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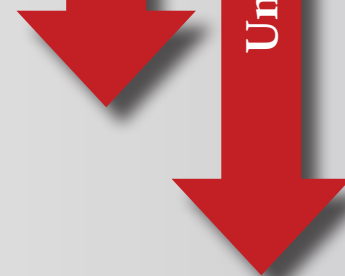
Fiscal Policies and Trade: On the existence of Nash equilibria

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Fiscal Policies and Trade: On the existence of Nash equilibria

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Abstract

This paper studies the existence of a fiscal Nash equilibrium in a multi-country model of trade where the relative prices are the channels of transmission of fiscal policies between countries. We stipulate the necessary conditions on goods, consumptions and trade that allow for the existence of a Nash equilibrium in the fiscal game. In the particular case of homotethic utility functions, we show that conditions on the shape and the degree of the curvature of the relative prices functions are sufficient to guarantee the existence of the Nash equilibrium

Keywords: Fiscal policy, Trade, Nash equilibrium

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1   Introduction

Numerous papers have examined the properties of the Nash equilibrium in a two-country or multi-country model of fiscal policy. Most of these analysis focus on the welfare implication of fiscal policy interactions. In doing so, they traditionally compare the Nash equilibrium to the cooperative equilibrium in order to point out the role of fiscal agreements between countries. Few of these models have explored the role of fiscal policy when the terms of trade are the link between countries giving rise to 'international transmission' of fiscal policy (Turnovsky (1988), Chari and Kehoe (1990), Devereux (1991), Sorensen (1996)). Most of these papers have stipulated the existence of the Nash equilibrium without checking it before. The objective of this paper is to fill this gap which is of high importance to understand the mechanisms of the fiscal game in real trade models.

The first way to check the existence of a Nash equilibrium is to stipulate particular welfare functions, as in Devereux (1991) who uses the logarithm of a Cobb Douglas function in private and public consumption. But when welfare functions are not specified as in Chari and Kehoe (1990), it is necessary to introduce conditions to check that the Nash equilibrium may exist. In a general standard multi-country model of trade, we point out the sufficient conditions on goods, consumptions and trade that allow for the existence of the fiscal policy Nash equilibrium.

The role of fiscal policy in an international context is generally studied through two main channels of transmission: capital flows and the terms of trade. When articles focus on capital as the mechanism of fiscal policy transmission, the tax competition mechanism is at work (Wilson (1985, 1986), Wildasin (1988)). In a recent paper Bayindir-Upmann and Ziad (2005) show the conditions that guarantee in a standard model of tax competition (Wildasin (1988)) the existence of a second-order locally consistent equilibrium which corresponds to a local Nash equilibrium. By definition, this is a weaker concept than the Nash equilibrium. These conditions can be applied in particular contexts such as homogeneous countries, a world reduced to two countries, or when capital demand is concave. Previously, Laussel & Le Breton (1998) have proved the existence and uniqueness of the Nash equilibrium in a tax competition model with two countries. However, their powerful result is obtained under rather strong assumptions such as the absence of capital owners and the linearity of each region's welfare. Recently, Taugourdeau and Ziad (2011) have gone further by studying the existence of Nash equilibria in a tax competition model where asymmetries between regions is allowed.

The existence of the Nash Equilibrium in a trade model has also been examined by Wong (2004). His article is based on a tariff retaliation model in which two countries set tariffs strategically in an exchange economy. The author shows that the concavity of the offer curve function to the origin is a sufficient condition to ensure the existence of the Nash equilibrium. But in this model, the author confines himself to a two-country model and to the case of homothetic and quasi-linear welfare functions.

In our paper, we develop a general trade model in which the channels of transmission of fiscal policies between countries are the relative prices. We basically build our framework on the models by Turnovsky (1988) and Chari and Kehoe (1990) that we extend in several ways. We depart from Turnovsky (1988) by extending the framework to a n-country model, but in line with Chari and Kehoe, and contrary to Turnovsky, we state that agents benefit from national public goods only. We also extend Chari and Kehoe (1990) since they assume that there are only two private goods in the whole economy composed by I countries. In our model, each country produces a specific good.

We first exhibit a counter-example in which the game has no Nash equilibrium in pure strategies. Then, without any specification of the welfare functions, we determine conditions which guarantee the existence of a Nash equilibrium in our general framework. We show that under the standard assumption of normality of goods, the only conditions which guarantee the existence on the Nash equilibrium concern the demand function of the private goods: demand functions of the private goods should be decreasing and concave functions of the national tax. When we concentrate our analysis on the particular case of homothetic functions, the necessary conditions ensuring the existence of a Nash equilibrium reduces to conditions on the bilateral terms of trade functions. If the relative prices are convex with respect to the consumptions ratio, there always exists a Nash equilibrium. When the bilateral terms of trade functions are concave, then the degree of concavity must not be to high to be able to implement a Nash equilibrium. Furthermore, when we limit the welfare functions to homothetic functions, identical for each country, we show that the n conditions on the terms of trade merge to a single condition.

The paper is organized as follows: the second section presents the n-country model. The third section deals with a counter-example showing that the existence problem may fail. In the fourth section, we stipulate conditions which guarantee the existence of the Nash equilibrium, first un a general set-up, and second, using homothetic welfare functions. Examples are also derived in a last subsection. The last section concludes.

2 The model

We consider a world economy composed by n countries. Each country is specialized in the production of one private good which is supposed to be traded between the n countries. In each country, public expenditures are financed through lump-sum taxation. They are decided by each government and are used for purchasing the national good only. Each government acts as a benevolent and maximizes the country's representative consumer's welfare function. Production of any good is fixed and given. Countries are identical in size and there is a large number of consumers in each country.

We denote by c_i^l quantities consumed by country l representative consumer of good i.

The available (given) quantity of good i produced in country i is denoted by y_i. The equilibrium conditions on the goods markets read:

$$y_i = \sum_{l=1}^{n} c_i^l + g_i, \ \forall_i$$

All households are supposed identical. The utility function of each representative consumer depends on the consumption of the national public good, the private good provided in the country and the imported goods: $U^i \left(c_1^i, ..., c_n^i, g_i \right)^1$. We assume that the utility functions are identical, continuous and twice differentiable functions.

