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To Sponsor or Not to Sponsor: Sponsored Search Auctions with Organic Links

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To Sponsor or Not to Sponsor: Sponsored Search Auctions with Organic Links

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Abstract

In 2010 sponsored search advertisements generated over \$12 billion in revenue for search engines in the US market and accounted for 46% of online advertising revenue. A substantial portion of this revenue was generated by the sale of search keywords using an auction mechanism. We analyze a game-theoretic model to understand the interplay between organic and sponsored links in keyword auctions. Our model allows both the relevance of the advertising firm as well as the position of its sponsored link to impact click-through-rates. Our results demonstrate how the presence of organic links (links generated by the search engine algorithm) may lead to either more or less aggressive bidding for sponsored link positions depending on consumer attitudes toward sponsored links and the extent to which sponsored and organic links are complements or substitutes. In contrast to equilibrium results in existing literature, the firm with the highest value per click does not necessarily win the first spot in the sponsored search listings. It also may be optimal for a firm to bid an amount greater than the expected value (or sale) from a click.

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1 Introduction

Consumers often access information about firms in online markets through a commercial search website such as Google.com, Yahoo.com or Bing.com. Sponsored search advertising enables firms to display sponsored ads above organic results produced by the search engine (*SE*) to improve visibility to customers.¹ Sponsored search provides a balance between several concerns. Because sponsored links are displayed together with organic links, from the user's perspective they appear less intrusive than other types of ads such as pop-up windows or E-mail advertising. From the advertiser's perspective, sponsored search provides the ability to target customers based on a search query which results in more qualified traffic viewing sponsored ads. Finally, sponsored search typically entails a cost-per-click payment under which advertisers only incur a charge if a consumer clicks on the sponsored link.² In 2010 sponsored search advertisements generated over \$12 billion in revenue for search engines in the US market and accounted for 46% of online advertising revenue (Source IAB). Many of these advertisements are sold through keyword auctions.

Our paper examines how organic and sponsored links impact the equilibrium bidding strategies in keyword auctions when customers can access a firm's website by clicking on either a sponsored or an organic link (generated by the search engine algorithm) appearing in the search results produced by the *SE*. To incorporate the impact of the relevance of each firm to searching consumers, the probability that a searching customer will click on either a sponsored or organic link differs for each firm. Our analysis highlights two effects of sponsored links. A location effect shifts clicks away from organic links to sponsored links because sponsored links are placed at the top of the search results page, and they move organic links further down the page. The location effect may be exacerbated or offset by the degree to which sponsored and organic links are perceived by consumers as substitutes or complements. Regardless of location, the nature of the keyword (generic, brand-specific, popular, niche) and the text accompanying the sponsored link may affect the relationship between sponsored and organic links. For some keywords, sponsored links serve as a substitute for organic links, whereas for other keywords, sponsored links complement organic links and lead customers

¹See Evans(2007, 2011) for a survey on the economics of online advertising.

²See Taylor(2010) for an economic rationale for current payment schemes in the online advertising industry. See also Moon and Kwon (2011) and Zhu and Wilbur (2010) for the advantages of cost-per-click (CPC) vs cost-per-thousand impressions (CPM) schemes for the advertiser and the publisher.

to click on both the sponsored and organic link to a given firm. Our analysis characterizes how the relevance (to searching consumers) of organic and sponsored links, and the expected value of a click to each firm interact to determine equilibrium bidding strategies.

We find that the presence of organic links in the search results can lead to more aggressive bidding in keyword auctions if sponsored and organic links are sufficiently strong complements. We also show that the firm which is most relevant or has the highest value per click does not necessarily win the first spot in the sponsored search listings. Under certain conditions, a less popular firm may use a sponsored link to increase its traffic while a more popular firm relies only on its organic links to attract customers. In addition, for some parameterizations of our model, firms adopt mixed strategies. In contrast to previous papers, the mixed strategies apply not to the bids submitted by each website but to the decision about whether or not to participate in the keyword auction. Our results imply that equilibrium outcomes for a given keyword can generate no sponsored links, a single sponsored link, or multiple sponsored links. The extent to which sponsored and organic links are complements or substitutes along with location effects also creates an important role for the reservation price (minimum cost-per-click) established by the search engine.

Our game-theoretic approach models the interplay between organic and sponsored links without restrictive assumptions on the characteristics of keywords or consumer preferences, and incorporates a range of possible consumer perceptions of sponsored and organic links consistent with empirical studies. For example, Ghose and Yang (2009) find that retailer-specific and brand-specific information in a sponsored link increases the efficiency of online advertising. Yang and Ghose (2010) also show that organic and sponsored links tend to be positively interdependent. In particular, total click-through rates, conversions rates, and revenues are significantly higher when both sponsored and organic links to the firm appear on the search results page. However, Reiley, Li and Lewis (2010) find that sponsored links may substitute for organic links. Agarwal *et. al.* (2008) also show that while the click-through-rate decreases with position, the conversion rate first increases and then decreases with position for longer keywords. They conclude that the top positions in sponsored search advertisements are not necessarily the optimal positions for advertisers. Complementary to these studies, Rutz and Bucklin (2011) investigate the interactions between several types of keywords (generic *versus* branded keywords), and find that generic keywords may induce

positive spillovers on the effectiveness (measured by click-through rate) of branded keywords. Similarly, Jeziorski and Segal (2009) and Chiou and Tucker (2010) show the prevalence of externalities across ads meaning that the click-through-rate on a given ad in a given position depends on which ads are shown in other positions as well as the words used in the text of these ads. Finally, Edelman and Gilchrist (2011) find that click-through-rates are influenced by the labeling of paid links (for example replacing “sponsored link” with “ad” or “paid advertisement”). Our theoretical model is built on this empirical literature and aims at understanding how consumer attitude towards sponsored links (i.e. clicking behavior) influences competition in search advertising.

Our results characterize bidding strategies in a “generalized second-price” (GSP) keyword auction with a positive reserve price when the relevance of both the organic and sponsored links (and the corresponding probabilities that consumers click on these links) differs across firms, and the value of a click also can differ across firms. Following the seminal papers of Edelman, Ostrovski & Schwartz (2007) and Varian (2007), a growing literature has analyzed keyword auctions and search advertising strategies (Athey & Ellison, 2011; Agarwal et al. 2006; Animesh et al. 2010; Chen, De & Whinston, 2009; Katona & Sarvary, 2009; Taylor, 2009; Xu, Chen & Whinston, 2009; Zhang & Feng, 2011). Our analysis incorporates organic links in the framework of Edelman et. al. and Varian and is closest in spirit to recent work by Katona and Sarvary (2009) and Xu et al. (2009). Katona and Sarvary (2009) show that under certain conditions a less relevant firm may outbid more relevant firms to win the top position in the sponsored listings. In contrast to their analysis in which all firms participate in the keyword auction, the firm’s decision to participate in a keyword auction is endogenous in our model and depends upon the minimum cost per click established by the *SE* as well as the relevance of both organic and sponsored links to searching consumers. Xu et al. (2009) analyze two asymmetric firms that differ with respect to their organic ranking and compete in the product market. Like us, they find that bidding strategies depend on the relevance of the firm’s organic and sponsored listings, but the keyword auction they consider awards the first sponsored link for a fixed payment (and awards the second sponsored position at no charge). In contrast, our model considers the impact of cost-per-click pricing on bidding behavior and incorporates a minimum cost-per-click for any sponsored link, consistent with the policies of search engines like Google and Yahoo! Our model also explores the impact

of the specific channel (organic or sponsored link) through which customers visit a site on equilibrium bidding decisions, rather than considering only the total probability of attracting a customer when sponsoring a link. Because an advertising firm incurs a cost for each click on its sponsored link, while clicks on organic links are free, directly accounting for the channel through which a customer reaches a firm’s website is integral to equilibrium bidding behavior.

Section 2 presents the model. Section 3 explains the nature of keyword auctions and analyzes bidding strategies. Section 4 characterizes the different equilibrium outcomes. Section 5 illustrates our results with a numerical example. Section 6 discusses our results and concludes.

2 The Model

We consider a duopoly market with sponsored search advertising with two firms $i = 1, 2$ and a single search engine. We extend models of position auctions (Edelman et. al. (2007) and Varian (2007)) to incorporate organic search listings and to allow firm relevance to differ for organic and sponsored links. In our duopoly setting, there are $n = 4$ possible positions on the search results page. Each position $k \in \{1, 2, 3, 4\}$ has a position specific parameter x_k that measures the quality of this position, where $x_k \geq x_{k+1}$, and $1 \geq x_k > 0$.

2.1 Search Results and Click-Through-Rates

To incorporate differences in firm relevance, the position specific parameter is adjusted by a firm specific factor to determine the click-through-rate (*CTR*) for each firm/position specific combination. In particular, let β_i denote the firm relevance factor for an organic link to firm i , where $1 \geq \beta_1 > \beta_2 > 0$. Because firm 1 is more relevant than firm 2, the organic (unsponsored) results produced by the search engine always list firm 1 before firm 2. When neither firm sponsors a link, firm 1’s organic link appears in position 1, firm 2’s organic link in position 2, and the *CTRs* are $\beta_1 x_1$ and $\beta_2 x_2$ for firms 1 and 2, respectively.

In addition to providing organic search listings, the search engine conducts an auction to sell sponsored links which are listed in the highest positions. If only one of the two firms wins a sponsored link, then this link appears in the first position and the organic listings to firm 1 and then firm 2 appear in the second and third positions. Because the relevance of links may differ for sponsored and organic links, let δ_i denote the firm specific effect of a

sponsored link to firm i where $1 \geq \delta_1 \geq \delta_2 > 0$.³ If firm i has a sponsored link in position $k \in \{1, 2\}$, then the *CTR* for that link is $\delta_i x_k$. If both firms sponsor links, then the *SE* must determine which sponsored link appears first. The sponsored links occupy the first two positions followed by the organic links to firm 1 and then firm 2 in the third and fourth positions.

To allow for the possibility that sponsored links might be either complements or substitutes for organic links, let γ_i be a firm specific adjustment to the relevance of the organic link to firm i when that link appears after a sponsored link to firm i ; the *CTR* for an organic link to firm i in position k following a sponsored link to firm i is $\beta_i \gamma_i x_k$. If $\gamma_i < 1$, then the sponsored link serves as a substitute for the organic link in the sense that the presence of a sponsored link reduces the firm-specific relevance of the organic link. Similarly, $\gamma_i > 1$ if the sponsored link complements firm i 's organic link by increasing its relevance. Our assumptions about consumer behavior generate click through rates which depend upon the number of sponsored links and the position of each firm in the sponsored links as depicted in Table 1. The number in parentheses denotes the firm located in the corresponding position.

Position	Firms Sponsoring Links				
	None	Firm 1	Firm 2	Both, Firm 1 First	Both, Firm 2 First
1	$\beta_1 x_1$ (1)	$\delta_1 x_1$ (1)	$\delta_2 x_1$ (2)	$\delta_1 x_1$ (1)	$\delta_2 x_1$ (2)
2	$\beta_2 x_2$ (2)	$\beta_1 \gamma_1 x_2$ (1)	$\beta_1 x_2$ (1)	$\delta_2 x_2$ (2)	$\delta_1 x_2$ (1)
3	-	$\beta_2 x_3$ (2)	$\beta_2 \gamma_2 x_3$ (2)	$\beta_1 \gamma_1 x_3$ (1)	$\beta_1 \gamma_1 x_3$ (1)
4	-	-	-	$\beta_2 \gamma_2 x_4$ (2)	$\beta_2 \gamma_2 x_4$ (2)

Figure 1 below illustrates search engine results for the case in which both firms have sponsored links and firm 2 is awarded the first sponsored link. In this case, the overall *CTR* for firm 1 is $\delta_1 x_2 + \beta_1 \gamma_1 x_3$ and for firm 2 is $\delta_2 x_1 + \beta_2 \gamma_2 x_4$.

³We make no assumption about the relationship between δ_i and β_i . If firms employ effective targeted marketing with specific phrases included in the sponsored link, then it is likely that $\delta_i > \beta_i$. However, if consumers have a distaste for sponsored links, then it is possible that $\delta_i < \beta_i$.



