# Documento de Trabajo

# The Role of Social Networks on Regulation in the Telecommunication Industry

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# THE ROLE OF SOCIAL NETWORKS ON REGULATION IN THE TELECOMMUNICATION INDUSTRY

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# The Role of Social Networks on Regulation in the Telecommunication Industry

by

Rodrigo Harrison, Gonzalo Hernandez and Roberto Muñoz [*]

## Abstract

This paper studies the welfare implications of equilibrium behavior in a market characterized by competition between two interconnected telecommunication firms, subject to constraints: the customers belong to a social network. It also shows that social networks matter because equilibrium prices and welfare critically depend on how people are socially related. Next, the model is used to study effectiveness of alternative regulatory schemes. The standard regulated environement, in which the authority defines interconnection access charges as being equal to marginal costs and final prices are left to the market, is considered as a benchmark . Then, we focus on the performance of two different regulatory interventions. First, access prices are set below marginal costs to foster competition. Second, switching costs are reduced to intensify competition. The results show that the second strategy is more efective to obtain equilibrium prices closer to Ramsey's level.

JEL codes: C70, D43, D60

Keywords: Access charges, social networks, random regular graphs.

## 1    Introduction

Over the last years several articles have been focused on the study of the equilibrium interconnection strategies in telecommunication markets, in a framework where heterogeneity of consumers is recognized (see for example Dessein (2004) and Hahn (2004),

among others).[1] This approach has been a significant improvement in the effort to obtain more realistic models. However, the social network structure among consumers has been mostly ignored, and heterogeneity has been usually motivated on the grounds of different propensities to make calls accross consumers. In this paper, we recognize that the position in a social network affects the amount of calls a typical consumer can make. Naturally, a more connected individual would make more calls than a hermit. Moreover, the number of calls to any particular member in the network should depend not only on prices, but also on how close they are in social terms.

In this context, we study a market characterized by competition between two interconnected telecommunication firms, with particular emphasis on the constraint that customers belong to a social network. We consider, as usual, that a firm A has two sources of revenues: its customer' payments and the access charges that a rival firm B pays to A in order to complete calls originated in B but terminated in A. Our benchmark case consists of the standard regulated environment where interconnection access charges are defined by the authority as equal to marginal costs, while final prices are left to the market. In this environment we show that social structure matters, because equilibrium prices, consumer surplus and producer surplus depend on network characteristics. Then, we study the efectiveness of two alternative regulatory interventions. First, access prices are set below marginal costs to enhance competition. Second, switching costs are reduced to intensify competition, while access charges remain at marginal costs levels. The simplest model corresponds to the case when both firms are single service providers and they are constrained to offer linear prices schemes in a nondiscriminatory way. In this sense, the closest model corresponds to Harrison *et al.* (2006), where the differential effect of regulated v/s unregulated frameworks is studied. Our analysis, however, differs from theirs in several important aspects. First, the initial focus here is on the role of social networks over the outcome, keeping the regulatory environment constant. Second, we introduce the study of welfare effects of competition policy among interconnected firms in the presence of a social network among customers. Third, we study regulatory interventions which -though they depart from the benchmark case- are feasible under standard regulatory environments.

Interestingly, our results show that a regulatory intervention focused on reducing access charges below marginal costs enhances welfare. However, the alternative policy intervention focused on reducing switching costs is much more effective. Welfare also increases when the social network is more dense, but this characteristic of the network can not be subject to policy implications.

The welfare effect of both policies is compared to another benchmark given by the Ramsey solution, which characterize the second best scenario (see Laffont et al. (1998a)). A comparison with expected results in a collusive scenario is also discussed and it shows

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[1]For an excellent review of the literature see Armstrong (2002). The seminal papers are Laffont *et al.* (1998a,b) and Armstrong (1998).

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that regulation is mandatory. In fact, it is more neccesary when the social network is more dense.

The rest of the paper is organized as follows: In section 2 we develop the economic model, including the agent's demand, the firms' problem and the game played by the two firms. In section 3 we establish the welfare considerations for the analysis. In section 4 we report numerical results and the main conclusions are stated in section 5.

## 2   The Economic Model

In the model we assume the existence of a social network, represented by graph $g$. Nodes in the graph represent agents (indexed by $i \in I$) and a link between a pair of agents represent a social connection between them. The graph $g$ is generated using random regular graphs (see Bollobas (2001)), where the *connectivity degree* $d$ of graph $g$ represents the average number of social connections accross agents.

There are two firms, $A$ and $B$, offering horizontally differentiated communication services (for example two wireless companies) and consumers have to decide which firm to subscribe to. In order to make the affiliation decision, agents take into account the pricing schemes offered by each firm and her own preferences for the services provided. It is assumed that the firms' pricing schemes are constrained to be linear and nondiscriminatory. On the other hand, the preferences are modeled in a way similar to a standard Hotelling horizontally differentiated model: each agent $i$ in the social network (i.e. each node in $g$) is endowed with a realization of a taste variable $x_i$, randomly assigned from a uniform grid with support in $[0, 1]$. In what follows we assume that firm $A$ is "located" in 0 and firm $B$ in 1. None of them provide the "ideal service" to agent $i$, positioned in $x_i$, unless $x_i$ itself be zero or one.