The consumer of country i is endowed with a positive amount of the national private good and is taxed τ_i units of this good. His budget constraint is:

$$y_i - \tau_i = \sum_{l=1}^{n} \frac{p_l}{p_i} c_l^i \tag{1}$$

whereas the budget constraint of government i writes:

$$\tau_i = g_i \tag{2}$$

The trade equilibrium for country i is given by:

$$\sum_{k=1, k \neq i}^{n} p_k c_k^i = p_i \sum_{k=1, k \neq i}^{n} c_i^k \ \forall i \tag{3}$$

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[1]The assumption that public goods benefit only residents of the country is a standard assumption in the literature on inter-regional or international fiscal policies (see e.g., Chari and Kehoe (1990)). In this case, it refers to national public goods rather than global public goods.

The maximization problem of a representative consumer reduces to the choice of consumption levels for each good, given the level of national public expenditures. The representative household in country i has to solve:

$$\max_{c_1^i,...,c_n^i} U^i\left(c_1^i, ..., c_n^i, g_i\right)$$
$$\text{s.t. } (1)$$

Which solutions are:

$$\frac{\partial U^i/\partial c_i^i}{\partial U^i/\partial c_j^i} = \frac{p_i}{p_j} \ \forall i,j \tag{4}$$

The equilibrium equation for trade (3), given the consumption levels c_j^i and c_i^i obtained by (4), implies that the Marshallian demand for good k in country i is given by

$$c_k^i = F_k\left(\frac{p_1}{p_i}, ., \frac{p_k}{p_i}, .., \frac{p_n}{p_i}, R_i, g_i\right) \tag{5}$$

where $R_i = y_i - \tau_i$ is the net income of tax. Combining each demand for goods in the good market equilibria, we obtain that each relative price between two goods depends of the vector of fiscal policies in the economy:

$$\frac{p_i}{p_j} \equiv \sigma_{ij}\left(\tau_1, \tau_2, ..., \tau_i, ...\tau_n\right) = \sigma_{ij}\left(\boldsymbol{\tau}\right) \tag{6}$$

where $\boldsymbol{\tau}$ represents the vector of fiscal policies.

Expression (6) shows that the terms of trade between two countries depend on the fiscal policies of the n countries and then constitute the channel of transmission of national fiscal policies. As a consequence, the existence of a Nash equilibrium crucially depends on the response of the bilateral terms of trade to a change in fiscal policies.

Governments are supposed to choose their level of public expenditures before the consumers' choices of consumption. The government i's objective is to maximize the welfare of its representative consumer, then for each country, the optimal level of fiscal policy is obtained differentiating U^i with respect to τ_i according to constraint (2), (5) and (6).

We obtain:

$$\frac{\partial U^i}{\partial g_i}\left(\sum_{l=1}^{n} \frac{1}{\frac{\partial U^i/\partial g_i}{\partial U^i/\partial c_l^i}}\frac{\partial c_l^i}{\partial \tau_i} + 1\right) = 0 \Longrightarrow \sum_{l=1}^{n}\frac{\partial c_l^i/\partial \tau_i}{\frac{\partial U^i/\partial g_i}{\partial U^i/\partial c_l^i}} = -1 \tag{7}$$

or equivalently from (1) and (4)

$$\frac{\partial U^i}{\partial c_i^i}\left[-1 - \sum_{l=1,l\neq i}^{n} c_l^i \frac{\partial p_l/p_i}{\partial \tau_i} + \frac{\partial U^i/\partial g_i}{\partial U^i/\partial c_i^i}\right] = 0 => \frac{\partial U^i/\partial g_i}{\partial U^i/\partial c_i^i} = 1 + \sum_{l=1,l\neq i}^{n} c_l^i \frac{\partial p_l/p_i}{\partial \tau_i} \tag{8}$$

The optimal level of public expenditure is obtained when the marginal rate of substitution between the public good and the local private good equals the relative price between these goods (i.e unity) adjusted by the $n-1$ price substitution effects between the n goods.

A variation in public spending has two effects: a direct income effect, and an indirect price effect which modifies the composition of the basket of private goods.

3 Counter-example: non-existence of a Nash equilibrium in pure strategies

We assume two countries A and B. Country A is specialized in production of good a, and country B in production of good b. The utility functions are given by:

$$U^A\left(c_a^A, c_b^A, g_A\right) = \ln c_a^A + c_b^A.g_A \tag{9}$$

$$U^B\left(c_b^B, c_a^B, g_B\right) = \ln c_b^B + \ln c_a^B + \ln g_B \tag{10}$$

The equilibrium conditions on the goods markets read $y_A = c_a^A + c_a^B + g_A$ for good a and $y_B = c_b^A + c_b^B + g_B$ for good b. The trade balance is $\sigma c_b^A = c_a^B$ with $\sigma = \frac{p_b}{p_a}$. The budget constraint of the representative consumer in country A is:

$$y_A - \tau_A = c_a^A + \sigma c_b^A \tag{11}$$

and the budget constraint of the representative consumer in country B is:

$$y_B - \tau_B = \sigma^{-1} c_a^B + c_b^B \tag{12}$$

We assume $1 \leq y_A.y_B \leq 2$.

The solution of the maximization problem for each representative agent gives the expression of each private consumption as a function of both the taxes and the relative price:

$$c_b^A = \frac{1}{\sigma}(y_A - \tau_A) - \frac{1}{\tau_A}, \qquad c_a^A = \frac{\sigma}{\tau_A}$$

$$c_a^B = \frac{\sigma\left(y_B - \tau^B\right)}{2}, \qquad c_b^B = \frac{(y_B - \tau_B)}{2} \qquad (13)$$

Given the consumption levels c_b^A and c_a^B, the trade equilibrium implies that the relative price between goods is given by the following equation:

$$\sigma \equiv \sigma\left(\tau_A, \tau_B\right) = \frac{2\left(y_A - \tau_A\right)\tau_A}{2 + \tau_A\left(y_B - \tau_B\right)} \qquad (14)$$

which enables us to write the private consumptions as functions of the taxes:

$$c_b^A = c_b^B = \frac{(y_B - \tau_B)}{2}$$

$$c_a^B = \frac{(y_B - \tau_B)(y_A - \tau_A)}{2 + \tau_A(y_B - \tau_B)}\tau_A \qquad c_a^A = \frac{2(y_A - \tau_A)}{2 + \tau_A(y_B - \tau_B)} \qquad (15)$$

