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AUCTIONS WITH RESALE MARKET AND ASYMMETRIC INFORMATION

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INDEX

Auctions with Resale Market and Asymmetric Information

Rodrigo Harrison [*] Roberto Muñoz [†] Felipe Varas [‡]

January 30, 2008

Abstract

In this paper we study the role of resale opportunities in secondary markets over the bidding process in first and second price auctions. This trade opportunity arises owing to the presence of two factors. On the one hand, after receiving the object, the winner obtains new information about the object's value and on the other hand, the winner may suffer a liquidity shock that force him to sell the object regardless of his valuation. The buyer in the secondary market, however, does not know if the good is being sold because the new information reveals bad news regarding the object's valuation, or because a liquidity shock affected the seller. Our results show that revenue equivalence still holds, and bids are usually lower than those observed in the absence of liquidity shocks.

Keywords: Auctions; Resale Market; Adverse Selection.

JEL Classification Numbers: D44; L1.

1 Introduction

In this paper we study the effect of resale opportunities in a secondary market over the bidding process in first and second price auctions. Unlike previous literature, we analyze a resale market with adverse selection problems.

Over the last years there has been an increasing interest in the study of auctions with resale. Though previous literature has modeled resale opportunities in many different ways it has not considered the problem of adverse selection. For example, Gupta and Lebrun (1999), and Hafalir and Krishna (2006) consider an asymmetric environment, in which resale

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opportunities could arise because the optimal mechanism is not efficient. Garratt and Tröger (2006) also study a model in which resale opportunities arise due to inefficiencies. They analyze auctions with speculation, in which there are some inefficient equilibria where the speculator wins, even though he has no valuation for the object. In a more general setting, Zheng (2002) identifies conditions under which the outcome of the optimal auction (Myerson (1981)) is implementable even with resale opportunities.

We consider a setting in which the resale opportunity arises because of the presence of two factors. First, after receiving the object, only the winner obtains new information about the object's value. Second, the winner may suffer a liquidity shock that forces him to sell regardless of his valuation. However, the buyer in the secondary market does not know whether the first-stage winner is selling because he received bad news, or because he suffered a liquidity shock. This rises an adverse selection problem at the resale stage.

In order to analyze this problem, we consider a simple two-stage game. The game is played by two symmetric and risk neutral buyers with interdependent valuation and privately observed signals. The timing of the game is as follows: first each buyer observes his private signal and submits a bid; then, the auctioneer reveals the bids, and the object is delivered to the agent with the highest bid. Second, after receiving some news - regarding the value of the object - the winner may go to the resale market.

This work is closely related to Haile (2002), who analyzes a model in which resale opportunities arise because of the arrival of new information to bidders after the auction has been held. This new information may generate new trade opportunities. The main differences in the setting under study are the following: i) the resale opportunity is not necessarily an option for the second stage seller on account of the presence of a liquidity shock; ii) in the resale market the owner of the object is better informed than the potential buyers; therefore, there is an asymmetric information problem, even though there were not any informational advantages before the assignment of the object; iii) the new information is revealed to the winner as a consequence of owning the object.

We find conditions for the existence of a separating equilibrium, in which the first-stage auction winner voluntarily sells only if he receives bad news. Furthermore, an equilibrium condition with monotone increasing bid functions is found only if the likelihood of receiving bad news or having a liquidity shock is sufficiently low. The results in this setting show that bids are usually lower than those observed in standard models without liquidity shocks and resale opportunities. In addition, we obtain that the revenue equivalence still holds.

The paper is organized as follows. Section 2 develops the model, section 3 contains the main results, and section 4 concludes. All proofs are relegated to the appendix.

2 The Model

We consider a simple two stage game played by two risk neutral buyers, indexed by $i \in \{1,2\}$. In the first stage the players face an auction for an indivisible object which is assigned to who offers the highest bid. Ties are solved by randomization. In the second stage, once the object has been assigned to the winner bid, bidders have the opportunity to bargain in a resale market. This trade opportunity arises due to the presence of two factors. On the one hand, after receiving the object, the winner obtains new information about the object value. On the other hand, the winner may suffer a liquidity shock, with probability $\lambda > 0$, that forces him to sell the object regardless of its valuation.

