# Technological Change, Investment in Human Capital, and Economic Growth
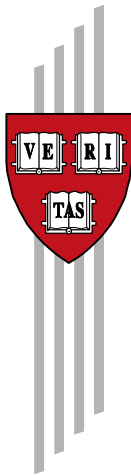
Yong Jin Kim and Jong-Wha Lee

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# Technological Change, Investment in Human Capital, and Economic Growth

Yong Jin Kim and Jong-Wha Lee*

## Abstract

This paper presents a theoretical model to analyze the effects of technology change on growth rates of income and human capital. We set up an overlapping generations model in which young agents invest in both width and depth of human capital in order to adopt new technologies. The model develops explicitly the micro-mechanism of the role of human capital in adopting new technologies as well as that of the process of human capital production. In our model an increase in the technology uncertainty decreases growth rates of income and human capital by lowering efficiencies both in creating new knowledge and in adopting new technologies. We also show that, depending on the initial structure of human capital and the uncertainty about the nature of new technologies, an economy can have multiple growth paths. Hence, increased inflows of new technologies with more uncertain characteristics may affect human capital accumulation and income growth adversely, leading the economy to a low growth trap.

**Keywords**: education, endogenous growth, human capital investment, technology adoption.

**JEL Classification Codes**: J24, O33

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# Technological Change, Investment in Human Capital, and Economic Growth

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## I.      Introduction

In this paper, we investigate how investments in human capital respond to the arrival of a new technology, and how long-term growth is determined by the adoption of the new technology in an economy. We explore these questions by focusing on the specific mechanism of the role of human capital in adopting new technology, and that of the process of human capital production.

The effect of technology progress on human capital investments has increasingly become an important topic. The literature shows that the effects of technology change on demand for human capital depend on whether there is technology-skill substitutability or complementarity. In a case of technology-skill complementarity, technology progress changes the relative demand for skill towards skilled and educated workers, and hence increases the investment in human capital. But, technological change can occur in a direction to decrease requirements for education and training, which is a case of technology-skill substitutability. Empirical evidence seems to support that technology-skill complementarity, rather than substitutability, is a dominant feature. Bartel and Lichtenberg (1987), Autor, Katz and Krueger (1997) and Bartel and Sicherman (1998) show that industries with higher rates of technology change experienced increases in the demand for more educated and skilled workers. But, Goldin and Katz (1996) provide a historical example of technology-skill substitutability in which the transition from the artisan shop to the factory decreased the overall demand for skill.

Some earlier studies explored the effects of uncertainty on the investment for education, considering that higher rates of technological changes are in general associated with greater uncertainty about the characteristics of future technologies. In this context, Lavhari and Weiss (1974), Paroush (1976), and Eaton and Rosen (1985) show that uncertainty has an ambiguous effect on human capital investments. Increased uncertainty in future earnings will decrease demand for education under the standard assumption of a risk aversion of workers. But, agents can also increase investment in human capital because human capital can help facilitate adjustments to future shocks.

While these previous studies attempted to assess the effects of technological change on human capital investment in various ways, they do not provide an integrated model that analyzes the effects of technological change on human capital accumulation and income growth. The purpose of this paper is to present a model that characterizes a general equilibrium path of human capital investment and economic growth for an economy experiencing technological changes. Stokey (1991), Lucas (1993) and Parente (1994) develop various types of models in which technology adoption is determined endogenously by the interaction with human skills. One of the main features of this paper, which differentiates our model from the existing ones, is that we explicitly model the micro-mechanism of human capital accumulation and technology adoption in uncertain environments. The key idea of the micro-mechanism is that the more closely one agent's knowledge is related to the knowledge of a new technology (or to the new knowledge to be created), the less time the agent spends in adopting the technology (or in creating the knowledge).

The model is an overlapping generations model where human capital plays an essential role in adopting new technologies. We assume that agents, when young, make investments in human capital and old agents adopt new technologies by using the human capital stock that they have accumulated when young. A distinguishing feature of the model is that it explicitly models the different roles of width and depth of human capital structure in adopting new technologies and in creating new human capital stock. In the model, the width of human capital determines the cost of technology adoption. Width of human capital represents the number of various specific knowledge points that human capital contains. Because each knowledge point helps to decipher and understand various characteristics of the future technology, wider human capital structure lowers the cost of adopting technologies. In contrast, the depth of human capital determines the level of the technology that can be adopted. In other words, higher quality human capital can adopt higher level technology. When an agent holds a certain level of depth of human capital regarding a specific type of knowledge, she acquires an ability to understand and adopt that specific type of knowledge.

The model emphasizes that technology adoption is endogenously determined by the expected cost of technology adoption and the uncertainty related to technology shocks. If the adoption cost is low, or if the level of uncertainty related to future technology shocks is small, agents make more investment in human capital when young, leading to adopting all the new

technology, when old. This occurs because adopting new technology is always more profitable than using old technology in this case. In contrast, if the adoption cost is high and thereby technology adoption is more likely to be unprofitable in the second period, then young agents make smaller investment in human capital, resulting in lower equilibrium growth rates of human capital and income.

The model presents several interesting implications about the effects of technological change on human capital accumulation and income growth. We show that higher expected rates of a technology advance increase investment in human capital and thus growth rates of income and human capital in the economy. Technology advance also changes the structure of human capital, depending on whether the technology adoption in the second period is certain or not. If agents know that all the new technologies will be adopted in the next period, investments in both width and depth of human capital increase. But, if technology adoption is not always an optimal strategy even with a new technology shock because of the high adoption cost, agents increase investment only in the width of human capital.

An increase in the technology uncertainty, on the contrary, decreases growth rates of income and human capital by lowering the efficiencies both in the creation of new knowledge that is used to adopt the new technologies and in the adoption of new technologies. An increase in the technology uncertainty decreases the efficiency of the creation of new knowledge, which is used to adopt the new technologies, because the new knowledge is so remotely related to the current knowledge that it becomes more difficult to identify and create this knowledge using the current knowledge. An increase in the uncertainty about the characteristics of future technologies will also decrease the efficiency in technology adoption because the cost of adopting the new technologies increases.

The model also shows that, depending on the initial structure of human capital stock, and the speed and uncertainty of new technology advances, the economy can have multiple growth paths. If the adoption cost is low due to the initial conditions, the agents always adopt the new technology and increase investment in human capital, resulting in higher growth rates of income and human capital. The economy follows a sustained balanced long-run growth path. But, if the adoption cost is too high and technology adoption is uncertain, the economy can show decelerating growth rates of human capital and income over time, and eventually be trapped in poverty with no human capital accumulation and no technology adoption, thus leading to slower

growth.[1] This implies that an increased inflow of advanced technologies caused by trade openness and FDI may have an adverse effect on human capital accumulation and income growth in a host economy, leading the economy to a low growth trap.[2] It is because the globalization increases not only the probability of having a new technology shock but also the level of the uncertainty about the characteristics of new technologies.

This paper is organized as follows. Section II describes and solves the basic model of certain technology adoption. In this economy, agents adopt all the new technologies when they occur. In Section III, a more general model of technology adoption, in which technology adoption is uncertain, is presented. In this economy agents adopt new technologies only when it is profitable. Additionally, the possible existence of multiple equilibria of the model is analyzed. Implications of these results are pursued in Section IV, and Section V concludes.

## II.    Basic Model

This section describes technology and environments of the economy, and the formal maximization problem of a representative agent living for two periods in an overlapping generations setting.

### 2.1 Technology and Environments of the Economy

The economy is described by an overlapping generations model with one commodity. The model economy consists of identical agents living for two periods, young and old. To maximize their utility, they decide how to allocate their endowed two units of time, one unit for each period, between work and education when young, and between work and technology adoption when old.

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[1] Previous studies point out that technology advance can cause temporary economic recession, although contributing to growth in the long-run. For example, Helpman and Rangel (1998) show that a recession is unavoidable with technology change when human capital of experienced workers becomes obsolete as they move from the old sector to the new sector.

[2] Borensztein, De Gregorio and Lee (1998) show that FDI inflows had an adverse effect on per capita income growth of the less developed countries which did not have a minimum threshold stock of human capital. Flug, Spilimbergo and Wachtenheim (1998) show that higher volatility of income has a negative effect on human capital accumulation. For the discussion of the empirical evidence on the effect of trade on growth, see Sachs and Warner (1995) and Rodriguez and Rodrik (1999).

**Technology and Adoption Cost**

In this economy, a new and advanced technology $A_t$ is assumed to occur with a probability of $P$ in each period.[3] The characteristics of a new technology are represented by a point on a continuous technology space of the real line $[0, S]$. Hence, this economy has two sources of uncertainty involved with the occurrence of a new technology: $(1-P)$ representing the uncertainty of having a technological advance and $S$ the uncertainty about the characteristics of a new technology.

Old agents adopt a new technology when a technology shock occurs.[4] The old agents spend a certain fraction of the endowed one unit of time to adopt the new technology, utilizing human capital stock they have accumulated through education when young. The rest of the time will be used to earn wage income. However, without a technology shock, old agents use all the endowment of time and human capital stock only for production and wage income.