Thus, indirect utility functions are given by:

$$V^A\left(\tau_A, \tau_B\right) = \ln\frac{2\left(y_A - \tau_A\right)}{2 + \tau_A\left(y_B - \tau_B\right)} + \frac{y_B - \tau_B}{2}\tau_A \qquad (16)$$

$$V^B\left(\tau_A, \tau_B\right) = \ln\frac{y_B - \tau_B}{2} + \ln\left(\frac{(y_B - \tau_B)(y_A - \tau_A)}{2 + \tau_A(y_B - \tau_B)}\tau_A\right) + \ln\tau_B \qquad (17)$$

The first derivative of the indirect utility function with respect to the tax is:

$$\frac{\partial V^A}{\partial \tau_A} = \frac{-4 - 2\tau_A\left(y_B - \tau_B\right) + \left(y_B - \tau_B\right)\left(y_A - \tau_A\right)\left(2 + \tau_A(y_B - \tau_B)\right)}{\left(y_A - \tau_A\right)\left(2 + \tau_A\left(y_B - \tau_B\right)\right)}$$

If $\tau_B = y_B$, then $\frac{\partial V^A}{\partial \tau_A} < 0$.

If $\tau_B < y_B$, the first derivative vanishes if and only if the following quadratic equation vanishes:

$$F\left(\tau_A\right) = -\left(y_B - \tau_B\right)^2\left(\tau_A\right)^2 + \tau_A\left(y_B - \tau_B\right)\left[y_A\left(y_B - \tau_B\right) - 2\right] - 4 = 0$$

which discriminant is

$$\triangle\left(\tau_B\right) = \left(y_B - \tau_B\right)^2 \left(y_A\left(y_B - \tau_B\right) - 6\right)\left(y_A\left(y_B - \tau_B\right) + 2\right)$$

As $1 \leq y_A.y_B \leq 2$, then for each $t_B \in [0, y_B[$, $\triangle\left(\tau_B\right) < 0$. Therefore the sign of $\frac{\partial V^A}{\partial \tau_A}$ is the sign of $-\left(y_B - \tau_B\right)$ which is negative. As a result, for each $\tau_B \in [0, y_B]$, the best reply for A is to choose $\tau^A = 0$. But $V^B \underset{\tau_A=0}{\rightarrow} -\infty$ then there does not exist any best response τ_B in this game: this game does not exhibit any Nash equilibrium in pure strategy.

4   Nash equilibrium

In this section, we determine sufficient conditions that guarantee the existence of the Nash equilibrium. To do so, we check if the government payoffs are concave, that is, if the result of the reaction functions compilation is a maximum. We assume twice-continuously differentiable utility functions.

   We will prove our result by assuming some conditions on the primitive of the model. Let introduce the following conditions.

   (C$_1$): $\frac{\partial}{\partial c_l^i}\left(TMS_{il}\right) \geq 0$ and $\frac{\partial}{\partial g_i}\left(TMS_{il}\right) \leq 0$ where $TMS_{il} = \frac{\partial U^i/\partial g_i}{\partial U^i/\partial c_l^i}$.

   Condition $(\mathbf{C_1})$ states that the marginal rate of substitution between the public good and a private good is decreasing with the public good and increasing with the private good. This is a standard normality condition requiring that both private and public goods are normal goods[2].

Theorem 1 *Assume twice-continuously differentiable utility functions, if (C_1) holds, then if private consumptions are a decreasing and concave function of the national tax, the fiscal policy game between the n countries possesses a fiscal Nash equilibrium*

   Before proving the result, let derive several Lemma:
   At the optimum, the second derivative of the utility function with respect to the local tax rate is given by:

$$\frac{\partial^2 U^i}{\partial \tau_i^2} = \frac{\partial U^i}{\partial g_i}\left[\sum_{l=1}^{n} \frac{\frac{\partial^2 c_l^i}{\partial(\tau_i)^2}TMS_{il} - \frac{\partial c_l^i}{\partial \tau_i}\frac{\partial TMS_{il}}{\partial \tau^i}}{\left(TMS_{il}\right)^2}\right] \tag{18}$$

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[2]This asumption is usually used in the models of fiscal interactions (see e.g., Bucovestky, 1991, Bayindir-Upmann and Ziad, 2005, Bloch and Zenginobuz, 2007).

Lemma 1 *The marginal rate of substitution between the local private good and the national public good is a non increasing function in the local rate tax when private consumptions decrease with the local tax rate..*

Proof:
From conditions $(\mathbf{C}_1)$, $\frac{\partial g_i}{\partial \tau_i} = 1$, and the assumption that private consumptions decrease with respect to the local tax rate, we get

$$\frac{\partial TMS_{il}}{\partial \tau^i} = \sum_{l=1, l \neq i}^{n} \frac{\partial TMS_{il}}{\partial c_l^i} \frac{\partial c_l^i}{\partial \tau_i} + \frac{\partial TMS_{il}}{\partial c_i^i} \frac{\partial c_i^i}{\partial \tau_i} + \frac{\partial TMS_{il}}{\partial g_i} \leq 0$$

Lemma 2 *In any tax rate satisfying the first order condition, then if private consumptions are a decreasing and concave function of the national tax,$\frac{\partial^2 U^i}{\partial \tau_i^2} < 0$*

Proof:
Immediate from (18) together with Lemma 1.

Lemma 2 precludes the existence of any minimum since any minimum requires $\frac{\partial^2 U^i}{\partial \tau_i^2} > 0$ for any τ_i satisfying $\frac{\partial U^i}{\partial \tau_i} = 0$. The absence of a minimum precludes also the existence of multiple maxima. The argumentation is the following:

Assume the existence of two best replies in the large. This implies the existence of two maxima and one minimum $\tau_{\min}$ between them. At this point, we should have

$$\frac{\partial U^i}{\partial \tau_i} (\tau_{\min}, \boldsymbol{\tau}_{-i}) = 0 \text{ (extremum)}$$

and

$$\frac{\partial^2 U^i}{\partial t^2} (\tau_{\min}, \boldsymbol{\tau}_{-i}) > 0 \text{ (minimum)} \tag{19}$$

stating that the indirect utility function is locally concave at each point satisfying $\frac{\partial U^i}{\partial t} = 0$, this implies $\frac{\partial^2 U^i}{\partial t^2} (\tau_{\min}, \boldsymbol{\tau}_{-i}) < 0$ which is contradictory with (19). This demonstration precludes the existence of any minimum between two maxima and then the existence of two (or more) best replies. The best reply is unique in the large.

