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Abstract

In the paper we introduce technological comparative advantage and transaction costs into the Heckscher-Olin (HO) model and refine the HO theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and factor equalization theorem. The refined core theorems can be used to accommodate recent empirical evidence that is at odds with the core theorems.

Keywords: H-O theorem, factor equalization theorem, Stolper-Samuelson theorem, Rybczynski theorem

JEL classification: F10, F11

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1. Introduction

This paper introduces technological comparative advantage and transaction costs into the Heckscher-Olin model and refines the four core theorems in trade theory, namely, the Heckscher-Olin (HO) theorem, Stolper-Samuelson (SS) theorem, factor equalization theorem, and Rybczynski (RY) theorem[1].

In the standard HO model, it is assumed that trading countries share identical technologies. This assumption is obviously inconsistent with empirical observation, and it has contributed to the poor empirical performance of the HO theorem. According to Trefler (1995), the HO theorem is consistent with empirical findings only 50% of the time. Despite the unsatisfactory performance, the HO theorem has retained its dominance in international economics simply because economists have not found anything that performs better (Bowen et. al., 1987). In a recent attempt to improve the performance of the HO theorem, Trefler (1995) demonstrated empirically that a modification is desirable that allows for consumption bias and technology difference between countries.

To complement Trefler's work, we introduce technology differences between countries to the traditional HO model. We shall show that if the trading countries differ in both productivity and factor endowments, the equilibrium trade pattern may be opposite to what the traditional HO theorem predicts; and we shall propose a refined HO theorem.

Recently reviving interests in the effects of international trade on domestic income distribution (see, for instance, Krugman, 1995 and Sachs, 1996) motivate our refinement of the SS theorem in an extended HO model. The SS theorem was used to justify tariffs as an instrument to improve domestic income distribution (Samuelson, 1953). However, our extended

[1] The background of the SS theorem and related core trade theorems, their extensions, empirical tests, reflections about them, and a comprehensive annotated bibliography can be found in Deardorff and Stern (1994). The research to extend the HO model by introducing more goods, factors, and countries can be found from Melvin (1968) and Ethier (1974).

model shows that within a certain parameter subspace, the common sense is closer to reality than the SS theorem. The common sense indicates that tariff and associated transaction costs may marginally improve domestic income distribution, but it may inframarginally cause the general equilibrium to jump from a trade structure with high trade dependence and high productivity to a structure in which productivity is low and home residents receive little gains from trade. The net effect of the transaction cost depends on whether the marginal effect dominates inframarginal effect.

The four core theorems are derived from the traditional 2×2×2 HO model. The model has some standard neoclassical assumptions (such as perfect competition and constant return to scale) and several somewhat restrictive assumptions (which will be discussed later). Given its assumptions, the theorems do not require specific functional forms; yet, they are able to identify several regularities in general equilibrium comparative statics.

But our extended model explicitly specifies the Cobb-Douglas utility and production functions. There are two reasons for assuming specific functional forms. First, the trade off between tractability, generality of functional forms, and generality of other aspects of the model implies that introduction of technological comparative advantage and transaction costs must be at the cost of generality of functional form. Second, a well-known theorem in general equilibrium theory states that in the absence of explicit model specifications, we can say nothing about the properties of the equilibrium comparative statics except that Walras' law holds, and that the excess demand function is homogenous of degree zero (See Sonnenschein, 1973, Mantel, 1974, and Debreu, 1974). We will use the HO model with Cobb-Douglas utility and production functions to show that some of the core theorems may not hold even in the original HO model with no technological comparative advantage and transaction costs.

When some economists prove the SS and RY theorems, they assume that the trading countries are small, thus each country is a price taker, and the equilibrium prices of goods can be treated as exogenous. Sometimes the equilibrium prices of factors are treated as exogenous when the HO theorem is proved (Dixit and Norman, 1980 and Jones, 1965). Since exogenous product or factor prices exclude from the analysis the interactions between prices and other parameters (such as endowment), the general equilibrium comparative statics become less unambiguous. However, it is unjustified to infer exogenous prices of goods or factors from the small country assumption for the same reason that perfect competition (price taking behavior) cannot be used to justify exogenous equilibrium prices in a general equilibrium model. In this paper prices of goods and factors in the HO model are endogenously determined. With endogenous prices, the core theorems of trade theory may or may not hold; certainly they cannot be derived in the same way as in the traditional way.[2]

As depicted in Figure 1, there are 8 possible trade structures in the HO model. Only the first 2 structures involve incomplete specialization for both countries. The last 6 structures involve complete specialisation in at least one country. We can refer to the first 2 structures as interior structures since the output choices of each goods for both countries are strictly positive, ie, they are based on interior solutions. And the last 6 structures can be referred to as corner structures as at least one country chooses zero value of output level of one good, ie, corner solutions are involved.

To find the general equilibrium of the model, we need to know which of the 8 structures (or trade patterns) occurs within which parameter subspace and also the prices and quantities in that structure. Correspondingly, the comparative statics analysis of general equilibrium should investigate not only marginal changes of quantities and prices in response to parameter changes within each structure, but also inframarginal changes (discontinuous jumps) of trade patterns across structures as parameters reach some critical values (or as parameter values shift between parameter subspaces that demarcate the structures). The comparative statics that relate to changes within a given structure are referred to as marginal comparative statics and those related to changes between structures are referred to as inframarginal comparative statics.

For some purposes, inframarginal comparative statics are more important than marginal comparative statics since the latter involve only marginal changes in quantities and prices within a trade structure, while the former involve discontinuous jumps of all endogenous variables including prices and quantities as well as changes of trade structure. For instance, marginal comparative statics may indicate that a tariff benefits labor which is a scarce factor in home country, but inframarginal comparative statics may indicate that the tariff may cause

[2] Wong (1995, pp. 91-97) uses the RY theorem to prove the HO theorem with no assumption of exogenous factor prices. But he proves the RY theorem using the assumption of exogenous product prices. As we will show later that it is not legitimate to assume exogenous product prices in order to work out comparative statics of general equilibrium and to prove the RY theorem.

inframarginal jump of trade structure, so that the cost of the tariff to workers may outweigh its benefit.

In some work, inframarginal changes of trade structure (shifts of equilibrium from or to the diversification cone) are explained by changes in prices. Since comparative statics of general equilibrium explain changes of equilibrium values of all endogenous variables including prices by changes in parameters, it is not legitimate to explain inframarginal changes of trade structure by changes in prices which themselves should be explained by parameter changes. In this paper, we will explicitly solve for inframarginal comparative statics of general equilibrium by partitioning parameter space into subspaces. [3]

The parameter subspace within which an unambiguous negative sign of the derivative of the equilibrium value of an endogenous variable with respect to a parameter occurs may have no intersection set with the parameter subspace within which the trade pattern concerned is the general equilibrium. This implies that identifying the sign of the derivative is not enough and the partition of the parameter space is essential for working out the comparative statics of general equilibrium. But the implications of the partition of the parameter space did not receive deserved attention when the four core theorems were proved.

The main findings of this paper are: (1) the HO theorem continues to hold when prices of goods and factors are endogenized and inframargianl comparative statics of general equilibrium are considered, though it needs to be refined when transaction costs or differences in technology are introduced; (2) the SS theorem remains valid within the diversification cone if the changes in prices are due to a change in taste or endowment, but no longer holds if the changes in prices are due to changes in production or transaction cost parameters; (3) the part of the RY theorem which states that an increase in a factor endowment leads to an expansion of the sector that uses the factor intensively remains valid, but the other part which states that such an increase leads to a contraction of the other sector is no longer true; (4) the factor price equalization theorem does not always hold within the diversification cone if transaction costs and differences in technology are introduced.

[3] The distinction between comparative statics of decision and comparative statics of equilibrium implies that comparative statics of equilibrium should not explain changes in equilibrium by changes in prices despite the legitimacy of explaining quantities demanded by prices.

