The Effect of Connectivity, Proximity and Market Structure on R&D Networks*

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Abstract

In a seminal paper, Goyal and Moraga-Gonzalez (2001) use an undirected network to characterize knowledge flows between firms engaging in research in an oligopolistic market. In their paper, firms are regarded as inhabiting a research joint venture (RJV), if they share the same edge of the network. These firms are allowed an R&D spillover of 1; the outside firms (firms not sharing an edge in the network) are permitted a constant knowledge spillover that is less than one. We begin our paper by showing that this last assumption has important consequences when dealing with R&D networks of size greater than or equal to six firms. We present examples of topologically non-equivalent networks that have the same degree of connectivity and generate identical outcomes in terms of R&D effort, firm profits and total welfare. We then modify their model so that R&D spillovers decrease as the number of shortest paths increases between any two firms. We show that under product differentiated Cournot and Bertrand competition, we have different outcomes for all economic variables. We also show that R&D effort increases with respect to the number of collaborative links if firms are in a weakly competitive market, whereas it declines if firms are in a more competitive market where products are closer substitutes. We also find that in more competitive markets there is a conflict between the stability and the efficiency of RJVs.

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Key Words: R&D Networks; R&D Spillovers, Degree of Connectivity, Shortest Path, Market Structure, Product Differentiation

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1 Introduction

There is a vast theoretical and empirical literature in industrial organization focusing on the incentives for and welfare implications from innovation stemming from the seminal paper by Arrow (1962).[1] Among the theoretical models used in this literature, one important sub-family are the class of models due to D'Aspremont and Jacquemien (1988, 1990) and Kamien, Muller and Zang (1992).[2] These models present a situation in which there is a possibility that firms can free-ride off the knowledge generated as a by-product of the research and development (R&D) activities of competing firms in the same industry. As such the money invested in cost-saving process innovation is far below the level that is socially optimal. The solution presented in these papers was for firms to share knowledge generated from R&D and/or coordinate their costs of investment. One of the criticisms of these models has been the way in which these knowledge spillovers are bundled into costs. This has lead to a growing literature that has focused on augmenting the models of D'Aspremont and Jacquemien and Kamien, Muller and Zang in order to unbundle the "black box" of knowledge spillovers and better understand how these spillovers lead to gains in terms of the tangible outcomes of profit and welfare gains.[3]

One approach for doing this was set down in an important paper by Goyal and Moraga-Gonzalez (2001), which extended the D'Aspremont and Jacquemien model to incorporate an undirected network as a means of characterizing knowledge flows between firms networked into a research joint venture (RJV).[4] In that paper, they model an R&D network for a market in which firms selling a homogeneous good engage in Cournot competition. Our paper provides an extension of this model. We argue that the definition of R&D effective effort that was used in their paper is not sufficient when the number of firms increases to six and beyond. We present examples of pairs of topologically non-equivalent R&D networks for a product differentiated Cournot oligopoly with six firms. We show that when Goyal and Moraga-Gonzalez (2001) is applied to each of the examples, we obtain identical outcomes in terms of R&D effort, firm profits and total welfare for each of the non-equivalent R&D networks. We show that this changes when we modify the way that R&D spillover term is defined, so that it depends directly on the size of the R&D network and how firms are placed within this network in relation to the placement of other firms.

Specifically, in Goyal and Moraga-Gonzalez (2001) this spillover term is a constant, which is set exogenously and is identical for all firms not directly engaged in a RJV with any one firm

[1] An excellent summary of this literature can be found in De Bondt (1996).

[2] Other important papers in this literature include Leahy and Neary (1997), Petit and Tolwinski (1999), Poyago-Theotoky, (1995), Spence (1984) and Suzumura (1984).

[3] Important papers include those by Amir and Wooders (1999), Bloch (1995), Kultti and Takalo (1998), Legros and Matsushima (1991), Legros and Mathews (1993), Long and Soubeyran (1999), Martin (1994), Salant and Shaeffer (1998, 1999), Veugelers and Kasteloot (1996).

[4] Other papers in this literature on R&D networks, with a slightly narrower focus concentrating on formation of RJVs, include Goyal and Joshi (2003), Yi (1998) and Yi and Shin (2000).

placed with the R&D network. Hence, the size of a firms effective R&D effort increases with the number of direct collaborative connections. By comparison, in our model, knowledge spillovers to any one firm increases with the number of its cooperative links, as well as the number of shortest paths between this firm and others that are not directed linked to it. In this way, as the length of the shortest path increases between this firm and other firms, then the marginal gain in effective R&D will decline. By applying our new model to the non-equivalent network examples, we show different outcomes for all economic variables.

The second point of departure between our paper and Goyal and Moraga-Gonzalez (2001) is that they study both symmetric and asymmetric R&D networks, considering only the case where firms are Cournot oligopolists producing a homogeneous good. In the first case, symmetric (or regular) networks means that each firm has the same number of links (neighbors). In the second case, asymmetric networks, firms have different numbers of links. Their most general results, for the case of n firms, focus exclusively on symmetric networks. They only consider asymmetric networks when there are three firms present in the market. The main finding in Goyal and Moraga-Gonzalez (2001), in the case of symmetric networks, is that although the complete R&D network is stable (in which all firms are engaged in a single RJV), there exist intermediate levels of collaboration that are more efficient. They also find that this result also holds for asymmetric R&D networks of size three.

By contrast, we will concentrate only on asymmetric R&D networks and apply our new model to examine the impact of Cournot and Bertrand competition under the assumption of product differentiation. This will allow us to examine the impact of market structure on the R&D effort generated from this underlying network. For both types of competition, Bertrand and Cournot, we examine the impact of different R&D networks for the case where there are three and six firms, allowing for the possibility of asymmetric networks. The most significant results are that the effort of firms in R&D increases with respect to the collaborative links if firms are in weakly competitive markets, whereas it declines if firms are in a more competitive market (as characterized by the degree of substitutability between products). Likewise, the R&D effort of neighbors of cooperative firms (i.e. firms not directly participating in a RJV) is also similarly affected.

The conflict between the stability and efficiency of networks reported in Goyal and Moraga-Gonazles (2001) sometimes appears, but not always. We find that the complete network is the unique stable network as well as the unique efficient network if firms are not in a strongly competitive market for R&D networks consisting of three firms. By contrast, for R&D networks composed of six firms, we find that the complete network is the unique stable network as well as the unique efficient network if firms are in a weakly competitive market. However, in more competitive markets (where goods are close substitutes), there is always a trade-off between stability and efficiency as the network becomes more connected, which is consistent with results in Goyal and Moraga-Gonzalez (2001). This suggests that product differentiation and type of competition exhibited between firms have an impact on the stability and efficiency of RJVs.

This paper is organized as a follows. In the second section we introduce the model of Goyal and Moraga-Gonzalez (2001). We explain a problem with this model by giving some examples of non-equivalent networks of six firms, where in each network firms have same degree distribution. We show that when this is the case the R&D network will produce identical outcomes in terms of R&D effort, firm profits and total welfare for each of the non-equivalent R&D networks. In the third section of this paper, we introduce our model for R&D networks consisting of three firms and four distinct network topologies and are able to provide a complete characterization of the impact of market structure on the R&D network. In sections four and five, we study the impact of this R&D network for the case where there are three and six firms, respectively. Unfortunately, when there are six firms there are too many R&D networks to provide a complete characterization. Hence, in section five, we concentrate will on the most significant networks in these two section. In the last section we conclude our work by summarizing the most important aspects of the R&D network model presented in this paper.

