Sequencing Renewables: Groundwater, Recycled Water, and Desalination

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Abstract

Optimal recycling of minerals can be thought of as an integral part of the theory of the mine. In this paper, we consider the role that wastewater recycling plays in the optimal extraction of groundwater, a renewable resource. We develop a two-sector dynamic optimization model to solve for the optimal trajectories of groundwater extraction and water recycling. For the case of spatially increasing recycling costs, recycled water serves as a supplemental resource in transition to the steady state. For constant unit recycling cost, recycled wastewater is eventually used as a sector-specific backstop for agricultural users, while desalination supplements household groundwater in the steady state. In both cases, recycling water increases welfare by shifting demand away from the aquifer, thus delaying implementation of costly desalination. The model provides guidance on when and how much to develop resource alternatives.

1 Introduction

The theory of optimal ordering of resource extraction is well established in the nonrenewable resource literature (e.g. Chakravorty and Krulce, 1994; Chakravorty et al., 2005; Chakravorty et al., 2008). With multiple demands and multiple resource stocks, the welfare-maximizing solution entails extracting according to a least-price rule, where the optimal shadow price is given by the sum of extraction cost, conversion cost, and the endogenous marginal user cost.[1] The theory of renewable resource ordering, however, is still emerging. For example, Zeitouni and Dinar (1997) consider two adjacent sources of groundwater but do not derive a generalized rule to determine the optimal order of extraction. Horan and Shortle (1999) develop a theoretical framework for the optimal management of multiple Mink-Whale stocks but do not solve for the transitional dynamics. More recently, Costello and Polasky (2008) determine the pattern of optimal harvest closures in a stochastic spatial dynamic multiple fisheries model. Although their results could be extended to a more general renewable resource setting, the analysis is restricted to a single sector and requires the assumption of state-independent control, i.e. that marginal returns to harvest are independent of the amount harvested. This may be appropriate for some fisheries, but downward sloping demand curves and stock-dependent extraction costs are common in other renewable-resource contexts. We seek to extend the theory of optimal sequencing with multiple renewables and demand sectors, with particular attention to groundwater and wastewater recycling.

Water scarcity has long been an important issue in many regions around the world, and the threat of climate change has recently brought it even further to the forefront of policy discussions (United Nations, 2006). Cost-effective reduction of scarcity requires optimal timing of supply-side development, such as wastewater recycling (reclaimed water)[2] and desalination. But while the theory of optimal resource-sequencing is known for nonrenewables, it has not been fully developed for renewable resources such as groundwater. As discussed below, this requires extending the least-opportunity-cost first principle to multiple resources. We find that optimal development of recycled water gradually extends the network of non-potable users up to the steady state. We also find that optimal development of reclaimed water promises substantial welfare gains, especially if the potential non-potable-using sector is relatively large.

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[1]An exception is Gaudet et al.'s (2001) application to spatially differentiated resource sites and users. They show that the least-price rule need not hold in the presence of setup costs.

[2] Water reclamation or the recycling of urban wastewater refers to the process of using treated wastewater from homes and businesses for various purposes, including artificial recharge of groundwater basins, irrigation for landscaping and agriculture, and industrial processes that do not require potable water such as cooling (US EPA, 2010).

Our optimal groundwater extraction and wastewater recycling problem also belongs to another general family of problems. Although recycling is a term often viewed in the context of materials like aluminum cans and glass bottles, the concept of reusing post-consumer material is analogous for water. Some previous studies have examined the role of recycling in reducing negative externality generating garbage stock (e.g. Plourde, 1972). Others have focused on the interdependence between nonrenewable resource scarcity and recycling without modeling garbage (e.g. Weinstein and Zeckhauser, 1974). Smith (1972) combines resource extraction, recycling, and garbage, and characterizes the welfare-maximizing trajectories of each. A more recent strand of the literature considers the imperfect substitutability between virgin and recycled materials in production (Conrad, 1999; Huhtala, 1999; Andre and Cerda, 2006).

Our analysis is similar in spirit to that of Weinstein and Zeckhauser (1974), extending their framework to allow for multiple demand sectors and resource growth. We show that their result about the scarcity of mineral resources and the optimal timing of recycling extends to renewable resources. In our model, recycled water has either perfect or zero substitutability with virgin groundwater, depending on the sector. In general, wastewater can be treated to varying degrees, and the resulting level of quality ultimately constrains the recycled water to particular end-uses. Higher substitutability in either sector increases welfare in both sectors by allowing for a longer transitional phase before a backstop becomes necessary.

Analyses in the engineering literature have begun to incorporate recycling as an option in large portfolios of water management strategies, but most of these studies do not optimize water use in a truly economic sense. The CALVIN (California value integrated network) model, for example, allocates water statewide within physical, environmental, and selected policy constraints, but its objective is to "maximize the year 2020 net economic benefits of water operations and allocations to agricultural and urban water users" (Jenkins et al., 2001; Draper et al., 2003; Jenkins et al., 2004), not the present value of the stream of net benefits accruing now and in the future as is generally the practice in resource economics. Wilkinson and Groves (2006) also develop a large-scale model whose purpose is to consider the "impact of alternative levels of groundwater conjunctive use and municipal wastewater reuse on long-term supply and demand balance in the region." The model allows a planner to consider the effects of various programs through specification of scenario-specific parameters but does not solve for the optimal economic allocation.

While demand for water continues to grow in most countries, several demand- and supply-side management strategies are being considered, including expansion of reservoir capacity, watershed conservation, efficient conjunctive use of ground and surface water, new pricing structures, voluntary or mandatory quantity restrictions, wastewater recycling, and desalination (United Nations 2006). The economic problem is how to time the development of various supply-side technologies in combination with demand-side management.

In the absence of recycled water, demand growth necessitates the eventual implementation of a costly but abundant *backstop* resource such as desalination, even if existing water resources are allocated optimally over time to maximize net social benefits (Krulce et al., 1997). The concept of a backstop technology is already established in the groundwater economics literature, even for the case of multiple water-using sectors (Koundouri and Christou, 2006). However, little attention has been paid to economic optimization with recycled water and its potential role as a supplemental resource or sector-specific backstop. Inasmuch as different demand sectors require different qualities of water (e.g. potable vs. non-potable), different resources can serve as backstops or partial substitutes for each respective sector.

In developing and solving a dynamic groundwater-economics model to optimize water extraction for two demand sectors, we characterize recycled water as either a supplemental resource (in the case of rising unit recycling cost) or a sector-specific backstop (in the case of constant unit recycling cost). The general specification allows for increasing unit recycling costs to implicitly incorporate infrastructure expansion costs for spatially differentiated users. The optimal order of resource extraction for each demand sector follows a least-price-first rule where the shadow price of the resource is given by its unit extraction cost, distribution cost, and (endogenous) marginal user cost. Recycled water serves as a supplemental resource for non-potable water users in transition to the desalination steady state. We find that in general, the transitional period is characterized by an endogenously expanding network of recycled water. When restricting our analysis to a single time period, the result is similar to that of Chakravorty and Umetsu's (2003) spatial analysis of conjunctive use of ground and surface water. In their model, the boundary of surface water users is determined endogenously, taking into account groundwater scarcity and distribution costs. Our model's integration of a dynamic component with an implicit spatial structure can be viewed accordingly as an extension of their general framework.

We also consider constant unit recycling costs as a special case of the model. In some situations, it may make sense to amortize capital costs to determine a single constant unit cost of recycling. For constant unit recycling costs, recycled wastewater serves as a sector-specific backstop for users who can substitute non-potable water. This allows the household to specialize in using groundwater supplemented by desalination in the steady state solution. In both cases, the rising and constant unit costs cases, recycling water increases welfare by shifting demand away from the aquifer, thus delaying costly desalination. On the other hand, immediate implementation of water recycling may reduce welfare.

