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Welfare Analysis of Free Entry in a Dynamic General Equilibrium Model*

Koichi Futagami[†], Tatsuro Iwaisako[‡§], and Makoto Okamura[¶]

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Abstract

This paper presents a welfare analysis of free entry equilibrium in dynamic general equilibrium environments with oligopolistic competition. First, we show that a marginal decrease in the number of firms at the free entry equilibrium improves social welfare. Second, we show that if a government can control the number of entrants intertemporally so as to maximize the level of social welfare, the number of entrants under free entry may be less than the second-best number of entrants. Capital accumulation plays an important role in determining whether excess entry occurs.

JEL Classification: D43; L50; O41

Keywords: Excess entry; Oligopolistic competition; Dynamic general equilibrium

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1 Introduction

One purpose of deregulation policies is to promote the entry of firms into industries that have been protected either by government policies or the entry barriers of incumbents. Governments of many developed countries often implement deregulation policies. Typical examples are as follows: the Government of the United States deregulated the airline industry with the Airline Deregulation Act of 1978. In 1986 the Government of the United Kingdom undertook the deregulation of the securities market, the so called 'Big Bang'. Such deregulation policies are regarded as increasing the economic performance of countries.

Deregulation policies that intend to increase the number of firms have significant impacts on the economy. First, if the production of goods is undertaken under increasing returns to scale due to the existence of fixed costs, this increases the number of firms and promotes competition, decreases the output level of each entrant, and leads to a rise in average costs. This has a negative effect on the level of welfare. Second, an increase in the number of firms raises the intensity of competition and thus reduces price–cost markups . This increases the factor demand, especially the demand for capital. Consequently the rental of capital rises and capital accumulation is promoted. Because an acceleration of capital accumulation causes an increase in output, an increase in the number of firms can improve welfare. In order to examine whether the deregulation policies improve social welfare, we examine which effect overcomes the other in a dynamic general equilibrium model with oligopolistic competition.

In the present paper, we examine the above welfare effects of deregulation, that is, increasing the number of entrants in a dynamic general equilibrium model with oligopolistic competition. First, we investigate how a marginal decrease in the number of firms at the free entry equilibrium affects social welfare. The government decreases the number of entrants by collecting a tax or a franchise fee from firms. A marginal decrease in the number of firms temporarily raises the consumption volume of households, thus increasing the level of welfare. However, reduction of capital thereafter reduces the welfare level. We can show that the former positive effect overcomes the latter negative effects and thus a marginal decrease in the number of firms can improve social welfare. Second, we show that the number of entrants under free entry may be less than the second-best number of entrants if a government can control the number of entrants so as to maximize the level of social welfare . That is, an insufficient entry occurs at the free entry equilibrium.

As for the first effect described above, Perry (1984), Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) show the excess entry theorem. The excess entry theorem states that a marginal decrease in the number of firms at the free entry equilibrium improves welfare if firms produce goods of strategic substitutes and compete in the Cournot fashion and if there exist fixed set-up costs. This theorem provides a counter example to the common ideas of the traditional theory of industrial organization.[1]

However, the above research was restricted to partial equilibrium analyses and there has been no research that examines the excess entry theorem by taking into account capital accumulation over time.[2] Because the theorem places attention on the long-run equilibrium, the disregard of capital accumulation by the theorem seems surprising. If the promotion of entry reduces an incentive for capital accumulation, such a promotion might decrease social welfare. In fact, as this paper's analyses show, capital accumulation plays an important role and can overturn the result of the excess entry theorem.

There are some studies of macroeconomic dynamics that incorporate oligopolistic competition with free entry. Gali and Zilibotti (1995) and Kuhry (2001) and Dos Santos Ferreira and Lloyd-Braga (2005) investigated the dynamic characteristics of the free entry equilibrium in dynamic general equilibrium models with Cournot oligopoly.[3] However, these researches did not examine welfare aspects of the free entry equilibrium. Therefore, the present analysis is the first one to explore the welfare properties of the free entry equilibrium in the dynamic general equilibrium framework with Cournot competition. [4]

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[1] Thereafter, Besley and Suzumura (1992) and Okuno-Fujiwara and Suzumura (1993) extended the analysis based on the excess entry theorem to a dynamic game setting, that is, a two-stage game setting. In the game, firms undertake cost-reducing investment in the first stage of the game and compete in a Cournot fashion in the second stage of the game. They derive the subgame perfect Nash equilibrium of the two-stage game. These papers examined the roles of the commitment of firms in the first stage.

[2] Konishi et al. (1990), examine the excess entry theorem in a general equilibrium model, however, their model is a static model.

[3] Some studies investigated the dynamic properties of equilibrium paths in dynamic models with monopolistic competition. See Benhabib and Gali (1995) and Gali (1994, 1995, 1996).

[4] Wu and Zhang (2000) examined the welfare effects of income taxation in a dynamic general equilibrium model under imperfect competition. However, this research did not examine welfare aspects of the free entry equilibrium. In addition, their model incorporated not oligopoly but monopolistic competition.

The structure of the paper is as follows. In section 2, we set up the dynamic general equilibrium model. In section 3, we derive the steady state equilibrium with free entry. In section 4, we examine whether or not a marginal decrease in the number of entrants at free entry equilibrium improves social welfare. We employ Judd's (1982) method to evaluate the welfare effect by a marginal change of the number of entrants. In section 5, we construct a government's optimal entry regulation problem and derive the steady state equilibrium under the second-best policy.[5] Section 6 provides an analytical comparison of the steady state equilibrium under free entry with the steady state equilibrium under the second-best policy. Section 7 concludes the paper.

2 The model

We consider a basic representative agent model, that is, the closed economy consists of a continuum of consumers populating L, who live infinitely. Each consumer supplies one unit of labor inelastically at each point of time. In order to analyze the entry of firms, the present model has some important features in the production structure as follows. There are one kind of final good, which can be devoted to both consumption good production and investment good production. The final good is produced using intermediate goods. In each intermediate goods sector, the firms behave as Cournot-Nash competitors and the number of entrants is determined by the free entry condition. In order to analyze the welfare effects of regulation and deregulation of entry, we assume that a government impose a tax or a franchise fee on entry firms and it can regulate the number of entrants indirectly by changing this tax or franchise fee.

2.1 Firms

The final goods sector is produced by the use of intermediate goods. In a similar way as in Ethier (1982), production technology of the final goods sector is specified as a CES production function:

$$Y = \left(\int_0^1 X_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma \geq 0, \tag{1}$$

[5]Devereux and Lee (2001) examine the gains from trade under imperfect competition in a similar dynamic general equilibrium model. They compare the free entry equilibrium with the social planning outcome, that is, the first-best outcome. In particular, excess entry occurs in a natural setting of parameters in their model. However, their focus is on the effects on countries opening up to international trade.

where Y denotes an amount of final goods, X_i denotes the quantity of the intermediate input $i \in [0, 1]$, and σ represents the elasticity of substitution among the intermediate inputs.

We assume that perfect competition prevails in the final goods market and let the final goods be the numeraire. The first-order conditions for the profit maximization of the final goods sector are given by the following:

$$Y^{\frac{1}{\sigma}} X_i^{-\frac{1}{\sigma}} = P_i, \quad i \in [0, 1], \tag{2}$$

where P_i denotes the price of intermediate goods i.

Each intermediate good is produced by using capital and labor. Firms face the technology given by

$$x_{ij} = k_{ij}^{\alpha} \ell_{ij}^{1-\alpha} - \phi, \tag{3}$$

where x_{ij}, k_{ij}, ℓ_{ij}, and ϕ denote an amount of output, capital and labor inputs, and the fixed cost to firms j in the intermediate goods sector i, respectively. We assume that in each intermediate goods sector Cournot competition and free entry prevail. We can split the firms' profit maximization problem into two steps. In the first step, solving the firms' cost minimization, we can get the labor and capital demands and the cost function as follows:

$$\ell(w, r; x_{ij}) = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \left(\frac{w}{r}\right)^{-\alpha} (x_{ij} + \phi), \tag{4}$$

$$k(w, r; x_{ij}) = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w}{r}\right)^{1-\alpha} (x_{ij} + \phi), \tag{5}$$

$$\Omega(x_{ij}; w, r) = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} r^{\alpha} w^{1-\alpha} (x_{ij} + \phi) \equiv q(w, r)(x_{ij} + \phi), \tag{6}$$

where $q(w, r)$ denotes the unit cost function, and r and w denote the rental rate of capital and the wage rate respectively. In the second step, using this unit cost function, we can write the profit of firm j in the intermediate goods sector i as follows:

$$\pi_{ij}(x_{ij}, x_{i,-j}) = P_i(\sum_j x_{ij}) x_{ij} - q(w, r)(x_{ij} + \phi),$$

where $P_i(\sum_j x_{ij})$ denotes the inverse demand function of intermediate good i. Substituting the demand function of intermediate goods i, (2) into this profit, the profit-maximizing condition of firm j is given by

$$\left(1 - \frac{x_{ij}}{\sigma X_i}\right) P_i(\sum_j x_{ij}) = q(w, r).$$

Therefore, the amount of the firms' output in intermediate goods sector i in a symmetric Cournot-Nash equilibrium, $x_{ij} = x_i$ satisfies the following condition:

$$P_i(N_i x_i) = \mu(N_i) q(w, r), \tag{7}$$

where N_i denotes the number of firms that enter the market of intermediate goods i and $\mu(N_i)$ denotes the markup of price over cost given by

$$\mu(N_i) = \frac{1}{1 - \frac{1}{\sigma N_i}}, \qquad i \in [0, 1].$$

These show that an increase in the number of entrants reduces the markup.

