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Debt and International Finance

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Threshold Effects in International Lending

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This dynamic model for international lending predicts that when production in the debtor country exhibits increasing returns, new money is a rational response by creditors to a debt crisis.

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Debt and International Finance

This paper — a product of the Debt and International Finance Division, International Economics Department — is part of a larger effort in PRE to examine the relationship between external debt and economic growth in developing countries. This work includes theoretical and empirical work on the possibility of a debt overhang, the relationship between external debt and internal debt, and the sectoral and factor of production burden of adjustment to a debt crisis. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Sheilah King-Watson, room S8-025, extension 31047 (39 pages).

Spiegel's dynamic model of international borrowing subject to a credit constraint was developed for an economy with increasing returns to physical capital.

Increases in the capital stock within the nonconvex range increase debtor borrowing opportunities. Conversely, a temporary liquidity shock may permanently lower the economy's growth path.

Introducing aggregate nonconvexities also has different implications for policy on debt overhangs.

In particular, the model allows for rational relending by creditors. It also predicts that new money (or interest capitalization) is in the interest of creditors and will be part of a debt restructuring strategy — as it was recently for Mexico and the Philippines.

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1. Introduction

1.1 Motivation

Recent theoretical contributions to the theory of growth [Romer (1986), Lucas (1988)] have examined the ability of increasing returns to scale models to explain "anomalous" observed growth patterns. For example, the well-known "Solow residual" observed empirically in growth studies is more consistent with a model of increasing returns to scale than the standard neoclassical growth model. In addition, the neoclassical growth model has been shown to predict movements in the behavior of aggregate variables, such as interest rates, which have not been substantiated by empirical observation [King and Rebelo (1988)]. Although the source of increasing returns within non-neoclassical models is still a quite controversial topic, non-convex models appear to be superior in explaining some stylized facts concerning growth.

It seems natural to look towards this new growth framework when attempting to explain certain anomalies in the recent borrowing experience of highly indebted countries (HIC's). One anomaly concerns the persistence of credit difficulties for countries experiencing what were apparently temporary shocks.[1] A well-known adage in international finance maintains that countries should borrow in response to temporary shocks, and adjust to permanent ones [Sachs (1981)]. This contention does not seem consistent with the recent HIC

[1]Of course, not all shocks in the 1980's were temporary. The fall in oil prices may be interpreted as a "permanent" negative shock to Mexico. However, this leaves a large number of net oil-importing HICs' experiences to be explained.

experience, where countries experiencing apparently temporary liquidity shortages found themselves unable to borrow further in capital markets.

Secondly, the divergent experiences of relatively similar countries during this period seems puzzling. Many seemingly similar countries, in terms of country risk characteristics such as debt-export or debt-GDP ratios, had quite different debt experiences in the 1980's [Krueger (1987)]. Although one must always take care to allow for political and social explanations, as well as differences in commodity baskets, when assessing the experiences of a cross-section of nations, a partial explanation for large differences in the experiences of relatively similar developing countries may stem from non-convexities in these countries' production functions. In the presence of aggregate non-convexities, countries exhibiting quite similar characteristics in levels may find themselves on quite different dynamic paths. To the extent that credit is constrained by the growth rates of debtors, as in Cohen and Sachs (1986), differences in growth paths may affect borrowing opportunities.

Finally, the introduction of non-convexities into the production process may motivate some beliefs about debt overhang policies. A result which differentiates a non-convex model of debt from its constant-returns-to-scale counterpart concerns the possibility of rational levels of "new money" from the creditor's viewpoint. In addition, the minimum rational levels of new lending derived below may help to explain why countries failing to acquire sufficiently large levels of new lending may be unable to acquire any new funds at all. These intuitive outcomes are contradictory to either neoclassical Solow-type or constant-returns-to-scale models of production, and provide further incentives for exploring non-standard production alternatives.

1.2 Framework

In this paper, I introduce a model of international lending under sovereign risk for a debtor nation whose aggregate production function is non-convex over physical capital within some range. The basic result of the paper is that non-convexities in an HIC's aggregate production function lead to non-convexities in its borrowing opportunities as well. The model yields a critical "threshold" capital stock, located within the range of increasing returns. Countries endowed with capital stocks above this critical level find themselves on a "high growth path," converging to a high steady state level of growth and capital accumulation. This growth process is aided by active participation of the debtor nation in foreign borrowing activities. Countries finding themselves below the critical capital stock level respond optimally by pursuing a consumption pattern which leads to a depletion of the capital stock to a low constant returns to scale range. At sufficiently low levels of marginal capital productivity, the HIC capital stock converges asymptotically to the origin.

It is assumed that the marginal product of capital within the debtor nation lies above the world interest rate. The ability to participate in international capital markets therefore plays an important role in determining the dynamic growth path of the debtor nation. As a result, the "critical capital stock" necessary to attain the high growth path is lower when the debtor has access to capital markets. HIC's with capital stocks sufficient to lead to the high growth path may find themselves below the critical capital stock level in the event of an international liquidity crisis. Moreover, countries experiencing temporary liquidity shocks may find themselves on permanently lower growth paths.

1.3 Previous Literature

A growth model with constant returns to scale has been applied by Cohen and Sachs (1986) to sovereign debt. They develop a model in which default decisions are based upon comparisons of costs of debt service with standard "default Penalty" [Eaton and Gersovitz (1981)] specifications.

Increasing returns to scale models based upon capital accumulation have been studied by Romer (1986), Lucas (1988), and Azariadis and Drazen (1988). These "production externality" models have helped to explain "Rostow-type" (1960) patterns of take off stages in empirically-observed growth patterns. While Romer has stressed increasing returns to physical capital, through expansion of production processes, the Lucas and Azariadis and Drazen approaches have stressed human capital accumulation as the engine of growth.

Deterministic models of dynamic optimization with an "S-shaped production function" for physical capital have been studied by Majumdar and Mitra (1982,1983) and by Dechert and Nishimura (1983). The analysis was extended to the stochastic case by Majumdar, Mitra and Nyarko (1988). Their models show that varieties of discounting magnitudes and capital stocks can yield alternative equilibrium growth paths for optimizing agents.

1.4 Organization

The paper is organized as follows: Section 2 lays out the basic model of borrowing with a non-convex production function. Section 3 investigates the critical mass argument associated with aggregate non-convexities. Section 4 introduces the problem of a debtor facing a "debt overhang" within the non-convex range, and examines the relative merits of debt policies such as debt forgiveness or "new money" in a non-convex framework. Section 5 examines the possibility of both HIC-specific and international liquidity shortages. Lastly, section 6 provides some conclusions and possibilities for extension.

2. The Model

2.1 Notation

The model is one of a representative agent who faces a credit constraint in international capital markets. The extensive form of the game has four stages: First, creditors determine the level of the ceiling, $h(k_t)$ on new credit extension, based upon full knowledge of the debtor's reaction function. Secondly, the debtor government chooses whether or not to service the outstanding debt burden, $\hat{r}D_t$, where $\hat{r}$ is equal to 1+r, the exogenous interest rate charged on lending, and D_t is the nominal amount of outstanding debt in period t. It is assumed that this ceiling is binding, so that the level of new credit extended is equal to this ceiling, $\hat{r}D_t$-$h(k_t)$. Thirdly, the debtor produces output, $Q(k_t)$, given the current capital stock, k_t. In the fourth stage, the debtor chooses consumption, c_t, and investment, I_t.

