

# The Analysis of Forward-looking Monetary Policy in a SVAR Framework

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*Abstract:* This paper analyzes forward-looking monetary policy rules in structural VAR's. First, an approach for modeling a monetary policy which aims at a strict medium term inflation or output growth target is developed. Second, the ex ante inflation-output-growth volatility trade-off for a forward-looking policy aiming at a convex combination these strategies is derived. Finally, an illustration of our approach using Swiss data is given.

*Keywords:* Structural VAR, forward-looking monetary policy, efficiency frontier

*JEL classification:* E52; E58

First draft June 2004

Revised March 2005

## 1. Introduction

In the last ten years forecast oriented monetary policy strategies became increasingly important. Most central banks now base current monetary policy implicitly or explicitly on a forecast of inflation and other key macroeconomic variables and in this way take into account the lags in the effects of monetary policy. This procedure is made very explicit by the inflation targeting strategy pioneered by the Bank of England, which aims at an inflation with a horizon one to two years. The same applies to the new monetary policy framework of the Swiss National Bank (SNB) adopted at the end of 1999. This framework replaced a money stock growth target by an inflation forecast over the next three years as the main indicator to guide monetary policy decisions. Besides the expected future inflation development, the medium term output growth forecast plays an important role. No central bank would hit its inflation target or keep medium term inflation in the stability range when this leads to a large deterioration of output growth performance. The trade-off between inflation and output growth is mostly analyzed by using Taylor rules in DSGE models or traditional macroeconometric models. This paper proposes an alternative approach that allows the analysis of forward-looking monetary policy in structural VARs, which are an often used tool to estimate the dynamic effects of monetary policy. First, an approach for modeling a monetary policy which aims at a strict medium term inflation or output growth target is developed. Second, the ex ante inflation-output-growth volatility trade-off for a forward-looking policy aiming at a convex combination of these strategies is derived. Finally, an illustration of our approach is given using Swiss data.

## 2. A SVAR Analysis of Monetary Policy

In this section, we develop a method for the analysis of monetary policy in a SVAR framework. We proceed on the assumption that the VAR model includes a vector of quarterly changes in log output, short term interest rate, log real money stock and log price level:  $X_t' = (\Delta \log y_t, \Delta r_t, \Delta \log(m_t / p_t), \Delta \log p_t)$ . This framework assuming I(1) level series and no cointegration is the specification we adopted in our empirical illustration, but many other models differing with respect to number of variables and order of integration are possible. There are only two restrictions which are fundamentally necessary in our framework: first inflation and GDP-growth have to be included in the model and these variables should be stationary. Second, the VAR model has to be specified in a way that allows the identification of a monetary policy shock. The number of other variables included

and their integration and cointegration properties as well the nature of the short and or long run restrictions adopted is only important for the way the impulse response coefficients used in the following exercise are estimated. In our empirical applications we used long-run neutrality restrictions in order to identify structural shocks with variances normalized to 1 for the VAR defined above but other identification schemes could be applied as well. In our application we get impulse responses for three non-monetary policy shocks  $u_{1t}, u_{2t}, u_{3t}$  and the monetary policy shock  $u_{4t}$ . The former shocks can be interpreted as aggregate supply shock, IS shock and money demand shock, respectively. However, this interpretation is not essential, as only the joint effect of these three shocks on future inflation and output growth is needed with our approach.

Alternative strategies for monetary policy can now be analyzed by deriving conditional forecasts from the SVAR model. Specifically, we determine a sequence of policy shocks required to satisfy such conditions as an average inflation or output growth target over a two- or three-year period. Now consider a monetary policy strategy based on an average inflation forecast for the next  $K$  quarters. Take a monetary policy reacting symmetrically to positive and negative deviations from the inflation target  $\pi^*$  measured at a quarterly rate. For such a monetary policy, we get conditional forecasts in the following way: First, we define the expected deviation, as of time  $t$ , of the average inflation from its target for horizon  $K$

$$d_p(K, t) = K\pi^* - (E_t \log p_{t+K} - \log p_t), \quad (1)$$

Next, we have to determine the sequence of monetary shocks from  $t+1$  to  $t+K$  that leads to an expected average inflation which is equal to the target  $\pi^*$ . There is an infinite number of ways to calculate these shocks. Leeper and Zha (1999) show that policy shocks in VAR-models have to be modest and least disturbing in order not to violate the validity of the simulations. We therefore minimize the sum of the squared shocks subject to the restriction so that the average inflation rate is on target:

$$\sum_{i=1}^K u_{4t+i}^2 \rightarrow \min \quad s. \quad t. \quad \sum_{i=1}^K AA_{44}(K-i)u_{4t+i} - d_p(K, t) = 0 \quad (2)$$