The rest of the paper is organized as follows. Section 2 presents the HO model that incorporates technology differences and transaction costs and checks the validity of the HO theorem. Section 3 discusses the concept of the diversification cone and analyzes the conditions for factor price equalization in the extended model. Sections 4 and 5 check the Stolper-Samuelson theorem and the Rybczynski theorem, respectively, in the extended model. Section 6 analyzes effects of the introduction of transaction costs on the core theorems. Section 7 concludes the paper.

2. The HO model with transaction costs and differences in technology

In this section, we develop an HO model with two countries differing in production technology and transaction conditions. The assumptions are similar to those in a standard 2×2×2 HO model, namely, that perfect competition prevails in both goods and factor markets; that factors are mobile within a country but immobile between countries; that factors are fully employed; and that the production technology exhibits constant returns to scale.

We start by finding the autarky product price in each country as reference, and then proceed to solve for the trade equilibrium.

2.1. Autarky

Assume that country i (i =1, 2) is endowed with labor L_i and capital K_i, which can be used to produce two consumer goods X and Y. In autarky, the decision problem of a representative consumer in country i is

$$\begin{aligned} &\max_{x_i, y_i} U_i = x_i^{q} y_i^{1-q} \\ &s.t.\ px_i + y_i = w_i L_i + r_i K_i \end{aligned} \tag{1}$$

where p is the price of good X in terms of good Y; w_i and r_i are wage rate and rental of capital, respectively.

Assume that the production functions for X and Y in country i are:

$$x_i = a_{ix} L_{ix}^{a} K_{ix}^{1-a}\ ; \qquad y_i = a_{iy} L_{iy}^{b} K_{iy}^{1-b} \quad . \tag{2}$$

where $a_{ij}\ (i=1,2; j=x,y)$ is the total factor productivity coefficient. Since a_{ij} is country specific, it captures the productivity difference between the two countries. Constrained by the production technology, the representative firm producing X in country *i* maximizes its profit, i.e.,

$$\max_{L_{ix},K_{ix}} p_{ix} = px_i - w_i L_{ix} - r_i K_{ix} = pa_{ix} L_{ix}{}^{a} K_{ix}{}^{1-a} - w_i L_{ix} - r_i K_{ix}. \qquad (3)$$

The decision problem for a firm producing Y is similar to (3).

From the first order conditions of the firms' decisions problems, we obtain:

$$\frac{K_{ix}}{L_{ix}} = [p_i \frac{a_{ix}}{a_{iy}} (\frac{a}{b})^{b} (\frac{1-a}{1-b})^{1-b}]^{\frac{1}{a-b}}$$

$$\frac{K_{iy}}{L_{iy}} = \frac{a(1-b)}{b(1-a)} \frac{K_{ix}}{L_{ix}}$$

It is easy to see that $\frac{K_{ix}}{L_{ix}} > \frac{K_{iy}}{L_{iy}}$ (that is, the X industry is capital intensive and the Y industry is labor intensive) if and only if $a < b$. Without loss of generality, we assume that the X industry is capital intensive, ie., $a < b$.

Using (4) and the market clearing conditions for factors and goods in each country, we obtain the autarky price in country *i* (p_i):

$$p_i = \frac{a_{iy}}{a_{ix}} [\frac{L_i}{K_i} \frac{(1-a)q + (1-b)(1-q)}{aq + b(1-q)}]^{b-a} \frac{b^{b}(1-b)^{1-b}}{a^{a}(1-a)^{1-a}} \qquad (4)$$

Clearly, $p_1 < p_2$ if and only if $(a_{2y}a_{1x}/a_{2x}a_{1y})^{1/(\beta-\alpha)}(K_1L_2/K_2L_1) > 1$ under our assumption that $a < b$, or iff the product of the degree of comparative technological comparative advantage $a_{2y}a_{1x}/a_{2x}a_{1y}$ and the degree of comparative endowment advantage K_1L_2/K_2L_1 is greater than 1. Suppose also that there is no comparative advantage in production technology in the two country, (ie., $\frac{a_{1x}}{a_{1y}} = \frac{a_{2x}}{a_{2y}}$), then $p_1 < p_2$ if and only if $\frac{K_1}{L_1} > \frac{K_2}{L_2}$. In other words, if country 1 is capital abundant, it will have comparative advantage in the capital intensive good X. This is the content of the HO theorem. However, if there is a comparative technological difference between the two countries, which country has comparative advantage in what good depends on *both* relative factor endowments and relative technological difference. In this case the traditional HO theorem may give the wrong prediction about comparative advantage.

Autarky price differences provide a clue for the direction of trade flows between the 2 countries, but to examine the exact trade pattern, we need to look at the trade equilibrium.

2.2 Trade equilibrium

With international trade, the nature of the decisions for consumers and firms is similar to that with autarky except that there are transaction costs for good imported from the foreign country.

Assume that international trade incurs ice-berg transaction costs. For each unit of goods purchased by a consumer in country i from a foreign country, the fraction $1-k_i$ disappears in transit because of transaction costs. Because of transaction costs, the price of the same good may differ between the two countries. In an interior structure, a consumer's decision problem in country i is:

$$\text{Max } U_i = (x_i+ k_i x_{ji})^{\theta} (y_i+k_i y_{ji})^{1-\theta}, \qquad \text{s.t. } p_{ix} x_i + p_{jx} x_{ji}+p_{1y} y_1+p_{jy} y_{ji} = w_1 L_1 + r_1 K_1 \qquad (5)$$

where p_{st} is the price of good t in country s, x_{st} and y_{st} are the amounts of the two goods, respectively, delivered from country s to country t, w_s and k_s are wage rate and rental of capital in country s, respectively. Because of transaction costs, the prices of the same good may differ between the two countries. We first consider structure (XY)YX in which x_{12}, $y_{21}> 0$ and $x_{21} = y_{12}= 0$. The first order conditions for the decision problems of representative consumers in the two countries yield: $p_{2y} = k_1 p_{1y} = k_1$, $p_{1x} = k_2 p_{2x}$. We assume that good Y produced in country 1 is the numeraire, so that $p_{1y} = 1$ and denote $p = p_{1x}$, so that $p_{2x} = p/k_2$ and $p_{2y} = k_1$.

Using the first order conditions for the decision problems of two types of firms (producing X or producing Y) in each country and the market clearing conditions for factors in each country, we can solve for factor prices and relative factor allocation as functions of product prices. We then use the world market clearing condition for goods to solve for the equilibrium values of relative prices.

The equilibrium is summarized as follows:

$$X_i = Aa_{ix}(a_{ix}/a_{iy})^{\alpha/(\beta-\alpha)}[\beta K_i -(1-\beta)L_i(a_{iy}/a_{ix})^{1/(\beta-\alpha)}/\gamma](\gamma)^{\alpha}/(\beta-\alpha), \qquad (6a)$$

$$Y_i = Ba_{iy}(a_{iy}/a_{ix})^{(1-\beta)/(\beta-\alpha)}[(1-\alpha)(L_i/\gamma)-\alpha K_i(a_{ix}/a_{iy})^{1/(\beta-\alpha)}](\gamma)^{\beta}/(\beta-\alpha),$$

$$L_{ix} = [\alpha/(\beta-\alpha)][(\beta K_i r_i/w_i)-(1-\beta)L_i], \qquad (6b)$$

$$L_{iy} = [\beta/(\beta-\alpha)][(1-\alpha)L_i-(\alpha K_i r_i/w_i)],$$

$$K_{ix} = (1-\alpha)L_{ix}w_i/r_i\alpha, \qquad K_{iy} = (1-\beta)L_{iy}w_i/r_i\beta.$$