In general, said immediacy depends on initial conditions. It is possible that implementation is already too late. In the Hawai‘i case illustrated, the optimal sequence involves immediate implementation of demand-side management, followed by recycling, and ultimately by use of desalination as a backstop technology.

In the following section, we develop an analytical, two-sector model to address the issue of optimal water recycling when groundwater is scarce. We then characterize the steady state and the transitional dynamics leading up to the steady state for several sets of initial conditions. We show that the optimal stages of resource use are dictated by the marginal opportunity cost of each resource and hence vary with initial conditions. The next section illustrates how the network of recycled users expands endogenously as groundwater becomes scarcer, i.e. that recycled water serves as a supplemental resource in transition to the steady state, wherein the marginal opportunity cost of water is given by the unit desalination cost. We then consider a special case, in which unit water recycling cost is constant. The results of the model suggest that under such an assumption, recycled water serves as a sector-specific backstop in the steady state. We apply the model, using data from the Pearl Harbor aquifer in Hawai‘i, to numerically illustrate our theoretical results. The final section discusses key findings, as well as directions for further research for both water management, and more generally, the role of recycling in resource economics.

2 The model

Groundwater is modeled as a renewable resource in the presence of an abundant yet costly substitute. Coastal aquifers, often characterized by a “Ghyben-Herzberg” lens (Mink, 1980) of freshwater floating on a layer of seawater, are “renewable” in that net recharge to the aquifer varies with the groundwater stock. The upper surface of the freshwater lens sits above sea level due to the difference in density between the freshwater and displaced saltwater. The head level (*h*), or the distance between the top of the lens and mean sea level is a measure of the aquifer stock. Although the stored volume is technically a function of rock porosity, lens geometry and other hydrologic parameters, the head-volume relationship can be approximated as linear (e.g. Krulce et al. 1997; Pitafi and Roumasset, 2009). Thus, as the stock declines, the head level falls proportionately, and groundwater extraction becomes more costly, inasmuch as water must be lifted a longer distance to the surface. Unit groundwater extraction cost is a non-negative, decreasing, convex function of head: $c_G(h_t) \geq 0$, $c_G'(h_t) < 0$, and $c_G''(h_t) \geq 0$.

Leakage from a coastal aquifer is also a function of the head level. In many coastal aquifer systems, low permeability sediment deposits bound the freshwater lens along the coast, but pressure from the lens causes some freshwater to leak or discharge into the ocean as springflow and submarine groundwater discharge. As the head level declines, leakage decreases both because of the smaller surface area along the ocean boundary and because of the decrease in pressure due to the shrinking of the lens. Thus, leakage is a positive, increasing, convex function of head: $L(h_t) \geq 0$, $L'(h_t) > 0$, and $L''(h_t) \geq 0$. Infiltration to the aquifer from precipitation and adjacent water bodies is fixed at a constant rate (*I*). Although a 3-D groundwater model[3] would be better equipped to determine the optimal distribution of groundwater pumping over space (taking into account subsurface flow, cones of depression, and upconing), we abstract from some hydrogeological detail in order to facilitate transparency of water management options. Such simplifications are often key to successful transdisciplinary research (see e.g. Duarte et al., 2010).

Inasmuch as water demand is multifaceted, from a long-term perspective, infrastructure choice should match the varying characteristics of water required for different end-users in terms of quantity and quality. The cost of distributing ground or surface water to users located far from the reservoir or groundwater facility can be non-trivial, but additional infrastructure is only required if new users are beyond the existing network of pipes for potable water conveyance. Non-potable recycled water, on the other hand, requires its own pipes and meters, regardless of the location. Thus, if recycled water users are highly spatially differentiated, infrastructure and distribution costs can quickly escalate with distance from the treatment facility, making recycled water a less cost-effective and hence less desirable resource for distant users.

Properly characterizing the cost of recycled water requires incorporating infrastructure investment into the optimization model. Lumpy investment could be introduced explicitly, but the same general insights can be obtained by assuming that the unit cost of recycling is an increasing and convex function of recycled water, i.e. $c_R(q_t^R) > 0$, $c_R'(q_t^R) > 0$, and $c_R''(q_t^R) \geq 0$. Implicitly, treatment facilities are first constructed near agricultural or industrial centers, i.e. where the concentration of potential recycled water users is highest. The distribution network endogenously expands over time, until eventually it becomes beneficial to build additional treatment plants or to supplement with an alternative resource. For a continuum of non-potable water users, cost increases convexly with units of recycled water. Since more energy is required to pump water a

[3] See Brozovic et al. (2010) for an economic-hydrologic framework, wherein spatial dynamic groundwater flow equations corresponding to a multi-cell aquifer model are coupled with a resource economics model.

greater distance, additional treatment facilities may need to be constructed, and costly pipes and meters must be installed for each additional consumer.[4]

Generally, with multiple end-uses or demands and varying qualities of water, users are naturally classified into categories by quality requirements. In some cases, benefits for certain uses may vary by input water quality so that optimality would not always necessitate using the minimum allowable quality for each use. To make the model more tractable and transparent, however, we aggregate non-potable uses into a single demand category (agriculture), and there is no additional benefit to using higher quality water than necessary. Groundwater is the primary source of high quality (potable) water. No surface water is available, but lower quality (non-potable) water can be obtained from wastewater recycling. In addition, desalinated seawater serves as a high quality backstop resource. High quality water can be utilized for both potable and non-potable uses, but recycled water cannot supply the residential/household demand sector.

The water manager chooses the rates of groundwater extraction for the household sector (q_t^{GH}) and the agricultural sector (q_t^{GA}), the rates of desalination for household (q_t^{BH}) and agricultural use (q_t^{BA}), and the rate of wastewater recycling (q_t^{RA}) to maximize the present value of net social benefit, measured as gross consumer surplus less total costs:

$$(1) \qquad \underset{q_t^{GH}, q_t^{BH}, q_t^{GA}, q_t^{RA}, q_t^{BA}}{Max} \int_0^\infty e^{-\delta t} \left\{ \begin{array}{l} \int_0^{q_t^{GH}+q_t^{BH}} D_H^{-1}(x,t)dx + \int_0^{q_t^{GA}+q_t^{RA}+q_t^{BA}} D_A^{-1}(x,t)dx - \\ (q_t^{GH}+q_t^{GA})c_G(h_t) - (q_t^{BH}+q_t^{BA})c_B - q_t^{RA}c_R(q_t^{RA}) \end{array} \right\} dt$$

subject to $\qquad \gamma \dot{h}_t = I - L(h_t) - (q_t^{GH} + q_t^{GA})$,

h_0 given, $h_t \geq 0$,

$q_t^{GH} \geq 0$, $q_t^{BH} \geq 0$, $q_t^{GA} \geq 0$, $q_t^{RA} \geq 0$, $q_t^{BA} \geq 0$,

[4] If recycling and groundwater infrastructure were both already in place, then recycling would be constrained by the quantity of wastewater input and efficiency of treatment. Since our problem involves recycling infrastructure planning, however, the cost of recycling implicitly includes a minimum cost investment without lumpiness. Consequently, recycling cost is a function of quantity recycled and investment. Given that optimal investment can be written as a function of water quantity, we need only solve for water, and investment is then determined.