$AA(j)$  is the 4x4 matrix of the impulse response, cumulated over  $j$  periods. Thus, the element 4,4 of this matrix gives the  $j$  period cumulated response of inflation to a monetary shock. The solution of this minimization problem is obtained as

$$u_{4t+i} = \frac{AA_{44}(K-i)}{\sum_{j=0}^{K-1} AA_{44}^2(j)} d_p(K,t) = g_{pi} d_p(K,t), \quad K = 1, \dots, K \quad (3)$$

Of course, we can apply the same approach using the average output growth as a target of monetary policy. Assume that the targeted output growth rate is denoted by  $\gamma^*$ . Again we define first the deviation of the unconditional forecast of the output growth from target  $K$  periods ahead as  $d_y(K,t) = K\gamma^* - (E_t \log y_{t+K} - \log y_t)$  and obtain the following optimal (in the least squares sense) policy shocks for  $t+1$  to  $t+K$ :

$$u_{4t+i} = \frac{AA_{14}(K-i)}{\sum_{j=0}^{K-1} AA_{14}^2(j)} d_y(K,t) = g_{yi} d_y(K,t), \quad K = 1, \dots, K \quad (4)$$

In what follows, we call a rule based exclusively on an inflation (output growth) target a *strategy of strict medium-term inflation (output growth) targeting*. Next, we consider the trade-off faced by monetary policy in the framework of our SVAR model. To this end, we consider the variability of inflation and output growth implied by different degrees of medium-term inflation and output growth targeting over the  $K$ -period horizon. To start with, we define a convex combination of the monetary policy shocks for strict medium-term inflation or output growth targeting derived in the last section:

$$u_{4t+i} = \alpha g_{pi} d_p(K,t) + (1-\alpha) g_{yi} d_y(K,t), \quad i = 1, \dots, K \quad (5)$$

This is the situation of a monetary policy board, where the decision is taken by consensus and according to the average preferences of its members. The board members have either the preference for pure inflation targeting or pure output growth targeting in the medium-term. The parameter  $\alpha$  thus reflects the fraction of the inflation hawks in the board and  $1-\alpha$  is the fraction of the inflation doves. This strategy corresponds to the goal of minimizing the weighted sum of the conditional variability of the expected medium term inflation and growth rate.

The proposed formalization of the monetary policy strategy is appropriate in the sense that it captures very well the focus of medium term orientation of monetary policy on inflation and GDP growth perspectives we see in many countries. Indeed, our experience in particular with Switzerland tells us that the discussion about monetary policy exactly focuses on

medium term inflation and GDP growth outlooks and a main issue is how much a GDP growth target should be taken into account. However, our framework for monetary policy analysis differs from the two approaches usually adopted in the literature, namely the Taylor rule and the optimizing framework. It may be argued that our framework lies somehow between these two approaches. Equation (5) shares some features with the Taylor rule framework: it implies, for instance, a negative value for the monetary policy shock when the medium term inflation forecast and/or GDP growth forecast is above target which leads to an increase of the interest rate. The weights  $\alpha$  and  $(1-\alpha)$  play a similar role as the coefficients of the inflation and output gap term in a Taylor rule, respectively. By contrast to an ad hoc Taylor rule, the policy reaction in our framework aims at a medium-term target taking fully into account the expected dynamic effects according to a fully specified empirical model. This feature of our model is shared with the optimizing framework. However, the latter is more general in the sense that a discounted weighted average of all future expected inflation and output variability is taken into account. With our approach policy makers are more “simple minded” as they care only about conditional medium term inflation and GDP growth volatility.