We suppose that a government imposes a tax on each entrant, which is proportional to the set-up cost, $q(w, r)\phi$. Letting m denote the tax rate on the entry, the government can regulate the number of entrants by controlling this tax rate. Using (7), the profit of the firm in intermediate good sector i in symmetric Cournot-Nash equilibrium is given by

$$\begin{aligned} \pi_i(N_i) &= P_i(N_i x_i) x_i - q(w, r)(x_i + \phi) - m q(w, r)\phi \\ &= \{[\mu(N_i) - 1] x_i - (1 + m)\phi\} q(w, r). \end{aligned} \tag{8}$$

Firms are symmetric in each intermediate good sector and furthermore the intermediate good sectors are symmetric, we obtain $k_{ij} = K_t/N_t$ and $l_{ij} = L/N_t$. Therefore, the amount of intermediate goods that one firm supplies, x_t is given by

$$x_t = \frac{{K_t}^{\alpha} L^{1-\alpha}}{N_t} - \phi. \tag{9}$$

Because the intermediate goods sectors are symmetrical, $Y_t = X_t$. From this and (2), we get $P_i = 1$. Substituting this into (7), we get $\mu(N)q(w, r) = 1$. Combining this with the cost minimizing conditions (4) and (5), equilibrium factor prices are given by

$$r_t = \frac{1}{\mu(N_t)} \alpha {K_t}^{\alpha-1} L^{1-\alpha}, \tag{10}$$

$$w_t = \frac{1}{\mu(N_t)} (1 - \alpha) {K_t}^{\alpha} L^{-\alpha}. \tag{11}$$

The number of entrants will adjust until the profit becomes zero, that is, $\pi(N_t) = 0$. Substituting (9) into (8), we obtain the zero profit condition as follows:

$$[\mu(N_t) - 1] \left(\frac{{K_t}^{\alpha} L^{1-\alpha}}{N_t} - \phi \right) = (1 + m)\phi, \tag{12}$$

which determines the number of firms in each intermediate goods sector at time t. Solving this equation for N_t yields the equilibrium number of firms under the entry tax m, N_t as follows:

$$N(K_t; m) = \frac{m}{2(1+m)\sigma} + \left\{ \left[\frac{m}{2(1+m)\sigma} \right]^2 + \frac{{K_t}^{\alpha} L^{1-\alpha}}{(1+m)\sigma\phi} \right\}^{\frac{1}{2}}. \tag{13}$$

Total differentiation of (12) yields

$$\frac{\partial N(K,m)}{\partial m} = -\frac{N(\sigma N - 1)}{1 + (2\sigma N - 1)(1+m)} < 0, \tag{14}$$

$$\frac{\partial N(K,m)}{\partial K} = \frac{N(\sigma N - 1)}{1 + (2\sigma N - 1)(1+m)} > 0, \tag{15}$$

Therefore, it is straightforward to show that raising the tax on entries discourages the entry and thus decreases the number of entrants for any constant value of K. Moreover, (15) shows that an increase in capital stock allows more firms to enter the market.

2.2 Households

We consider an economy populated with L households, who each supply one unit of labor inelastically. Each household seeks to maximize the lifetime utility

$$U = \int_0^\infty e^{-\rho t} \frac{{c_t}^{1-\theta} - 1}{1-\theta} dt, \tag{16}$$

where, c_t is consumption per household and ρ is the discount rate. The revenue earned by tax on entries is distributed equally among consumers through lump-sum transfer. Thus the household's intertemporal budget constraint is given by

$$\dot{a}_t = r_t a_t + w_t - c_t + \frac{N_t m q(w_t, r_t)\phi}{L}, \tag{17}$$

$$\lim_{t\to\infty} \exp\left(-\int_0^t r_s ds \right) a_t \geq 0, \tag{18}$$

where a_t denotes asset holdings per household. The dynamic optimization of the utility function, (16), subject to the intertemporal constraint, (17) and (18), yields the Euler equation and the transversality condition

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r_t - \rho), \tag{19}$$

$$\lim_{t\to\infty} e^{-\rho t} {c_t}^{-\theta} a_t = 0. \tag{20}$$

3 Market equilibrium with free entry

In this section, we derive the steady state under the free entry equilibrium in the long run, that is, the steady state with no entry tax, $m = 0$. For simplicity, we assume no depreciation. Then the equilibrium condition of final goods market is given by $Y_t = C_t + \dot{K}_t + \phi N_t$. Combining this, (19), (10), and (13), the dynamics of this economy can be summarized by the following two differential equations:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}\left[\frac{1}{\mu(N(K_t))}\alpha {K_t}^{\alpha-1}L^{1-\alpha} - \rho\right], \tag{21}$$

$$\dot{K}_t = {K_t}^{\alpha}L^{1-\alpha} - \phi N(K_t) - C_t, \tag{22}$$

where

$$N(K_t) = \left(\frac{{K_t}^{\alpha}L^{1-\alpha}}{\sigma\phi}\right)^{\frac{1}{2}}. \tag{23}$$

First, we derive the conditions under which there exist steady states of the free entry equilibrium. We obtain the following proposition:

Proposition 1. *The free entry equilibrium has steady states, if, and only if the following inequality is satisfied:*

$$\frac{\alpha^2}{2-\alpha}\left\{\left[\frac{2(1-\alpha)}{2-\alpha}\right]^2\left(\frac{\sigma L}{\phi}\right)\right\}^{\frac{1-\alpha}{\alpha}} > \rho. \tag{24}$$

Proof. First, we must investigate the character of the interest rate. The interest rate is the function of K, and is given by

$$r(K) \equiv \left[1 - \left(\frac{\phi}{\sigma}\frac{1}{K^{\alpha}L^{1-\alpha}}\right)^{\frac{1}{2}}\right]\alpha K^{\alpha-1}L^{1-\alpha}. \tag{25}$$

Differentiating $r(K)$ with K, we obtain

$$\frac{dr(K)}{dK} = \left[\frac{2-\alpha}{2}\left(\frac{\phi}{\sigma K^{\alpha}L^{1-\alpha}}\right)^{\frac{1}{2}} - (1-\alpha)\right]\alpha K^{\alpha-2}L^{1-\alpha}. \tag{26}$$

We let $\bar{K}$ denote the value of K that satisfies $\{dr(K)\}/(dK) = 0$ and from (26), we obtain

$$\bar{K} = \left\{\left[\frac{2-\alpha}{2(1-\alpha)}\right]^2\left(\frac{\phi}{\sigma L^{1-\alpha}}\right)\right\}^{\frac{1}{\alpha}}. \tag{27}$$

The graph of $r(K)$ has the unique maximum value, $r(\bar{K})$ as depicted in Figure 1. Here, the parameters must satisfy $r(\bar{K}) > \rho$ for the existence of a steady state. From (25) and (26), there exist steady states if, and only if the following inequality is satisfied:

$$\frac{\alpha^2}{2-\alpha}\left\{\left[\frac{2(1-\alpha)}{2-\alpha}\right]^2\left(\frac{\sigma L}{\phi}\right)\right\}^{\frac{1-\alpha}{\alpha}} > \rho. \tag{28}$$

□

If this inequality does not hold, the steady states of the free entry equilibrium do not exist. Thus, we assume that (28) holds true in the rest of this paper.

There are two values of K that satisfy $r(K) = \rho$ when (28) holds as depicted in Figure 1. However, the lower value of K is unstable, and therefore the unique stable steady state of the free entry equilibrium is the higher value of K that satisfies $r(K) = \rho$.

From (21) and (22), the phase diagram of this dynamic system is depicted in Figure 2. As shown in section 4, the steady state, (K^*, C^*) is a saddle point; thus there exists a unique equilibrium path converging to this steady state.[6]

4 The welfare effects of regulation of entry

In the rest of this paper, we examine whether free entry maximizes social welfare, that is, the representative agent's lifetime utility. In this section, in particular, we examine whether a marginal decrease in the number of entrants improves social welfare.

First, we suppose that the economy is on the steady state of the free entry equilibrium path initially. Then we examine how regulating entry affects the welfare. More concretely, we examine how an increase in the entry tax rate, m affects the welfare. Alternatively, we can assume that the government can control the number of entrants directly. In this case, we can interpret that the government regulates the entry so that the profit flow that each entrant can earn is equal to $mq(w,r)\phi$.

From (8), the number of entrants satisfies

$$\left\{[\mu(N_t)-1]\left(\frac{{K_t}^{\alpha}L^{1-\alpha}}{N_t}-\phi\right)-\phi\right\} = m\phi.$$

[6] The economy has the equilibrium path converging to the origin, that is, a poverty trap. For more information on this point, see Gali and Zilibotti (1995).