If the debtor services the debt, the game is repeated as before. However, if the debtor has defaulted, he suffers a loss in productivity and is barred from future activity in international capital markets, as in the standard sovereign debt literature [Eaton and Gersovitz (1981)]. Credit extensions are constrained to the level at which the debtor is indifferent between defaulting and servicing his debt obligations. It is assumed that the debtor chooses to service the debt at this indifference level.

2.2 The Production Process

I introduce a production process which is non-convex at low levels of the capital stock, k_0, exhibiting increasing returns to scale, as in Dechert and Nishimura (1983). This range of increasing returns to scale is bounded by two regions which exhibit constant returns to scale in physical capital. The production process satisfies:

1. Q is twice continuously differentiable on $[0,\infty)$, with $Q'>0$ and $Q(0)=0$;
2. There exists a high capital stock, k_H, and a low capital stock, k_L, such that $Q''(k_0)=0$ if $k_0 \geq k_H$ or $k_0 \leq k_L$.
3. A region of increasing marginal productivity of physical capital exists over the range $k_L<k_0<k_H$, such that $Q''(k_0)>0$ within this range.

These assumptions result in a production function similar to the "S-shaped" production function utilized by Dechert and Nishimura, bounded by two regions of constant returns to scale similar to the production function found in Cohen and Sachs (1986). The distinction between the "S-shaped" production function above and its counterpart in the standard neoclassical growth model is that it is convex on the interval $[k_L,k_H]$ and exhibits constant returns to capital in intervals $(-\infty,k_L]$ and $[k_H,\infty)$.

The increasing returns to scale range can be interpreted as a takeoff stage in which production externalities are prevalent, bounded by two alternative equilibrium paths of steady state endogenous growth.[2] This

[2]See Azariadis and Drazen (1988) for a similarly-specified production externality model.

specification allows for continued endogenous growth, while distinguishing a "takeoff" range in the development process. Moreover, constraints faced in international capital markets will have implications for the growth paths of the developing countries.

Even under this novel production function, however, the activities of creditors are going to be qualitatively similar to constant-returns-to-scale growth models of debt [Cohen and Sachs (1986)]. Creditors limit the amount of new lending such that $V^d(k_0)$, the discounted value of current and future utility given default, is less than $V^r(k_0,D_0)$, the discounted value of current and future utility given debt service and the stock of outstanding debt. Let $h(k_0)$ be the resulting credit ceiling. It satisfies the condition:

$$(2.1) \quad \hat{r}D_0 = h(k_0) \text{ iff } V^d(k_0,D_0) = V^r(k_0,D_0).$$

2.3 Borrower decision problem

It is assumed that aggregate production economies are external to private agents, who face a private constant-returns-to-scale production function. The debtor government exogenously allocates its borrowing to its private constituents. Default on these private loans is ruled out for simplicity.[3]

Individual consumption and investment decisions are made to maximize private utility. It is assumed that private investors are price takers in all markets. The utility function in each period, $u(c_t)$, is twice continuously differentiable, where $u'>0$, $u''<0$, and $\lim_{c\to 0} u'(c) = +\infty$.

[3]As long as capital flight is prohibited, this assumption is valid since domestic defaults merely redistribute domestic income, leaving the creditworthiness of the debtor nation unchanged.

An explicit investment cost is required within the range of constant returns to avoid infinite investment or disinvestment. Let J_t, the flow of investment expenditure, be related to new investment by the relationship:

$$(2.2) \quad J_t = J_t(I_t, K_t) = I_t + (1/2)\phi[I_t/K_t] \ .$$

The decision problem faced by a private agent within the debtor nation is to maximize:

$$(2.3) \quad u_0 = \Sigma_0^{\infty} \beta^t u(c_t).$$

subject to:

a. $c_t + J_t(I_t, k_t) = f(k_t) - d_{t+1}$

b. $k_{t+1} = k_t(1-\delta) + I_t$

c. $k_t \geq 0$

where d_{t+1} is the endowment of new money distributed to the private sector by the government, which is serviced at identical exogenous rates $\hat{r}d_t$. δ is the rate of depreciation on old capital. Assuming that the non-negativity constraint is not binding, and that the transversality condition holds in the limit, I obtain the Euler condition faced by private consumer/investors:

$$(2.4) \quad f'(k_t) = \frac{u'[f(k_{t-1}) + D_t - J_{t-1}(I_{t-1}, k_{t-1}) - \hat{r}D_{t-1}]}{\beta u'[f(k_t) + D_{t+1} - J_t(I_t, k_t) - \hat{r}D_t]} \ .$$

Dechert and Nishimura show that the optimal path solution to this type of problem is monotonic, ie if k_0 and k_0' represent initial capital stocks with $k_0'>k_0$, then either $k_1'>k_1$ or $k_1'<k_1$ always, depending upon whether k_0 and k_0' are above or below the critical capital stock described below. Moreover, they show that every optimal path converges to a steady state.

It can be seen that the dynamic path is not invariant to the level of discounting of the debtor nation. It is convenient to distinguish between three ranges of discounting: "Mild discounting," defined as $\beta f' \geq 1$, was shown by Majumdar and Mitra (1982) to converge to a steady state level of capital k^*. "Strong discounting," defined as $\beta f(k)<k$, was shown by the same authors to have an optimal path leading to a steady state at the origin.[4] Holding β constant, equation (2.4) determines the conditions under which growth takes place.

The critical capital stock, k_c, will be that which satisfies $\beta\hat{f}(k_c)=1$, where $\hat{f}(k_c)$ represents the "borrowing-enhanced" private production function: $\hat{f}(k_c) = f(k_c) + D_{t+1} - \hat{r}D_t$. For aggregate capital stock below k_c, the equilibrium path for the capital stock converges to the origin. This involves running the capital stock down to zero over some period of time. Above k_c, the economy converges to an upper steady state, k^*. A lower steady state, k_*, also exists in the Dechert and Nishimura model. However, this steady state is locally unstable. Moreover, the authors show that this steady state is not attained, even for $k_0 > k_*$.[5]

The introduction of high and low level constant return ranges does nothing to affect results concerning the existence of a critical capital

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[4]In the case of a concave utility function, the capital stock only converges to the origin asymptotically. Therefore actual depletion of the capital stock is never reached.

[5]The exceptional case is $k_0=k_*$, which is an unstable equilibrium in the Dechert and Nishimura model.

stock, k_c. For $k_0<k_c$, the marginal product of capital is monotonically declining towards the constant return range $f'(k)=f_L'(k)$, while for $k_0>k_c$, the marginal product of capital is monotonically increasing towards the constant return range $f'(k)=f_H'(k)$. This simple setup allows for the incorporation of a critical capital stock in a model of endogenous growth with alternative potential equilibrium growth paths.