Another modeling issue is raised by the Lucas critique. Of course, our approach would be invalid if we consider changes in  $\alpha$  as regime changes. We see “regime changes” as a fundamentally new orientation of the goals of monetary policy as the transition from an environment of high and volatile inflation to a policy of keeping inflation low as many countries experienced in the last 15 years. In our model inflation (and a fortiori growth) always returns to its long-run equilibrium irrespective of the value adopted for  $\alpha$  and we consider such changes in  $\alpha$  therefore as “modest” and non-fundamental.

Now, let us see to what extent the planned sequence of monetary policy shocks is able to close the deviation of average inflation and output growth rate from their targets. The remaining gap  $rp_{t+K}$  (measured as deviation from target) corresponds to the impact of the policy shocks minus the forecasted deviation from target  $d_p(K,t)$  induced by the three non-policy shocks at time  $t$ . These effects can be calculated using the corresponding cumulated impulse responses and equation (5) as well as the formulae for the shocks of the two strict targeting strategies given by equation (3) and (4), respectively:

$$\begin{aligned}
rp_{t+K} &= \sum_{i=1}^K AA_{44}(K-i)u_{4t+i} - d_p(K,t) \\
rp_{t+K} &= \sum_{i=1}^K (1-\alpha) \frac{AA_{14}(K-i)AA_{44}(K-i)}{\sum_{j=0}^{K-1} AA_{14}^2(j)} d_y(K,t) + \alpha d_p(K,t) - d_p(K,t) \\
rp_{t+K} &= (1-\alpha)G_y d_y(K,t) - (1-\alpha)d_p(K,t) = -(1-\alpha)[d_p(K,t) - G_y d_y(K,t)] \tag{6}
\end{aligned}$$

$$\begin{aligned}
ry_{t+K} &= \sum_{i=1}^K AA_{14}(K-i)u_{4t+i} - d_y(K,t) \\
ry_{t+K} &= \sum_{i=1}^K \alpha \frac{AA_{14}(K-i)AA_{44}(K-i)}{\sum_{j=0}^{K-1} AA_{44}^2(j)} d_p(K,t) + (1-\alpha)d_y(K,t) - d_y(K,t) \\
ry_{t+K} &= \alpha G_p d_p(K,t) + (1-\alpha)d_y(K,t) - d_y(K,t) = -\alpha[d_y(K,t) - G_p d_p(K,t)] \tag{7}
\end{aligned}$$

For the economic interpretation of the two expressions derived above, we briefly consider the (expected) response of the inflation rate to the forecast-oriented monetary policy. If  $\alpha$  is equal to one (strict medium-term inflation targeting), we expect to hit the average inflation target exactly and the remaining gap  $rp_{t+K}$  is zero. Otherwise, we expect the medium-term inflation rate to deviate from the target. This deviation is determined by the expression  $[d_p(K,t) - G_y d_y(K,t)]$ . The second term in the brackets reflects the influence of the reaction of monetary policy to the output growth target. Of course, the latter effect depends on the corresponding deviation from the output target  $d_y$  and the co-movement of output and prices in reaction to a monetary policy shock  $G_y$ . In fact,  $G_y$  can be interpreted as the population regression coefficient of the  $(K-i)$ -period cumulated monetary policy response of inflation on the  $(K-i)$ -period cumulated monetary policy response of output growth. Thus, the size of the remaining gap due to not following a strict medium-term inflation targeting rule, depends on the sign and size of  $G_y$ , as well as on the difference in sign and size between  $d_p(K,t)$  and  $d_y(K,t)$ . The closer  $G_y$  is to 1 and the closer  $d_p(K,t)$  is to  $d_y(K,t)$ , the smaller the remaining gap  $rp_{t+K}$  will be. Of course, the interpretation for the remaining output growth gap  $ry_{t+K}$  corresponds exactly to that of the inflation gap.