### 2.1   The Agent Demand

Consider the affiliation decision problem of agent $i$. If she decides to subscribe firm $l = A, B$ then we will say that she belongs to the set $I_l \subseteq I$ of subscribers to $l$. Agent $i$'s demand for calls is represented by the vector $q_i = (q_{ij})_{j \in I, j \neq i}$, where the generic element $q_{ij}$ is the number of calls that agent $i$ makes to agent $j$. Then the gross utility of agent $i$ can be described as follows:[2]

$$U_i(q_i) = \sum_{j \in I, j \neq i} \delta^{t_{ij}} u(q_{ij}) \qquad \text{with} \qquad u(q_{ij}) = \frac{q_{ij}^{1-1/\eta}}{1 - 1/\eta} \tag{1}$$

where:

---

[2]It is convenient to note that the funtional form of $u(\cdot)$ is standard in the literature (see Laffont et al (1998 a,b).

$\delta$ : represents a utility discount factor when agent $i$ calls other agents located farther in network $g$. Accordingly, it satisfies $0 < \delta < 1$.

$t_{ij}$ : it is the shortest distance (in terms of links) connecting agents $i$ and $j$ with $i \neq j$. We consider $t_{ij} \in \mathbb{N}$ so that if the agents are direct neighbors, the discount factor is $\delta^0 = 1$. On the other hand if agents $i$ and $j$ are not connected, then $t_{ij} \equiv \infty$.

$\eta$ : is a constant parameter that represents elasticity of demand, which is assumed to be greater than 1 and independent of $j$.

Suppose that after observing the prices offered by the firms, $p_A$ and $p_B$, agent $i$ has to decide which firm to become affiliated with. In order to make that decision, she needs to figure out her net consumer surplus in both scenarios. If she decides to affiliate with firm $A$, the vector of calls $q_i = (q_{ij})_{j \in I, j \neq i}$ to all her contacts in the social network $g$ is defined by:

$$V_i(p_A) = \max_{q_i} \left\{ U_i(q_i) - p_A \sum_{j \in I, j \neq i} q_{ij} \right\} \tag{2}$$

Solving this maximization problem, we obtain her demand's components:

$$q_{ij}(p_A) = \left( \frac{p_A}{\delta^{t_{ij}}} \right)^{-\eta} \tag{3}$$

Intuitively, for the same price $p_A$, agent $i$ makes more calls to contacts located closer in the social network $g$ than to those farther in it. Therefore, plugging into equation 2 we get the indirect utility function:

$$V_i(p_A) = \sum_{j \in I, j \neq i} \delta^{\eta t_{ij}} \frac{p_A^{1-\eta}}{\eta - 1} \tag{4}$$

and an analogous result arises for firm $B$.

Consider the parameter $t$ that represents the unit cost that agent $i$, located in $x_i$, has to incur in order to become affiliated with firm A located in 0 or firm B located in 1. Accordingly, the total cost of selecting a service, eventually different from $i$'s preferred one, is assumed to be $x_i t \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ if agent $i$ select firm A or $(1 - x_i) t \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ if firm B is preferred. It is important to note that in this model we assume that agent $i$ incurs in a discounted disutility for calls due to the imperfect matching between her preferences and the service provided, where the discount appears because the imperfection is more annoying the closer agent $j$ is to $i$ in the social network. The total cost of imperfect matching is the sum of all the pairwise discounted costs. In addition, note that the cost to agent $i$ of an imperfect service to call agent $j$ is assumed to be independent of the number

of calls.[3]

The decision of affiliation to A or B depends on whether $x_i$ is to the right or to the left of a critical value $x_i^*$ given by:

$$V_i(p_A) - tx_i^* \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}} = V_i(p_B) - t(1 - x_i^*) \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$$

If $x_i < x_i^*$, that means that agent $i$ prefers firm A even considering that firm A does not provide her with the ideal service (and has to pay $tx_i \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}$ due to imperfect matching). Solving for $x_i^*$, we got:

$$x_i^* = \frac{1}{2} + \sigma_i \frac{(p_A^{1-\eta} - p_B^{1-\eta})}{\eta - 1} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} \qquad \left(\text{with } \sigma_i = \frac{1}{2t \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}}}\right)$$

So if $x_i < x_i^*$ (resp. $x_i > x_i^*$) then player $i$ joins firm $A$ (resp. $B$).

## 2.2 The Firm's Problem

The firms select their prices, $p_A$ and $p_B$, simultaneous and independently, in order to maximize profits. However, they know that, after observing prices, consumers are going to make optimal affiliation decisions, so the number of clients for each firm is endogenous. In addition, when they decide prices the level of access charges, $a_A$ and $a_B$, are taken as given. To illustrate the role of these payments, let us discuss $a_A$. Access charge $a_A$ is a unitary fee paid by firm B to firm A so that A may complete a call made by a client of B to a client of A (analogous for $a_B$). We assume that access charges are defined in a first stage, either by the regulatory authority or the firms. In the last case, access charges can be the result of a competitive process or a collusive agreement among operators.