A distinguishing feature of the model is that it provides a micro-mechanism of the role of human capital stock in adopting technologies, emphasizing two aspects of human capital in the adopting process. Human capital structure (or knowledge structure) consists of two dimensions: the width and the depth of human capital. The width of human capital represents flexibility, adaptability, and the feature of the allocative role of general human capital in adopting a new technology.[5] We denote the level of width of education as $N_t$ representing a set of $N$ units of different knowledge points in the technology space $[0, S]$. To adopt a new technology, an agent uses the knowledge that is most closely related to the characteristics of this technology. The

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[3] The probability, $P$, can represent the probability of success in adopting technologies due to limited information. In the case of the R&D model, $P$ represents the probability of success of R&D investments.

[4] In this basic model, all the new technologies are adopted whenever they occur irrespective of their profitability. And to make the model time consistent, we will impose certain adoption restrictions on the values of parameters of the model such that the second period utility with the adoption is higher than without it with these parameter values satisfying this restriction. Later in Section III, we will present a more general model in which agents maximize their utility by considering that they have the option of adopting new technologies depending on their profitability in the second period.

[5] There are several studies focusing on this type of role of human capital. For example, Nelson and Phelps (1966) argue that education increases the ability of workers to adjust to changing conditions by adopting new technologies. Welch (1970) also notes that education may improve workers' ability to acquire and decode information about the productivity and cost of the inputs. Schultz (1963) has written that schooling can strengthen society's ability to take advantage of new job opportunities associated with economic growth. Schultz (1971) also emphasizes that a primary role of education is to reinforce society's ability to adjust to disequilibria and to changing socio-economic conditions As for empirical research, Welch (1970), Bartel and Lichtenberg (1987), and Foster and Rosenzweig (1996), Benhabib and Spiegel (1994) find complementary relationships between human capital and technology investment, using different data sets.

bigger the width of the accumulated knowledge is, the higher is the probability that the new technology can be more easily understood and adopted. The wider human capital leads to a lower adoption cost because the characteristics of a new technology can be more probably located at a point closer to the knowledge points already accumulated.

The depth of human capital measures the size of each specific unit of knowledge that human capital stock contains. It represents the size of specific human capital stock or the quality of human capital, which has a strong complementarity with a new technology. We assume that the depth of human capital determines the level of absorption capacity that determines the level of technology to be adopted. For instance, if the agent accumulated a broad set of knowledge on many concepts at a shallow level because of either little experience in adopting technologies or the poor quality of the knowledge itself, she could not adopt higher level technologies. Therefore, the new technology can not be adopted above the level of the depth of human capital ($Q_t$). For simplicity, we assume that $A_t = Q_t$.

While the depth of human capital picks out the highest level of a new technology to be adopted, the width of human capital determines the adoption cost. To adopt a technology whose knowledge point is located at $x$ with the level of new technology A, agents are assumed to spend the adoption time of

(1) $$l_A = a|x - s| \cdot A,$$

where $s$ denotes the location of the knowledge that an agent uses to adopt a technology with a knowledge point $x \in [0, S]$, and is located closest to point $x$ among the agent's $N_t$ number of invested knowledge points.

This specification of adoption cost implies: To adopt a new technology with a higher level, agents should pay a higher adoption cost.[6] And the adoption time cost increases

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[6] Note that equation (1) becomes $l_A = a|x - s| \cdot Q$ because the depth of human capital determines the quality of a new technology A to be adopted. We specify that technology adoption time is determined by the level of the width of human capital and the quality of new technology to be adapted. In addition, we can allow that the depth of human capital also determines adoption time. A higher depth of human capital can, by lowering the gap between existing specific human skills and the new technology, decrease the adoption time. We can specify this as (1a) $l_A = a|x - s| \cdot (A/Q) \cdot A$. This equation implies that with given A, higher Q decreases the adoption time. Then, (continued)

proportionally to the distance between two knowledge points ($x$ and $s$). Here, this occurs because this distance represents the degree of similarity between these two pieces of knowledge.

To minimize the expected adoption cost, the $N$ knowledge points must be equally distributed over the knowledge space (technology space) as in Figure 1.[7] Figure 1 depicts the relationship between the adoption cost and the location of the characteristics (knowledge point) of a new technology represented by $x$, when $N=3$. $N=3$ implies that agents invested in three knowledge points at n1, n2 and n3 on the knowledge space [0, $S$], which are located at $\frac{S}{6}$, $\frac{3S}{6}$ and $\frac{5S}{6}$, respectively.[8]

## Human Capital Accumulation

After an old agent adopts a new technology with a level of quality $Q$, the agent's level of specific human capital (i.e., the depth of the human capital held by the old agent) becomes $Q$:

$$(2) \qquad\qquad H_{ot} = Q_t,$$

This equation implies that the old agent forms her specific human capital by adopting technology ($Q$) and uses it for production. Here, this human capital is fully embodied into the adopted technology.[9] And because we assume that there is no investment in human capital when the agent is old, the old agent holds the same width of human capital of her first period.

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note that with the assumption that the level of new technology is determined by the level of the depth of human capital, that is, $A_t = Q_t$, the specification (1a) is expressed again as $l_A = a|x-s| \cdot Q$. Although in the specification of (1a), the agents can choose the level of $Q$ above the level of A to minimize adoption cost, we do not allow this by assuming that it is not realistic that the existing level of human skills already exceeds the level of new technology.

[7]The structure of this minimization problem is identical to Baumol and Tobin's inventory model of money demand. In their model, "$N$" represents the number of trips to a bank. As in Baumol and Tobin's model, we treat $N$ as a variable that can take any positive continuous value on the real line.

[8] This modeling technique for technology adoption is similar to the one in Eaton and Schmitt (1994).

[9] In this model, a new technology is more like a new piece of an idea about the ways to do certain things. Thus, agents can accumulate specific human capital by utilizing a new technology.

We also assume that a certain fraction of the technology, once adopted and being currently used, can also be used by young agents without cost due to the spillover effect. The young agents' specific human capital at time $t$ becomes:

(3) $$H_{yt} = d\overline{Q} ,$$

where $\overline{Q}$ denotes the current old generation's amount of specific human capital and $\delta$ measures the spillover effect, $0 < \delta \leq 1$.

A young agent, by investing a certain fraction of her time in education, builds her human capital stock for the second period. As already emphasized above, the human capital stock has two dimensions, width ($N$) and depth ($Q$). We assume that the young agent invests an amount of time $l_E$ in education and builds her human capital stock of $NQ$:

(4) $$N_t Q_t = b \cdot l_E \cdot \overline{NQ} ,$$

where the bar on the variable denotes belonging to 'old agents of the previous generation', and the parameter $b$, decreasing in $S$, measures the efficiency of human capital formation. [10] And $b > 1$ implies that human capital stock can increase over time if the agent invests a certain fraction of her time in education such that $b \cdot l_E > 1$.

This equation of education production implies that the more education old agents of the previous generation have, the more human capital young agents can accumulate with any fixed input of time investment in education. It also implies that because the agent can not increase both $N$ and $Q$ simultaneously with a given time investment in education, there exists a tradeoff

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[10] See Appendix A for the micro-mechanism behind this equation of human capital formation through education. Following Appendix A, $b \cong \dfrac{2d}{k(1 + e + ed)S}$. Note that the parameter $b$ measuring the efficiency of human capital formation decreases in $S$.

between $N$ and $Q$.[11] Without time investment in education, the human capital stock can not grow over generations in this economy.

**Production Technology**

The old agent is endowed with one unit of time and allocates it among technology adoption ($l_A$), and work (1-$l_A$). Young agents invest $l_E$ in education and work for the rest of the time, (1- $l_E$).

The representative firm employs young and old workers together, with the production function of

$$(5) \qquad y = H_{yt} \cdot (1 - l_E) + H_{ot-1} \cdot (1 - l_A),$$

Here, we assume a linear production technology, which uses specific human capital as the only input. We also assume that the competitive wage rate per one unit of labor supply adjusted by human capital is determined at one. This production function also implies that young and old agents' human capital stocks are perfect substitutes for each other. This simple assumption enables us to focus on main features of the model without excessive complexities.

**2.2. Equilibrium**

This subsection characterizes the equilibrium of the model economy. Recall that we assume in this basic model that once a new technology shock occurs in the second period, the agents will always adopt the new technology. We assume that the utility of the second period with the

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[11] We can relate $NQ$ to conventional measures of human capital such as quantity and quality of schooling (see Barro and Lee (1996)). Let us take an example of the elementary, high school and college educational system. Elementary school teaches students, for example, the knowledge located at 1/2 on the knowledge space [0,1], while high school teaches the knowledge at 1/4 and at 3/4 on [0,1]. College teaches the knowledge at 1/8, at 3/8, at 5/8 and at 7/8. In other words, as the level of education increases, the distance between adjacent knowledge points that students acquired becomes smaller and smaller (i.e., the knowledge points becomes more finely distributed on [0,1]). In this context, an increase in the education level (years of schooling) increases $N$ (the number of knowledge points). In this model, $Q$ represents the depth of each piece of knowledge point. Therefore, Q can be interpreted as the investment per each piece of knowledge. In this context, $Q$ can be measured by the public plus private expenditures per one year of schooling. And $NQ$ represents total expenditures on schooling. In this context, an increase in the length of years of schooling increases the ratio of $\dfrac{N}{Q}$, if the public plus private expenditures per one year of schooling increases less than proportionally.

technology adoption is always higher than that without by imposing time consistent restrictions on the parameter values of the model so that the adoption of the new technology is always certain.[12]

## Maximization Problem of a Representative Agent

A representative agent maximizes her two period discounted utility by allocating time between education and work while young, and between technology adoption and work with a new technology shock while old, with wage rates equal to one given exogenously.

