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Berliant, Marcus and Tabuchi, Takatoshi

Washington University in St. Louis, University of Tokyo

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Marcus Berliant†

Department of Economics, Washington University in St. Louis

Takatoshi Tabuchi‡

Faculty of Economics, University of Tokyo

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Abstract

We consider information aggregation in national and local elections when voters are mobile and might sort themselves into local districts. Using a standard model of private information for voters in elections in combination with a New Economic Geography model, agglomeration occurs for economic reasons whereas voter stratification occurs due to political preferences. We compare a national election, where full information equivalence is attained, with local elections in a three district model. A stable equilibrium accounting for both the economic and political sectors is shown to exist. Restricting to an example, we show that full information equivalence holds in only one of the three districts when transport cost is low. The important comparative static is that full information equivalence is a casualty of free trade. When trade is more costly, people tend to agglomerate for economic reasons, resulting in

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†Department of Economics, Washington University, Campus Box 1208, 1 Brookings Drive, St. Louis, MO 63130-4899 USA. Phone: (314) 935-8486. Fax: (314) 935-4156. E-mail: berliant@artsci.wustl.edu

‡Faculty of Economics, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan. Phone: (+3) 5841-5603. Fax: (+3) 5841-5521. E-mail: ttabuchi@e.u-tokyo.ac.jp

full information equivalence in the political sector. Under free trade, people sort themselves into districts, most of which are polarized, resulting in no full information equivalence in these districts. We examine the implications of the model using data on corruption in the legislature of the state of Alabama and in the Japanese Diet.

Keywords and Phrases: information aggregation in elections, informative voting, new economic geography, local politics

JEL Classification Numbers: D72, D82, R13

1 Introduction

We seek to address questions at the boundary of politics and geography: How much information is revealed in local as opposed to national elections? Does the mobility of voters help or hinder information aggregation in local elections? Of course, the electorate is generally smaller in local as opposed to national elections, but does voter migration for economic reasons result in polarization of local elections?

For an empirical viewpoint, we examine officials that are elected and later found to have received outside money that might compromise their votes. Consider the following data, collected by Couch et al. (1992) on whether Alabama state elected officials receive income from serving on boards of local state-funded universities. House districts are evidently smaller.

Table 1: 2×2 Contingency Table for Alabama's Legislature

Alabama	No Outside Income	Outside Income
Senate	31 (88.6%)	4 (11.4%)
House	77 (73.3%)	28 (26.6%)

Sources: Couch et al. (1992), http://www.legislature.state.al.us/

Note that House districts are not necessarily subsets of Senate districts.

$\chi^2 = 3.46$

Degrees of Freedom = 1

Probability = 0.063

From this table we can see that the likelihood that House and Senate members differ in their receipt of outside income is large but not definitive. Could it be that some elections for the House imply more information aggregation than others?

Next consider members of the Diet in Japan. It is bicameral, the House of Councilors having fewer members than the House of Representatives.

Table2: 2×2 Contingency Table for Japan's Diet[1]

Japan	No Allegations	Resigned Under Duress or Convicted
House of Councilors	145 (99.3%)	1 (0.7%)
House of Representatives	290 (96.7%)	10 (3.3%)

Sources: http://www.notnet.jp/data04index.htm

Note that House of Representatives districts are subsets of House of Councilors districts.

$\chi^2 = 2.86$

Degrees of Freedom = 1

Probability = 0.091

Again, there appears to be more corruption in elections involving smaller districts, but this is not definitive.

To address the theoretical questions we have posed as well as to explain the data, we formulate a model of politics and information aggregation in elections where voters are also economic agents and mobile. Geography and politics interact and feed back in interesting ways: On the one hand, economic factors might cause agglomeration of agents, thus affecting the polarization of districts, the aggregation of information in local elections, and the outcomes of local elections. On the other hand, the outcomes of elections in localities might affect the agglomeration of agents into these localities. This interplay leads us into the introduction of geography into models of politics, in particular those associated with the Condorcet jury theorem such as Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). It also leads us to introduce politics into models of stratification or agglomeration, such as Krugman (1991). In this respect, we could have used a model of local public goods for this purpose, but find the New Economic Geography model from urban economics to be both more tractable and less biased toward stratification. For example, in the US context, local education and quality of schools, along with property taxes, are the most important criteria used by consumers/voters for determining location of residence. Tiebout

[1] In the House of Councilors of Japan's Diet, 146 of the 242 seats are elected in single-seat districts and 96 by proportional representation. In the House of Representatives, 300 of the 480 seats are elected in single-seat districts and 180 by proportional representation.

sorting models will generally lead directly to stratification by type of consumer in equilibrium, implying a failure of full information equivalence in the various districts. In summary, we could use a model of equilibrium in a local public goods economy in place of the New Economic Geography part of our model, but we conjecture that results would be similar. In general, New Economic Geography models lead to agglomeration, but not directly to stratification.

Our main findings are summarized as follows. We compare a national election, where the same outcome is attained whether voters know everyone's private information or not (called *full information equivalence*[2] in the political science literature), with local elections in a three district model. A stable equilibrium accounting for both the economic and political sectors in the local model is shown to exist. Restricting to an example, we show that full information equivalence holds in only one of the three districts when transport cost is low. The important comparative static is that *full information equivalence is a casualty of free trade.* When trade is more costly, people tend to agglomerate for economic reasons, resulting in full information equivalence in the political sector. Under free trade, people sort themselves into districts, two of which are polarized, resulting in no full information equivalence in these districts. The remaining district still satisfies full information equivalence. Thus, if the signals voters receive concern the conflict of interest or corruption of candidates in their district, it is expected that elections in districts with smaller populations (local elections) will result in a higher proportion of compromised elected officials. This might even happen if the electorate is large, as in our model. But some of these districts will still satisfy full information equivalence, so the correlation between size of electorate and information aggregation in elections is imperfect.