In order to prove the existence of at least one Nash equilibrium, we consider the best response of each jurisdiction and prove that they are functions and not a correspondence. Then by continuity assumptions, we can use a fixed point theorem (Brouwer's theorem) to prove the existence of a Nash equilibrium.

Proof of Theorem 1:

Let denote the indirect utility function $V^i(\boldsymbol{\tau})$ when conditions (4) and (6) are introduced in the utility function U_i.

Let us fix one player i, and let τ^{-i} be the strategy of the rest of players. Consider the best reply of player i,

$$BR(\tau_{-i}) = \{\tau_i' \in [0, T_i] : V_i(\tau_i', \tau_{-i}) = \sup_{\tau_i \in [0, T_i]} V_i(\tau_i, \tau_{-i})\}$$

Let us discuss two cases:

1. If for τ_{-i}, we have $\frac{\partial V^i}{\partial T^i}(\tau_i, \tau_{-i}) \neq 0$ for each τ_i, which means that the payoff function is a monotonic function (the payoff function is of course smooth). Therefore the best reply strategy is unique.

2. If for τ_{-i}, we have $\frac{\partial V^i}{\partial T^i}(\tau_i, \tau_{-i}) = 0$ for some τ_i, then from the property L_i is strictly positive, the best reply strategy is unique.

Therefore

$$BR(\tau_1, \tau_2, \ldots, \tau_n) = \prod_{i=1}^{n} BR_i(\tau_{-i})$$

where $\prod_{i=1}^{n} BR_i(\tau_{-i}) = \prod_{i=1}^{n}\{\tau_i^* \in [0, T_i] : V_i(\tau_i^*, \tau_{-i}) = \sup_{\tau_i} V_i(\tau_i, \tau_{-i})\}$ is a function (single valued) from $\prod_{i=1}^{n} [0, T_i]$ into $\prod_{i=1}^{n} [0, T_i]$. By Brouwer's theorem there exist a fixed point of $BR(.)$. It is easy to check that the fixed point is also a Nash equilibrium.

In Theorem 1, the reaction of the private demand function to a change in the fiscal policy is a key variable. From the demand function (5), the reaction of the private consumption demand depends on three effects as described below: a price effect, an income effect and a preference effect:

$$\frac{\partial c_k^i}{\partial \tau_i} = \sum_k \frac{\partial c_k^i}{\partial \sigma_{ki}} \frac{\partial \sigma_{ki}}{\partial \tau_i} - \frac{\partial c_k^i}{\partial R_i} + \frac{\partial c_k^i}{\partial g_i}$$

The first term characterizes the price effect: an increase in tax of the country modifies the relative prices and the demand for good k in country i. The sign of the effect depends on both the effect on the relative price and the nature of the goods (if they are complement or substitute). The second term highlights the income effect $\left(-\frac{\partial c_k^i}{\partial R_i}\right)$ which is negative. Finally, the third

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term stands for an arbitrage between the public and the private good in the preferences of the agents. If we consider that public and private goods are complement, this effect is positive. If both goods are supposed to be substitute, the effect is negative.

According to that comment, the private demand functions are decreasing in the tax rate if the three effects work in the same way or if the income effect dominates the two others.

4.1   Homothetic functions

In this section, we focus on homothetic utility functions. Our objective is to refine the previous analysis to obtain more tractable conditions. The representative agent's maximization program becomes:

$$\frac{c_l^i}{c_k^i} = \Phi\left(\frac{p_k}{p_l}\right) = \frac{c_l^j}{c_k^j} \tag{20}$$

with $\Phi'\left(\frac{p_k}{p_l}\right) > 0$ and $\Phi(1) = 1$.

Let assume an additional condition on the demand functions:

$(\mathbf{C_2})$: The income effect dominates the price effect of the tax on the imported goods demand.

Note that under the assumption of homothetic welfare functions, the preference effect on the private good consumption described in the previous section disappears. Only the income and relative prices effects remain.

In this framework, we are the able to derive two theorems

Theorem 2 *Assume identical homothetic utility functions, if (C_1) and (C_2) hold, if private consumptions are a concave function of the national tax, then the fiscal policy game between the n countries possesses a fiscal Nash equilibrium.*

Theorem 3 *Assume identical homothetic utility functions, if (C_1) and (C_2) hold, then, if the bilateral terms of trade are a convex function of the consumptions ratio, the fiscal policy game between the n countries possesses a fiscal Nash equilibrium*

In order to rpove both results, we first derive two lemmas:

Lemma 3 *With homothetic welfare functions, the bilateral terms of trade are given by:*

$$\frac{p_k}{p_i} = \Phi^{-1}\left(\frac{y_i - \tau_i}{y_k - \tau_k}\right) \tag{21}$$

And

$$\frac{\partial p_k/p_i}{\partial \tau_i} = \frac{-1}{y_k - \tau_k}\left(\Phi^{-1}\right)'\left(\frac{y_i - \tau_i}{y_k - \tau_k}\right) < 0$$

$$\frac{\partial^2 p_k/p_i}{\partial \tau_i^2} = \frac{1}{(y_k - \tau_k)^2}\left(\Phi^{-1}\right)''\left(\frac{y_i - \tau_i}{y_k - \tau_k}\right)$$

Lemma 3 implies that a bilateral term of trade $\left(\frac{p_k}{p_l}\right)$ only depends on the taxes applied in countries k and l. With homothetic utility functions it is easier to understand the different effects of tax τ_i on consumptions. Combining the budget constraint condition with Equation (20), we obtain

$$c_k^i = \frac{(y_i - \tau_i)}{\sum\limits_{l}\frac{p_l}{p_i}\Phi\left(\frac{p_k}{p_l}\right)}$$

The income effect appears at the numerator: an increase of τ_i diminishes the income and then the consumption of any goods of the representative agent i.

