Asymmetric Shocks, Long-term Bonds and Sovereign Default

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Abstract: We present a sovereign default model with asymmetric shocks and long-term bonds, and solve the model using discrete state dynamic programming. As result, our model matches the Argentinean economy over period 1993Q1-2001Q4 quite well. We show that our model can match high default frequency, high debt/output ratio and other cyclical features, such as countercyclical interest rate and trade balance in emerging countries. Moreover, with asymmetric shocks we are able to match high sovereign spread level and low spread volatility simultaneously in one model, which is till now not well solved. As another contribution of our paper, we propose a simulation-based approach to approximate transition function of output shocks between finite states, which is an indispensable step in discrete state dynamic programming. Comparing to Tauchen's method, our approach is very flexible in transforming various econometric models to finite state transition function, so that our approach can be widely used in simulating different kinds of discrete state shocks.

JEL Classification: E44, F32, F34
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1 Introduction

Sovereign default risk is intensively studied in academic research and policy analysis. Regarding the present debt crisis in several European countries and the fact that most of developed economies are heavily indebted, the research on sovereign default models has important implication in fiscal policy.

The current sovereign default models start from the seminal work of Eaton and Gersovitz (1981), which analyzes the relationship between output shocks and endogenous default probability. This model and most of the following sovereign default models apply discrete state dynamic programming methods to solve recursive competitive equilibrium. Thus solution approach relies on transition function between discrete states to simulate exogenous output shocks and examine its impact on default probability. In previous studies, researchers use quadrature-based methods introduced by Tauchen (1986, 1990) and Tauchen and Hussey (1991) to obtain transition function, with the assumption that the shocks are autoregressive. As far as we know, there are very few studies dealing with non-AR type shocks.

However, one limitation of Tauchen's method is that it can only be applied on AR type output shocks. While in the real world, the output shocks are widely considered as asymmetric and have a time-varying volatility structure. This paper introduces a simulation-based method to obtain the transition function, analyzes the effect of Markov-switching type asymmetric output shocks on sovereign default, and provide a model which matches the empirical data quite well, better as the recent working paper by Chatterjee and Eyigungor (2009). Additionally, our model answers the question why the inclusion of asymmetric shocks can result in better match of the data.

Among the influential sovereign default models introduced recently, Aguiar and Gopinath (2006) analyze the effect of transitory and permanent shocks on default probability in emerging countries, and find that shocks to trend enable their model to match the empirical data better. Arellano (2008) shows one model with better fit regarding the recent Argentina default by assuming that the output loss in default is more sensitive to output shocks. In a quantitative analysis, her model improves greatly in matching the default probability, countercyclical interest rate and current account in Argentine. However, this model delivers much lower average spread level than the empirical data. Hatchondo and Martinez (2010) show that the assumption of multiple-periods bonds instead of one-period bonds will solve this anomaly, but they do not provide a model matching empirical moments. Chatterjee and Eyigungor (2009) introduce another assumption on multiple-periods bonds, and provide a model which matches the empirical moments well. However, they can not simultaneously match the relative high spread level and low spread volatility, which is frequently observed in emerging countries, and the default frequency in their model is lower than the empirical one, regarding to the estimated Argentinean default frequency in our model.

In this paper, we make two contributions to the literature. One contribution is that we provide a model that matches the data even better than Chatterjee and Eyigungor (2009), by assuming Markov-switching type asymmetric shocks and long-term bonds. In our model, emerging economy has positive probability to run into recession in each period, which enables the default probability to be significantly positive even in economic boom periods. An overall positive default probability drives the default spread level high. What's more, with asymmetric shocks the sovereign stays in normal growth states in most of the periods, which reduces the spread volatility greatly. So our model can incorporate high spread level and low spread volatility simultaneously.

In order to show the contribution of our assumption on asymmetric shocks, we compare our model with another model of the same structure and with the same parameters, except that the output shocks is symmetric AR type, which is frequently used assumption in previous studies. Quantitative results show that with asymmetric shocks the default frequency, debt/output ratio and spread level all increase significantly, while the spread volatility decline dramatically, and the correlation coefficients between output, spread and trade balance are closer to data. This result indicates that the asymmetric shocks assumption enhances the performance of our model greatly.

As another contribution of our paper, we propose a simulation-based method to approximate the transition function between discrete states. Our method is not limited to AR type shocks, and can be applied to transform many kinds of econometric models to discrete state transition function. So our approach enables the DSGE models to incorporate different types of shocks, instead of limited on AR ones. Our simulation-based model is so flexible that it can transform almost all econometric models to finite state Markov chains, if only we can simulate data series with these models. In this paper, we present the resulting difference in transition function, with our simulation-based method and that Tauchen's method, if AR type of shocks is assumed. And we describe how to approximate Markov-switching model with transition function.

With simulation-based approach, we can simulate more realistic shocks, and analyze their impact on different DSGE models. In this paper, we present application in sovereign default model. Generally, the simulation-based approach can be applied in other macroeconomic models, wherever discrete state dynamic programming as a solution method is used.

Our model is similar to Chatterjee and Eyigungor (2009). However, comparing to their model, we provide modifications in the following three ways. Firstly, we assume asymmetric output shocks which reflect the real output dynamic process. We will discuss this point more in detail in section 3. Secondly, we make little modification in computational algorithm, so that we can solve the convergence problem showed in Chatterjee and Eyigungor (2009). With this modification, we avoid the redundant and confusing assumption on minimum consumption requirement in Chatterjee and Eyigungor (2009), which adds no additional value except helping reach convergence in computation. Thirdly, we interpret the payment structure of long-term bonds differently. We assume sovereign promises coupon payment and principal repayment each period. Although this assumption is essentially the same as that in Chatterjee and Eyigungor (2009), our interpretation can be more easily understood, and it is also consistent with the bond payment structure in reality. Moreover, the parameters on long-term bonds assumption can be more easily calibrated in our model. In sensitivity analysis, we also show that the principal repayment ratio is critical for the sovereign default model to match the data.