Rewriting this, we obtain

$$[1 + (\sigma N_t - 1)(1 + m)]\, \phi N_t = K_t^{\alpha} L^{1-\alpha}. \tag{29}$$

We let $N(K, m)$ denote the function that satisfies (29). Totally differentiating (29), we obtain

$$\frac{\partial N(K, m)}{\partial m} = -\frac{N(\sigma N - 1)}{1 + (2\sigma N - 1)(1 + m)} < 0. \tag{30}$$

Therefore, it is straightforward to show that $N(K, m)$ is a decreasing function of m for any constant value of K. This means that a government must reduce the number of entrants to keep the profit flow high. The resource constraint is given by

$$\dot{K}_t = {K_t}^{\alpha} L^{1-\alpha} - \phi N(K_t, m) - C_t.$$

The market equilibrium path is characterized by the following two differential equations:

$$\dot{K}_t = F(K_t, m) - C_t, \tag{31}$$

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}\left[r(K_t, m) - \rho\right], \tag{32}$$

where

$$F(K_t, m) \equiv {K_t}^{\alpha} L^{1-\alpha} - \phi N(K_t, m), \tag{33}$$

$$r(K_t, m) \equiv \left[1 - \frac{1}{\sigma N(K_t, m)}\right] \alpha {K_t}^{\alpha-1} L^{1-\alpha}. \tag{34}$$

Because we will need the partial derivatives of $F(K, m)$ and $r(K, m)$, we calculate them in advance. Partially differentiating (33) and (34) with respect to K and m, we obtain

$$F_K(K, m) = \alpha K^{\alpha-1} L^{1-\alpha} - \phi N_K(K, m) > 0, \tag{35}$$

$$r_K(K, m) = \left[\frac{N_K(K, m)}{\sigma[N(K, m)]^2} - \frac{(1-\alpha)(\sigma N - 1)}{K\sigma N(K, m)}\right] \alpha K^{\alpha-1} L^{1-\alpha}, \tag{36}$$

$$F_m(K, m) = -\phi N_m(K, m) > 0, \tag{37}$$

$$r_m(K, m) = \frac{N_m(K, m)}{\sigma[N(K, m)]^2} \alpha K^{\alpha-1} L^{1-\alpha} < 0, \tag{38}$$

where, hereafter a variable with the subscript denotes its partial derivative with respect to change in the subscript; e.g. $F_K(K, m) = [\partial F(K, m)]/[\partial K]$. Linearizing (31) and (32) around the steady state yields

$$\begin{bmatrix} \dot{K}_t \\ \dot{C}_t \end{bmatrix} = \begin{bmatrix} F_K(K^*, m) & -1 \\ \frac{C^*}{\theta} r_K(K^*, m) & 0 \end{bmatrix} \begin{bmatrix} K_t - K^* \\ C_t - C^* \end{bmatrix}, \tag{39}$$

Here, we let J denote the Jacobian matrix and let ν_1 and ν_2 denote the negative and positive eigenvalues respectively as shown below. The determinant of this Jacobian matrix can be calculated as follows:

$$\text{det}J = \frac{C^*}{\theta} r_K(K^*, m) < 0, \tag{40}$$

where, we use the fact that $r_K(K^*, m) < 0$. By (40), we have shown that the steady state is a saddle point.

Next, we examine how the change in the number of entrants affects the steady state values of capital stock and consumption. From (31) and (32), these steady state values satisfy the following equations:

$$F(K^*, m) = C^*, \tag{41}$$

$$r(K^*, m) = \rho. \tag{42}$$

Differentiating the both sides of (42) with respect to m, we obtain

$$\frac{dK^*}{dm} = -\frac{r_m(K^*, m)}{r_K(K^*, m)}. \tag{43}$$

Because both $r_m(K^*, m)$ and $r_K(K^*, m)$ are negative, $(dK^*)/(dm)$ is negative. This means that a marginal decrease in the number of entrants impedes capital accumulation in the long run. Moreover, differentiating both sides of (41) with respect to m and substituting (43) into (44), we obtain

$$\frac{dC^*}{dm} = F_m(K^*, m) - F_K(K^*, m)\frac{r_m(K^*, m)}{r_K(K^*, m)}. \tag{44}$$

To find the sign of (44), we must examine whether $F_K(K^*, m)\ [-r_M(K^*, m)]\ -F_m(K^*, m)$ $[-r_K(K^*, m)]$ is positive or negative. Substituting these partial derivatives into (44) and using (29) and (42), we can summarize the results in the following proposition:

Proposition 2. *A marginal decrease in the number of entrants necessarily reduces the steady state level of capital stock. A marginal decrease in the number of entrants reduces the steady state level of consumption if the number of entrants at the steady state satisfies the following inequality:*

$$N^* \leq \frac{1-\alpha}{\sigma(1-2\alpha)}. \tag{45}$$

As shown in section 6, higher elasticity of substitution among intermediate goods, higher entry cost, and a lower subjective discount rate tend to make the steady state value of the number of entrants in the free entry equilibrium smaller. According to this relationship between the number of entrants and the parameters, we can understand Proposition 2 as follows: a decrease in the number of entrants tends to make consumption lower in the long run in the economy with higher elasticity of substitution among intermediate goods, higher entry cost, and a lower subjective discount rate. This implies that a marginal decrease in the number of entrants aggravates social welfare in the long run.

In the final part of this section, we analyze the welfare effect by a marginal decrease in the number of entrants. For simplicity, we suppose that the instantaneous utility function takes the logarithmic form ($\theta = 1$). From (19), the consumption path is given by

$$c(t,m) = c(0,m) \exp \int_0^t \left[r(K(v,m),m) - \rho \right] dv, \tag{46}$$

where, $c(t,m)$ and $K(t,m)$ denote per capita consumption and capital stock at time t when the profit flow is $mq\phi$ respectively. As shown in Appendix 1, a marginal decrease in the number of entrants raises the initial level of consumption. Therefore a marginal decrease in the number of entrants improves social welfare in the short run.

Substituting (46) and $C(t,m) = c(t,m)L$ into (16) gives the indirect utility function as follows:

$$U(m) = \frac{1}{\rho}[\log C(0,m) - \log L] + \int_0^\infty \left\{ \int_0^t \left[r(K(v,m),m) - \rho \right] dv \right\} e^{-\rho t} dt, \tag{47}$$

Using Judd's (1982) method, we differentiate (47) with respect to m and obtain the formula for the welfare effect caused by a change in the profit rate, that is, the number of entrants is

$$\frac{dU(m)}{dm} = \frac{1}{\rho} \frac{C_m(0,m)}{C(0,m)} + \int_0^\infty \left\{ \int_0^t \left[r_K(K^*,m) K_m(v,m) + r_m(K^*,m) \right] dv \right\} e^{-\rho t} dt. \tag{48}$$

The purpose of this section is to examine whether a marginal decrease in the number of entrants improves social welfare, and therefore we assume that the economy is initially in the free entry equilibrium, that is, $m = 0$. As shown in Appendix 1, (48) can be rewritten as

$$\frac{dU(m)}{dm}\Big|_{m=0} = \frac{1}{\rho} \left[\frac{C_m(0,0)}{C(0,0)} + \frac{r_m(K^*,0)}{\rho - \nu_1} \right]. \tag{49}$$

(49) shows that we can separate the welfare effect of reducing the number of entrants into two subeffects. The first term of the right-hand side of (49) represents the effect on the initial

consumption. Because a decrease of the number of entrants saves using inputs for fixed cost, this effect raises the initial consumption and social welfare. This short-run effect corresponds to the jump from E to E' as depicted in Figure 3. Next, the second term on the right-hand side of (49) represents the effect on capital accumulation. A decrease in the number of entrants reduces the demand for capital and the rental rate, r, and impedes capital accumulation and lowers the steady state level of capital stock. Therefore, this effect reduces the consumption level and social welfare in the long run. This long-run effect corresponds to the transition from E' to E'' as depicted in Figure 3. To sum up, if the former positive effect dominates the latter negative effect, a marginal decrease in the number of entrants improves social welfare. As shown in Appendix 1, $\frac{dU(m)}{dm}|_{m=0} > 0$. Hence the increase in welfare due to an increase of the initial consumption is larger than the decrease of welfare due to a decrease of the steady state level of capital stock. Therefore, we obtain the following proposition:

Proposition 3. *Suppose that the economy is initially in the steady state of the free entry equilibrium. A marginal decrease in the number of entrants improves social welfare.*

This proposition implies that free entry results in excess entry also in the dynamic general equilibrium. In other words, the excess entry theorem holds true in the dynamic general equilibrium model.

5 Second-best equilibrium

In the previous section, we examine only how a marginal decrease in the number of entrants affects social welfare. However, as mentioned in Introduction, governments change policies of regulation of entry over time depending on the market size of economies in a real world: for most of developed countries, the governments regulate entry in the early stages of their development and relax the regulation of entry gradually after the economies developed fully. In this section, in accordance with the actual tendency of the behaviors of governments, we assume that a government can control the number of entrants intertemporally so as to maximize social welfare and examine how many firms a government allows to enter the market in the long run.