The benevolent debtor government acts as the international borrowing agent for the private sector. Given the credit constraint, $D_t \leq h(k_t)$, the debtor government borrows up to the credit ceiling and distributes the funds to its private sector. It then collects past due loan payments and decides whether to service debt or default. This choice is made by comparison of aggregate discounted utility in the default and repayment regimes. In the event of default, it returns revenues from private debt service to its domestic citizens. It maximizes aggregate utility as a function of aggregate consumption. The debtor government chooses to default if:

$$(2.5)\quad V^r(k_t,D_t) < V^d(k_t)$$

where $V^r(k_t,D_t)$ represents the discounted value of current and future income subject to the choice of debt service:

$$(2.6)\quad V^r(k_t,D_t) = \max \Sigma_{t=0}^{\infty} \beta^t u(c_t)$$

subject to (2.4) and the aggregate budget constraints:

$$\text{a.}\quad D_{t+1} = Q(K_t) - c_t - \hat{r}D_{t-1} - J_t(I_t,k_t)$$

$$\text{b.}\quad k_{t+1} = (1-\delta)k_t + I_t.$$

$Q(k_t)$ represents the aggregate production function, which incorporates the production externalities found in the non-convex range. $Q'(k_t)>f'(k_t)$ for two reasons: First, private agents take their borrowing opportunities as exogenous. Secondly, private agents fail to incorporate the externalities associated with capital accumulation in the non-convex range. As will be shown below, the credit ceiling will be increasing in the capital stock in all ranges of the production function, although at a more rapid rate within the "take-off" range.

2.4 Default Solution

Following Cohen and Sachs (1986), assume that the debtor is barred from future access to capital markets in the event of a default and that a penalty, $\theta Q(k_t)$, is experienced as well, due to a loss in productivity. The existence of a default penalty is necessary to allow for rational satisfaction of the transversality condition, but is specified so as to drive none of the results below.[6]

The optimization problem faced by a private agent given default by the debtor government satisfies:

$$(2.6) \quad V^d(k_t) = \max \Sigma_{i=t}^{\infty} \beta^i u[c_t]$$

Subject to:

[6]The specification of the penalty function as increasing in debtor nation output yields the results that the credit ceiling is increasing in the capital stock even within the constant returns to scale ranges of the production function.

a. $(1-\theta)f(k_t) - c_t + J_t(I_t,k_t) \geq 0$

b. $k_{t+1} = k_t(1-\delta) + I_t$

c. $k_t \geq 0$

Private maximization within the default state leads to the Euler condition:

$$(2.7)\quad f'(k_t) = \frac{u'[(1-\theta)f(k_{t-1}) - J_{t-1}(I_{t-1},k_{t-1})]}{\beta u'[(1-\theta)f(k_t) - J_t(I_t,k_t)]} .$$

or:

$$(2.8)\quad c_t < c_{t+1} \text{ iff } \beta f'(k_t) < 1$$

Notice that the non-convex range of the production function will contain a similar "critical capital stock" to that found in the repayment regime. The debtor nation in default therefore faces two alternative steady state outcomes: Achieving the "takeoff" capital stock level, $k_0 \geq k_c$, leads the debtor nation to the high growth rate path. However, countries experiencing even temporary movements below k_c find themselves on a dynamic path with k_t converging to zero asymptotically.

2.5 Debt Service Solution

Since the model is deterministic, the creditor will always adjust the level of lending so that default does not occur. In this manner, as in Cohen and Sachs, the value function of the debtor nation under default affects the credit ceiling faced by the debtor, even though actual default never takes place.

Given the rate of interest, $\hat{r} = 1+r$, the debtor chooses to service his debt burden when $V^r(k_0,D_0) \leq V^d(k_0)$. The Bellman equations for these value functions satisfy:

$$(2.9) \quad V^r(k_0) = u\Big[\, Q(k_0) + D_1 - J_1(k_1) - \hat{r}D_0 \,\Big] + \beta[V^r(k_1)]$$

$$(2.10) \quad V^d(k_0) = u[(1-\theta)Q(k_0) - J_1(k_1)] + \beta[V^d(k_1)]$$

The reward for debt service in this model is avoiding the production penalty and the exclusion from future borrowing. The benefits from current and future borrowing opportunities, which are contingent upon debt service, are increasing in the current marginal product of capital $Q'(k_t)$. As k_t increases within the take-off range, the marginal product of all forms of investment, including debt service, will rise. For k_0 in the interval $k_L<k_0<k_H$, it follows that the rising marginal product of capital brings with it a rise in the returns to debt service. Beyond k_H, however, the marginal benefits of debt service relate linearly to the capital stock.

The new level of lending chosen, D_1, leaves the debtor nation with a "borrowing enhanced" production function, $\hat{Q}(k_t)$, where the original production function, $Q(k_t)$, is augmented by the ability of the debtor nation to borrow from abroad at rates below its domestic marginal product of capital. Note that $\hat{Q}(k_t)$ is also totally determined by the capital stock, whether the debt ceiling is binding or not, since D^*_t, the level of optimal borrowing by the debtor, is completely determined by k_t. Therefore, we can write the "borrowing-enhanced" production function as:

$$(2.11) \quad \hat{Q}(k_t)=Q(k_t) + D_{t+1} - \hat{r}D_t.$$

The characteristics of the solution found by the creditor are stated as the first theorem:

Theorem 1: If $Q(k_t)$ is an "S-shaped" production function satisfying the assumptions above, then the "borrowing enhanced" production function, $\hat{Q}(k_t)$, where $\hat{Q}(k_t)=f(k_t) + D_t - \hat{r}D_{t-1}$, satisfies the credit constraint, $h(k_t)\geq\hat{r}D_t$ and is also S-shaped.

This theorem is proven in three parts. I consider separately the cases: $k_0\geq k_H$, $k_L>k_0>k_H$, and $k_0\leq k_L$, where k_H and k_L represent the high and low points of inflection in the nation's production function.

2.5.1 Case 1: $k_0 > k_H$

Within the high constant returns to scale range, the basic results of Cohen and Sachs go through: At "high" capital stock levels, there is a binding debt ceiling. Moreover, this ceiling grows at the same rate as the economy. Although repudiation never occurs in this deterministic framework, threat of repudiation results in a lower level of growth than that consistent with maximization of productive wealth. Given a binding level of lending, D_t, which is a function of the capital stock, k_0, it follows that there is also a binding debt-to-capital stock ratio D_t/k_t. I state this result as a Lemma:

Lemma 1: The credit constraint faced by the debtor, D_t, in the constant returns to scale range $k_0 \geq k_i$ grows with the capital stock k_t, and the income level, $\hat{Q}(k_t)$.

The proof can be found in Cohen and Sachs (1986), and is summarized in the appendix. I am implicitly assuming that k_H is sufficiently large so that the debtor nation avoids the range of "non-binding" credit constraint identified by those authors.