The price effects impact the denominator, directly through a change of each relative price $\left(\frac{p_l}{p_i}\right)$ and indirectly through the impact of the change of the relative price on the composition of the good basket $\Phi\left(\frac{p_i}{p_l}\right)$, but only for the local private good i. The direct effect of relative prices implies a positive effect on consumptions. Conversely, the composition of basket good effect implies a negative effect on the local private consumption.

Lemma 4 *With homothetic welfare functions, condition* $\mathbf{C_2}$ *insures that* $\frac{\partial c_l^i}{\partial \tau_i} \leq 0$ *for each l.*

Proof: We have

$$\frac{c_l^i}{c_k^i} = \Phi\left(\frac{p_k}{p_l}\right) = \frac{c_l^j}{c_k^j}$$

From previous Lemma we have

$$\frac{p_k}{p_l} = \Phi^{-1}\left(\frac{y_l - \tau_l}{y_k - \tau_k}\right)$$

Then

$$\frac{c_l^i}{c_k^i} = \frac{y_l - \tau_l}{y_k - \tau_k} = \frac{c_l^j}{c_k^j}$$

replacing in (1) gives

$$y_i - \tau_i = c_k^i \sum_{l=1}^{n} \frac{p_l}{p_i} \cdot \frac{y_l - \tau_l}{y_k - \tau_k} \implies c_k^i = \frac{y_i - \tau_i}{\sum\limits_{l=1}^{n} \Phi^{-1}\left(\frac{y_i - \tau_i}{y_l - \tau_l}\right) \cdot \frac{y_l - \tau_l}{y_k - \tau_k}}$$

Where the numerator represents the income effect of the tax which is negative, whereas the denominator represents the substitution effect which is positive. Condition $(\mathbf{C_2})$ (the income effect dominates the substitution effect) enables us to state that the total effect of the tax τ_i on both the imported goods demand and the local good demand is negative: $\frac{\partial c_k^i}{\partial \tau_i} < 0 \ \forall k$.

Proof of theorem 2:
Straightforward according to Theorem 1 and Lemma 3

Proof of theorem 3:
The first order condition of each government's program may be rewritten as:

$$\frac{\partial U^i}{\partial c_i^i}\left[-1 - \sum_{l=1, l\neq i}^{n} c_l^i \frac{\partial p_l/p_i}{\partial \tau_i} + \frac{\partial U^i/\partial g_i}{\partial U^i/\partial c_i^i}\right] = 0$$

At the optimum, the second order condition is given by:

$$\frac{\partial^2 U^i}{\partial \tau_i^2} = \frac{\partial U^i}{\partial c_i^i}\left[\frac{\partial}{\partial \tau_i}\left(-1 - \sum_{l=1, l\neq i}^{n} c_l^i \frac{\partial \sigma_{li}}{\partial \tau_i}\right) + \frac{\partial}{\partial \tau_i}(TMS_{ii})\right] < 0$$

$$\iff$$

$$L_i = \frac{\partial}{\partial \tau_i}\left(-1 - \sum_{l=1, l\neq i}^{n} c_l^i \frac{\partial \sigma_{li}}{\partial \tau_i}\right) + \frac{\partial}{\partial \tau_i}(TMS_{ii}) < 0 \qquad (22)$$

From condition $(\mathbf{C}_1)$ and Lemma 4, we are able to sign the impact on the tax rate on the marginal rate of substitution between the public an the national private good:

$$\left(\frac{\partial}{\partial \tau_i} TMS_{ii}\right) = \sum_{l\neq i}^{n} \frac{\partial}{\partial c_l^i}\left(\frac{\partial U^i/\partial c_i^i}{\partial U^i/\partial g_i}\right)\frac{\partial c_l^i}{\partial \tau_i} + \frac{\partial}{\partial c_i^i}\left(\frac{\partial U^i/\partial c_i^i}{\partial U^i/\partial g_i}\right)\frac{\partial c_i^i}{\partial \tau_i} + \frac{\partial}{\partial g_i}\left(\frac{\partial U^i/\partial g_i}{\partial U^i/\partial c_i^i}\right) < 0$$

Then, according to Equation (22), a sufficient condition to ensure the locally second order condition is then

$$\frac{\partial}{\partial \tau_i}\left(-1 - \sum_{l=1,l\neq i}^{n} c_l^i \frac{\partial \sigma_{li}}{\partial \tau_i}\right) \leq 0$$

This condition stipulates that the variation of the n goods expenditures of country i consumers via the consumption effect decreases with the rise of the public expenditure. When τ_i increases, two effects on the consumer $i's$ expenditures are at work: a quantity effect and a price effect. The concavity of the government pay-off necessitates the decrease of the quantity effect.

The conditions in our n-country model reduce to:

$$H^i = \sum_{l=1,l\neq i}^{n} c_l^i \frac{\partial^2 \sigma_{li}}{\partial \tau_i^2} + \sum_{l=1,l\neq i}^{n} \frac{\partial c_l^i}{\partial \tau_i}\frac{\partial \sigma_{li}}{\partial \tau_i} > 0 \ . \tag{23}$$

From Lemmas 3 and 4, we know:

$$\sum_{l=1,l\neq i}^{n} \frac{\partial c_l^i}{\partial \tau_i}\frac{\partial \sigma_{li}}{\partial \tau_i} > 0 \ .$$

and H^i to be positive needs

$$H_0^i = \sum_{l=1,l\neq i}^{n} \frac{c_l^i}{(y_l - \tau_l)^2}\left(\Phi^{-1}\right)''\left(\frac{y_i - \tau_i}{y_l - \tau_l}\right) \geq 0$$

which is feasible for convex bilateral terms of trade.∎

The curvature of the function $\Phi^{-1}(\cdot)$ helps to guarantee the existence of a Nash equilibrium. If $\Phi^{-1}(\cdot)$ is convex, then the existence of a fiscal Nash equilibrium is ensured. Since agents have identical preferences, a single condition on function $\Phi^{-1}(\cdot)$ is necessary to ensure the existence of the Nash equilibrium. If $\Phi^{-1}(\cdot)$ is concave, condition (23) states that the degree of concavity must not be to high to guarantee the existence of a Nash equilibrium.