$$p = B\gamma^{\beta-\alpha}/A, \qquad (6c)$$

$$\gamma \equiv \{(a_{1y}^{1-\alpha}/a_{1x}^{1-\beta})^{1/(\beta-\alpha)}[\theta+(1-\beta)/(\beta-\alpha)]L_1+ [(k_1 k_2 a_{2y})^{1-\alpha}/a_{2x}^{1-\beta}]^{1/(\beta-\alpha)}[(\theta/k_2)+(1-\beta)/(\beta-\alpha)]L_2\}\div \{(a_{1x}^{\beta}/a_{1y}^{\alpha})^{1/(\beta-\alpha)}[\beta/(\beta-\alpha)-\theta]K_1+ [a_{2x}^{\beta}/(k_1k_2a_{2y})^{\alpha}]^{1/(\beta-\alpha)}[\beta/(\beta-\alpha)-(\theta/k_2)]K_2\}$$

$$r_1/w_1 = (Apa_{1x}/Ba_{1y})^{1/(\beta-\alpha)}, \quad r_2/w_2 = (Apa_{2x}/k_1k_2Ba_{2y})^{1/(\beta-\alpha)} \qquad (6d)$$

where $A \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}$, $B \equiv \beta^{\beta}(1-\beta)^{1-\beta}$, X_i, and Y_i are total amounts of two goods consumed in country i, which include quantities purchased from domestic and foreign markets. $X_1 = x_1$, $X_2 = x_2 + k_2x_{12}$, $Y_2 = y_2$, $Y_1 = y_1 + k_1y_{21}$,

L_{ix} and L_{iy} can be greater or equal to zero. Correspondingly, there are 8 possible trade structures as depicted in Figure 1.

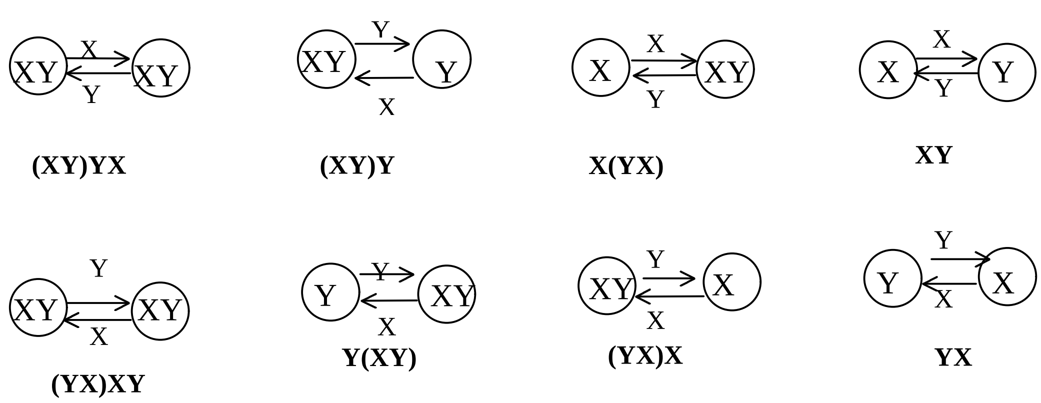

Figure 1: Possible Trade Structures

where (XY)YX denotes that each country produces two goods, country 1 exports good X and country 2 exports good Y. XY denotes that each country produces only one good and country 1 exports X and imports Y. Other notations for structures in Fig. 1 have similar meaning.

Let relevant variables in (6b), which are functions of thirteen parameters, be greater than or equal to 0, we can partition the thirteen dimension space of parameters β, α, θ, K_i, L_i, a_{iy} , a_{ix}, k_i into subspaces. There may be a parameter subspace within which a particular structure is the general equilibrium structure. For instance, let L_{ix} , $L_{iy} > 0$ for i=1,2, we can identify the parameter subspace for the interior structure, which is associated with the diversification cone, to be the general equilibrium structure, and let L_{ix}, $L_{1y} > 0$ and $L_{2y} = 0$, we can identify the parameter subspace for structure (YX)X to be the general equilibrium structure.

Throughout the paper, we assume

$$\mu = (a_{2y}a_{1x}/a_{2x}a_{1y})^{1/(\beta-\alpha)}(K_1L_2/K_2L_1) > 1 \qquad (7)$$

which, together with the assumption $\beta > \alpha$, implies that the autarky price of X in terms of Y is lower in country 1 than in country 2. Considering $a_{2y}a_{1x}/a_{2x}a_{1y}$ as the degree of technological comparative advantage and K_1L_2/K_2L_1 as the degree of endowment comparative advantage, this condition means that there exist net comparative advantages of the two types. (7) holds if there are comparative technological and endowment advantages, or if comparative advantages of one type dominate comparative disadvantages of the other type.

Under this assumption, we can show that the parameter subspace for structure YX to occur in general equilibrium is empty. In structure YX, $L_{1x} = 0$, $L_{1y} > 0$, $L_{2x} > 0$,and $L_{2y} = 0$ which requires that $(a_{2y}a_{1x}/a_{2x}a_{1y})^{1/(\beta-\alpha)}(K_1L_2/K_2L_1) < (k_1k_2)^{1/(\beta-\alpha)}\alpha(1-\beta)/\beta(1-\alpha)$ that violates the assumption (7) and $\beta > \alpha$ if $k_i \in (0, 1)$. Similarly, we can show that the parameter subspaces for structures (YX)X and Y(XY) to occur in equilibrium are empty if (7) holds.

In order to rule out structure (YX)XY, we assume x_{12}, $y_{21}= 0$ and x_{21}, $y_{12} > 0$ in (5). Following the same procedure for solving for the local equilibrium in structure (XY)YX, we can show that the equilibrium prices in this structure are $p_{1y}=1$, $p_{1x} = p$, $p_{2x} = k_2$, $p_{2y}= k_1p$ and that domestic excess demand for X in country 1 and that for Y in country 2 are positive only if

$$\mu = (a_{2y}a_{1x}/a_{2x}a_{1y})^{1/(\beta-\alpha)}(K_1L_2/K_2L_1) < (k_1k_2)^{1/(\beta-\alpha},$$

which is incompatible with the assumption (7) for $k_i \in [0, 1]$ and $\beta > \alpha$. This implies that the parameter subspace for structure (YX)XY to occur in equilibrium is empty or this structure is ruled out from the set of candidates for equilibrium structures.

Now we consider the dividing line between structure (XY)YX and autarky. Since both structures satisfy $L_{ij} > 0$ for i = 1, 2 and j = x, y, we have to calculate the domestic excess demand for X and Y to identify the dividing line. Inserting the equilibrium prices into the domestic excess demand for goods yields

$$x_1^s \geq X_1^d \text{ iff } \gamma \geq \gamma_1 \equiv [(\beta-\alpha)\theta+1-\beta](a_{1y}/a_{1x})^{1/(\beta-\alpha)}L_1/K_1[\beta(1-\theta)+\alpha\theta]$$

$$y_2^s \geq Y_2^d \text{ iff } \gamma \leq \gamma_2 \equiv [(\beta-\alpha)\theta+1-\beta](k_1k_2a_{2y}/a_{2x})^{1/(\beta-\alpha)}L_2/K_2[\beta(1-\theta)+\alpha\theta]$$

The two semi-inequalities imply that country 1 exports X and country 2 exports Y, if and only if

$$\gamma_2 > \gamma > \gamma_1 \text{ which holds only if } \gamma_2 > \gamma_1$$

It is not difficult to see that $\gamma_2 > \gamma_1$ if and only if

$$\eta \equiv (k_1k_2)^{1/(\beta-\alpha}\mu = (k_1k_2a_{2y}a_{1x}/a_{2x}a_{1y})^{1/(\beta-\alpha)}(K_1L_2/K_2L_1) > 1,$$

where η is a product of transaction efficiencies in the two countries and comparative technological and endowment advantages and μ represents the degree of net comparative technological and endowment advantages. It is obvious for $k_1k_2 = 1$, $\gamma_2 > \gamma_1$ if and only if (7) holds. Hence, if (7) holds and transaction costs in two countries do not outweigh net comparative advantages of two types, then structure (XY)YX may occur in equilibrium.