An intuitive argument for Lemma 1 can be obtained by considering the creditor's lending decision. Given the debtor country production function, $Q(k_t)$, a debt ceiling will exist for any k_t, based upon the debtor agents choosing their utility-maximizing consumption path $\{C_t\}$. This debt ceiling function, $D_t \leq h(k_t)$, defines an optimal investment program for debtors with access to capital markets. The value of remaining a debtor in good standing satisfies equation (2.9) above, where D_1 is constrained by the credit ceiling. On the other hand, the value of defaulting in any given period satisfies equation (2.10). Cohen and Sachs show that in the range of constant returns to scale, the credit constraint will be linear in the debtor nation capital stock: $h^*(k_t) = c \cdot k_t$, where c is a constant and $D_t \leq h^*(k_t)$ if and only if $V^r(k_t, D_t) \geq V^d(k_t, D_t)$. This linear debt ceiling thus satisfies the debt constraint criteria.

The solution found within this range looks much like the Cohen and Sachs model at high capital stock levels. As in their model, $h(k_t)$ is increasing in $f'(k_t)$, the marginal product of capital, β, the rate of discount of the debtor nation, and θ, and the "default penalty," while h is decreasing in r, the world rate of interest.

2.5.2 Case 2: $k_L<k_0<k_H$

The difficulty connected with the determination of an analytical solution for the credit constraint in a non-linear model has been documented in the case of diminishing returns by Cohen and Sachs. The problem is that the value function under repayment is no longer separable in k_t and D_t. Non-linearity implies that both output and the level of credit constraint tomorrow will depend upon investment today.

Additional complications are associated with the dynamic programming problem in the presence of non-convexities in the aggregate production function. When $k_L<k_0<k_H$, an increase in the capital stock will raise the marginal product of capital within the debtor nation. The difficulties of finding an analytical solution are compounded by the non-uniqueness of "optimal investment paths" [Majumdar and Mitra (1982)]. A number of potential optimal investment paths exist from any capital stock k_t. This problem is solved here through the externality assumption. Since private borrowing opportunities are taken as invariant to investment decisions, the solution is unique.

Previous studies of investment under non-convexity [Dechert and Nishimura (1983)] have shown that a monotonic path of positive or negative capital accumulation will emerge under autarky. The problem here is to determine how this monotonic path of capital accumulation under autarky will correspond to a model with borrowing opportunities.

Since the marginal product of capital is increasing in k_t, our intuition would lead us to believe that the ratio of the credit ceiling to the capital stock should also be increasing in k_t. The intuition behind this conjecture lies in the borrower's valuation of arriving in the following period as a debtor in good standing. The "reward" for debt service is the ability to borrow at world rates, adjusted for default premia and invest domestically with an expected marginal product return.[7] An increase in the capital stock within this range increases this reward, resulting in an increase in the credit ceiling. This is stated as a lemma:

Lemma 2: Given $k_L<k_0<k_H$, the ratio of the credit ceiling, $h(k_t)$, to the capital stock, k_0, is increasing in k_0.

Lemma 2 is proven in the appendix. Consider the credit constraint, $h^*(k_0)$, which satisfies:

(2.12) $V^r(k_0,D_0) = V^d(k_0)$

Totally differentiating this condition twice with respect to D_0 and k_0 yields the result that $\partial^2 D_0/\partial k_0^2>0$ within the range of increasing marginal physical product. Intuitively, the marginal benefits of being able to borrow at world rates and invest domestically are increasing within this range in the debtor nation's capital stock. It follows that the penalty of exclusion from international capital markets is also increasing in k_0. As a result of the increased default penalty, the debtor will choose debt service over default for larger values of D_0. Therefore, in the range in which $f(k_0)$ is non-convex, $k_L<k_t<k_H$, the credit ceiling will be increasing in k_0.

[7]The qualitative results in the non-convex range are invariant to whether or not the production penalty is increasing in the capital stock. The current specification was chosen to satisfy the transversality condition.

Since $h(k_0)$ is increasing in k_0 within this range, it follows that $\hat{Q}''(k_0)>Q''(k_0)$. This result is also proven in the appendix. The easing of the credit constraint within the "take-off" range results in an additional source of increase in the marginal physical product of capital. The ability to borrow internationally subject to a sovereign risk credit ceiling increases the importance of non-convex technologies in determining the marginal product of capital. Differentiating the "borrowing-enhanced production function with respect to k_1 yields:

$$(2.13)\quad \hat{Q}'(k_1) = Q'(k_1)[h_1'(k_1)/J_0(I_0,k_0)] - \hat{m}h_1'(k_1) > 0.$$

Note that $\hat{Q}'(k_1) > 0$, since:

$$(2.14)\quad Q'(k_1)/J_0(I_0,k_0)] - \hat{r} > 0,$$

must be satisfied for rational borrowing.

Notice the second term in equation (2.13). By borrowing at world interest rates and investing domestically, international borrowing becomes a source of domestic growth. Moreover, within the range of increasing returns to physical capital, the increase in the debt ceiling associated with an increase in the capital stock, $\partial D_t/\partial k_t$, is an important part of the aggregative marginal product of capital. Differentiating (2.13) with respect to k_1 yields:

$$(2.15)\quad \hat{Q}''(k_1) = Q''(k_1)[h_1'(k_1)/J_0)] + [(Q'(k_1)/J_0) - \hat{r}]h_1''(k_1).$$

Equation (2.15) is unambiguously positive within the range of increasing marginal product of physical capital. It follows that the marginal productivity of capital in terms of the "borrowing-enhanced" production function will be increasing within this range as well. Moreover, because of the second order effects of easing the credit ceiling, the marginal productivity of the borrowing-enhanced production function increases at a faster rate than the original production function.

2.5.3 Case 3: $k_0<k_L$

Since the production function exhibits constant returns to scale in the range $k_0<k_L$, the analysis here will be similar to Case 1:

Lemma 3: The credit constraint faced by the debtor, $D_t \leq h(k_t)$, in the constant returns to scale range, $k_0<k_L$, grows with the capital stock, k_t and the income level, $\hat{Q}(k_t)$.

The proof of Lemma 3 is identical to that of Lemma 1 and is summarized in the appendix. However, the distinction for the "low" constant returns to scale range lies in the specification that $k_t<k_c$, ie that the domestic capital stock lies below the "borrowing-enhanced" critical value. Given this situation, and assuming zero outward capital mobility, the results of Dechert and Nishimura go through that the capital stock will continue to be depleted at some rate. As in the "high" constant returns to scale section, the constant marginal product of capital implies that D_t/k_t will remain constant, implying that the credit ceiling will fall with declines in k_t. The low

marginal productivity of capital implies a negative growth path, with the capital stock asymptotically approaching zero.

The resulting "borrowing-enhanced" production function, taking these three stages together, satisfies the "S-shape" criteria in Theorem 1. In the two constant return to scale ranges, the linear increase in the credit constraint yields the result that $\hat{Q}'(k_t)$ will be greater than $Q(k_t)$, but will exhibit constant returns to scale. Between these regions lies the "take-off" stage in which increases in the physical capital stock both raise the marginal product of capital and the credit ceiling. As in the constant marginal product ranges, increases in output will raise the default penalty, raising the credit ceiling as well.