Theorem 3 gives a condition which is, as we will see in the next section devoted to examples, easy to check. If this condition is not satisfied, then conditions of Theorem 2 should be verified.

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4.2 Examples

4.2.1 Cobb-Douglas utility function

Let illustrate the paper by a Cobb-Douglas utility function example.

$$U^i = (g^i)^\gamma \cdot \prod_l (c_l^i)^{\alpha_l}$$

the maximization program of the consumer gives:

$$\frac{\frac{\partial U^i}{\partial c_l^i}}{\frac{\partial U^i}{\partial c_k^i}} = \frac{\alpha_l}{\alpha_k} \frac{c_k^i}{c_l^i} = \frac{p_l}{p_i}$$

and

$$\frac{\partial U^i / \partial g_i}{\partial U^i / \partial c_i^i} = \frac{\gamma}{\alpha_i} \frac{c_i^i}{g^i}$$

which immediately implies that $\mathbf{C}_1$ and $\mathbf{C}_2$ are verified:

$$\frac{\partial \frac{\partial U^i / \partial g_i}{\partial U^i / \partial c_i^i}}{\partial c_i^i} > 0; \quad \frac{\partial \frac{\partial U^i / \partial g_i}{\partial U^i / \partial c_i^i}}{\partial g_i} < 0 \text{ and } \frac{\partial \frac{\partial U^i / \partial g_i}{\partial U^i / \partial c_i^i}}{\partial c_l^i} = 0$$

We have

$$\frac{p_l}{p_i} = \Phi^{-1} \left(\frac{y_i - \tau_i}{y_l - \tau_l} \right) = \frac{\alpha_l}{\alpha_i} \frac{y_i - \tau_i}{y_l - \tau_l}$$

which implies $(\Phi^{-1})'' = 0$.

Finally c_k^i is given by

$$c_k^i = \frac{y_i - \tau_i}{\sum\limits_{l=1}^{n} \Phi^{-1} \left(\frac{y_i - \tau_i}{y_l - \tau_l} \right) \cdot \frac{y_l - \tau_l}{y_k - \tau_k}} = \frac{y_k - \tau_k}{n \frac{\alpha_l}{\alpha_i}} => \frac{\partial c_k^i}{\partial \tau_i} = 0 \text{ and } \frac{\partial c_i^i}{\partial \tau_i} = -\frac{1}{n} < 0$$

and C_0 is verified.

The requirements of Theorem 3 are satisfied, then there exists at least one Nash equilibrium.

4.2.2 CES utility function

The utility functions are supposed to be of the form:

$$U^i = \left[\left(\sum_k \left(c_k^i \right)^\delta \right)^{\frac{\gamma}{\delta}} + \left(g^1 \right)^\gamma \right]^{\frac{1}{\gamma}} \tag{24}$$

where $\delta \equiv 1 - \frac{1}{\eta_1} < 1$ and $\eta_1 > 1$ characterizes the elasticity of substitution between private goods whereas $\gamma \equiv 1 - \frac{1}{\eta_2} < 1$ and $\eta_2 > 1$ stands for the elasticity of substitution between the private good basket and public good. For simplicity we assume that $y_i = y_j = y$.

The marginal rates of substitution between the public good and the private goods are given by

$$TMS_{il} = \frac{\partial U^i / \partial g_i}{\partial U^i / \partial c_l^i} = \frac{\left(g^i \right)^{\gamma - 1}}{\left(c_l^i \right)^{\delta - 1} \left(\sum_k \left(c_k^i \right)^\delta \right)^{\frac{\gamma}{\delta} - 1}}$$

which immediately implies that $\mathbf{C}_1$ is verified since

$$\frac{\partial TMS_{il}}{\partial g^i} = (\gamma - 1) \frac{\left(g^i \right)^{\gamma - 2}}{\left(c_l^i \right)^{\delta - 1} \left(\sum_k \left(c_k^i \right)^\delta \right)^{\frac{\gamma}{\delta} - 1}} < 0$$

$$\frac{\partial TMS_{il}}{\partial c_l^i} = - \left(g^i \right)^{\gamma - 1} \frac{(\delta - 1) \sum_{k \neq l} \left(c_k^i \right)^\delta + (\gamma - 1) \left(c_l^i \right)^\delta}{\left(c_l^i \right)^{\delta + 1} \left(\sum_k \left(c_k^i \right)^\delta \right)^{\frac{\gamma}{\delta}}} > 0$$

The marginal rate of substitution between two private goods is given by

$$\frac{\partial U^i / \partial c_l^i}{\partial U^i / \partial c_k^i} = \frac{p_l}{p_k} = \left(\frac{y_k - \tau_k}{y_l - \tau_l} \right)^{1 - \delta}$$

The analysis of the terms of trade function does not ensure that a Nash equilibrium exists since we have

$$\frac{\partial^2 \sigma_{li}}{\partial \left(\tau_i \right)^2} = \frac{\partial^2 p_l / p_i}{\partial \left(\tau_i \right)^2} = - (1 - \delta) \, \delta \frac{\left(y_k - \tau_k \right)^{-\delta - 1}}{\left(y_l - \tau_l \right)^{1 - \delta}} < 0$$

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and the relative price are then a concave function of the national tax. We are not able to use Theorem 3 in this case.

We have then to check the reaction of the private consumption to the national tax. The marginal rate of substitution between private goods allows us to rewrite any private consumption as:

$$c_l^i = \frac{y_i - \tau_i}{\displaystyle\sum_k \left(\frac{y_i - \tau_i}{y_k - \tau k}\right)^{1-\delta} \left(\frac{y_k - \tau_k}{y_l - \tau_l}\right)}$$

then

$$\frac{\partial c_l^i}{\partial \tau_i} = -\delta\left(y_l - \tau_l\right)\left(y_i - \tau_i\right)^{\delta-1} \frac{\displaystyle\sum_{k\neq i}\left(y_k - \tau_k\right)^{\delta}}{\left(\displaystyle\sum_k\left(y_k - \tau_k\right)^{\delta}\right)^2} < 0$$

and

$$\frac{\partial^2 c_l^i}{\partial\left(\tau_i\right)^2} = -\frac{\delta\left(y_l - \tau_l\right)\left(\displaystyle\sum_{k\neq i}\left(y_k - \tau_k\right)^{\delta}\right)^2\left(y_i - \tau_i\right)^{\delta-2}}{\left(\displaystyle\sum_k\left(y_k - \tau_k\right)^{\delta}\right)^2}\left(2\delta\left(y_i - \tau_i\right)^{\delta} + \left(1 - \delta\right)\sum_k\left(y_k - \tau_k\right)^{\delta}\right) < 0$$

The private consumptions are increasing and concave functions of the national tax. Since condition $\mathbf{C}_1$ is satisfied, Theorem 2 ensures that there exists at least one Nash equilibrium in this fiscal game.