The expected deviation of the  $K$ -period ahead log price and log output level from the target path is revised in period  $t$  according to the non-monetary policy shocks  $u_{1t}, u_{21t}, u_{3t}$  hitting the economy. For the sake of simplicity, we assume that we are in equilibrium in time  $t-1$  in the sense that we expect to hit both targets in the period  $t$  to  $t+K-1$ . However, the

non-monetary policy shock of period  $t$  lead to deviations from the targets, which in turn needs a revision of the planned monetary policy shock sequence. We first note that the deviations of the unconditional forecasts from their targets are given by:

$$d_p(K, t) = -\sum_{l \neq 4} (AA_{4l}(K) - AA_{4l}(0))u_{lt} \quad (8)$$

$$d_y(K, t) = -\sum_{l \neq 4} (AA_{1l}(K) - AA_{1l}(0))u_{lt} \quad (9),$$

where we assume that the policy shock is equal to zero as of time  $t$ . Note that the non-monetary policy shocks have an impact effect on prices and output which has to be subtracted as it has no influence on future inflation and growth.

Substituting these expressions into the responses of the price and output levels to the policy shocks

$$rp_{t+K} = (1 - \alpha) \sum_{l \neq 4} [AA_{4l}(K) - AA_{4l}(0) - G_y(AA_{1l}(K) - AA_{1l}(0))]u_{lt} \quad (10)$$

$$ry_{t+K} = \alpha \sum_{l \neq 4} [AA_{1l}(K) - AA_{1l}(0) - G_p(AA_{4l}(K) - AA_{4l}(0))]u_{lt} \quad (11)$$

and calculating conditional variances (given information of time  $t$ ) provides the following expressions:

$$Var_t(rp_{t+K}) = (1 - \alpha)^2 \sum_{l \neq 4} [AA_{4l}(K) - AA_{4l}(0) - G_y(AA_{1l}(K) - AA_{1l}(0))]^2 \quad (12)$$

$$Var_t(ry_{t+K}) = \alpha^2 \sum_{l \neq 4} [AA_{1l}(K) - AA_{1l}(0) - G_p(AA_{4l}(K) - AA_{4l}(0))]^2 \quad (13)$$

This implies a linear trade-off in the standard deviations of the output growth and inflation medium-term responses. Of course, this conditional variance is zero for inflation (output growth) when  $\alpha$  is 1 (0), otherwise both variances are larger than zero. The reader has to be reminded that these conditional variances are with respect to the  $K$ -period ahead expected values in  $t$ . Finally, it should be mentioned that the approach outlined above is straightforwardly modified to a policy aiming at an inflation or growth rate for some future period  $j$  to  $K$ , i.e. from 5 to 8 quarters: We have simply to replace  $AA_{il}(0)$  by  $AA_{il}(j)$  in equations (8) and (9).

Before turning to our empirical application, we have briefly to address the question why we do not incorporate the policy reaction in the VAR coefficients as it is done with

structural models instead of focusing on the policy shocks. This alternative approach is, however, in general not feasible in our VAR framework. In order to demonstrate this let us assume for the sake of simplicity that the vector  $X_t' = (\Delta \log y_t, \Delta r_t, \Delta \log(m_t / p_t), \Delta \log p_t)$  follows a VAR(1) model and that policy reacts to the expected medium term inflation and growth rate according to a Taylor rule with idiosyncratic policy shock  $\xi_t$ :

$$\begin{aligned} X_t &= AX_{t-1} + \varepsilon_t \\ \Delta r_t &= E_t(\gamma_1 \sum_{i=1}^K \Delta \log p_{t+i} + \gamma_2 \sum_{i=1}^K \Delta \log y_{t+i}) + \xi_t \end{aligned} \quad (14)$$

The cross equations restriction implied by the Taylor rule for the VAR coefficients can be derived as follows: Let us define the selection vectors for the interest rate, inflation and growth in the vector  $X$

$$\begin{aligned} h &= (0,1,0,0) \\ g_1 &= (0,0,0,1) \\ g_2 &= (1,0,0,0) \end{aligned} \quad (15)$$

Considering the time  $t$  expected value of the Taylor rule for period  $t+1$  with the future expected values replaced by their VAR forecast we obtain

$$hE_t X_{t+1} = \gamma_1 \sum_{i=1}^K g_1 A^i E_t X_{t+1} + \gamma_2 \sum_{i=1}^K g_2 A^i E_t X_{t+1} \quad (16)$$