Figure 1: Example of search engine results when firm 2 is awarded the first sponsored link

2.2 Firm profit

Firms are interested in maximizing profit generated by the search channel. Let $p_{i,k}$ denote the cost-per-click (*CPC*) paid by firm i when firm i sponsors a link in position k . Firms only incur the *CPC* if a customer reaches the firm through its sponsored link. Let $\pi_i^{r,s}$ denote the profit of firm i when, at most, one of the firms sponsors a link, firm 1 adopts a strategy r , firm 2 adopts a strategy s , and $r, s \in \{A, N\}$, where A is a strategy of advertising a sponsored link and N is a strategy of not advertising. Finally, let π_i^{AAk} denote the profit for firm i when both firms have sponsored links and firm i 's sponsored link appears in position k . The expected value to firm i of a customer who clicks on a link to firm i is v_i .⁴

Expected profits for each firm under each possible strategy profile can be determined using the click-through-rates in Table 1. If neither firm advertises, then

$$\begin{aligned}\pi_1^{NN} &= \beta_1 x_1 v_1 \\ \pi_2^{NN} &= \beta_2 x_2 v_2.\end{aligned}$$

If only firm 1 advertises, then

$$\begin{aligned}\pi_1^{AN} &= \delta_1 x_1 (v_1 - p_{1,1}) + \beta_1 x_2 \gamma_1 v_1 \\ \pi_2^{AN} &= \beta_2 x_3 v_2.\end{aligned}$$

⁴This expected value is the product of the conversion rate (the probability the consumer makes a purchase after clicking on the sponsored link) and the average purchase amount. We make no assumptions on the ordering of v_1 and v_2 . Firm 2 is assumed less relevant ($\beta_2 < \beta_1$), but it may provide a more valuable product ($v_2 > v_1$).

If only firm 2 advertises, then

$$\begin{aligned}\pi_1^{NA} &= \beta_1 x_2 v_1 \\ \pi_2^{NA} &= \delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_3 \gamma_2 v_2.\end{aligned}$$

If both firms advertise, then each firm's profit depends upon the placement of its sponsored listing. If firm 1 is listed first, then

$$\begin{aligned}\pi_1^{AA1} &= \delta_1 x_1 (v_1 - p_{1,1}) + \beta_1 x_3 \gamma_1 v_1 \\ \pi_2^{AA2} &= \delta_2 x_2 (v_2 - p_{2,2}) + \beta_2 x_4 \gamma_2 v_2.\end{aligned}$$

If firm 2 is listed first when both firms advertise, then

$$\begin{aligned}\pi_1^{AA2} &= \delta_1 x_2 (v_1 - p_{1,2}) + \beta_1 x_3 \gamma_1 v_1 \\ \pi_2^{AA1} &= \delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_4 \gamma_2 v_2.\end{aligned}$$

3 The Keyword Auction

Consider a position auction of the form analyzed by Edelman et. al. (2007) and Varian (2007) in which firms also have the option of not bidding and relying only on organic links to access customers, and the search engine establishes a minimum *CPC* of $c > 0$.⁵ Each firm can submit a bid b_i which represents the maximum *CPC* that firm i can be assessed for a sponsored link (in either position 1 or position 2). We restrict attention to a generalized second price auction in which the *CPC* $p_{i,k}$ paid by firm i for a sponsored link in position k is not a function of the bid submitted by firm i . Furthermore, we assume firm i pays a *CPC* of c for its sponsored link if i is the only firm with a sponsored link or if both firms have sponsored links and i is listed second.

3.1 Optimal bidding strategies

The profit functions for each firm determine firm strategies for the game in which the firms simultaneously choose advertising strategies and bids. There will be an equilibrium in which

⁵Google previously imposed a uniform minimum *CPC* of \$.05. Currently, Google's minimum *CPC* can vary across keywords and firms.

neither firm chooses to advertise if $\pi_1^{NN} > \pi_1^{AN}$, and $\pi_2^{NN} > \pi_2^{NA}$. Noting that $p_{i,1} = c$ if only one firm advertises, these restrictions require

$$\begin{aligned}\beta_1 x_1 v_1 &\geq \delta_1 x_1 (v_1 - c) + \beta_1 x_2 \gamma_1 v_1, \text{ and} \\ \beta_2 x_2 v_2 &\geq \delta_2 x_1 (v_2 - c) + \beta_2 x_3 \gamma_2 v_2\end{aligned}$$

which imply that firm 1 will prefer not to advertise conditional on firm 2 not advertising if $c \geq v_1 (1 - \beta_1/\delta_1 + \beta_1 \gamma_1 x_2/\delta_1 x_1) \equiv c_1$, and firm 2 will prefer not to advertise conditional on firm 1 not advertising if $c \geq v_2 (1 - \beta_2 x_2/\delta_2 x_1 + \beta_2 \gamma_2 x_3/\delta_2 x_1) \equiv c_2$.

Proposition 1 *If $c > \max\{c_1, c_2\}$, then there is an equilibrium in which neither firm bids on a sponsored link.*

If $c < \max\{c_1, c_2\}$, then at least one firm will advertise with strictly positive probability. In addition, while a high minimum CPC of $c > \max\{c_1, c_2\}$ ensures existence of an equilibrium in which neither firm chooses to sponsor a link, as shown in Section 4 below, it does not rule out the possibility of an equilibrium in which both firms sponsor a link.

To determine optimal strategies for each firm when at least one firm bids on a sponsored link, we begin by assuming that one firm j definitely bids on a sponsored link. Three conditions determine the optimal response by firm i . Conditions 1.1 and 2.1 correspond directly to the upper bound envy-free equilibrium bidding condition in Edelman et. al. and Varian for the case of two firms with $c > 0$. As Varian (2007 p. 1168) argues, this upper bound is a more compelling determinant of bidding behavior, so our analysis focuses on equilibrium behavior in which bids and the decision of whether or not to bid are based on the upper bound of possible equilibrium bids. The second and third conditions are no-regret conditions which ensure that advertising is preferred to the alternative of encountering customers through the organic channel alone.

- Condition 1.1. Firm 1 prefers to advertise and be listed first over advertising and being listed second (given firm 2 does advertise) if $\pi_1^{AA1} > \pi_1^{AA2}$ which implies

$$\delta_1 x_1 (v_1 - p_{1,1}) + \beta_1 x_3 \gamma_1 v_1 > \delta_1 x_2 (v_1 - c) + \beta_1 x_3 \gamma_1 v_1$$

or

$$p_{1,1} < v_1 (1 - x_2/x_1) + c x_2/x_1 \equiv \tilde{p}_1.$$

Note that \tilde{p}_1 is increasing in c , and $\tilde{p}_1 < v_1$ if and only if $c < v_1$.

- Condition 1.2. Firm 1 prefers to advertise and be listed first over not advertising given firm 2 does advertise if $\pi_1^{AA1} > \pi_1^{NA}$ which implies

$$\delta_1 x_1 (v_1 - p_{1,1}) + \beta_1 x_3 \gamma_1 v_1 > \beta_1 x_2 v_1$$

or

$$p_{1,1} < v_1 (1 - \beta_1 x_2 / \delta_1 x_1 + \beta_1 \gamma_1 x_3 / \delta_1 x_1) \equiv \hat{p}_1.$$

Note that $\hat{p}_1 \leq v_1$ if and only if $\gamma_1 \leq x_2 / x_3$.

- Condition 1.3. Firm 1 prefers to advertise and be listed second over not advertising given firm 2 advertises if $\pi_1^{AA2} > \pi_1^{NA}$ which implies

$$\delta_1 x_2 (v_1 - c) + \beta_1 x_3 \gamma_1 v_1 > \beta_1 x_2 v_1$$

or

$$c < v_1 (1 - \beta_1 / \delta_1 + \beta_1 \gamma_1 x_3 / \delta_1 x_2) \equiv \tilde{c}_1.$$

Note that $\tilde{p}_1 = \hat{p}_1$ when $c = \tilde{c}_1$. If $c > \tilde{c}_1$, then $\tilde{p}_1 > \hat{p}_1$, and if $c < \tilde{c}_1$, then $\tilde{p}_1 < \hat{p}_1$. In addition, $\tilde{c}_1 \leq v_1$ if and only if $\gamma_1 \leq x_2 / x_3$.

One implication of the above conditions is that with sufficiently strong complementarities ($\gamma_1 > x_2 / x_3$), firm 1 is willing to incur a *CPC* exceeding v_1 to sponsor a link provided firm 2 also sponsors a link.⁶

Similar conditions also apply to firm 2.

- Condition 2.1. Firm 2 prefers to advertise and be listed first over advertising and being listed second (given firm 1 advertises) if $\pi_2^{AA1} > \pi_2^{AA2}$ which implies

$$\delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_4 \gamma_2 v_2 > \delta_2 x_2 (v_2 - c) + \beta_2 x_4 \gamma_2 v_2$$

or

$$p_{2,1} < v_2 (1 - x_2 / x_1) + c x_2 / x_1 = \tilde{p}_2.$$

Note \tilde{p}_2 is increasing in c and that $\tilde{p}_2 < v_2$ if and only if $c < v_2$.

⁶Because $\gamma_1 > x_2 / x_3$ implies $\tilde{c}_1 > v_1$, condition 1.3 implies firm 1 will pay a *CPC* up to $\tilde{c}_1 > v_1$ for a sponsored link in the second position. Furthermore, conditions 1.1 and 1.2 imply that if $\tilde{c}_1 > c > v_1$, then $\hat{p}_1 > \tilde{p}_1 > v_1$ and firm 1 is willing to pay a *CPC* of $\tilde{p}_1 > v_1$ for the first sponsored link. If $c > \tilde{c}_1 > v_1$, then $\tilde{p}_1 > \hat{p}_1 > v_1$ and firm 1 is willing to pay a *CPC* of $\hat{p}_1 > v_1$ to sponsor a link in the first position but is not willing to sponsor a link in the second position.

- Condition 2.2. Firm 2 prefers to advertise and be listed first over not advertising given firm 1 advertises if $\pi_2^{AA1} > \pi_2^{AN}$ which implies

$$\delta_2 x_1 (v_2 - p_{2,1}) + \beta_2 x_4 \gamma_2 v_2 > \beta_2 x_3 v_2$$

or

$$p_{2,1} < v_2 (1 - \beta_2 x_3 / \delta_2 x_1 + \beta_2 \gamma_2 x_4 / \delta_2 x_1) \equiv \hat{p}_2.$$

Note that $\hat{p}_2 \leq v_2$ if and only if $\gamma_2 \leq x_3 / x_4$.

- Condition 2.3. Firm 2 prefers to advertise and be listed second over not advertising given firm 1 advertises if $\pi_2^{AA2} > \pi_2^{AN}$ which implies

$$\delta_2 x_2 (v_2 - c) + \beta_2 x_4 \gamma_2 v_2 > \beta_2 x_3 v_2$$

or

$$c < v_2 (1 - \beta_2 x_3 / \delta_2 x_2 + \beta_2 \gamma_2 x_4 / \delta_2 x_2) \equiv \tilde{c}_2.$$

Note that $\tilde{p}_2 > \hat{p}_2$ if and only if $c > \tilde{c}_2$ and that $\tilde{c}_2 \leq v_2$ if and only if $\gamma_2 \leq x_3 / x_4$. As was true for firm 1, if there are strong complementarities between firm 2s sponsored and organic links so that $\gamma_2 > x_3 / x_4$, then conditional on firm 1 advertising, firm 2 is willing to sponsor a link at a *CPC* exceeding v_2 .

The above conditions enable us to specify the optimal bidding behavior of a given firm i given both firms bid on sponsored links. Because we assume the search engine holds a second price auction in which the *CPC* for the sponsored link in the first position is not a function of the bid submitted by the firm awarded this position, and the *CPC* for the sponsored link in the second position is c , standard arguments imply that bidding the maximum willingness to pay per click (*e.g.*, \tilde{p}_i , or \hat{p}_i), depending upon which of the conditions above applies, is a weakly dominant strategy.