We assume symmetric bidders with interdependent valuation given by $v(x_1, x_2, \theta) = \theta + \frac{1}{2}(x_1 + x_2)$, where $x_i \in [0,1]$ are independent privately observed signals according to a distribution $F(\cdot)$ that admits a continuously differentiable density $f(\cdot)$. The variable $\theta \in \{\underline{\theta}, \overline{\theta}\}$ is a shock observed only by the winner after the object has been assigned. We assume that $0 < \underline{\theta} < \overline{\theta}$ and the prior probability of $\overline{\theta}$ is $\alpha \in (0,1)$.

The timing of the game is as follows. First each player observes his private signal x_i and submits a bid b_i. Then, the auctioneer reveals the bids vector $\mathbf{b} = (b_1, b_2)$, and the object is assigned and delivered to the highest bidder. Second, after observing θ he may go to the resale market. This could happens because he observed $\underline{\theta}$ or because he suffered a liquidity shock. The liquidity shock is independent of $\mathbf{x} = (x_1, x_2)$ and θ. Therefore, at the moment of the resale the buyer is not able to distinguish whether the winner is selling the object because he observed a low value of θ or because he suffered a liquidity shock.

In the case of resale, a bargaining process is assumed between the bidders.[1] In order to analyze the adverse selection problem we focus on a separating equilibrium, in which the winner does not sell the object if he observes $\overline{\theta}$ and no liquidity shock affects him. We model the outcome of the resale process as a fraction of the value conditional on all public information, therefore the transaction price is $p(\mathbf{z}) = \xi\left(E^{\pi}[\theta] + \frac{1}{2}(z_1 + z_2)\right)$. Where $\mathbf{z} = (z_1, z_2)$ are the inferred signals from the bids in the first stage auction; $E^{\pi}[\theta] = \pi\overline{\theta} + (1-\pi)\underline{\theta}$ is the expected value of θ conditional on the event that the winner is selling the object; and $\xi \in [0,1]$ is the bargaining power of the seller. In this sense the probability of $\overline{\theta}$ is denoted by π in the case of resale. It is easy to see, using Bayes' rule, that in a separating equilibrium π is equal to

$$\frac{\alpha\lambda}{1 - \alpha + \alpha\lambda}$$

When player i becomes the winner bidder, his object's valuation before the resale, is given by:

[1] For the sake of simplicity, we assume that in the resale market only participate the original bidders.

$$w_i(x_i, z_j, \theta) = \theta + \frac{1}{2}(x_i + z_j)$$

On the other hand, if player i is the loser at the auction then, at the moment of the resale, he only has access to public information and his own signal, therefore, his object's valuation is:

$$l_i(x_i, z_j) = E^\pi[\theta] + \frac{1}{2}(x_i + z_j)$$

Since $\underline{\theta} < E^\pi[\theta] < \overline{\theta}$, for all $\mathbf{x}$, $w_i(x_i, x_j, \underline{\theta}) < l_j(x_j, x_i) < w_i(x_i, x_j, \overline{\theta})$.

The following lemma states a necessary condition over ξ, the seller's bargaining power, for the existence of a separating equilibrium in the asymmetric information resale problem.

Lemma 1. *There is a separating equilibrium for every* $\mathbf{x}$ *only if:*

$$\frac{\underline{\theta}}{E^\pi[\theta]} \leq \xi \leq 1$$

Proof. See appendix. □

In a separating equilibrium the first stage expected valuation for bidder i is

$$u_i(x_i, \mathbf{z}) = \begin{cases} W_i(x_i, \mathbf{z}) & \text{if bidder } i \text{ wins} \\ L_i(x_i, \mathbf{z}) & \text{if bidder } i \text{ lose} \end{cases} \tag{1}$$

where $W_i(x_i, \mathbf{z}) = \alpha(1-\lambda)w_i(x_i, z_j, \overline{\theta}) + (1-\alpha+\alpha\lambda)p(\mathbf{z})$ represent the expected bidders object valuation when he wins. With probability $\alpha(1-\lambda)$ the winner will not receive a liquidity shock and will observe $\overline{\theta}$ retaining the object and obtaining $w_i(x_i, z_j, \overline{\theta})$. With probability $(1-\alpha+\alpha\lambda)$ he will resale the object in the second stage receiving $p(\mathbf{z})$.