As related literatures, Arellano and Ramanarayanan (2008) also provide one research on sovereign default model with long-duration bonds. However, their interest lies on the interconnection between optimal maturity structure and sovereign bond spread. They show that rising interest rate spread leads to shortening of the debt maturity, which results in higher spread on short-term bonds than that on long-term bonds. Mendoza and Yue (2008) examine a sovereign default model with endogenous default cost, with the attempt to connect the sovereign default model and international business cycle models in one framework. They claim that with endogenous default cost the mean spread is higher, thus provide another explanation for high spread level in emerging countries.

Political factors are often considered in sovereign default model. Hatchondo et. al. (2011) provide an explanation to high spread level by introducing political risks. They show that with political turnovers between governments the mean spread level will be increased significantly.

Business cycle asymmetry is another stream of the literature related to our paper, and it is now not a new topic. More than 40 years ago, Friedman (1963) pointed out that the amplitude of contractions is strongly correlated with the amplitude of succeeding expansions in U.S. output, but that the amplitude of expansions was uncorrelated with that of subsequent contractions. Friedman (1963) called it asymmetry in business cycle. Among the early literatures, Neftci (1984) and Goodwin and Sweeney (1993) formally tested the asymmetry of economic time series. Now it is commonly accepted that output growth shows cyclical asymmetry in almost all countries.

Among variety of models, Markov switching model introduced by Hamilton (1989) is widely used to study the business cycle asymmetry. This seminal work describes the economy as constantly switching process between two states, namely recession and expansion. Moreover, the timing of business cycles in Hamilton (1989) fits the NBER results quite well. Diebold and Rudebusch (1996) introduce a regime-switching dynamic factor model to analyze business cycle asymmetry and comovement. Kim and Murray (2002) generalize the regime-switching dynamic factor model, and suggest a three-phase description of U.S. economy: recession, high-growth recovery and normal growth. In our model, we use conventional maximum likelihood method to estimate an MS-AR model[2].

The rest of paper is organized as follows. In section 2, we propose the simulation-based approach to approximate the transition function between finite states, show its performance and explain the difference comparing to Tauchen's method. In section 3, we describe the model environment and define the recursive competitive equilibrium. We present the main quantitative result in section 4, with reference to modification of the computational algorithm, and show the contribution of our assumption on asymmetric shocks. In section 5, we provide sensitivity analysis regarding to parameter values on default cost, principal repayment and exclusion periods following autarky state of the sovereign. Section 6 concludes.

2 Simulation-based method and asymmetric output shocks

Tauchen (1986) introduce a quadrature-based method to construct finite state Markov chain, which

[2] We use the Matlab code provided by Perlin (2008) to estimate the model.

approximates an AR model. Tauchen (1990) implement this method to form discretized recursive equilibrium model, and then solve the model with value-function iteration. However, with this method it is hard to construct discrete state space, if the exogenous variable is assumed to follow regime-switching process, which is commonly used to analyze asymmetry in business cycle.

We propose a simulation-based method to construct finite state Markov chain which approximating different kinds of models, e.g., Markov-switching models. In fact, our method is so general that it can simulate Markov chain for almost every time-series model, if only we can simulate data series with the underlying models. At the following, we describe this method, and the compare its performance with Tauchen's method.

At first, we need to assume the type of output shock y [3], which we aim to approximate with finite state transition function. The first step of our method it to estimate the econometric model under the predetermined assumption, and simulate series of data $\{\tilde{y}_t\}_{t=1}^T$ according to the estimated model. The second step is to construct the discrete state space of shocks $\{\overline{y}^i\}_{i=1}^n$, with *n* denoting the maximum grid points and $\overline{y}^1 < ... < \overline{y}^i < ... < \overline{y}^n$. We denote the interval length between two neighboring grid points to be *w*. While constructing the discrete state space of shocks, we should make sure that more of simulated data falling into the interval $[\overline{y}^1, \overline{y}^n]$.

The third step is to determine the state series $\{S_t\}_{t=1}^T$. Following Tauchen (1986), if $\overline{y}^i - w/2 < \tilde{y}_t < \overline{y}^i + w/2$, then the state in time *t* is *i*, that is $S_t = i$. The final step is to determine the transition function $f(y_{t+1}, y_t)$. As in discrete state, the transition function is actually the discrete transition probabilities between different states, that is

$$f(y_t, y_{t-1}) = p_{ij} = \Pr[y_t \approx \overline{y}^j \mid y_{t-1} \approx \overline{y}^i],$$

with p_{ij} indicating the transition probability from state *i* to *j*. The calculation of transition probabilities proceed as follows. As $\sum_{j=1}^n p_{ij} = 1$, and $p_{ij} > 0$, then $(p_{i1}, ..., p_{in})$ follow the Dirichlet distribution. The density function of $p_i = (p_{i1}, ..., p_{in})$ is given by

$$f(p_i; a_1, ..., a_n) \sim p_{i1}^{a_{i1}-1} ... p_{in}^{a_{in}-1}, \tag{1}$$

where $a_{ij} = \sum_{t=1}^T 1(S_t = j \mid S_{t-1} = i)$.

a_{ij} is the number of times the simulated data series transitions from state *i* to state *j* during *T* periods. With simulated data series $\{\tilde{y}_t\}_{t=1}^T$, the correspondent state series $\{S_t\}_{t=1}^T$ can be easily

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[3] Low case denotes log value of variable.

determined, and then each value of a_{ij} can be counted.

Given the parameters $(a_{i1},...,a_{in})$, $(p_{i1},...,p_{in})$ can be sampled with the density function of Dirichlet distribution. The frequently used sampling method is to generate *n* independent random variables $(p_{i1}^*,...,p_{in}^*)$ from the Gamma distribution $p_{ik}^* \sim Gamma(a_{ik},1)$, $k=1,...,n$, and then normalize the resulting variables with $p_{ik} = p_{ik}^* / \sum_{l=1}^{n} p_{il}^*$ (Fruehwirth-Schnatter, 2006, pp.432 -433). With this method, we sample p_{ij} many times, and average the results to obtain the final transition function in form of a probabilities matrix. Of course, we can get more accurate approximations if we increase the sampling times of the probabilities.