Assuming that the opposing firm j bids on a sponsored link, the optimal strategy of firm i can be depicted as a function of c and depends on the degree to which a sponsored link substitutes for or complements the firm's organic link. First suppose $\gamma_i < x_{i+1} / x_{i+2}$ (sponsored and organic links are substitutes or weak complements). In this case $\tilde{c}_i < \hat{p}_i < v_i$ and three ranges for c are possible. First, if $c \leq \tilde{c}_i$, then $\hat{p}_i \geq \tilde{p}_i$ and firm i bids \tilde{p}_i . Conditions *i.1* and *i.2* imply that the firm prefers to win a sponsored link in the first position at any

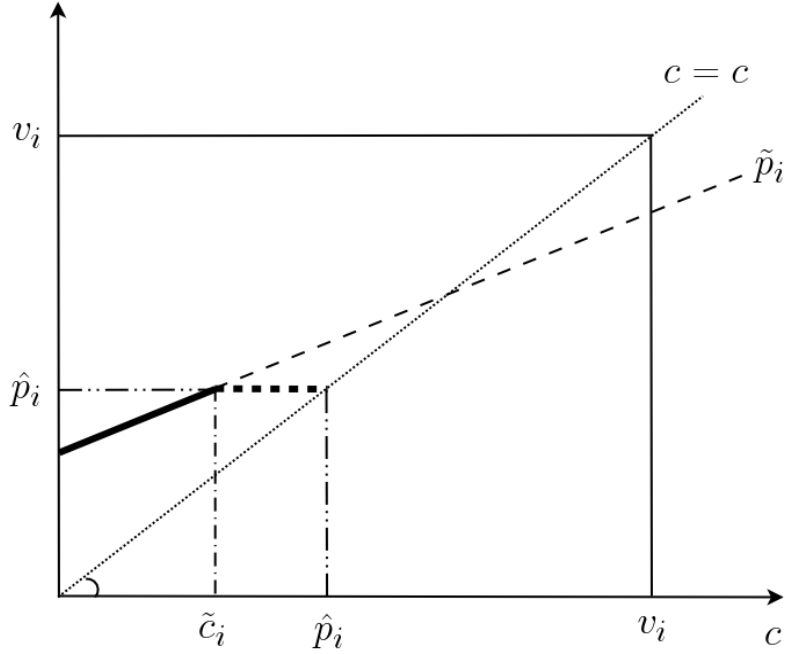


Figure 2: Optimal bids when $\gamma_i < x_{i+1}/x_{i+2}$

CPC up to \tilde{p}_i , and also is willing to sponsor a link in the second position at a CPC of c . Therefore, the firm should bid \tilde{p}_i . Second, if $\tilde{c}_i \leq c \leq \hat{p}_i$, then $\tilde{p}_i \geq \hat{p}_i$ and conditions $i.1$ and $i.2$ imply that firm i is willing to pay a CPC of \hat{p}_i to sponsor a link in the first position, but condition $i.3$ implies that firm i is not willing to sponsor a link in the second position. Therefore, for $\tilde{c}_i \leq c \leq \hat{p}_i$, firm i should bid \hat{p}_i if and only if this bid will win the auction. Finally, if $c > \hat{p}_i$, then firm i is not willing to sponsor a link in either position. The optimal bid for firm i given $\gamma_i < x_{i+1}/x_{i+2}$ and firm j sponsors a link is depicted in figure 2. The dashed section of the bid function for values of $c \in (\tilde{c}_i, \hat{p}_i]$ indicates that for values of c in this range firm i is only willing to bid \hat{p}_i if it is certain to win the first position.

The optimal strategy for firm i when $\gamma_i > x_{i+1}/x_{i+2}$ (sponsored and organic links are strong complements) is depicted in figure 3. In this case $v_i < \hat{p}_i < \tilde{c}_i$. Furthermore, $c > \tilde{p}_i > v_i$ for $c \geq v_i$, and $\tilde{p}_i \geq \hat{p}_i$ for $c \geq \tilde{c}_i$. Assuming that firm j bids on a sponsored link, four ranges of c must be considered. First, if $c \leq v_i$, then firm i bids \tilde{p}_i . Second, if $\hat{p}_i \geq c > v_i$, then $c > \tilde{p}_i$, so firm i prefers to be listed second instead of first and should bid c . However, if both firms submit a bid of c , then firm i may be randomly awarded the first sponsored link. This is acceptable as long as $c \leq \hat{p}_i$ (condition $i.2$ implies firm i prefers to

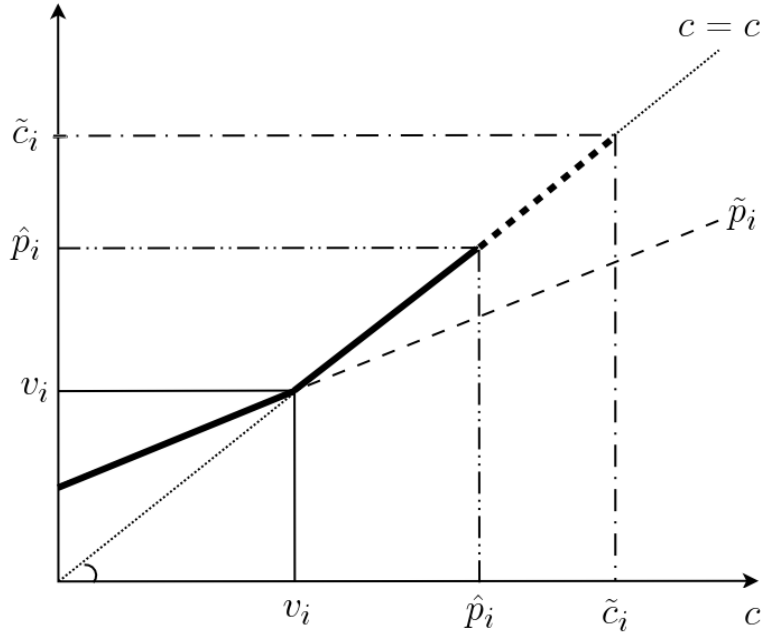


Figure 3: Optimal bids when $\gamma_i > x_{i+1}/x_{i+2}$

be listed first over not advertising as long as $p_{i,1} < \hat{p}_i$). Thus, for $\hat{p}_i \geq c > v_i$, firm i will submit a bid of c . Because $c > v_i$, the benefit of the sponsored link accrues only through the complement effect of increasing clicks on the organic link. The firm prefers the second sponsored link position in order to realize the complementarity while minimizing the number of clicks on the sponsored link. However, firm i is still better off sponsoring a link in the first position than not sponsoring as long as $c \leq \hat{p}_i$.

Third, if $\tilde{c}_i \geq c > \hat{p}_i$, then firm i is only willing to sponsor a link if it is in the second sponsored position with a *CPC* of c . In figure 3 the dashed bid function for values of c satisfying $\tilde{c}_i \geq c > \hat{p}_i$ indicates that firm i is only willing to bid for values of c in this range if its sponsored link is guaranteed to appear in the second position. Finally, if $c > \tilde{c}_i$, then firm i is always better off not sponsoring a link given firm j does sponsor.

The discussion of optimal bidding strategies for firm i in the previous paragraphs is summarized in Lemmas 2 and 3. Note that these results assume that the competing firm j sponsors a link. If firm j does not sponsor a link, then firm i will sponsor a link if and only if $c < c_i$. In section 4 we consider conditions under which firms choose to bid on sponsored links or rely only on organic links in equilibrium.

Lemma 2 Assume $\gamma_i \leq x_{i+1}/x_{i+2}$ and that firm j participates in the auction for sponsored links. If $c \leq \tilde{c}_i$, then firm i bids \tilde{p}_i . If $\hat{p}_i \geq c > \tilde{c}_i$, then firm i participates and bids \hat{p}_i if and only if firm i will secure the first position in the sponsored links with a bid of \hat{p}_i . If $c > \hat{p}_i$, then firm i will not sponsor a link.

Lemma 3 Assume $\gamma_i > x_{i+1}/x_{i+2}$ and that firm j participates in the auction for sponsored links. If $c \leq v_i$, then firm i bids \tilde{p}_i . If $v_i < c \leq \hat{p}_i$, then firm i bids c . If $\hat{p}_i < c \leq \tilde{c}_i$, then firm i participates and bids c if and only if firm i will secure the second position in the sponsored links with a bid of c . If $c > \tilde{c}_i$, then firm i will not sponsor a link.

Note that $\min(\tilde{p}_i, \hat{p}_i) = \tilde{p}_i$ if $c \leq \tilde{c}_i$, and $\min(\tilde{p}_i, \hat{p}_i) = \hat{p}_i$ if $c > \tilde{c}_i$. Thus, lemmas 2 and 3 imply the following proposition.

Proposition 4 If $c > \max\{\hat{p}_i, \hat{p}_j\}$, then there is no equilibrium in which both firms bid on sponsored links.

The above analysis provides insight into how the presence of organic links impacts bids placed on sponsored links. In the absence of organic links, $\beta_i = 0$, and the upper bound equilibrium bid for each firm is defined by $b_i^* = \tilde{p}_i$.⁷ From lemma 2 it follows that if organic and sponsored links are substitutes or sufficiently weak complements, and c is sufficiently small for both firms ($c \leq \tilde{c}_i$), then the presence of organic links has no impact on equilibrium bids. Both firms bid \tilde{p}_i , which is independent of β_i and δ_i , so bids do not decrease as the relevance β_i of the organic link increases or increase as the relevance δ_i of sponsored links increases. However, because \hat{p}_i is decreasing in β_i when $\gamma_i < x_{i+1}/x_{i+2}$, if the relevance of a firm's organic link is sufficiently high (in particular, if $\beta_i > \left(\frac{v_i - c}{v_i}\right) \left(\frac{\delta_i x_2}{x_{i+1}} / \left(1 - \gamma_i \frac{x_{i+2}}{x_{i+1}}\right)\right)$, which is equivalent to $c > \tilde{c}_i$), then the presence of organic links alters equilibrium bidding behavior. In this case, assuming firm i chooses to bid, firm i bids \hat{p}_i which is less than \tilde{p}_i , and the equilibrium bid of firm i is decreasing in β_i and increasing in δ_i . In addition, because the maximum *cpc* \tilde{c}_i that firm i is willing to pay for a sponsored link in the second position is decreasing in β_i (increasing in δ_i), as the relevance of organic links increases (relevance of the sponsored links decreases), the *SE* must lower the minimum *cpc* in order to induce both firms to participate in the keyword auction.

⁷The value \hat{p}_i for each firm is equivalent to the upper bound established in equation (10) in Varian.

The impact of organic links on the upper bound of the equilibrium bids is reversed if organic and sponsored links are sufficiently strong complements and $c > v_i$. As per lemma 3, assuming $\gamma_i > x_{i+1}/x_{i+2}$ and that firm j bids on sponsored links, if $c \leq v_i$, then firm i bids \tilde{p}_i which is equivalent to the bid firm i would submit in the absence of organic links. If $c > v_i$, then firm i is willing to bid more than \tilde{p}_i provided $c < \hat{p}_i$. Furthermore, the maximum cpc that the SE can extract is \hat{p}_i which is increasing in β_i and decreasing in δ_i . For $c > \hat{p}_i$, firm i will only participate in the auction if $c \leq \tilde{c}_i$ and firm i wins the second sponsored position, but the maximum cpc \tilde{c}_i that firm i is willing to pay for the second position still is increasing in β_i and decreasing in δ_i .

The previous two paragraphs consider the impact of organic links assuming both firms bid on sponsored links. However, it is possible that only one firm will choose to bid on a sponsored link. The maximum cpc that a given firm i is willing to pay for a sponsored link when firm i is the only firm sponsoring a link is c_i .⁸ The impact of organic links on c_i depends upon how γ_i compares to x_i/x_{i+1} (as opposed to x_{i+1}/x_{i+2} when both firms bid on sponsored links). Intuitively, when only firm i bids on a sponsored link, the location effect moves firm i 's organic link from position i to $i + 1$, while, if firm j is sponsoring a link, a decision to sponsor by firm i will move firm i 's organic link from position $i + 1$ to $i + 2$. If organic and sponsored links are substitutes or weak complements (so $\gamma_i < \min(x_i/x_{i+1}, x_{i+1}/x_{i+2})$), then (provided firm i is not bidding \tilde{p}_i) an increase in β_i reduces the amount firm i is willing to pay for a sponsored link. Similarly, if $\gamma_i > \max(x_i/x_{i+1}, x_{i+1}/x_{i+2})$, then an increase in β_i increases (or at least never reduces) the amount firm i is willing to pay for a sponsored link. If the complement effect of sponsored and organic links and the location specific effects x_k are such that $x_{i+1}/x_{i+2} > \gamma_i > x_i/x_{i+1}$, then an increase in the relevance β_i of organic links to firm i would reduce the amount firm i is willing to pay for a sponsored link if firm j also sponsors, but increase the amount firm i is willing to pay if firm i is the only firm with a sponsored link.