The literature on information aggregation in elections has a focus on an electorate that is exogenously given and thus is immobile. Austen-Smith and Banks (1996) presented the seminal work on the Condorcet jury theorem, showing in a game-theoretic context that for some states of nature, not all the information of voters is revealed in Nash equilibrium even if they all have the same objective functions and priors. Feddersen and Pesendorfer (1997) find sufficient conditions for which full information equivalence holds at Nash equilibrium, and that is the framework we

[2] Equivalently, it can be said that full information aggregation occurs in the election.

employ below.

The literature on economic geography has almost no focus on voting, particularly when there is asymmetric information about candidates or ballot measures.

The outline for the balance of the paper is as follows. Section 2 gives the model and definitions of equilibrium and stability. Section 3 provides the basic results on existence of a stable equilibrium and proofs. Section 4 contains the comparative statics of the model with a focus on local politics. Section 5 discusses the general implications of the model, returning to our discussion of the data. Finally, Section 6 gives our conclusions.

2 The model

The spatial structure of the model consists of three districts indexed by $i = 1, 2, 3$, located at each vertex of a regular triangle. These can be cities, regions or jurisdictions within a city. There is an exogenously given mass $L > 0$ of consumers, each of whom supplies one unit of labor inelastically. Let the population of district i be denoted by L_i.

The model has a political as well as an economic sector. Overall utility is given by the sum of subutilities from the two sectors. The utility from the economic sector for a resident of district i is given by u_i, whereas the utility from the political sector is given by v. The total utility is given by

$$U_i \equiv u_i + v.$$

We will describe these subutility functions, including their domains, in detail. We begin by describing the economic sector.

2.1 The economic sector

Preferences are defined over a continuum of varieties of a horizontally differentiated good. The preferences of a typical resident of district i are represented by the following CES utility:

$$u_i = \left[\sum_{j=1}^{3} \left(\int_{\Omega_j} d_{ji}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega \right) \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $d_{ji}(\omega)$ is the consumption in district i of variety ω produced in district j, and Ω_j is the set of varieties produced in district j with $j = 1, 2, 3$. The parameter $\varepsilon > 1$ measures both the constant own-price elasticity of demand for any variety, and the elasticity of substitution between any two varieties. Unlike standard models in the tradition of New Economic Geography, there is no freely traded homogeneous good. The freely traded homogeneous good is unrealistic and its presence might not be innocuous (Davis, 1998).

To explain how the economic sector works, first fix the locations of consumers.

Production of any variety of the differentiated good takes place under increasing returns to scale by a set of monopolistically competitive firms. This set is endogenously determined in equilibrium by free entry and exit. In what follows, we denote by n_i the mass of firms located in district i. Production of each variety requires both a fixed and a constant marginal labor input requirement, denoted by $\bar{c}$ and c respectively. As for transportation costs, inter-district shipments of any variety are subject to iceberg transportation costs: $\tau_{ij} \geq 1$ units have to be shipped from district i to district j for one unit to reach its destination.

Given our assumptions, in equilibrium firms differ only by the district in which they are located. Accordingly, to simplify notation, we drop the variety label ω from now on. Then, the maximization of (1) subject to the budget constraint

$$\sum_{j=1}^{3} n_j p_{ji} d_{ji} = w_i \tag{2}$$

yields the following individual demand in district i for a variety produced in district j:

$$d_{ji} = \frac{p_{ji}^{-\varepsilon}}{P_i^{1-\varepsilon}} w_i, \tag{3}$$

where w_i is the wage in district i, p_{ji} is the delivered price of the variety from district j to district i, and P_i is the CES price index in district i defined by:

$$P_i \equiv \left(\sum_{k=1}^{3} n_k p_{ki}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{4}$$

Because of the iceberg transport cost assumption, a typical firm established in district i has to produce $q_{ij} = \tau_{ij} d_{ij} L_j$ units to satisfy final demand d_{ij} in district j, where

L_j is the number of consumers in district j. The firm takes (3) into account when maximizing its profit given by:

$$\Pi_i = \left(\sum_{j=1}^{3} p_{ij} d_{ij} L_j \right) - w_i \left(c \sum_{j=1}^{3} q_{ij} + \overline{c} \right). \tag{5}$$

Profit maximization with respect to p_{ij}, taking the price index P_j as given because of the continuum of varieties, then implies that the price per unit delivered is:

$$p_{ij} = \frac{\varepsilon c}{\varepsilon - 1} \tau_{ij} w_i = \tau_{ij} w_i. \tag{6}$$

Due to free entry and exit, profits must be non-positive in equilibrium. Then (5) and (6) imply that firms' equilibrium scale of operation in country i must satisfy $\Pi_i = 0$, which is rewritten as:

$$(p_{ii} - c w_i) \sum_{j=1}^{3} \tau_{ij} d_{ij} L_j = w_i \overline{c}. \tag{7}$$

Because the labor input is given by $c \sum_{j=1}^{3} q_{ij} + \overline{c}$ in (5), the labor market clearing conditions are given by:

$$n_i \left(c \sum_{j=1}^{3} \tau_{ij} d_{ij} L_j + \overline{c} \right) = L_i. \tag{8}$$

Eliminating p_{ij} and $\sum_{j=1}^{3} \tau_{ij} d_{ij} L_j$ from (6), (7) and (8), we get:

$$n_i^* = \frac{L_i}{\varepsilon \overline{c}}. \tag{9}$$

That is, the number of firms in a district is proportional to the number of workers in that district at equilibrium.

Substituting (4), (6) and (9) into the zero profit condition (7), we have:

$$\sum_{j=1}^{3} \frac{\phi_{ij} w_j L_j}{\sum_k n_k^* w_k^{1-\varepsilon} \phi_{kj}} = w_i^{\varepsilon}. \tag{10}$$

Due to the geographically symmetric location of the districts, we set

$$\phi_{ij} \equiv \tau_{ij}^{1-\varepsilon} = \begin{cases} \phi \in [0,1] & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases},$$

which is a measure of how free trade is. Its value is one when trade is free and zero when trade is prohibitively costly. There are three equilibrium conditions (10) and three unknowns: w_1, w_2 and w_3. However, one of the three equations in (10) is redundant by Walras' law. We set $w_1 = 1$ by choosing the wage in district 1 as the numéraire. As is standard in the New Economic Geography literature, it can be shown that there is a unique solution, namely $(w_1, w_2, w_3) = (1, w_2^*, w_3^*)$.