We illustrate our method with a simulation exercise, and compare the results with that of Tauchen's method. In the exercise, we assume the underlying model is AR model:

$$y_t = \mu(1-\rho) + \rho y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma^2).$$

with the true parameters $\hat{\mu}, \hat{\rho}$ and $\hat{\sigma}$. We calculate the transition probabilities matrixes with two methods, and then we simulate 2 time series each consisting of *T* data. Then we estimate AR parameters. Table 1 shows the true parameters, along with the AR parameters with Tauchen's method, and our simulation-based method.

Table 1: Simulation-based method

	True value	Tauchen	Simulation-based method		
			T=1000	*T*=10000	*T*=100000
μ	-0.0002	0.0004	-0.0020	-0.0001	-0.0001
ρ	0.9	0.8806	0.8810	0.8902	0.8927
σ	0.02	0.0208	0.0201	0.0207	0.0210

With table 1, we conclude that the performance of our method is at least not worse than Tauchen's method. At the following, we use the simulation-based method to approximate Markov-switching model, so that we can incorporate asymmetric shocks in sovereign default model.

Figure 1: GDP growth, consumption growth and interest rate in Argentine

It is commonly accepted that economic variables show asymmetric movements through different growth phases. Figure 1 presents quarterly output growth, consumption growth and interest of Argentine from 1993 to 2007. The data series come from IMF International Financial Statistics. In figure 1, output growth occasionally moves downward in recessions, and quick recovery follows the recession before the economy steps into normal growth phase.

In the following, we estimate Markov switching models for Argentine economy, and then apply the simulation-based method to derive the transition probability matrix which will be used in value function iteration of a recursive equilibrium model in section 3.

We use the Argentine quarterly log GDP data from 1993 Q1 to 2007 Q4, adjust the seasonal effects and derive HP filtered cyclical data series y_t. While estimating Markov switching models, we try different assumptions on number of states and AR lag numbers. As result, we obtain the MS(3)-AR(1) model, which has the largest likelihood value. The estimated parameters in each state are as following.

Table 2: Parameters of Markov switching model MS(3)-AR(2)

	State 1	State 2	State 3
μ	0.0074 (0.0013)	-0.0390 (0.0084)	-0.0461 (0.0048)
ρ	0.7898 (0.0404)	0.5427 (0.0973)	1.2568 (0.1330)
σ	0.0086 (0.0009)	0.0085 (0.0027)	0.01 (0.0035)

Notice: with std. errors in parenthesis.

The estimated transition probabilities matrix is

$$P = \begin{bmatrix} 0.96(0.14) & 0.00(-) & 0.04(0.03) \\ 0.40(0.21) & 0.60(0.37) & 0.00(-) \\ 0.00(-) & 0.45(0.30) & 0.55(0.38) \end{bmatrix}.$$

Clearly, the Argentine GDP growth shows significant asymmetries between different states. Similarly as Kim and Murray (2002), we describe three states in Argentine business cycle as normal growth phase, recovery and recession, respectively. Given the transition probabilities matrix, the duration of three economic phases can be calculated as 25, 2.5 and 2.22 periods. While the Argentine economy stays in normal growth phase in 84% of the time, it has 4% probability to step into recession abruptly. If the economy is in recession, it has 45% probability to enter the recovery phase, yet there is no probability for the economy to transit from recession directly into normal growth phase. In recovery phase, the prospect of the economy looks well, for the chance of getting better is 40%, and double recessions does not happen. The interpretation of Argentine business cycle based on Markov switching model fit well with the real economy, which supports our assumption on asymmetric output shocks.

With the estimated parameters, we can easily simulate output shock data series. We simulate 100 samples, with each sample containing 100,000 data. After dropping the extreme data[4], we check the mean value and standard variation of each sample, so that the moments of every sample are in comparable with that in real business cycle. Then we derive $\{p_{ik}\}_{i,k=1}^{n}$ using the method we propose, delete the rows and columns that consist all zeros or NaNs, take average of all sample transition matrixes and obtain the final transition function $P(y_t \mid y_{t-1})$.

In this paper, we set number of states in transition function $n = 25$, construct the state space grid using $\rho = 0.9055$ and $\sigma = 0.0181$, which is the estimated AR model parameters for Argentine economy between 1993Q1 to 2007Q4, and finally we obtain one transition probabilities matrix with 23 dimensions. The resulting matrix's dimensions are less than 25, because the simulated data in Markov-switching model are rarely above 9%, which is the value of 24th grid in state space, so that the last 2 rows and columns are either 0 or NaNs. We delete these rows and columns to obtain the final result.

In order to show the differences between our transition matrix with that by Tauchen's method, we derive $\{p_{ik}\}_{i,k=1}^{n}$ by Tauchen's method with the same state space construction. Figure 2 shows the differences between these two probability matrixes.

Figure 2 shows that the left panel reflects significant asymmetry, which mirrors the asymmetric features in Markov-switching model. These asymmetric features include: the shocks are more persistent if the economy is experiencing large positive or negative ones, and there are strong evidences for recovery if the economy is in recession. Moreover, the matrix in left panel shows slightly greater than 0 probabilities for the economy to transit from growth periods to recession, but this is not clearly identifiable in the figure.