3.2 Search Engine Allocation of Sponsored Links

The search engine must determine which firm to list first when both firms submit bids exceeding the reservation price c . The expected revenue to the search engine from listing

⁸In the absence of organic links $\beta_i = 0$ and $c_i = v_i$.

firm 1 first is

$$\delta_1 x_1 p_{1,1} + \delta_2 x_2 p_{2,2}$$

and from listing firm 2 first is

$$\delta_2 x_1 p_{2,1} + \delta_1 x_2 p_{1,2}.$$

The price paid by any firm cannot exceed that firm's bid, so $p_{i,k} \leq b_i$, and the *CPC* for the firm listed second is $p_{i,2} = c$ for $i = 1, 2$. Thus, assuming that the search engine maximizes its revenues, it will list firm 1 first if

$$\delta_1 x_1 b_1 + \delta_2 x_2 c \geq \delta_2 x_1 b_2 + \delta_1 x_2 c$$

or

$$b_1 \geq \frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1} \quad (1)$$

and will list firm 2 first otherwise. While equation (1) determines the optimal position assignments based on bids submitted, it does not provide a general rule for determining the price paid by the firm listed in the first sponsored link position. Assuming it is optimal for the search engine to award firm 1 the first sponsored link, any price $p_{1,1}$ satisfying $b_1 \geq p_{1,1} \geq \frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1}$ would be possible. However, because we are considering a second price auction rule and because setting $p_{1,1}$ strictly greater than $\frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1}$ would result in a suboptimal assignment of firms to sponsored link positions by the search engine if $b_1 \in \left(\frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1}, p_{1,1} \right)$, we assume the auction rule establishes a price $p_{i,1}$ for the first sponsored link position that is just sufficient to ensure positions are awarded based on the condition (1).⁹ In particular, if bids are such that equation (1) is satisfied, then firm 1 is awarded the first sponsored position and pays a cost-per-click (*CPC*) of

$$p_{1,1} = \max \left(\frac{\delta_2}{\delta_1} b_2 + \frac{x_2 c (\delta_1 - \delta_2)}{\delta_1 x_1}, c \right) < b_2,$$

where the inequality follows from the assumptions that $b_2 > c$, $x_1 \geq x_2$, and that the firm in the second sponsored position pays a *CPC* of c . If bids are such that equation (1) is violated, then firm 2 is awarded the first sponsored position with a *CPC* of

$$p_{2,1} = \frac{\delta_1}{\delta_2} b_1 - \frac{x_2 c (\delta_1 - \delta_2)}{\delta_2 x_1} > b_1.$$

⁹Establishing a lower minimum price would clearly not maximize search engine expected revenue. Establishing a higher minimum price would create incentives for advertisers to repeatedly lower their bids in an attempt to learn the lowest price the search engine would accept. As Edelman et. al. note, similar behavior under first-price auction rules in early sponsored search markets was a significant reason for the transition to the use of a generalized second price auction.

Because $b_1 > c$ by assumption, firm 2 incurs a *CPC* greater than c if it is awarded the first sponsored link position. Given the assumptions that $\delta_1 > \delta_2$ and $x_1 \geq x_2$, firm 1 may be listed first even if it bids less than firm 2, and firm 2 must bid strictly more than firm 1 in order to be listed first. Note that if $c = 0$, then a rule of ranking firms by their quality adjusted bids $\delta_i b_i$ is optimal and the second price auction rule charges the firm i listed first a *CPC* equal to the quality adjusted bid $b_j \delta_i / \delta_j$ of the firm listed second.¹⁰ However, casual observations from Google searches indicate that for many keywords some sponsored link positions often go unsold, so understanding the implications of a binding minimum cost-per-click, $c > 0$ is important.

The premium $(p_{2,1} - b_1)$ that firm 2 must pay in order to be listed first is increasing in δ_1 (the relevance of the sponsored link to firm 1), and decreasing in δ_2 (the relevance of the sponsored link to firm 2). Similarly, the discount $(b_2 - p_{1,1})$ for firm 1 is decreasing in δ_1 and increasing in δ_2 . In the limiting case in which $\delta_1 \rightarrow \delta_2$, so that firms are equally relevant to consumers who utilize sponsored links, the search engine will simply rank the firms according to their bids, and the firm submitting the highest bid pays a *CPC* equal to the second highest bid.

The premium paid by firm 2 and the discount to firm 1 are both decreasing in the ratio x_2/x_1 . As this ratio increases, the incremental gain in the click-through-rate from being in the first as opposed to the second sponsored position decreases. This lessens the search engine incentive to provide a discount to firm 1 and lessens its ability to extract a premium from firm 2 for the first sponsored position. Similar logic explains why both the premium to firm 2 and discount to firm 1 are decreasing in c .

In the equilibrium analysis below, it will be useful to define

$$h_i(b_j) = \frac{\delta_j}{\delta_i} b_j + \frac{(\delta_i - \delta_j) c x_2}{\delta_i x_1}.$$

Note that if both firms submit bids (by assumption, these bids will be at least as large as the minimum *CPC* of c), then the search engine will award the first sponsored position to

¹⁰It is easily shown that such an ordering rule generalizes to the case of N bidders with K sponsored link positions provided either $c = 0$, or $c > 0$ and $N > K$ and $\delta_{g(K+1)} b_{g(K+1)} > \delta_{g(K)} c$, where $g(k)$ is the identity of the firm awarded a sponsored link in position $k < K$ and $g(K+1)$ is the identity of the highest ranked firm not awarded a sponsored link. (In this case the firm awarded the last sponsored link position pays a *CPC* of $\delta_{g(K+1)} b_{g(K+1)} / \delta_{g(K)} > c$.) However, if the firm awarded the last sponsored link position pays c , either because $N < K$ or because $N > K$ and $\delta_{g(K+1)} b_{g(K+1)} < \delta_{g(K)} c$, then the search engine should award higher sponsored link positions based on quality adjusted bids plus a premium (or discount if the less relevant firm is ranked first) which depends upon the minimum *CPC*, c .

the firm i for which $b_i > h_i(b_j)$. The firm i that is awarded the first sponsored position will be charged a *CPC* of $\max\{h_i(b_j), c\}$, and the firm that is awarded the second sponsored position will be charged a *CPC* of c . For simplicity, we assume that if the search engine is indifferent between which firm to list first, then the first sponsored position is awarded to the more relevant firm 1.

4 Equilibrium Participation and Bidding Outcomes

We now present equilibrium results for the case in which sponsored links substitute for or weakly complement organic links, so $\gamma_i \leq x_{i+1}/x_{i+2}$ for $i = 1, 2$ and for the case in which sponsored links strongly complement organic links so $\gamma_i > x_{i+1}/x_{i+2}$. The analysis of the case in which $\gamma_i \leq x_{i+1}/x_{i+2}$ for one firm i while $\gamma_j > x_{j+1}/x_{j+2}$ is similar and is discussed briefly in the appendix. For all ranges of γ_i and γ_j the equilibria fall into one of four categories; both firms bid on sponsored links with probability 1; one firm bids and the other does not; neither firm bids on sponsored links; the firms adopt mixed strategies in which the decision of whether or not to bid is determined randomly. The mixed strategy equilibria arise if market parameters are such that it is optimal for one firm i to bid on a sponsored link if and only if firm j does not bid, and it is optimal for firm j to bid if and only if firm i does bid. Propositions 5 through 8 below briefly summarize the equilibrium results.¹¹ A more detailed presentation of equilibrium results and the derivation of equilibrium mixed strategies are provided in the appendix.

Proposition 5 *Suppose $\gamma_i \leq x_{i+1}/x_{i+2}$ for $i = 1, 2$.*

1. *If $c \leq \min(\tilde{c}_1, \tilde{c}_2, \max(c_1, c_2))$, then there is a unique equilibrium in which each firm submits a bid of \tilde{p}_i for sponsored links.*
2. *If $\max(c_1, c_2) < c < \min(\tilde{c}_1, \tilde{c}_2)$, then there are two pure strategy equilibria; in one equilibrium, neither firm bids on sponsored links, and in the other equilibrium each firm submits a bid of \tilde{p}_i .*

¹¹Note that propositions 5 and 6 do not provide a comprehensive treatment of all possible equilibrium outcomes. Because of the large number of possible parameter combinations, we have presented a subset of equilibrium results that summarize all the types of possible outcomes. Characterization of equilibria for other possible ordering of the model parameters can be similarly derived using the approach presented here.

3. Suppose $\max(c_i, \hat{p}_i) < c$ for one firm i . If $c < c_j$, then there is a unique equilibrium in which only firm j sponsors a link. If $c > c_j$, then neither firm sponsors a link.
4. Suppose $\max(c_i, \tilde{c}_i) < c < \hat{p}_i$, and $c < \tilde{c}_j$. If $\hat{p}_i > h_i(\tilde{p}_j)$, then there is a unique equilibrium in which firm i bids \hat{p}_i , firm j bids \tilde{p}_j , and firm i is awarded the first sponsored link. If $\hat{p}_i < h_i(\tilde{p}_j)$ and $c < c_j$, then there is an equilibrium in which firm j bids \tilde{p}_j and firm i does not bid, and if $\hat{p}_i < h_i(\tilde{p}_j)$ and $c > c_j$, then neither firm bids on sponsored links.
5. Suppose $c_i < c < \tilde{c}_i$, and $\tilde{c}_j < c < c_j$. If $\hat{p}_j > \max\{h_j(\tilde{p}_i), c\}$, then there is a unique equilibrium in which firm i bids \tilde{p}_i , firm j bids \hat{p}_j , and firm j is awarded the first sponsored link. If $\hat{p}_j < \max\{h_j(\tilde{p}_i), c\}$, then there is a mixed strategy equilibrium as described in the appendix.
6. If $c > \max(c_1, c_2)$ and $c > \min(\tilde{c}_1, \tilde{c}_2)$ then neither firm bids on sponsored links.

Proof. The first, third, fourth and sixth results follow directly from comparison of firm profit from advertising versus not advertising for c in the specified range. The second result follows from Proposition 1 if $c > \max(c_1, c_2)$, and from lemma 2 and the fact that if firm j advertises, then cases $i.1$, $i.2$, and $i.3$ imply that firm i should advertise and bid \tilde{p}_i if $c < \min(\tilde{c}_1, \tilde{c}_2)$. For the fifth result note that c in the specified range implies $c < \tilde{p}_i < \hat{p}_i$ and $c < \tilde{c}_i$, so conditions $i.1$, $i.2$, and $i.3$ imply firm i will bid \tilde{p}_i if firm j bids. Furthermore, $c > \tilde{c}_j$ implies $\hat{p}_j < \tilde{p}_j$, so firm j will only advertise if it wins the first sponsored link at a CPC up to \hat{p}_j . Now consider the case of $c \leq \hat{p}_j$ and $\hat{p}_j > \frac{\delta_i}{\delta_j} \tilde{p}_i + \frac{x_2 c (\delta_j - \delta_i)}{\delta_j x_1}$. Because firm j will be listed first with a bid of \hat{p}_j , there is an equilibrium in which firm i bids \tilde{p}_i and firm j bids \hat{p}_j . Furthermore, because it is optimal for each firm to bid if the other firm bids, and $c < c_j$ implies firm j will bid if firm i does not, this equilibrium is unique. Proofs for mixed strategy results are presented in the appendix. ■

Statements 1, 3, 4, and 6 in the above proposition generate straightforward equilibrium outcomes. Not surprisingly, the minimum CPC must be sufficiently low to induce both firms to submit bids, and if c is too large, then neither firm will participate in the auction. Under the conditions of statement 2 firms engage in a coordination game in which equilibrium entails matching the strategy adopted by the competing firm. Statement 5 illustrates that a mixed strategy equilibrium can exist. An interesting outcome occurs if $c_j > c > \hat{p}_j$. In this

case firm j will bid in the keyword auction despite the fact that the minimum CPC exceeds the highest amount, \hat{p}_j , that firm j is willing to pay for the first sponsored position when both firms sponsor links. Firm j is willing to submit such a high bid because of the fact that firm i may choose not to bid and if firm i does not bid, then firm j is willing to incur a CPC up to c_j to sponsored a link.