However, the "take-off" stage will be characterized by more rapid increases in the credit ceiling per unit of capital accumulation. In addition to the output effect on the default penalty, increases in the marginal product of capital raise the default penalty per unit of capital. The existence of a take-off stage in production therefore implies a "take-off" stage in borrowing as well. Debtors in this take-off stage will experience an increase in D_t/k_t, their debt-to-capital stock ratios, as credit ceilings are adjusted to the increased $f'(K_t)$. This results in $\hat{Q}''(k_t)$ exceeding $Q''(k_t)$. Moreover, the "borrowing-enhanced" production function will be "S-shaped" within this range, as suggested by Theorem 1.

This last point is of some interest, since many point to increased foreign borrowing as a source of debtor difficulties in the early 1980's. An alternative interpretation of this period, consistent with increasing marginal product of physical capital would be that the sub-optimal credit ceilings faced by borrowers in international markets, were eased in the 1970's as countries accumulated sufficient capital to enter the non-convex range of the growth path. Under increasing marginal product of physical capital, rapid increases in the debt to capital stock ratio may be consistent with rational behavior on the part of foreign lenders if production externalities made these debtor nations more creditworthy.

3. Critical Capital Stock Mass

Dechert and Nishimura have shown that in an autarky model, non-convex production technologies can lead to two alternative steady states in the presence of "mild discounting."[8] The results above allow us to form similar conclusions for a developing nation with foreign borrowing opportunities. The suggestion is made in the following theorem:

Theorem 2: Under "mild discounting," there exists a critical level of capital stock, k_c, below which private investment decisions, given the credit constraint faced by the debtor, will run the debtor nation capital stock asymptotically to the origin.

The proof is relegated to the appendix. Intuitively, the argument can be understood as follows: It has been shown that under "mild discounting" and an S-shaped production function $Q=f(k_t)$, there is a critical stock mass, k_c, below which the optimal debtor response will be to consume the capital stock asymptotically, driving it to zero in the limit.[9] By Theorem 1, foreign

[8]The conditions for mild discounting in the presence of borrowing are shown in the appendix.
[9]Note that actual depletion of the capital stock will not take place in finite time. Although a long-run capital stock of measure zero seems implausible, one can interpret this equilibrium as a secular decline in the capital stock within a relevant time frame seems more intuitive.

borrowing opportunities are consistent with an S-shaped "borrowing-enhanced" production function. An additional source of productivity stems from the impact of capital stock increases on the credit ceiling. Assuming $c_t=0$ for simplicity, the production function $\hat{Q}(k_t)$ will satisfy:

$$(3.1)\ \hat{Q}(k_1) = Q\left[h_1(k_1)/J_0(I_0,k_0) + (1-\delta)k_0 \right] - \hat{m}h_1(k_1)$$

It has been shown that with relatively few assumptions on debtor behavior, and the fact that $f(k_t)$ is S-shaped, $\hat{Q}(k_t)$ will be S-shaped as well, and therefore subject to the same critical mass argument as the autarky model. In the deterministic model addressed here, repudiation does not occur, and the debtor chooses a utility maximizing path subject to the constraints of this "borrowing-enhanced" production function.

The critical capital stock, k_c, however, will be smaller in the case of a borrowing country than one existing in autarky. This follows from the additional capital obtainable by the debtor through his borrowing activities. Essentially, the ability to borrow from abroad for a country with a marginal product of capital which exceeds world interest rates allows the debtor to "stretch" his capital stock with additional foreign capital inflows. In the presence of increasing returns to physical capital, the increased ability to borrow will increase the marginal product of capital.

4. Debt overhang and non-convex technologies

The analysis above seems to fit well with the empirical evidence concerning creditor responses to debtor nation difficulties in the 1980's. In particular, a non-convex growth model may prove superior in explaining the "debt overhang" experience of these borrowing countries. I examine the implications of a debt overhang in this deterministic model by conducting the following theoretical exercise: Suppose that a nation enters period t with a stock of outstanding debt, D_t, that exceeds the debt ceiling such that $\hat{r}D_t>h(k_t)$, although the remaining periods are assumed to be deterministic.[10]

Since $V^r(k_t,D_t) < V^d(k_t)$, the debtor would choose default over debt service. One can then question the implications of four possible creditor strategies: 1. No additional lending or debt relief, 2. Rescheduling, 3. "New Money," and 4. Outright Debt Forgiveness. I examine each in turn.

4.1 No creditor response

In the absence of any creditor response, the outcome will depend upon the severity of the debt overhang and the capital stock of the debtor nation. This is stated as a proposition:

Proposition 1: Let the "debt overhang" be described by the vector DO = $\{D_t,k_t\}$. In the absence of creditor intervention, for all $D_t > h(k_t)$, there exists a critical capital stock, k^*, below which the debtor nation capital stock will converge asymptotically to zero, while for $k_t \geq k^*$, the debtor nation will converge to a growth path below that which would be attainable if the credit ceiling were satisfied.

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[10]Of course, a loan for which $D_t>h(k_t)$ would never be made in a deterministic framework. However, I examine this as a benchmark case. The qualitative distinctions in a stochastic framework with risk-neutral agents would be minimal.

Since the credit constraint is violated, $V^r(k_t,D_t)<V^d(k_t)$. It follows that debtors will choose to default in the absence of creditor intervention. Once they have defaulted, debtors are assumed to be precluded from participation in international capital markets. The result is that they produce under an autarky S-shaped production function, subject to the productivity loss associated with the default penalty, $(1-\theta)Q(k_t)$. Given this S-shaped production function, Theorem 2 suffices to motivate a critical capital stock.

Moreover, magnitude of the critical capital stock, k_c, will be dependent upon the regime in place. k_c will be greater under default than repayment. Recall that k_c satisfies $\beta\hat{f}(k_t) = 1$. Because of the combination of suspension of borrowing opportunities and the default penalty, the k_t necessary to satisfy this constraint will be larger in the default regime. By the Euler equation associated with the default regime (2.7), it follows that the level of investment chosen will be less than that which would have been chosen in a repayment regime given the same capital stock, k_t.

Hence, it is possible that a debtor nation may have a capital stock sufficient to lead towards positive growth only in the presence of foreign borrowing. Should such a nation enter a default regime, its capital stock would no longer exceeds the higher k_c which will be encountered in the default state. Non-convexities can lead to quite disparate experiences for similarly endowed countries. It follows that in the neighborhood of the critical capital stock, the potential benefits of keeping the debtor nation out of the default state may be large.