where $D_j^{-1}(\bullet)$ is the inverse demand function for sector $j \in \{H, A\}$, δ is the positive discount rate, c_B is the constant unit cost of desalinating seawater,[5] and γ is a head-to-volume conversion factor. We assume that demand for water in each sector is a positive, bounded, continuous, strictly decreasing function of price, i.e. $D_j(\bullet) > 0$, $D_j'(\bullet) < 0$, and $\int_0^\infty D_j(p,t)dp < \infty$. The corresponding current value Hamiltonian is

$$(2) \qquad \Lambda = \begin{cases} \int_0^{q_t^{GH}+q_t^{BH}} D_H^{-1}(x,t)dx + \int_0^{q_t^{GA}+q_t^{RA}+q_t^{BA}} D_A^{-1}(x,t)dx - (q_t^{GH}+q_t^{GA})c_G(h_t) \\ -(q_t^{BH}+q_t^{BA})c_B - q_t^{RA}c_R(q_t^{RA}) + \gamma^{-1}\lambda_t[I - L(h_t) - (q_t^{GH}+q_t^{GA})], \end{cases}$$

and the Maximum Principle requires that the following conditions hold:

$$(3) \qquad \frac{\partial \Lambda}{\partial q_t^{GH}} = D_H^{-1}(q_t^{GH}+q_t^{BH},t) - c_G(h_t) - \gamma^{-1}\lambda_t \le 0 \qquad \text{if} < \text{then } q_t^{GH} = 0$$

$$(4) \qquad \frac{\partial \Lambda}{\partial q_t^{BH}} = D_H^{-1}(q_t^{GH}+q_t^{BH},t) - c_B \le 0 \qquad \text{if} < \text{then } q_t^{BH} = 0$$

$$(5) \qquad \frac{\partial \Lambda}{\partial q_t^{GA}} = D_A^{-1}(q_t^{GA}+q_t^{RA}+q_t^{BA},t) - c_G(h_t) - \gamma^{-1}\lambda_t \le 0 \qquad \text{if} < \text{then } q_t^{GA} = 0$$

$$(6) \qquad \frac{\partial \Lambda}{\partial q_t^{RA}} = D_A^{-1}(q_t^{GA}+q_t^{RA}+q_t^{BA},t) - c_R(\bullet) - q_t^{RA}c_R' \le 0 \qquad \text{if} < \text{then } q_t^{RA} = 0$$

$$(7) \qquad \frac{\partial \Lambda}{\partial q_t^{BA}} = D_A^{-1}(q_t^{GA}+q_t^{RA}+q_t^{BA},t) - c_B \le 0 \qquad \text{if} < \text{then } q_t^{BA} = 0$$

[5] The assumption of constant unit desalination costs is meant to be an approximation. Energy costs are likely to be slowly rising in the long run, and advancements in desalination technology are likely to be pushing the cost in the opposite direction. The two forces are small and offsetting, but we cannot assert that either is dominating a priori. See Chakravorty et al. (1997) for an analogous case in the context of oil. We expect that rising (falling) costs will tend to shift the entire efficiency price path up (down), implying an earlier (later) transition to the backstop steady state, but we leave the exploration of those cases to further research (see e.g. Fischer and Salant, 2010).

(8) $$\dot{\lambda}_t - \delta\lambda_t = -\frac{\partial \Lambda}{\partial h_t} = (q_t^{GH} + q_t^{GA})c_G'(h_t) + \gamma^{-1}\lambda_t L'(h_t)\,.$$

Inequalities (3)-(7) describe the optimal rules for resource use in each demand sector. If groundwater extraction for use in the household sector is optimally positive in period *t*, (3) holds with equality and indicates that the marginal benefit (MB), measured by the inverse demand function $D_H^{-1}(\bullet)$, is equal to the *marginal opportunity cost* (MOC) of groundwater, which is the sum of the marginal extraction cost and the marginal user cost ($\gamma^{-1}\lambda_t$), i.e $\pi_t^{GH} \equiv c_G(h_t) + \gamma^{-1}\lambda_t$. The marginal user cost (MUC) is defined as the loss in present value of using one unit of the resource today. The complementary slackness condition says that extraction is optimally zero if instead MB<MOC. The interpretation of condition (5) is analogous, except that the MB is determined by the inverse demand of the agricultural sector, $D_A^{-1}(\bullet)$.

Since the unit cost of desalinated water is constant and the MUC of a backstop resource is zero by definition, the MOC of desalinated water is equal to c_B for both demand sectors, i.e. $\pi_t^{BH} = \pi_t^{BA} = c_B$. Conditions (4) and (7), therefore, say that if desalination is optimally used in either the household or agricultural sector, MB=MOC of desalinated water in that sector. Otherwise, it must be that MB<MOC.

The optimality condition (6) is specific to the agricultural sector because of our assumption that recycled water has zero substitutability with groundwater for potable uses. The MOC of recycled water is $\pi_t^{RA} \equiv c_R(q_t^{RA}) + q_t^{RA}c_R'$, i.e. not just $c_R(q_t^{RA})$, inasmuch as the unit cost is quantity dependent. If recycling is positive in any period, then the equimarginality condition (6) determines the optimal treatment quantity. When recycling is not optimal, it must be that MB<MOC. Conditions (1)-(7) are jointly summarized in proposition 1.

Proposition 1: *In a given period t, for each demand sector* $j \in \{A, H\}$*, only the resource(s)* $i \in \{G, R, B\}$ *with the lowest MOC is/are used, where the MOC of resource i for use in sector j is denoted as* π_t^{ij}.

Proof: Suppose instead that for some sector *j*, $q_t^{xj} > 0$ even though $\exists$ a resource $y \neq x$ such that $\pi_t^{yj} < \pi_t^{xj}$. If *x* is being used and *y* is not, conditions (3)-(7) require that $D_j^{-1}(\bullet) = \pi_t^{xj}$ and $D_j^{-1}(\bullet) < \pi_t^{yj}$, or equivalently $\pi_t^{yj} > \pi_t^{xj}$, a contradiction.

If both x and y are being used simultaneously, conditions (3)-(7) require that $D_j^{-1}(\bullet) = \pi_t^{xj} = \pi_t^{yj}$, a contradiction to the supposition that $\pi_t^{yj} < \pi_t^{xj}$. That leaves only the possibility that $q_t^{yj} > 0$ and $q_t^{xj} = 0$, i.e. least-MOC is optimal. □

The costate equation (8) in combination with the equation of motion for the head level ensures that the solution is also optimal dynamically. For ease of interpretation, we rearrange condition (8) as follows:

$$(9) \qquad \dot{\lambda}_t - (q_t^{GH} + q_t^{GA})c_G'(h_t) = \delta\lambda_t + \gamma^{-1}\lambda_t L'(h_t).$$

Equation (9) says that the marginal benefit should be equated to the cost of the marginal conserved unit of groundwater in every period, taking into account stock-dependent extraction costs and leakage. The marginal benefit includes the increase in the in situ value of the resource by not using it, as well as the decrease in extraction costs resulting from a higher head level. The marginal cost is the forgone interest from the scarcity rent had the unit of the resource been extracted, plus the lost value resulting from increased leakage at a higher head level. The costate variable (λ_t) is by definition the increase in net present value resulting from an additional unit of the groundwater stock. From a cost perspective, it is the loss in value when the stock is reduced by one unit, or the marginal user cost as noted previously.