A representative young agent optimally decides the width ($N$) and depth ($Q$) of human capital to maximize her log utility preference as[13]

$$\max_{N,Q} \quad U(c_{yt}, c_{ot}) = \log c_{yt} + \frac{1}{1+r} E[\log c_{ot}]$$

$$s.t. \quad c_{yt} = d\overline{Q}(1 - \frac{NQ}{b\overline{N}\overline{Q}}),$$

(P1)

$$c_{ot} = (1 - l_A)Q \quad with \quad a \quad tech. \quad change,$$

$$c_{ot} = d\overline{Q} \quad without \quad it, \quad and$$

$$l_A = \min_{x \in \{N \cdot po \text{ int } s\}} a|x - s| \cdot Q.$$

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[12] We will relax this assumption in Section III. This assumption implies that in equation (6), $Q(1 - aQ\frac{S}{2N}) > d\overline{Q} \Rightarrow 1 - \frac{d\overline{Q}}{Q} > \frac{aS}{2}\frac{Q}{N}$. Then, in the second period agents always adopt every new technology whenever it occurs, because the second period utility with the adoption of new technologies always exceeds that without the adoption. Note that this condition can be satisfied more probably when either δ or S is smaller. Note also that if $d$ is sufficiently large, then it is better for agents to keep an old technology rather than to adopt a new technology even when a technology shock occurs. We will identify this restriction in Section IV more formally.

[13] The solutions of the model with a CRRA utility function are not qualitatively different from those with a logarithmic case assumed here. Even though the risk aversion parameter interacts with decision variables in a very complicated nonlinear way, this does not alter the qualitative nature of the implications we explore.

where $\dfrac{NQ}{b\overline{NQ}}$ represents the time investment for education $(l_E)$[14], and $l_A$ the time cost of technology adoption. Note also that wage rate is equal to one due to the linear technology of (5). With this setup of the model, we solve the utility maximization problem as described below.

Second Period Expected Utility

Note that with a technology shock occurring with the probability $P$, agents can adopt the new advanced technology with the quality level of $Q$. Note also that without a technology shock agents must keep their previously inherited technology, working the whole one unit of time without incurring any technology adoption cost. From (1) to (5), the expected utility of an agent of generation t in his second period is

$$
\begin{aligned}
(6) \quad \frac{P}{1+r}E[\log c_{ot}] &= \frac{P}{1+r}2\frac{N}{S}\int_0^{\frac{S}{2N}}\log(Q(1-aQx))dx+\frac{1-P}{1+r}\log(d\overline{Q}) \\
&= \frac{P}{1+r}\frac{1}{S}\int_0^S\log(Q(1-aQ\frac{y}{2N}))dy+\frac{1-P}{1+r}\log(d\overline{Q})
\end{aligned}
$$

First Period Maximization

Using (6), the first period maximization problem with a logarithmic utility can be described by

$$
(7)\quad \max_{N,Q}\quad \log c_{yt}+\frac{1}{1+r}E[\log c_{ot}]=\log(1-\frac{NQ}{b\overline{NQ}})+\frac{P}{1+r}\frac{1}{S}\int_0^S\log[Q(1-\frac{aQy}{2N})]dy+const
$$

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[14] With the more general description of the human capital accumulation equation of (4) as $NQ = d\overline{NQ} + bl_E\overline{NQ}$ ($l_E = \dfrac{NQ}{b\overline{NQ}} - \dfrac{d_H}{b}$), where $d_H$ represents the fraction of old agents' human capital that young agents inherit through spillover. However, this more general representation will not change the above maximization problem (P1) if we assume that young agents are endowed with one plus $\dfrac{d_H}{b}$ units of time instead of one unit as assumed in the case of (4). Thus, we can easily see that the results will not change qualitatively with a slightly different functional form of (4).

## Equilibrium Balanced Growth

We will derive the FOC's with respect to $N$ and $Q$ respectively as

(8)
$$-\frac{Q}{b\overline{NQ}-NQ}-\frac{P}{1+r}\frac{1}{N}+\frac{P}{1+r}\frac{1}{S}\int_0^S \frac{1}{N-\frac{aQy}{2}}dy$$
$$=-\frac{Q}{b\overline{NQ}-NQ}-\frac{P}{1+r}\frac{1}{N}-\frac{2P}{(1+r)aQS}(\log(N-\frac{aSQ}{2})-\log N)=0$$,

(9)
$$-\frac{N}{b\overline{NQ}-NQ}+\frac{P}{1+r}\frac{1}{Q}+\frac{P}{1+r}\frac{1}{S}\int_0^S \frac{-\frac{ay}{2}}{N-\frac{aQy}{2}}dy$$
$$=-\frac{N}{b\overline{NQ}-NQ}+\frac{2P}{1+r}\frac{1}{Q}+\frac{2PN}{(1+r)aQ^2S}(\log(N-\frac{aSQ}{2})-\log N)=0$$

Subtracting (8), multiplied on both sides by $N$, by (9) multiplied on both sides by $Q$, and additional algebra yield

(10)
$$\frac{3}{2}(z-1)=\log z,$$

where $z=1-\frac{aS}{2}\frac{Q}{N}$.

The relationship of (10) is very simple. We can easily prove that there exists a unique solution for z satisfying (10), as depicted in Figure 2. Let us call this solution as $z^*$. Then we can easily find $z^*$ to be a constant of about 0.417 through computer simulation. Utilizing the relationship of $z=1-\frac{aS}{2}\frac{Q}{N}$, the ratio of depth to width of human capital can be represented by

(11)
$$\frac{Q}{N}=\frac{2(1-z^*)}{aS}.$$

It is easy to solve for $N$ and $Q$ in the equilibrium. In the equilibrium, $N$ and $Q$ grow at the same rate of $g$ as (11) implies. We can derive easily that the equilibrium growth rate of income is equal to the growth rate of $Q$. Thus, the equilibrium is a balanced growth path.

By utilizing the relationship of $\dfrac{N}{\overline{N}} = \dfrac{Q}{\overline{Q}} = 1 + g$ derived from (11),[15] the education time of young agents can be denoted as $l_E = \dfrac{NQ}{b\overline{NQ}} \Rightarrow l_E = \dfrac{(1+g)^2}{b}$.

Using this and substituting (11) into (8) multiplied by $N$ yield the growth rate of income ($g$) of

(12)
$$-\frac{NQ}{b\overline{NQ} - NQ} - \frac{PS}{1+r} - \frac{2PN}{(1+r)aQS}(\log(N - \frac{aSQ}{2}) - \log N) = 0.$$
$$\Rightarrow -\frac{(1+g)^2}{b - (1+g)^2} - \frac{PS}{1+r} - \frac{2PN}{(1+r)aQS}\log z^* = 0.$$
$$\Rightarrow -\frac{(1+g)^2}{b - (1+g)^2} + \frac{P}{2(1+r)} = 0.$$
$$\Rightarrow 1 + g = \sqrt{\frac{Pb}{2(1+r)+P}}$$

Then, (12) yields the expected growth rate of income and human capital as

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[15] From (11), we can see that this steady state relationship does not hold on the date when the parameter value of $a$ or $S$ changes and then it holds from one period after this date. However, even with any change of the other parameter values, this relationship still holds. Form (9), (10) and (11), we can easily solve for $Q$ and $N$ as:

$Q = \sqrt{\dfrac{2(1-z^*)bP\overline{NQ}}{(2+2r+P)aS}}$ and $N = \sqrt{\dfrac{abPS\overline{NQ}}{2(1-z^*)(2+2r+P)}}$. From this we can infer that even on the date

when $S$ changes, the growth rate of $N$ does not change in $S$, while that of $Q$ decreases in $S$, considering that $bS$ does not change in $S$ (see Footnote 10). Thus, we can infer that when $S$ changes once-and-for-all, the growth rate of $N$ does not change now and decreases a little next period, and stays at this rate afterwards; while the growth rate of $Q$ decreases on the date of change, it rises to a growth rate a little lower than the previous steady state growth rate next period, and stays at this rate afterwards. We can also easily see that an increase in $P$ raises $N$ and $Q$ by the same proportion, and that the growth rate jumps once-and–for-all on the date of change, and stays at this rate afterwards, as (12) implies.

(13) $$E[1+g] = P(1+g) + (1-P)d.$$

(12) and (13) imply that the expected growth rate increases in $P$, but does not change in S.

(11) yields the expected technology adoption time of

(14) $$E[l_A] = \frac{P}{S} \int_0^s \frac{aQy}{2N} dy = \frac{P}{2}(1-z^*).$$

## 2.3. Technology Change and Human Capital Investment

The characterizations of the equilibrium described above provide several interesting implications on human capital accumulation. In particular, the effects of uncertainty about new technologies on human capital accumulation are analyzed below:

### Probability of Having a Technology Shock ($P$)

An increase in the probability of having a technological advance increases human capital and thus income growth rates. Therefore, more certain occurrence of future technology shocks contributes to more rapid human capital accumulation.

From (11) and (12), we know that an increase in $P$ increases growth rates of income and education level by offering more opportunities to upgrade their technology. However, note that it does not change the relative investment size of width to depth of education.