In particular, we consider the following circumstance: a government can control the number of firms, however, a government cannot control the pricing behavior of the firms; in this sense, the equilibrium in this circumstance is second-best one. In second-best equilibrium, a government chooses path of the number of entrants, $\{N_t\}_{t=0}^{\infty}$, to maximize the lifetime utility of the representative household (16) subject to Euler equation (19), rental rate of capital (10) and resource constraint. Therefore, the dynamic optimal entry problem by a government can be formulated as follows: [7]

$$\max_{\{N_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{(C_t/L)^{1-\theta} - 1}{1-\theta} dt,$$

subject to

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}\left[\frac{1}{\mu(N_t)} \alpha {K_t}^{\alpha-1} L^{1-\alpha} - \rho\right],$$

$$\dot{K}_t = {K_t}^{\alpha} L^{1-\alpha} - \phi N_t - C_t,$$

$$\lim_{t\to\infty} e^{-\rho t} {C_t}^{-\theta} K_t = 0.$$

The current–value Hamiltonian for a government's problem is given by

$$H_t = \frac{C_t^{1-\theta} L^{\theta-1} - 1}{1-\theta} + \lambda_{1,t}\frac{1}{\theta}\left[\left(1-\frac{1}{\sigma N_t}\right)\alpha K_t^{\alpha-1} L^{1-\alpha} - \rho\right] C_t + \lambda_{2,t}\left(K_t^{\alpha} L^{1-\alpha} - \phi N_t - C_t\right), \tag{50}$$

where λ_1 and λ_2 are the costate variables for C and K respectively. The necessary conditions for this dynamic optimization problem are the following:

$$\frac{\partial H_t}{\partial N_t} = \lambda_{1,t}\frac{1}{\theta \sigma {N_t}^2}\alpha K_t^{\alpha-1} L^{1-\alpha} C_t - \lambda_{2,t}\phi = 0, \tag{51}$$

$$\frac{\partial H_t}{\partial C_t} = C_t^{-\theta} L^{1-\theta} + \lambda_{1,t}\frac{1}{\theta}\left[\left(1-\frac{1}{\sigma N_t}\right)\alpha K_t^{\alpha-1} L^{1-\alpha} - \rho\right] - \lambda_{2,t} = \rho\lambda_{1,t} - \dot{\lambda}_{1,t}, \tag{52}$$

$$\frac{\partial H_t}{\partial K_t} = -\lambda_{1,t}\frac{1}{\theta}\left(1-\frac{1}{\sigma N_t}\right)\alpha(1-\alpha) K_t^{\alpha-2} L^{1-\alpha} C_t + \lambda_{2,t}\alpha K_t^{\alpha-1} L^{1-\alpha} = \rho\lambda_{2,t} - \dot{\lambda}_{2,t}, \tag{53}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{1,t} C_t = 0,$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{2,t} K_t = 0.$$

$$\lambda_{1,0} = 0.$$

In the steady state equilibrium, $\dot{C} = 0$. From (21), the following equation holds in the steady state:

$$\left(1-\frac{1}{\sigma N}\right)\alpha K^{\alpha-1} L^{1-\alpha} = \rho. \tag{54}$$

[7] A government regulates the entry of firms in the case of excess entry. On the other hand, in the case of insufficient entry, a government gives entrants subsidies to enhance the entry of firms by imposing lump-sum taxes on households.

In addition, because $\dot{\lambda}_2 = 0$ in the steady state, we obtain $-\lambda_1\lambda_2^{-1}\theta^{-1}\left[1-(\sigma N)^{-1}\right]\alpha(1-\alpha)K^{\alpha-2}L^{1-\alpha}C + \alpha K^{\alpha-1}L^{1-\alpha} = \rho$. From this equation and (51), we can obtain

$$-(1-\alpha)\phi\sigma\frac{N^2}{K}\left(1-\frac{1}{\sigma N}\right) + \alpha K^{\alpha-1}L^{1-\alpha} = \rho. \tag{55}$$

Moreover, we examine whether there exist optimal paths converging to the steady state. Concerning the stability of the steady state, according to Appendix 2, we can show that there exist parameter values such that the steady state is stable.

6 The comparison of the free entry equilibrium and the second-best equilibrium

In this section, we compare the number of firms in the steady state under the free entry equilibrium with the number of firms in the steady state under the second-best policy. From (21) and (23) in Section 3, N and K in the steady state under the free entry equilibrium satisfy the following equations:

$$\left(1-\frac{1}{\sigma N}\right)\alpha K^{\alpha-1}L^{1-\alpha} = \rho. \tag{56}$$

$$\phi\sigma N^2 = K^{\alpha}L^{1-\alpha}. \tag{57}$$

From (54) and (55) in Section 4, N and K in the steady state under the second-best policy satisfy the following equations:

$$\left(1-\frac{1}{\sigma N}\right)\alpha K^{\alpha-1}L^{1-\alpha} = \rho. \tag{58}$$

$$-(1-\alpha)\phi\sigma\frac{N^2}{K}\left(1-\frac{1}{\sigma N}\right) + \alpha K^{\alpha-1}L^{1-\alpha} = \rho. \tag{59}$$

Moreover, we can rewrite (55) by using (54) as follows:

$$\frac{1-\alpha}{\alpha}\phi\sigma(\sigma N-1)N^2 = K^{\alpha}L^{1-\alpha}. \tag{60}$$

In order to compare the number of firms in the steady state under the free entry equilibrium, N_{FE} with the number of firms in the steady state of the second-best policy, N_{SB}, we depict graphs of the relations that satisfy the above equations in the N-K space. First, call the relation that satisfies (56) and (58) the Steady State line. They can be depicted in the N-K plane as shown in Figure 4. Next, call the relation that satisfies (57) and the relation that satisfies (60) Free Entry line and Second-best line respectively. They can be depicted in the N-K plane as shown in Figure 4. The number of firms in the free entry equilibrium, N_{FE} is determined by the intersection of the Steady State line and the Free Entry line. In the same way, the number of firms in the second-best equilibrium, N_{SB} is determined by the intersection of the Steady State line and the Second-best line.

Using these figures, we have the following proposition:

Proposition 4. *If* $\alpha^2\left[(1-\alpha)^2\sigma\frac{L}{\phi}\right]^{\frac{1-\alpha}{\alpha}} < \rho < \frac{\alpha^2}{2-\alpha}\left\{\left[\frac{2(1-\alpha)}{2-\alpha}\right]^2\left(\frac{\sigma L}{\phi}\right)\right\}^{\frac{1-\alpha}{\alpha}}$*, the number of firms in the free entry equilibrium is less than the number of firms in the second-best equilibrium. In other words, the free entry equilibrium leads to insufficient entry. If* $\alpha^2\left[(1-\alpha)^2\sigma\frac{L}{\phi}\right]^{\frac{1-\alpha}{\alpha}} > \rho$*, the number of firms in the free entry equilibrium is more than the number of firms in the second-best equilibrium. In other words, the free entry equilibrium leads to excess entry.*

Proof. First, we assume that the values of the parameters satisfy (28), that is,

$$\frac{\alpha^2}{2-\alpha}\left\{\left[\frac{2(1-\alpha)}{2-\alpha}\right]^2\left(\frac{\sigma L}{\phi}\right)\right\}^{\frac{1-\alpha}{\alpha}} > \rho. \tag{61}$$

Inspecting Figures 4 and 5, we find that $N_{FE} > N_{SB}$ when the intersection of the Free Entry line and the Second-best line is above the Steady State line. Let $(\hat{K}, \hat{N})$ denote the intersection of the Free Entry line and the Second-best line. From (21), (57) and (60), we obtain

$$\hat{K} = \left[\frac{\phi}{\sigma(1-\alpha)^2 L^{1-\alpha}}\right]^{\frac{1}{\alpha}}, \quad \hat{N} = \frac{1}{\sigma(1-\alpha)}. \tag{62}$$

From (62) and (21), the condition for $N_{FE} < N_{SB}$ is given by

$$\alpha^2\left[(1-\alpha)^2\sigma\frac{L}{\phi}\right]^{\frac{1-\alpha}{\alpha}} < \rho \tag{63}$$

Here, all the values of the parameters must satisfy (28). Therefore we derive the values of the parameters that satisfy both (28) and (63). Such parameters meet the following inequalities:

$$\alpha^2\left[(1-\alpha)^2\sigma\frac{L}{\phi}\right]^{\frac{1-\alpha}{\alpha}} < \rho < \frac{\alpha^2}{2-\alpha}\left\{\left[\frac{2(1-\alpha)}{2-\alpha}\right]^2\left(\frac{\sigma L}{\phi}\right)\right\}^{\frac{1-\alpha}{\alpha}}. \tag{64}$$

To show that the values that satisfy (64) exist, we derive the condition that the first term of (64) is less than the third term of (64).

$$\alpha^2 \left[(1-\alpha)^2\sigma\frac{L}{\phi}\right]^{\frac{1-\alpha}{\alpha}} < \frac{\alpha^2}{2-\alpha}\left\{\left[\frac{2(1-\alpha)}{2-\alpha}\right]^2\left(\frac{\sigma L}{\phi}\right)\right\}^{\frac{1-\alpha}{\alpha}},$$
$$2-\alpha < 4^{1-\frac{1}{2-\alpha}}. \tag{65}$$

This inequality holds strictly true for all $\alpha \in (0,1)$ and therefore we have shown that there exist the values of $\sigma,\ \phi$, and, L that satisfy (64) for all $\alpha \in (0,1)$. In other words, insufficient entry can occur for any value of $\alpha \in (0,1)$.