Assuming that access charges are given by $a_A$ and $a_B$, firm $A$ (resp. $B$) will select its price $p_A$ (resp. $p_B$) such that:

$$\max_{p_A \geq 0} \pi_A(p_A, p_B, a_A, a_B) =$$
$$\sum_{i \in I_A} \left\{ \sum_{\substack{j \neq i \\ j \in I_A}} q_{ij}(p_A)(p_A - c_A^o - c_A^f) + \sum_{j \in I_B} q_{ij}(p_A)(p_A - c_A^o - a_B) - f \right\} + \qquad (5)$$
$$\sum_{i \in I_B} \sum_{j \in I_A} q_{ij}(p_B)(a_A - c_A^f)$$

---

[3] Alternative approaches would be to make the transportation cost dependent on the utility obtained from the calls or dependent on the number of calls. Our selection is consistent with Laffont *et al.* (1998a). They do not consider, however, a discount factor because in their model agents are not connected through a social network.

where:[4]

$f$ : is the fixed cost incurred by a firm when it affiliates a new subscriber.

$c_A^o$ : is the cost of originating a call for firm $A$ ($c_B^o$ is defined analogously).

$c_A^f$ : is the cost of terminating or finishing a call for firm $A$ ($c_B^f$ is defined analogously).

$a_A$ : is the price or access charge that firm $A$ charges firm $B$ in order to terminate a call from a subscriber of $B$ to a subscriber of $A$ ($a_B$ is defined analogously).

At this stage it is clear that we are facing a non standard optimization problem for the firms, because sets $I_A$ and $I_B$ in equation (5), and which contain the consumers affiliated to A and B, respectively, depend on the prices selected. In order to deal with this complexity, we need to write the problem (5) using nonlinear programming schemes that are as standard as possible. Let us define $\alpha_i = 0$ if agent $i$ affiliates network A and $\alpha_i = 1$ if agent $i$ affiliates network B. Using this notation, our goal will be to write firm A's problem as:

$$\underset{p_A \geq 0}{Max}\ \pi_A(p_A, p_B, a_A, a_B; \alpha) \tag{6}$$

$$s.t. \quad H\alpha \leq z, \quad \alpha \in \{0,1\}^I$$

Note that the previous structure is not warranted in general, because we are requiring that affiliation decisions be represented by a linear constraint. The gains from obtaining such a neat representation of the problem are very important. First, we can bound the level of complexity of the problem; second, we will be able to expand the aplicability of our model without changing this structure, and third, it helps us to figure out an algorithm to solve it. With this goal in mind, we separate the problem in two parts. First, we should write the vector of optimal affiliation decisions as the solution of a linear inequality constraint ($H\alpha \leq z$, $\alpha \in \{0,1\}^I$) and then, we should write the objective function as in (6), so that we make explicit the dependance of the objective function on the vector of affiliation decisions $\alpha$. The following sections are devoted to these tasks.

### 2.3 The Constraint

Using the definition of $\alpha_i$ the optimal affiliation decision can be written as:

$$\alpha_i = \begin{cases} 0 & \text{if } x_i < x_i^* \\ 0 \text{ or } 1 & \text{if } x_i = x_i^* \\ 1 & \text{if } x_i > x_i^* \end{cases}$$

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[4]In what follows, we always assume that firms select prices while consumers make optimal affiliation decisions. Access prices are defined by the regulatory authority unless we explicitly consider a different scenario. The rest of the variables involved are exogenous parameters in the problem.

where

$$x_i^* = \frac{1}{2} + \sigma_i \frac{(p_A^{1-\eta} - p_B^{1-\eta})}{\eta - 1} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}}$$

Noting that the values of $x_i^*$ do not depend on the affiliation decisions of agents other than $i$,[5] it is easy to see that the previous expression has the following structure:

$$\alpha_i = \begin{cases} 0 & \text{if} \quad b_i < 0 \\ 0 \text{ or } 1 & \text{if} \quad b_i = 0 \\ 1 & \text{if} \quad b_i > 0 \end{cases} \tag{7}$$

where $b_i \in \mathbb{R}$ with:

$$b_i = x_i - \frac{1}{2} - \mathbf{1}^t e_{-i}(p_A, p_B)$$

where:

$$e_{-i}(p, q) = \begin{pmatrix} e_{i,1}(p, q) \\ \vdots \\ e_{i,i-1}(p, q) \\ e_{i,i+1}(p, q) \\ \vdots \\ e_{i,I}(p, q) \end{pmatrix}_{I-1} \qquad \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{pmatrix}_{I-1}$$

$$e_{i,j}(p, q) = \frac{\sigma_i \delta^{\eta t_{ij}}}{\eta - 1} \left( p^{1-\eta} - q^{1-\eta} \right)$$

The optimal affiliation decisions are then formally characterized, but they are still nonlinear. In order to linearize them, consider $M \in \mathbb{R}_+$ sufficiently high such that, for given $i$, constraint (7) is equivalent to the following couple of inequations:[6]

$$\begin{aligned} 0 &\geq b_i - M\alpha_i \\ 0 &\leq b_i + M(1 - \alpha_i) \end{aligned} \tag{8}$$

---

[5] In a companion paper we study the discriminatory case, where $x_i^*$ actually depends on the affiliation decisions of all the agents, and the problem becomes much more complicated.