Proposition 6 presents similar results for the case in which organic and sponsored links are complements.

Proposition 6 *Suppose $\gamma_i > x_{i+1}/x_{i+2}$ for $i = 1, 2$.*

1. *If $c \leq \min(\hat{p}_1, \hat{p}_2, \max(c_1, c_2))$, then there is a unique equilibrium in which each firm submits a bid of $\max\{\tilde{p}_i, c\}$ for sponsored links.*
2. *If $\max(c_1, c_2) < c < \min(\hat{p}_1, \hat{p}_2)$, then there are two pure strategy equilibria; in one equilibrium, neither firm bids on sponsored links, and in the other equilibrium each firm submits a bid of $\max\{\tilde{p}_i, c\}$.*
3. *If $\max(c_i, \tilde{c}_i) < c < c_j$ for some i and j , then there is a unique equilibrium in which only firm j sponsors a link.*
4. *Suppose $\max(c_i, \hat{p}_i) < c < \tilde{c}_i$ and $c < \hat{p}_j$. If $c < h_i(\max\{\tilde{p}_j, c\})$, then there is a unique equilibrium in which firm i bids c , firm j bids $\max\{\tilde{p}_j, c\}$, and firm j is awarded the first sponsored link. If $c > h_i(\max\{\tilde{p}_j, c\})$ and $c < c_j$, then there is an equilibrium in which firm j bids $\max\{\tilde{p}_j, c\}$ and firm i does not bid, and if $c > h_i(\max\{\tilde{p}_j, c\})$ and $c > c_j$, then neither firm bids on sponsored links.*
5. *Suppose $\hat{p}_i < c < \min\{c_i, \tilde{c}_i\}$ and $\max(c_j, \hat{p}_j) < c < \tilde{c}_j$. If $c < h_i(c)$, then there is a unique equilibrium in which firm i bids c , and firm j does not bid. If $c > h_i(c)$, then there is a mixed strategy equilibrium as presented in the appendix.*
6. *Suppose $c_i < c < \hat{p}_i$ and $\hat{p}_j < c < c_j$. If $c < \tilde{c}_j$ and $c < h_j(\max\{\tilde{p}_i, c\})$, then there is a unique equilibrium in which firm i bids $\max\{\tilde{p}_i, c\}$, firm j bids c , and firm i is awarded the first sponsored link. Otherwise, there is a mixed strategy equilibrium as presented in the appendix.*
7. *If $c > \max(c_1, c_2)$ and $c > \min(\hat{p}_1, \hat{p}_2)$ then neither firm bids on sponsored links.*

Proof. The first, third, fourth and seventh results follow directly from comparison of firm profit from advertising versus not advertising for c in the specified range. The second result follows from Proposition 1 if $c > \max(c_1, c_2)$, and from lemma 2 and the fact that if firm j advertises, then cases $i.1$, $i.2$, and $i.3$ imply that firm i should advertise and bid \tilde{p}_i if $c < \min(\tilde{c}_1, \tilde{c}_2)$. For the fifth result note that c in the specified range implies $c > \hat{p}_i > \tilde{p}_i$ and $c < \tilde{c}_i$ so conditions $i.1$, $i.2$, and $i.3$ imply that if firm j bids, then firm i will bid c only if it will be awarded the second sponsored link, and because $c > c_j$ and $\hat{p}_j < c < \tilde{c}_j$, firm j will bid only if firm i also bids and firm j wins the second sponsored position. Thus, if $c < h_i(c)$, so firm i loses the auction and is awarded the second sponsored position when both firms bid, firm j will never bid. However, if $c > h_i(c)$, so firm j would lose the auction, then there is no pure strategy equilibrium because it is optimal for firm i to bid only if firm j does not, and it is optimal for firm j to bid if firm i does. For the sixth result, because $c_i < c < \hat{p}_i$, it is optimal for firm i to bid if firm j bids, regardless of the sponsored position it wins, but it is not optimal for firm i to bid if firm j does not. Similarly, because $c < c_j$, it is always optimal for firm j to bid if firm i does not, but because $c > \hat{p}_j$, it is only optimal for firm j to bid when firm i also bids if $c < \tilde{c}_j$ and firm j wins the second sponsored position. Thus if $c < \tilde{c}_j$ and $c < h_j(\max\{\tilde{p}_i, c\})$, so firm j wins the second sponsored position when both firms bid, then there is a unique equilibrium in which firm i bids $\max\{\tilde{p}_i, c\}$, firm j bids c . Otherwise, there is a mixed strategy equilibrium as presented in the appendix. ■

The following two propositions demonstrate the important role that γ_i , the degree of complementarity or substitutability between sponsored and organic links, plays in determining whether firms participate in the keyword auction.

Proposition 7 *If $\gamma_i < (\beta_i x_i - \delta_i x_1) / \beta_i x_{i+1}$ for $i = 1, 2$, then for any $c \geq 0$ there is an equilibrium in which neither firm bids on sponsored links.*

Proof. This follows from part 2 of proposition 5 and the fact that $c_1 < 0$ and $c_2 < 0$ given the values of δ_1 , δ_2 , γ_1 , and γ_2 in the statement of the proposition. ■

Note that because $\gamma_i > 0$, the conditions of proposition 7 require $\delta_1 < \beta_1$ and $\delta_2 < \beta_2 x_2 / x_1$. These conditions imply that consumers are averse to sponsored links – they are more likely to click on the organic link to firm i when presented with both a sponsored and an organic link to the firm. Thus, proposition 7 implies that if consumers are sufficiently

averse to sponsored links and sponsored links are not strong complements to organic links, then firms may choose not to participate in keyword auctions even if the search engine offers to provide sponsored links at no cost to the advertising firm. Note that this result can apply even if sponsored links are strict complements to organic links so that $\gamma_i > 1$.¹²

Under the conditions of Proposition 7, an equilibrium in which neither firm bids on sponsored links exists for any $c \geq 0$, but this equilibrium may not be unique. In particular, it is possible that $\tilde{c}_1 > 0$ and $\tilde{c}_2 > 0$, and a second equilibrium in which both firms participate in the auction with bids of \tilde{p}_i exists if $c < \min\{\tilde{c}_1, \tilde{c}_2\}$. However, if δ_1 , δ_2 , γ_1 , and γ_2 are all sufficiently small, then there is a unique equilibrium in which neither firm participates in the keyword auction for any $c \geq 0$.¹³

Alternatively, as demonstrated in proposition 8 below, sufficiently strong complementarities between sponsored and organic links give the search engine significant market power regardless of the relationship between the relevance β_i of organic links and δ_i of sponsored links.

Proposition 8 *If $\gamma_i > x_{i+1}/x_{i+2}$ for $i = 1, 2$, then for any $c \leq \min\{\hat{p}_1, \hat{p}_2\}$ there is an equilibrium in which both firms participate in the keyword auction and submit bids of $\max\{\tilde{p}_i, c\}$. In addition, if $\gamma_i > (x_i - x_{i+1})/(x_{i+1} - x_{i+2}) \equiv \bar{\gamma}_i$ for $i = 1, 2$, then the equilibrium is unique.*

Proof. This follows directly from lemmas 2 and 3 and the fact that $c_i > \hat{p}_i$ if $\gamma_i > \bar{\gamma}_i$, so that $c \leq \min\{\hat{p}_1, \hat{p}_2\}$ implies $c < c_i$ and part 1 of proposition 6 applies.

Proposition 8 implies that if complementarities between sponsored and organic links are sufficiently strong for both firms, then the search engine can extract all of the surplus directly generated by the sponsored link from the firm with the lower expected value v_i of a click-through by setting $c = v_i$. In fact, because both firms participate in the auction for any $c \leq \min\{\hat{p}_1, \hat{p}_2\}$ and because $\gamma_i > x_{i+1}/x_{i+2}$ implies $\hat{p}_i > v_i$, the search engine can extract

¹²For example, if $\delta_1 < \beta_1(x_1 - x_2)/x_1$, then $(\beta_1 - \delta_1)x_1/\beta_1x_2 > 1$, so $c_1 < 0$ will hold for some $\gamma_1 > 1$. More generally, for any $c \geq 0$, $c_1 < c$ and $c_2 < c$ will both hold, so there is an equilibrium in which neither firm participates in the keyword auction, if $\gamma_1 < (\beta_1x_1 + \delta_1x_1(c - v_1)/v_1)/\beta_1x_2$ and $\gamma_2 < (\beta_2x_2 + \delta_2x_1(c - v_2)/v_2)/\beta_2x_3$.

¹³In particular, $\max(c_1, \tilde{c}_1, c_2, \tilde{c}_2) < 0$ is a sufficient condition for a unique equilibrium in which neither firm participates for any $c \geq 0$. If $\delta_1 < \beta_1$ and $\gamma_1 < \min\{(\beta_1 - \delta_1)x_2/\beta_1x_3, (\beta_1 - \delta_1)x_1/\beta_1x_2\}$, then both $c_1 < 0$ and $\tilde{c}_1 < 0$. Similarly, if $\delta_2 < \min\{\beta_2x_2/x_1, \beta_2x_3/x_2\}$ and $\gamma_2 < \min\{(\beta_2x_2 - \delta_2x_1)/\beta_2x_3, (\beta_2x_3 - \delta_2x_2)/\beta_2x_4\}$, then both $c_2 < 0$ and $\tilde{c}_2 < 0$.

a premium in excess of v_i . This result holds even if the probability $\delta_i x_k$ that a consumer clicks on the sponsored link to firm i in position k is very small. Proposition 8 indicates that when there are sufficiently strong complementarities between sponsored and organic links, the driving force in a firm's decision to participate in a keyword auction is not the net profit generated by clicks on the sponsored link, but rather the increased traffic that the sponsored link generates through the firm's organic link for which the firm pays nothing to the search engine. The positive externality accrued through the complementarity justifies purchase of a sponsored link even at a *CPC* of $c > v_i$. Furthermore, because this result depends only on the extent of the complementarity γ_i and the ratio x_k/x_{k+1} of the position specific click-through rates of adjacent search listings, even a small complementarity (γ_i greater than but close to 1) can convey substantial market power to the search engine if the difference in click-through-rates for adjacent positions is small so that $x_k/x_{k+1} \approx 1$. However, under currently utilized pricing practices the search engine's ability to exert this market power is limited to the minimum *CPC* c . In particular, the search engine is unable to extract rents generated by the positive externality the sponsored link creates by increasing traffic to the organic link.¹⁴

Propositions 7 and 8 demonstrate the important role that organic links as well as the extent to which sponsored links complement or substitute for organic links play in determining equilibrium outcomes in keyword auction markets. At one end of the spectrum, the presence of organic links combined with consumer resistance to sponsored links (low δ_i) and crowding out effects (low γ_i) can lead to equilibria in which firms are unwilling to utilize sponsored links even if they are available at no charge. At the other end of the spectrum, relatively small complementarities between sponsored and organic links can induce firms to participate in a keyword auction even if the search engine establishes a minimum *CPC* which exceeds the expected value v_i of a customer. The roles of organic links and the parameter γ_i in keyword auction markets are further illustrated in the following example.

¹⁴This suggests that the search engine may be able to increase profits by implementing alternative pricing schemes such as a two part tariff.

5 Example

Several of the more interesting results predicted by the model can be illustrated through a simple example. The example presented here demonstrates outcomes in which both firms bid amounts exceeding the expected value v of a click, in which the firm submitting the lower bid is listed first in the sponsored links, and in which only the less relevant firm participates in the auction. Which of these outcomes applies depends on the relevance of each firm, the extent to which organic and sponsored links are substitutes or complements, and the minimum cost-per-click established by the search engine. The example also enables us to explore the optimal minimum cost-per-click c for the search engine. For example, if there are strong complementarities, the search engine should set c sufficiently low so that both firms participate in the auction if the difference in the position specific location effects x_1 and x_2 is sufficiently small, while it maximizes profit by setting c so that only firm 1 participates if the difference in these position specific location effects is large.