2 Motivation

In this section we introduce the R&D network model in Goyal and Moraga-Gonzalez (2001). In this model there are n firms competing as Cournot oligopolists, with each firm selling an homogeneous good and engaging in cost reducing R&D. (Goyal and Moraga-Gonzalez (2001) assume a linear demand equation for this market). As in the standard R&D spillovers model firms choose with whom they wish to collaborate (which determines the R&D network topology and the R&D spillover levels accruing to each firm), each firm i then chooses their level of cost reducing R&D effort x_i, which in turn influences their output quantity q_i. The costs associated with R&D effort are quadratic γx_i^2, where $\gamma > 0$ indicates the effectiveness of R&D expenditure.

In Goyal and Moraga-Gonzalez (2001) the effective R&D effort for each firm is described by the following equation

$$X_i = x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k, \quad i = 1, \ldots, n, \tag{1}$$

where x_i denotes R&D effort of firm i, N_i is the set of firms participating in a joint venture with firm i and $\beta \in [0, 1)$ is an exogenous parameter that captures knowledge spillovers acquired from firms not engaged in a joint venture with firm i. The effective R&D effort is cost reducing in the sense that it reduces firm i's marginal cost of production:

$$c_i = \bar{c} - x_i - \sum_{j \in N_i} x_j - \beta \sum_{k \notin N_i} x_k, \quad i = 1, \ldots, n, \tag{2}$$

where $\bar{c}$ is the marginal cost of production unadjusted for R&D effort. One can see that effective R&D effort (and marginal cost of production) will increase (decrease) as the degree of connectivity in the R&D network increases.

We will now show that this creates a problem when the number of firms $n \geq 6$. We will do this by comparing topologically non-equivalent networks with identical degree connectivity. We will show that, when this is the case, these two R&D networks will be identical in terms of the R&D effort, profit and welfare they generated. The reason this does not occur when $n = 3$ or $n = 4$ is that there are no non-equivalent networks that have the same degree distribution. We were unable to verify whether this was the case for $n = 5$, as there are too many non-equivalent networks.[5]

We now present two examples for the case where there are six possible firms. In these two examples we show that by using the approach in Goyal and Moraga-Gonzalez (2001) for computing effective R&D effort, we generate identical outcomes for pairs of non-equivalent R&D networks.

Example 1 (R&D Networks of Degree Distribution 2) *Consider the following two R&D networks, G_4 and G_5.[6] (Shown in Figure 1). In term of economics, the two networks should lead to different outcomes: Network G_4 models two competing RJVs (Kamien and Zang (1993)), while G_5 models a situation where there are overlapping RJVs.*

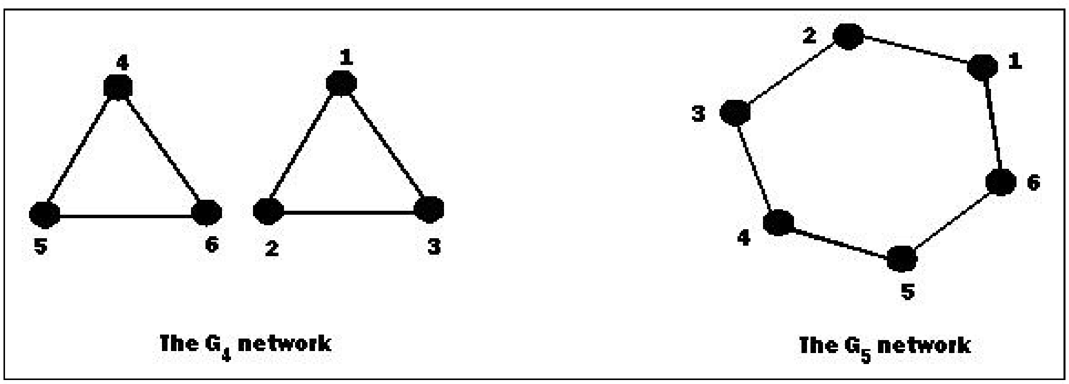

Figure 1: Two non-equivalent networks G_4 and G_5 with the same degree distribution.

From the perspective of network theory, each network has six nodes and six links. Within each network, each node has two links, which implies that each node has the same degree. Hence G_4

[5] The number of non-equivalent networks for $n = 3$, 4 and 5 are 4, 11 and 34. Hence at $n = 5$ there are too many networks to able to explore the full range of possible collaborative structures. Note that when $n = 6$, there will be 156 non-equivalent networks.

[6] Regarding the notation we use in Examples 1 and 2, the reader should consult Figure 10. G_n is a placeholder used to denote one of the networks listed in this figure.