[6] A feasible definition of $M$ is given in Appendix I.

In effect, when $b_i < 0$ holds, agent $i$ is forced to choose $\alpha_i = 0$ otherwise (i.e. by selecting $\alpha_i = 1$) the second inequality in (8) is violated. An analogous argument applies when $b_i > 0$. In the case when $b_i = 0$ the inequalities in (8) hold with $\alpha_i = 0$ or $\alpha_i = 1$. As a result, the vector of affiliation decisions must satisfy the following system of linear inequations:

$$H\alpha \leq z$$

where:

$$H = \begin{bmatrix} -M & & \\ M & & \\ & -M & \\ & M & \\ \vdots & \vdots & \vdots \\ & & -M \\ & & M \end{bmatrix}_{2I \times I} \qquad z = \begin{bmatrix} -b_1 \\ b_1 + M \\ -b_2 \\ b_2 + M \\ \vdots \\ -b_I \\ b_I + M \end{bmatrix}_{2I \times 1} \qquad \alpha = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_I \end{pmatrix}_{I \times 1}$$

It is convenient to emphasize that $H$ is independent of a particular vector of prices $(p_A, p_B)$. On the other hand, $z$ depends on the vector of prices because $b_i$ does so for each $i$. Accordingly we should write the constraint as: $H\alpha \leq z(p_A, p_B)$.[7]

## 2.4 The Objective Function

Consider the problem for firm $A$ established in equation (5). By replacing the optimal values for $q_{ij}$ defined in equation (3) it becomes:

$$\max_{p_A \geq 0} \pi_A(p_A, p_B, a_A, a_B) = (p_A - c_A^o - c_A^f)p_A^{-\eta} \sum_{i \in I_A} \sum_{\substack{j \neq i \\ j \in I_A}} \delta^{\eta t_{ij}} + (p_A - c_A^o - a_B)p_A^{-\eta} \sum_{i \in I_A} \sum_{j \in I_B} \delta^{\eta t_{ij}}$$

$$- \sum_{i \in I_A} f + (a_A - c_A^f)p_B^{-\eta} \sum_{i \in I_B} \sum_{j \in I_A} \delta^{\eta t_{ij}}$$

It is important to remember that the previous structure of the objective function is inadequate because the sets $I_A$ and $I_B$ represent the group of consumers affiliated to the

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[7] In a companion paper we study the effect of price discrimination depending on the destiny of the call. Interestingly, in such case we also obtain a linear representation for the affiliation decisions, but matrix $H$ becomes more complicated because of the presence of network externalities.

corresponding firms, which are endogenous to the vector of prices $(p_A, p_B)$. The objective function can be simplified by incorporating the variables $\alpha_i$ identifying the affiliation decisions. If we include the fact that affiliation decisions are also optimal for consumers, we have that firm A's problem is given by:

$$
\max_{p_A \geq 0} \pi_A(p_A, p_B, a_A, a_B; \alpha) = (p_A - c_A^o - c_A^f) p_A^{-\eta} \sum_{i \in I} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} (1 - \alpha_i)(1 - \alpha_j) \tag{9}
$$

$$
+ (p_A - c_A^o - a_B) p_A^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} (1 - \alpha_i) \alpha_j
$$

$$
- \sum_{i \in I} (1 - \alpha_i) f + (a_A - c_A^f) p_B^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} \alpha_i (1 - \alpha_j)
$$

$$
s.t. \quad H\alpha \leq z(p_A, p_B), \quad \alpha \in \{0, 1\}^I
$$

where $H$, $z$ and $\alpha$ were defined in the previous subsection. It is clear that problem (9) has the structure required in (6).

The analogous problem for Firm B is trivially given by:

$$
\max_{p_B \geq 0} \pi_B(p_A, p_B, a_A, a_B; \alpha) = (p_B - c_B^o - c_B^f) p_B^{-\eta} \sum_{i \in I} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} \alpha_i \alpha_j \tag{10}
$$

$$
+ (p_B - c_B^o - a_A) p_B^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} \alpha_i (1 - \alpha_j)
$$

$$
- \sum_{i \in I} \alpha_i f + (a_B - c_B^f) p_A^{-\eta} \sum_{i \in I} \sum_{j \in I} \delta^{\eta t_{ij}} (1 - \alpha_i) \alpha_j
$$

$$
s.t. \quad H\alpha \leq z(p_A, p_B), \quad \alpha \in \{0, 1\}^I
$$

Note that the constraint is the same that in equation (9), even when the objective function changes according to the definition of $\alpha_i$'s.

## 2.5   The Regulatory Interventions

In the subsequent analysis we consider a benchmark case associated with the standard regulatory approach, where access charges are defined by the authority, and only the final prices are the result of market interactions. In this case, the authority selects access charges as equal to marginal termination costs (i.e. $a_A = c_A^f$ and $a_B = c_B^f$). For simplicity we also assume symmetric firms so that $c_A^f = c_B^f$. Departing from this benchmark, we have two alternative regulatory interventions:

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1. The authority can set access charges below marginal termination costs to enhance competition. Under that condition, the firms have an additional incentive to reduce prices, because a net outflow of calls is more profitable than a balanced pattern.