The indirect equilibrium utility (with a fixed distribution of consumers) is given by:

$$u_i^* = \frac{w_i^*}{P_i^*} = \frac{w_i^*}{\left[\frac{1}{\varepsilon \bar{c}} \sum_{j=1}^{3} \phi_{ji} \left(w_j^*\right)^{1-\varepsilon} L_j\right]^{\frac{1}{1-\varepsilon}}}.$$

It can also be shown that if $L_i > L_j$, then $w_i^* > w_j^*$ and $u_i^* > u_j^*$. This is called the market size effect.

2.2 The political sector

There are two types of elections, namely national elections and local elections. For national elections, every consumer votes. For local elections, the alternatives are chosen in each district independently. Only the residents of a district vote in the elections for that district. We formulate two models, one with only a national election, and one with only local elections. We adopt the framework of Feddersen and Pesendorfer (1997) for the political sector. There are two alternatives in any election, A and Q. Let $\alpha \in \{A, Q\}$. A preference parameter for a voter is given by $x \in [-1, 1]$, whereas the state is given by $s \in [0, 1]$. The set of voter types is denoted by $X = [-1, 1]$.[3] The probability distribution over consumer types is given by F (if it has a density, call it f), whereas the common prior over states is given by $G(s)$ (if it has a density, call it g). Define the utility from the political sector of type x from alternative α in state s to be $v(\alpha, s, x)$. We assume that $\Delta v \equiv v(A, s, x) - v(Q, s, x)$ is continuous and increasing in s and x.

The total utility of a consumer in district i of type x is abbreviated as

$$U_i^x \equiv u_i + v.$$

[3] In the terminology of Feddersen and Pesendorfer (1997), there is only one information service.

Each voter receives a signal $\sigma \in \{1, ..., M\}$ at the beginning of the political stage, before voting, but after the economic stage. Denote by $p(\sigma \mid s)$ the probability that a consumer receives signal σ in state s.

2.3 Timing of the game

All players have perfect foresight. The timing of the game is as follows. First, the firms and consumers locate themselves in the three districts, knowing what lies ahead. The agents cannot relocate after this step. Then economic equilibrium in the districts is achieved. Next, each consumer receives a signal about the alternatives in the political sector. Then they simultaneously vote over the two alternatives, the winner determined by majority rule. For national elections, the outcome is independent of the district of residence. For local elections, the outcome is specific to each of the three districts. This is equilibrium in the political sector. Finally, all players receive their utility payoffs. We seek the subgame perfect Nash equilibria of this game.

Notice that for national elections, only the economic sector matters in the choice of location, so the game reduces to a standard New Economic Geography model. Hence, we focus on local elections.

2.4 Equilibrium

Definition 1 *A **strategy profile** is a measurable map $\pi = (\pi_1, \pi_2)$ where $\pi_1 : X \longrightarrow \{1, 2, 3\}$ and $\pi_2 : X \times \{1, 2, 3\} \times \{1, ..., M\} \longrightarrow [0, 1]$. Here, π_1 denotes the strategy at stage 1, the economic stage, whereas π_2 denotes the strategy at stage 2, the political stage. In general, the range of π_2 denotes a mixed strategy where 0 is a pure strategy vote for A whereas 1 is a vote for Q.*

In stage 1, each consumer (of any type) chooses a location. In stage 2, they vote.

We face a technical issue here that is faced by most working on information aggregation in elections. In general, models with a finite number of voters are used due to division by zero in applying Bayes' rule when there is a continuum of voters. In other words, the event that a person is pivotal when there is a continuum of voters often has probability zero, so conditioning on this event is not possible. One option to address this problem is to use regular conditional probabilities, but that is not possible in our context. The alternative that we (and the literature) use is specified as follows.

The first stage of the game proceeds as an economy and game with a continuum of players. This yields a population distribution in each of the three districts. For national elections, votes from both districts are counted. For local elections, only votes from a district are counted for the election in that district. When there are local elections, there is an outcome for each district.

Fix population distributions F_1, F_2, F_3 in districts 1, 2, and 3, respectively (if there is a density f for F, then F_1 has density f_1, F_2 has density f_2, and f_3 has density F_3). For local elections (national elections follow in an obvious way) we draw randomly and independently N voters from each district using the appropriate district-wide distribution, where N is exogenous. Focus on a district i and a symmetric strategy profile for the district π_2. Following Feddersen and Pesendorfer (1997, p. 1034), for each state $s \in [0, 1]$ we can calculate the updated posterior for the state, conditional on a voter being pivotal, on the signal they receive, and on others' strategies. For simplicity, we denote this by $\rho(s \mid \pi_2, \sigma, k)$. Using this posterior, we can compute expected utility from the two alternatives, namely $E[v(A, s, x) \mid \pi_2, \sigma]$ and $E[v(Q, s, x) \mid \pi_2, \sigma]$. A voter can choose Q or A. If the proportion of voters who choose Q is larger than $1/2$, then Q is the outcome. Otherwise, A is the outcome.

A **second stage N-equilibrium** is a symmetric Bayesian Nash equilibrium in this second stage of the game, where no consumer/voter uses a weakly dominated strategy. Proposition 1 (actually the proof in the appendix) of Feddersen and Pesendorfer (1997) shows that such an equilibrium exists under their Assumption 1.

A **second stage equilibrium** is any limit point of second stage N-equilibria where N tends to infinity. Such exist if second stage N-equilibrium exists for each N due to the following argument. Let π_2^N be a second stage N-equilibrium with N voters drawn from F_i. If necessary, draw a converging subsequence so that for $i = 1, 2, 3$: $\int_X \pi_2^N(x, i, \sigma) dF_i$ converges for each σ.[4] This yields the expected number of votes for Q given σ at equilibrium. Then apply Fatou's lemma in several dimensions (see

[4]Notice that for each N this is just a list of real numbers of fixed, finite length, so such a converging subsequence exists.