On the other hand $L_i(x_i, \mathbf{z}) = (1-\alpha+\lambda\alpha)[l_i(x_i, z_j) - p(\mathbf{z})]$ represent the expected bidder's object valuation when he loses. With probability $(1-\alpha+\lambda\alpha)$ he buys the object in the resale market and receives $l_i(x_i, z_j) - p(\mathbf{z})$.

3 Equilibrium

In this section we attempt to identify symmetric strictly increasing equilibrium bid functions for first and second price auctions with resale. The following assumptions establish sufficient conditions for the existence of such functions.

Assumption 1. *The density function* $f(\cdot)$ *satisfies:* $f(x)^2 - f'(x)\,[F(x) - 1/2] \geq 0 \quad \forall x \in (0, 1)$.

Assumption 1 is required to assure that the bid functions are strictly increasing. This assumption is not very demanding; for example, it holds for many commonly used distributions such as: uniform, exponential and any symmetric single peaked density function.

Assumption 2. *The probability of resale in the secondary market is strictly lower than* $1/2$*, i.e.* $1 - \alpha + \alpha\lambda < 1/2$.

Assumption 2 is required to assure that the candidate bid functions indeed constitute a Bayesian Nash equilibrium.[2] In this model, this requirement is equivalent to imposing that the marginal benefit of winning the auction is greater than the marginal benefit of waiting for the resale market, i.e. $\frac{\partial W_i(x_i,\mathbf{z})}{\partial x_i} > \frac{\partial L_i(x_i,\mathbf{z})}{\partial x_i}$.

In what follow we develop the second and first price auctions cases. We find explicit expressions for the equilibrium bid functions, and we also prove that revenue equivalence holds. In order to provide some intuition for the results, we consider, as benchmark models, the cases of first and second price sealed bid auctions in the same setting but without resale opportunities and liquidity shocks.

3.1 Second Price Auction

In a second price auction, the expected payoff for a bidder with signal x_i who bids $\beta^{II}(z_i)$ is:

$$U_i^{II}(x_i, z_i) = \int_0^{z_i} (W_i(x_i, z_i, x_j) - \beta^{II}(x_j))dF(x_j) + \int_{z_i}^1 L_i(x_i, z_i, x_j)dF(x_j). \qquad (2)$$

This expected payoff is composed by two terms: the first one consists of the expected payoff of winning the auction, and the second term is the expected payoff of losing the auction. In the standard case without resale the second term is zero; however, with resale opportunities there is some probability of getting the object on the resale market.

The equilibrium strategy is stated in the following proposition.

Proposition 1. *Suppose that Assumptions 1 and 2 hold. The equilibrium bid in the second price auction is given by:*

$$\beta^{II}(x_i) = \mu + \zeta x_i + \kappa(x_i) \qquad (3)$$

where

$$\mu = \alpha(1-\lambda)\overline{\theta} + (1-\alpha+\alpha\lambda)(2\xi - 1)E^{\pi}[\theta]$$
$$\zeta = \alpha(1-\lambda) + (1-\alpha+\alpha\lambda)\xi$$

[2] This assumption is a sufficient condition to meet the single crossing property in Athey (2001).

and

$$\kappa(x_i) = (1 - \alpha + \alpha\lambda)\xi \left(\frac{F(x_i) - \frac{1}{2}}{f(x_i)} \right)$$

Proof. See appendix. □

To provide some intuition let us consider the case where the winner bidder has all the bargaining power in the second stage, i.e. $\xi = 1$. In this case $\mu = E^{\alpha}[\theta]$, and $\zeta = 1$. Considering that the symmetric equilibrium strategies in the benchmark case are given by $\gamma^{II}(x_i) \equiv E^{\alpha}[\theta] + x_i$, then we have:

$$\beta^{II}(x_i) - \gamma^{II}(x_i) = (1 - \alpha + \alpha\lambda) \left(\frac{F(x_i) - \frac{1}{2}}{f(x_i)} \right). \tag{4}$$

The difference in the bids arises due to the strategic effect of the bids over the resale price. Note that for a signal below the median of the distribution, the bidder faces a greater probability of losing the auction, so the strategic effect leads him to bid lower than in the case without resale. To the contrary, for a signal above the median the opposite effect occurs.