2. The authority can implement policies aimed at reducing switching costs, that is $t$, which intensify rivalry to affiliate consumers.

We are going to describe how equilibrium is affected under each regulatory intervention and then we will perform a comparative analysis for the welfare achieved in both of them and in relation to reference cases.

## 3   The Welfare Analysis

The welfare analysis can be constrained to a simple comparison between the results of both regulatory interventions, but it is illustrative to compare those results with clear common benchmarks. The first benchmark considered is given by the standard regulation, where access charges are fixed at marginal termination values and final prices result from competition between the firms. A second one is a Ramsey approach, where consumer surplus is maximized subject to break even constraints. In the last benchmark we consider the collusive scenario characterized by the monopoly outcome, where competition is absent but the standard access charge regulation is in place. In this section we discuss the two last benchmarks.

For any pair of prices $(p_A, p_B)$ we can evaluate consumer surplus as:

$$CS(p_A, p_B) = \sum_{i \in I_A} V_i(p_A) + \sum_{i \in I_B} V_i(p_B) - t \left[ \sum_{i \in I_A} \left( x_i \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}} \right) + \sum_{i \in I_B} \left( (1 - x_i) \sum_{\substack{j \neq i \\ j \in I}} \delta^{t_{ij}} \right) \right]$$

(11)

Accordingly, total welfare could be defined by:

$$W(p_A, p_B) = CS(p_A, p_B) + \pi_A(p_A, p_B) + \pi_B(p_A, p_B)$$

And we could evaluate how close is welfare obtained in equilibrium $W(p_A^*, p_B^*)$ from the maximum achievable welfare given by:

$$\underset{p_A, \ p_B}{Max} \ W(p_A, p_B)$$

Unfortunately, consumer surplus can not be directly added to profits, because the multiple ways to consider transportation costs in an horizontally differentiated model

implies multiple measures for consumer surplus.[8] An alternative approach that permit us to avoid this problem is the second best solution associated to the Ramsey problem:[9]

$$\underset{p_A,\ p_B \geq 0}{Max}\ CS(p_A, p_B) \tag{12}$$

$$s.t.$$

$$\pi_A(p_A, p_B) + \pi_B(p_A, p_B) = 0$$

Where access charges have been set as equal to marginal termination costs. In this approach we can compare the Ramsey solution, given by equation (12), with the values obtained in equilibrium -for both regulatory interventions- for prices and consumer surplus, i.e., $p_A^*$, $p_B^*$, $CS(p_A^*, p_B^*)$.

Finally, it is also illustrative to use the monopoly case as another benchmark. In this case the affiliation decision is irrelevant and the firm simply solves:

$$\max_{p \geq 0} \pi_M(p) = \left\{ (p - c_M^o - c_M^f)p^{-\eta} \sum_{i \in I} \sum_{\substack{j \neq i \\ j \in I}} \delta^{\eta t_{ij}} - \sum_{i \in I} f \right\}$$

Where the subindex $M$ denotes monopoly levels.

## 4    Results

In order to study the role of social networks on welfare in telecommunication markets with a view to evaluate the relative performance of the two regulatory interventions, we establish the basic parameters for our simulations in Table 1.

Table 1: Basic Parameters

| elasticity of demand | $-\eta = -1.2$ |
|---|---|
| discount factor | $\delta = 0.9$ |
| origination cost | $c_A^o = c_B^o = c_M^o = 0.75$ |
| termination cost | $c_A^f = c_B^f = c_M^f = 0.75$ |
| fixed cost | $f = 0.5$ |
| number of individuals | $I = 1000$ |
| transportation cost | $t = 0.5$ |

---

[8] See the different options mentioned in footnote 3.
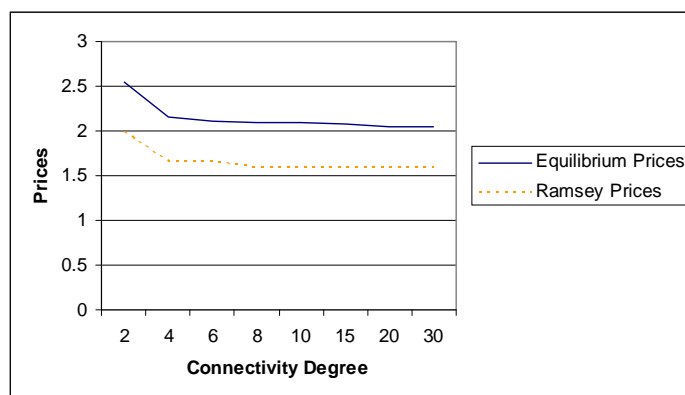[9] A similar approach is followed by Laffont *et al.* (1998a).