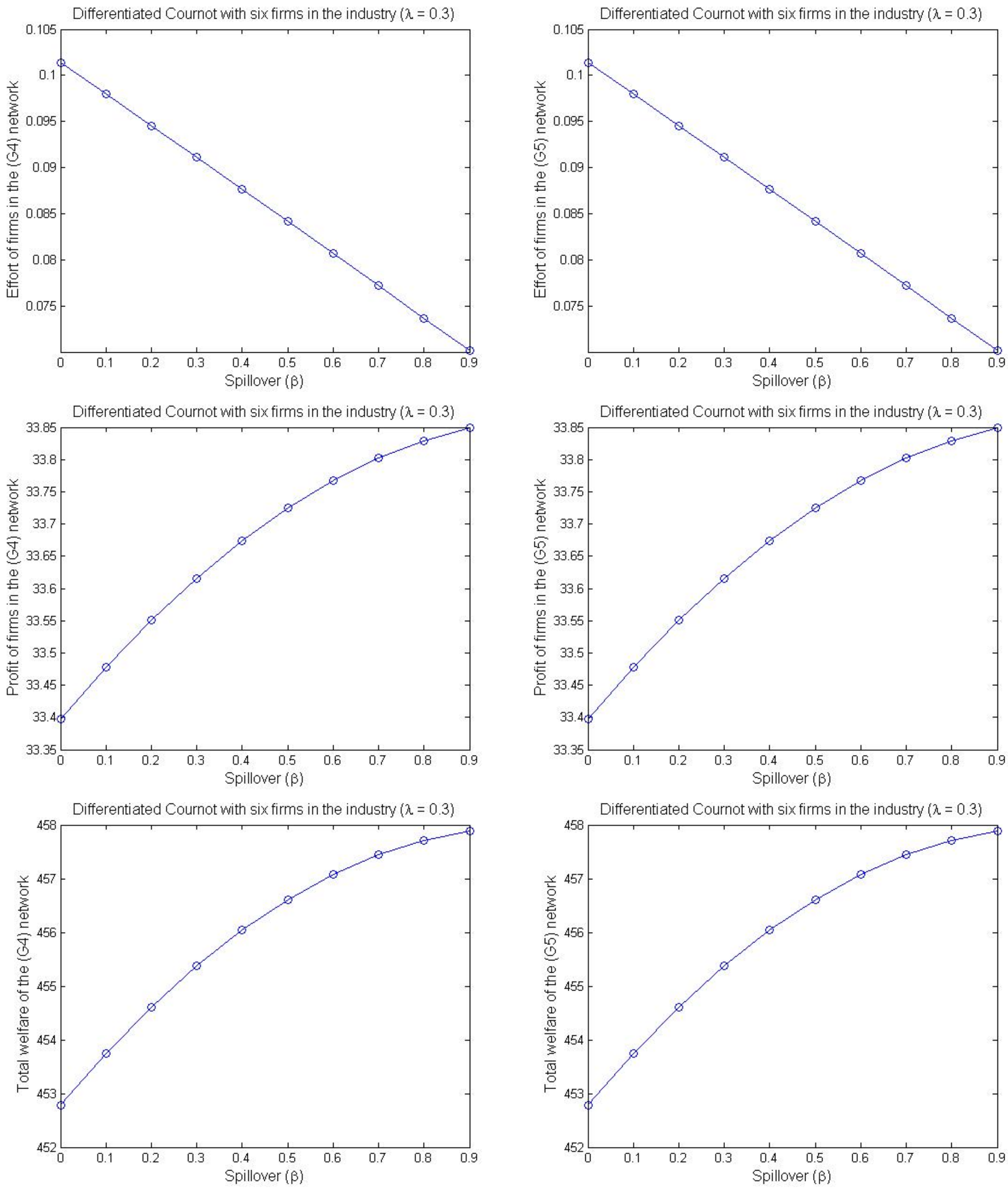

Figure 2: R&D effort, profits and total welfare for networks G_4 and G_5, $\lambda = 0.3$.

and G_5 have the same degree distribution. However, the networks are non-equivalent from the perspective of network theory as G_4 has two cycles and G_5 has one cycle. Figures 2 shows plots that show aggregate R&D effort, firm level profits and total welfare are identical for networks G_4 and G_5

In the next example, we will compare two networks which are non-equivalent and topologically more complex than networks G_4 and G_5.

Example 2 (R&D networks with six firms and degree distribution three) *Consider the following two R&D networks. The first network is called G_{11} and the second is G_{12}. (See Figure 3).*

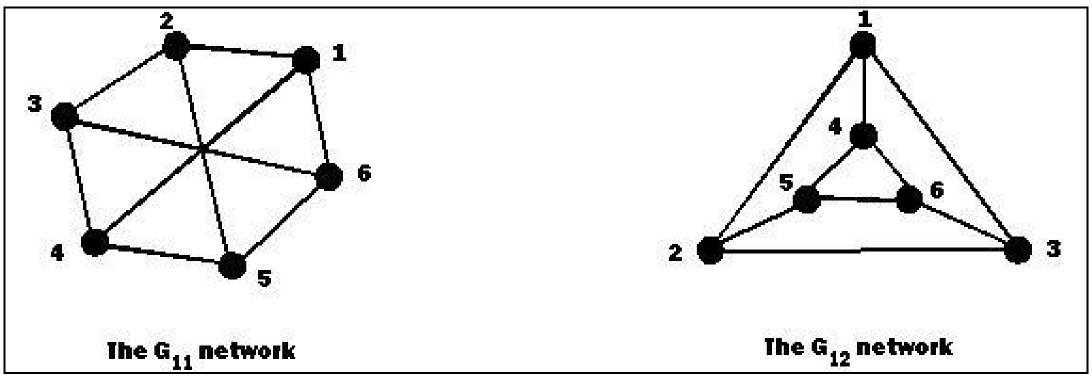

Figure 3: Two non-equivalent networks G_{11} and G_{12} with identical degree and more complex structure.

Both networks have same degree distribution where all firms are of degree three. The two networks are not equivalent because the second network, G_{12}, has two cycles of length three, whereas G_{11} network, which has multiple cycles, has no one cycles of length three. The following three outcomes can be observed (see Figure 4): R&D effort, total profit of firms and total welfare are identical. This is because the degree distribution of these two networks is the same, even though the network topologies differ greatly in all other aspects.

3 The Model

In Goyal and Moraga-Gonzales (2001), effective effort for each firm depends on the number of direct links that firm has to other firms in the network (i.e., its degree of connectivity). If the number of these direct links increases, then its effective R&D effort will grow. However, the degree connectivity can be quite misleading in some situations and because of that we use an idea of shortest path. To overcome this problem, we will make the size of spillover dependent on

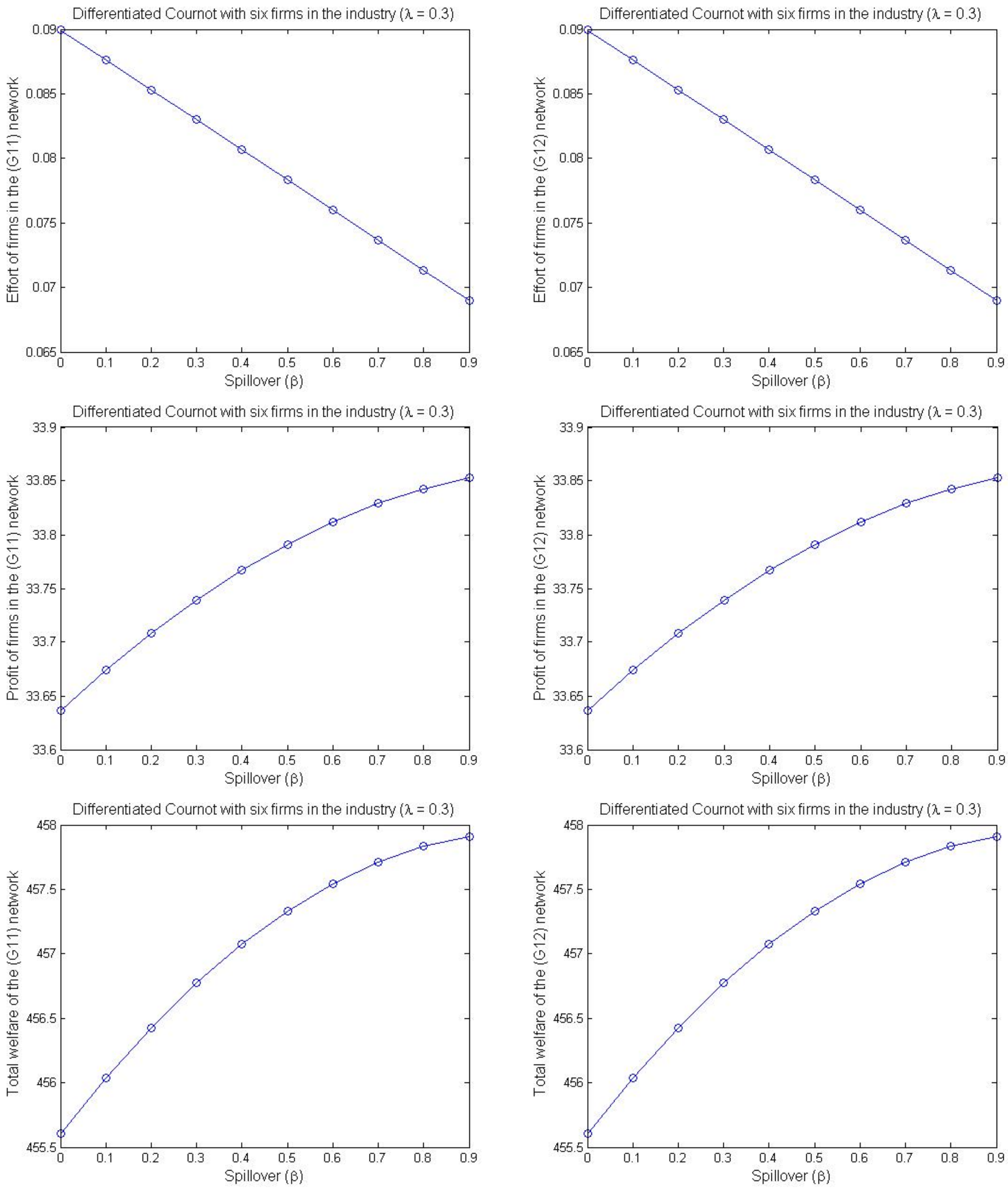

Figure 4: R&D effort, profits and total welfare for networks G_{11} and G_{12}, $\lambda = 0.3$.