Finally, when $\xi < 1$, in addition to the strategic effect, we have to correct the other terms in the bid function because of the asymmetric information problem. The modified terms satisfy $\mu \leq E^{\alpha}[\theta]$ and $\zeta \leq 1$. When the winner does not have all the bargaining power, i.e. $\xi < 1$, the strategic effect is less important, and the two first terms are lower than in the case with $\xi = 1$.[3]

3.2 First Price Auction

In a first price auction with resale the expected payoff for a bidder with signal x_i who bids $\beta^I(z_i)$ is:

$$U_i^I(x_i, z_i) = \int_0^{z_i} (W_i(x_i, z_i, x_j) - \beta^I(z_i))dF(x_j) + \int_{z_i}^1 L_i(x_i, z_i, x_j)dF(x_j). \tag{5}$$

The equilibrium strategy is stated in the following proposition.

Proposition 2. *Suppose that Asumptions 1 and 2 hold. The equilibrium bid in the first price auction is then given by:*

$$\beta^I(x_i) = \mu + \zeta E[x|x_i > x] + E[\kappa(x)|x_i > x] \tag{6}$$

Proof. See appendix. □

[3] When $\xi < 1$ the expression $\beta^{II}(x_i) - \gamma^{II}(x_i)$ could be either negative or positive.

As before, it is convenient to compare this result with the corresponding benchmark model, where the equilibrium bid functions are given by $\gamma^I(x_i) \equiv E^\alpha[\theta] + E[x|x_i > x]$. Hence, depending on the value of $E[\kappa(x)|x_i > x]$ the equilibrium bids in the presence of resale opportunities will be lower or higher than in the model without resale. Integrating by parts we obtain:

$$E[\kappa(x)|x_i > x] = (1 - \alpha + \alpha\lambda)\xi \left(\frac{x_i}{F(x_i)} (F(x_i) - 1/2) - E[x|x_i > x] \right). \tag{7}$$

It follows from equation 7 that $E[\kappa(x)|x_i > x] < 0$ for all x_i lower than the median of the distribution. However, it can take positive values for sufficiently high values of x_i. The following corollary establishes sufficient conditions under which $E[\kappa(x)|x_i > x]$ is non-positive for all x_i, implying that the equilibrium bid in the first price auction is always lower with resale opportunities.

Corollary 1. *Suppose that Assumptions 1 and 2 hold, and $E[x] \geq 1/2$.*

1. If $E[x] > 1/2$ then $\beta^I(x_i) < \gamma^I(x_i) \ \forall \ x_i \in [0, 1]$.
2. If $E[x] = 1/2$ then $\beta^I(x_i) < \gamma^I(x_i) \ \forall \ x_i \in [0, 1)$ and $\beta^I(1) = \gamma^I(1)$.

Proof. See appendix. □

Note that in a first price auction over-bidding is costly for both the winner and the loser. On the one hand, the winner has to pay higher in the first stage when he submits a higher bid; on the other hand, the loser increases the price he will pay in the second stage. Accordingly, the strategic effect is non-positive, and the bids are always lower than in the benchmark case without resale.

3.3 Revenue Equivalence

Despite the fact that equilibrium strategies are quite different than in the benchmark models, revenue equivalence still holds.

Proposition 3. *Suppose that Assumptions 1 and 2 hold, then the expected revenue in a first price auction is the same as in a second price auction.*

Proof. See appendix. □

The reason why revenue equivalence holds in this setting is because the existence of a secondary market only affects the structure of the valuations, but not the informational structure. In fact, the equilibrium would be the same in a private value model with independent signals, where bidders' valuations are given by $v(x_i) = \mu + \zeta x_i + \kappa(x_i)$.

Proposition 3 permit us to focus on only one of the auction mechanism to discuss the seller's revenue. For example, consider the second price auction, which involves four equally likely cases, depending on whether the two bidders receive signals above or below the median. Only when both receive signals above the median, the price in the second price auction with resale could be higher than in the benchmark case without resale. The following corollary states sufficient conditions for having an unambiguous effect of resale over expected revenue.

Corollary 2. *Suppose that Assumptions 1 and 2 hold; and $E[x] \geq 1/2$, then the expected revenue in the auction with resale is always lower than the expected revenue in the corresponding benchmark auction.*

The previous corollary trivially follows from Corollary 1 and Proposition 3.