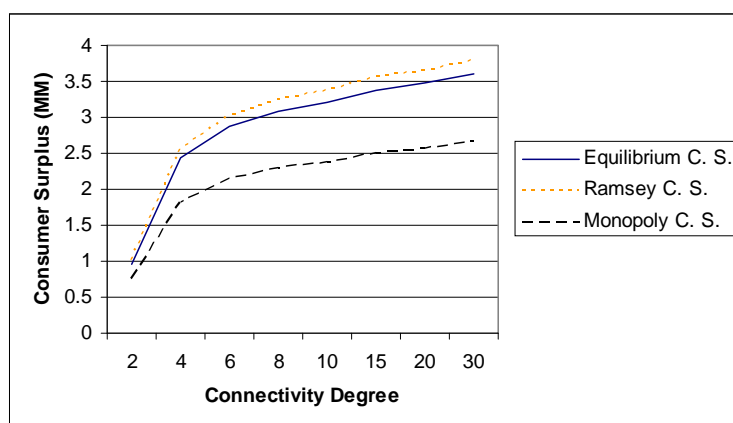Figure 1: The impact of Connectivity degree ($d$) on Equilibrium prices



Figure 2: The impact of Connectivity degree on Consumer Surplus (CS).

All the numbers in Table 1 were selected trying to conform a reasonable setting. For example, Ingraham and Sidak (2004) have estimated that the elasticity of demand in US for wireless services is between -1.12 and -1.29. The fixed cost ($f$) has been selected in order to represent 10% of ARPU (Average Revenue per User). On the other hand, origination, termination and transportation costs are in the same order of magnitude than those reported by De Bijl and Peitz (2002) in their simulations. The main algorithm is reported in Appendix II.

Figure 1 shows how the connectivity degree $d$ affects both equilibrium prices as well as Ramsey prices, mainly in the case of low levels of connectivity. However, Figure 2 shows that connectivity degree is an important factor affecting consumer surplus for all
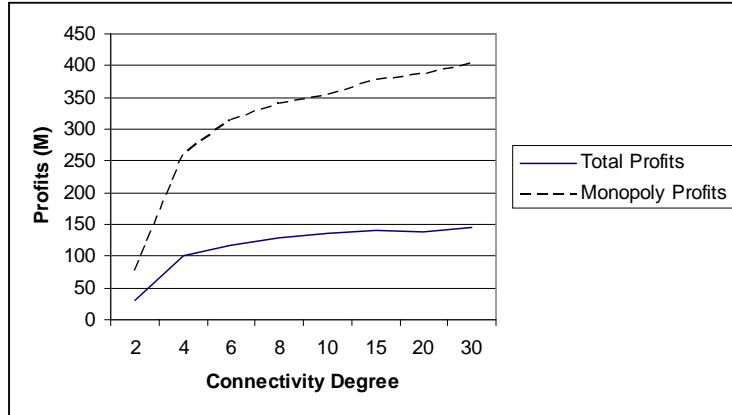
Figure 3: The impact of Connectivity degree on Producer Surplus.

$d$. Although the authority can not read this result as implying a policy intervention to increase $d$, it is clear that the gap between the equilibrium and the Ramsey benchmark can be reduced through regulation, and this is especially relevant for high values of $d$. On the other hand, the gap between the Ramsey benchmark and the monopoly case increase in $d$, showing that the relevance of the underlying social network, and the importance of regulation, increases when societies becomes more complex. Finally, Figure 3 shows the gap between total profits in competition and monopoly, both of them under the standard regulated environment. It is clear that firms have a higher incentive to collude when the connectivity degree increases.

For the analysis of the regulatory interventions, Table 2 summarize the setting for the parameters. The first column corresponds to the standard case where regulatory authorities set access prices as equal to marginal termination costs. The second column contains the parameters for the Ramsey approach, while the last two columns contain the settings where the intervention occurs in access charges (scheme 1) and in switching costs (scheme 2), respectively.

| Table 2: Basic Parameters under Regulatory Interventions | | | | |
| --- | --- | --- | --- | --- |
| Parameters | Standard | Ramsey | Scheme 1 | Scheme 2 |
| access charges $(a_A = a_B)$ | 0.75 | 0.75 | $< 0.75$ | 0.75 |
| transportation cost $(t)$ | 0.5 | 0.5 | 0.5 | $< 0.5$ |

We already mentioned that competition in retail markets increases when access charges
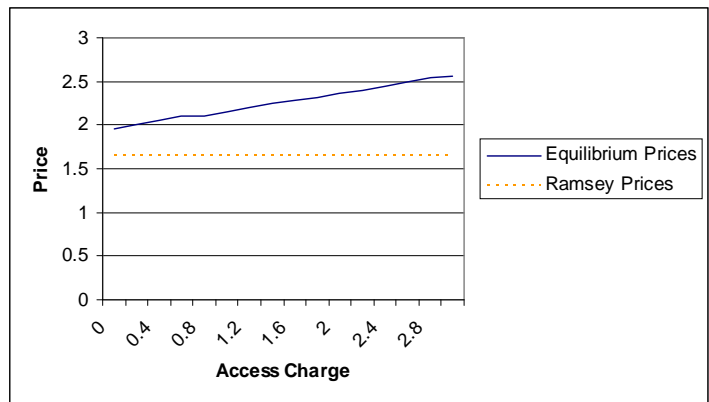
Figure 4: The effect of Access Charges on the Gap between Equilibrium and Ramsey Prices.
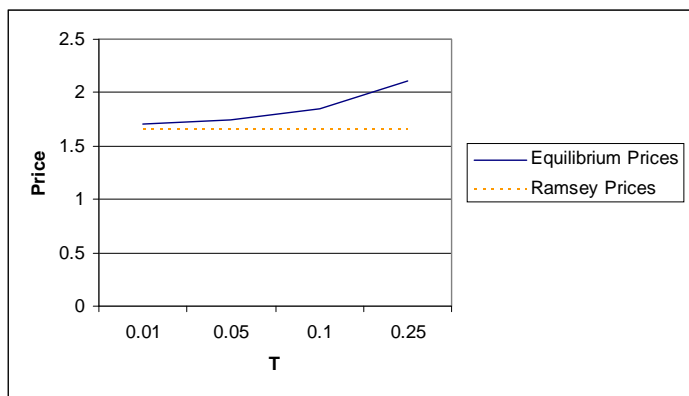