Using the least-MOC resource(s) is always optimal because the benefit from the marginal unit of water is the same regardless of the source. In other words, the net social benefit is maximized by minimizing MOC in each period. In most cases, an internal solution is not satisfied simultaneously for every resource in periods leading up to the steady state. In the steady state, however, all resources are optimally used, given reasonable assumptions about the initial conditions of the problem (proposition 2). We define the *efficiency price* for each sector as that which induces the optimal trajectory of water consumption, i.e. the marginal benefit along the optimal paths. For $p_t^H \equiv D_H^{-1}(\bullet)$ and $p_t^A \equiv D_A^{-1}(\bullet)$, conditions (3)-(7) can be simplified to

$$(10) \qquad p_t^H = \min\{\pi_t^{GH}, \pi_t^{BH}\}$$

$$(11) \qquad p_t^A = \min\{\pi_t^{GA}, \pi_t^{RA}, \pi_t^{BA}\}$$

The price of water for household use is determined by the lower of either the MOC of groundwater or the MOC of desalination. Similarly, the price of water for agricultural use is the minimum of the MOC of groundwater, recycled wastewater, and desalinated seawater. Thus, the least-MOC-first rule for the optimal extraction of nonrenewable resources (e.g. Chakravorty and Krulce, 1994; Chakravorty et al., 2005; Chakravorty et al., 2008) extends to the management of renewable resources with multiple demand sectors. Although the MOC of desalination is constant in both sectors, the MOCs of groundwater and recycled water are variable. In particular, unit groundwater extraction cost rises as the head level declines, and marginal user cost rises as the resource becomes scarcer. For the reasons previously discussed, unit recycling cost varies with the quantity recycled.

2.1 Steady state

If we assume positive demand growth, use of desalination is ensured in the steady state for both sectors, and $p_T^H = p_T^A = c_B$ (proposition 3). That the steady state requires $\dot{h} = 0$ means groundwater extraction must be positive and exactly equal to $I - L(h)$, where leakage is determined endogenously by the optimal steady state head level. Combining conditions (3) and (4) yields $\lambda_T = c_B - c_G(h_T)$. Taking $\dot{\lambda} = 0$ and plugging λ_T into condition (8) results in a single equation that can be solved for the unique[6] steady state aquifer head level (h_T^*):

$$(12) \qquad c_B = c_G(h_T) - \frac{\gamma^{-1} c_G'(h_T)[I - L(h_T)]}{\delta + \gamma^{-1} L'(h_T)}.$$

Proposition 2: *For $\pi_0^{GA} < \pi_0^{RA} < \pi_0^{BA}$ and unbounded demand growth over time, each resource $i \in \{G, R, B\}$ is used in the optimal steady state.*

Proof: Suppose that the steady state optimally begins at period T and that $q_T^{GA} = 0$. If groundwater extraction is zero, the head level must be growing, which contradicts the supposition that a steady state is obtained at *T*, since a steady state requires that $\dot{h}_T = 0$. Thus it must be that $q_t^{GA} > 0$ in the steady state.

Suppose instead that $q_t^{BA} = 0$ for $t \geq T$. q_t^{GA} must be constant to ensure that $\dot{h}_t = 0$ for $t \geq T$, so further demand growth needs to be met entirely by recycled water. If the unit recycling cost is increasing in quantity, π_t^{RA} necessarily increases over time. At some point, π_t^{RA} would exceed π_t^{BA}, thus violating the

[6] See the appendix for a proof of this result.

least-MOC rule. Even prior to that point, $\pi_t^{GA} < \pi_t^{RA}$, even though both resources are being used simultaneously, a contradiction to the least-MOC rule. Thus it must be that $q_t^{BA} > 0$ in the steady state.

Now suppose that $q_t^{RA} = 0$ for $t \geq T$. The least-MOC rule requires that $\pi_t^{RA} > \pi_t^{GA} = \pi_t^{BA}$. Since π_t^{RA} is only dependent on q_t^{RA} (i.e. not state-dependent) and π_t^{BA} is positive and constant by assumption, the inequality cannot be satisfied for a non-positive q_t^{RA} given our assumption that $\pi_0^{GA} < \pi_0^{RA} < \pi_0^{BA}$. Thus, in the steady state, q_t^{RA} must be positive and is chosen to satisfy $\pi_t^{RA} = \pi_t^{GA} = \pi_t^{BA}$. □

The proof is analogous for the household sector and will not be repeated here.

Proposition 3: *If* $q_T^{Bj} > 0$ *in the optimal steady state, it must be that* $p_T^j = c_B$.

Proof: Suppose instead that $p_T^j > c_B$ in the steady state. Since desalinated water is an unlimited resource, one could lower costs (increase PV welfare) by raising the share of household demand supplied by desalination. But that means groundwater extraction would be declining, and hence the head level would be rising, a contradiction to the assumption that a steady state is reached.

Now suppose that $p_T^j < c_B$ in the steady state. Since the efficiency price (p_T^j) is determined by the resource with the lowest marginal opportunity cost, $q_T^{Bj} > 0$ means that the least-MOC rule is being violated, i.e. the described steady state cannot be optimal. □

2.2 Order of resource use

In this section, we consider three variations based on the assumption that the unit cost of desalination is greater than the initial MOC of wastewater treatment. Although multiple combinations of initial conditions are possible, we focus on the case where desalination is relatively costly. In Hawai‘i, for example, the unit cost of desalination based on reverse osmosis of seawater has been approximated at $7 per thousand gallons (Pitafi and Roumasset, 2009), whereas industrial users in the Pearl Harbor area pay $4.00-$5.00 per thousand gallons for recycled wastewater. Scenario (a) depicts the prototypical case wherein groundwater is sufficiently abundant that its MOC is less than the initial MOC of recycled water. Scenario (c) concerns the opposite case wherein groundwater has been depleted to the extent that its MOC is higher than the unit cost of desalination. Scenario (b) is the intermediate case wherein MOC is greater than the MOC of recycling water but less than that of desalination.

Table 1. Stages of Resource Use				
Scenario	**Sector**	**Stage 1**	**Stage 2**	**Stage 3**
a	Household	GW	GW	GW + DW
	Agriculture	GW	GW + RW	GW + RW + DW
b	Household	GW	GW + DW	
	Agriculture	RW (+ GW)	GW + RW + DW	
c	Household	DW	GW + DW	
	Agriculture	RW (+ DW)	GW + RW + DW	
Note: GW = groundwater, DW = desalinated water, RW = recycled water.				

Table 1 summarizes the results. In case (a), groundwater optimally supplies both sectors in the initial stage of extraction. The MOC of groundwater rises over time until it reaches the MOC of the first unit of recycled water. Groundwater continues to supply both sectors, but as the MOC of groundwater continues to rise, more of the water consumed by the agricultural sector is supplied by recycling, i.e. the network of recycling infrastructure is endogenously expanded as more users optimally switch to the lower quality source. This result parallels the conjunctive water use literature, wherein the area serviced by groundwater is increasingly taken over by surface water as the water table declines (e.g. Chakravorty and Umetsu, 2003). In the steady state, all water resources are used; recycled water is used for the agricultural sector, and desalinated water and groundwater are used for both sectors.

For scenario (b), wherein the initial groundwater MOC is greater than the initial MOC for recycled water but less than the unit cost of desalination, groundwater is extracted exclusively for the household sector and at least some water is recycled for the agricultural sector from the outset. If the agricultural sector is sufficiently large, recycled water will be used in the initial period until the marginal production and distribution cost of recycled water, $c_R + q_t^{RA} c_R'$, equals the MOC of groundwater. The equality determines the outer boundary of agricultural land irrigated by treated wastewater. The remainder is irrigated with groundwater. Recycled water is used exclusively for the agricultural sector if the MOC of recycled water at the demand curve is below the MOC of groundwater. As the MOC of groundwater rises over time, more of the agricultural sector's demand is met by recycled water. In the steady state, all water resources are used.