distance between any two firms, where we will model distance in the graph theoretic sense (i.e., in terms of the number of hops or links between any two firms (nodes) in the R&D network) (Jackson 2008; Newman 2003). We do this so that knowledge spillovers will be increasing as the relative distance between any two firms in the network decreases. In this way knowledge spillovers will be highest between any two firms when they share the same edge of the network. Since there may be many potential shortest paths between any two firms located in the R&D network, the distance between these two firms will be characterized in terms of the shortest path between them.

In this way the effective R&D effort applied by any one firm can be characterized by the following equation:

$$X_i = x_i + \sum_{j \in N_i} x_j + \beta \sum_{l \notin N_i} T_{il} x_l \tag{3}$$

where

$$T_{il} = \frac{m_{il} C_d(i)}{L_{il}}, \quad i = 1, \ldots, n. \tag{4}$$

Here m_{il} is number of shortest paths between firms i and j, $C_d(i) = deg(i)/(n-1)$ is the degree centrality of the firm i , where $deg(i)$ denotes the degree of node i (i.e., the number links that connect to that node) and L_{il} is length of the shortest path (i.e. smallest number of steps or jumps) between firms i and j. Thus T_{ij} (equation (4)) measures the fall in graph theoretic distance.

We note that T_{ij} has the following properties:

1. The fraction T_{ij} takes its highest value when $m_{ij} = n-2$, $C_d(i) = \frac{n-2}{n-1}$ and $L_{ij} = 2$, where then $T_{ij} = \frac{(n-2)^2}{2(n-1)}$.

2. The fraction T_{ij} takes its lowest value when $m_{ij} = 1$, $C_d(i) = \frac{1}{n-1}$ and $L_{ij} = n-1$ where then $T_{ij} = \frac{1}{(n-1)^2}$.

3. When the fraction T_{ij} is more than one ($T_{ij} > 1$), we assume that the spillover between firms i and j is not restricted $\beta \in [0, 1)$.

4. In disconnected networks, we need to ensure low knowledge spillover between non-connected firms by assuming $T_{ij} = \frac{1}{n(n-1)}$ between non-connected firms.

5. We assume $T_{ij} = \frac{1}{n^2}$ in the empty network to make the knowledge spillover lower in the empty network smaller than in disconnected networks.

Our analysis differs from that of Goyal and Moraga-Gonzales (2001) in the sense that we assume firms sell differentiated products. Hence we assume that the utility function of consumers is given by the following quadratic function:

$$U = a\sum_{i=1}^{n}(q_i) - \frac{1}{2}\left(\alpha\sum_{i=1}^{n}q_i^2 + 2\lambda\sum_{j\neq i}(q_iq_j)\right). \tag{5}$$

Here the demand parameters $a > 0$ denotes the consumers' willingness to pay and $\alpha > 0$ is the diminishing marginal rate of consumption, while q_i is quantity consumed of good i. Without loss of generality, we will assume $\alpha = 1$ to simplify the analysis. The parameter λ captures the marginal rate of substitution between different products.

We use the utility function to find the inverse demand function (D_i^{-1}) by calculating $\frac{\partial U}{\partial q_i} - p_i = 0$ where p_i is the price of the firm producing good i. Then,

$$D_i^{-1} = p_i = a - q_i - \lambda\sum_{j\neq i}q_j \quad i = 1, \ldots, n, \tag{6}$$

where q_i is quantities of goods produced by firm i and λ determines the degree of product differentiation. If $\lambda > (< 0)$, products are substitutes (complements) and if $\lambda = 0$, products are independent.

As in Goyal and Moraga-Gonzales (2001), firms are assumed to be operating in the market with the same constant marginal cost of production, $\overline{c}$. R&D effort level X_i, which is defined in equations (3) and (4), describes how much the marginal cost ($\overline{c}$) of production is reduced as a consequence direct R&D expenditure x_i by firm i and knowledge spillovers generated from other firms in the R&D network. Hence, the marginal cost of production for each firm i, denoted by c_i, is expressed as follows:

$$c_i = \overline{c} - X_i, \quad i = 1, \ldots, n \tag{7}$$

where $\overline{c} \geq X_i$. Upon substituting equation (3) into equation (7), we see that the marginal cost function for firm i takes the following formula

$$c_i = \overline{c} - x_i - \sum_{j\in N_i}x_j - \beta\sum_{l\notin N_i}T_{il}x_l, \quad i = 1\ldots n \tag{8}$$

The effort is assumed to be costly and the function of the cost is quadratic function. so that the cost of R&D is γx_i^2, where $\gamma > 0$ indicates the effectiveness of R&D expenditure. The profit function for firm i is given by this formula

$$\pi_i = \left(a - q_i - \lambda\sum_{j\neq i}^{n}q_j - c_i\right)q_i - \gamma x_i^2 \quad i = 1, \ldots, n \tag{9}$$

The total welfare is given by the following expression

$$TW = \underbrace{\frac{(1-\lambda)}{2}\sum_{i=1}^{n} q_i^2 + \frac{\lambda}{2}\left(\sum_{i=1}^{n} q_i\right)^2}_{Consumer\ surplus(CS)} + \underbrace{\sum_{i}^{n} \pi_i}_{Industry\ profit} \tag{10}$$

The sequence of play between competing firms in this model follows that of Goyal and Moraga-Gonzales (2001) and proceeds as follows:

The first stage: Each firm chooses its research partners. Firms collaborate by forming bilateral (or pairwise) links between themselves and other firms. The firms and the cooperative links together characterize a network of cooperation in R&D.

The second stage: Given the R&D network, each firm chooses the amounts of investment (efforts) in R&D simultaneously and independently in order to reduce the cost of production. The R&D effort across the network determines the effective R&D of each firm.

The third stage: Given the R&D investments of each firm and the effective R&D effort (as determined by the R&D network), firms now compete in the product market by setting quantities (Cournot competition) or prices (Bertrand Competition) in order to maximize their profits.