4 Conclusion

In this article we have analyzed the role of liquidity shocks and resale opportunities on the outcome of both first and second price auctions, in the presence of adverse selection problems. In relation to a benchmark -without those effects- we found that equilibrium bids in the second price auction are more dispersed, while in the first price auction they are lower. In addition, we proved that revenue equivalence still holds and expected revenue is lower.

Relative to policy implications, differently from Haile (2002), to postpone the auction is no longer useful, because no information is revealed until the object is assigned. In contrast, two other issues arise regarding auction design. First, the signaling effect implies that bids disclose policy matters. For example, it could be convenient for the seller to reveal only the winner's bid in order to avoid a strategic underbidding effect by a potential loser. Second, not only auction design matters, but also the design of the object being auctioned. For example, in spectrum auctions a license can be considered as a bundle of rights, which is usually designed to prevent or reduce resale possibilities. Interestingly, reducing the rights could increase seller revenues.

5 Appendix

Proof of Lemma 1. The existence conditions of a separating equilibrium require that:

$$w_i(x_i, z_j, \overline{\theta}) \geq p(\mathbf{z})$$
$$w_i(x_i, z_j, \underline{\theta}) < p(\mathbf{z})$$

and

$$l_i(z_i, x_j) \geq p(\mathbf{z}).$$

Which are only satisfied for every $\mathbf{x}$ only if

$$\frac{\theta}{E^{\pi}[\theta]} \leq \xi \leq 1.$$

$\square$

Proof of Proposition 1. The expected utility in a second price auction when the bidders follow $\beta^{II}(z_i)$ is

$$U_i^{II}(x_i, z_i) = \int_0^{z_i} (W_i(x_i, z_i, x_j) - \beta^{II}(x_j))dF(x_j) + \int_{z_i}^1 L_i(x_i, z_i, x_j)dF(x_j). \tag{8}$$

Differentiating with respect to z_i and replacing the equilibrium condition $z_i = x_i$ we find the unique candidate equilibrium strategy

$$\beta^{II}(x_i) = \mu + \zeta x_i + \kappa(x_i). \tag{9}$$

First, we must verify that $\beta^{II}(x_i)$ is strictly increasing. $\zeta > 0$ implies that it is sufficient to show that $\kappa(x_i)$ is nondecreasing. Differentiating $\kappa(x_i)$ with respect to x_i we find that $\kappa(x_i)$ is nondecreasing only if

$$f(x_i) - \frac{f'(x_i)}{f(x_i)} \left(F(x_i) - \frac{1}{2} \right) \geq 0.$$

Second, we need to show that the first-order condition indeed characterizes the optimum. For this it is sufficient to show that $\frac{\partial U_i^{II}(x_i,z_i)}{\partial z_i \partial x_i} > 0$. Integrating by parts in (8):

$$\begin{aligned} U_i^{II}(x_i, z_i) = &\left(W_i(x_i, z_i, z_i) - \beta^{II}(z_i)\right) F(z_i) + L_i(x_i, z_i, 1) - L_i(x_i, z_i, z_i)F(z_i) \\ &- \int_0^{z_i} \left(\frac{1}{2} \left(\alpha(1-\lambda) - (1-\alpha+\alpha\lambda)\xi\right) - \beta^{II}(x_j) \right) F(x_j)dx_j \\ &- \frac{1}{2}(1-\alpha+\alpha\lambda)(1-\xi) \int_{z_i}^1 F(x_j)dx_j. \end{aligned} \tag{10}$$

Differentiating (10) we obtain

$$\frac{\partial U_i^{II}(x_i, z_i)}{\partial z_i \partial x_i} = \frac{f(z_i)}{2} \left[\alpha(1-\lambda) - (1-\alpha+\alpha\lambda)\right].$$

Hence, by Assumption 2 the first order condition indeed characterizes the equilibrium. □

Proof of Proposition 2. The expected utility in a first price auction when the bidders follow $\beta^I(z_i)$ is

$$U_i^I(x_i, z_i) = \int_0^{z_i} (W_i(x_i, x_j, z_i) - \beta^I(z_i))dF(x_j) + \int_{z_i}^1 L_i(x_i, x_j, z_i)dF(x_j). \tag{11}$$