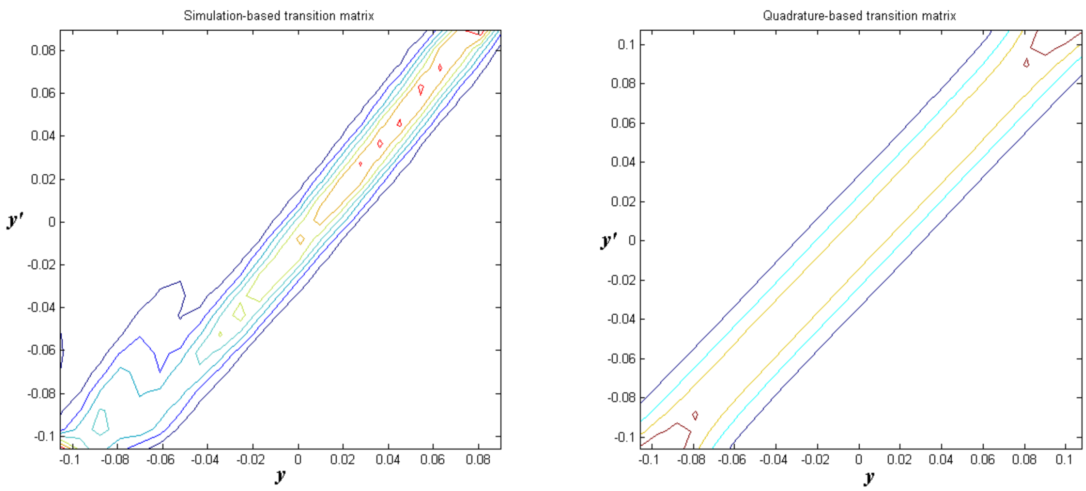

Figure 2: Transition probabilities with simulation-based method and Tauchen's method

[4] As table 1 shows, ρ >1, which means the shock in state 3 is explosive. We do not worry about the stationarity of the whole model, because state 3 has average duration of 2.22 periods only. However, in data simulation, the shock will be extreme negative if it stays in state 3 for relative long periods. As the output shock is normally greater than -0.15 in Argentine real business cycle, we eliminate the 15 periods before and after the date where extreme negative shocks appear.

3 The Model

Our sovereign default model follows the seminal work of Eaton and Gersovitz (1981) and the extended models introduced recently. The main difference is that we introduce asymmetric output shocks, analyze it effect on sovereign default model, and provide better match to empirical moments. Moreover, we show a simpler interpretation towards long-term bonds, and provide another assumption on multi-periods bond, instead of the mature probability concept in Chatterjee and Eyigungor (2009), and the decreasing coupon payment assumption in Hatchondo and Martinez (2010).

We consider a small open economy, which receives stochastic endowment y_t each period. The endowment shock is assumed to be asymmetric, and characterized as a regime-switching process. We apply the simulation-based method introduced in section 2 to derive a finite Markov chain with the transition function $f(y, y') = \Pr(y_{t+1} = y' \mid y_t = y)$. This function characterizes the dynamic process of endowment shocks in our model.

The sovereign maximize the expected present utility over consumption stream c_t, namely

$$E[\sum_{t=0}^{\infty} u(c_t)],$$

where β is the discount factor, and $0 < \beta < 1$. The benevolent sovereign has access to international credit market, and it can invest in or issue long-term bonds b_t. In each period, the sovereign has option to default, if it holds net foreign debt in the market. The consumption of private household c_t depends on the stochastic endowment y_t, debt or investment volume of the sovereign and its default decision in period t.

The international credit market is competitive that there are a larger number of foreign creditors. The creditors are risk neutral in pricing the defaultable sovereign bonds, and they generate zero profit in the competitive market. In each period, the sovereign starts with b_t foreign bonds. After the sovereign and foreign creditors observe the endowment shock y_t, the sovereign decides whether to default or continue to stay in the contract. If the sovereign default, it erases the debt obligations contracted in the past, but this decision will cause output loss in next period, and the sovereign will be excluded from international credit market for random periods. If the sovereign decides to stay in the contract, the bond price $q(b_{t+1}, y_t)$ which depends on sovereign's decision of bond holding in next period and the zero-profit condition in competitive credit market. The sovereign choose b_{t+1}, taking the bond price function $q(b_{t+1}, y_t)$ as given, and the creditors accept b_{t+1}.

3.1 Principal repayment and assumption on long-term bond

In order to model long-term bonds, we need to assume a specified repayment structure that does not increase the dimensionality of the state space. Hatchondo and Martinez (2010) makes the assumption that all previous bonds promise a same coupon payment which decrease at the same rate, and Chatterjee and Eyigungor (2009) assume "each unit of outstanding debt matures next period with a constant probability". However, both of these assumptions do not represent the bond payment structure in the real world.

In contrast, we make a simpler assumption on long-term bonds. We assume, in each period, the outstanding bonds promise a coupon payment equal to cb, and the sovereign make principal repayment γb. If the sovereign do not default, the outstanding bonds in next period $b' = (1-\gamma)\ b + i$, where i denotes the new issued bonds.

With this assumption, (c, γ) is enough to describe the payment structure each period. This assumption is essentially the same as that in Chatterjee and Eyigungor (2009). We only interpret it differently, reflecting the payment structure of bonds in real world.

We assume that the foreign creditors can borrow or lend as much as they need in the market with world interest rate $r*$, and we denote the default decision of sovereign as $d(y', b')$, which equal to 1 in default and 0 without default, then the bond price is

$$q(b', y) = \{\sum_{y'}[1 - d(b', y')]\frac{\gamma + (1-\gamma)[c + q(b'', y')]}{1 + r*}\}P(y' \mid y), \tag{2}$$

where $P(y' \mid y)$ denotes the transition probability between discrete states of output shock.

3.2 Recursive competitive equilibrium

As Arellano (2008), we denote $V^o(b, y)$ as the value function of the sovereign with default, and this value function satisfies

$$V^o(b, y) = \max_{\{c,d\}}\{V^c(b, y), V^d(y)\}, \tag{3}$$

where $V^c(b, y)$ is the value if sovereign promise the repayment of the debt, and $V^d(y)$ denotes the value in default.