4.2 Rescheduling

Since creditors are assumed to gain nothing from penalizing debtors, it follows that an equilibrium in which the debtor nation is penalized would not exist in this deterministic model. The creditor can always do better than allowing the debtor to default by rescheduling the amount the debtor is not willing to pay. This "rescheduling strategy" consists of relending the difference:

$$(4.1) \qquad D_{t+1} = \hat{r}D_t - h(k_t).$$

It is easy to verify that rescheduling the amount above is more desirable to both creditors and debtors than allowing a default. On the debtor side, given current required debt service of $h(k_t)$, the debtor is indifferent between debt service and default by definition. On the creditor side, the stream of payments exceeds zero. Since the creditor is assumed to gain nothing from penalizing the debtor, this stream is always going to be preferred. The present value of pursuing the rescheduling strategy is:

$$(4.2) \qquad h(k_t) + \Sigma_{i=t+1}^{\infty} (\hat{r})^{-1}h(k_i)$$

It is even possible for the rescheduling strategy to eradicate the debt overhang. In each period, The creditor loans out D_{t+1}, according to equation (4.1), and receives $h(k_t)$. With sufficiently high growth in the debtor nation, the increase in the penalty function can result in $h(k_t)$ growing faster than D_t. The rescheduling strategy can be successful in eliminating the debt overhang if:

$$(4.3) \qquad \Sigma_{i=t}^{\infty} (\hat{r})^{-(i-t)}[\hat{r}D_t - h(k_t)] \leq 0.$$

Equation (4.3) clearly requires a late period in which $\hat{r}D_t \leq h(k_t)$, i.e. in which new lending is growing at a slower pace than the credit constraint. Moreover, this future "repayment" period will be more discounted than the initial rescheduling period. It follows that it is quite unlikely even if the debtor nation's credit constraint is growing that a relending strategy will be able to secure all of the outstanding nominal debt of the debtor nation.

The fact that rescheduling will take place in this deterministic model is not surprising. Since creditors do not gain from penalizing the debtor nation, they will obviously prefer the rescheduling option. It is important to note however, that creditors experience losses even under this strategy, so the ability to reschedule is ex-ante a zero probability event in a deterministic model, where the debt has already been adjusted to satisfy $\hat{r}D_t \leq h(k_t)$.

4.3 "New Money"

The question of actual "new money," or net capital flows from creditors to debtors, is more interesting. Arguments for new money are commonly associated with the original Baker plan for dealing with debt difficulties of the developing nations. However, the relending argument proposed in the Baker plan has been highly criticized, both in the popular press and in the literature. Lindert (1987) shows in a static model that "... whatever is wrong with old loans will be wrong with new." Essentially, the Lindert argument maintains that by increasing the future debt burden of the debtor nation in a neoclassical framework, one is merely substituting future defaults for current ones.

The Lindert argument is based upon the specification that the default penalty is invariant to the capital stock of the debtor nation. Alternatively, models such as Cohen and Sachs (1986) have the penalty increasing in the debtor nation's capital stock. However, with constant returns to scale, there will still be no reason to extend new capital to a debtor nation already in difficulty. By definition, a debtor nation facing an overhang has $D_t \geq h(k_t)$. In order for new lending to be viable alternative for the creditor:

$$(4.4) \qquad \Sigma_{i=t}^{\infty} \quad (\hat{r})^{-(i-t)} \frac{\partial h(k_t)}{\partial \hat{D}_{t+1}} \geq 1$$

must hold, where $\hat{D}_{t+1} = D_{t+1} - [\hat{r}D_t - h(k_t)]$ is the initial amount of "new money. New money provides an increase in the debtor nation's capital stock, which increases the default penalty in each period, and hence the debtor nation's credit constraint, $h(k_t)$.

With either constant or decreasing returns to scale, new money will not be a viable alternative. Since $\hat{r}D_t < h(k_t)$, it follows that some improvement in old loans is necessary to motivate the extension of new loans. However, with constant marginal product, the value of old claims will be invariant to new money. I assume the best possible case for new money, ie that for which all of the new money granted to the debtor nation is invested rather than consumed. We know that $V^r(k_t) \geq V^d(k_t)$ for all k_t, since the debtor always has the option of defaulting. Therefore, a sufficient condition for debt service to take place is:

$$(4.5) \qquad \theta Q(k_t) + D_{t+1} \geq \hat{r}D_t.$$

It follows that a sufficient condition for new lending to be rational is that the θ-weighted marginal product of capital exceed the interest rate:

$$(4.6) \quad \theta Q'(k_t) \geq \hat{r}.$$

The fact that new lending to a debtor nation in difficulty cannot be rational in a constant returns to scale framework is motivated by the fact that since the debtor is credit-constrained, initial borrowing will have taken place up to the point where (4.6) is already violated. Since (4.6) implies that additional lending is possible, equilibrium under constant returns to scale implies that it must be violated. In other words, since lenders lent initially up to the point at which the credit constraint was binding, the adverse impact to the capital stock in no way allowed for additional lending since $Q'(k_t)$ is invariant in k_t. With constant marginal product of capital, the initial equilibrium requires that:

$$(4.7) \quad \Sigma_{i=t}^{\infty} \frac{\partial h(k_t)}{\partial \hat{Q}(k_t)} \cdot \frac{\partial \hat{Q}}{\partial Kt_t} \cdot \frac{\partial I^*}{\partial J_t} \cdot \frac{\partial J_t}{\partial D_{t+1}} > 0.$$

Once non-convexity is introduced into the aggregate production function, the potential for rational relending improves. Moreover, new money may not only improve debt service on old loans and be in the creditors interest, it may move a problem debtor from the low growth path back to the high growth path. This argument is stated more formally in the following proposition:

Proposition 2: Given $Q''(k_t) > 0$, a parameter space exists in which new money in the presence of a debt overhang $DO = \{D_t, k_t\}$ may be a Pareto-improving activity. Moreover, a unique level of rational new lending, D^*, will exist such that all rational new loans satisfy $D_t \geq D^*$.

Consider the case of a debt overhang $DO = \{D_t, k_t\}$ which satisfies $k_L < k_0 < k_H$. Levels of rational new lending, $\hat{D}_{t+1}$ may exist for which D_t satisfies $D_t \leq h(k_t)$. The distinction with the constant marginal product case is that the creditor has the ability to affect the performance of his outstanding loans through the extension of new money. By increasing the debtor's marginal product of capital, The creditor increases the default penalty and hence lowers the magnitude of the debt overhang. Given that case, the creditor would benefit by issuing a loan D_{t+1} which led to a smaller future discounted debt overhang than the current one.

The equilibrium pattern will be one in which the creditor extends an unsustainable loan in each period, in the sense that $\hat{r}D_t > h(k_t)$, with the knowledge that in the following period, he will pursue the same strategy. Two possible outcomes of this strategy exist: If the range of increasing returns to scale is large enough, the strategy may eliminate the debt overhang, leaving the debtor free to renew regular borrowing activities. Alternatively, the borrower may attain $k_0 \geq k_H$ with a remaining debt overhang. At this stage, as in the Lindert case, there is no reason to relend to a borrower in difficulty and the debt must be rescheduled to avoid borrower default.

The return from new lending of amount D^*_{t+1} satisfies:

$$(4.8) \quad \frac{\partial V^c}{\partial D_{t+1}} = - D_{t+1} + \Sigma_{i=t}^{\infty} (\hat{r})^{-(i-t)} \frac{\partial h(k_t)}{\partial \hat{Q}(k_t)} \left[\frac{\partial^2 \hat{Q}}{\partial k_t^2} \cdot \frac{\partial k_t}{\partial D_{t+1}} + \frac{\partial \hat{Q}}{\partial Kt_t} \cdot \frac{\partial I_t}{\partial J_t} \cdot \frac{\partial J_t}{\partial D_{t+1}} \right]$$

Returns from new money in the non-convex range stem from two sources: First, the new money will be invested domestically, increasing the default penalty by increasing output. However, as in the constant returns to scale range, this alone would be insufficient to motivate new money. Secondly, the marginal product of both old and new capital will be increasing due to the positive impact of increased investment. This secondary effect creates the potential for rational new lending. To paraphrase Lindert, not only might new loans be viable in the face of bad old loans, they may improve the quality of those loans as well.