As k_1 and k_2 decrease from 1, γ_2 tends to γ_1. If k_1 and k_2 are sufficiently small, it will be true that $\gamma_2 = \gamma_1$ which implies that the domestic excess demand in both countries is 0, or autarky occurs in equilibrium. Hence, the dividing line between structure (XY)YX and autarky is

$$\eta = (k_1k_2a_{2y}a_{1x}/a_{2x}a_{1y})^{1/(\beta-\alpha)}(K_1L_2/K_2L_1) = 1.$$

For $\eta \leq 1$, autarky occurs in equilibrium for $\eta > 1$, structure (XY)YX may occur in equilibrium. The other necessary conditions for structure (XY)YX to occur in equilibrium are $1 > L_{ij} > 0$ for i = 1, 2 and j = x, y. The conditions hold only if

$$\beta(1-\alpha)/\alpha(1-\beta) > \eta > 1,$$

which implies that the transaction efficiencies in the two countries and net comparative technological and endowment advantages are neither great nor small.

Alternatively, we can verify that as k_1 or k_2 becomes sufficiently close to 0, autarky occurs in equilibrium. First, note the fact that $d\gamma/d\, k_1 > 0$ and for $k_1k_2 = 1$, $\gamma > \gamma_1$ where γ is given in (7). This implies that as k_1 decreases from 1 to 0, γ monotonously decreases from a value greater than γ_1 toward γ_1. As γ has reached γ_1, the equilibrium price in structure (XY)YX becomes the same as that in autarky (see (4) and (7)). Hence, as k_1 becomes sufficiently close to 0, country 1 chooses autarky and therefore the general equilibrium is autarky. Similarly, we can prove that as k_2 becomes sufficiently close to 0, country 2 chooses autarky and therefore the general equilibrium is autarky. Hence, if transaction efficiency in either country is too low, the general equilibrium will be autarky. As transaction efficiencies increase, the general equilibrium jumps from autarky to a structure with trade.

The parameter subspace for structure X(YX) to occur in equilibrium is given by

$(1-\alpha)L_1(a_{1x}/a_{1y})^{1/(\beta-\alpha)}/\alpha K_1 < \gamma < (1-\alpha)L_2(a_{2y}k_1 k_2/ a_{2x})^{1/(\beta-\alpha)}/\alpha K_2$ and

$(1-\alpha)L_2(a_{2y}/ a_{2x})^{1/(\beta-\alpha)}/\alpha K_2 > \gamma/(k_1k_2)^{1/(\beta-\alpha)} > (1-\beta)L_2(a_{2y}/ a_{2x})^{1/(\beta-\alpha)}/\beta K_2$.

The first expression holds only if k_1k_2 is sufficiently large, compared to other parameters. Since $d[\gamma/(k_1k_2)^{1/(\beta-\alpha)}]/dk_2 < 0$, the second expression implies that k_2 is neither too great nor too small, compared to other parameters. Hence, for give values of other parameters, the general equilibrium occurs in structure X(XY) if transaction efficiency is neither too high nor too low in country 2, while it is high in country 1.

The parameter subspace for structure (XY)Y to occur in equilibrium is given by

$(1-\beta)L_1(a_{1x}/a_{1y})^{1/(\beta-\alpha)}/\beta K_1 < \gamma < (1-\alpha)L_1(a_{1y}/ a_{1x})^{1/(\beta-\alpha)}/\alpha K_1$ and

$(1-\alpha)L_2(a_{2y}k_1k_2/ a_{2x})^{1/(\beta-\alpha)}/\alpha K_2 > \gamma > (1-\beta)L_1(a_{1y}/ a_{1x})^{1/(\beta-\alpha)}/\beta K_1$.

The second expression holds only if k_1k_2 is sufficiently large, compared to other parameters. Since $d\gamma/dk_1 > 0$, the first expression implies that k_1 is neither too great nor too small, compared to other parameters. Hence, for give values of other parameters, the general equilibrium occurs in structure (XY)Y if transaction efficiency is neither too high nor too low in country 1, while it is high in country 2.

The inframarginal comparative statics are summarized in Table 1.

Table 1: General Equilibrium and Its Inframarginal Comparative Statics

η	< 1	$\in [1, \beta(1-\alpha)/\alpha(1-\beta)]$		
k	k_1 and/or k_2 are small	k_1 and k_2 are neither great nor small	k_1 is great and k_2 is neither great nor small	k_2 is great, k_1 is neither great nor small
Equilibrium structure	Autarky	(XY)YX	X(YX)	(XY)Y

η	$> \beta(1-\alpha)/\alpha(1-\beta)$		
k	k_1 is great and k_2 is neither great nor small	k_1 and k_2 are great	k_2 is great and k_1 is neither great nor small
Equilibrium structure	X(YX)	XY	(XY)Y

where $\eta \equiv (k_1k_2)^{1/(\beta-\alpha}\mu = (k_1k_2a_{2y}a_{1x}/a_{2x}a_{1y})^{1/(\beta-\alpha)}(K_1L_2/K_2L_1)$, and $\beta(1-\alpha)/\alpha(1-\beta) > 1 > \alpha(1-\beta)/\beta(1-\alpha) > 0$. η is a product of transaction efficiencies in the two countries and net comparative technological and endowment advantages and μ represents the degree of net comparative technological and endowment advantages.

Table 1 indicates that under our assumption (7), autarky and four different trade patterns can occur in equilibrium depending on parameter values. All the trade patterns feature country 1 exporting good X. (It can be shown that if we reverse our assumption (7), the other four trade patterns will occur in equilibrium each featuring country 1 exporting good Y).[4]

Our assumption (7) holds if we assume that country 1 is capital abundant ($K_1/L_1>K_2/L_2$) and X is capital intensive ($\beta>\alpha$) and that country 1 has no comparative technological disadvantage in producing X, ($a_{1x}/a_{1y} \geq a_{2x}/a_{2y}$). Hence, the results in Table 1 are consistent with the traditional HO theorem, that is, country 1 exports the good (X) that uses its abundant factor (K) intensively, if $a_{1x}/a_{1y} \geq a_{2x}/a_{2y}$. However, if country 1 has comparative technological disadvantage in producing X (ie, $a_{1x}/a_{1y} < a_{2x}/a_{2y}$), then whether the traditional HO theorem

[4] Drabicki and Takayama (1979) use a model of three goods to show that a country may import the good that is cheaper under autarky (or export the good that is more expensive under autarky). The world commodity price rations need not fall between the corresponding price ranges under autarky. A similar example is also provided in Dixit and Norman (1980, pp. 95-96).

holds depends on whether country 1's comparative endowment advantage dominates the comparative technological disadvantage in producing X.

Hence, our model shows that the HO theorem can stand the test of the endogenization of prices of goods and factors as well as the test of inframarginal comparative statics analysis of general equilibrium. However, it needs to be refined if there are transaction costs and/or differences in production technology. The refined HO theorem can take the following form.

Proposition 1: *Provided transaction efficiencies are not too low in the two countries, capital abundant country exports capital intensive good if it has no comparative technological disadvantage in producing this good or if the technological disadvantage is dominated by its comparative endowment advantage. Otherwise the country exports labor intensive good. If transaction efficiency is very low in either country, the general equilibrium occurs in autarky. As transaction efficiencies in both countries are slightly improved, the equilibrium jumps to the structure in which each country produces two goods and trades with the other country. As transaction efficiency in one country is further improved, the equilibrium jumps to a structure in which the country with higher transaction efficiency completely specializes and receives most of gains from trade, while the other country produces two goods. As transaction efficiencies in both countries are further improved, the equilibrium jumps to the structure in which each country completely specializes. For given transaction conditions, an increase in the degree of comparative technological and/or endowment advantage will shift the equilibrium from autarky to a low level of division of labor and trade, then to a high level of trade and complete specialization of both countries.*

For convenience of presentation, we assume k_I to be 1 and focus the effects of comparative technological advantage on the other core theorems in sections 3, 4, and 5. We shall then examine the effects of transaction costs on the theorems in section 6.