We assume finite states in stochastic endowment shocks, and the transition probabilities between states are $P(y' \mid y)$, then the value functions in default and without default are as follows.

$$V^c(b, y) = \max_{b'}\{u(c) + \beta\sum_{y'}V^c(b', y')P(y' \mid y)\}, \tag{4}$$

with the budget constraint $c \leq y + \gamma b + c(1-\gamma)b - q(b', y)[b' - (1-\gamma)b]$.

$$V^d(y) = u(c^{def}) + \beta\sum_{y'}[\theta V^o(b', y') + (1-\theta)V^d(y')]P(y' \mid y), \tag{5}$$

where θ is the probability of re-entry into the international credit market, and the budget constraint is

$$c^{def} = \min\{\overline{y}, y\}\,. \tag{6}$$

with $\overline{y}$ denotes the reduced consumption in default, serving as parameter for output loss in case of default. With equation (6), the consumption in default takes the lower value between $\overline{y}$ and y , thus it is sensitive to output realization. Quantitative simulations show that this assumption is essential to generate countercyclical interest rate and current account.

4 Quantitative Analysis

We discretize the stochastic output shock and generate transition function with the simulation-based method we introduce in previous section, so that we incorporate the asymmetric shocks in modeling. The model is solved numerically using discrete state space dynamic programming, which is also used in other sovereign default models.

Chatterjee and Eyigungor (2009) points out the computation difficulty in sovereign default model with long-term bonds, that small change of pricing function trigger discrete jump of sovereign's decision on bond holding next period, which again change the future value of outstanding bonds and current bond price. This process leads to a convergence difficulty for the algorithm.

In our model, we assume the bond price in first period and in second period is the same, that is, we let $q(b', y) = q(b'', y')$ in equation (2). We take this modification as reasonable, as in equilibrium the price function is stationary across time. In computation, we iterate value function and price function in the same loop algorithm[5], till the average value and price differences between two iterations are smaller than one predetermined tolerance value. In our case, we set this tolerance threshold to be 1e-06[6]. As result, the algorithm converges well.

4.1 Quantitative results

Arellano (2008) select β , $\overline{y}$ and θ to match the probability of default, the volatility of the trade balance and the ratio of debt to GDP. Chatterjee and Eyigungor (2009) choose β and $\overline{y}$ to match the average spread and 70% of the average debt-to-output ratio. We follow similar calibration approach. The risk aversion coefficient is set to 2, which is very common. The probability of re-entry into credit market following default is 10%, which means sovereigns are excluded from international borrowing for 10 quarters averagely in case of default. The risk-free interest rate is set to 1%. The coupon rate of the bond is assumed to be 3%. The above parameters are the same as that in Chatterjee and Eyigungor (2009). The differences of our model lie in asymmetric shocks

[5] According to Hatchondo et. al. (2010), one single loop algorithm increases the calculation speed in sovereign default model.

[6] However, the convergence seems to be related to the parameter choice. If β is selected to be larger than 0.98, the convergence comes slowly, and discrete jumps that Chatterjee and Eyigungor (2009) point out effect the convergence speed significantly. Occasionally, it results in the difficulties in convergence.

and principal repayment ratio. We explain the simulation-based approach to calculate transition function associated with asymmetric shocks in section 2. As to principal payment ratio, we set this value to 2%, for Argentine reports annual principal repayment of 7.26% averagely according to IMF statistics.

Table 3: Parameters

parameters	values
δ	2
θ	10%
r^*	1%
c	3%
γ	2%
β	0.9675
$\overline{y}$	0.89

As Chatterjee and Eyigungor (2009), we calibrate β and $\overline{y}$ to match empirical moments. We target to match sovereign bond spread and spread volatility, without deteriorating correlation and ratio of standard deviation between sovereign interest rate r_t and y_t. As results, we present the selected moments of our models in table 2. In fact, Chatterjee and Eyigungor (2009) fit the empirical data very well. One contribution of our paper is to fit spread volatility σ(r-r*), while keep other moments close to the empirical data too. As to default frequency, we argue that the "correct" default frequency is actually higher than commonly accepted 3% in recent literature of sovereign default models.

What is the 'correct' default frequency for Argentine? Arellano (2008) and Aguiar and Gopinath (2006) suggest the default probability to be 3%, circa 3 defaults in 100 years. Arellano (2008) refers to Beim and Calomiris (2001) that Argentine defaulted on its foreign debt for 3 times in 1956, 1982 and 2001, respectively. Chatterjee and Eyigungor (2009) calculate the default frequency conditional on the sovereign in good standing. According to a recent study, Reinhart and Rogoff (2010) state that Argentine defaulted 5 times in the last century: 1951, 1956, 1982, 1989 and 2001, and stay in default for 26.36%[7] time, so we have a default frequency of 6.78%. Considering that these 5 defaults all took place after World War II, the real default frequency may be higher than this ratio. One advantage of our model lies in the capability to allow much higher default frequency. We will show that the default frequency with assumption of asymmetric shocks rises for c.a. 40%, comparing to that with symmetric shocks.

Clearly, according to the above calculation of default frequency, the 7.2% in our model is closer to the data. Additionally, we can match even higher default frequency, by decreasing the time preference parameter β. According to our simulation exercise, with β equals 0.95 we can obtain a default frequency of over 10%. Such a high probability can be hardly reached with the models of Arellano (2008) and Chatterjee and Eyigungor (2009). According to the fact that Argentine defaulted 5 times after World War II, it is perhaps important to match a default frequency of over 10%, for theoretical modeling or empirical implication.

We have no intention to match debt/output ratio. However, with a result of 0.87, our model seems to be close to the empirical ratio of 100%. Comparing to the 5.95% debt/output in Arellano (2008), and 10-20% in Aguiar and Gopinath (2006), our result is more realistic. Yet, we accept that the assumption of long-duration bonds contributes the most to the match of this ratio, as Chatterjee

7 According to Reinhart and Rogoff (2010), the duration of 5 defaults is totally 29 years, so Argentine spent 26.36% of the last 110 years in default or restructuring.

and Eyigungor (2009) and Hatchondo and Martinez (2010) point out. Our assumption on asymmetric shocks also improves this ratio, but the improvement is moderate.