Finally, the unique D_{t+1}^{*} can be found by setting equation (4.8) equal to zero. One can verify that the solution is a maximum since $Q'''(k_t)<0$ even in the non-convex range.

4.3 Debt forgiveness

Debt forgiveness has been motivated in the literature in terms of the implications they have for debtor investment decisions [Helpman (1987)]. These models include some implicit tax on debtor nation output which is increasing in the nominal debt burden of the debtor nation. However, when willingness to pay is the binding criteria rather than seizure, the desirability of debt forgiveness becomes questionable from the creditor's point of view, both relative to rescheduling and relending within the non-convex range:

Proposition 3: For all debt overhangs DO = (D_t, k_t), outright debt forgiveness will be dominated by a rescheduling strategy.

I distinguish between two types of debt forgiveness. Nominal debt write-downs consist of write-downs to levels above $h(k_t)$. To induce debt service, these "small" debt write-downs must be supplemented by additional rescheduling. The amount rescheduled satisfies:

$$(4.9) \qquad D_{t+1} = \hat{r}D_t - x - h(k_t)$$

where x is the nominal debt write-down.

These small debt write-downs will never be preferred to complete rescheduling, since creditors are giving up their claims on future debt service payments.[11] Hence small debt write-downs of magnitude x, where $x<\hat{r}D_t - h(k_t)$ will never be optimal from a creditor's point of view.

Large debt write-downs, however, will not induce debt service either. The relative ineffectiveness of debt forgiveness in this overhang model stems from the fact that $h(k_t)$ is invariant to the nominal outstanding debt. Once faced with this overhang, both creditors and debtors know that the creditors can only obtain $h(k_t)$ in each period, so that their best strategy is one of rescheduling the overhang, with possible relending in the non-convex range as discussed above.

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[11]There may of course be small debt write-downs which would never be binding. Both creditors and debtors would obviously be indifferent to these trivial write-downs.

5. Liquidity traps

It is easy to imagine a case of collective action difficulty in which a debtor may be solvent, in the sense that an injection of new money would be rational from the point of view of creditors as a whole, but "temporary market difficulties"[12] preclude a loan of amount D_t^*. In this case, since the debtor will choose to default in the absence of outside intervention, no new lending will be forthcoming. Within the range $\hat{Q}_{kk}>0$, the non-convexity of the aggregate production function may play a perverse role in the debtor's growth outcome.

A temporary interruption from capital markets would lower the debtor's capital stock. In a neoclassical model, this would increase the debtor's marginal product of capital and leave the debtor more creditworthy in the next period. In a non-convex model, the "temporary" capital shock lowers the credit constraint, which may leave the capital stock even lower in the following period. As a result, temporary liquidity problems can become permanent. This conjecture is stated as the final Proposition:

Proposition 4: Given $\hat{Q}''(k_t)>0$, a "temporary" liquidity shortage can lead a debtor to a permanent low growth path.

The proof follows from Theorem 2. Given $k_0<k_c$, a debtor nation will be on the low growth path. Recall that k_c is higher in the default regime than within the repayment regime, due to the production penalty and the

[12]An empirical example may be Brazil in August 1982. Due to the Mexican default, all credit extended towards Latin America was lowered. Presumably, this was initially a temporary response, but Brazil's difficulties have lingered and increased.

exclusion from future borrowing. It follows that a range exists in which a temporary liquidity shock which leaves the debtor in default and below the default regime critical capital stock, k_c, lowers the marginal product of capital in the debtor nation sufficiently to preclude positive capital accumulation in the debtor nation. From the default regime Euler equation (2.7), optimization leaves the capital stock even lower in the next period. Hence, the marginal product of capital is lowered again.

This represents a case where either debtor-government intervention, in the form of forced investment of its private citizens, or a transfer from an external source is necessary if the debtor is to once again attain the high growth path.[13] Private borrowing by the debtor government will be constrained below that level which would allow the debtor to achieve $k_t \geq k_c$. Unlike a neoclassical model, temporary shocks can lead to permanent growth changes. It follows that the potential for Pareto-improving official intervention is enhanced in a non-convex growth model with borrowing opportunities.

However, the conditions for a "creditor panic" should not be ignored. If any individual bank was willing to lend the amount necessary for the debtor to achieve the solvent growth path, the high growth equilibrium would emerge. In other words, the absence of such a bank requires a richer model of both risk aversion and collective action difficulties among banks, as in Sachs (1984). Non-convexity in the aggregate production function alone is not a sufficient condition for temporary liquidity shocks to lead to permanent growth effects.

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[13]Note that within the class of increasing returns to scale models, the possibility exists that depletion of the capital stock is socially, as well as privately, optimal. In these cases, the motivation for government intervention is unclear.

7. Conclusion

In this paper, a model of lending under sovereign risk with non-convexity in the aggregate production function was introduced. The introduction of this non-standard technology was shown to lead to quite different conclusions concerning both the relationship between borrowing and the domestic marginal product of capital, and the proper policy prescriptions concerning a country facing a debt overhang. For example, the model above yielded predictions concerning the viability of relending policies, and the relative desirability of relending vs. debt forgiveness from the point of view of creditors, which run counter to those found in a neoclassical model of debt and growth. It should be stressed that the theoretical results above require only that the marginal product of capital be increasing in the capital stock, and do not require the more stringent coefficient restrictions of constant returns to scale necessary for balanced growth.

An interesting empirical note concerning the dominance of relending emerges from the 1989 write-down deal negotiated with Mexico. Given the choice between nominal write-downs and relending, a portion of the banks voluntarily chose a combination of new money and forgiveness, even though the level of credit extended towards that country appears likely to result in future reschedulings. Perhaps the non-convex production technology results helps to explain the willingness of some creditors to participate in new money arrangements.[14]

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[14]OHowever, new money participation was not universal. The actions of banks such as J.P. Morgan to limit their exposure may indicate that even if some creditors believe in increasing returns to capital in Mexico, the belief is hardly universal.

APPENDIX

In this appendix I define the conditions for "mild discounting" in a Dechert and Nishimura sense which allows for a critical capital stock in a model with borrowing possibilities. I then prove theorem 1, proving Lemmas 1,2, and 3, and theorem 2.

I. Conditions for mild discounting

Dechert and Nishimura show that for "intermediate discounting," $\beta Q'(0) < 1 < \beta \max[Q(k)/k]$, a critical capital stock exists, even for concave utility functions. In order to adapt this criterion to one with borrowing opportunities, the credit constraint faced by the debtor nation at $k_t=0$ must be specified. Within the range of constant returns to scale, Cohen and Sachs have shown that the credit constraint increases linearly with the capital stock. I assume that the capital stock at zero is within the constant returns to scale range such that:

(A.1) $\quad [h(k_t)\,|\,k_t=0] = 0.$

Then the condition for intermediate discounting in the presence of borrowing opportunities consistent with the conclusions of Dechert and Nishimura satisfies:

(A.2) $\quad \beta Q(0) < 1 \leq \beta \hat{Q}(k_0)/k_0$

for all k_0.