Hildenbrand, 1974, p. 69) to obtain a limit. The law of large numbers implies that if this number exceeds $\frac{1}{2}$ in district i, then given σ, the winner is Q. Otherwise, it is A. Notice that the limit is not necessarily an equilibrium of the limiting game, due to problems with division by zero mentioned above. Rather, it is the limit of a sequence of equilibria for games with finitely many players, where the number of players tends to infinity.

Fix a strategy profile $\pi = (\pi_1, \pi_2)$. Fix a district i. At stage 2, the posterior over states conditional on being pivotal in that district and observing signal σ is denoted by $\beta_i(s \mid piv, \pi_2, \sigma)$. Then the explicit derivation of β_i can be found in Feddersen and Pesendorfer (1997, p. 1034) and below. The objective of a voter of type $x \in X$ is given by

$$\max_{\alpha \in \{A,Q\}} \int_0^1 v(\alpha, s, x) \cdot \beta_i(s \mid piv, \pi_2, \sigma) ds.$$

An **equilibrium** is the limit point of a sequence of subgame perfect, symmetric Bayesian Nash equilibria in this two stage game, where (almost) no consumer/voter in the sequence of games uses a weakly dominated strategy.

Informally, an equilibrium is said to satisfy **full information equivalence in district *i*** if the alternative that wins the election in that district is almost surely the one that would have been chosen if the electorate in that district were fully informed about the state s. The formal definition of full information equivalence is technical because it relies on statements about the asymptotic properties of large but finite elections, and can be found in Feddersen and Pesendorfer (1997, p. 1042).

2.5 Stability

To ease notation, we define $\lambda_i = \frac{L_i}{L}$. Take an equilibrium population distribution $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$ with indirect economic utility u_i^* and with indirect political expected utility $Ev_i^*(x)$ for $i = 1, 2, 3$. Let f_i^* be the equilibrium density of types in district i, and let S_i^* be its support. We say that the equilibrium is *stable* if

$$\frac{d\left(U_j^* - U_i^*\right)}{d\lambda_j} = \frac{d\left(Ev_j^* - Ev_i^*\right)}{dx} \bullet \frac{dx}{d\lambda_j} + \frac{d\left(u_j^* - u_i^*\right)}{d\lambda_j}\Bigg|_{x \in S_i^* \cap S_j^*} < 0 \qquad \text{for } i, j = 1, 2, 3;\ i \neq j. \tag{11}$$

Here we are assuming that the economic equilibrium *does* change at the margin. However, the marginal change in the distribution of voters in the districts *does not* change the political equilibrium in either their origin or destination. The reason for this asymmetry between sectors is as follows. On the one hand, in the economic sector, even though no single consumer can affect prices, the consumers who are moved to a new district can observe that equilibrium prices, and thus their indirect utility, actually change. We take the limit of the change in utility divided by the measure of consumers moved as the measure of consumers goes to zero, resulting in the derivative of indirect utilities with respect to population. On the other hand, for the political sector, we are taking a different kind of limit, namely the limit of voting equilibria when there are random draws from the electorate as the size of the draw becomes large. When the distribution F has a density f, the probability that any particular person is even chosen as a member of a finite draw is zero. Thus, each individual agent does not think that their move to another district will affect the political outcome in either their origin or destination. (One can move a set of positive measure between districts and take limits as both the size of the draw and the measure of the set moved tend to zero. In that case, the order of limits is important. Since the limit of the equilibria as the size of the draw tends to infinity is not necessarily an equilibrium of the limiting game, we must focus on a fixed, finite size of draw and take the limit as the measure of agents moved tends to zero first, then focus on the limit of such equilibria as the sample size tends to infinity. In essence, we are testing for stability of the equilibria of the games with finite random draws of the electorate from the distribution rather than stability of the limit game. The latter has ill-defined conditional probabilities of being pivotal.)

If the supports don't overlap on an open set, $\frac{dx}{d\lambda_j} = \frac{1}{f_j^*}$. If they overlap on an open set, then on that open set, $\frac{dx}{d\lambda_j} = 0$.

3 Existence of Stable Equilibrium

Theorem 2 *Assume that the political sector of the national model, with all agents present and voting, satisfies either Assumptions 1-8 of Feddersen and Pesendorfer*

(1997)[5] **or** *assume that full information equivalence holds. Then a national election equilibrium in mixed strategies exists, and it satisfies full information equivalence. Let* $v(\alpha, s, \cdot)$ *and* f *be symmetric about* 0*. For every set of parameter values, there exists a local election equilibrium. This equilibrium features pure strategies, except for the second stage (political) equilibrium strategies in the one district with full information equivalence, where mixed voting strategies might be used. It also features a population distribution symmetric around* $x = 0$*. Generically, at least one such equilibrium is stable.*

Proof. The part of the Theorem concerning national elections follows from Feddersen and Pesendorfer (1997) and the argument that a second stage equilibrium exists if a second stage N-equilibrium exists for each N. The part of the proof concerning local elections proceeds as follows. First, we find a candidate symmetric allocation. Then we prove that it is an equilibrium. Finally, we prove that it is stable.

Let $\lambda \in [0, \frac{1}{2}]$ represent the populations of districts 1 and 3, namely $\lambda = \lambda_1 = \lambda_3$, so $1 - 2\lambda$ is the population of district 2. We will *guess* that district 1 always votes unanimously for Q, district 3 always votes unanimously for A, and district 2 satisfies full information equivalence, so the state-dependent outcome in district 2 is the same as the outcome with no uncertainty. For notational purposes, define that outcome to be $\alpha^*(s)$.