Table 4: Selected moments of our models and Chatterjee

		Chatterjee	Our Model	Benchmark
		$\gamma = 0.05$	γ=0.02	γ=0.02
		AR shocks	Asymmetric	AR shocks
Def. Frq.	6.78%	5.94%	7.2%	4.35%
Spread	0.1025	0.0877	0.1105	0.0613
Debt/Output	1.00	0.70	0.86	0.48
σ (c)/ σ(y)	1.09	1.10	1.11	1.05
σ(NX)	1.50	1.02[8]	1.40	1.04
σ(r-r*)	2.85	5.60	2.80	3.57
σ(c,y)	0.98	0.97	0.99	0.99
σ(NX,y)	-0.86	-0.33	-0.74	-0.39
σ(r-r*,y)	-0.77	-0.68	-0.78	-0.65
σ(NX,r-r*)	0.70[9]	-	0.87	0.55

Additionally, our model also shows countercyclical spread, countercyclical current account and larger volatile consumption in emerging countries. Most of these simulated moments are closer to empirical data, comparing the result in Chatterjee and Eyigungor (2009).

Our major contribution is that we simulate relative low spread volatility and high spread level simultaneously, as table 3 shows, which is not solved in previous researches. In Arellano (2008) , varying default probability is the only driving force to the spread volatility, yet it can not account for the level of spread. Chatterjee and Eyigungor (2009) point out that with long-duration bonds, the model can significantly raise the average spread level, yet the spread volatility in their model is larger than empirical data. Although they explain that their model matches the spread volatility in non-default periods well, the only shortcoming is that volatility seems very sensitive to the default scenario.

With asymmetric shocks, we bring another explanation. Business cycle switches between different states through time. In each state, there is a positive probability for the economy to run into negative output shocks, or recessions. In fact, it often appears that emerging countries run into recession abruptly, sometimes even after a series of economic growth. As an example, “East Asia miracle” suddenly turns into “East Asia financial crisis” in 1997. As explanations, Calvo and Reinhart (2000) suggest “sudden stop” concept, and Boz (2007) relies on sudden change of investor’s belief as driving force behind the abrupt outbreak of crisis. We have no intention to explain the reason behind asymmetric switch between different states, and we take it as exogenous and incorporate the asymmetric switching shocks into the computational process of the model. As

8 As in Charterjee and Eyigunor (2009), we exclude the first 20 periods following autarky periods after each default in calculating the moments in table 4. We will provide sensitivity analysis regarding the length of excluding periods. Moreover, Charterjee and Eyigunor (2009) do not provide σ(y), and we estimate this value by taking their σ(y) moment to be 4%. The same approach is taken calculating the σ(r) for Charterjee and Eyigunor (2009).

9 Correlation between sovereign quarterly interest rate and current account comes from Arellano (2008).

result, probability to run into recession depends on the current state of the economy. However, in almost every state, our model shows one small and positive probability that the economy will be in recession in the next period. Considering that default occurs in recession, our model presents positive default probability even in economy boom periods. That is the major difference, comparing the previous default models. The existence of positive default probability in each period guarantees a high spread level.

Our model also shows relative low spread volatility, because the default probability changes little between different growth periods, and the sensitivity of spread volatility towards default scenarios is also low, because it is in some sense anticipated with the existence of positive default probability. Long-duration bonds alone can significantly increase the spread level, but at the same time spread volatility. It is hard to match these 2 moments simultaneously with long-duration bonds alone. With asymmetric shocks, we provide a concept to match these 2 moments simultaneously in a model.

4.2 The effect of asymmetric shocks on sovereign default

As Chatterjee and Eyigungor (2009) already give an excellent model in fitting the empirical moments, the question raised automatically is: what is the contribution of considering asymmetric shocks instead of AR symmetric shocks? We provide the selected statistics of a benchmark model, which has the same parameters as our model, except the normal shock assumption. We use Tauchen's method to calculate the finite state transition function. The underlying AR process is estimated using quarterly real GDP data over the period 1993:1-2007:4. The estimated parameters are $\rho = 0.9055$ and $\sigma = 0.0181$. The calculated moments of benchmark model are presented in 5th column of table 4.

Comparing the statistics between our model and the benchmark model, with asymmetric shocks the default frequency is 65% higher, debt/output ratio is 80% higher and spread level is almost double as that with normal shocks, while the spread volatility is much lower. The 3 correlation coefficients in table 4 change relatively less. As results, default frequency, debt/output ratio, spread level and volatility are all sensitive to the type of shocks.

As explanations, the growth prospect in emerging economies is optimistic in normal state, which consists over 80% period of time. With long-duration of growth, the sovereign can bear more debt, thus in our model the debt/output ratio is much higher in equilibrium than the benchmark model. Considering that the output shock volatility in normal growth periods is also lower than that in recession and recovery, the spread volatility which is calculated excluding the recession and recovery periods[10] is lower too.

In order to show the differences of spread more vividly, we simulate spread and output shocks in models with asymmetric shocks and normal AR shocks, excluding default periods and debt

[10] As we mentioned earlier, we calculate the statistics excluding the debt accumulation phase following the re-entry of sovereign into international credit market. As default takes place mostly in recession, the debt accumulation phase is also the recession and recovery periods.

accumulation periods following end of autarky periods. As these variables are all discrete spaced numbers, we add one small random number to each simulated data with the condition that the random numbers do not affect the original relationship between these variables. We show the results in the following scatter plot.