Consider an example with $\gamma_1 = \gamma_2 = \gamma$, $v_1 = v_2 = v$, $\frac{x_{k+1}}{x_k} \equiv \alpha$, and $\frac{\beta_1}{\delta_1} = \frac{\beta_2}{\delta_2} \equiv \theta$. The parameter assumptions imply that the two firms are identical with the exception that firm 1 is more relevant than firm 2 ($\beta_1 > \beta_2$ by assumption). However, the relative relevance β_i/δ_i of sponsored and organic links is identical for both firms. In addition, the ratio x_{k+1}/x_k of the location specific effect of adjacent positions is constant and equal to α .

The parameter assumptions imply $c_1 = \tilde{c}_1 = v(1 - \theta(1 - \gamma\alpha))$ and $c_2 = \tilde{c}_2 = v(1 - \alpha\theta(1 - \gamma\alpha))$. Moreover, $\tilde{p}_1 = \tilde{p}_2 = v(1 - \alpha) + c\alpha$, $\hat{p}_1 = v(1 - \alpha\theta(1 - \gamma\alpha))$, $\hat{p}_2 = v(1 - \alpha^2\theta(1 - \gamma\alpha))$, and $\hat{p}_1 = \tilde{c}_2$. Because $\alpha \leq 1$ by assumption, $\gamma > 1/\alpha$ implies $c_1 = \tilde{c}_1 > c_2 = \tilde{c}_2 > v$ and $\hat{p}_1 > \hat{p}_2$, and $\gamma < 1/\alpha$ implies $c_1 = \tilde{c}_1 < c_2 = \tilde{c}_2 < v$ and $\hat{p}_1 < \hat{p}_2$. The parameter assumptions rule out the possibility of mixed strategy equilibria and enable a straightforward characterization of equilibria for ranges of the minimum *CPC* c .

Suppose $\gamma > \frac{1}{\alpha}$ (sponsored links strongly complements organic links).

- If $c < v$, then part 1 of proposition 6 applies and both firms participate with equilibrium bids $(\tilde{p}_1, \tilde{p}_2)$.
- If $v < c < \hat{p}_1$, then because $\hat{p}_1 > \hat{p}_2$, part 2 of proposition 6 implies that if $c < \hat{p}_2$, then there is an equilibrium in which both firms bid c because $c > \tilde{p}_1 = \tilde{p}_2$. Furthermore, if c satisfies $\hat{p}_2 < c < \hat{p}_1 = \tilde{c}_2$, then both firms still bid on sponsored links because

$\hat{p}_1 > \frac{\delta_2}{\delta_1}\hat{p}_2 + \frac{c(\delta_1 - \delta_2)}{\delta_1}\alpha$ always holds. Thus, firm 2 is awarded the second sponsored link when both firms bid and part 4 of proposition 6 applies.

- if $c_2 = \tilde{c}_2 < c < \tilde{c}_1 = c_1$, then part 3 of proposition 6 implies that only firm 1 bids on the sponsored link.
- if $c > \tilde{c}_1 = c_1$, then neither firm bids on a sponsored link.

Note that $\gamma > 1/\alpha$ implies that both \hat{p}_1 and \hat{p}_2 are greater than v , so there are equilibria in which both firms are willing to pay a cost-per-click exceeding v in order to be listed in the sponsored links. In this case, both firms bid exactly the same amount c , but the more relevant firm (firm 1) is always listed first.

Suppose $\gamma < \frac{1}{\alpha}$.

- If $c < c_1$, then part 1 of proposition 5 implies that both firms bid $\tilde{p}_1 = \tilde{p}_2$, and firm 1 wins the first sponsored position.
- If $c > c_2$, then part 6 of proposition 5 implies that neither firm participates in the keyword auction.
- If $c_1 = \tilde{c}_1 < c < c_2 = \tilde{c}_2$, then firm 2 bids \tilde{p}_2 . In addition, part 4 of proposition 5 implies that firm 1 will bid \hat{p}_1 only if this bid results in firm 1 being listed first in the sponsored listings. This requires

$$\hat{p}_1 > \frac{\delta_2}{\delta_1}\tilde{p}_2 + \frac{c(\delta_1 - \delta_2)}{\delta_1}\alpha = h_i(\tilde{p}_2).$$

Substituting expressions for \hat{p}_1 and \tilde{p}_2 and simplifying yields

$$c < \frac{v(1 - \alpha\theta(1 - \alpha\gamma) - \frac{\delta_2}{\delta_1}(1 - \alpha))}{\alpha} \equiv \bar{c}. \quad (2)$$

If $c < \bar{c}$, then firm 1 announces a lower bid than firm 2 ($\hat{p}_1 < \tilde{p}_2$), but is listed first and pays a *CPC* equal to $\frac{\delta_2}{\delta_1}v(1 - \alpha) + c\alpha$. This *CPC* increases in the value v of the product and in the minimum *CPC* set by the search engine. If condition (2) does not hold, then firm 1 does not participate in the keyword auction and firm 2 bids \tilde{p}_2 and pays a *CPC* of c . Recall that α indicates how consumer interest in a link depreciates for positions sequentially lower on the search engine results page. As α increases,

the four top positions are viewed as more similar (ignoring differences in firm specific relevance) by consumers. As $\alpha \rightarrow 0$, the first position provides a kind of winner-take-all advantage, and $c_1 \rightarrow v(1 - \frac{\beta_1}{\delta_1})$, $c_2 \rightarrow v$, and condition 2 becomes $v(1 - \frac{\delta_2}{\delta_1}) > 0$ which is always satisfied. This implies that if the location advantage is extreme for the first position and the minimum *CPC* is between c_1 and c_2 , then both firms bid on keywords and firm 1 secures the first position even though it bids less aggressively than firm 2. As $\alpha \rightarrow 1$, (no location advantage), $c_1 \rightarrow c_2$ and an outcome in which firm 1 bids \hat{p}_1 is less likely. However, firm 1 is more likely to participate if organic and sponsored links are not strong substitutes and the sponsored link of firm 1 is highly relevant (δ_1 close to 1). More generally if the minimum *CPC* increases in the interval $[c_1, c_2]$, then firm 1 is less likely to bid.

If $\gamma < 1/\alpha$, then the less relevant firm 2 (which is always listed second in the organic links) has a stronger incentive to participate in the keyword auction than firm 1. In particular, firm 2 always bids \tilde{p}_2 , while firm 1 only bids if $c < \bar{c}$. Sponsored links will rebalance positions between competing firms on the search engine if $c > \bar{c}$ and either sponsored links substitute for organic links (*i.e.*, γ is small) or if being listed in a higher sponsored link position do not provide a significant advantage (*i.e.*, α is small), so only firm 2 sponsors a link in equilibrium. In addition, if $c < \bar{c}$, then the search engine will award the first sponsored position to the firm submitting the lower bid in the keyword auction.

In the context of this example, we also can determine the optimal *CPC* established by the search engine. If $\gamma > \frac{1}{\alpha}$, then it is clearly optimal for the search engine to establish a minimum *CPC* of at least $\tilde{c}_2 = \hat{p}_1 > v$. At this *CPC* both firms participate and are charged strictly more than v for each click. Whether establishing an even higher *CPC* of c_1 is optimal depends on whether the profit $\delta_1 x_1 c_1$ with only firm 1 sponsoring a link exceeds the profit $(\delta_1 x_1 + \delta_2 x_2) \tilde{c}_2$ from establishing a minimum *CPC* sufficiently low so that both firms sponsor links. Substituting the values of c_1 and \tilde{c}_2 into the profit expressions it can be shown that if the difference α in position specific click-through rates is sufficiently high (in particular, if $\alpha \geq \delta_1 x_1 / (\delta_1 x_1 + \delta_2 x_2)$), then it is optimal for the search engine to set $c = \tilde{c}_2$ so that both firms participate in the auction. However, if $\alpha < \delta_1 x_1 / (\delta_1 x_1 + \delta_2 x_2)$, then a minimum *CPC* of $c = \tilde{c}_2$ is optimal if γ is sufficiently close to $1/\alpha$, while if γ is sufficiently large, then it is optimal for the search engine to set $c = c_1$ so that only firm

1 participates in the keyword auction. These results imply that the search engine’s ability to optimally extract surplus in the keyword auction depends upon both the magnitude of the complementarity γ and the location specific effect α . If the location effect is small (α is close to 1), the search engine is able to extract sufficient surplus from both firms as the complementarity γ increases, that including both firms is always optimal. However, if the location effect is large (so that $\alpha < \delta_1 x_1 / (\delta_1 x_1 + \delta_2 x_2)$), then as the magnitude of the complementarity increases, the surplus generated by the first position is substantially higher than that generated by the second position. Because keywords are being allocated through a second price auction, setting $c = \tilde{c}_2$ to induce both firms to participate would generate a price for the first sponsored position which is much lower than the surplus that position generates. As a result, for γ sufficiently large, the search engine should set $c = c_1$.

If $\gamma < \frac{1}{\alpha}$, then as long as $c \leq c_1$, both firms participate with bids of \tilde{p}_i , and the the *CPC* paid by each firm increases with c . Furthermore, for $c_1 < c < \bar{c}$ the bid \tilde{p}_2 submitted by firm 2 is strictly increasing in c , so both the *CPC* paid by firm 1, which is awarded the first sponsored link if $c \leq \bar{c}$, and the minimum *CPC* paid by firm 2 are increasing in c . This implies the search engine will never set $c < \bar{c}$. Alternatively, the search engine could set $c = \tilde{c}_2$ so that only firm 2 participates. However, it can be shown that setting $c = \bar{c}$ is always optimal if $\gamma < 1/\alpha$. Following the intuition above, if organic and sponsored links are substitutes or only weak complements, then the search engine extracts maximal surplus by always inducing both firms to participated in the keyword auction, regardless of the magnitude of the location effect α .

6 Managerial implications and concluding remarks

This paper investigates strategic behavior of firms that utilize sponsored search ads to attract customers who may reach the firm through either sponsored or organic links. We develop a model which allows for asymmetry between these firms in several dimensions. Firms may differ in the relevance to consumers of both their organic and their sponsored links (*i.e.*, consumers may be more likely to follow a link to one firm than another or to follow an organic versus sponsored link). Firms also may differ in the expected value of a consumer visit to their website – one firm may offer products that are more valuable to consumers than another firm even though consumers reach both firms by initially searching the same

keyword. Our model also integrates several empirically observed aspects of consumer search behavior including the possibility that sponsored links may be substitutes or complements to organic links, and that consumers may have either a preference for or aversion to sponsored links which influences the probability they click on either sponsored or organic links.

Our model analyzes how the above characteristics determine each firm’s equilibrium bidding strategy in a sponsored search keyword auction. Our results highlight the important role that the relationship between organic and sponsored links and the relevance of these links to consumers plays in firm bidding decisions. To the extent that these parameters can be manipulated by bidding firms or the search engine, our results have important implications for management strategy. For example, the relevance β_i to consumers of a firm’s organic link and the relevance δ_i of a firm’s sponsored link both directly impact the firm’s decision to participate in the keyword auction. However, contrary to what one might anticipate, a firm’s optimal bid for a sponsored link does not necessarily decrease as the organic link to the firm becomes more relevant to consumers (as β_i increases). Nor does a firm’s optimal bid necessarily increase if consumers are more likely to click on a sponsored link to the firm. Rather these relationships depend on the extent to which sponsored and organic links are substitutes or complements and on the minimum cost-per-click established by the search engine. For example, if the minimum cost-per-click is sufficiently small and organic and sponsored links are not strong substitutes (so the conditions of proposition 7 do not apply), then the optimal bid \tilde{p}_i for each firm is independent of the relevance to consumers of the firm’s sponsored or organic links. The low minimum cost-per-click induces both firms to bid in equilibrium, so, although changes in β_i and δ_i impact the firm’s total profit, neither affects the marginal profit from a change in firm i ’s sponsored link position (from first to second or second to first). Therefore, neither change has any impact on the optimal bid.¹⁵

Furthermore, while it seems obvious that the managerial objective of increasing the relevance β_i and δ_i of a firm’s organic and sponsored links will increase firm i ’s profit, our analysis demonstrates this is not necessarily the case. An increase in β_i does, in fact, always increase firm i ’s profit because increased clicks on the firm’s organic link generate additional expected sales while imposing no additional costs (per-click) on the firm. However, an increase in δ_i may actually reduce firm profit. In particular, if organic and sponsored links

¹⁵This is evident from conditions 1.1 and 2.1 which generate the expressions for \tilde{p}_i which are independent of β_i and δ_i .

are strong complements (so $\gamma_i > x_{i+1}/x_{i+2}$ in the model),¹⁶ then firm i is willing to pay a cost-per-click which exceeds the expected sale amount v_i generated by a customer who visits the firm's site. If the search engine operator understands the nature of the complementarity, it will exploit this fact by establishing a minimum cost-per-click which exceeds v_i . As a result, additional clicks on the sponsored link caused by an increase in δ_i will actually reduce firm profit. Despite the negative expected profit generated by each click on the firm's sponsored link, participation in the keyword auction may still be optimal because the strong complementarity implies that the increased visibility created by the sponsored link also leads to a substantial increase in clicks on the firm's organic link which generate sales with no cost-per-click.