### Uncertainty about the Characteristics of Future Technology ($S$)

An increase in S[16] represents an increase in the uncertainty about the characteristics of future technologies that agents will adopt in the next period. In other words, an increase in $S$ implies that the knowledge space to which knowledge points of the future technology belongs increases.

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[16] An increase in $S$ represents that every interval between any two adjacent knowledge points increases by the equal amount. Another comment on the exercises of changing $S$ is: A change in $S$ has the identical effects with that of $a$ throughout this paper, where $a$ represents the inefficiency of adoption or "barriers to technology adoption" as in Parente and Prescott [1994]. This occurs because $S$ appears always with $a$ in every equation of this paper as in the form of $aS$. Exercises on $S$ (or $a$) provide the identical implications with those of Parente and Prescott.

Thus, with an increase in $S$, agents must increase their ratio of width to depth of knowledge to lower the expected technology adoption cost.

An increase in $S$ decreases growth rates of human capital and income, as we can infer from Footnote 15. This occurs because it decreases the efficiency of human capital formation $b$, as stated in Footnote 10 and Appendix A. Also an increase in $S$ raises the relative investment size of width to depth of education, as we can see from (11). It is very intuitive that agents will increase their adaptability or flexibility by investing relatively more resources in width of education in the face of increased uncertainty about future technology. The following lemma summarizes these findings.

**Lemma 1**: In the basic model, an increase in the probability of having a technology shock ($P$) increases the growth rates of both width and depth of human capital and income, not affecting the relative investment size of width to depth of education. However, an increase in the uncertainty about the characteristics of future technologies ($S$) decreases the growth rates of both width and depth of human capital, and income, and increases the relative investment size of width to depth of education.

## III.    An Extended Model with a Choice of Technology Adoption

The basic model presented in the previous section shows the equilibrium dynamics of human capital and income with the assumption that once the technology shock occurs in the second period, the agents always adopt it.  The basic model assumes that the second period utility with technology adoption is always higher than that without it.

This section characterizes an extended model in which agents maximize their utility considering that agents will decide whether to adopt the new technology depending on the size of the adoption cost whenever the new technology shock occurs.[17] If the adoption cost is too large, the agents will use the old technology rather than adopt a new technology. This can happen when the knowledge of the new technology is remotely located from the knowledge points invested.

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[17] Given a set of initial parameter values, the utility of this extended model is always higher than or equal to that of the basic model. This occurs because, while the extended model does not have any market failing features, the basic
(continued)

We can calculate the threshold level of the location of knowledge ($s^*$) such that if the knowledge of the new technologies is located above this level, then adopting new technology becomes less profitable than sticking to old technology. This threshold value of $s^*$ satisfies

(15) $$Q(1-aQs) = d\overline{Q} \implies \frac{Q-d\overline{Q}}{aQ^2} = s^* \in (0, \frac{S}{2N}),^{18}$$

which implies that the second period utility with technology adoption equals that without it at the threshold value of $s^*$.

## 3.1. The Maximization Problem

A representative young agent with a logarithmic utility function solves the following maximization problem.[19]

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model has a restriction on decision variables such that agents should "adopt every new technology irrespective of its profitability."

[18] In the previous section, we solved the case in which $\frac{Q-d\overline{Q}}{aQ^2} = s^* > \frac{S}{2N} \implies 1 - \frac{d\overline{Q}}{Q} > \frac{aS}{2}\frac{Q}{N}$. (11) and (12)

transform this relationship into $\sqrt{\frac{Pb}{2(1+r)+P}} > \frac{d}{z^*} \implies P > \frac{2d^2(1+r)}{bz^{*2}-d^2}$. In other words, under this condition in the basic model of Section II, agents in their second period will also adopt every new technology, even if they are not constrained to adopt every technology. This behavior is time consistent because their second period utility with the adoption is greater than that without it.

[19] This maximization problem needs the restriction that $s^* \leq \frac{S}{2N}$. However, to simplify the solution process, we first solve the model without this restriction. And then if the decision variable $s^*$ is found to be bigger than the boundary value (i.e., $s^* > \frac{S}{2N}$) without this restriction, we have only to solve the basic model of Section II after setting $s^*$ to be $\frac{S}{2N}$. The theoretical justification for this solution process is given in Lemma 3. We call the condition to produce the relationship of $s^* < \frac{S}{2N}$ as the uncertain adoption condition of the extended model.

$$\text{(P3)} \qquad \max_{N,Q} \ \log(1 - \frac{NQ}{b\overline{\overline{NQ}}}) + \frac{2NP}{S(1+\boldsymbol{r})} \{ \int_0^{s^*} \log[Q(1-aQx]dx + \int_{s^*}^{\frac{S}{2N}} \log \boldsymbol{d}\overline{Q}\, dx\}$$

$$+ \log \boldsymbol{d}\overline{Q} + \frac{1-P}{1+\boldsymbol{r}} \log \boldsymbol{d}\overline{Q} \Rightarrow$$

$$\max_{N,Q} \ \log(1 - \frac{NQ}{b\overline{\overline{NQ}}}) + \frac{2PN}{(1+\boldsymbol{r})S} \{(s^* - \frac{1}{aQ}) \log(1-aQs^*) - s^* + s^* \log Q)$$

$$- \frac{P}{1+\boldsymbol{r}} \frac{2N}{S} s^* \log \boldsymbol{d}\overline{Q} + \log \boldsymbol{d}\overline{Q} + \frac{1}{1+\boldsymbol{r}} \log \boldsymbol{d}\overline{Q}$$

.

(15) simplifies this maximization problem as

$$\max_{N,Q} \ \log(1 - \frac{NQ}{b\overline{NQ}}) + \frac{2PN}{(1+\boldsymbol{r})S} \{ \frac{1}{aQ} (\log \frac{Q}{\boldsymbol{d}\overline{Q}} + \frac{\boldsymbol{d}\overline{Q}}{Q} - 1) \} + \log \boldsymbol{d}\overline{Q} + \frac{1}{1+\boldsymbol{r}} \log \boldsymbol{d}\overline{Q}$$

The first order conditions with respect to $N$ and $Q$ are as follows:

$$\text{(16)} \qquad -\frac{Q}{b\overline{NQ} - NQ} + \frac{2P}{(1+\boldsymbol{r})S} \{ \frac{1}{aQ} (\log \frac{Q}{\boldsymbol{d}\overline{Q}} + \frac{\boldsymbol{d}\overline{Q}}{Q} - 1) \} = 0,$$

$$\text{(17)} \qquad -\frac{N}{b\overline{NQ} - NQ} + \frac{2PN}{(1+\boldsymbol{r})S} \{ -\frac{1}{aQ^2} (\log \frac{Q}{\boldsymbol{d}\overline{Q}} + \frac{2\boldsymbol{d}\overline{Q}}{Q} - 2) \} = 0.$$

Subtracting (17) times $Q$ from (16) times $N$, and simple algebra lead to

$$\text{(18)} \qquad \log \frac{\boldsymbol{d}\overline{Q}}{Q} = \frac{3}{2} (\frac{\boldsymbol{d}\overline{Q}}{Q} - 1).$$

Note that the structure of the above equation is exactly identical to (10). Therefore, by replacing $\frac{\boldsymbol{d}\overline{Q}}{Q}$ by $z*$ and using the results of (10), we obtain

$$\text{(19)} \qquad Q = \frac{\boldsymbol{d}}{z^*} \overline{Q}.$$

17

where $z*$ is a constant with the value of about 0.417. Equations (15) and (19) show that $Q$ and $s^*$ are functions only in $d\overline{Q}$, and also that $Q$ is always greater than $d\overline{Q}$, leading to $s^*>0$.

### 3.2. Equilibrium

**Human Capital Investment**

Substituting (18) into (16) and simple algebra yield

(20)
$$1+g_H = \frac{Pb\frac{2N}{S}s^*}{2(1+r)+P\frac{2N}{S}s^*},$$

where $g_H \equiv \frac{NQ}{\overline{NQ}}-1$ and $s^* = \frac{Q-d\overline{Q}}{aQ^2}$.

(20) implies that the growth rate of human capital stock ($g_H$) in the case of the uncertain adoption case of the extended model (i.e., $s^* < \frac{S}{2N} \Rightarrow \frac{2N}{S}s^* <1$) is always smaller than that in the certain adoption case (i.e., $s^* = \frac{S}{2N} \Rightarrow \frac{2N}{S}s^* =1$) of the basic model.

Equation (19) shows the equilibrium growth rate of the depth of human capital stock ($g_Q$), in this extended model, is given by

(21)
$$(1+g_Q) = \frac{Q}{\overline{Q}} = \frac{d}{z^*}.$$

Therefore, the depth of human capital may increase or decrease over time depending only on the value of $d$. If there exists a rather strong spillover of technology over generations (i.e., $d > z^* \cong 0.417$), the depth of human capital always increases over time.

We can compare the equilibrium growth rate of $\dfrac{\overline{\overline{Q}}}{Q}$ in this extended model with that in the certain technology adoption case of the basic model. In Section II, we solved the certain technology adoption model in which $\dfrac{Q - d\overline{Q}}{aQ^2} = s^* > \dfrac{S}{2N} \Rightarrow 1 - \dfrac{d\overline{Q}}{Q} > \dfrac{aS}{2}\dfrac{Q}{N}$. By (11) and (12), this relationship leads to $\sqrt{\dfrac{Pb}{2(1+r)+P}} > \dfrac{d}{z^*}$. From this, (12), and (19), we can easily see that the growth rate of $Q$ with the certain adoption of technology is always higher than that with the uncertain adoption of the extended model. By the same logic, the growth rate of income is also always higher with the certain adoption of technology than that with the uncertain adoption.