□

Proposition 4, means that in contrast to the results of the partial equilibrium analyses, the number of entrants under free entry may be less than the second-best number of entrants in dynamic general equilibrium environments; that is, insufficient entry can occur. In the static general equilibrium analysis of Konishi et al.(1990), the existence of the factor intensity twist between the oligopolistic and the competitive sectors plays a crucial role in determining whether excess entry occurs. It should be noted that this insufficient entry occurs although there is no such factor intensity twist in the present framework, that is, the dynamic general equilibrium analysis.

Why may the number of entrants under the second-best equilibrium be larger than one under the free entry equilibrium? The reason is as follows: an increase in the number of entrants raises the intensity of competition and thus increases the demand of capital. This raises the rental of capital and thus promotes capital accumulation. When this positive effect dominates any other negative effect, an increase in the number of entrants improves social welfare. It should be noted that this positive effect of increasing the number of entrants appears only when considering capital accumulation.

Moreover, Proposition 4 shows that higher elasticity of substitution among intermediate goods, larger population, lower entry cost, and a lower discount rate tends to lead the free entry equilibrium to excess entry. Here, we let L and α be constant, and focus our attention on the elasticity of substitution, σ and the fixed cost, ϕ, and then investigate what values of σ and ϕ lead to excess entry or insufficient entry. We depict the line of the values of σ and ϕ that satisfy the equality of (28) in the ϕ-σ space as depicted in Figure 6. The region below this line corresponds to the values of ϕ and σ that do not satisfy (28); that is, an economy with such values has an interest rate which is too low, and no steady states. In the same way, we depicted the values of σ and ϕ that satisfy the equality of (63) in the ϕ-σ space. The region below this line corresponds to the values of ϕ and σ that satisfy (63); that is, insufficient entry occurs in an economy with such values. On the other hand, in an economy with the values in the region above the line, excess entry occurs.

7 Conclusion

This paper have presented a welfare analysis of free entry equilibrium in dynamic general equilibrium environments with Cournot competition.

First, in section 4, we have shown that a marginal decrease in the number of firms at the free entry equilibrium improves social welfare. This result is consistent with one of the partial equilibrium analyses, that is, the excess entry theorem, which is shown by Perry (1984), Mankiw and Whinston (1986) and Suzumura and Kiyono (1987).

Second, in sections 5 and 6, we have assumed that a government can control the number of entrants intertemporally so as to maximize the level of social welfare and derived the number of entrants in the steady state under this second-best equilibrium. Consequently, we have shown that the number of entrants under free entry may be less than the second-best number of entrants, that is, free entry may lead to insufficient entry in contrast to the result of partial equilibrium analyses. Why may an increase in the number of entrants improve social welfare in a dynamic model with capital accumulation? The reason is as follows: an increase in the number of entrants raises the intensity of competition and thus increases the demand of capital. This raises the rental of capital and thus promotes capital accumulation. When this positive effect dominates any other negative effect, an increase in the number of entrants improves social welfare.

The results of this paper indicate that capital accumulation plays an important role when considering deregulation policies. A government must take care to check not only whether its policies promote entry into industries but also whether these policies stimulate capital formation.

A Appendix 1

In this appendix, following Futagami et al.(1993), we use Judd's (1982) method to prove Proposition 1. First, we examine how a marginal decrease in the number of entrants affects initial consumption. Differentiating (31) and (32) with respect to m in the neighborhood of the steady state, we obtain

$$\begin{bmatrix} \dot{K}_m \\ \dot{C}_m \end{bmatrix} = \begin{bmatrix} F_K(K^*, m) & -1 \\ \frac{C^*}{\theta} r_K(K^*, m) & 0 \end{bmatrix} \begin{bmatrix} K_m \\ C_m \end{bmatrix} + \begin{bmatrix} F_m(K^*, m) \\ \frac{C^*}{\theta} r_m(K^*, m) \end{bmatrix}. \tag{66}$$

Here, we let $\widetilde{K}(s)$ and $\widetilde{C}(s)$ denote the Laplace transformation of K_t and C_t, respectively; e.g. $\widetilde{K}(s) \equiv \int_0^\infty K_t e^{-st} dt$. Taking the Laplace transformations of (66), we obtain

$$\begin{bmatrix} s\widetilde{K}_m \\ s\widetilde{C}_m - C_m(0, m) \end{bmatrix} = \begin{bmatrix} F_K(K^*, m) & -1 \\ \frac{C^*}{\theta} r_K(K^*, m) & 0 \end{bmatrix} \begin{bmatrix} \widetilde{K}_m \\ \widetilde{C}_m \end{bmatrix} + \begin{bmatrix} \frac{1}{s} F_m(K^*, m) \\ \frac{1}{s}\frac{C^*}{\theta} r_m(K^*, m) \end{bmatrix}.$$

where we use the fact that $K_m(0, m) = 0$. We can reduce this to

$$\begin{bmatrix} \widetilde{K}_m \\ \widetilde{C}_m \end{bmatrix} = (sI - J)^{-1} \begin{bmatrix} \frac{1}{s} F_m(K^*, m) \\ C_m(0, m) + \frac{1}{s}\frac{C^*}{\theta} r_m(K^*, m) \end{bmatrix}. \tag{67}$$

Here, we can rewrite the inverse of the matrix, $(sI - J)^{-1}$, as follows:

$$(sI - J)^{-1} = \frac{1}{\det(sI - J)} \text{adj}(sI - J) = \frac{1}{(s - \nu_1)(s - \nu_2)} \text{adj}(sI - J), \tag{68}$$

where adj$(sI - J)$ is the adjoint matrix of $(sI - J)$, that is,

$$\text{adj}(sI - J) = \begin{bmatrix} s - J_{22} & J_{12} \\ J_{21} & s - J_{11} \end{bmatrix}.$$

Substituting (68) into (67), we can rewrite (67) as follows:

$$\begin{bmatrix} \widetilde{K}_m \\ \widetilde{C}_m \end{bmatrix} = \frac{\begin{bmatrix} s - J_{22} & J_{12} \\ J_{21} & s - J_{11} \end{bmatrix} \begin{bmatrix} \frac{1}{s} F_m(K^*, m) \\ C_m(0, m) + \frac{1}{s}\frac{C^*}{\theta} r_m(K^*, m) \end{bmatrix}}{(s - \nu_1)(s - \nu_2)}. \tag{69}$$

The solutions for $K_m(t, m)$ and $C_m(t, m)$ must be bounded. If $s = \nu_2 > 0$, $\det(sI - J) = 0$ and the denominator of the left-hand side of (69) becomes 0. Therefore, the numerator of the left-hand side of (69) must also be 0 in order to obtain bounded solutions. Therefore the following condition must be satisfied:

$$\frac{\nu_2 - J_{22}}{\nu_2} F_m(K^*, m) + J_{12}\left[C_m(0, m) + \frac{1}{\nu_2}\frac{C^*}{\theta} r_m(K^*, m)\right] = 0. \tag{70}$$

Because $J_{22} = 0$ and $J_{12} = -1$, we obtain

$$C_m(0, m) = F_m(K^*, m) - \frac{1}{\nu_2}\frac{C^*}{\theta} r_m(K^*, m) > 0, \tag{71}$$

where we use $F_m(K^*, m) > 0$ and $r_m(K^*, m) < 0$. Consequently, we have shown that a marginal decrease in number of entrants raises initial consumption. Next, we examine how a marginal decrease in the number of entrants affects the lifetime utility of the representative agent. In the rest of the paper, we assume that $\theta = 1$ for simplicity. The effect of the decrease in the number of entrants on $K(t, m)$ is given by

$$K_m(t, m) = \left(1 - e^{\nu_1 t}\right) {K_m}^*, \tag{72}$$

where we use $K_m(0, m) = 0$. Substituting (72) into (48) yields

$$\begin{aligned} \frac{dU(m)}{dm} &= \frac{1}{\rho}\frac{C_m(0, m)}{C(0, m)} \\ &\quad + \int_0^\infty \int_0^t \{[r_K(K^*, m){K_m}^* + r_m(K^*, m)] \\ &\quad - r_K(K^*, m){K_m}^* e^{\nu_1 v}\} dv \, e^{-\rho t} dt, \end{aligned} \tag{73}$$

Because we assume that the economy is initially in the steady state, we obtain

$$r_K(K^*, m){K_m}^* + r_m(K^*, m) = 0. \tag{74}$$

Substituting (74) into (73), we obtain

$$\frac{dU(m)}{dm} = \frac{1}{\rho}\left[\frac{C_m(0, m)}{C(0, m)} + \frac{r_m(K^*, m)}{\rho - \nu_1}\right]. \tag{75}$$

The first term of the right-hand side of (75) represents the effect of a decrease in the number of entrants on the initial consumption, which is positive. The second term of the right-hand side of (75) represents the effect of a decrease in the number of entrants on the growth rate of consumption, which is negative.