Our analysis also demonstrates how organic links impact a firm's decision to participate in the keyword auction. This decision relies on a cost-per-click threshold representing the maximum cost-per-click that a firm is willing to pay for a sponsored link.¹⁷ In order for a firm to participate, the minimum cost-per-click established by the search engine must not exceed this threshold. Suppose organic and sponsored links are substitutes or weak complements. If a firm i is optimally participating in the auction, then an increase in the relevance δ_i of its sponsored link increases firm i 's profit and raises its threshold cost-per-click threshold, which ensures that participation remains optimal. Similarly, an increase in β_i reduces firm i 's threshold cost-per-click. As β_i increases, the marginal profit accrued through organic links when the firm does not participate in the auction is greater than the marginal profit accrued from organic links when the firm does sponsor a link because of consumer substitution to clicks on the sponsored link instead of the organic link. Thus, while increasing β_i raises the profit of the firms (regardless of whether it participates in the auction), efforts by the search engine to increase β_i can be counter productive if organic and sponsored links are substitutes.

If organic and sponsored links are strong complements, on the other hand, then an

¹⁶This case is presented in the discussion of Figure 3.

¹⁷Recall from the analysis in subsection 3.1 that this threshold for firm i is either \hat{p}_i or \tilde{c}_i if both firms bid on sponsored links, and is c_i if the competing firm j does not bid. It is easily shown that both \hat{p}_i and \tilde{c}_i are decreasing in β_i and increasing in δ_i if organic links and sponsored links are substitutes or weak complements (so $\gamma_i < x_{i+1}/x_{i+2}$). Similarly, \hat{p}_i and \tilde{c}_i are increasing in β_i and decreasing in δ_i if organic links and sponsored links are strong complements (so $\gamma_i > x_{i+1}/x_{i+2}$). If the competing firm j does not participate in the auction, then firm i will only participate if the minimum cost-per-click c established by the search engine does not exceed the threshold c_i which is decreasing in β_i and increasing in δ_i if $\gamma_i < x_i/x_{i+1}$ and increasing in β_i and decreasing in δ_i if $\gamma_i > x_i/x_{i+1}$.

increase in δ_i reduces the threshold cost-per-click threshold for which participation in the keyword auction is optimal for firm i . Thus, in the case of strong complementarities, efforts by the search engine to increase the relevance of sponsored links to consumers may backfire and cause firms to stop participating in the auction unless the search engine simultaneously reduces the minimum cost-per-click. Whether reducing the minimum cost-per-click to ensure that both firms participate in the auction is optimal for the search engine depends on a variety of factors as discussed in section 5. An increase in the relevance β_i of the organic link has the opposite effect. If organic and sponsored links are strong complements, then an increase in β_i raises firm i 's threshold cost-per-click and enables the search engine to extract greater surplus through the keyword auction. Thus, in the case of strong complementarities, both the search engine and the firms bidding for sponsored links have an incentive to increase the relevance β_i of organic links.

We conclude with an observation on recent changes to Google's AdWords design. Our analysis highlights the importance of the relevance of organic and sponsored links as well as the extent to which sponsored and organic links are substitutes or complements to outcomes in sponsored search auctions. The effect of search engine and firm strategies to alter these parameters can impact firm participation and bidding decisions and search engine profit in a variety of ways. The ongoing modifications by search engines of their sponsored search auction policies suggest attempts to better understand these complex relationships and generate greater surplus. For example, Google has recently introduced a new feature in AdWords which enables a firm to locate a sponsored link adjacent to its organic link. Our analysis implies that such a strategy can increase total surplus and make both the search engine and the firm better off if the sponsored link placement simultaneously makes organic and sponsored links stronger complements and increases the relevance organic links.

7 Appendix

7.0.1 Proof of equilibrium outcomes

The appendix presents results for mixed strategy equilibria corresponding to conditions in propositions 5 and 6 as well as a proposition characterizing the equilibrium if sponsored and organic links are strong complements for one firm j (so $\gamma_j > x_{j+1}/x_{j+2}$) and either substitutes or weak complements for the other firm i (so $\gamma_i < x_{i+1}/x_{i+2}$). We first present notation regarding equilibria with mixed strategies. Consider the case of $\gamma_i \leq x_{i+1}/x_{i+2}$ and assume $\hat{p}_i > c > \tilde{c}_i$ for both firms. Under these conditions each firm only wants to bid if it will be listed first, and each firm will bid \hat{p}_i if that bid will win the first sponsored link position. However, unless $\hat{p}_1 = \hat{p}_2$, one of the two firms will win the first position with probability 1 and the other firm will choose not to bid. Suppose $\hat{p}_i > \hat{p}_j$ and $c < c_i$. If firm j sponsors a link, then firm i will sponsor a link because it wins the first sponsored position, and if firm j does not sponsor a link, firm i will sponsor because $c < c_i$. Therefore, firm j knows that it will be in the second sponsored position if it chooses to bid, so it will not bid. However, if $c > c_i$ and firm j decides not to bid because $\hat{p}_i > \hat{p}_j$, then firm i is better off not sponsoring a link and also will choose not to bid. If $c > c_j$, then the fact that firm i chooses not to bid will not change firm j 's decision – the conditions of part 4 of proposition 5 are satisfied. However, if $c_i < c < c_j$, then a decision by firm i to not bid makes bidding \hat{p}_j a best response for firm j . But if firm j bids \hat{p}_j , then it is optimal for firm i to bid \hat{p}_i and win the first sponsored link position, which then makes not bidding a best response for firm j ; there is no equilibrium in pure strategies. However, there will be an equilibrium in mixed strategies in which a given firm i bids on a sponsored link with probability q_i such that given the other firm bids with probability q_j , firm i 's expected profit from a bid of \hat{p}_i is equal to firm i 's expected profit from not sponsoring a link. In particular, if $\hat{p}_i > \hat{p}_j$, then q_j and q_i satisfy

$$q_j \pi_i^{AA1} + (1 - q_j) \pi_i^{AN} = q_j \pi_i^{NA} + (1 - q_j) \pi_i^{NN}, \quad (3)$$

and

$$q_i \pi_j^{AA2} + (1 - q_i) \pi_j^{NA} = q_i \pi_j^{AN} + (1 - q_i) \pi_j^{NN} \quad (4)$$

where the profits are as defined in subsection 2.2 with a *CPC* of c if only one firm bids on a sponsored link and the *CPC* for each firm determined as described in subsection 3.2

if both firms bid on sponsored links. More generally, a mixed strategy equilibrium will exist whenever conditions are such that one firm i will bid on a sponsored link given firm j bids on a sponsored link, but firm j will not bid on a sponsored link if firm i bids, and $c_j > c > c_i$. To calculate the mixed strategy equilibrium, define π_k^{r,s_j} as the profit of firm $k = i, j$ when firm i adopts strategy r and firm j adopts strategy s , where profits are as defined in subsection 2.2, and the equilibrium CPC is c when only one firm sponsors a link and CPC amounts are determined by the bids as specified in subsection 3.2 when both firms sponsor links. If firm i advertises with probability q_i , then if firm j advertises, firm j earns an expected profit of $q_i \pi_j^{AAm} + (1 - q_i) \pi_j^{N_i A_j}$, where $m \in \{1, 2\}$ denotes the sponsored position that firm j wins in the auction when both firms bid. If firm j does not advertise, then it earns an expected profit of $q_i \pi_j^{A_i N_j} + (1 - q_i) \pi_j^{N_i N_j}$. Equating these expected profit amounts and solving for the equilibrium advertising probability of firm i yields $q_i = \left(\pi_j^{N_i A_j} - \pi_j^{N_i N_j} \right) / \left(\pi_j^{N_i A_j} - \pi_j^{N_i N_j} + \pi_j^{A_i N_j} - \pi_j^{AA2} \right)$. Note that $0 < q_i < 1$ because $c < c_j$ implies $\pi_j^{N_i A_j} > \pi_j^{N_i N_j}$, and the fact that firm j prefers not to advertise over bidding and being awarded position 2 in the sponsored links implies $\pi_j^{A_i N_j} > \pi_j^{AA2}$. Similar analysis yields $q_j = \left(\pi_i^{N_i N_j} - \pi_i^{A_i N_j} \right) / \left(\pi_i^{N_i N_j} - \pi_i^{A_i N_j} + \pi_i^{AA1} - \pi_i^{N_i A_j} \right)$. Depending on the bids submitted by each firm, the sponsored positions awarded to firms i and j could be reversed from those in equations (3) and (4), *e.g.*, π_i^{AA1} in equation (3) could be π_i^{AA2} , and π_j^{AA2} in equation (4) could be π_j^{AA1} . To allow for all possible outcomes, define

$$q_{km} \equiv \left(\pi_{-k}^{N_i A_j} - \pi_{-k}^{N_i N_j} \right) / \left(\pi_{-k}^{N_i A_j} - \pi_{-k}^{N_i N_j} + \pi_{-k}^{A_i N_j} - \pi_{-k}^{AAm} \right),$$

and

$$\tilde{q}_{km} \equiv \left(\pi_{-k}^{N_i N_j} - \pi_{-k}^{N_i A_j} \right) / \left(\pi_{-k}^{N_i N_j} - \pi_{-k}^{N_i A_j} + \pi_{-k}^{AAm} - \pi_{-k}^{A_i N_j} \right)$$

for $k = 1, 2$. Thus, q_{km} is the advertising probability for firm k that makes the other firm, $-k$, indifferent between bidding and not bidding given $c < c_{-k}$ (so that firm $-k$ prefers to sponsor a link over not sponsoring a link given firm k does not sponsor, *i.e.*, so $\pi_{-k}^{N_i A_j} > \pi_{-k}^{N_i N_j}$) and firm $-k$ is awarded sponsored position m if both firms bid, and \tilde{q}_{km} is the advertising probability for firm k that makes firm $-k$ indifferent between advertising and not advertising given $c > c_{-k}$ (so $\pi_{-k}^{N_i A_j} < \pi_{-k}^{N_i N_j}$), and firm $-k$ is awarded sponsored position m if both firms bid.

Proposition 9 *Mixed strategy equilibrium when $\gamma_i \leq x_{i+1}/x_{i+2}$ for $i = 1, 2$ with $c_i < c < \tilde{c}_i$,*

and $\tilde{c}_j < c < c_j$, and $\hat{p}_j < \max \{h_j(\tilde{p}_i), c\}$.

1. If $c < \hat{p}_j < h_j(\tilde{p}_i)$, then there is a mixed strategy equilibrium in which firm i bids \tilde{p}_i with probability q_{i2} and firm j bids \hat{p}_j with probability \tilde{q}_{j1} , and if both firms bid, then firm i is awarded the first sponsored link position with a CPC of $\max \{c, h_i(\hat{p}_j)\}$ and firm j is awarded the second sponsored link position with a CPC of c .
2. If $\hat{p}_j < c < h_j(\tilde{p}_i)$, then there is a mixed strategy equilibrium in which firm i bids \tilde{p}_i with probability q_{i2} and firm j bids c with probability \tilde{q}_{j1} , and if both firms bid, then firm i is awarded the first sponsored link position with a CPC of c and firm j is awarded the second sponsored link position with a CPC of c .
3. If $\hat{p}_j < c$, and $h_j(\tilde{p}_i) < c$, then there is a mixed strategy equilibrium in which firm i bids \tilde{p}_i with probability q_{i2} if $i = 1$ and q_{i1} if $i = 2$ and firm j bids c with probability \tilde{q}_{j1} if $i = 1$ and \tilde{q}_{j2} if $i = 2$, and if both firms bid, then firm 1 is awarded the first sponsored link position with a CPC of c and firm 2 is awarded the second sponsored link position with a CPC of c .