This implies the following 4 cross equations restrictions depending on the policy reaction parameter for the VAR coefficients

$$h = \gamma_1 \sum_{i=1}^K g_1 A^i + \gamma_2 \sum_{i=1}^K g_2 A^i \quad (17)$$

If the Taylor equations coefficients are known we can test the restrictions given by equation (17) and can get restricted estimates of the VAR coefficient matrix using the approaches developed for VAR models of the term structure of interest rates<sup>1</sup>. However, this does not help in our context as we should find a reparametrization of the VAR coefficients involving the Taylor equation coefficients. This means that we have to find solutions of (17) for 4

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<sup>1</sup> An outline of this approach can be found in Kugler (1990), for instance.



elements of the  $A$ -matrix depending on the remaining 12 elements of  $A$  as well as of  $\gamma_1$  and  $\gamma_2$ . Thus the parameters of our restricted VAR-model consists of 12 unrestricted VAR coefficients as well as  $\gamma_1$  and  $\gamma_2$ . If this approach were possible we could estimate the model in this new form and then calculate the  $K$ -period forecasting variance decomposition for different values of  $\gamma_1$  and  $\gamma_2$  in order to show the trade-off between inflation and GDP-growth variability. However, the required reparametrization is not possible in general (except for  $K=1,2$ ), as polynomials of order  $K$  in the  $A$ -coefficients are involved on the right-hand side of (17) and no explicit solution for 4 elements of the  $A$ -matrix is therefore available.

### 3. An Application to Swiss Data

This section provides an illustration of our approach using Swiss data. Switzerland is an interesting example for our approach as there was no fundamental regime change since the breakdown of the Bretton Woods system in 1973. We consider the change in the SNB's monetary policy strategy mentioned in the introduction as non-fundamental as price stability remained the ultimate objective of Swiss monetary policy throughout the sample period. Moreover, although the SNB adjusted its operating procedures at the end of 1999, this modification did not cause a break in the time series process of the variables considered in our SVAR model: The level of bank reserves, used as the main policy instrument before 1999, and the interest rate on repos, the principal new instrument, are not included in our VAR system.