Substituting (71) and $C^* = F(K^*, m)$ into (75) yields

$$\frac{dU(m)}{dm} = \frac{1}{\rho}\left[\frac{F_m(K^*, m)}{F(K^*, m)} + \left(\frac{1}{\rho - \nu_1} - \frac{1}{\nu_2}\right) r_m(K^*, m)\right]. \tag{76}$$

Substituting (37) and (38) into (76), we obtain

$$\frac{dU(m)}{dm} = \frac{-N_m(K^*,m)}{\rho}\left\{\frac{\phi}{F(K^*,m)} - \left(\frac{1}{\rho-\nu_1} - \frac{1}{\nu_2}\right)\frac{\alpha K^{*\alpha-1}L^{1-\alpha}}{\sigma[N(K^*,m)]^2}\right\}. \quad (77)$$

In order to examine whether a marginal decrease in the number of entrants improves social welfare when the economy is in the free entry equilibrium, we substitute $m = 0$ into (77). Because we assume that the economy is initially in the free entry equilibrium, from (21) and (23), K^* and N^* satisfy

$$\left(1 - \frac{1}{\sigma N^*}\right)\alpha K^{*\alpha-1}L^{1-\alpha} = \rho, \quad (78)$$

$$\phi\sigma N^{*2} = K^{*\alpha}L^{1-\alpha}. \quad (79)$$

From (78), we obtain $\alpha K^{*\alpha-1}L^{\alpha-1} = \frac{\rho\sigma N^*}{\sigma N^*-1}$. Moreover, using (79), we obtain $F(K^*,m) = K^{*\alpha}L^{1-\alpha} - \phi N^* = \phi N^*(\sigma N^* - 1)$. Substituting these terms into (77), we obtain

$$\begin{aligned}\frac{dU(m)}{dm}\Big|_{m=0} &= -\frac{N_m(K^*,0)}{\rho N^*(\sigma N^*-1)}\left[1 - \left(\frac{1}{\rho-\nu_1} - \frac{1}{\nu_2}\right)\rho\right] \\ &= -\frac{N_m(K^*,0)}{\rho N^*(\sigma N^*-1)}\frac{\rho^2 - \nu_1(\nu_2+\rho)}{\nu_2(\rho-\nu_1)} \\ &> 0.\end{aligned}$$

where we use $\nu_1 < 0 < \nu_2$. This proves Eq. (49).

B Appendix 2

In this appendix, we prove that there exist parameter values such that the steady state is stable.

By linearizing the system of the differential equations, we obtain the following:

$$\begin{pmatrix} \dot{k} \\ \dot{c} \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{pmatrix} = A \begin{pmatrix} k - k^* \\ c - c^* \\ \lambda_1 - \lambda_1^* \\ \lambda_2 - \lambda_2^* \end{pmatrix},$$

where the entries, a_{ij} of the Jacobian matrix, A are given by

$$a_{11} = \alpha k^{\alpha-1} + \tfrac{(1-\alpha)\widetilde{\phi}N}{2k}, \quad a_{12} = -\left(1 + \tfrac{\widetilde{\phi}N}{2c}\right), \quad a_{13} = \tfrac{\widetilde{\phi}N}{2\lambda_1}, \quad a_{14} = -\tfrac{\widetilde{\phi}N}{2\lambda_2},$$

$$a_{21} = -\frac{c}{\theta}\left(1 - \frac{1}{2\sigma N}\right)(1-\alpha)\alpha k^{\alpha-2}, \quad a_{22} = \frac{1}{2\sigma N\theta}\alpha k^{\alpha-1},$$
$$a_{23} = -\frac{c}{\theta}\frac{1}{2\sigma N}\frac{1}{\lambda_1}\alpha k^{\alpha-1}, \quad a_{24} = \frac{c}{\theta}\frac{1}{2\sigma N}\frac{1}{\lambda_2}\alpha k^{\alpha-1},$$

$$a_{31} = \lambda_1(1-\alpha)\alpha k^{\alpha-2} - \frac{c}{\theta}\lambda_2\left[\frac{1}{2\sigma N}\frac{1}{\lambda_1}(1-\alpha)\alpha k^{\alpha-2} + \left(1 - \frac{1}{\sigma N}\right)(2-\alpha)(1-\alpha)\alpha k^{\alpha-3}\right]$$
$$a_{32} = \frac{1}{\theta}\lambda_2\left(1 - \frac{1}{2\sigma N}\right)(1-\alpha)\alpha k^{\alpha-2}, \quad a_{33} = \rho - \alpha k^{\alpha-1} - \frac{c}{\theta}\frac{1}{2\sigma N}\frac{\lambda_2}{\lambda_1}(1-\alpha)\alpha k^{\alpha-2},$$
$$a_{34} = \frac{c}{\theta}\left(1 - \frac{1}{2\sigma N}\right)(1-\alpha)\alpha k^{\alpha-2}$$

$$a_{41} = \frac{\lambda_2}{\theta}\left(1 - \frac{1}{2\sigma N}\right)(1-\alpha)\alpha k^{\alpha-2}, \quad a_{42} = -\theta c^{-\theta-1} - \frac{1}{\theta}\frac{\lambda_2}{c}\frac{1}{2\sigma N}\alpha k^{\alpha-1},$$
$$a_{43} = 1 + \frac{1}{\theta}\frac{\lambda_2}{\lambda_1}\frac{1}{2\sigma N}\alpha k^{\alpha-1}, \quad a_{44} = \rho - \frac{1}{\theta}\frac{1}{2\sigma N}\alpha k^{\alpha-1},$$

where $\widetilde{\phi} \equiv \phi/L$. From (49), (50), and (53)-(55), the following conditions hold at the steady state:

$$\left(1 - \frac{1}{\sigma N}\right)\alpha k^{\alpha-1} = \rho, \tag{80}$$

$$k^{\alpha} = \widetilde{\phi}N + c, \tag{81}$$

$$\alpha k^{\alpha-1} - \rho = \frac{c}{\theta}\left(1 - \frac{1}{\sigma N}\right)\frac{\lambda_2}{\lambda_1}(1-\alpha)\alpha k^{\alpha-2}, \tag{82}$$

$$\lambda_1 + \rho\lambda_2 = c^{-\theta}, \tag{83}$$

$$\frac{c}{\theta}\frac{1}{\sigma N}\alpha k^{\alpha-1} = \frac{\lambda_1}{\lambda_2}\widetilde{\phi}N. \tag{84}$$

From (80) and (82), we obtain

$$1 = \frac{c}{\theta}(\sigma N - 1)\frac{1-\alpha}{k}\frac{\lambda_2}{\lambda_1}. \tag{85}$$

From (80) and (84), we obtain

$$\rho\frac{c}{\theta}\frac{1}{(\sigma N - 1)} = \frac{\lambda_1}{\lambda_2}\widetilde{\phi}N. \tag{86}$$

These two equations (85) and (86) result in

$$\rho k = (1-\alpha)\widetilde{\phi}N(\sigma N - 1)^2. \tag{87}$$

This equation and (80) determine the steady state values of k and N.

From (83) and (86), we obtain

$$\lambda_1 = \frac{c^{-\theta}c}{\theta\widetilde{\phi}(\sigma N - 1) + c}, \tag{88}$$

$$\lambda_2 = \frac{c^{-\theta}\theta\widetilde{\phi}(\sigma N-1)}{\rho[\theta\widetilde{\phi}(\sigma N-1)+c]}. \tag{89}$$

By using (80). (81), and (87), we obtain

$$c = \widetilde{\phi}N\left[\frac{1-\alpha}{\alpha}\sigma N(\sigma N-1)-1\right]. \tag{90}$$

Based on (80), (87), (88), and (89), we can rewrite the Jacobian matrix, A as follows:

$$\begin{pmatrix} \rho\frac{\sigma N}{\sigma N-1}+\frac{\rho}{2(\sigma N-1)^2} & -1-\frac{\widetilde{\phi}N}{2c} & \frac{[\theta\widetilde{\phi}(\sigma N-1)+c]\widetilde{\phi}N}{2c^{-\theta}c} & -\rho\frac{[\theta\widetilde{\phi}(\sigma N-1)+c]}{2c^{-\theta}\theta(\sigma N-1)} \\ -\rho^2\frac{c}{\theta}\frac{2\sigma N-1}{2\widetilde{\phi}N(\sigma N-1)^3} & \rho\frac{1}{\theta}\frac{1}{2(\sigma N-1)} & -\rho\frac{[\theta\widetilde{\phi}(\sigma N-1)+c]}{2c^{-\theta}\theta(\sigma N-1)} & \rho^2\frac{[\theta\widetilde{\phi}(\sigma N-1)+c]c}{2c^{-\theta}\theta^2\widetilde{\phi}N(\sigma N-1)^2} \\ a_{31} & \rho\frac{c^{-\theta}}{[\theta\widetilde{\phi}(\sigma N-1)+c]}\frac{2\sigma N-1}{2(\sigma N-1)^2} & -\rho\frac{2\sigma N-1}{2(\sigma N-1)^2} & \rho^2\frac{c}{\theta}\frac{2\sigma N-1}{2\widetilde{\phi}N(\sigma N-1)^3} \\ \rho\frac{c^{-\theta}}{[\theta\widetilde{\phi}(\sigma N-1)+c]}\frac{2\sigma N-1}{2(\sigma N-1)^2} & a_{42} & 1+\frac{\widetilde{\phi}N}{2c} & \rho-\rho\frac{1}{\theta}\frac{1}{2(\sigma N-1)} \end{pmatrix},$$

where $a_{31} = \rho^2\frac{1}{\widetilde{\phi}(\sigma N-1)^3}\frac{c^{-\theta}c}{[\theta\widetilde{\phi}(\sigma N-1)+c]}\left(\sigma-\frac{1}{N}\frac{2-\alpha}{1-\alpha}\right)-\frac{\rho}{2(\sigma N-1)^2}$ and $a_{42} = -\theta c^{-\theta-1}-\frac{c^{-\theta-1}\widetilde{\phi}N}{2[\theta\widetilde{\phi}(\sigma N-1)+c]}$.