Proof. For cases 1 and 2 where $c < \hat{p}_j < h_j(\tilde{p}_i)$ or $\hat{p}_j < c < h_j(\tilde{p}_i)$, note that firm i prefers to bid if and only if firm j bids (if firm j does not bid, then $c > c_i$ implies firm i prefers not to bid), while firm j prefers to bid if and only if firm i does not ($c > \tilde{c}_j$ implies that if firm i bids, then a bid by firm j is optimal only if firm j will win the first position which will not happen because $c < \tilde{c}_i$ implies firm i will bid \tilde{p}_i if firm j bids, and $\hat{p}_j < h_j(\tilde{p}_i)$ ensures firm i wins the first sponsored position when both firms bid). Thus, there is no pure strategy equilibrium. To calculate the mixed strategy equilibrium, define $\pi_k^{r_i s_j}$ as the profit of firm $k = i, j$ when firm i adopts strategy r and firm j adopts strategy s , where profits are as defined in subsection 2.2 and the equilibrium CPC is c when only one firm sponsors a link and CPC amounts are determined by the bids as specified in subsection 3.2 when both firms sponsor links. If firm i advertises with probability q_i , then if firm j advertises, firm j earns an expected profit of $q_i \pi_j^{AA2} + (1 - q_i) \pi_j^{N_i A_j}$ and if firm j does not advertise, then it earns an expected profit of $q_i \pi_j^{A_i N_j} + (1 - q_i) \pi_j^{N_i N_j}$. Equating these expected profit amounts and solving for the equilibrium advertising probability of firm i yields $q_i = \left(\pi_j^{N_i A_j} - \pi_j^{N_i N_j} \right) / \left(\pi_j^{N_i A_j} - \pi_j^{N_i N_j} + \pi_j^{A_i N_j} - \pi_j^{AA2} \right) = q_{i2}$. Similar analysis yields

$q_j = \left(\pi_i^{N_i N_j} - \pi_i^{A_i N_j} \right) / \left(\pi_i^{N_i N_j} - \pi_i^{A_i N_j} + \pi_i^{AA1} - \pi_i^{N_i A_j} \right) = \tilde{q}_{j1}$. The third case of $c > \hat{p}_j$ and $c > h_j(\tilde{p}_i)$ is identical except that when firm j bids c and i bids \tilde{p}_i , firm j is listed first (note that $\gamma_i < x_{i+1}/x_{i+2}$ and $c < \tilde{c}_i$ imply $c < \tilde{p}_i$, so \tilde{p}_i is the optimal bid for firm i , while c is the optimal bid for firm j because firm j only wants to sponsor a link if firm i does not), so the equilibrium mixed strategy probabilities are q_{i2} , and \tilde{q}_{j1} if $i = 1$ (because firm 1 is awarded the first position when both firms bid), and q_{i1} and \tilde{q}_{j2} if $i = 2$. ■

Proposition 10 *Mixed strategy equilibrium when $\gamma_i > x_{i+1}/x_{i+2}$ for $i = 1, 2$.*

1. *Suppose $\hat{p}_i < c < \min\{c_i, \tilde{c}_i\}$ and $\max(c_j, \hat{p}_j) < c < \tilde{c}_j$. If $c > h_i(c)$, then there is a mixed strategy equilibrium in which firm i bids c with probability \tilde{q}_{i2} , firm j bids c with probability q_{j1} , and firm i is awarded the first sponsored position if both firms bid.*
2. *Suppose $c_i < c < \hat{p}_i$ and $\hat{p}_j < c < c_j$.*
 - (a) *If $h_j(\max\{\tilde{p}_i, c\}) < c < \tilde{c}_j$, then there is a mixed strategy equilibrium in which firm i bids $\max\{\tilde{p}_i, c\}$ with probability q_{i1} and firm j bids c with probability \tilde{q}_{j2} , and if both firms bid, then firm j is awarded the first sponsored link and firm i is awarded the second sponsored link position both with a CPC of c .*
 - (b) *If $\tilde{c}_j < c < h_j(\max\{\tilde{p}_i, c\})$, then there is a mixed strategy equilibrium in which firm i bids $\max\{\tilde{p}_i, c\}$ with probability q_{i2} and firm j bids c with probability \tilde{q}_{j1} , and if both firms bid, then firm i is awarded the first sponsored link position with a CPC of $h_i(c)$ and firm j is awarded the second sponsored link position with a CPC of c .*
 - (c) *If $c > \tilde{c}_j$ and $c > h_j(\max\{\tilde{p}_i, c\})$, then there is a mixed strategy equilibrium in which firm i bids $\max\{\tilde{p}_i, c\}$ with probability q_{i1} and firm j bids c with probability \tilde{q}_{j2} , and if both firms bid, then firm i is awarded the second sponsored link and firm j is awarded the first sponsored link position both with a CPC of c .*

Proof. For case 1 note that $\hat{p}_i < c < \min\{c_i, \tilde{c}_i\}$ implies that it is only optimal for firm i to bid when firm j bids if i will be awarded the second sponsored position, and it is always optimal for i to bid if j does not. Similarly, $\max(c_j, \hat{p}_j) < c < \tilde{c}_j$ implies that it is optimal for firm j to bid when firm i bids if firm j will be awarded the second sponsored position,

and it is never optimal for firm j to bid if firm i does not. Also, it because $c > \hat{p}_i$ and $c > \hat{p}_j$, both firms will bid c if they bid. Because $c > h_i(c)$ implies that firm i will win the first sponsored position when both bid, firm i never wants to bid if firm j bids, and because $c > c_j$, firm j never wants to bid if firm i does not bid. It follows that no pure strategy equilibrium exists, and because firm i wins the first position when both bid and $c_j < c < c_i$, the equilibrium mixed strategies are \tilde{q}_{i2} and q_{j1} . For case 2 note that $c_i < c < \hat{p}_i$ implies that it is always optimal for firm i to bid if firm j bids and not optimal for firm i to bid if firm j does not. Similarly $\hat{p}_j < c < c_j$ implies it is only optimal for firm j to bid when firm i bids if firm $c < \tilde{c}_j$ and firm j is awarded the second sponsored position and it is always optimal for firm j to bid if firm i does not. Because $c < \hat{p}_i$, firm i will bid $\max\{\tilde{p}_i, c\}$ when it bids, and because $c < \hat{p}_j$, firm j will bid c when it bids. For case 2a, $c > h_j(\max\{\tilde{p}_i, c\})$ implies firm j wins the first position when both bid, so there is no pure strategy equilibrium and because $c_i < c < c_j$, the equilibrium mixed strategies are \tilde{q}_{j1} and q_{i2} . In case 2b, firm j wins the second sponsored position when both firms bid, but because $c > \tilde{c}_j$, firm j is better off not bidding at all when firm i bids, so again there is no pure strategy equilibrium and the equilibrium mixed strategies are q_{i2} and \tilde{q}_{j1} . Finally, in case 2c firm j wins the first position when both bid, so the equilibrium is the same as in part 2a. ■

Finally, similar arguments to those presented above imply the following proposition for the case in which organic and sponsored links are strong complements for one firm j and either substitutes or weak complements for the other firm i .

Proposition 11 *Suppose that $\gamma_i < x_{i+1}/x_{i+2}$ for one firm i and $\gamma_j > x_{j+1}/x_{j+2}$ for the other firm j .*

1. *If $c \leq \min(c_i, \tilde{c}_i, c_j, \hat{p}_j)$, then there is a unique equilibrium in which firm i bids \tilde{p}_i and firm j bids $\max\{\tilde{p}_j, c\}$ for sponsored links.*
2. *If $\max(c_1, c_2) < c < \min(\tilde{c}_i, \hat{p}_j)$, then there are two pure strategy equilibria; in one equilibrium, neither firm bids on sponsored links, and in the other equilibrium firm i bids \tilde{p}_i and firm j bids $\max\{\tilde{p}_j, c\}$.*
3. *If $\max(c_i, \hat{p}_i) < c < c_j$, then firm j sponsors a link and firm i does not. If $\max(c_j, \tilde{c}_j) < c < c_i$, then firm i sponsors a link and firm j does not.*

4. Suppose $c_i < c < c_j$ and $\hat{p}_j < c < \hat{p}_i$. If $c < \tilde{c}_j$, and $c < \frac{\delta_i}{\delta_j} \min\{\tilde{p}_i, \hat{p}_i\} + \frac{x_2 c(\delta_j - \delta_i)}{\delta_j x_1}$, then there is a unique equilibrium in which firm i bids $\min\{\tilde{p}_i, \hat{p}_i\}$, firm j bids c , and firm i is awarded the first sponsored link. If $c \geq \tilde{c}_j$ and $c < \frac{\delta_i}{\delta_j} \min\{\tilde{p}_i, \hat{p}_i\} + \frac{x_2 c(\delta_j - \delta_i)}{\delta_j x_1}$, then there is a mixed strategy equilibrium in which firm i bids $\min\{\tilde{p}_i, \hat{p}_i\}$ with probability q_{i2} and firm j bids c with probability q_{j1} , and if both firms bid, then firm i is awarded the first sponsored link position with a CPC of $\max\left\{c, \frac{\delta_j}{\delta_i} c + \frac{x_2 c(\delta_i - \delta_j)}{\delta_i x_1}\right\}$ and firm j is awarded the second sponsored link position with a CPC of c . If $c < \tilde{c}_i$ (and $c < \tilde{c}_j$ or $c > \tilde{c}_j$), and $c > \frac{\delta_i}{\delta_j} \min\{\tilde{p}_i, \hat{p}_i\} + \frac{x_2 c(\delta_j - \delta_i)}{\delta_j x_1}$, then there is a mixed strategy equilibrium in which firm i bids $\min\{\tilde{p}_i, \hat{p}_i\}$ with probability q_{i1} and firm j bids c with probability q_{j2} , and if both firms bid, then firm j is awarded the first sponsored link and firm i is awarded the second sponsored link position both with a CPC of c . If $c > \tilde{c}_i$ (and $c < \tilde{c}_j$ or $c > \tilde{c}_j$), and $c > \frac{\delta_i}{\delta_j} \min\{\tilde{p}_i, \hat{p}_i\} + \frac{x_2 c(\delta_j - \delta_i)}{\delta_j x_1}$, then only firm i bids and submits a bid of c . Suppose $c_j < c < c_i$ and $\hat{p}_i < c < \hat{p}_j$. If $c < \tilde{c}_j$ and $\min\{\tilde{p}_i, \hat{p}_i\} > \frac{\delta_j}{\delta_i} c + \frac{x_2 c(\delta_i - \delta_j)}{\delta_i x_1}$, then firm i bids $\min\{\tilde{p}_i, \hat{p}_i\}$, firm j bids c , and firm i is awarded the first sponsored link. If $\min\{\tilde{p}_i, \hat{p}_i\} < \frac{\delta_j}{\delta_i} c + \frac{x_2 c(\delta_i - \delta_j)}{\delta_i x_1}$, then only firm i bids. Proof: for these values of c , firm i only wants to bid if it will be listed first and firm j only wants to bid if it will be second. Furthermore, if firm j does not bid, firm i will bid to have the only sponsored link because $c < c_i$, while if firm i does not bid, firm j will not bid because $c > c_j$. Because $c < \hat{p}_i$, firm i bids $\min\{\tilde{p}_i, \hat{p}_i\}$ when it does bid, and because $c > \hat{p}_j$, firm j bids c when it does bid. If $\min\{\tilde{p}_i, \hat{p}_i\} > \frac{\delta_j}{\delta_i} c + \frac{x_2 c(\delta_i - \delta_j)}{\delta_i x_1}$, then firm i is first when both bid, so it is optimal for both firms to bid. If $\min\{\tilde{p}_i, \hat{p}_i\} < \frac{\delta_j}{\delta_i} c + \frac{x_2 c(\delta_i - \delta_j)}{\delta_i x_1}$, then firm j would be first, so firm j will not bid if firm i bids, but it is optimal for firm i to bid when firm j does not, so firm i bids and firm j does not.
5. If $c > \max(c_1, c_2)$ and $c > \min(\hat{p}_1, \hat{p}_2)$ then neither firm bids on sponsored links.

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