Figure 5: The Effect of reducing Switching Costs on the Gap between Equilibrium and Ramsey Prices.

are settled below marginal costs. Figure 4 provides support for this policy recomendation, showing that equilibrium prices can get closer to Ramsey levels when access charges are reduced. It is clear that, according to our simulations, lowering access charges even below marginal termination costs permits us to increase social welfare. However, this is not the only policy intervention that can be evaluated. Figure 5 shows the effect of an alternative policy intervention where the authority reduces transportation costs, making it easier to switch from one service provider to another. For example, one of such policy interventions would be the implementation of number portability policies, that permit a consumer to switch the phone service provider keeping the same phone number. Our simulations shows that this policy intervention is even more effective than access charge regulation in generating equilibrium prices closer to the Ramsey benchmark case.

## 5  Conclusion

In this paper we studied the competition between two interconnected firms providing communication services in a context where consumers are related through a social network. Taking as a benchmark the standard regulatory case, where access charges are fixed at marginal termination cost levels, we showed that network structure affects consumer and producer surplus. In this setting we have performed a welfare comparative analysis between two different regulatory interventions departing from the standard regulation. The policies considered were: (1) Setting access charges below marginal termination costs and, (2) Reducing switching costs between service providers.

The main difference with the existing literature is that the analysis was performed using a model where rational consumers are related through a social network, and then the number of calls between any pair of them depends not only on prices and transportation costs, but also on how socially close they are in the network.

The results showed that equilibrium prices, consumer surplus and producer surplus depend on the connectivity parameter $d$, showing that social networks matter in the way how markets perform and also how regulation should be accomplished. For example, regulation seems to be mandatory, but its importance depends on the social network characteristics, because the collusive scenario, associated to monopoly outcomes, is more profitable and has higher impact on consumer surplus for higher connectivity degrees in the social network.

In relation to the regulatory interventions, our results showed that setting access charges below marginal costs have a positive impact on competition, reducing equilibrium prices to consumers. However, an alternative policy intervention, oriented to reduce switching costs, was much more effective, because it brought final prices closer to a second best solution, the Ramsey approach. In this line, policies such as number portability appear as highly desirable in telecommunication markets.

## 6 Appendix

### 6.1 AI: Definition of M

In this section we want to define a valid upper bound $M$ such that (8) be an alternative representation of (7). We need to consider two cases:

Case I. Consider first the case when $b_i < 0$. In such a case we want agent $i$ to have the incentive to choose $\alpha_i = 0$. According to (8) if he or she chooses $\alpha_i = 1$ the second inequation in (8) is violated. But we also need to be sure that by choosing $\alpha_i = 0$ the binding constraint is the first one. In order to do that we need to select an $M$ sufficiently high such that the second inequation in (8) is always satisfied when $\alpha_i = 0$. In effect:

$$0 \leq b_i + M \quad \Leftrightarrow \quad -b_i \leq M \quad \text{but:}$$

$$-b_i = -x_i + \tfrac{1}{2} + \mathbf{1}^t e_{-i}(p_A, p_B)$$

$$\leq \tfrac{1}{2} + \tfrac{\sigma_i}{\eta-1}\left(p_A^{1-\eta} - p_B^{1-\eta}\right)\sum_{j \neq i} \delta^{\eta t_{ij}} \quad \text{using that } 0 < \delta < 1 \text{ and } \eta > 1 \text{ we have:}$$

$$\leq \tfrac{1}{2} + \tfrac{\sigma_i}{\eta-1}\left[p_A^{1-\eta} - p_B^{1-\eta}\right]\sum_{j \neq i} \delta^{\eta 0}$$

$$\leq \tfrac{1}{2} + \tfrac{\sigma_i}{\eta-1}(I - 1)\left[p_A^{1-\eta} - p_B^{1-\eta}\right]$$

Assuming that individual $i$ is connected to the network $\left(\sum_{j \neq i} \delta^{t_{ij}} \geq 1\right)$ we have:[10]

$\sigma_i \leq \sigma \equiv \frac{1}{2t}$ and considering the extreme possible values for prices we have:

$$M_{CaseI} = \tfrac{1}{2} + \tfrac{\sigma}{\eta-1}(I - 1)\left[\underline{p}_A^{1-\eta} - \overline{p}_B^{1-\eta}\right]$$