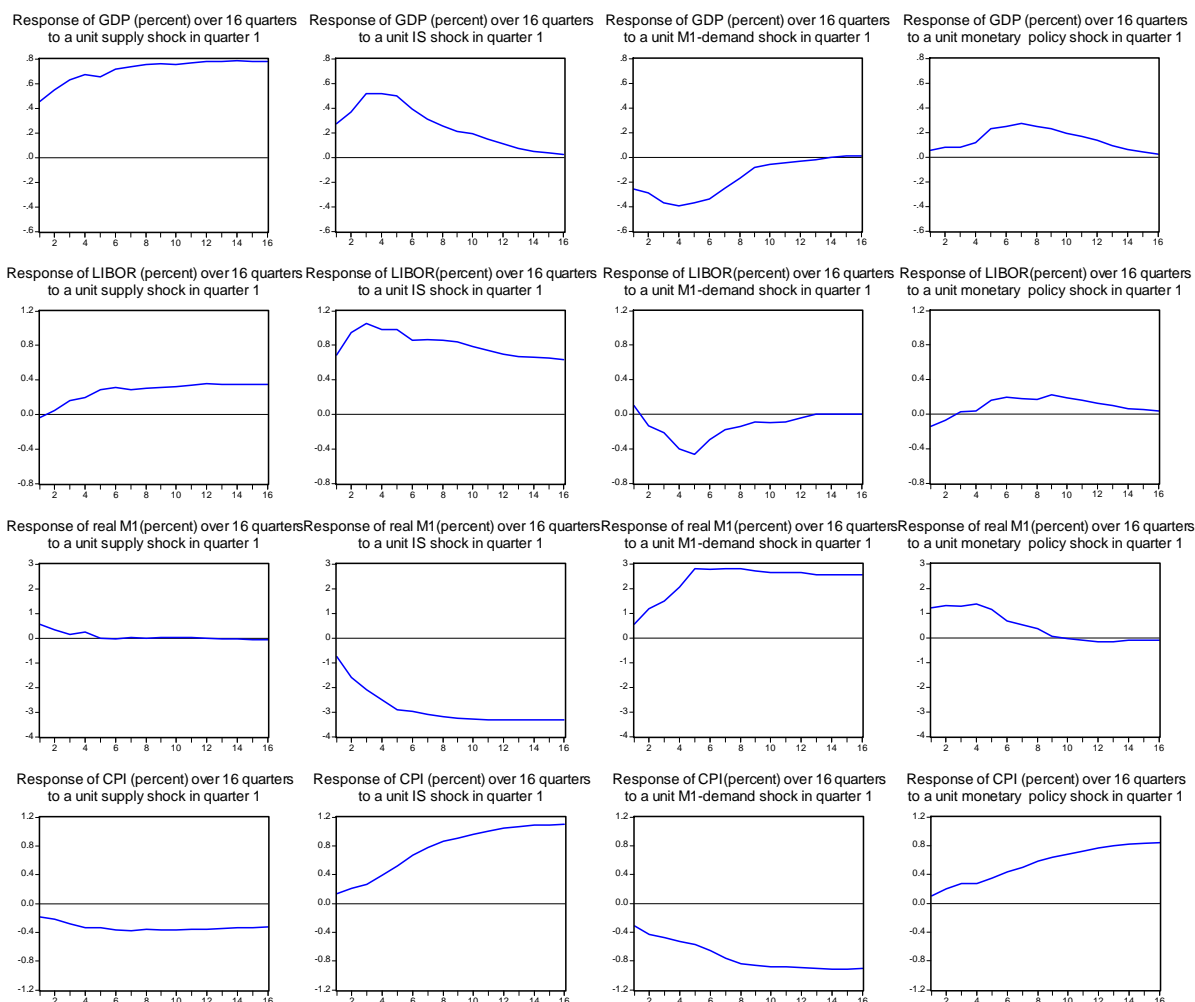
Figure 1 displays the impulse response estimate for a SVAR model for Switzerland corresponding to that used in Section 2, which is exactly identified by six long-run triangularity restrictions. The variable  $y$  is GDP in 1990 in Swiss francs,  $p$  denotes the consumer price index,  $m$  the money stock M1 and  $r$  the quarterly average of the three-month Swiss franc Libor rate of interest and the VAR lag-length is 5. In order to keep the model as lean as possible, the exchange rate is excluded from the vector  $X$ , as the transmission of monetary policy via the exchange rate is indirectly captured by the impulse responses of the VAR model. Explicit inclusion of the exchange rate would be necessary if this variable had influenced SNB behavior in a systematic way and, therefore, were required to identify a monetary policy shock. Exchange rate considerations played no important role for the bulk of the sample period, the exception being in particular the years 1978/79. Note that we do not

select a monetary aggregate with a stable long-run money demand function in levels such as M3. We are only interested in a money stock concept providing a lot of information for the identification of a monetary policy shock. The monetary base was not used as the introduction of the electronic Swiss Interbank Clearing System and the relaxation of banks' liquidity requirements strongly distorted even the rates of change in this aggregate. However, we should also mention that the results are robust with respect to the in- or exclusion of the money stock series: a three variables VAR without money produces essentially the same shape of the impulse responses to the monetary policy shock as the four variable system. Finally, we ought to mention that the standard unit-root and co-integration tests support the first-difference specification adopted in this paper.<sup>2</sup> More information on the application of SVAR models to Swiss data is provided by Kugler and Jordan (2004) and Kugler and Rich (2002).

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<sup>2</sup> The results with respect to the interest rate are ambiguous: we cannot reject the unit root hypothesis (ADF test) as well as the stationarity hypothesis (KPSS test). Thus, we proceed on the I(1) hypothesis which is more convenient in our framework for identification of a monetary policy shock. Of course, this implies that the real rate of interest rate is non-stationary what is clearly doubtful in the long-run. However, for the medium-term forecasting horizon considered in this paper this assumption is deemed acceptable.