5   Concluding remarks

In this methodological paper, we have exhibited the conditions allowing for the existence of a Nash equilibrium in a multi-country model of trade when fiscal policies are the strategic tools of the game. Although the question of existence of a fiscal Nash equilibrium in a model of trade has not still been investigated in the literature, the high number of papers analyzing the properties of the equilibrium justifies such a study. Moreover, the exhibition of a counter-example in which the game has no Nash equilibrium in pure strategies reinforces the appropriateness of such a question.

In a very general set-up where goods are normal, we show that the reaction of the private demand functions to a change in fiscal policy is the key point

of our analysis. Decreasing and concave private demand functions allow for the existence of a Nash equilibrium in this context. Since the relative prices represent the channels of transmission of the fiscal policies between countries, when we restrict our analysis to homothetic functions, we show that a sufficient condition to allow for the existence of a Nash equilibrium is that the bilateral terms of trade should be a convex functions of the consumptions ratio. When implementing identical homothetic welfare functions, we show that the sufficient conditions come down to a single condition on the shape of the terms of trade.

Several extensions to this model should be explored. The main interesting idea would consist in mixing both channels of transmission (exchange rate and capital flow).

6 Appendix : Proof of Lemma 3:

From the agents' decisions we have:

$$\frac{c_l^i}{c_k^i} = \Phi\left(\frac{p_k}{p_l}\right) = \frac{c_l^j}{c_k^j}$$

Replacing c_l^i in the budget constraint holds:

$$p_i\left(y_i - \tau_i\right) = c_k^i \sum_{l=1}^{n} p_l \cdot \Phi\left(\frac{p_k}{p_l}\right)$$

then

$$c_k^i = \frac{p_i\left(y_i - \tau_i\right)}{\sum\limits_{l=1}^{n} p_l \cdot \Phi\left(\frac{p_k}{p_l}\right)}$$

The trade balance for country i gives :

$$\sum_{k=1,k\neq i}^{n} p_k c_k^i = p_i \sum_{k=1,k\neq i}^{n} c_i^k$$

replacing c_k^i and c_k^i gives:

$$p_i \sum_{k=1,k\neq i}^{n} p_k \frac{\left(y_i - \tau_i\right)}{\sum\limits_{l=1}^{n} p_l \cdot \Phi\left(\frac{p_k}{p_l}\right)} = p_i \sum_{k=1,k\neq i}^{n} \frac{p_k\left(y_k - \tau_k\right)}{\sum\limits_{l=1}^{n} p_l \cdot \Phi\left(\frac{p_i}{p_l}\right)}$$

Which gives after manipulations

$$\frac{\sum\limits_{k=1}^{n} p_k \left(y_k - \tau_k\right)}{p_i \left(y_i - \tau_i\right)} = \sum_{k=1}^{n} \frac{p_k}{p_i} \cdot \frac{\sum\limits_{l=1}^{n} p_l \cdot \Phi\left(\frac{p_i}{p_l}\right)}{\sum\limits_{l=1}^{n} p_l \cdot \Phi\left(\frac{p_k}{p_l}\right)}$$

or

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_i}{p_k}\right) - \frac{\left(y_k - \tau_k\right)}{\left(y_i - \tau_i\right)} \right) = 0$$

Since we get

$$\frac{\Phi\left(\frac{p_i}{p_l}\right)}{\Phi\left(\frac{p_k}{p_l}\right)} = \frac{\frac{c_l^i}{c_i^i}}{\frac{c_l^i}{c_k^i}} = \frac{c_k^i}{c_i^i} = \Phi\left(\frac{p_i}{p_k}\right)$$

The expression

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_i}{p_k}\right) - \frac{\left(y_k - \tau_k\right)}{\left(y_i - \tau_i\right)} \right) = 0$$

gives for $i = 1$

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_1}{p_k}\right) - \frac{\left(y_k - \tau_k\right)}{\left(y_1 - \tau_1\right)} \right) = 0$$

for $i = 2$

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_2}{p_k}\right) - \frac{\left(y_k - \tau_k\right)}{\left(y_2 - \tau_2\right)} \right) = 0$$

....
for $i = n$

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_n}{p_k}\right) - \frac{\left(y_k - \tau_k\right)}{\left(y_n - \tau_n\right)} \right) = 0$$

Since we get

$$\frac{\Phi\left(\frac{p_i}{p_l}\right)}{\Phi\left(\frac{p_k}{p_l}\right)} = \frac{\frac{c_l^i}{c_i^i}}{\frac{c_l^i}{c_k^i}} = \frac{c_k^i}{c_i^i} = \Phi\left(\frac{p_i}{p_k}\right)$$

we rewrite the expressions as:

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for $i = 1$

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_1}{p_k}\right) - \frac{(y_k - \tau_k)}{(y_1 - \tau_1)} \right) = 0$$

for $i = 2$

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_2}{p_1}\right) \Phi\left(\frac{p_1}{p_k}\right) - \frac{(y_k - \tau_k)}{(y_1 - \tau_1)} \frac{(y_1 - \tau_1)}{(y_2 - \tau_2)} \right) = 0$$

....

for $i = n$

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_n}{p_1}\right) \Phi\left(\frac{p_1}{p_k}\right) - \frac{(y_k - \tau_k)}{(y_1 - \tau_1)} \frac{(y_1 - \tau_1)}{(y_n - \tau_n)} \right) = 0$$