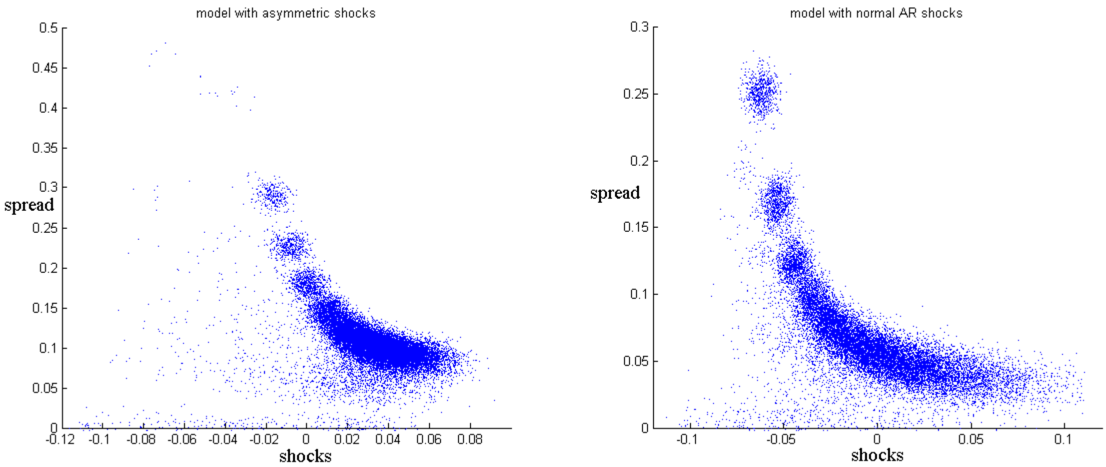

Figure 3: Spread level in model with asymmetric shocks and normal AR shocks

The left panel of figure 3 shows that most of the shocks happen in growth periods, which is an asymmetric feature of the real business cycle. Comparing with the right panel, the mean spread level in models with asymmetric shocks is almost double high as that with normal AR shocks. The selected moments in table 4 demonstrate the same result. As the spreads in left panel are more concentrated in positive shock areas, the spread volatility is lower than that in the right panel. The extreme negative shocks take place very rare with asymmetric shocks, but if it occurs, the spread level is more widely dispersed than that with normal AR shocks.

Similarly, we provide justification for higher default frequency with asymmetric shocks in the following section. According to the assumption of asymmetric shocks, it is possible that the sovereign run into recession abruptly in each period, including in economic boom periods. This feature increases the default frequency significantly, along with higher spread level. Figure 4 shows the relationship between default probabilities and output shocks, with left panel presenting the results in asymmetric shocks and the right panel normal AR shocks. The approach to simulate the relevant data is similar to figure 3.

As in figure 3, output shocks concentrate in positive area, which corresponds to normal growth periods of the emerging economy. The mean default probability is higher with asymmetric shocks in positive shock area, while with negative shocks the default probabilities are more scattered in left panel.

It is at the first sight confusing, why the default probabilities do not increase monotonically with decreasing output shocks in the left panel? Through simulation, we know that if the economy run into recession, the default will occur in the first periods of the recession with over 50% probability. With other words, this means every 2 recessions there is 1 default. This simulated result is consistent with the Argentinean economy over periods 1993Q1 to 2001Q4. During these periods Argentine experienced 2 recessions and default once. With simulation-based method in calculating transition function between different states, we incorporate this information into the model building, thus the simulated result reflect the empirical happenings of Argentinean economy between 1993Q1 to 2001Q4.

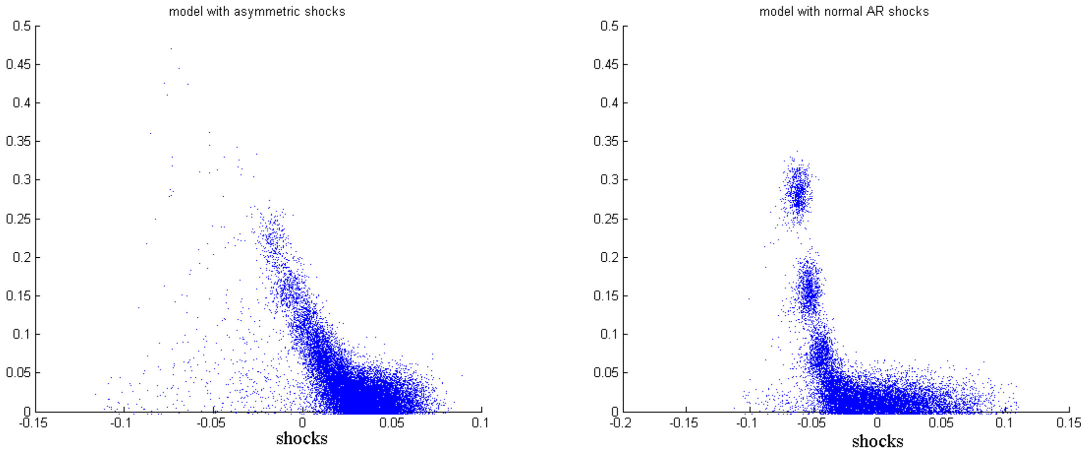

Figure 4: Default probabilities in model with asymmetric shocks and normal AR shocks

Moreover, why do defaults occur in the first period of recession in our model, which does not match the empirical results? As in other sovereign default models, we assume that the sovereign and creditors observe the current output and they know the stochastic transition function between different output states so that they can evaluate the sovereign default probabilities in the following periods. As AR model does not fit the empirical stochastic output very well, the forecast based on AR model is less accurate. With Markov-switching model and correspondent asymmetric shocks, the agents in our model can forecast the future output development more accurate. Moreover, if the economy runs into recession, the recession will persistent for 2 or 3 periods. At the beginning of the recession, the sovereign will evaluate whether they can go through the recession without default. If they can not, they will prefer to default earlier, because with earlier default they can spend less for interest rate and principal repayment, and the default cost is relatively indifferent to when to default.

However, in reality the sovereign rarely default in the first period of recession. Usually, they will first try their best to avoid the default. Our model does not match the default timing of the sovereign, because of the almost perfect forecast of the future output. One possible modification to our model lies on changing the form of expectation of the agents on future output. In our model, although we do not explicitly explain the expectation form, but our model structure implies one rational expectation. Abandoning the rational expectation may probably improve to match the default timing.

Moreover, figure 3 and figure 4 seem very similar in that the stochastic process between default spread and default frequency have a stable relationship. This is proved in Chatterjee and Eyigungor (2009), that $r(y,b')-r^{*}\approx\pi(y,b')$, with π denoting the default frequency. Our simulation results show that this relationship also holds with asymmetric shocks.

5 Sensitivity Analyses

In this section, we present sensitive analysis of our results to different values of principal repayment γ , default cost $\overline{y}$ and exclusion periods following re-entry in moments calculation.