Under Cournot competition, the equilibrium output and profit for each firm are given by

$$q_i^* = \frac{1}{4+2(n-2)\lambda-(n-1)\lambda^2}\left[(2-\lambda)(a-\bar{c}) + \left(2+\lambda\left((n-2)-\beta\sum_{i\neq j} T_{ji}\right)\right)x_i \right.$$
$$\left. +(2+(n-2)\lambda)\beta K_{-i} - \lambda\left(\sum_{j\neq i} x_j + \beta\sum_{j\neq i} K_{-j}\right)\right], \quad i = 1, \ldots, n \tag{11}$$

and

$$\pi_i^* = \frac{1}{(4+2(n-2)\lambda-(n-1)\lambda^2)^2}\left[(2-\lambda)(a-\bar{c}) + \left(2+\lambda\left((n-2)-\beta\sum_{i\neq j} T_{ji}\right)\right)x_i \right.$$
$$\left. +(2+(n-2)\lambda)\beta K_{-i} - \lambda\left(\sum_{j\neq i} x_j + \beta\sum_{j\neq i} K_{-j}\right)\right]^2 - \gamma x_i^2 \quad i = 1, \ldots, n, \tag{12}$$

where $K_{-i} = \sum_{j\neq i}^{n} T_{ij}x_j$ denotes the effort of all of the other $n-1$ firms in the R&D network, with the excluding firm i.

Under Bertrand competition, the equilibrium output and profit for each firm are given by

$$q_i^* = \left(\frac{1+(n-2)\lambda}{1+(n-2)\lambda-(n-1)\lambda^2}\right)\left[\frac{(1-\lambda)((2(n-2)+1)\lambda+2)(a-\bar{c})+Y_1x_i+Y_2\beta K_{-i}}{(2n-3)(n-3)\lambda^2+6(n-2)\lambda+4}\right.$$
$$\left. - \frac{\lambda(\lambda(n-2)+1)\left[\sum_{j\neq i}x_j+\beta\sum_{j\neq i}K_{-j}\right]}{(2n-3)(n-3)\lambda^2+6(n-2)\lambda+4}\right] \quad (13)$$

and

$$\pi_i^* = \left(\frac{1+(n-2)\lambda}{1+(n-2)\lambda-(n-1)\lambda^2}\right)\left[\frac{(1-\lambda)((2(n-2)+1)\lambda+2)(a-\bar{c})+Y_1x_i+Y_2\beta K_{-i}}{(2n-3)(n-3)\lambda^2+6(n-2)\lambda+4}\right.$$
$$\left. - \frac{\lambda(\lambda(n-2)+1)\left[\sum_{j\neq i}x_j+\beta\sum_{j\neq i}K_{-j}\right]}{(2n-3)(n-3)\lambda^2+6(n-2)\lambda+4}\right]^2 - \gamma x_i^2, \quad i=1,\ldots,n, \quad (14)$$

where $Y_1 = 2+3(n-2)\lambda+(n^2-5n+5)\lambda^2-\lambda(\lambda(n-2)+1)\beta\sum_{j\neq i}^n T_{ji}$ and $Y_2 = 2+3(n-2)\lambda+(n^2-5n+5)\lambda^2$.

It can be seen that the equilibria and equilibrium profits under Cournot and Bertrand competition depend on X_{-i}, the effective R&D effort of firm i's competitors. In our model, this depends on the entire structure of the R&D network, not on the number of direct links as is the case in Goyal and Moraga-Gonzalez (2001). As such, it is not possible to generalize our model to the extent of the analysis constructed in Goyal and Moraga-Gonzalez (2001) for symmetric R&D networks and derive a general formula for equilibrium R&D effort and equilibrium output and prices. However, we will apply our model to illustrate what effect of changes in R&D network topology will have on oligopolies consisting of three and six firms under Cournot and Bertrand competition with differentiated products. This will be done in sections four and five of our paper.

To show that we obtain different results the reader should consult Figures 5. Figure 5 shows a comparison of R&D effort, firm level profit and total welfare for networks G_4 and G_5 (left side) and networks G_{11} and G_{12} (right side). For both pairs of R&D networks in Figures 5 we assumed Cournot competition and used the same parameters, as were used in Examples 1 and 2 that generated Figures 2 and 4, respectively. The only difference was in the way that the spillovers were calculated for firms not directly involved in a RJV. Figures 2 and 4 of Example 1 and 2 were generated from the model of Goyal and Moraga-Gonzales (2001), while Figures 5 uses our model of R&D spillovers.

The reader will immediately that these results for R&D effort, firm level profit and total welfare are different for the comparisons of networks G_4 and G_5 and G_{11} and G_{12}. In fact we later show in Sections 4 and 5, for R&D networks comprised of six firms. Furthermore, R&D effort and profits among firms within each R&D network differs substantially. The only time that

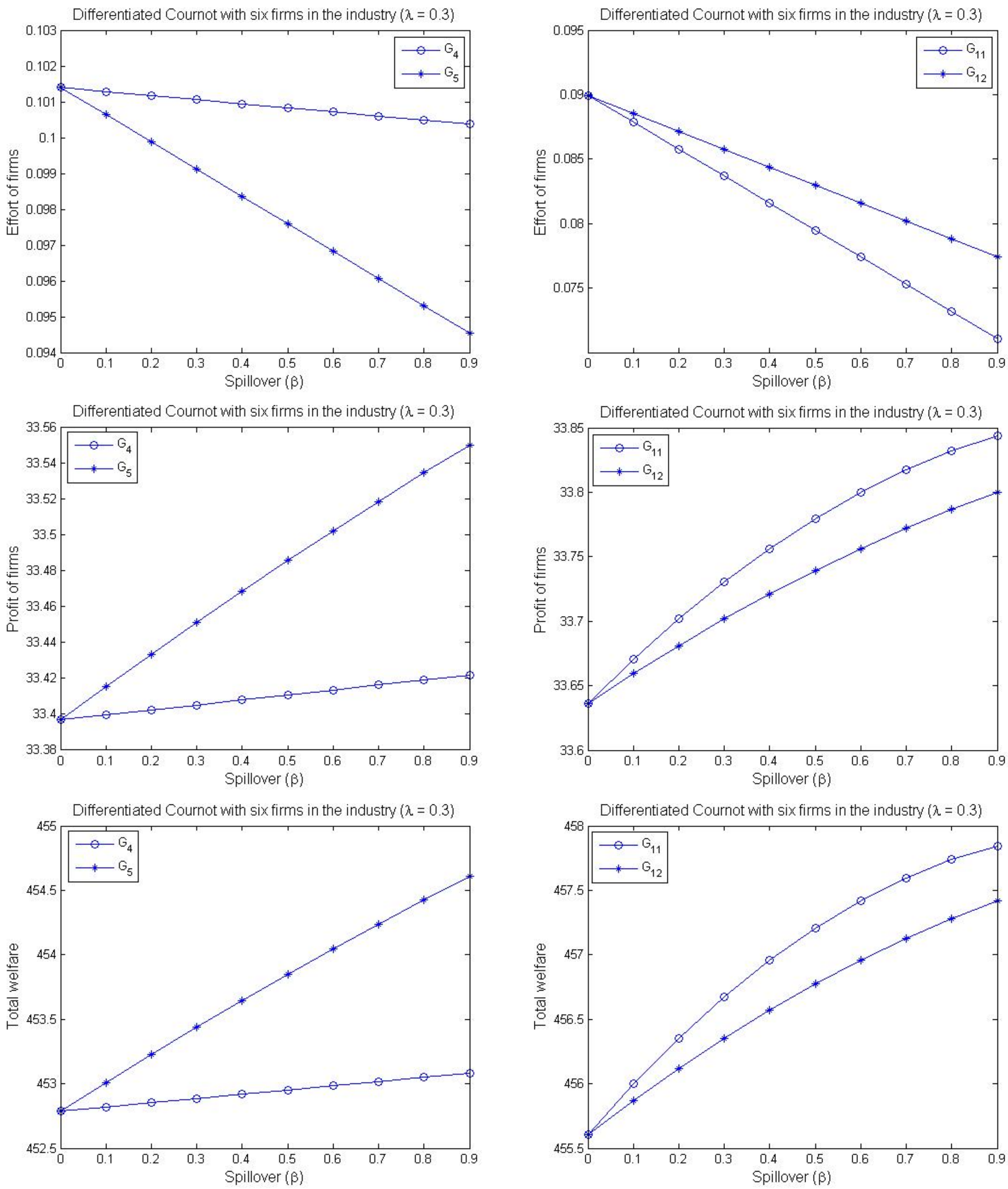

Figure 5: R&D effort firm profits and total welfare for the networks in Example 1 (left hand side) and Example 2 (right hand side).

networks generate symmetric equilibria (in which prices, output, R&D effort and profits are identical) is when an R&D network is regular, i.e. when each node has the same number of links or degree (Jackson 2009, Newman 2003).