By summing all the expressions we get:

$$\sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_1}{p_k}\right) - \frac{(y_k - \tau_k)}{(y_1 - \tau_1)} \right) + \sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_2}{p_1}\right) \Phi\left(\frac{p_1}{p_k}\right) - \frac{(y_k - \tau_k)}{(y_1 - \tau_1)} \frac{(y_1 - \tau_1)}{(y_2 - \tau_2)} \right) +$$

$$... + \sum_{k=1}^{n} p_k \left(\Phi\left(\frac{p_n}{p_1}\right) \Phi\left(\frac{p_1}{p_k}\right) - \frac{(y_k - \tau_k)}{(y_1 - \tau_1)} \frac{(y_1 - \tau_1)}{(y_n - \tau_n)} \right) = 0$$

or

$$\left(\Phi\left(\frac{p_1}{p_1}\right) + \Phi\left(\frac{p_2}{p_1}\right) + ... + \Phi\left(\frac{Pn}{p_1}\right) \right) \sum_{k=1}^{n} p_k \Phi\left(\frac{p_1}{p_k}\right) =$$

$$\left(\frac{(y_1 - \tau_1)}{(y_1 - \tau_1)} + \frac{(y_1 - \tau_1)}{(y_2 - \tau_2)} + ... + \frac{(y_1 - \tau_1)}{(y_n - \tau_n)} \right) \sum_{k=1}^{n} p_k \frac{(y_k - \tau_k)}{(y_1 - \tau_1)}$$

From (**??**) we know that $\sum_{k=1}^{n} p_k \Phi\left(\frac{p_i}{p_k}\right) = \sum_{k=1}^{n} p_k \frac{(y_k - \tau_k)}{(y_i - \tau_i)}$ then

for $i = 1$

$$\left(1 + \Phi\left(\frac{p_2}{p_1}\right) + ... + \Phi\left(\frac{Pn}{p_1}\right) \right) = \left(1 + \frac{(y_1 - \tau_1)}{(y_2 - \tau_2)} + ... + \frac{(y_1 - \tau_1)}{(y_n - \tau_n)} \right)$$

and for each i

$$\left(\Phi\left(\frac{p_1}{p_i}\right) + \Phi\left(\frac{p_2}{p_i}\right) + ... + \Phi\left(\frac{Pn}{p_i}\right) \right) = \left(\frac{(y_i - \tau_i)}{(y_1 - \tau_1)} + \frac{(y_i - \tau_i)}{(y_2 - \tau_2)} + ... + \frac{(y_i - \tau_i)}{(y_n - \tau_n)} \right)$$

$$= (y_i - \tau_i) \left(\frac{1}{(y_1 - \tau_1)} + \frac{1}{(y_2 - \tau_2)} + ... + \frac{1}{(y_n - \tau_n)} \right)$$

Taking the relations for $i = 1$ and $i = 2$. We can write:

$$\frac{\left(1 + \Phi\left(\frac{p_2}{p_1}\right) + ... + \Phi\left(\frac{p_n}{p_1}\right)\right)}{(y_1 - \tau_1)} = \left(\frac{1}{(y_1 - \tau_1)} + \frac{1}{(y_2 - \tau_2)} + ... + \frac{1}{(y_n - \tau_n)}\right)$$

and

$$\frac{\left(\Phi\left(\frac{p_1}{P2}\right) + 1 + ... + \Phi\left(\frac{Pn}{P2}\right)\right)}{(y_2 - \tau_2)} = \left(\frac{1}{(y_1 - \tau_1)} + \frac{1}{(y_2 - \tau_2)} + ... + \frac{1}{(y_n - \tau_n)}\right)$$

and then

$$\frac{\left(1 + \Phi\left(\frac{p_2}{p_1}\right) + ... + \Phi\left(\frac{Pn}{p_1}\right)\right)}{(y_1 - \tau_1)} = \frac{\left(\Phi\left(\frac{p_1}{P2}\right) + 1 + ... + \Phi\left(\frac{Pn}{P2}\right)\right)}{(y_2 - \tau_2)}$$

or

$$\frac{(y_2 - \tau_2)}{(y_1 - \tau_1)} = \frac{\left(\Phi\left(\frac{p_1}{P2}\right) + 1 + ... + \Phi\left(\frac{Pn}{P2}\right)\right)}{\left(1 + \Phi\left(\frac{p_2}{p_1}\right) + ... + \Phi\left(\frac{Pn}{p_1}\right)\right)}$$

multiplying the numerator and the denominator by $\Phi\left(\frac{p_2}{p_1}\right)$ on the expression at right yields:

$$\frac{(y_2 - \tau_2)}{(y_1 - \tau_1)} = \frac{\left(1 + \Phi\left(\frac{p_2}{p_1}\right) + ... + \Phi\left(\frac{Pn}{P1}\right)\right)}{\Phi\left(\frac{p_2}{p_1}\right)\left(1 + \Phi\left(\frac{p_2}{p_1}\right) + ... + \Phi\left(\frac{Pn}{p_1}\right)\right)}$$

which gives:

$$\Phi\left(\frac{p_2}{p_1}\right) = \frac{(y_1 - \tau_1)}{(y_2 - \tau_2)}$$

etc...for every couple and we can then write that for each i and k we get:

$$\Phi\left(\frac{p_i}{p_k}\right) = \frac{(y_k - \tau_k)}{(y_i - \tau_i)}$$

and

$$\frac{p_i}{p_k} = \Phi^{-1}\left(\frac{y_k - \tau_k}{y_i - \tau_i}\right) \quad \text{and} \quad \frac{p_k}{p_i} = \Phi^{-1}\left(\frac{y_i - \tau_i}{y_k - \tau_k}\right)$$

with $(\Phi^{-1})'\left(\frac{y_k - \tau_k}{y_i - \tau_i}\right) > 0$.

Then

$$\frac{\partial p_k / p_i}{\partial \tau_i} = \frac{-1}{y_k - \tau_k}(\Phi^{-1})'\left(\frac{y_i - \tau_i}{y_k - \tau_k}\right)$$

and

$$\frac{\partial^2 p_k / p_i}{\partial \tau_i^2} = \frac{1}{(y_k - \tau_k)^2}(\Phi^{-1})''\left(\frac{y_i - \tau_i}{y_k - \tau_k}\right)$$

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