Substituting (18) and (19) into (16) yields

(22)
$$
\begin{aligned}
(1+g_N) = \dfrac{\overline{\overline{N}}}{N} &= \dfrac{bz^*}{d} - \dfrac{(1+r)S}{P}\dfrac{1}{s^*} \\
&= \dfrac{bz^*}{d} - \dfrac{ad(1+r)S}{z^*(1-z^*)P}\dfrac{\overline{\overline{Q}}}{\overline{N}}, \quad if \quad \dfrac{2N}{S}s^* < 1 \\
(&= \sqrt{\dfrac{Pb}{2(1+r)+P}}, \qquad if \quad \dfrac{2N}{S}s^* = 1).
\end{aligned}
$$

(22) implies that the growth rate of the width of human capital ($\dfrac{\overline{\overline{N}}}{N}$) is a function of several parameters, while that of the depth is a function only of $d\overline{Q}$. This also implies that while an increase in $P$ increases $N$, an increase in $S$ decreases $N$.[20] (22) also implies that the growth rate of width of human capital increases in $\dfrac{\overline{N}}{\overline{Q}} = \dfrac{d\overline{N}}{z^*\overline{Q}}$. That is, an increase in $\overline{N}$ decreases the adoption cost of new knowledge points in the human capital formation process, while an increase in $Q$ raises it, based on our interpretation of (4) presented in Appendix A. We can summarize the results related to the two types of uncertainty in the following lemma.

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[20] This result is quite different from the result in Section II in which an increase in $S$ raises $N$ as well as $Q$.

**Lemma 2**: In the uncertain adoption case ($s^* < \dfrac{S}{2N}$), an increase in the probability of having a technology shock ($P$) increases the width of human capital but does not affect the depth, resulting in higher level of human capital stock and higher growth rates of human capital stock and the expected income. And a <u>decrease</u> in uncertainty about the characteristics of new technologies ($S$) shows the identical effects on these variables.

**Growth Rate of Income**

The expected growth rate of income can be calculated as[21]

(23)
$$1 + g_I^E = P(1 - \frac{(\frac{S}{2N} - s^*)}{\frac{S}{2N}})\frac{Q}{\overline{\overline{Q}}} + (1 - P + \frac{\frac{S}{2N} - s^*}{\frac{S}{2N}}P)\boldsymbol{d}$$

$$= P(\frac{2Ns^*}{S})\frac{Q}{\overline{\overline{Q}}} + (1 - \frac{2Ns^*}{S}P)\boldsymbol{d}$$

where $s^* = \dfrac{Q - \boldsymbol{d}\overline{Q}}{aQ^2} = \dfrac{z^*(1 - z^*)}{a\boldsymbol{d}\overline{Q}}$.

Using (20), (22) and $s^* = \dfrac{z^*(1 - z^*)}{a\boldsymbol{d}\overline{Q}}$, we can solve $\dfrac{2N}{S}s^*$ as

(24)
$$\frac{2N}{S}s^* = \frac{2}{S}\{\frac{bz^{*2}(1 - z^*)}{a\boldsymbol{d}^2}\frac{\overline{N}}{\overline{\overline{Q}}} - \frac{(1 + \boldsymbol{r})S}{P}\} < 1$$

$$\Rightarrow \frac{\overline{N}}{\overline{\overline{Q}}} < (\frac{S}{2} + \frac{(1 + \boldsymbol{r})S}{P})\frac{a\boldsymbol{d}^2}{bz^{*2}(1 - z^*)} \equiv \boldsymbol{G}$$

---

[21] The variable $\dfrac{2Ns^*}{S}$, increasing in $\dfrac{\overline{N}}{\overline{\overline{Q}}}$, represents the proportion of the band of specialized knowledge points in the whole knowledge space. The band of specialized knowledge points denotes that agents adopt only the new technologies whose characteristics fall on this band. Thus, it is quite intuitive that an increase in this band increases the growth rate of income since the broader band implies the more frequent technology adoption.

This is the condition, under which the extended model belongs to the uncertain adoption case. In other words, if this condition is not satisfied, the basic model instead of (P3) should be solved.

**Dynamics of Human Capital Accumulation and Income Growth**

Equations (20), (22), (23) and (24) show that growth rates of human capital stock ($g_H$), width of human capital ($g_N$) and income ($g$) change over time depending on the dynamics of the ratio of width to depth of human capital ($\dfrac{\overline{N}}{\overline{Q}}$).[22] If $\dfrac{\overline{N}}{\overline{Q}}$ increases over time, the growth rates of $N$, $NQ$ and income increase unambiguously over time, and vice versa.

Using (19) and (22), the dynamics of $\dfrac{N}{Q}$ over time can be described by a difference equation of

(25)
$$\frac{N}{Q} = \frac{bz^{*2}}{d^2}\frac{\overline{N}}{\overline{Q}} - \frac{a(1+r)S}{(1-z^*)P}.$$

With these results, we obtain the following Propositions 1 and 2.

**<u>Proposition 1</u>**: If $\dfrac{d^2}{z^{*2}} > b$, the ratio of width of human capital to depth, and the growth rates of human capital and expected income continuously decline over time, resulting in no technology adoption in the extended model.

(Proof) The dynamics of $\dfrac{N}{Q}$ in (25) implies that with the above condition in this proposition, $\dfrac{N}{Q}$ decreases continuously over time, forcing the economy to remain in the uncertain adoption mode. Additionally, (24) implies that the band of the specialized knowledge (technology)

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[22] Recall that $Q$ always grows at a constant growth rate of $\dfrac{d}{z^*}$.

represented by $\dfrac{2N}{S}s^*$, increasing in $\dfrac{\overline{N}}{Q}$, will decline over time. Then, (23) and (24) imply that the expected income growth rate also declines.///

This proposition implies that the economy can undoubtedly result in a poverty trap if the economy has a low efficiency of human capital production ($b$) caused by an increase in $S$, or a large spillover of the existing technology ($\boldsymbol{d}$). In this economy, investment in the width of human capital becomes smaller and smaller over time, and fewer and fewer new technologies are adopted. Eventually, $N$ goes to zero and the economy will be trapped in the old technology.

**<u>Lemma 3:</u>** If $\dfrac{\boldsymbol{d}^2}{z^{*2}} < b$ and $\Gamma > \Phi = \dfrac{a\boldsymbol{d}^2(1+\boldsymbol{r})S}{(1-z^*)(bz^{*2}-\boldsymbol{d}^2)P} \Rightarrow P > \dfrac{2\boldsymbol{d}^2(1+\boldsymbol{r})}{bz^{*2}-\boldsymbol{d}^2}$, then the economic system has three different equilibria (one stable and two unstable ones) depending on the initial value of $\dfrac{N}{Q}$ (denoted simply by $\dfrac{\overline{N}}{Q}$ in the below). If $\dfrac{\overline{N}}{Q}$ equals exactly to a critical value ($\boldsymbol{F}$), which increases in $\dfrac{S}{P}$, then the economy follows the steady-state balanced growth path.[23] In this steady-state equilibrium, the ratio of width to depth of human capital ($\dfrac{N}{Q}$), human capital growth rate ($g_H$), and the expected income growth rate ($g_I^E$) remain constant over time. If $\dfrac{\overline{N}}{Q} > \boldsymbol{F}$, $\dfrac{N}{Q}$, $g_H$, and $g_I^E$ increase over time. Hence, the economy will eventually move into the certain technology adoption economy. In contrast, if $\dfrac{\overline{N}}{Q} < \boldsymbol{F}$, then $\dfrac{N}{Q}$, $g_H$, and $g_I^E$ decline over time, eventually leading to a poverty trap with no technology adoption.[24]

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[23] In the case with an additional condition of (24) holding with equality, the solution equals that of the certain adoption case of the basic model. This is intuitively obvious because this condition implies that agents adopt all technologies that occur.

[24] For the difference equation of (25), there exists one unstable equilibrium. However, for the economic system, there exist two long run stable steady state equilibria, poverty trap and certain adoption equilibrium as described in this proposition. The case of the unstable equilibrium of the difference equation system represents the long run (continued)

(Proof) See Appendix B.///

The inequality of $\dfrac{d^2}{z^{*2}} < b$ implies that the possibility of new technology adoption is more profitable than resting on the inherited old technology, resulting in positive investment in human capital. It is because $b$ denotes the efficiency of human capital formation to help adopt future technologies, while $d$ denotes the fraction of the old technology inherited from the old generation through spillover. In this sense the case with this inequality is more interesting and realistic than that with the reversed inequality with which Proposition 1 deals. These characterizations of the equilibrium growth paths are summarized in the following proposition.