Differentiating with respect to z_i and using the equilibrium condition $z_i = x_i$, we find that the unique candidate equilibrium strategy must satisfy:

$$\frac{d\beta(x_i)F(x_i)}{dx_i} = V_i(x_i)f(x_i) + (1 - \alpha + \alpha\lambda)\xi \left(F(x_i) - \frac{1}{2} \right),$$

with boundary condition $\beta^I(0) = V(0)$, where $V_i(x_i) = W_i(x_i, x_i, x_i) - L_i(x_i, x_i, x_i)$. It follows that the equilibrium strategy is

$$\beta^I(x_i) = \mu + \zeta E[x|x_i > x] + E[\kappa(x)|x_i > x]. \tag{12}$$

First, we must verify that $\beta^I(x_i)$ is an increasing function. Comparing equations (9) and (12) it follows immediately that $\beta^I(x_i) = E[\beta^{II}(x)|x_i > x]$. Hence, $\beta^I(x_i)$ is increasing because $\beta^{II}(x_i)$ is increasing too. Second, we must show that the first-order condition indeed characterizes an optimum. For the latter it is sufficient to show that $\frac{\partial U_i^I(x_i,z_i)}{\partial z_i \partial x_i} > 0$. By integration by parts of equation (11) the expected utility is

$$\begin{aligned} U_i^I(x_i, z_i) = &\left(W_i(x_i, z_i, z_i) - \beta^I(z_i)\right) F(z_i) + L_i(x_i, z_i, 1) - L_i(x_i, z_i, z_i)F(z_i) \\ &- \frac{1}{2}\left(\alpha(1-\lambda) - (1 - \alpha + \alpha\lambda)\xi\right) \int_0^{z_i} F(x_j)dx_j \\ &- \frac{1}{2}(1 - \alpha + \alpha\lambda)(1 - \xi) \int_{z_i}^1 F(x_j)dx_j. \end{aligned} \tag{13}$$

Differentiating equation (13) we obtain

$$\frac{\partial U_i^I(x_i, z_i)}{\partial z_i \partial x_i} = \frac{f(z_i)}{2} \left[\alpha(1-\lambda) - (1 - \alpha + \alpha\lambda)\right].$$

Hence, by Assumption 2 the first order condition indeed characterizes the equilibrium. □

Proof of Corollary 1. The inequalities $\mu \leq E^\alpha[\theta]$ and $\zeta \leq 1$ imply that it is sufficient to show that $E[\kappa(x)|x_i > x] = (1 - \alpha + \alpha\lambda)\frac{\xi}{F(x_i)} \int_0^{x_i} \left(F(t) - \frac{1}{2}\right) dt \leq 0$.

Denoting the median by x^*, it follows immediately that $\int_0^{x_i} \left(F(t) - \frac{1}{2}\right) dt \leq 0$ for all $x_i \leq x^*$. At the same time, for $x_i > x^*$, $\int_0^{x_i} \left(F(t) - \frac{1}{2}\right) dt$ is a monotone increasing; therefore, it is sufficient to show that $\int_0^1 \left(F(t) - \frac{1}{2}\right) dt \leq 0$ to prove that $\int_0^{x^*} \left(F(t) - \frac{1}{2}\right) dt \leq 0$ for all $x_i > x^*$. Integrating by parts

$$\int_0^1 \left(F(t) - \frac{1}{2}\right) dt = \frac{1}{2} - E[x].$$

Hence, $E[x] \geq 1/2$ implies that $\int_0^1 \left(F(t) - \frac{1}{2}\right) dt \leq 0$

□

Proof of Proposition *3.* On the one hand, the expected revenue of the first price auction is

$$R^I = 2\int_0^1 \beta^I(x)F(x)dF(x).$$

On the other hand, the expected revenue of the first price auction is

$$R^{II} = 2\int_0^1 \int_0^x \beta^{II}(t)dF(t)dF(x).$$

Moreover, Propositions 1 and 2 imply $\beta^I(x_i) = E[\beta^{II}(x)|x_i > x]$, which means that

$$R^I = 2\int_0^1 \int_0^x \beta^{II}(t)dF(t)dF(x).$$

□

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