log\left(1-\beta^2\right). \\ U_B^F - U_B^L &= -\frac{\beta^2(1+\beta\theta_A)}{1-\beta^2} - \log\left(1-\beta^2\right). \end{aligned}$$

  which yield

$$\begin{aligned} \forall \theta_A &> 0, \quad U_A^L < U_A^F \\ \forall \theta_A &> 1 > \theta^* = -\frac{\beta^2 + \left(1-\beta^2\right)\log\left(1-\beta^2\right)}{\beta^3}, \quad U_B^F > U_B^L. \end{aligned}$$

  Country A always has a second-mover-advantage, while country B has a first-mover advantage as soon as $\theta_A > 1$. Thus, we have without ambiguity: the SPE (*A Follows, B Leads*) Pareto-dominates (*A Leads, B Follows*).□