3. The Factor Price Equalization Theorem

The factor price equalization theorem predicts that international trade will equalize factor prices in the trading countries even though the factors are immobile across countries (Samuelson, 1948, 1953). This prediction has been mostly inconsistent with empirical evidence. There are various explanations for the inconsistence. We look at two here. The first explanation is the well-recognized result that if the general equilibrium occurs outside the diversification cone, the factor price equalization theorem does not hold. The second was that international productivity differences account for much of the differences in factor prices (Leontief, 1956 and Trefler, 1993). We analyze the two explanations in turn.

The concept of the diversification cone was developed in the 1950s (see Lerner, 1952, and McKenzie, 1956). It is defined as the range of factor endowments within which a country produces both goods for given prices. The focus of the concept on factor endowments is probably due to that factor endowments are the only exogenous variables besides commodity prices in those early trade models. In our model, however, the diversification cone should be understood as the parameter subspace within which both countries produce both goods in equilibrium. The parameters include relative consumer preference, technology and endowments. Specifically, the diversification cone in our model is defined by the following system of inequalities:

$$L_{1x} > 0;\ L_{1y} > 0;\ \ L_{2x} > 0; L_{2y} > 0\,.$$

Under our assumption (7), ie., $(\frac{a_{1x}}{a_{1y}})^{\frac{1}{b-a}}\ \frac{K_1}{L_1} > (\frac{a_{2x}}{a_{2y}})^{\frac{1}{b-a}}\ \frac{K_2}{L_2}$, the above inequalities imply:

$$[(1-\alpha)\beta/\alpha(1-\beta)](a_{2x}/a_{2y})^{1/(\beta-\alpha)}K_2/L_2 > (a_{1x}/a_{1y})^{1/(\beta-\alpha)}\ K_1/L_1 \tag{8a}$$

and $$\gamma\in((1-\beta)(a_{2y}/a_{2x})^{1/(\beta-\alpha)}L_2/\beta K_2,\ (1-\alpha)(a_{1y}/a_{1x})^{1/(\beta-\alpha)}L_1/\alpha K_1). \tag{8b}$$

where γ is given in (6). Note that in condition (8a), $[(1-\alpha)/\alpha]/[(1-\beta)/\beta] > 1$ iff $\beta>\alpha$ (iff good X is capital intensive). $[(1-\alpha)/\alpha]/[(1-\beta)/\beta]$ can be interpreted as labor intensity in the Y sector relative to the X sector, or the capital intensity in the X sector relative to the Y sector. We may measure technological comparative advantage by $a_{1x}a_{2y}/a_{1y}a_{2x}$ and comparative endowment advantage between the two countries by K_1L_2/L_1K_2. Condition (8a) implies each country's comparative advantage in producing the good that is intensive of the factor which is abundant in this country is not too great and/or the capital intensity of good X relative to good Y is great. (8b) implies that relative taste for two goods is in balance with relative population size and relative productivity between the two countries. If the comparative advantage is too great and/or the relative intensity is too small, equilibrium will be structure XY which is outside the diversification cone and involves complete specialization of each country. If the relative taste is not in balance with relative population size and relative productivity, one country will be completely specialized in structure X(YX) or (XY)Y. This condition in terms of a parameter subspace for the diversification cone is much more accurate than the conventional condition in terms of prices. It, together with (7), indicates that the parameter subspace for the diversification cone is very small. This subspace requires comparative advantage in technology and in endowment is nether too great nor too small.

Outside the diversification cone, the factor price equalization theorem does not hold. For instance, in structure XY, factor prices in country 1 are,

$$w_1 = \alpha\, pa_{1x}(K_1/L_1)^{1-\alpha},\ r_1 = (1-\alpha)\, pa_{1x}(K_1/L_1)^{-\alpha},$$

where $p = \theta a_{2y}K_2^{1-\beta}L_2^{\beta}/(1-\theta)a_{1x}\, K_1^{1-\alpha}L_1^{\alpha}$ is the equilibrium price of good X in terms of good Y.

Factor prices in country 2 are:

$$w_2 = \beta a_{2y}(K_2/L_2)^{1-\beta},\ r_2 = (1-\beta)a_{2y}(K_2/L_2)^{-\beta}.$$

Clearly, $w_1 \neq w_2$, $r_1 \neq r_2$, except for trivial razor edge cases.

Even within the diversification cone, the factor prices are still not equalized. The factor prices in country 1 and country 2 in structure (XY)(YX) are:

$$w_1 = \alpha\, pa_{1x}(K_{1x}/L_{1x})^{1-\alpha},\ r_1 = (1-\alpha)\, pa_{1x}(K_{1x}/L_{1x})^{-\alpha}, \tag{9a}$$

$$w_2 = \alpha\, pa_{2x}(K_{1x}/L_{1x})(a_{1x}a_{2y}/a_{2x}a_{1y})^{(1-\alpha)/(\beta-\alpha)},$$

$$r_2 = (1-\alpha)\, pa_{2x}(K_{1s}/L_{1s})^{-\alpha}\,(a_{1x}a_{2y}/a_{2x}a_{1y})^{(1-\alpha)/(\beta-\alpha)} \tag{9b}$$

where p is given in (6). A comparison between (9a) and (9b) shows

$$w_1 \le w_2 \text{ iff } (a_{1x}/a_{2x})^{(1-\beta)}(a_{2y}/a_{1y})^{1-\alpha} \ge 1 \tag{9c}$$

$$r_1 \ge r_2 \text{ iff } (a_{1x}/a_{2x})^{\beta}(a_{2y}/a_{1y})^{\alpha} \ge 1 \tag{9d}$$

(9c) and (9d) implies that the country that has comparative technological advantage in the capital intensive good is likely to have higher rental rate and lower wage rate than the other country. This is consistent with Leontief’s (1956) view that the difference between factor prices can be explained by productivity differences.

The analysis in this section is summarized in the following proposition.

Proposition 2 *The general equilibrium occurs within the diversification cone if each country’s comparative advantage is not too great, the relative intensity of capital to labor in X sector is* *sufficiently greater than in Y sector, and relative taste is in balance with the two country's relative population size and relative productivity. If comparative advantage is sufficiently great and/or the relative factor intensity is sufficiently small, the general equilibrium entails complete specialization of each country. If relative taste is not in balance with relative population size and relative productivity, one country completely specializes and the other produces both goods in equilibrium.*

Within the diversification cone, the factor price equalization theorem holds only if there is no productivity difference between the trading countries or the relative productivity between two countries in producing two goods is in balance according to equalities in (9c, d). If a country has comparative technological advantage in the capital (labor) intensive good, its rental (wage) is likely to be higher and its wage (rental) lower than in the other country.

4. The Stolper-Samuelson Theorem

The SS theorem states that if the price of the capital-intensive (or labor-intensive) good rises, the price of capital (or labor) rises, and in greater proportion to the commodity price increase; the price of labor falls, but necessarily in greater proportion to the commodity price increase (Stolper and Sammuelson, 1941). Since with the opening-up of international trade, the price of a country's comparative advantage good rises, a corollary of the SS theorem is that international trade benefits a country's abundant factor and hurts its scarce factor or a tariff benefits a country's scarce factor. [5]

We check the validity of the SS theorem in our model in 3 steps: we examine whether it holds (1) within the diversification cone; (2) outside the diversification cone; and (3) when the general equilibrium jumps from one structure to another.

(1) Does the SS theorem hold within the diversification cone?

First, consider an increase in total factor productivity of X in country 1 due to a neutral technological progress (ie., an increase in a_{1x}). Differentiation of (6c) with respect to a_{1x} yields

$$\mathrm{d}p/\mathrm{d}a_{1x} < 0 \quad \text{always holds} \tag{10a}$$

$$\mathrm{d}(r_1/w_1)/\mathrm{d}a_{1x} > 0 \text{ iff } [\gamma/a_{1x}(\beta-\alpha)]+(\mathrm{d}\gamma/\, a_{1x})>0. \tag{10b}$$

[5] Grossman and Levinsohn (1989) show that the specific factors model captures reality more closely than the SS theorem for many U.S. industries.