We next calculate the determinant of this matrix, $\det A$. Multiplying the components in the third column by $\rho\frac{c}{\theta\widetilde{\phi}N(\sigma N-1)}$ and adding them to the components in the fourth column, we obtain the following:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & \rho\left(1+\frac{c}{\theta\widetilde{\phi}N(\sigma N-1)}\right) \end{vmatrix}$$

$$= \rho\left(1+\frac{c}{\theta\widetilde{\phi}N(\sigma N-1)}\right)\begin{vmatrix} \rho\frac{\sigma N}{\sigma N-1}+\frac{\rho}{2(\sigma N-1)^2} & -1-\frac{\widetilde{\phi}N}{2c} & \frac{[\theta\widetilde{\phi}(\sigma N-1)+c]\widetilde{\phi}N}{2c^{-\theta}c} \\ -\rho^2\frac{c}{\theta}\frac{2\sigma N-1}{2\widetilde{\phi}N(\sigma N-1)^3} & \frac{\rho}{\theta}\frac{1}{2(\sigma N-1)} & -\rho\frac{[\theta\widetilde{\phi}(\sigma N-1)+c]}{2c^{-\theta}\theta(\sigma N-1)} \\ a_{31} & \frac{\rho c^{-\theta}}{[\theta\widetilde{\phi}(\sigma N-1)+c]}\frac{2\sigma N-1}{2(\sigma N-1)^2} & -\rho\frac{2\sigma N-1}{2(\sigma N-1)^2} \end{vmatrix}.$$

Moreover, multiplying the components in the second column of the above 3×3 determinant by $\frac{[\theta\widetilde{\phi}(\sigma N-1)+c]}{c^{-\theta}}$ and adding them to the components in the third column, we obtain the following:

$$\rho\left(1+\frac{c}{\theta\widetilde{\phi}N(\sigma N-1)}\right)\begin{vmatrix} \rho\frac{\sigma N}{\sigma N-1}+\frac{\rho}{2(\sigma N-1)^2} & -1-\frac{\widetilde{\phi}N}{2c} & -\frac{[\theta\widetilde{\phi}(\sigma N-1)+c]}{c^{-\theta}} \\ -\rho^2\frac{c}{\theta}\frac{2\sigma N-1}{2\widetilde{\phi}N(\sigma N-1)^3} & \frac{\rho}{\theta}\frac{1}{2(\sigma N-1)} & 0 \\ a_{31} & \frac{\rho c^{-\theta}}{[\theta\widetilde{\phi}(\sigma N-1)+c]}\frac{2\sigma N-1}{2(\sigma N-1)^2} & 0 \end{vmatrix}$$

$$= -\rho\left(1+\frac{c}{\theta\widetilde{\phi}N(\sigma N-1)}\right)\frac{[\theta\widetilde{\phi}(\sigma N-1)+c]}{c^{-\theta}}\begin{vmatrix} -\rho^2\frac{c}{\theta}\frac{2\sigma N-1}{2\widetilde{\phi}N(\sigma N-1)^3} & \frac{\rho}{\theta}\frac{1}{2(\sigma N-1)} \\ a_{31} & \frac{\rho c^{-\theta}}{[\theta\widetilde{\phi}(\sigma N-1)+c]}\frac{2\sigma N-1}{2(\sigma N-1)^2} \end{vmatrix}.$$

Therefore, because the coefficient of the above 2×2 determinant takes a negative value, the sign of $\det A$ is determined by this 2×2 determinant. We denote this 2×2 determinant by Γ. For simplicity, we set $\theta = 1$. Γ becomes

$$\begin{aligned}
\Gamma &= -\rho^3 \frac{(2\sigma N-1)^2}{2\widetilde{\phi} N(\sigma N-1)^5} \frac{1}{[\widetilde{\phi}(\sigma N-1)+c]} - \rho \frac{1}{2(\sigma N-1)} a_{31} \\
&= -\frac{\rho}{4(\sigma N-1)^3} \left[\frac{\rho(2\sigma N-1)^2 + 2(\sigma N-1)\left(\sigma N - \frac{2-\alpha}{1-\alpha}\right)}{\widetilde{\phi} N(\sigma N-1)^2[\widetilde{\phi}(\sigma N-1)+c]} - 1 \right] \\
&= -\frac{\rho}{4(\sigma N-1)^3} \left[\frac{\rho(2\sigma N-1)^2 + 2(\sigma N-1)\left(\sigma N - \frac{2-\alpha}{1-\alpha}\right)}{(\widetilde{\phi} N)^2(\sigma N-1)^2[(\sigma N-1)\left(\frac{1-\alpha}{\alpha}\sigma N+1\right)-1]} - 1 \right],
\end{aligned}$$

where we have used (90). Therefore if the numerator of the term in the brace is larger than the denominator of this term, Γ takes a negative value and thus $\det A$ takes a positive value. Consequently, the system of the differential equation can be stable.[8] In order to examine this, we draw the graphs of the numerator and the denominator. Consider the case where those graphs are depicted as in Figure 6. The numerator is a quadratic function of N and takes a positive value of $N = 1/\sigma$. The denominator is a 6th order equation of N and becomes 0 at $N = 1/\sigma$. Consequently Γ takes positive values in the interval, $[1/\sigma,\ N_{max}]$.

[8] See Dockner and Feichtinger (1991) and Kemp, Long, and Shimomura (1993).

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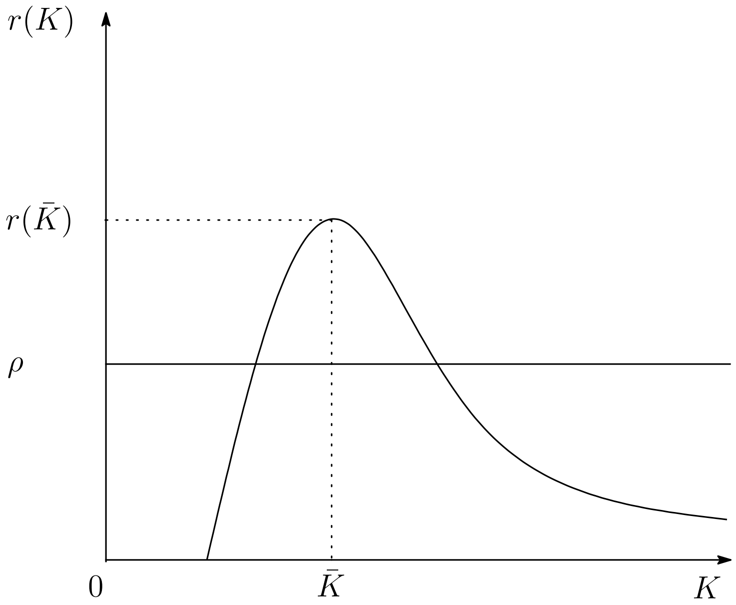