A third possibility is that the aquifer is sufficiently depleted such that the MOC of groundwater starts above the unit cost of desalination. Recycled water is used exclusively by the agricultural sector, unless the MOC exceeds the unit cost of desalination at the demand curve, in which case recycling is supplemented by desalination. Desalination is used exclusively by the household sector. The aquifer is allowed to fill until the MOC of groundwater falls to the unit cost of the backstop, at which point groundwater and desalination are used simultaneously. In the meantime, the number of recycled users steadily increases until the steady state. Further demand growth in the agricultural sector necessitates eventual supplementation by desalination. The stages of resource use for each scenario are summarized in table 1.

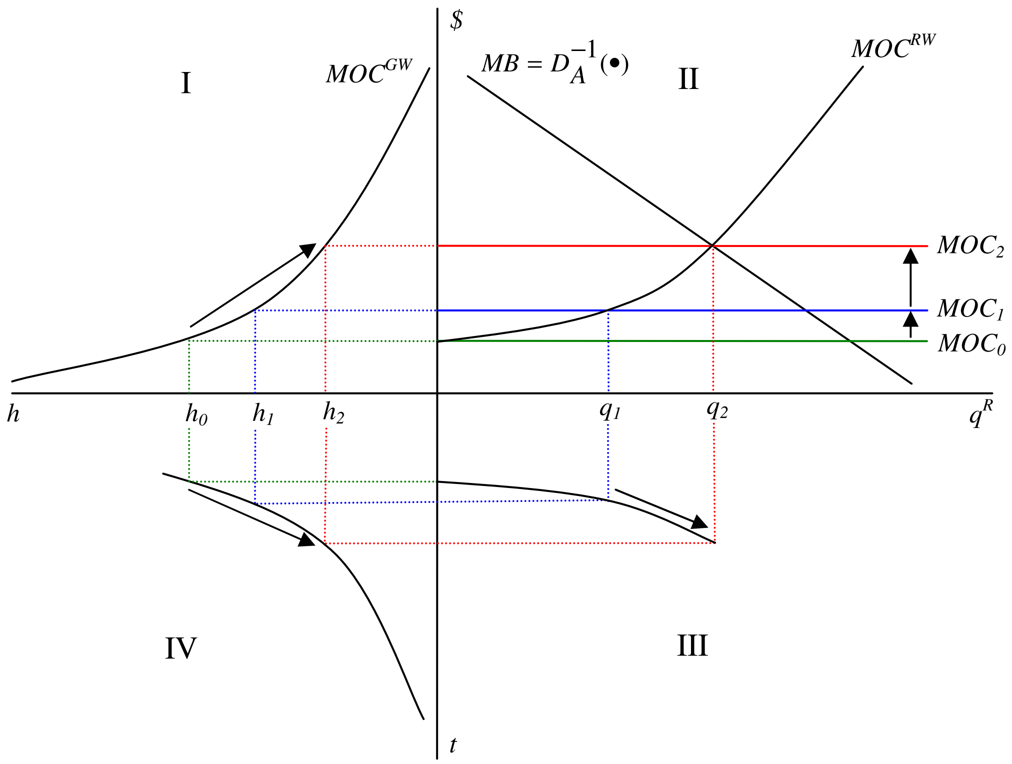


Figure 1. Network of recycled water users expands over time.

When water recycling is incorporated into an optimal groundwater management plan, the boundary of recycled-water users shifts out over time as the scarcity value of groundwater increases (figure 1).[7] Although the approach path may be non-monotonic, the aquifer head level is eventually drawn down toward its steady state level (quadrant IV in figure 1). As the aquifer is depleted, groundwater becomes scarcer, and its MOC shifts upward (quadrant I). Given the

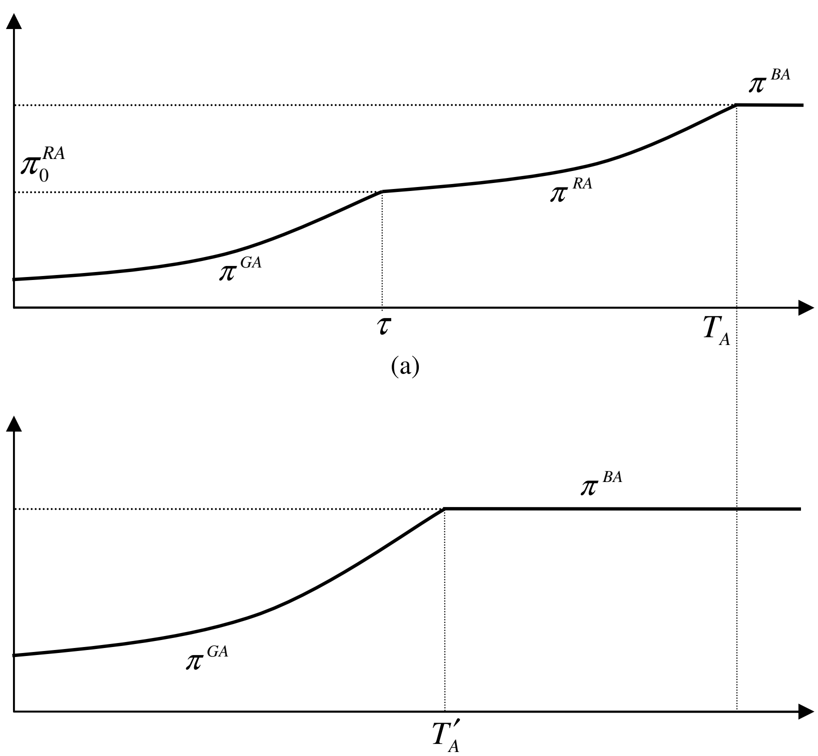
[7] To maintain graphical clarity, the demand curve is depicted as constant over time. Growing demand does not change the qualitative result that the network of recycled users expands over time.

choice between groundwater and recycled water for the agricultural sector, the source with the lowest MOC is used first. For the head level h_1, and the corresponding groundwater MOC_1, the optimal quantity of recycled water is q_1 (quadrant II). Up until that quantity, the MOC of recycled water is lower than the MOC of groundwater, i.e. $MOC^{RW} < MOC_1$ for $0 \leq q^R < q_1$. The remainder of the quantity demanded is met by groundwater at unit cost MOC_1. In later periods, the MOC of groundwater is even higher, which means more recycled water is used, and the boundary of recycled water users expands over time (quadrant III). Eventually, the system reaches a steady state, at which time expansion ceases and recycling infrastructure is sustained, while the remainder of consumption is met by desalination.

Another way to depict the stages of optimal resource use is to compare directly the time path of each resource's MOC for each demand sector. We again illustrate the optimal program for *scenario a* because it is the most complex of the three. Figure 2a portrays hypothetical MOC paths for the agricultural sector. Since groundwater is optimally used in every period in both sectors, the efficiency price path for agriculture is identical to that of the household sector (not illustrated). Marginal opportunity cost trajectories for the other two scenarios can be constructed in a similar manner. For $\pi_0^{GA} < \pi_0^{RA} < \pi_0^{BA}$ and $\pi_0^{GH} < \pi_0^{BH}$, groundwater is used initially in both sectors. As groundwater scarcity rises, it eventually becomes optimal to use recycled water in the agricultural sector starting from some year τ, i.e. $\pi_\tau^{GA} = \pi_0^{RA}$. Since the quantity of recycled water is zero prior to τ, the MOC of recycled water remains constant at π_0^{RA} during that period. The MOC of groundwater "kinks" at τ because recycling in the agricultural sector, by conserving on freshwater, lowers groundwater extraction costs from that point forward. Eventually, the MOC of groundwater and that of recycled water both rise to the MOC of desalination, and the system reaches a steady state, wherein all of the MOCs are equal. The efficiency price, determined by the minimum of the MOC curves for each resource in each period (equation 10), is illustrated by the heavy curve in figure 2a.[8] As drawn, the length of the first and second stages of extraction (τ and $T_A - \tau$ respectively) prior to the steady state are approximately equal, but that need not be the case. Depending on the application, the shape/slope of the MOC curves may also vary.