It can be shown that (10) and the condition for general equilibrium to occur within the diversification cone (condition 8a and 8b) can hold simultaneously if

$$\mu a_{1x}\{(a_{1y}{}^{1-\alpha}/a_{1x}{}^{1-\beta})^{1/(\beta-\alpha)}L_1K_1+$$
$$(a_{2x}{}^{\beta}/a_{2y}{}^{\alpha})^{1/(\beta-\alpha)}[(1-\beta)(a_{1y}/a_{1x})^{1/(\beta-\alpha)}L_1K_2+\beta(a_{2y}/a_{2x})^{1/(\beta-\alpha)}L_2K_1]\}$$
$$< (1-\alpha)(a_{1y}/a_{1x})^{1-\alpha/(\beta-\alpha)}L_1[(a_{1x}{}^{\beta}/a_{1y}{}^{\alpha})^{1/(\beta-\alpha)}K_1+(a_{2x}{}^{\beta}/a_{2y}{}^{\alpha})^{1/(\beta-\alpha)}K_2]^2/\alpha K_1 \quad (11)$$

where $\mu \equiv [(\beta-\alpha)\theta+1-\beta]/[\beta-\theta(\beta-\alpha)]$. (11) holds for a value of α that is sufficiently close to 0, since μ and the left hand side are limited positive values while the right hand side of (11) tends to infinity as α tends to 0. This means that there exists a parameter subspace within the diversification cone, defined by (11), such that an increase in a_{1x} leads to an increase in the price of capital intensive good (X), but a decrease in the rental for capital. This result is clearly inconsistent with the SS theorem. Similarly, we can prove that there exist parameter subspaces such that other non-neutral technical changes may generate changes in prices that are inconsistent with the SS theorem.

Next, consider a non-neutral technical change that raises the relative productivity of capital to labor in producing X (ie., an increase in α). To show the SS theorem may not hold within the diversification cone even in the original HO model with no technical difference between the countries, we assume $a_{ij} = 1$. The differentiation of (6c) with respect to α yields

$$dp/d\alpha > 0 \text{ iff } -\ln[\alpha/(1-\alpha)][\beta-\theta(\beta-\alpha)][(\beta-\alpha)\theta+1-\beta]>\theta A \quad (12a)$$

$$d(r/w)/d\alpha < 0 \text{ always holds.} \quad (12b)$$

It is easy to see that (12a) holds if α is sufficiently close to 0. We can also show that (12) and the condition for the general equilibrium to occur within the diversification cone hold simultaneously if

$$-\ln[\alpha/(1-\alpha)][\beta-\theta(\beta-\alpha)][(\beta-\alpha)\theta+1-\beta]>\alpha[(L_1+L_2)K_1/(K_1+K_2)L_1 \quad (13)$$

Inequality (13) holds if α is sufficiently close to 0. Hence, for a sufficiently small α, an increase in α raises the price of the capital intensive good and at the same time reduce the rental for capital within the diversification cone.

This result is intuitive. A change in α indirectly affects r/w through interdependence between r/w and p (a change in α affects p which in turn affects r/w), given in the first expression in (6c). This effect is counted by the SS theorem. However, the change in α has a direct effect on r/w too, as shown in the second expression in (6c), which is not counted by the SS theorem. If the direct and indirect effect have the same sign, then the SS theorem holds. But if the two effects are opposite, then the SS theorem does not hold when the direct effect dominates the indirect one. In other words, the SS theorem ignores some interdependencies and feedback loops between factor and commodity markets, between consumption and production, between prices and quantities, and between different agents' self-interested behaviors. The ignorance is due to the assumption of exogenous commodity prices when the SS theorem was proved.

We now consider a change in taste (θ) or endowments (L_i. K_i, i=1,2). From (6c), it is clear that the inter-relationship between *p* and *r/w* is independent of changes in taste or endowments, and that a change in taste or endowment affects *p* and *r/w* in the same direction. In other words, the SS theorem holds.

Summarizing the above analysis, we have

Proposition 3 *Within the diversification cone, price movements are consistent with the SS theorem if changes in prices are due to changes in taste or endowment; price movements may be inconsistent with the SS theorem if the changes in prices are due to changes in production parameters. This is true even if technological difference is absent.*

(2) Does the SS theorem hold outside the diversification cone?

The well known answer to this question is negative. We need to solve for the local equilibrium in each structure to formalize the answer. The approach is similar to the one we used to solve for the local equilibrium in the interior structure (the solution presented in (6)). The equilibrium in autarky and in structures (XY)Y, XY, X(YX) are summarized as follows:

Autarky: $p_i = (a_{iy}/a_{ix})(r_i/w_i)^{\beta-\alpha}B/A, \quad r_i/w_i = L_i\,\mu\,/K_i$ (14)

$x_i = a_{ix}A(\beta\mu-1+\beta)L_i^{\alpha}K_i^{\,1-\alpha}/(\beta-\alpha)\mu, \quad y_i = a_{iy}\,b(1-\alpha-\alpha\mu)L_i^{\beta}K_i^{\,1-\beta}/(\beta-\alpha)\mu$

$K_{ix} = (1-\alpha)(\beta\mu-1+\beta)K_i/(\beta-\alpha)\mu, \quad K_{iy} = (1-\beta)(1-\alpha-\alpha\mu)K_i/(\beta-\alpha)\mu$

$L_{ix} = \alpha K_{ix}\,r_i/w_i(1-\alpha), \quad L_{iy} = \beta K_{iy}\,r_i/w_i(1-\beta).$

where $\mu \equiv [(\beta-\alpha)\theta+1-\beta]/[\beta-\theta(\beta-\alpha)]$.

Structure XY: $p = \theta a_{2y}L_2^{\beta}K_2^{1-\beta}\,/a_{1x}L_1^{\alpha}K_2^{1-\alpha}(1-\theta)$, (15)

$r_1/w_1 = (1\text{-}\alpha)L_1/\alpha K_1$, $\quad w_1 = \alpha\theta a_{2y}L_2^{\beta}K_2^{1\text{-}\beta}/L_1\,(1\text{-}\theta)$,

$r_2/w_2 = (1\text{-}\beta)L_2/\beta K_2$, $\quad w_2 = \beta a_{2y}\,(K_2/L_2)^{1\text{-}\beta}$,

$x_1 = a_{1x}\,L_1^{\alpha}K_1^{1\text{-}\alpha}$, $\quad y_2 = a_{2y}L_2^{\beta}K_2^{1\text{-}\beta}$,

Structure (XY)Y: p is given by

$$F \equiv (a_{1y}B)^{-\alpha/(\beta-\alpha)}K_1(a_{1x}Ap)^{\beta/(\beta-\alpha)} - (1/\mu)(a_{1y}B/)^{(1-\alpha)/(\beta-\alpha)}L_1(a_{1x}Ap)^{(\beta-1)/(\beta-\alpha)}$$

$$-\{\theta(\beta\text{-}\alpha)/[\beta\text{-}\theta(\beta\text{-}\alpha)]\}a_{2y}L_2^{\beta}K_2^{1\text{-}\beta} = 0 \qquad (16a)$$

$$r_1/w_1 = (a_{1x}Ap/a_{1y}B)^{\,1/(\beta-\alpha)}, \quad w_1 = a_{1x}Ap(a_{1y}B/a_{1x}Ap)^{(1-\alpha)/(\beta-\alpha)}, \qquad (16b)$$

$$r_2/w_2 = (1\text{-}\beta)L_2/K_2\beta, \qquad w_2 = a_{2y}B\,[\beta K_2/L_2(1\text{-}\beta)]^{1\text{-}\beta}. \qquad (16c)$$

Structure X(YX): Symmetric to structure (XY)Y.