We define the potential equilibrium value $\lambda^* \in [0, \frac{1}{2}]$ to be the minimal value of λ such that the marginal consumer is indifferent between districts 1 and 2:[6]

$$u^*(\lambda) + \int_0^1 v(Q, s, F^{-1}(\lambda))dG(s) = u^*(1 - 2\lambda) + \int_0^1 v(\alpha^*(s), s, F^{-1}(\lambda))dG(s).$$

Next, we show that this is in fact an equilibrium. To accomplish this, we must simply consider the decision of one individual at this allocation. No individual can unilaterally affect the economic allocation in any district, no matter their action. Similarly, no individual can affect the political outcome in any district, no matter their action, since the probability that they are selected as a pivotal voter is zero. So it is simply a matter of showing that the agents we have assigned to each district are

[5] See the appendix for a precise statement of the assumptions.

[6] This condition is familiar from models of local public goods.

at least as happy with the outcomes in that district as they would be in any other. By symmetry, if the argument works for one side of the distribution, it works for the other. So we focus on the left side. Notice that since $\Delta v(s, x)$ is increasing in s and x, $v(\alpha^*(s), s, x) - v(Q, s, x)$ is non-decreasing in x for each s.

$$\begin{aligned} & u^*(\lambda^*) - u^*(1 - 2\lambda^*) \\ = & \int_0^1 [v(\alpha^*(s), s, F^{-1}(\lambda^*)) - v(Q, s, F^{-1}(\lambda^*))]dG(s). \end{aligned}$$

So for $x \geq F^{-1}(\lambda^*)$,

$$u^*(\lambda^*) - u^*(1 - 2\lambda^*) \leq \int_0^1 [v(\alpha^*(s), s, x) - v(Q, s, x)]dG(s)$$

and thus $U_1^x \leq U_2^x$. A similar argument works for $x \leq F^{-1}(\lambda^*)$, implying $U_2^x \leq U_1^x$. A symmetric argument holds for the boundary between districts 2 and 3. Notice that this argument also holds if $\lambda^* = \frac{1}{2}$, noting that $\alpha^*(s)$ is replaced by A in the expressions above.

Finally, we show that this equilibrium is generically stable. We must evaluate equation (11) for movement between the districts. There are two ways to attempt this. We could evaluate it directly for this system. Alternatively, we could apply the result in Tabuchi and Zeng (2004), reducing our work load. Although the latter approach is easier, and is the one we will use, the issue is that their framework is set up for homogeneous consumers. Obviously, we have heterogeneous consumer/voters. So in order to apply the result, we formulate an artificial model that gives all consumers in a district the utility of the consumer at the margin or boundary for that district. Then this artificial model fits into the framework of Tabuchi and Zeng (2004).

Define

$$\begin{aligned} \overline{U}_1(\lambda_1) &= u_1^*(\lambda_1) + \int_0^1 v(Q, s, F^{-1}(\lambda_1))dG(s) \qquad (12) \\ \overline{U}_2(\lambda_2) &= u_2^*(\lambda_2) + \int_0^1 v(\alpha(s), s, F^{-1}(\frac{1}{2} - \frac{\lambda_2}{2}))dG(s) \\ &= u_2^*(\lambda_2) + \int_0^1 v(\alpha(s), s, F^{-1}(\frac{1}{2} + \frac{\lambda_2}{2}))dG(s) \\ \overline{U}_3(\lambda_3) &= u_3^*(\lambda_3) + \int_0^1 v(A, s, F^{-1}(1 - \lambda_3))dG(s), \end{aligned}$$

where

$$\lambda_1 + \lambda_2 + \lambda_3 = 1. \tag{13}$$

The remainder of the proof consists of 3 steps. First, apply Tabuchi and Zeng (2004, Theorem 2) to the system defined by (12) and (13) to obtain generic existence of a stable equilibrium for this system. Second, we claim that there is no asymmetric equilibrium of the system (12) and (13), so the stable equilibrium must be symmetric, namely $\lambda_1 = \lambda_3$. This holds because if the equilibrium is not symmetric, then there is a discontinuity in equilibrium utility between some pair of districts, implying that it is not an equilibrium, a contradiction. Third, we claim that any stable equilibrium of the system (12) is also a stable equilibrium for the original system in the sense of equation (11). It is an equilibrium of the original system because the utilities of the consumers at the boundaries between districts are equated. Stability holds because the derivatives of the two systems, evaluated at equilibrium populations, are the same. This is easily verified for each part of the left hand side of inequality (11), where each part is evaluated at the boundary (in consumer/voters) between districts. ■

4 Comparative Statics

In order to study the comparative statics of equilibrium, we must be much more specific about the political sector. There are several reasons for this. First, since we want to be able to say something specific about the equilibrium distribution of population, we must know more about the equilibrium in the political sector for each given distribution of population, as agents can anticipate (at least in expectation) what will happen politically in each individual district, given the population distribution. The abstract framework of Feddersen and Pesendorfer (1997) tells us that equilibrium in *mixed* strategies exists and it has the form of a cutpoint equilibrium. But for our application, it is very useful to have an equilibrium in pure strategies. So we use one of their examples that does not fit their general framework, namely their example 2, where for any distribution of population, equilibrium in pure strategies exists, is unique, and (under some further conditions) satisfies full information equivalence. The drawback of using this example is that since it does not satisfy their assumptions, we cannot claim the same generality in our results as they do in their paper.

A related issue pertaining to the modeling strategy concerns the fact that we have made functional form assumptions for the New Economic Geography sector of the model, for reasons detailed in that literature. This allows us to find equilibrium in that sector explicitly. If we were to use the general functional form we have specified for the political sector, then although we could know about existence of equilibrium and perhaps its general properties, we would not be exploiting the specific functional form assumptions made in the economic sector, and thus we could not use this to find equilibrium explicitly. In other words, we waste the additional information provided by functional form assumptions in the economic sector. With functional form assumptions in the political sector as well, we have balanced the assumptions in the two sectors so that we can exploit all of the functional form assumptions we use to find equilibrium explicitly, and thus find comparative statics explicitly.