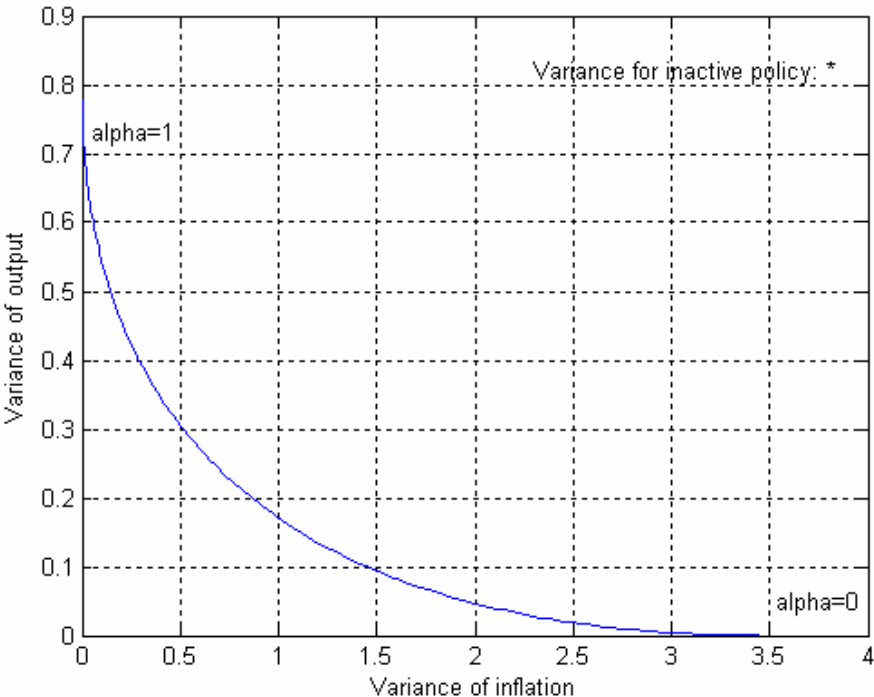
**Figure 1: Accumulated impulse responses of changes in log GDP, interest rate (3M-SFR-LIBOR), log real money stock (M1) and log prices (CPI) to structural shock, SVAR(5), quarterly data 1974/I-2002/IV**



In the first column we find the responses of all variables (in the order they appear in the vector  $X$ ) to the supply shock (Shock 1). Then, we have the responses to the IS-shock (Shock 2), to the money demand shock (Shock 3), and finally in the last column the response to the monetary policy shock (Shock 4), which is of most importance in the current context. First, there is evidence of a short-run negative liquidity effect on the interest rate extending over two quarters. The positive reaction in real GDP starts weakly and reaches its peak after five quarters and starts to peter out after another year. With respect to prices, it takes six quarters until a major positive effect is visible and 14 quarters are needed for full adjustment of prices. After about the fourth quarter, rising prices and inflation expectations cause the interest rate to overshoot temporarily its long-run equilibrium level.

Figure 2 shows the scatter diagram for the 8-period ahead inflation and output growth variance conditional on time  $t$  information obtained by using the impulse responses given in Figure 1. The 8-period time horizon is favorable to output growth targeting because the effect of a monetary shock on output is strong at this horizon whereas only half of the long-run effect on prices has occurred. We can see from Figure 2 that our SVAR model implies a standard convex efficiency frontier for the conditional variances of inflation and growth. The maximum variability of inflation is clearly higher than the maximum variability of output growth. This result is caused by the higher persistence of the impact of the shocks on inflation than on growth. It is interesting to compare the variances of the gaps achieved with an active monetary policy to a situation where monetary policy is not adopted to the time  $t$  structural shocks. Thus, the AS-shock, the IS-shock and the money demand shock influences medium-term inflation and GDP-growth without any reaction of the central bank to them. In Figure 2, we see that this point is far above the efficient frontier which indicates the substantial benefits of an active monetary policy that reacts to new information.

**Figure 2: 8-quarters-ahead conditional variance of inflation and output growth trade-off**  
 Structural shocks (dashed line) as well as structural shocks with varying  $\alpha$  and data revision errors (solid line), inflation and output growth measured in percent; alpha is the weight of medium term inflation target



## 4. Conclusions

In this paper we propose a new method to analyze forward-looking monetary policy in structural VAR's based on the parameters of the impulse response function. To this end we develop first an approach for representing a monetary policy which aims at a strict medium-term inflation or output growth target in a SVAR model. Second, we show how the ex ante inflation-output-growth volatility trade-off for a forward-looking policy aiming at a convex combination of a strict medium-term inflation and output growth targeting rule can be obtained in this framework. An illustration of this approach using Swiss data suggest that avoiding expected output growth variability has a high price in terms of expected inflation variability.

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