**Proposition 2:** If the efficiency of human capital formation is very low ( $\dfrac{d^2}{z^{*2}} \geq b$ ), the economy will be trapped in poverty with no investment in human capital. If $\dfrac{d^2}{z^{*2}} < b$ , various equilibrium paths are characterized as follows depending on the value of $\dfrac{\overline{N}}{Q}$ [25]:

<u>Case 1:</u> $\Phi < \Gamma \Rightarrow P > \dfrac{2d^2(1+r)}{bz^{*2} - d^2}$ [26]

(1) $\Phi < \Gamma \leq \dfrac{\overline{N}}{Q}$ :  Certain Adoption Equilibrium Path,

unstable steady state equilibrium of the economic system. (However, if $s^* = \dfrac{S}{2N}$ in (24), this equilibrium is identical to the certain adoption equilibrium, as stated in Footnote 19.)

[25] From (20), (23) and (24), we can see that this ratio is the key variable determining the dynamics of the economy. The reasoning is as follows: (20) and (23) imply that an increase in the variable $\dfrac{2Ns^*}{S}$ representing the band of the specialized knowledge points increases growth rates of income as well as human capital. This is because the broader band implies the more frequent technology adoption. (24) in turn implies that the band of the specialized knowledge points increases in $\dfrac{\overline{N}}{Q} = \dfrac{d\overline{N}}{z^*Q}$ . Here, an increase in $\overline{N}$ decreases the adoption cost of new knowledge points in the human capital formation process through education, while an increase in $Q$ raises it.

[26] $\Gamma$ is defined in (24). And in the following section, we will focus on this case, because this case is more realistic and has more interesting aspects.

(2) $\Phi < \dfrac{\overline{\overline{N}}}{Q} < \Gamma$: Higher Growth Equilibrium Path in Uncertain Adoption Case

     Converging to (1),

(3) $\dfrac{\overline{\overline{N}}}{Q} = \Phi < \Gamma$: Steady State Equilibrium Path in Uncertain Adoption Case

     Converging to (1),

(4) $\dfrac{\overline{\overline{N}}}{Q} < \Phi < \Gamma$: Lower Growth Equilibrium Path in Uncertain Adoption Case

     Converging to Poverty Trap.

<u>Case 2:</u> $\Phi \geq \Gamma \Rightarrow P \leq \dfrac{2d^2(1+r)}{bz^{*2} - d^2}$

(1) $\Gamma = \Phi \leq \dfrac{\overline{\overline{N}}}{Q}$: Certain Adoption Equilibrium Path,

(2) $\dfrac{\overline{\overline{N}}}{Q} < \Gamma \leq \Phi$: Lower Growth Equilibrium Path in Uncertain Adoption Case

     Converging to Poverty Trap.


     Following Proposition 2, as $S$ or $\dfrac{1}{P}$ increases, the equilibrium growth path moves down from (1) to (2), from (2) to (3), and from (3) to (4) in Case 1, and from (1) to (2) in Case 2, because $\Gamma$ and $F$ increase in $S$ or in $\dfrac{1}{P}$. In other words, as $S$ or $\dfrac{1}{P}$ increases, one equilibrium path will decrease to the lower and lower growth equilibrium path.


**<u>Proposition 3:</u>** An increase in $S$ or in $\dfrac{1}{P}$ moves the equilibrium path down to the lower growth equilibrium path.


Implications of these results will be pursued in the next section.

# IV.    Multiple Growth Paths and Implications for Public Policies

This section presents several implications related to the existence of multiple equilibria of an economy by utilizing the results of Sections II and III. The presence of multiple equilibria suggests a possible effectiveness of government intervention. In this context we illustrate the effects on economic growth of several government policies such as technology and education policy, open trade and FDI policy, and a coordination policy.

## 4.1. Technology Uncertainty and Multiple Equilibria

The comparison of Sections II and III shows that there exist two quite different growth paths of the economy. In the economy of the certain technology adoption of the basic model in Section II, agents maximize their utilities with the restriction that they must always adopt new technology once it occurs irrespective of the profitability. In this basic model, agents make more investment in human capital leading to higher growth than the uncertain adoption case of the extended model. The economy in the basic model follows a sustained balanced growth path as described in Lemma 1 in Section II.

In contrast, in the model of the uncertain adoption of the extended model, the agents are not sure in advance of whether they will adopt the new technology in the next period or not. If the adoption cost turns out to be too high and thus the technology adoption is not profitable in the second period, the agents will not adopt the new technology even when it occurs. With this uncertain adoption in the second period, the first-period human capital investment becomes smaller, and thus equilibrium growth rates of human capital and income are lower than those in the certain adoption case. The economy may eventually join the club of economies with the certain technology adoption as human capital increases over time, but can lead to a poverty trap as human capital continues to decrease, as described in Proposition 2 of Section III, depending on its initial conditions.

## Different Equilibria of Basic and Extended Models with Identical Initial Conditions

We can specify the condition that determines which growth path the economy will follow. The time consistent condition, enforcing agents adopt all the new technologies and leading the

economy to the certain technology adoption of the basic model in Section II, is that the second period utility with technology adoption is always higher than that without:

$$(26) \qquad Q(1 - aQ\frac{S}{2N}) \geq d\overline{Q} \Rightarrow 1 - \frac{d\overline{Q}}{Q} \geq \frac{aS}{2}\frac{Q}{N}.$$

The certain adoption condition of the basic model can be characterized, using (11) and (12) in Section II, as

$$(27) \qquad \begin{aligned} & 1 - \frac{d\overline{Q}}{Q} > \frac{aS}{2}\frac{Q}{N} \Rightarrow 1 - d\sqrt{\frac{2(1+r)+P}{Pb}} > 1 - z^* \\ & \Rightarrow P > \frac{2d^2(1+r)}{bz^{*2} - d^2} \end{aligned}.$$

This condition implies: The higher the probability of having new technologies ($P$) or the efficiency of human capital formation ($b$) is, the more probably this condition holds. Also the lower the technology spillover effect ($d$), the more probably this condition holds. Note that an increase in the level of uncertainty about the characteristics of future technologies ($S$), by lowering the efficiency parameter $b$, makes the inequality of (27) more likely to hold.

This condition is identical to that of Case 1 in Proposition 2 ($\Phi < \Gamma$). However, Case 1 includes various equilibrium paths of the certain adoption as well as the uncertain adoption of the extended model. This implies: Only with the certain adoption condition of the basic model of (27), adoption of all new technologies in the second period is optimal in the basic model of Section II. However, a stricter condition is necessary for the certain adoption case in the extended model. This is because agents invest more in human capital with the forced adoption restriction in the basic model, resulting in adopting more new technologies and leading to the higher growth rates of income and human capital than those in the extended model. The condition for the certain adoption is stronger in the latter than in the former model, as we can see in Proposition 2. Thus, even with the identical condition of (27), the equilibrium paths can be quite different depending on whether the model is basic or extended.

**Multiple Equilibria with Identical Initial Conditions**

Also even with an identical set of initial parameter values, we can show that there exist multiple equilibria situations by introducing an externality into the model.

As stated in Proposition 2, if the condition of $\dfrac{\overline{N}}{Q} < \Phi < \Gamma$ holds as in (3) of Case 1, then the low growth equilibrium path will lead to a poverty trap in the uncertain adoption model. And note that this equilibrium gives higher expected utility to agents than the high growth equilibrium of the certain adoption of the basic model in Section II.[27] Note also that if the above condition of $\dfrac{\overline{N}}{Q} < \Phi < \Gamma$ holds, the certain adoption condition of the basic model of $P > \dfrac{2d^2(1+r)}{bz^{*2} - d^2} > 0$ is also satisfied.

Now, a set of assumptions are in order to present a model with multiple equilibria with an identical set of initial parameter values satisfying the above condition. Assume first that all agents are identical. Assume also that there exists an externality: If all the other identical agents adopt new technologies, there will be a utility cost $L$ incurring to a deviating agent who does not adopt them. For example, this can happen in the real world if many firms are connected in the network of division of labor. There can exist many cases in which it will be more beneficial for them to use the same protocol of the identical technology. Assume also that the utility difference between the low growth path of the extended model and the high growth of the certain adoption of the basic model is less than $L$.

Then in this economy with the parameters satisfying the above conditions, there exist multiple equilibria. If all the other agents follow any one of the two equilibria (low growth path of the extended model and high growth path of the basic model), any agent will follow the same path in the Nash equilibrium setting. In other words, if all other agents make it a rule to adopt all new technologies, every agent must follow the others' strategies by solving the basic model with the restriction of the certain adoption of technologies. This occurs because following the low growth path of the extended model incurs a deviation cost such that this optimal strategy

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becomes sub-optimal. In this situation, government policies can do something depending on whether the government's objective is growth or welfare.


**Multiple Equilibria in the Uncertain Adoption Model**

As Proposition 2 stated, there exist multiple equilibria depending on the initial parameter values in the extended model of the uncertain technology adoption.

Once the economy adheres to the uncertain adoption case, the economy can follow either the higher or the lower growth path. The possibility of multiple equilibria in the economy with an uncertain technology adoption is intuitive. With the given level of income (or human capital; $d\overline{Q}$) of a young agent, this young agent can follow either of the two equilibrium paths. Along the high growth path, the agent makes high investment in the width of human capital ($N$), which allows more chances to adopt new technologies by lowering the technology adoption cost, and thus increases growth rates of income and human capital over time. The other path is low investment in human capital due to the expected high adoption cost. Along this lower growth path, the adoption cost continues to rise, lowering the possibility of technology adoption, which in turn lowers human capital investment and thus growth rates of income and human capital over time.