Assume that the political utility v is given by

$$v(\alpha, s, x) = K - \frac{1}{2}\left(x_\alpha - x\right)^2 - \left(x_\alpha + \frac{1}{2} - s\right)^2,$$

where $x_A = 1$ and $x_Q = -1$. Then,

$$\Delta v \equiv v(A, s, x) - v(Q, s, x) = 2\left(-1 + x + 2s\right),$$

which is similar to the examples of section 5 in Feddersen and Pesendorfer (1997). Also assume that $f(x)$ is uniform over $[-1, 1]$. Then, the probability that a randomly selected voter votes for Q in state s is

$$\begin{aligned} t(s, \overline{\pi}) &= \sum_{\sigma=1}^{2} p(\sigma \mid s) \int_X \overline{\pi}(x, \sigma) f(x) dx \\ &= \begin{cases} (1-\alpha) F(x_1) + \alpha F(x_2) & \text{if } s < 1/2 \\ \alpha F(x_1) + (1-\alpha) F(x_2) & \text{if } s > 1/2 \end{cases} \end{aligned} \tag{14}$$

from the definition of $p(\sigma \mid s)$. The probability that a vote is pivotal in state s is given by

$$\Pr\left(piv \mid s, \overline{\pi}\right) = \binom{n}{n/2} t(s, \overline{\pi})^{n/2} \left[1 - t(s, \overline{\pi})\right]^{n/2},$$

where $t(s,\overline{\pi})$ is given by (14).

Analogous to Feddersen and Pesendorfer (1997), let x_1 and x_2 be cutpoints, $x_1 > x_2$, namely for $x < x_2$ the voter always votes for Q, for $x > x_1$ the voter always votes for A, and for $x_2 \leq x \leq x_1$ the voter uses a state-dependent strategy. Because of the symmetric setting relative to $x = 0$, it must be that the cutpoints are symmetric: $x_1 + x_2 = 0$, implying that $\Pr(piv \mid s, \overline{\pi})$ as calculated above is constant for all s. Then, the probability distribution over states conditional on being pivotal, $\beta(s \mid piv, \overline{\pi})$, is also constant, and hence, the probability distribution over states conditional on being pivotal and observing signal σ is reduced to

$$\begin{aligned}\beta(s \mid piv, \overline{\pi}, \sigma) &= \frac{\beta(s \mid piv, \overline{\pi})\, p(\sigma \mid s)}{\int_0^1 \beta(w \mid piv, \overline{\pi})\, p(\sigma \mid w) dw} \\ &= \frac{p(\sigma \mid s)}{\int_0^1 p(\sigma \mid w) dw}.\end{aligned}$$

Because

$$\begin{aligned}\beta(s \mid piv, \overline{\pi}, 1) &= \begin{cases} 1-\alpha & \text{if } s < 1/2 \\ \alpha & \text{if } s > 1/2 \end{cases} \\ \beta(s \mid piv, \overline{\pi}, 2) &= \begin{cases} \alpha & \text{if } s < 1/2 \\ 1-\alpha & \text{if } s > 1/2 \end{cases},\end{aligned}$$

we have

$$E[s \mid piv, \overline{\pi}, \sigma] = \frac{\int_0^1 \beta(s \mid piv, \overline{\pi}, \sigma)\, s ds}{\int_0^1 s ds} = \begin{cases} \frac{1}{4}(1+2\alpha) & \text{if } \sigma = 1 \\ \frac{1}{4}(3-2\alpha) & \text{if } \sigma = 2 \end{cases}.$$

Solving

$$\begin{aligned}E[v(x_1, s) \mid piv, \overline{\pi}, 1] &= -1 + 2x_1 + 2E[s \mid piv, \overline{\pi}, 1] = 0 \\ E[v(x_2, s) \mid piv, \overline{\pi}, 2] &= -1 + 2x_2 + 2E[s \mid piv, \overline{\pi}, 2] = 0\end{aligned}$$

respectively, we obtain the two cutpoints:

$$x_1 = \frac{1}{2} - \alpha \qquad \text{and} \qquad x_2 = \alpha - \frac{1}{2}.$$

Plugging them into (14) yields

$$\left| t(s, \overline{\pi}^n) - \frac{1}{2} \right| = \frac{1}{4}(1-2\alpha)^2.$$

Hence, the political expected utilities are computed as

$$E\left[v(Q,s,x)\right] = \int_0^1 v(Q,s,x)ds = K - \frac{1}{12}\left(6x^2+12x+19\right)$$
$$E\left[v(A,s,x)\right] = \int_0^1 v(A,s,x)ds = K - \frac{1}{12}\left(6x^2-12x+19\right).$$

In the case of full information equivalence,

$$E\left[v(\alpha(s),s,x)\right] = \int_0^{1/2} v(Q,s,x)ds + \int_{1/2}^1 v(A,s,x)ds = K - \frac{1}{12}\left(6x^2+13\right).$$

See Figure 1 for these political expected utilities.

For simplicity assume an axisymmetric distribution: $(L_1, L_2, L_3) = (\lambda, 1-2\lambda, \lambda)\cdot L$. District 1 always votes unanimously for Q and district 3 always votes unanimously for A, whereas alternative Q is elected for $s < 1/2$ and alternative A is elected for $s > 1/2$ in district 2, i.e.,

$$U_1^x = u_1 + E\left[v(Q,s,x)\right]$$
$$U_2^x = u_2 + E\left[v(\alpha(s),s,x)\right]$$
$$U_3^x = u_3 + E\left[v(A,s,x)\right].$$

Due to symmetry, the necessary condition for interior equilibrium is given by

$$\Delta U(\lambda) \equiv U_2^x - U_1^x|_{x=1-\lambda} = 0.$$

(i) Full agglomeration at district 2 ($\lambda = 0$)

Suppose all individuals are agglomerated at district 2. Plugging $\lambda = 0$ into (10), we have $w^* = \phi^{-1/\varepsilon}$, and hence,

$$\Delta U(0) = \left(\frac{L}{\varepsilon\overline{c}}\right)^{\frac{1}{\varepsilon-1}}\left(1-\phi^{\frac{2\varepsilon-1}{\varepsilon(\varepsilon-1)}}\right) - \frac{1}{2}.$$