II. Proof of Theorem 1

A credit constraint similar to that proposed in Lemmas 1 and 3 is derived in Cohen and Sachs. I summarize the derivation here. Let $V^r(k_0,D_0)$ represent the discounted value to the debtor nation of remaining in the repayment regime. Since $V^r(k_0,D_0)$ is strictly decreasing in D_0, there is a unique $h(k_0)=D_0/k_0$ which satisfies

(A.3) $\quad V^r(k_0,D_0) \geq V^d(k_0).$

Moreover, Cohen and Sachs show that if $D_0/k_0=h$, ie if the credit constraint is binding, $\partial V^r/\partial h<0$. It follows that there exists a unique h^* for which $V^r(k_0,D_0)=V^d(k_0)$.

Given the existence and uniqueness of a credit constraint within the range of constant returns to scale, it is straightforward to show that the level of the credit constraint, D_0, will be increasing in k_0. Totally differentiating (A.3) with respect to D_0 and k_0 yields:

(A.4) $$\frac{\partial D_0}{\partial k_0} = \frac{\partial V^d(k_0)/\partial k_0 - \partial V^r(D_0,k_0)/\partial k_0}{\partial V^r(D_0,k_0)/\partial D_0} > 0$$

In both the constant returns to scale and the increasing returns to scale ranges, $\partial D_0/\partial k_0>0$ since V^r is decreasing in D_0. However, Lemma 2 claims that within the non-convex portion of the production function, an increase in the capital stock raises the credit constraint at a greater rate than constant rate. To prove this Lemma, differentiate equation (A.4) with respect to k_0, obtaining:

$$\text{(A.5)} \quad \frac{\partial^2 D_0}{\partial k_0{}^2} = \frac{\partial V^{d2}(k_0)/\partial k_0^2 - \partial V^{r2}(D_0,k_0)/\partial k_0^2}{\partial V^{r2}(D_0,k_0)/\partial D_0^2}$$

Recall that $Q'(k_t)>0$, but within the range of increasing returns, $Q''(k_t)>0$ as well. Holding D_0 constant, the sign of $\partial V^{r2}(k_0)/\partial k_0^2$ will depend upon the magnitude of $Q''(k_t)$. Since $Q''(k_t)$ is equal to zero within the range of constant returns, it follows that $\partial V^{r2}(k_0)/\partial k_0^2 = 0$ as well. Within the range of aggregative production externalities, however, $\partial V^{r2}(k_0)/\partial k_0^2$ will be positive. An identical argument goes through for $\partial V^{d2}(k_0)/\partial k_0^2$, so that the numerator of equation (A.7) will be zero within the CRS range. The denominator is unambiguously negative since $u''(c_t)<0$. It follows that $\partial^2 D_0/\partial k_0{}^2$ is equal to zero within the CRS range.

For the range of increasing returns to scale, the sign will depend upon the relative magnitudes of $\partial V^{d2}(k_0)/\partial k_0^2$ and $\partial V^{r2}(D_0,k_0)/\partial k_0^2$, both of which are positive. Since a change in k_0 will only enter through its effect upon output, it is sufficient to compare Q_{kk} for the two regimes. The default penalty yields a disparity, ie $Q_{kk}=Q''(k_0)>0$ in the repayment regime, while $Q_{kk}=(1-\theta)Q''(k_0)>0$ in the default regime. Holding k_0 constant, it is clear that $Q^r_{kk} > Q^d_{kk}$. It follows that $\partial V^{d2}(k_0)/\partial k_0^2 < \partial V^{r2}(D_0,k_0)/\partial k_0^2$. Hence, $\partial^2 D_0/\partial k_0{}^2$ is positive within the range of increasing returns to scale.

Given that $h(k_0)=D_0/k_0$ is a constant in the CRS ranges and increasing in the range of production externalities, one need only observe the "borrowing-enhanced" production function subject to $h(k_t)$ to complete the proof of theorem 1. For simplicity, assume zero consumption. The production function can then be written:

$$\text{(A.6)} \quad \hat{Q}(k_1) = Q\Big[\, h_1(k_1)/J_0(I_0,k_0) + (1-\delta)k_0 \,\Big] - \hat{r}h_1(k_1)$$

Differentiating with respect to k_1 yields:

$$\text{(A.7)} \quad \hat{Q}'(k_1) = Q'(k_1)[h_1'(k_1)/J_0(I_0,k_0)] - \hat{r}h_1'(k_1) > 0.$$

$\hat{Q}'(k_1) > 0$ since $Q'(k_1)/J_0(I_0,k_0)] - \hat{r} > 0$, must be satisfied for rational borrowing. Differentiating (A.9) with respect to k_1 yields:

$$\text{(A.8)} \quad \hat{Q}''(k_1) = Q''(k_1)[h_1'(k_1)/J_0)] + [Q'(k_1)/J_0) - \hat{r}]h_1''(k_1).$$

Within the range of CRS, the second term will be equal to zero since $h_1''(k_1)=0$. This additional term becomes positive within the range of aggregate production externalities, increasing $\hat{Q}''(k_1)$. This completes the proof of Theorem 1.

III. Proof of Theorem 2

The proof of Theorem 2 depends upon the result that $\hat{Q}(k_0)$ is an S-shaped production function, as shown in Theorem 1. The results of Dechert and Nishimura for optimal investment relative to an S-shaped production function now hold for the "borrowing-enhanced" production function $\hat{Q}(k_0)$. Since the capital stock of the debtor nation, k_0, completely describes the current state of the debtor nation given some stock of debt, D_0, I continue the proof in terms of $\hat{Q}(k_0)$ assuming some D_0.

Dechert and Nishimura show that a k_c exists for an S-shaped production function for which every optimal path starting from $k_0 > k_c$ converges to a steady state at which $k=k^*$. Similarly, for $k_0<k_c$, all optimal paths starting from k_0 converge to the origin.

The only distinction in the current specification, once the S-shape of the borrowing-enhanced production function has been established, is that the high growth path in the current model converges to a level of steady-state growth in the capital stock, rather than a constant capital level. However, this is a trivial modification.

Consider the optimal path discussed above. The steady state capital stock, k^*, is never one of maximum marginal product [See Majumdar and Mitra]. Consider two capital stocks, k_0 and k_1, which satisfy $k_c<k_0<k_1$ and $\hat{Q}'(k_0)<\hat{Q}'(k_1)$. Suppose that $k_0=k^*$. Then $\hat{Q}'(k^*)<\hat{Q}'(k_1)$. But if this is true, then k^* is not a steady state [see Majumdar and Mitra (1982)]. Therefore, k^* cannot lie within the range of increasing returns to scale.

Having shown that k^* does not lie in increasing returns range, and given that the optimal path is monotonic in capital, it follows that the optimal path when $k_0>k_c$ will enter the high CRS range. Once in this range, a steady-state will exist with $h^*(k_0)=D_0/k_0$, as discussed in the text. This completes the proof of Theorem 2.

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