Figure 1: The interest rate

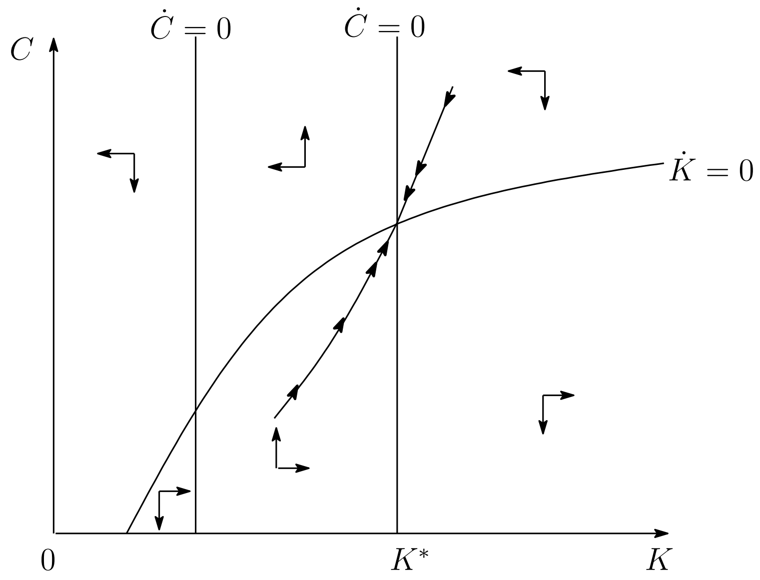

Figure 2: The phase diagram

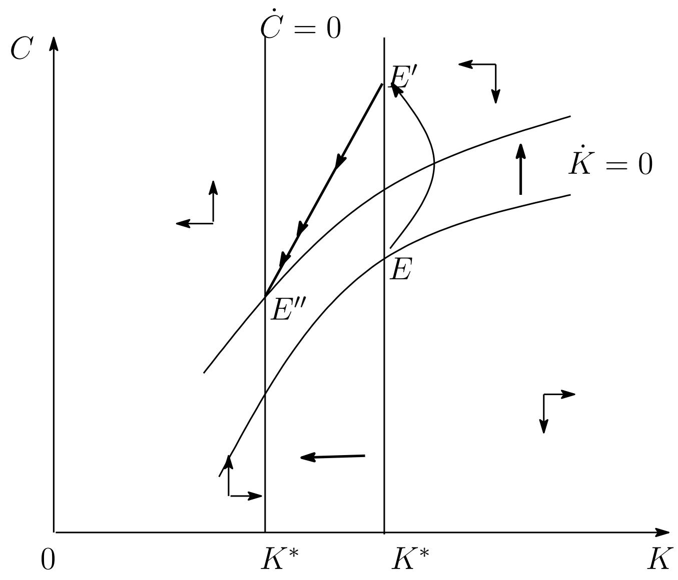

Figure 3: The effect of a marginal decrease of the number of entrants

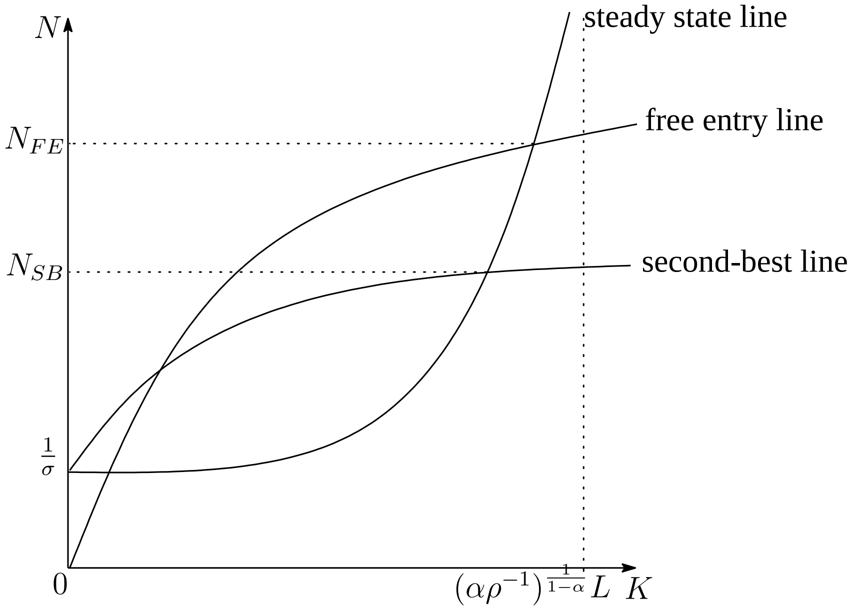

Figure 4: The case of excess entry

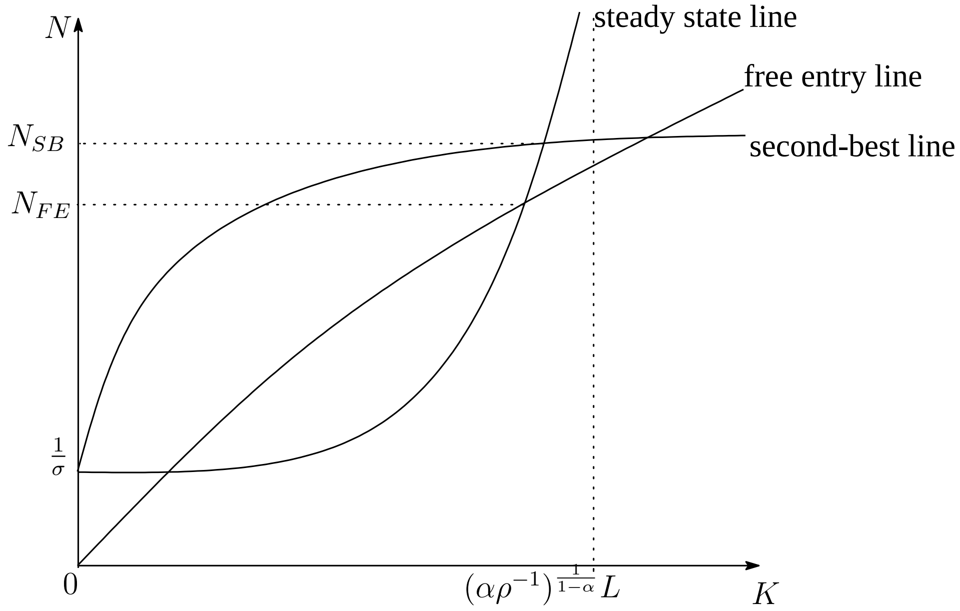

Figure 5: The case of insufficient entry

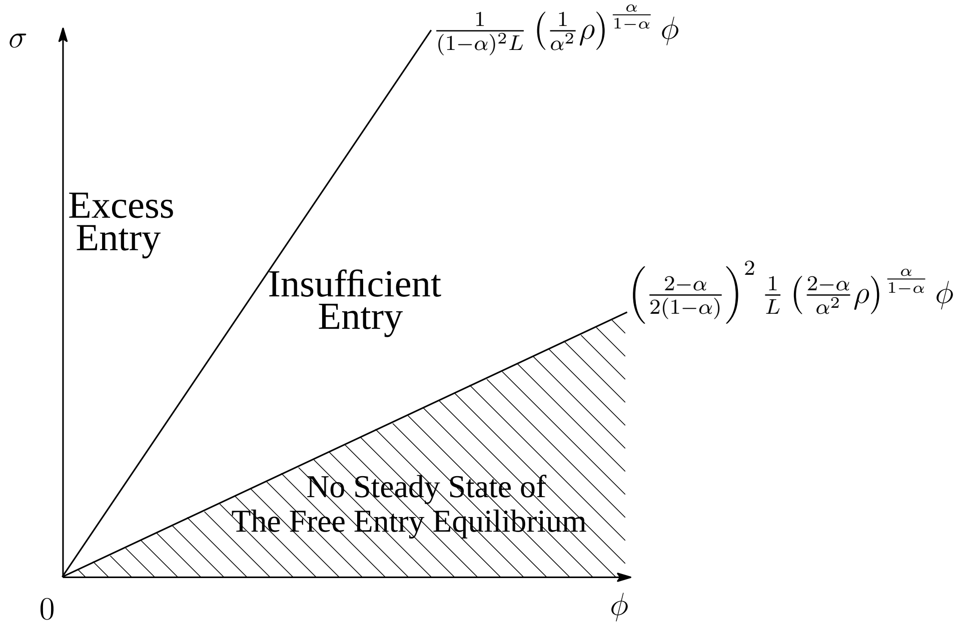

Figure 6: The values of the parameters

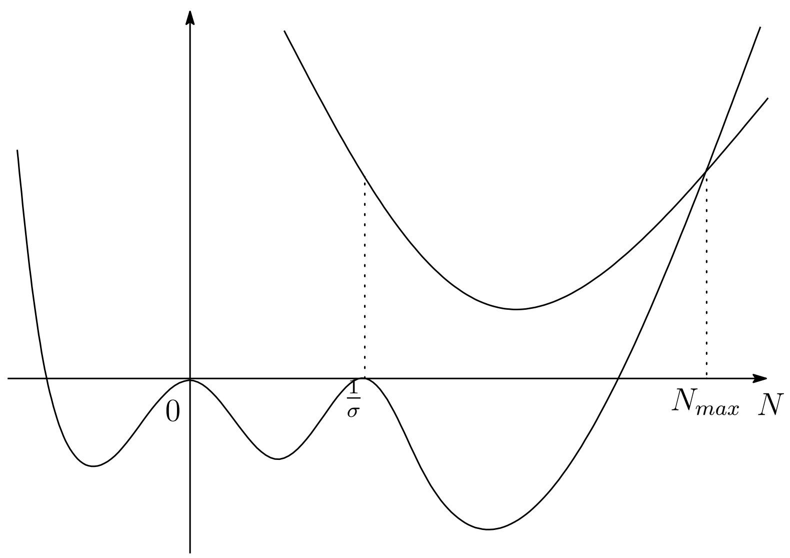

Figure 7: The values of N when the second-best equilibrium is stable