[8] Similar to the optimal solution for extracting multiple nonrenewable resources, the efficiency price for our problem is graphically represented by the lower envelope of the MOC curves within each sector. However, while nonrenewable MOC curves generally cross (e.g. Chakravorty and Krulce, 1994; Chakravorty et al., 2005; Chakravorty et al., 2008), that need not be the case for renewable resources. This follows from the fact that after each switchpoint (e.g. τ), resources are simultaneously used to maintain equality of the MOCs.

The qualitative welfare implications of the optimal recycling program are revealed when comparing the MOC trajectories to those that would obtain under groundwater optimization alone (figure 2b). Without recycling, groundwater is used exclusively until the steady state. Consequently, extraction costs rise more rapidly, as does groundwater scarcity, meaning desalination must be implemented earlier in both sectors. Implementation of optimal wastewater recycling increases the present value net benefit to society, inasmuch as the lower extraction path allows for an extended period of drawdown before implementation of costly desalination in the steady state.



(a) (b)

Figure 2. Hypothetical time paths of MOCs: (a) Agricultural sector with recycling, (b) Agricultural sector without recycling.

In figure 3, we illustrate the resource quantities corresponding to the MOCs and efficiency price path depicted in figure 2a. Groundwater extraction provides all of the water consumed in the agricultural sector in the first stage, i.e. prior to time τ. Thereafter, water recycling increases in every period, although the total water consumption declines in response to the rising efficiency price. Consequently, recycled water, as a percentage of the total, increases until the steady state. When desalination optimally comes online in period T_A , the quantity of groundwater jumps down to its steady state level, where at extraction is exactly equal to net recharge. The total quantity of recycled water remains constant thereafter, and the remaining steady state quantity demanded is met by desalination.

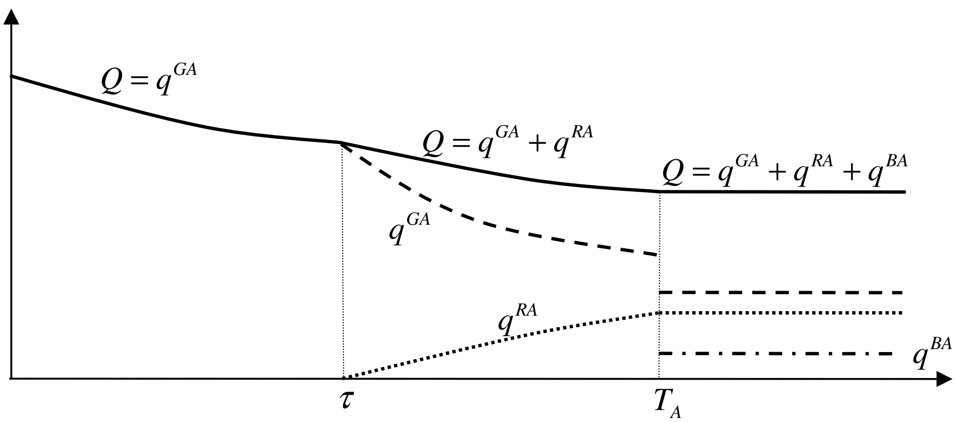


Figure 3. Total water in the agricultural sector (Q) vs. quantities of groundwater (q^{GA}), recycled water (q^{RA}), and desalinated water (q^{BA}).

2.3 Implications of a binding state-space constraint

In the previous sections, we characterized the problem under the implicit assumption that the optimal head level in every period is a positive, interior solution. In this section we briefly discuss the implications of a binding head constraint. If, for example, the resource manager is concerned about saltwater intrusion, he/she may choose to impose a lower limit on the head level ($h_{\min}$). In that case, the maximization problem (1) would be modified to include the constraint. Letting μ_t denote the multiplier for the constraint, the standard expression for the marginal user cost of groundwater must also be modified as follows:

(13) $$\frac{\dot{p}_t^{Gj} - \gamma^{-1}c_G'(h_t)[I - L(h_t)] + \gamma^{-1}\mu_t}{\delta + \gamma^{-1}L'(h_t)}$$

for sector *j*. Since the MUC is defined as the change in PV from using an additional unit of the resource today, it makes sense that μ_t enters (13) positively. Whenever the head level constraint is binding, groundwater extraction must remain constant. Therefore, the result only applies in the steady state. The optimal least-MOC rule leading up to the steady state remains unchanged. Moreover, the positive multiplier ensures that the least-MOC rule remains satisfied even in the steady state. Without $\mu_t > 0$, the steady state $h_{\min}$ would not likely correspond to the MOC of groundwater that is exactly equal to the MOCs of recycled and desalinated water.

2.4 Possible solution methods

Although the optimal order of resource use is theoretically governed by a straightforward least-MOC rule, the MOCs of ground and recycled water are endogenous. For nonrenewable resources with constant extraction costs, the entire path of a particular resource's shadow price is determined once the initial value is specified. For a renewable resource with stock-dependent extraction costs and growth, however, each feasible MOC path must be solved for in conjunction with the associated feasible path of the aquifer head level. In the previous section, we discuss the ordering of water resource use as if we already know the optimized MOC paths for each resource within each demand sector. However, solving the problem in practice involves consideration of multiple trial MOC paths, only one of which maximizes PV. The problem can be tackled using a gradient ascent, genetic, or neural algorithm, but implementation can prove challenging in the presence of multiple nonlinearities, and moreover it may be difficult to ensure convergence to a global rather than a local maximum.

In our numerical illustration that follows (section 3), we use a forward-shooting procedure to determine the optimal solution. We start by assuming a value for the initial shadow price of groundwater, and condition (8) allows one to determine the shadow price in the next period. The efficiency price for each sector can then be ascertained from the equations (10) and (11) for the current period. The price reveals the current-period rates of extraction, recycling, or desalination. The equation of motion for the aquifer generates the next-period head level, and the entire process is iterated over time. Eventually, one of the terminal conditions is reached; either the head level declines to h_T^* or one of the efficiency prices rises to c_B. If the conditions do not coincide, i.e. one is inconsistent, the trial value for the initial shadow price of groundwater is revealed as incorrect. The guess must be adjusted and the process repeated until all of the initial and terminal conditions are satisfied for the head level and the efficiency prices in each sector, so that the PV functional is maximized.

2.5 A special case: constant unit recycling cost

In the case that lumpy infrastructure investment timing is not as crucial (e.g. when the non-household sector is relatively small and stable, and a single treatment facility's capacity would be sufficient) one could use standard amortization methods to approximate a constant unit cost of recycling (c_R). Since recycled water is of less than potable quality, it is a reasonable assumption that the unit cost of wastewater recycling is less than the unit cost of desalinating seawater, i.e. $c_R < c_B$. Recycled water serves as a sector-specific backstop for the agricultural sector. Groundwater is used in every period for household consumption, but recycled water eventually serves the entire agricultural sector in the steady state. We contrast this with the traditional backstop steady state characterized by the general model, in which desalination eventually supplements both sectors. Analogous to the general case with rising unit recycling cost, stages of resource use leading to the steady state are determined by the ordering of the three MOCs in each of the demand sectors. Table 2 summarizes the stages of resource use for the same three scenarios discussed in the general cost case.