4 Cournot competition with product differentiation

In this section, we discuss the Cournot competition. First, we take the case of three firms and then we assume that there are six firms in the market.

4.1 Three firms in the market

When there are three firms in the market, we will have four possible R&D networks. These can be seen below in Figure 6. The network on the top left in Figure 6 is the complete network (which we denote by K_3), while the network on the bottom left is the empty network (denoted by $\overline{K_3}$ or E_3). The star network (S_3), where the hub firm is denoted by h and the two spoke firms are denoted by s is on to top right of Figure 6. The partial network (P_3) where the linked firms and the isolated firm denoted by ℓ and u, respectively, is on the bottom right of Figure 6.

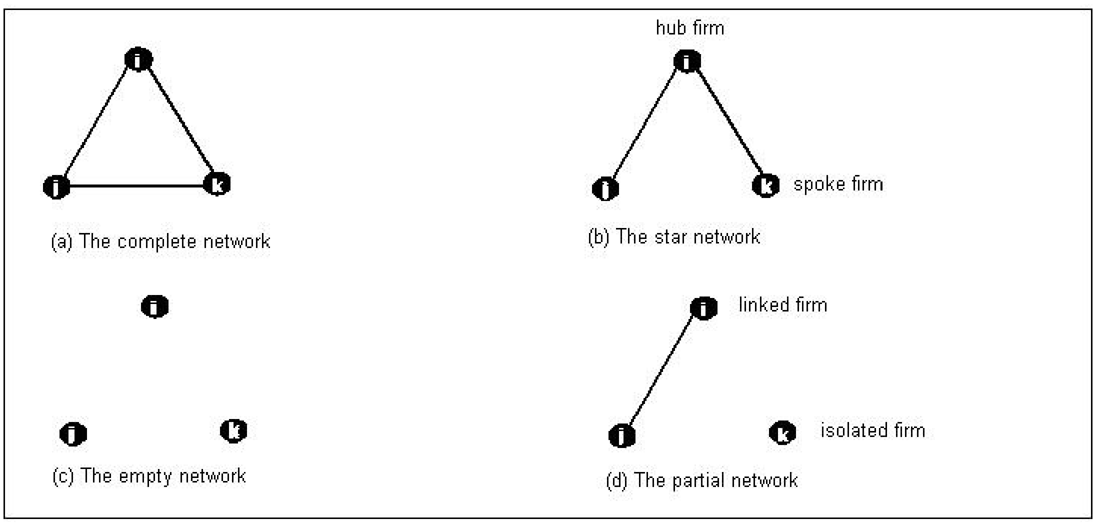

Figure 6: The four distinct R&D networks for a market consisting of three firms.

In this market the inverse demand function for firm i is given by

$$p_i = a - q_i - \lambda\left(q_j + q_k\right), \quad i, j, k = 1, 2, 3. \tag{15}$$

Then, the profit of the firm i is

$$\pi_i = (a - q_i - \lambda (q_j + q_k) - c_i)\, q_i - \gamma x_i^2, \quad i, j, k = 1, 2, 3, \tag{16}$$

where c_i the marginal cost of production is defined as in equation (7). The equilibrium quantity and profit derived for firm i in the product market stage game are derived as follows

$$q_i^* = \frac{(2-\lambda)a - (2+\lambda)c_i + \lambda(c_j + c_k)}{2(2+\lambda-\lambda^2)} \tag{17}$$

and

$$\pi_i^* = \left[\frac{(2-\lambda)a - (2+\lambda)c_i + \lambda(c_j + c_k)}{2(2+\lambda-\lambda^2)}\right]^2 - \gamma x_i^2, \quad i, j, k = 1, 2, 3. \tag{18}$$

The subgame prefect equilibrium firm level R&D effort and quantities are provided below for each of the four possible R&D networks.

1. **The complete network:**

$$x_k^* = \frac{(a-\bar{c})}{4(\lambda+1)^2\gamma - 3} \tag{19}$$

and

$$q_k^* = \frac{2\gamma(1+\lambda)(a-\bar{c})}{4(\lambda+1)^2\gamma - 3} \tag{20}$$

2. **The empty network:**

$$x_e^* = \frac{9(a-\bar{c})((2\beta-9)\lambda - 18)}{324\gamma\lambda^3 + (81 - 972\gamma - 4\beta^2)\lambda + 36\beta - 648\gamma + 162} \tag{21}$$

and

$$q_e^* = \frac{162\gamma(a-\bar{c})(\lambda+1)(\lambda-2)}{(324\gamma\lambda^3 + (81 - 972\gamma - 4\beta^2)\lambda + 36\beta - 648\gamma + 162)} \tag{22}$$

3. **The star network:**

   (a) **Hub firms:**

$$x_h^* = \frac{(a-\bar{c})(32\gamma\lambda^3 - 96\gamma\lambda^2 + (\beta^2 - 4\beta)\lambda - 8\beta + 128\gamma + 32)}{\lambda K_1 + \lambda^2 K_2 - \lambda^3 K_3 + K_4} \tag{23}$$

and

$$q_h^* = \frac{2\gamma(a-\bar{c})(\lambda+1)(32\gamma\lambda^3 - 96\gamma\lambda^2 + (\beta^2 - 4\beta)\lambda - 8\beta + 128\gamma + 32)}{\lambda K_1 + \lambda^2 K_2 - \lambda^3 K_3 + K_4} \tag{24}$$

(b) **Spoke firms:**

$$x_s^* = \frac{8\gamma(8-\lambda\beta)(a-\bar{c})(2+\lambda-\lambda^2)}{\lambda K_1 + \lambda^2 K_2 - \lambda^3 K_3 + K_4} \tag{25}$$

and

$$q_s^* = \frac{64\gamma^2(a-\bar{c})(\lambda+1)^2(\lambda-2)^2}{\lambda K_1 + \lambda^2 K_2 - \lambda^3 K_3 + K_4} \tag{26}$$

where $K_1 = 4\beta^2\gamma - \beta^2 - 16\beta\gamma + 4\beta + 1024\gamma^2$, $K_2 = 4\beta^2\gamma + 128\gamma^2 + 224\gamma$, $K_3 = 32\gamma + 16\beta\gamma + 640\gamma^2$, and $K_4 = 8\beta - 256\gamma - 32\beta\gamma + 512\gamma^2 - 128\gamma^2\lambda^4 + 128\gamma^2\lambda^5 - 32$.