First consider structure XY. Differentiation of prices in structure XY with respect to different parameters yields

$\mathrm{d}(r_i/w_i)/\mathrm{d}\theta = 0$ and $\mathrm{d}p/\mathrm{d}\theta > 0$.

$\mathrm{d}(r_i/w_i)/\mathrm{d}a_{ij} = 0$, $\mathrm{d}p/\mathrm{d}a_{2y} \neq 0$, and $\mathrm{d}p/\mathrm{d}a_{1x} \neq 0$;

$\mathrm{d}(r_1/w_1)/\mathrm{d}\alpha < 0$ and $\mathrm{d}p/\mathrm{d}\alpha > 0$ if $K_1 > L_1$;

$\mathrm{d}(r_1/w_1)/\mathrm{d}\,L_1 > 0$ and $\mathrm{d}p/\mathrm{d}L_1 < 0$.

None of the above relationships between r/w and p is consistent with the SS theorem.

Next consider structures (XY)Y and X(YX). Because of the symmetry between the two structures, we can focus on structure (XY)Y. Let's first look at country 1. (16b) indicates that the relationship between r_1/w_1 and p is independent of the taste or endowment parameter. Hence, any change in taste or endowment parameter will affect the prices of goods and the prices of factors in the same direction, that is, the SS theorem holds.

From (16a), we have

$\partial F/\partial p > 0$, $\partial F/\partial\theta > 0$, $\partial F/\partial a_{1x} > 0$, $\partial F/\partial a_{1y} < 0$, $\partial F/\partial a_{2y} < 0$

And the application of the implicit function theorem to the above yields

$$\mathrm{d}p/\mathrm{d}\theta = -(\partial F/\partial\theta)/(\partial F/\partial p) < 0,\ \mathrm{d}p/\mathrm{d}a_{1x} = -(\partial F/\partial a_{1x})/(\partial F/\partial p) < 0, \qquad (17a)$$

$$\mathrm{d}p/\mathrm{d}a_{1y} = -(\partial F/\partial a_{1y})/(\partial F/\partial p) > 0,\ \mathrm{d}p/\mathrm{d}a_{2y} = -(\partial F/\partial a_{2y})/(\partial F/\partial p) > 0.$$

Similarly, we can prove

$$\mathrm{d}p/\mathrm{d}K_i < 0,\ \mathrm{d}p/\mathrm{d}L_i > 0 \text{ for i} = 1, 2. \qquad (17b)$$

Following the method to prove proposition 3, we can prove that there exist parameter subspaces such that changes in a_{1x}, a_{1y}, α, or β will generate changes in prices that are inconsistent with the SS theorem.

For country 2, we have

$d(r_2/w_2)/d\theta = 0$, $d(r_2/w_2)/da_{1x} = 0$, $d(r_2/w_2)/da_{1y} = 0$, $d(r_2/w_2)/da_{2y} = 0$ (18)

$d(r_2/w_2)/dL_1 = 0$, $d(r_2/w_2)/dL_2 > 0$, $d(r_2/w_2)/dK_1 = 0$, $d(r_2/w_2)/dK_2 < 0$

(17) and (18) indicate that the changes in prices caused by changes in parameters of tastes, production technology, and endowments may be inconsistent with the SS theorem.

Summarising the above, we have

Proposition 4 *In the structure where both countries completely specialize, price movements are inconsistent with the SS theorem. In the structure where only one country completely specializes, price movements are inconsistent with the SS theorem for the country that completely specializes. For the country which produces both goods, price movements are inconsistent with the SS theorem if the price changes are due to production parameter changes, but consistent if the price changes are due to taste or endowment changes.*

The implications of this proposition are more important if deserved attention is paid to the fact that the parameter subspace for the general equilibrium to occur within the diversification cone is much smaller than the subspace within which the general equilibrium occurs outside the diversification cone.[6]

(3) Does the SS theorem hold when the general equilibrium jumps from one structure to another?

Now consider the case when general equilibrium jumps from structure (XY)YX where both countries produce both goods, to structure XY where both countries completely specialize.

Comparing the local equilibrium in structure (XY)YX with that in structure XY, we find that for country 1, the ratio of capital rental to wage rate is higher in structure XY than in the interior structure with trade iff

[6] For a more recent example in which the wage-rental ratios of two trading countries diverge in the presence of two factor internsity reversal, see Deardorf (1980). Thompson (1986) examines the possibility of factor price polarization in a three-factor, two-sector model.

$$\gamma < (1\text{-}\alpha)L_1(a_{1y}/a_{1x})^{1/(\beta\text{-}\alpha)}/K_1\alpha \qquad (19)$$

Also, the price of capital intensive good is lower in the former than in the latter iff

$$\gamma > A\theta a_{2y}L_2^{\beta}K_2^{1\text{-}\beta}/L_1^{\alpha}K_1^{1\text{-}\alpha}a_{1x}(1\text{-}\theta)B. \qquad (20)$$

(19) and (20) hold simultaneously if

$$(1\text{-}\alpha)a_{1y}^{1/(\beta\text{-}\alpha)} > \alpha a_{1x}^{(1\text{-}\alpha\text{-}\beta)/(\beta\text{-}\alpha)}a_{2y}L_2^{\beta}K_2^{1\text{-}\beta}K_1^{\alpha}/L_1^{1+\alpha}$$

This means that there exists a parameter subspace such that the ratio of capital rental to wage is higher and the price of capital intensive good is lower in country 1 in structure XY than in structure (XY)YX.

It can be shown that within an appropriate interval of parameter values, an increase in α, in K_1/L_1, or in $a_{1x}a_{2y}/a_{1y}a_{2x}$, a decrease in β or in K_2/L_2 will make the general equilibrium discontinuously jump from structure (XY)YX to structure XY. If parameter values are within the subspace that is defined by (20) and (21), then the changes in prices caused by the jump are inconsistent with the SS theorem.

The above analysis is summarized in the following propositions.

Proposition 5: *A change in parameters that causes a jump of general equilibrium from the structure with incomplete specialization for both countries to one with complete specialization of each country may generate price changes that are inconsistent with the SS theorem.*

Our analysis in this section suggests that the SS theorem cannot stand the test of the endogenization of commodity prices nor survive the inframarginal comparative statics analysis. The SS theorem holds only within the diversification cone when changes in prices are caused by changes in taste or endowment parameters.

5. The Rybczynski Theorem

The Rybczynski theorem states that *at given commodity price*, if the endowment of some resources increases, the industry that uses that resource most intensively will increase its output, while the other industry reduces its output (Rybczynski, 1955). We examine whether this theorem remains valid in our model where the commodity price is endogenous.

Consider the autarky structure. Without loss of generality, suppose the capital endowment in country i (K_i) increases. If the content of the Rybczynski theorem holds true, then the capital-intensive X industry would expand, (i.e., $\partial x_i/\partial K_i > 0$), and the labor-intensive Y industry would shrink (i.e., $\partial y_i/\partial K_i < 0$). The differentiation of (8) with respect to K_i yields $\partial x_i/\partial K_i > 0$ and $\partial y_i/\partial K_i > 0$. This implies that part of the RY theorem does not hold when commodity price is endogenized.