Full agglomeration is an equilibrium if $\Delta U(0) \geq 0$. Solving $\Delta U(0) = 0$, we get the agglomeration sustain point:

$$\phi_A = \begin{cases} \left[1-\frac{1}{2}\left(\frac{\varepsilon\overline{c}}{L}\right)^{\frac{1}{\varepsilon-1}}\right]^{\frac{\varepsilon(\varepsilon-1)}{2\varepsilon-1}} \in (0,1) & \text{if } 1 > \frac{1}{2}\left(\frac{\varepsilon\overline{c}}{L}\right)^{\frac{1}{\varepsilon-1}} \\ 0 & \text{if } 1 \leq \frac{1}{2}\left(\frac{\varepsilon\overline{c}}{L}\right)^{\frac{1}{\varepsilon-1}} \end{cases}.$$

Hence, full agglomeration emerges only if the fixed labor requirement is sufficiently small relative to the mass of workers ($1 > \frac{1}{2}\left(\frac{\varepsilon\bar{c}}{L}\right)^{\frac{1}{\varepsilon-1}}$) and the transport cost is large enough ($\phi \leq \phi_A$).

(ii) Stratified equilibrium with district 2 empty ($\lambda = 1/2$)

Substituting $\lambda = 1/2$ into (10) yields $w^* = \left(\frac{2\phi}{1+\phi}\right)^{\frac{1}{\varepsilon}}$. If

$$\Delta U(1/2) = \left(\frac{L}{\varepsilon\bar{c}}\right)^{\frac{1}{\varepsilon-1}} \left[\phi^{\frac{2\varepsilon-1}{\varepsilon(\varepsilon-1)}} \left(\frac{2}{1+\phi}\right)^{\frac{1}{\varepsilon}} - \left(\frac{1+\phi}{2}\right)^{\frac{1}{\varepsilon-1}}\right] + \frac{1}{2} \leq 0, \tag{15}$$

then a distribution that is symmetric between districts 1 and 3 is an equilibrium. Notice that the bracketed terms in (15) are non-positive and increasing in ϕ, reaching a maximum of 0 at $\phi = 1$. The symmetric equilibrium is stratified for $\phi < \phi_B$, where the stratification point ϕ_B is given by the unique solution to the equation $\Delta U(1/2) = 0$. Assume that the fixed labor requirement is small relative to the mass of workers so that $\Delta U(1/2) < 0$ holds at $\phi = 0$. Then, like the full agglomeration case, stratification emerges only if the fixed labor requirement is small relative to the mass of workers and the transport cost is large enough ($\phi < \phi_B$). Otherwise, individuals would migrate to district 2. Furthermore, at a stratified equilibrium, the stability condition between districts 1 and 3

$$\frac{d}{d\lambda_3}\left(U_3^* - U_1^*|_{\lambda_2=0,\lambda_1=1-\lambda_3,x=1-\lambda_3}\right)\bigg|_{\lambda_3=1/2} < 0 \tag{16}$$

should hold. This is satisfied when ε is not too small.

(iii) Partial agglomeration ($\lambda \in (0, 1/2)$)

In this case, solve (10) and $\Delta U(\lambda) = 0$ simultaneously with respect to λ and w. It can be shown numerically that $\lambda^* \in (0, 1/2)$ for large ϕ.

Figure 2 illustrates the equilibrium distribution $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$ as a function of trade freedom ϕ given $\varepsilon = 5$ and $L/\bar{c} = 100$.[7] Observe that there are multiple equilibria for small ϕ ($< \phi_B$).

The conclusion that should be drawn from this analysis is that for high and low freedom of trade, stable equilibria where not everyone is in the same district occur.

[7] Note that if $L/\bar{c}$ is sufficiently large, the "symmetric" equilibrium is unstable for small ϕ. This condition is somewhat similar to the black-hole condition that is standard in the New Economic Geography.

Higher freedom of trade means location is less important for economic welfare, and hence the equilibrium location of consumers is driven by the political sector. With low trade freedom, either everyone is agglomerated in the same district, or the electorate is polarized in two separate districts. For moderate trade freedom, everyone is agglomerated in the same district, and the political outcome is state dependent. For high trade freedom, all three districts are occupied in equilibrium. Two of the districts are polarized, always voting for the same candidate or outcome independent of state, whereas the occupants of the larger moderate district vote according to their information.

5 Information aggregation in local elections

Using Feddersen and Pesendorfer (1997) Theorem 3 or their Example 2, full information equivalence always holds in the *national elections* for this model, where every agent votes in the same election. Thus, elections aggregate information effectively, and we expect to see relatively few corrupt politicians elected.

On the other hand, *local elections* have different properties in this model with migration, where only the residents of a district have the opportunity to vote in that district's election. In this model with 3 districts and, for example, high trade freedom, only one of the 3 features full information equivalence at equilibrium. This is the largest district. The other two will always elect the same candidate, independent of individual signals and information. The conclusion is that elections in larger geographical districts, called national elections in our terminology, will lead to the election of less corrupt candidates in those districts, whereas elections in smaller geographical districts, called local elections in our terminology, will lead to less information aggregation, and thus will lead to the election of more corrupt candidates as representatives of those districts. This matches the empirical evidence used as motivation for our work in the introduction. Notice that the theory does not predict that corrupt officials will be elected in every local district in every state of the world, but rather only for certain states of the world in the more polarized districts. Thus, one cannot expect a high p-value for this test.

Ideally, we would want to use data from the US Congress to test this theory.

The reason is that Senate districts are quite large and contain the House districts as subsets. However, there are data issues with this idea. Criminal convictions of members of the US Congress for corruption, for example by the Public Integrity Section of the Criminal Division of the US Department of Justice, are few. Although they are made public in their annual reports, most of the convictions are of officials in other branches of the federal government or of local officials. One could weaken the standards and look only at ethics investigations by congressional committees, but information about this is primarily confidential or leaked. Actual data, for example from the group Citizens for Responsibility and Ethics in Washington, is consistent with our hypotheses, but rather imprecise.