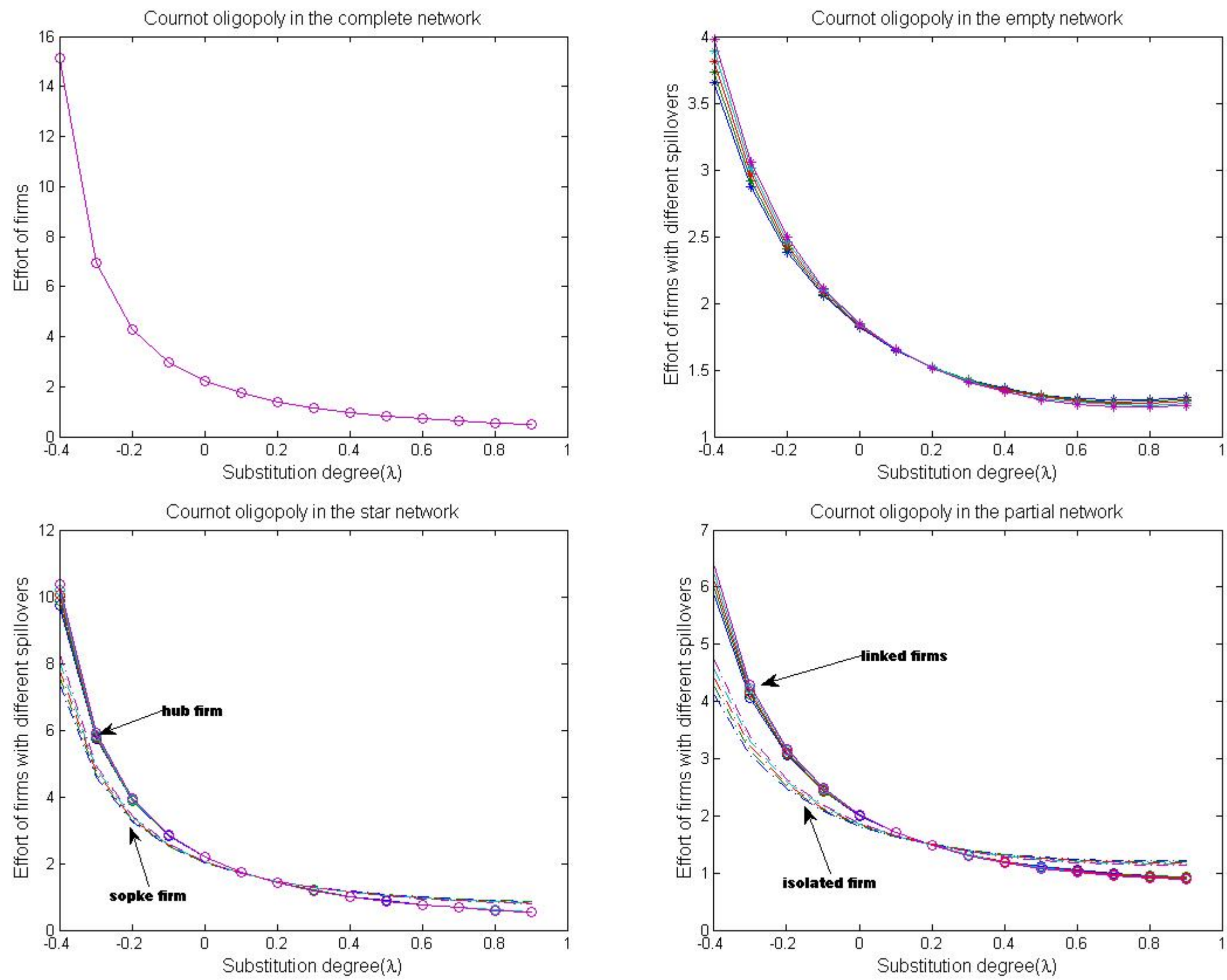

Figure 7: R&D effort as a function of degree of substitution (λ) for individual firms with in each of the four R&D networks

4. **The partial network:**

   (a) **Linked firms:**

$$x_\ell^* = \frac{3(\beta\lambda - 12)(a-\bar{c})(36\gamma\lambda^3 - 108\gamma\lambda^2 + \lambda(9\beta - \beta^2 - 18) - 6(6-\beta-24\gamma))}{\lambda M_1 + \lambda^2 M_2 + \lambda^3 M_3 + \lambda^4 M_4 + M_5} \tag{27}$$

   and

$$q_\ell^* = \frac{36\gamma(a-\bar{c})(\lambda^2 - \lambda - 2)(36\gamma\lambda^3 - 108\gamma\lambda^2 + \lambda(9\beta - \beta^2 - 18) - 6(6-\beta-24\gamma))}{\lambda M_1 + \lambda^2 M_2 + \lambda^3 M_3 + \lambda^4 M_4 + M_5} \tag{28}$$

   (b) **Isolated firms:**

$$x_u^* = \frac{6(a-\bar{c})(\beta\lambda - 3\lambda - 6)(36\gamma\lambda^3 - 108\gamma\lambda^2 + \beta\lambda(6-\beta) + 12(\beta + 12\gamma - 6))}{\lambda M_1 + \lambda^2 M_2 + \lambda^3 M_3 + \lambda^4 M_4 + M_5} \tag{29}$$

   and

$$q_u^* = \frac{36\gamma(a-\bar{c})(\lambda^2 - \lambda - 2)(36\gamma\lambda^3 - 108\gamma\lambda^2 + \beta\lambda(6-\beta) + 12(\beta + 12\gamma - 6))}{\lambda M_1 + \lambda^2 M_2 + \lambda^3 M_3 + \lambda^4 M_4 + M_5} \tag{30}$$

   where $M_1 = 36\beta^2 - 18\beta^3 - 3456\beta\gamma + 648\beta - 31104\gamma^2 + 12960\gamma - 1296$, $M_2 = \beta^4 - 3\beta^3 + 216\beta^2\gamma - 36\beta^2 - 2592\beta\gamma + 108\beta + 15552\gamma^2 - 3888\gamma$, $M_3 = 108\beta^2\gamma + 1296\beta\gamma + 28512\gamma^2 - 1944\gamma$, $M_4 = 432\beta\gamma - 108\beta^2\gamma - 7776\gamma^2 - 648\gamma - 648\gamma$, and $M_5 = 2592\gamma^2\lambda^6 + 72\beta^2 - 7776\gamma^2\lambda^5 - 20736\gamma^2 + 15552\gamma - 2592$.

We now provide a numerical example for an R&D network consisting of three firms. We assume that $a = 120$ and $\bar{c} = 100$. R&D effort, profit of firms and total welfare are plotted in the following figures. When $\lambda \in [-0.4, 1)$, we find that $\gamma = 3$ is sufficient to obtain non-negative results for all variables in all of the networks.[7] Figure 7 plots firm level R&D effort against the substitutability parameter λ. The first thing to observe is that for all R&D networks, there is a negative relationship between R&D effort of individual firms and λ the degree of substitutability between products, with the highest level of R&D conducted by firms selling complementary products.