Now look at structure XY. From (15), we have $\partial x_1/\partial L_1 > 0$, $\partial x_1/\partial L_2 = 0$, $\partial x_1/\partial K_1 > 0$, $\partial x_1/\partial K_2 = 0$, $\partial y_2/\partial L_1 = 0$, $\partial y_2/\partial K_1 = 0$, $\partial y_2/\partial L_2 > 0$, $\partial y_2/\partial K_2 > 0$. These are clearly inconsistent with the RY theorem. It can also be shown that there exists parameter subspace such that $\partial x_1/\partial L_1 > 0$, and $\partial x_1/\partial K_1 > 0$ in the structure (XY)YX. It is not difficult to find an example that a change in an endowment parameter causes a shift of general equilibrium from a structure to another and generates changes in outputs that are inconsistent with the RY theorem. Hence, we conclude:

Proposition 6: *The RY theorem cannot survive the test of endogenization of commodity prices. With endogenous commodity price, the output of both industries can increase in response to an increase in the endowment of some resource. The RY theorem cannot survive the test of inframarginal comparative statics analysis either.*

6. Transaction costs

In the absence of transaction cost, autarky cannot be the general equilibrium (or the parameter subspace for autarky to be equilibrium is empty). Hence, opening up of international trade is *ad hoc* and a shift of the general equilibrium from autarky to the division of labor and a high level of trade dependence is yet to be endogenized.

Proposition 1 summarizes inframarginal comparative statics of general equilibrium that relate to effects of changes in transaction conditions on trade structure. Table 1 partitions 13-dimension parameter space into many subspaces. The inframarginal comparative statics rigorously describe the degree to which changes in a parameter can be substituted by changes in another parameter in generating discontinuous jumps of trade structure. Hence, the statement that improvements in transaction conditions can increase level of division of labor and related degree of trade dependence is not a simple tautology. In particular, proposition 1 implies that as transaction efficiency is improved, total transaction cost may increase due to the increase in the equilibrium trade volume. This is supported by empirical evidence found in North (1986), which indicates that the employment share of the transaction sector increased as transaction conditions were improved in the USA in the last century.

Our iceberg transaction cost in this model is equivalent to a tax system that uses all tax revenue to pay bureaucrats who collect tax. Hence, our model can be used to analyze effects of tariff that incurs significant bureaucracy cost. We leave full general equilibrium analysis of tariff revenue transfer to future research. The SS theorem was originally motivated to investigate effects of tariff on income distribution. It seems to predict that as the home country increases tariff, the price of good that is intensive of the factor that is scarce in home country increases, so that the scarce factor is benefited. But this is not a general equilibrium analysis of tariff and transaction costs. First, it ignores possible inframarginal effect of tariff and transaction costs on a discontinuous shift of trade structure. As shown in Table 1, tariff and associated transaction costs may generate jumps between structures A, XY, and (XY)YX that may be inconsistent with the SS theorem if the inframarginal effects of tariff are opposite to marginal effects.

Second, proposition 1 implies that as the transaction efficiency in a country unilaterally decreases compared to another country, the general equilibrium may shift from structure XY to an asymmetric structure where this country produces two goods and most of gains from trade go to the other country. This is because in an asymmetric structure, terms of international trade are determined by terms of domestic trade in the country producing two goods. This formalizes the common sense that even if tariff may marginally change terms of trade and increase income of abundant factors, it may inframarginally hurt abundant factor by reducing level of trade and associated gains.

Finally, we prove that even within the diversification cone and $a_{ij} = 1$, the SS theorem may not hold if price movements are caused by changes in transaction conditions. Suppose that the equilibrium occurs in structure (XY)YX, so that the equilibrium prices are given by (6). Country 2's terms of trade is $P \equiv p_{2y}/p_{1x} = k_1/p$, where p_{2y} is the price of good Y in country 2 in terms of good Y in country 1, $p \equiv p_{1x}$ is the price of good X in terms of good Y in country 1. The differentiation of (6d) yields

$$\mathrm{d}(r_2/w_2)/\mathrm{d}k_2 = \partial(r_2/w_2)/\partial k_2 + [\partial(r_2/w_2)/\partial P](\mathrm{d}P/\mathrm{d}k_2)$$

where $\partial(r_2/w_2)/\partial k_2 < 0$, $\partial(r_2/w_2)/\partial P < 0$, and $\mathrm{d}P/\mathrm{d}k_2 = -\, k_1\mathrm{d}p/p^2\mathrm{d}k_2$ is ambiguous since $\mathrm{d}p/\mathrm{d}k_2$ is positive within a parameter subspace and negative within the other subspace. If $\mathrm{d}p/\mathrm{d}k_2 > 0$, the sign of $\partial(r_2/w_2)/\partial k_2$ is opposite to that of $[\partial(r_2/w_2)/\partial P](\mathrm{d}P/\mathrm{d}k_2)$. Therefore, the SS theorem may not hold if the indirect effect of a change in k_2 on r_2/w_2 dominates direct effect. The prediction of the SS theorem even within the diversification cone may be wrong if price movements are caused by changes in transaction conditions because some feedback loops between price differentials in two countries, transaction conditions, production and consumption in the two countries are ignored by the SS theorem.

Let $a_{ij} = 1$ in (6d). A comparison between relative factor prices in the two countries indicates that the relative factor prices are equal between the two countries only if $k_1k_2 = 1$, or only if transaction cost is zero. It is also obvious that the RY theorem may not hold within the diversification cone if there are transaction costs. The result is summarized as follows.

Proposition 7: *If price movements are caused by changes in transaction conditions, the SS theorem may not hold outside or within the diversification cone. The RY theorem and factor equalization theorem do not hold if there are transaction costs for international trade.*

7. Conclusion

In this paper, we have introduced technological comparative advantage and transaction costs into the HO model and conducted an inframarginal comparative statics analysis of general equilibrium. The HO theorem holds in all trade structures, but it needs to be refined to accommodate comparative technological advantage and transaction costs. The SS theorem holds only within the diversification cone when changes in prices are caused by changes in taste or endowment parameters. It may not hold if the changes are caused by changes in production parameters. This is true even if technical difference between countries and transaction costs are absent. The SS theorem may not hold if the changes in prices are caused by changes in transaction cost parameters. We have also refined the RY theorem and factor price equalization theorem.

Our exercise has highlighted the limitations of two types of partial equilibrium analysis. Type I partial equilibrium analysis is to assume exogenous prices of goods (or factors), then investigate changes in prices of factors (or output) in response to changes in prices of goods (or endowment parameters). This is a partial equilibrium analysis since in general equilibrium all prices are endogenized. Such a partial equilibrium analysis could be misleading since the model is not closed and some interdependencies and feedback loops between prices and quantities, between consumption and production, between the markets for goods and factors, and between different agents' self-interested decisions are ignored. Type II partial equilibrium analysis is confined within the interior structure, ignoring corner structures. It does not partition the parameter space into subspaces within each of which a particular interior or corner equilibrium is the general equilibrium; and it totally ignores the implications of the partition of the parameter space for comparative statics. Type II partial equilibrium analysis also ignores discontinuous jumps of general equilibrium between different trade patterns.

The two type partial equilibrium analyses differ from Marshallian partial equilibrium analysis which focuses on only one market. The two type partial equilibrium analyses consider all markets and some (but not all) interactions between the markets. Possibly because in many cases the two type partial equilibrium analyses provide a fuller picture than the Marshallian partial equilibrium analysis, their limitations have not received much attention, while the shortcomings of the Marshallian analysis is well-known.

We are cautious about implications of our results for policy purposes since our results are obtained from a specific model. If we change the functional forms, the results may change too. However, the value of our exercise lies that it has demonstrated that obsession with very general results can have high costs. As indicated in everything possible theorem (Sonnenschein, 1973, Mantel, 1974, and Debreu, 1974), the comparative statics of general equilibrium that are as general as the compensated demand law (comparative statics of decisions) are impossible to obtain in the absence of explicit specification of models. The comparative statics of general equilibrium, which are the main sources of the explaining power of economics, are model structure specific, functional form specific, and parameter value specific. Thus there is good reason to doubt the validity of some very general comparative statics results of general equilibrium in the absence of explicit specification of models. We should be very cautious when we make policy recommendations on the basis of some comparative statics of general equilibrium, which are valid only for a specific model structure, for specific functional forms, and for a specific subspace of parameter values.

Further research can explicitly specify tariff and introduce local increasing returns, which no doubt will enrich the implications of the model, but which is likely to make the model more difficult to manage.

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