6 Conclusions

Full information equivalence is a casualty of free trade. The reason is that under free trade, people sort themselves into districts, most of which are polarized. When trade is more costly, people tend to agglomerate for economic reasons, resulting in full information equivalence in the political sector.

It is interesting to discuss welfare in the context of this model. Originally, the New Economic Geography, representing the economic side of our model, was designed to answer the positive question: Why are there cities? The early literature shied away from normative questions, though more recent literature has examined efficiency. Similarly, the literature on information aggregation in elections also tends to focus on positive questions. There are reasons this has happened.

In the context of the Feddersen and Pesendorfer (1997) model, under assumptions that ensure full information equivalence, their model reduces to a standard political model where all policies, specifically A and Q, are Pareto optimal. As is standard in many political economy models, this represents a purely redistributive game, and thus welfare evaluation reduces to interpersonal utility comparisons. This is not desirable. Since our model is an adaptation of the Feddersen-Pesendorfer model, something similar happens here. Beyond that issue, when discussing allocations that Pareto dominate equilibrium allocations but might not be equilibrium allocations themselves, it is unclear what information structure to use for evaluation of the political sector, for instance full information or a structure less informative to agents.[8]

Finally, it is clear that welfare evaluations in our model will hinge on the relative weight given to the economic and political sectors in the utility functions.

For all of these reasons, we eschew explicit welfare comparisons using our model.

If we were to use a model of local public goods in place for our New Economic Geography model for the economic sector, it is likely that stratification would always occur in equilibrium. Thus, it is likely that full information equivalence would never hold in local elections.[9]

With only 2 instead of 3 districts, the comparative statics reduce to the left hand half of Figure 2. That is, when trade costs are high, there is an equilibrium with full agglomeration of agents in one district, and an equilibrium with half the population in each district, sorted by voter type. For lower trade cost, only the stratified equilibria survive. Thus, our main conclusion still holds. With more than 3 districts, it is difficult to calculate the second stage (political) equilibria in the districts.

Appendix: Assumptions in Feddersen and Pesendorfer (1997)

1. $\Delta v(x,s)$ is continuous and increasing with $|\Delta v(x,s) - \Delta v(x,s')| \geq \kappa\,|s - s'|$ and $|\Delta v(x,s) - \Delta v(x,s')| \geq \kappa\,|x - x'|$ for some $\kappa > 0$. Moreover, $\Delta v(-1,s) < 0$, $\Delta v(1,s) > 0$ for all s.

2. G has a density g and there is an $\alpha > 0$ such that $1/\alpha > g(s) > \alpha$ for all $s \in [0,1]$.

3. $F(x,k)$ is continuously differentiable in x and $f(x,k)$ denotes the derivative. There is an $\alpha > 0$ such that $\sum_{k=1}^{K} f(x,k) > \alpha$ for all $x \in X$.

4. If $\sigma > \sigma'$ and $s > s'$, then $p_k(\sigma' \mid s')p_k(\sigma \mid s) > p_k(\sigma \mid s')p_k(\sigma' \mid s)$ for all k.

5. There is an $\alpha > 0$ such that $p_k(\sigma \mid s) > \alpha$ for all (k,s).

6. nq is an integer.

7. $p_k(\sigma \mid s)$ is continuous in s for all k and for all σ.

[8] These ideas will not be novel to those who work in this literature, as they are part of the folklore.

[9] Such a model would predict p-values of 0 in our data.

8. For all k, $\frac{p_k(M|s)}{p_k(1|s)}$ is strictly increasing in s and $p_k(\sigma \mid s)$ satisfies the monotone likelihood ratio property.

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Figure 1: Political expected utilities with K=10

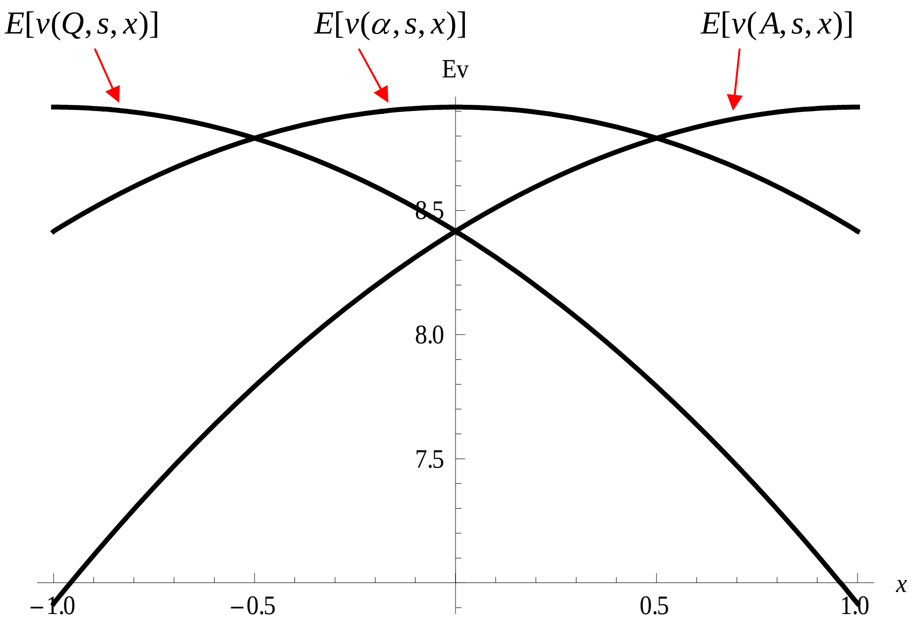

Figure 2: Equilibrium distributions when $\varepsilon = 5$ and $L/\bar{c} = 100$

Dotted arrow is L_1, solid arrow is L_2 and dashed arrow is L_3