As described in Proposition 2, the above growth path is determined by the ratio of width to depth of human capital ($\dfrac{\overline{N}}{\overline{Q}}$), in the uncertain technology adoption model. If $\dfrac{\overline{N}}{\overline{Q}}$ is larger than a critical value ($F$), which increases in $S$ or in $\dfrac{1}{P}$, the economy follows a higher growth path, and vice versa. This case has very interesting implications as follows.

First, if $P$ is large or $S$ is small, the economy is likely to follow the higher growth path and eventually becomes the certain technology adoption economy. Then technology change can lower growth rates of human capital and income. If it raises the uncertainty related to the characteristics of new technologies S sufficiently more than the possibility of having a new technology shock P, it can lower growth rates of income, as we see in Proposition 3.

Second, the model shows that with fixed values of $S$ and $P$, the country with more investment in the width of human capital relative to the depth can more probably show higher growth rates of human capital and income. This may be consistent with the empirical fact that

rapidly growing developing countries such as Korea invest more in the quantity side ($N$) of human capital than in the quality side ($Q$) of human capital. In the Barro and Lee (1996) data set, Korea is found to have behaved in this way. This story can also be applied to compare different economic growth performances between Great Britain and Germany in the late 19$^{th}$ and early 20$^{th}$ centuries, and between South American and East Asian countries in the past three or four decades. The fact that Germany and East Asian countries have shown higher growth rates of income than their respective counterparts can be explained by the former countries' higher investment in education, according to our framework. Again, this is because education increases the ratio of $\dfrac{\overline{N}}{\overline{Q}}$.[28]

The above discussion leads us to conclude that if the value of $P$, $\dfrac{1}{S}$ or $\dfrac{\overline{N}}{\overline{Q}}$ is high enough, the economy will eventually follow the balanced higher growth path of the certain technology adoption, resulting in higher growth rates of income and human capital.

## 4.2. Government Policies

In this subsection, we demonstrate the effects of several government policies, such as technology and education policy, open trade and FDI policy, and coordination policy, on the growth path of an economy.

### Public Policies of Education and Technology

We show that when the probability of having new technologies ($P$) or the efficiency of human capital formation ($b$) is high, the more the probably the economy will follow the higher growth

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[28] Refer to Footnote 11. In the labor economics literature, education is related more to the general purpose knowledge than on-the-job training or than learning-by-doing. As several economists such as Schultz and Welch noted, education is believed to increase the adaptability of economic agents, just like the effect of an increase in $N$ leading to the lower technology adoption cost. Also note that even though $\dfrac{\overline{N}}{\overline{Q}}$ is a decision variable for the old generation, this ratio can also be affected by the endowment of natural resources of one country and government policies such as various education subsidies.

path. Therefore, any public polices which increase either P or $b$ will contribute to the higher growth of human capital and income.[29]

The government can raise the probability of having a technology shock ($P$) also by increasing public investment in R&D or by providing subsidies to private R&D investment. The government can also increase growth rates of human capital and income by adopting technology policies that can lower the uncertainty about the characteristics of future technology shocks ($S$). Education policy can also move the economy from a lower to a higher growth path. An improvement in the efficiency of the education system can make the economy more likely to move to the higher growth path with the certain technology adoption.

**Open Trade and Foreign Investment Policy**

It is reasonable to assume that an increase in trade openness and foreign investment raises the parameter value of $P$. More open trade policy or more FDI inflows will increase the probability of having a technological change ($P$). The more openness enables agents to have more access to new technologies and make more accurate decisions about adopting new technologies because they can have more information about these new technologies.

The model shows that an increase in $P$ unambiguously raises growth rates of income and education level by offering higher returns to human capital investment. An increase in $P$ makes an economy more likely to move to the certain technology adoption mode or a higher growth path of the uncertain adoption case.

But, increased openness will also increase the uncertainty about the characteristics of new technologies ($S$) in addition to $P$. An increase in $S$, unlike $P$, has unfavorable effects on growth. A higher value of $S$ makes agents invest less in human capital, by lowering the profitability of human capital investment through raising the adoption cost, and thus more likely leading the economy to the lower growth path of the uncertain adoption case. Moreover, an increase in $S$ decreases the efficiency of human capital formation ($b$), leading to lower growth rates of income and human capital. Therefore, the net effects of increased openness on human capital accumulation and income growth are ambiguous, depending on the relative magnitude of changes in $P$ and $S$.

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[29] Recall that $b$ decreases with $S$.

**Coordination Policy**

If the cost of transition from the low investment to the high investment equilibrium is considerably high, one economy can be trapped in the low investment equilibrium. This transition cost may come from externalities and coordination failures in human capital production. Even though one young agent attempts to follow the high equilibrium, this agent alone cannot accomplish this if the other young agents follow the low equilibrium, or if the old agents had followed the low one. This situation persists because one single agent alone cannot adopt, learn, and utilize new technologies. In other words, there can exist externalities in human capital formation among young agents and also among young and old agents. In this case, certain government policies for better coordination among the agents can push the economy out of the low growth path into the high.

An externality story, for example, can be formalized by introducing a heterogeneity among the agents into the model. Suppose that in the education production function, the efficiency of human capital formation ($b$), differing across agents, can become lower, the higher is the gap between each individual's human capital stock and the average human capital stock. In other words, one individual's education efficiency will be lower if her inherited human capital stock is much higher than that of the average person because, for example, formal schooling is conducted in the standard national school system focusing on the student with average ability. Then, even though some agents want to follow a higher growth path by making higher investment in human capital, they can not do it as long as the others do not move together. With this type of externality, government policy such as education subsidies can increase welfare.[30]

# V.    Conclusion

In this paper we have explored the effects of technology change on growth rates of income and human capital, using an overlapping generations model in which identical agents invest in both the width and the depth of education under uncertain environments. For this, we model a micro-

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[30] We already provided another type of an externality model with multiple equilibria in the previous subsection. Also in this case, government policy can matter.

mechanism of the role of human capital in adopting new technologies as well as that of the process of human capital production.

The model presents some interesting implications about the effects of technological change on human capital accumulation and income growth. We show that higher expected rates of a technology advance increase investment in human capital and thus growth rates of income and human capital in the economy. An increase in the technology uncertainty, on the contrary, leads to lower growth rates of income and human capital by decreasing the efficiencies both in the creation of new knowledge and in the adoption of new technologies. The model also shows that, depending on the initial structure of human capital stock and the nature of new technologies, an economy can have multiple growth paths. Hence, our model offers some plausible explanations for the observed failure of development or lower growth performance in less developed economies which have experienced integration into the world economy by trade openness and increased FDI inflows. The model also emphasizes the important role of public policies in the area of education and technology in making the economy adjust to uncertain technology change.

Our model can be extended in several avenues. First, we can introduce a more flexible functional form of distribution of the characteristics of new technologies on knowledge space instead of the assumption of uniform distribution. A change in the distribution functional form can better represent a certain aspect of an increase in the uncertainty of new technologies. Second, we can model the technology adoption and the human capital formation process in a more delicate way. For example, we can introduce a fixed cost of information processing or consider interactions between width and depth of education more realistically. However, the implications will not be much different from ours.

## Appendix A:  A Micro-Mechanism of the Education Process

In this appendix, we will provide a simple micro-mechanism of the human capital formation through education compactly described by (4).

We assume that young agents accumulate their human capital by spending education time ($t_s$) in schools, and by paying tuition to old agents, compensating old agents' opportunity cost of teaching efforts. Further assume that old agents' teaching efforts consist of two parts: creating the human capital structure of $NQ$ that young agents want to have by spending time ($t_c$) and through using the old agents' human capital of $\overline{NQ}$; and delivering this created human capital to young agents by spending education time ($t_s$) in schools.

Now, we will describe the process of old agents' creation of human capital structure $NQ$ in the below discussion. This knowledge creation process is similar to the technology adoption process of (1). However, there are several differences. We assume that to create a piece of knowledge located at $x$ with depth of $Q$, old agents use the two neighboring pieces of knowledge points by spending the knowledge creation time of

$$t_c = \frac{k}{2}\{|x - s_1| + |x - s_2|\}\frac{Q}{\overline{Q}}$$

(A1)

where $s_i$'s denote the locations of the two neighboring knowledge points that are most closely located to $x$. Here, we further assume that if $x$ happens to be identical to $s_1$, then $s_2$ can be either one of the two neighboring points.

(A1) implies that the creation time is a sort of the average of two different adoption times using knowledge points of $s_1$ and $s_2$, respectively. This is because  knowledge creation needs to use the two neighboring knowledge points most closely located.

Another difference between (1) and (A1) is the multiplicative term of $\frac{Q}{\overline{Q}}$. This term implies that knowledge creation time increases proportionally with the size of the quality level of the knowledge to be created.

The relationship of $|x - s_1| + |x - s_2| = \frac{S}{\overline{N} + 1}$ simplifies (A1) as

(A2)
$$t_c = \frac{k}{2} \frac{S}{\overline{N}+1} \frac{Q}{\overline{Q}},$$

where $\dfrac{S}{\overline{N}+1}$ represents the distance between any two adjacent knowledge points of the old agents' human capital structure.

To calculate the equilibrium amount of tuition, we need two more assumptions. The education time in schools ($t_s$) needed to deliver the created $NQ$ to young agents is proportional to the old agents' total knowledge creation time ($t_c N$) as

(A3)
$$t_s = e t_c N,$$

and the old agents' opportunity cost per unit of time is the old agents' wage rate per unit of time of $\overline{Q}$. The old agents' wage rate will be $\overline{Q}$ due to the assumptions about the production technology described by (2) and (5).