Table 2. Stages of Resource Use (Constant Unit Recycling Cost)

Scenario	Sector	Stage 1	Stage 2	Stage 3
a	Household	GW	GW	GW + DW
	Agriculture	GW	RW	RW
b	Household	GW	GW + DW	
	Agriculture	RW	RW	
c	Household	DW	GW + DW	
	Agriculture	RW	RW	

Note: GW = groundwater, DW = desalinated water, RW = recycled water.

3 A numerical example: the Pearl Harbor aquifer

The study area chosen is the Pearl Harbor region on the island of O'ahu, Hawai'i. Following Krulce, Roumasset, and Wilson (1997), we assume that 78.149 billion gallons of freshwater are stored per foot of head, i.e. $\gamma = 1/78.149$, that recharge to the aquifer is approximately 220 million gallons per day (mgd), and that the leakage function is quadratic in the head level: $L(h_t) = 0.24972h_t^2 + 0.022023h_t$. The extraction cost function specified, $c_G(h_t) = \xi(e - h_t)$, is linear in lift, where lift is defined as the difference between the average ground surface elevation of the wells ($e = 272$ feet) and the head level. The energy-cost parameter ($\xi = 0.00121$) is calculated using the initial unit extraction cost of $0.31 per thousand gallons (tg) which is the volume-weighted average of unit extraction costs for all primary wells in the initial period, and the initial head level, which is estimated as roughly 16 feet for Pearl Harbor. We impose a constraint on the head level of 15.125 feet (Liu, 2006)[9] as a precaution against saltwater intrusion, and the unit cost of desalinating water is estimated at $7.43/tg (Pitafi and Roumasset, 2009).[10]

The demands for the household and agricultural sectors are modeled as constant elasticity functions: $D_j(x,t) = \alpha_j e^{g_j t}(x)^{-\eta_j}$ for $j \in \{H, A\}$. We assume that agricultural demand for water is more elastic than residential demand. Specifically, we assume elasticities of -0.2 and -0.6 for the household and agriculture sectors respectively. The demand coefficient for each sector is calculated using pumping data and the retail price of water for the year 2006. Total pumpage from the Pearl Harbor aquifer in 2006 is estimated at 103.46 million gallons per day (Wilson Okamoto Corp., 2008). For the county of Honolulu, domestic residential consumption makes up nearly 61% of municipal water use, non-residential domestic use add another 30%, and thus industry, agriculture and other non-domestic sectors account for 9% of the total water consumed in 2006. If we take these ratios as a rough approximation for the Pearl Harbor region, year 2006 estimated consumption in sectors *H* and *A* are 94.15 mgd and 9.31 mgd respectively. A retail price of $2.23/tg implies demand coefficient values of $\alpha_H = 115.05$ and $\alpha_A = 11.38$, as well as an average distribution cost of $2.23/tg, the latter of which was subsumed in the extraction cost function in the previous theoretical sections. We also assume exogenous rates of demand growth in both sectors of 1% ($g_H = g_A = 0.01$) and a discount rate δ =3%.

Reclaimed water is currently being used in the Pearl Harbor region by industry and golf courses. Several golf courses have entered into individual agreements with the Honolulu Board of Water Supply to purchase recycled water at rates ranging from $0.25/tg to $0.40/tg. However, the initial rates were set significantly below cost, so the current pricing structure does not reflect the true cost of recycled water. Companies in the nearby Campbell Industrial Park pay between $4.00 and $5.00/tg of recycled water, however, which may be more indicative of the true unit cost of recycling. We assume that c_R is constant and equal to $4.00/tg.

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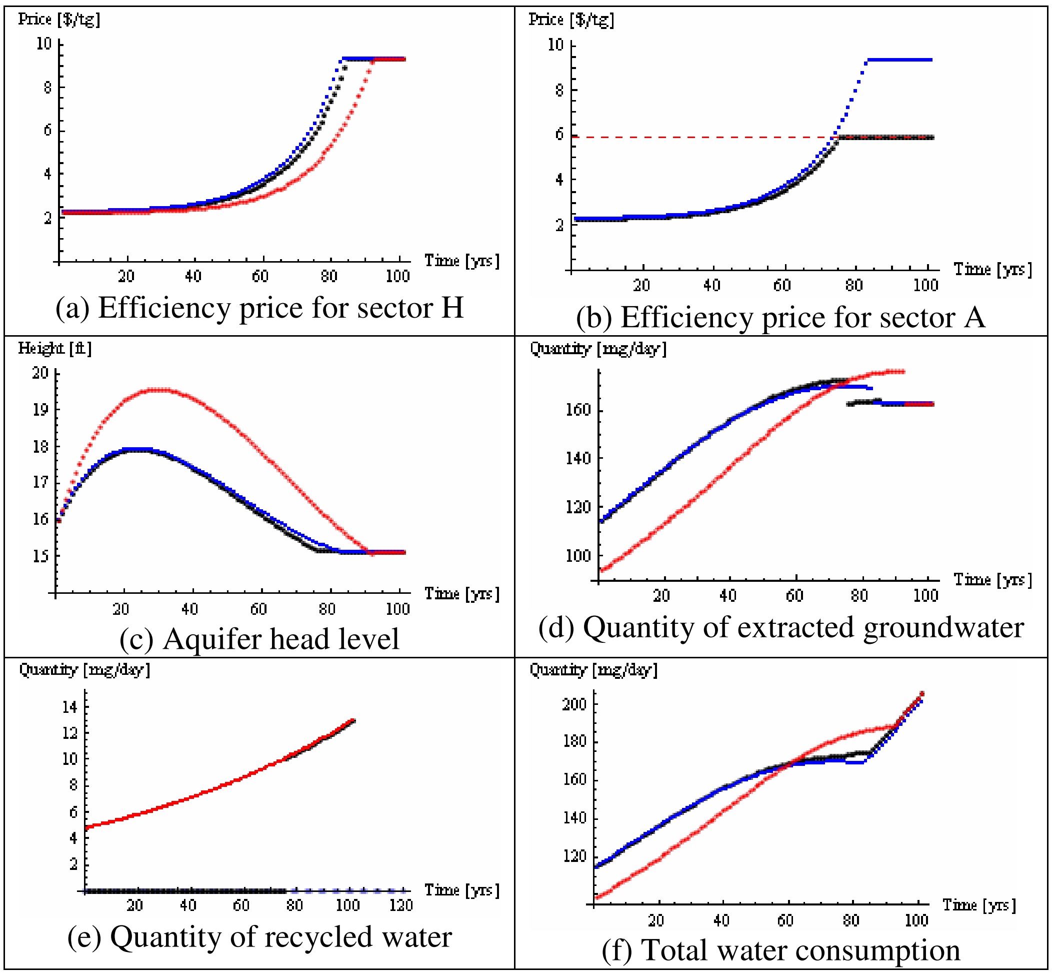
[9] The average well depth is about 200 feet below mean sea level, and upconing is estimated at 100 feet. Taking into account the thickness of the transition zone, the head level must be kept above 15.125 feet to avoid seawater intrusion of the wells.

[10] This figure is updated to account for increases in energy costs and inflation.

We solve the problem numerically using the solution method described in section 2.4. Since the Pearl Harbor aquifer is currently above its steady state level, the efficiency price (figures 4a and 4b) starts relatively low and groundwater is used initially for both sectors. After 75 years, however, the price rises to the unit cost of recycling water, and it becomes optimal to shift the agricultural sector to recycled water use. For the next decade, the household sector uses groundwater exclusively, while the agricultural sector relies entirely on recycled water. The price of groundwater reaches the unit cost of desalination at year 85, after which groundwater extraction is limited to net recharge and the remaining optimal quantity demanded is supplied by desalination.