5.1 Principal repayment ratio and default cost

We use the same computational approach to solve the recursive competitive equilibrium with different principal repayment parameters and default costs, and calculate the relevant moments via simulation. The results are presented in table 5.

Table 5: Model results with different principal repayment ratios and default costs

	γ=0.05, $\beta=0.96$			γ=0.02, $\beta=0.96$		
	$\overline{y}$=0.94	$\overline{y}$=0.92	$\overline{y}$=0.90	$\overline{y}$ = 0.90	$\overline{y}$ = 0.88	$\overline{y}$ = 0.87
Def. frequency	8.84%	7.72%	6.28%	8.52%	7.12%	6.52%
Spread level	0.1278	0.1117	0.097	0.1276	0.1113	0.1092
Debt/output	0.59	0.71	0.82	0.92	1.03	1.11
σ (c)/ σ(y)	1.15	1.16	1.16	1.12	1.13	1.10
σ(NX)	1.08	2.08	4.04	0.64	2.04	4.16
σ(r-r*)	4.03	4.27	4.70	2.81	2.98	3.09
σ(c,y)	0.99	0.97	0.92	1.00	0.97	0.90
σ(NX,y)	-0.64	-0.35	-0.21	-0.72	-0.21	-0.03
σ(r-r*,y)	-0.79	-0.72	-0.61	-0.80	-0.76	-0.58
σ(NX,r-r*)	0.77	0.54	0.43	0.84	0.35	0.32

Table 5 shows that model results are sensitive to principal repayment ratio and default costs, thus careful choice regarding to these 2 parameters is important. Additionally, time preference parameter β also influence the calculated moments, as Aguiar and Gopinath (2006) present. Low value of β means the agent is more inpatient, so that they value the current consumption more important, which leads to more default and raises default frequency. A detailed sensitivity analysis towards β is not considered here.

With smaller principal repayment ratio, the default frequency, spread level and debt/out ratio are all higher, and the correlation coefficients between sovereign spread, output and trade balance are more sensitive. Smaller principal repayment ratio means that the sovereign bonds have longer duration, and Hatchondo and Martinez (2009) explain that long-duration bonds are associated with higher default probabilities in the future periods, and these bonds are also more sensitive to output shocks, which lead to more volatile sovereign spread. Moreover, since with long-duration bonds the principal repayment in each period is lower, the sovereign will issue more debt to raise consumption, which leads to higher debt/output ratio.

As to default cost, it is clear that larger default costs (smaller $\overline{y}$), will decrease the probability that sovereign default in case of negative shocks. Since sovereign spread and default frequency hold a stable relationship, larger default cost will result in smaller spread level. With decreasing default probability, the creditors are willing to lend more to sovereign, thus increase the debt/output ratio.

5.2 Exclusion periods following re-entry

According to the assumption of our model, the sovereign repudiate the outstanding debt obligation in default, so that the model shows no debt position as the sovereign re-enters the international credit market. Chatterjee and Eyigungor (2009) states that the debt accumulation process just after the re-entry is not related to the output and interest rate. Thus we exclude the debt accumulation periods following the re-entry in calculating relevant statistic moments.

We denote the length of debt accumulation phase to be m periods. Chatterjee and Eyigungor (2009) set m to be 20, and in their model the calculated moments are not sensitive to the choice of m. In our model, however, we find that several statistic moments are very sensitive to m. As reasons, defaults take place mostly in recessions and in our model the economy moves quickly from recession to recovery. If m is chosen to be very larger, then the calculated volatilities will be too small, because the important dynamic process of recovery is not included in moments calculation, and as the GDP growth rate changes less in normal growth phase, the resulting volatility will be smaller, at the same time the negative correlation between net export and output, spread and output will be attenuated.

The calculation of default frequency is not related to exclusion periods, so we do not report the sensitivity test regarding default frequency. The other moments with different exclusion periods are presented in table 6.

Table 6: Moments with different exclusion periods

	m = 15	m = 20	m = 25
Spread level	0.1108	0.1104	0.1104
Debt/output	0.858	0.859	0.859
σ (c)/ σ(y)	1.13	1.11	1.11
σ(NX)	1.68	1.40	0.56
σ(r-r*)	2.89	2.82	2.82
σ(c,y)	0.97	0.99	0.99
σ(NX,y)	-0.28	-0.74	-0.81
σ(r-r*,y)	-0.78	-0.78	-0.78
σ(NX,r-r*)	0.32	0.87	0.96

Table 6 shows that trade balance is very sensitive to the choice of exclusion periods. With shorter exclusion periods, the trade balance is more volatile, and correlation coefficients with output and sovereign spread are less significant. This sensitivity can be explained with debt accumulation process following the re-entry into credit market. Yet, debt/output ratio remains stable in this process, which indicates that debt accumulates along with the increasing output at a similar speed. In this process, the volatility of sovereign spread is slightly higher than other periods, and the other moments remain stable with different choice of exclusion periods.

6 Results

We develop a sovereign default model by assuming asymmetric output shocks and long-term bonds. In quantitative analysis to Argentinean economy between 1993Q1 to 2001Q4, our model matches the data well. We can simulate countercyclical interest rate and trade balance, as long as high spread level and low spread volatility. Comparing to the models with normal symmetric shocks, our model can generate higher default frequency, higher debt/out ratio, higher spread level and low spread volatility.

As explanations, we point out that with asymmetric shocks the emerging countries may run into recession with positive probability in each periods, which keeps the default frequency stay in a high level. Additionally, the emerging countries meet with positive shocks in most of periods, and this lead to less volatile spread.

Moreover, we provide a simulation-based approach to approximate transition function between finite states. While Tauchen's method is limited to transform AR model to finite state Markov chain, our method is very general in converting various kinds of econometric models to Markov chain, which is indispensable in discrete state dynamic programming. Considering dynamic programming to be an important computational approach to modern macroeconomic models, our method can be widely used to achieve numerical analysis.

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