Considering that the tuition equals the total opportunity cost of the education activity of old agents, including knowledge creation and education activity in schools, in the competitive market, the tuition will be

(A4)
$$
\begin{aligned}
c_E &= (t_s + t_c N)\overline{Q} \\
&= (1+e)t_c N \overline{Q} \\
&= (1+e)(\frac{k}{2}\frac{S}{\overline{N}+1}QN)
\end{aligned}
$$

We can easily infer that this tuition is the market equilibrium price. This is so because if tuition is higher than this equilibrium price (i.e., the wage rate in schools is higher than $\overline{Q}$), then all old agents will specialize in education, resulting in no production in this economy. And if it is lower than the equilibrium price, no old agents will supply education services.

Considering (A4), the fact that young agents must spend the education time in schools ($t_s$) to learn and accumulate human capital, and that (3) and (5) say that young agents' wage rate is $d\overline{Q}$, then the total time cost for young agents to accumulate human capital of $NQ$ is

$$l_E = t_s + \frac{c_E}{d\overline{\overline{Q}}}$$

(A5)
$$= \frac{k}{2} \frac{S}{\overline{\overline{N}}+1} \frac{QN}{\overline{\overline{Q}}} (e + \frac{1+e}{d})$$

$$\cong \frac{kS(1+e+e\boldsymbol{d})}{2\boldsymbol{d}} \frac{NQ}{\overline{\overline{NQ}}},$$

where if $N$ and $\overline{N}$ are large enough, the approximation in the third line of (A5) holds.

Comparing (A5) and (4), the parameter $b$ in (4) can be described by

(A6)
$$b \cong \frac{2\boldsymbol{d}}{kS(1+e+e\boldsymbol{d})},$$

which implies that the efficiency of human capital formation decreases in $S$. This condition exists because an increase in $S$ increases the old agents' knowledge creation time.

One comment on the nature of the maximization problem with the existence of the education market described above is as follows. Even though the education activity continues following the above exposition, we do not have to consider it explicitly except the relationship of (A6). That is, the old agents' income per one unit of time remains identical and constant irrespective of whether they work in firms or in schools, as the production technology is linear as in (5) and old agents are completely compensated for their education-related activities through tuition from young to old agents.

Now, consider the case with the specification of knowledge creation similar to (1) as

(A7)
$$t_c = k|x - s| \frac{Q}{\overline{\overline{Q}}},$$

where $s$ is the location of the knowledge point most closely located to the characteristics of the new technology ($s$), and we assume that $N$ is large and grows realistically.

Then we can easily infer that this specification results in the average knowledge creation time being about the half of that of the above specification. This is because all the new knowledge points to be created will be almost uniformly distributed over the interval between the two neighboring old knowledge points, considering the relative position of each of the new knowledge points in this interval containing it, if $N$ is large. Then the parameter value of $b$ in this specification will be two times of that in the above specification of (A6). Thus, even with this specification, the results in this paper will remain intact.

# Appendix B: Proof of Lemma 3

Equation (25) implies that, with $\frac{d^2}{z^{*2}} < b$ and $\frac{\overline{N}}{\overline{Q}} > F = \frac{a(1+r)d^2}{(1-z^*)(bz^{*2}-d^2)}\frac{S}{P}$, $\frac{N}{Q}$ increases over

time to the point where (24) is violated. Thus, in this economy, the amount of investment in the width of human capital, the expected income growth rate, and the band of the specialized knowledge (technology) increase over time. We can also easily see that at the moment when (24) is violated first by the continuous increase in $\frac{N}{Q}$, the economy with a choice of adoption will move into the higher growth path of the certain adoption mode. The income growth rate will also move as described above.

At this point we must prove that the economic system converges to the certain adoption economy. In other words, we must prove that when $s^* > \frac{S}{2N}$ without the restriction of $s \leq \frac{S}{2N}$ in the extended model, the optimal s with this restriction will be $s^* = \frac{S}{2N}$. Then this will imply that if $s^* > \frac{S}{2N}$ when solving (P3) without the restriction, the basic model instead of (P3) should be solved again after setting $s^* = \frac{S}{2N}$. In other words, we should solve the certain adoption model in this case. For this, we will first prove that if the solution of $s$ with the above restriction is strictly less than $\frac{S}{2N}$, this will not satisfy the conditions for maximization by proving that the utility increases in $s$ if its value is less than $\frac{S}{2N}$. We will prove this below.

To prove that the utility increases in $s$, firstly we prove that an increase in $s$ implies a decrease in $Q$ by utilizing the fact that $z^*$ is approximately 0.417 less than 0.5. Then the question whether the utility increases in $s$ boils down to discerning the sign of the LHS of (17) with the appropriate value of $Q$ corresponding to the value of $s < s^*$, utilizing the relationship of (16), which holds due to the envelop theorem. The sign of the LHS of (17) represents the change of the utility with respect to a decrease in $s$ in the uncertain adoption case of the extended

model. This occurs because s is a decreasing function only in $Q$. It is easy to prove that the sign

is negative when $Q$ is larger than the optimal value of $Q$ (in other words, when $s$ is smaller

than $s^*$). This means that a decrease in $s$ decreases the utility. Thus, we can infer that if

$s^* > \dfrac{S}{2N}$ without the restriction, $s^* = \dfrac{S}{2N}$ will be the optimal solution for $s$ in the extended

model. The rest of the proof is easy. Thus, we can say that the high growth equilibrium

converges to the certain adoption case.

In contrast, if $\dfrac{\overline{N}}{\overline{Q}} < \Phi$ [31], $\dfrac{N}{Q}$ decreases over time. Then, these variables will move in the

opposite direction. The proof is basically identical to the above. Lastly, if this condition holds

with equality, then (25) says that $\dfrac{N}{Q}$ does not change over time.///

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[31] This inequality together with (24) implies $\sqrt{\dfrac{Pb}{2(1+\boldsymbol{r})+P}} > \dfrac{\boldsymbol{d}}{z^*} \Rightarrow P > \dfrac{2\boldsymbol{d}^2(1+\boldsymbol{r})}{bz^{*2} - \boldsymbol{d}^2}$. In other words, these

two inequalities together satisfy the condition in Footnote 18 under which agents will adopt all the new technologies
even with the option of no adoption in the second period, in the basic model of Section II.

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Adoption Cost of $x$

$$\frac{aSQ}{2N} = \frac{aSQ}{6}$$



**<Figure 1>**
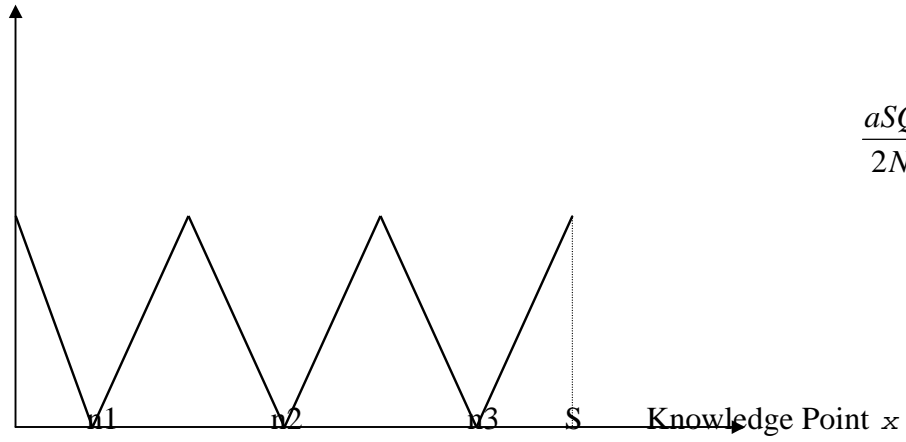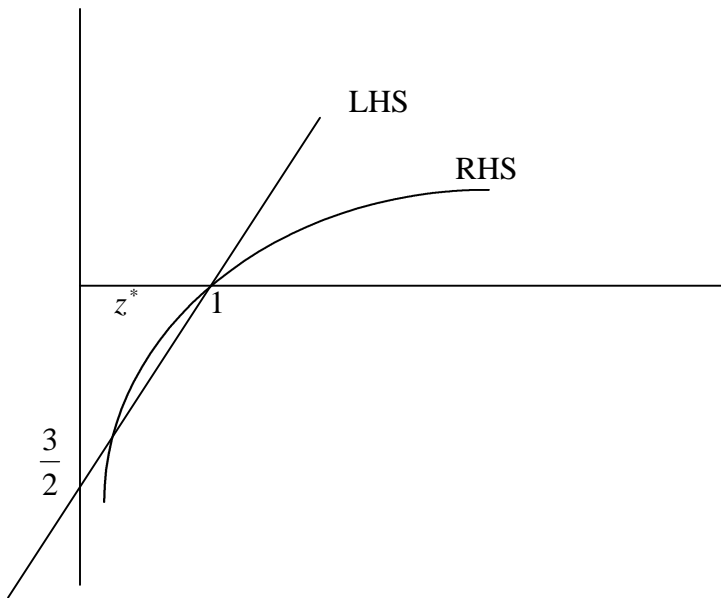


<**Figure 2> Existence of the Solution for z**