(a) Efficiency price for sector H

(b) Efficiency price for sector A

(c) Aquifer head level

(d) Quantity of extracted groundwater

(e) Quantity of recycled water

(f) Total water consumption

Figure 4. Dynamic paths: optimal recycling (black), no recycling (blue), premature recycling (red)

Since demand is growing in both sectors, the aquifer head level is allowed to rise initially in anticipation of future scarcity. After around 20 years, the head level begins to decline as extraction exceeds net recharge to the aquifer. After the first switch-point, the rate of decline decreases, since from that point groundwater is not used for the agricultural sector. After the second switch-point, the aquifer reaches its steady state head level (figure 4c).

The net present value (NPV) of social benefits derived from the optimal water management program is approximately \$11.73 billion. If recycled water is not considered as a potential resource to draw from in the planner's management strategy, then optimal extraction from the aquifer still entails solving a dynamic optimization problem, albeit with only a single groundwater alternative (desalination). In that case, the aquifer is optimally drawn down more rapidly, inasmuch as groundwater supplies both sectors all the way until the backstop. Desalination is implemented two years sooner (figure 4), and the NPV of social benefits totals \$11.66 billion. The welfare gain from optimal recycling is therefore \$70 million or 0.6%. Alternatively, if recycling is implemented (non-optimally) at time zero and groundwater is extracted optimally for the household sector, then the NPV of social benefits is \$11.23 billion and desalination is implemented at year 86. Although implementation of the costly backstop is delayed slightly, the NPV is \$500 million or 4% less than the optimum welfare because costly recycled water is used prematurely.

4 Conclusion

Efficient management of renewable resources may require optimization over multiple margins, including the development of supplementary resources. This is exemplified by wastewater recycling and its role in determining optimal groundwater extraction and the development of a desalination alternative. Inasmuch as different demand sectors require different qualities of water, it is natural to think of recycled water as a supplemental resource for those users with low water quality requirements. When unit recycling cost is constant, recycled water serves as a sector-specific backstop. Implementation of the model provides guidance on the appropriate timing and size of backstop and recycling infrastructure.

We provide an operational model that simultaneously determines optimal groundwater extraction and wastewater recycling. We show that incorporating an optimal recycling program lowers the price of groundwater in the present and increases the present value of welfare relative to the status quo. If water recycling and groundwater extraction are managed independently, then the benefits of recycling will be overestimated and recycling may be recommended prematurely. One does not need to entirely reject the "3Rs of sustainability" -- reduce, reuse, recycle -- but the question is when and how much. For sustainable development in the economic sense, resources should be managed in accordance with dynamic efficiency (Stavins et al. 2003, Ravago et al. 2010). In the present case, dynamic efficiency provides a schedule of reduced resource extraction and the timing of water reuse and recycling. As in the numerical application, the optimal solution may involve delaying wastewater recycling. This underscores the cautionary tale that following the symbolic edicts of "sustainability" may be economically wasteful and undermine the need for more thorough analyses of resource management and sustainable development.

The necessary conditions derived from the optimal control problem accord with a least-marginal-opportunity-cost-first extraction rule, where the marginal opportunity cost of a particular resource is comprised of its extraction cost and endogenous marginal user cost. Inasmuch as the marginal user cost of groundwater is stock-dependent and the marginal cost of recycled water is increasing in quantity produced, various stages of extraction are possible, depending on initial values and other parameters in the actual application. For example, groundwater may be used exclusively in all sectors for a finite period of time, or it may be that recycled water or desalinated water optimally supplements groundwater in any given period. Although recycled water can never serve as a true backstop for the agricultural sector if demand is growing, it eases the transition of usage from groundwater to desalinated water. More specifically, it increases the present value to society by allowing an extended period of drawdown before implementation of the ultimate backstop.

While the literature on optimal timing of alternative resources primarily focuses on nonrenewables, the current paper considers a renewable resource along with one recycled and one backstop resource, where demands for different resource qualities are distinguishable. Our analysis also serves as a variation and extension of Chakravorty and Umetsu's (2003) framework for conjunctive water use, by incorporating dynamic and implicitly spatial components into a conjunctive use model. Although increasing the model's complexity increases the difficulties of implementation,[11] incorporating spatial heterogeneity more explicitly into a dynamic framework may be a worthwhile direction for further research.

Integrating our results on ordering multiple resources over multiple demand sectors with previous analyses on recycling and the theory of the mine (Smith, 1972; Weinstein and Zeckhauser, 1974) would provide a general model applicable to many problems involving management of either renewable or nonrenewable resources. Particularly for nonrenewable resources, an even more

[11] See, e.g. Rausser et al. (1983) for an early discussion of solution algorithms appropriate for complex systems models of renewable and non-renewable resources.

complete theory could be achieved by incorporating landfills (e.g. Plourde, 1972; Smith, 1972), such that the management problem involves optimally allocating waste between landfills and recycling, while simultaneously extracting the scarce resource(s) to maximize welfare. We expect that our key qualitative result about the timing of recycling would extend to this more general framework. As landfill space decreases and resource scarcity increases, recycling would become more attractive, but the key question is when this would occur. Recycling can be implemented too soon or too late and optimal timing varies across the material being recycled.

We assumed that fixed capital costs (e.g. facility construction costs) were divisible and straightforward to amortize, thus allowing us to work with a single unit cost function for wastewater recycling. While incorporating lumpy investment would likely not change the qualitative results on resource ordering, the optimal timing of implementation would be affected (e.g. Gaudet et al., 2001). More specifically, lumpy investment may lead to the delay of recycling investment, i.e. allowing the marginal opportunity cost of one resource to "overshoot" relative to the continuous solution. That translates to a relatively longer period of groundwater use in the non-household sector prior to implementation of wastewater recycling. Once investment is initiated, it may be that excess capacity is built in, anticipating the growth in optimal usage in future periods. Given that groundwater extraction for each sector is determined simultaneously, the entire optimal management trajectory (i.e. for both demand sectors) would be altered.

Water quality is another aspect of the model that could be generalized in future research. The current model only differentiates between potable and non-potable water, instead of the many levels of treated water that may be suitable for different uses. While the lowest quality recycled water is acceptable for uses such as industrial cooling, water used for food crops generally requires additional treatment. It remains to be seen whether introducing more finely differentiated categories of end-uses as well as multiple qualities would change the qualitative results presented here.

Appendix

The steady state condition relating price and aquifer head is

$$c_B = G(h_T) \quad \text{where} \quad G(h_T) \equiv c_G(h_T) - \frac{\gamma^{-1} c_G'(h_T)[I - L(h_T)]}{\delta + \gamma^{-1} L'(h_T)}.$$

Since the unit cost of desalination is constant, the head level that solves the steady state condition is unique if G is strictly monotonic in h and $c_B < G(0)$. We prove that this is indeed the case by showing that the derivative of the right hand side with respect to h is negative for any value of h. Applying the quotient rule and the chain rule to differentiate the term yields the following result:

$$G'(h_T) = c_G'(h) -$$

$$\frac{[\delta + \gamma^{-1}L'(h)]\{-\gamma^{-1}c_G'(h)L'(h) + \gamma^{-1}c_G''(h)[I - L(h)]\} - \gamma^{-1}c_G'(h)[I - L(h)]\gamma^{-1}L''(h)}{[\delta + \gamma^{-1}L'(h)]^2}.$$

That the term is negative for any positive h follows from the assumed characteristics of the leakage and extraction cost functions.

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