Case II. An analogous argument leads us to show that when $b_i > 0$, the constraint is equivalent to select $\alpha_i = 1$ when $M$ is sufficiently high, and a feasible selection of $M$ is obtained from:

$$0 \geq b_i - M \quad \Leftrightarrow \quad M \geq b_i \quad \text{but:}$$

$$b_i = x_i - \tfrac{1}{2} - \mathbf{1}^t e_{-i}(p_A, p_B)$$

$$\leq \tfrac{1}{2} - \tfrac{\sigma_i}{\eta-1}\left(p_A^{1-\eta} - p_B^{1-\eta}\right)\sum_{j \neq i} \delta^{\eta t_{ij}}$$

$$\leq \tfrac{1}{2} + \tfrac{\sigma_i}{\eta-1}\left[p_B^{1-\eta} - p_A^{1-\eta}\right]\sum_{j \neq i} \delta^{\eta 0}$$

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[10] If individual $i$ is disconnected from the social network, then, whithout loss of generality, he can be removed from the problem.

$$\leq \tfrac{1}{2} + \tfrac{\sigma_i}{\eta-1}(I-1)\left[p_B^{1-\eta} - p_A^{1-\eta}\right]$$

Assuming again that individual $i$ is connected to the network and considering the extreme possible values for prices we have:

$$M_{CaseII} = \tfrac{1}{2} + \tfrac{\sigma}{\eta-1}(I-1)\left[\underline{p}_B^{1-\eta} - \overline{p}_A^{1-\eta}\right]$$

So when the upper and lower bounds for the feasible prices for firms $A$ and $B$ are the same, $M = M_{CaseI} = M_{CaseII}$ otherwise $M$ is selected as the biggest of them.

## 6.2  AII: A Feasible Algorithm

The algorithm developed permit us to study a more general case than the one used in the paper where both, access charges and final prices, are defined by the equilibrium solution of a two stage noncooperative game $\Gamma$. In the first stage the firms simultaneously and non cooperatively select access charges in a range $[a_{\min}, a_{\max}]$ defined by the regulatory authority, while in the second, firms compete in prices in the presence of the selected access charges, with payoff functions given in (9) and (10) and subject to the constraint that customers are making optimal affiliation decisions. Of course, the regulated environment is a particular case where the authority defines $a_{\min} = a_{\max}$.

Given the set of parameters and the realization of the taste random variables $(x_i)_{i \in I}$, a feasible procedure to get the Nash equilibrium of the two stage game $\Gamma$ consists in finding the second stage reaction functions. The main difficulties to apply standard optimization tools is that the constraint is discrete and that the vector $z$ in equations (9) and (10) depends on the vector of prices.

The numerical study was performed by medium scale simulations. The social network was modeled by random regular graphs (see Bollobas (2001)). The access charges and prices varied in a predefined grid $G$ in the range: $[a_{\min}, a_{\max}], [p_{\min}, p_{\max}]$. The methodology for the simulations was the following:

0) Constant and parameter definitions: $I, d, f, \eta, \delta, c_A^o, c_B^o, c_A^f, c_B^f$.

1) Generate random graph $g$ with fixed degree $d$.

2) Generate random vector of network preferences: $x = (x_i)_{i \in I}$.

3) For each pair of access charges $a_A, a_B$ in the grid over $[a_{\min}, a_{\max}]$:

    (a) For each pair of prices $p_A, p_B$ in the grid over $[p_{\min}, p_{\max}]$:

        i) Compute indifference points: $x^*(p_A, p_B) = (x_i^*(p_A, p_B))_{i \in I}$.

i. Select $\alpha(p_A, p_B)$ satisfying the constraint in (9).

ii) For the selected $\alpha$ in (ii) we can write the profits for firms A and B as:
$\pi_A(p_A, p_B; a_A, a_B), \pi_B(p_A, p_B; a_A, a_B)$.

(b) Compute prices response functions: $p_A^*(p_B; a_A, a_B), p_B^*(p_A; a_A, a_B)$.

(c) Compute Nash equilibrium prices in the second stage: $p_A^*(a_A, a_B), p_B^*(a_A, a_B)$.

(d) Compute $V_A(p_A^*, p_B^*), V_B(p_A^*, p_B^*)$.

(e) Compute consumer surplus $CS(p_A^*, p_B^*)$.

4) From $\pi_A(p_A^*(a_A, a_B), p_B^*(a_A, a_B); a_A, a_B), \pi_B(p_A^*(a_A, a_B), p_B^*(a_A, a_B); a_A, a_B)$ compute access charges response functions $a_A^*(a_B)$ and $a_B^*(a_A)$.

5) Compute Nash equilibrium access charges $a_A^*, a_B^*$.

6) Compute indifference values in equilibrium: $x^*(a_A^*, a_B^*) = (x_i^*(a_A^*, a_B^*))_{i \in I}$.

7) Compute equilibrium profits for firms A and B: $\pi_A(p_A^*(a_A^*, a_B^*), p_B^*(a_A^*, a_B^*); a_A^*, a_B^*)$, $\pi_B(p_A^*(a_A^*, a_B^*), p_B^*(a_A^*, a_B^*); a_A^*, a_B^*)$.

8) Compute consumer surplus $CS(p_A^*(a_A^*, a_B^*), p_B^*(a_A^*, a_B^*))$.

9) Repeat steps 1) to 8) for a statistically significant number of graphs.

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