[7] Values of γ must be large enough to find non-negative results for all economic variables. For this to be achieved in the case of the complete network and others are similar, the following two conditions must hold:

1. The effort of firms in the complete network must be non-negative which means that from equation (19), we have $\gamma > \frac{3}{4(1+\lambda)^2}$.
2. The cost function equation of K_3 is given by $c^* = \frac{4(\lambda+1)^2\gamma\bar{c} - 3a}{4(\lambda+1)^2\gamma - 3}$, from this we have $\gamma > \frac{3a}{4(1+\lambda)^2\bar{c}}$.

To obtain non-negative results for independent products (i.e $\lambda = 0$), γ should satisfy this condition $\gamma > max(\frac{3a}{4\bar{c}}, \frac{3}{4})$

The second thing to note from Figure 7 is that, with the exception of the complete R&D network, K_3, the size of each firm R&D effort depends on their location within the R&D network relative to other firms contained within this network. This is a direct consequence of the R&D network being asymmetric and emerges because of the way in which R&D spillovers are formulated in our model. A natural question to ask is what degree do the R&D investment decision of one firm impact on other firms' investment decisions within the R&D network and whether this is influenced by the topology of the R&D network and firms' location within it?

Figure 8 shows that, for our numerical example, when firms sell either complementary or independent goods (i.e, when $\lambda \leq 0$), an increase in R&D investment by any firm will always lead to other firms' raising their R&D efforts ($\partial x_i/\partial x_j > 0$ for all firms i and j). This relationship holds for all R&D networks. When $\lambda > 0$ and small and the spillover β is high, then for the empty network $\bar{K}_3$, the star network S_3 and the partial network P_3 we also find that $\partial x_i/\partial x_j > 0$ for all firms i and j. However, for other values of λ and β, the R&D investment decisions of firms that are not linked will always have a negative impact on the R&D effort other firms in the network.

Specifically we find for the star network S_3 that spoke firms $\partial x_j s/\partial x_k s < 0$ and for the partial network P_3 that $\partial x_\ell/\partial x_u < 0$). For the empty network $\overline{K_3}$ this negative relationship holds for all firms. We find that when firms are linked, so that are participating in a RJV with some or all firms in the market, than for all $\lambda > 0$, participating firms encourage each other to invest in R&D effort. As a general rule, λ is positive and not close to zero ($\lambda > 0.1$), R&D effort of firms declines with an increase in the number of collaborative links between firms. This would be expected as R&D spillovers are higher when the number of connections increases across the network.

As a consequence we can say that when goods are substitutes firms are more willing to free ride off the R&D effort invested by other firms. However, we find that the degree to which this happens depends on the size of the spillover parameter β. We find that when the spillover β increases in substitute goods (i.e. $\lambda > 0$), the efforts of all firms declines (i.e., $\partial x_i/\partial \beta < 0$). This relationship holds for each of the three networks mentioned above[8]. Although the effect of the R&D spillover is negative for substitute goods, it impacts positively for complementary and independent goods ($\lambda \leq 0$), where the incentive of firms to invest in R&D increases with increasing the spillover ($\partial x_i/\partial \beta > 0$ for each firm i).

Figure 9 shows that the profit of firms increase with respect to the number of the cooperative links and that this holds for all values of λ that makes the complete network stable network (consistent with Goyal and Morga-Gonzales (2001)). When $\lambda < 0.1$ the profits of firms in the complete R&D network K_3 dominate the profits earned by firms in all other networks. We

[8]Note that the R&D effort of firms in the complete network remains constant with increasing the spillover as $X_i = x_i + x_j + x_k$ for all i, j, k.

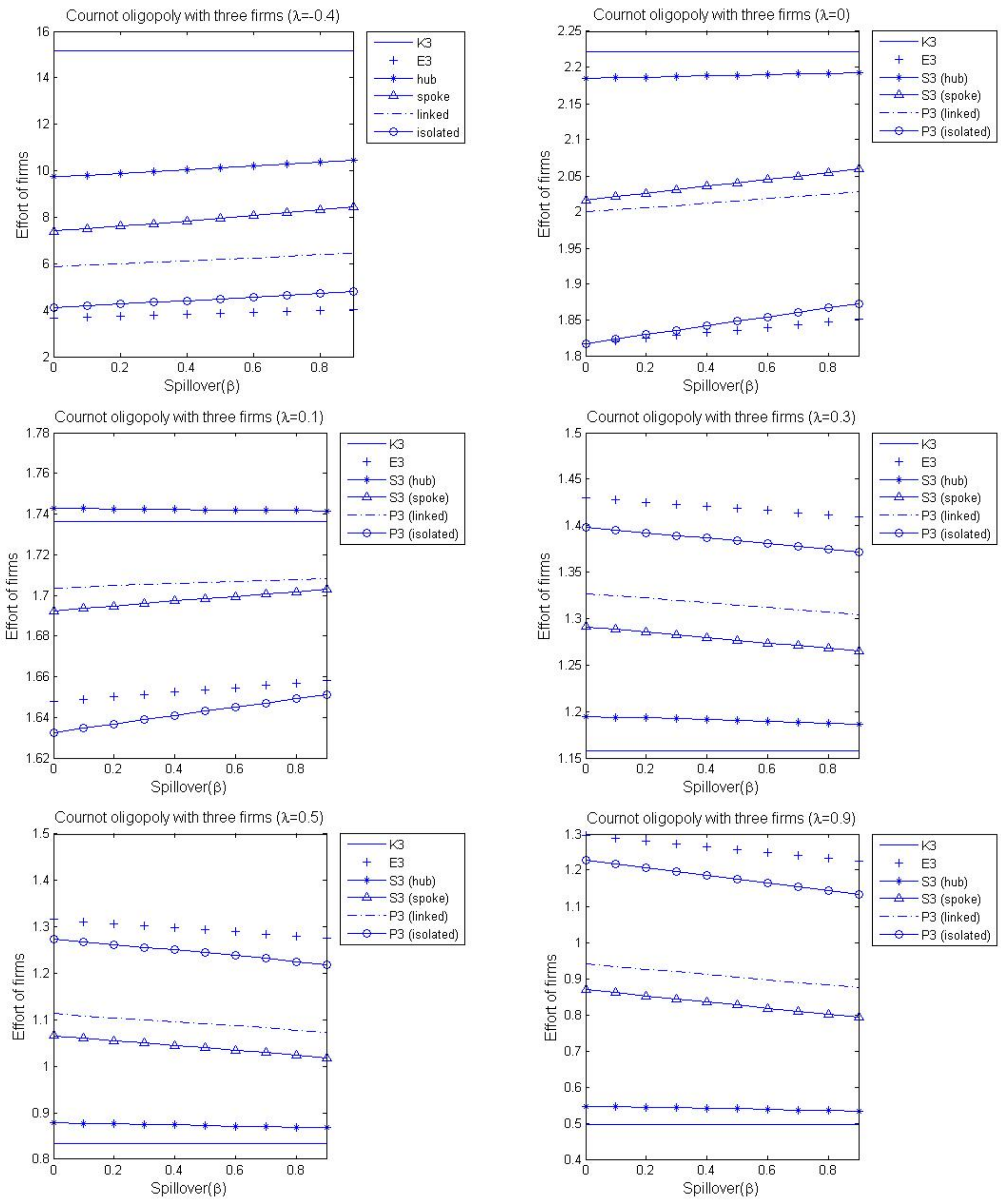

Figure 8: R&D effort of the four networks for $\lambda = -0.4, 0, 0.1, 0.3, 0.5, 0.9$, respectively.

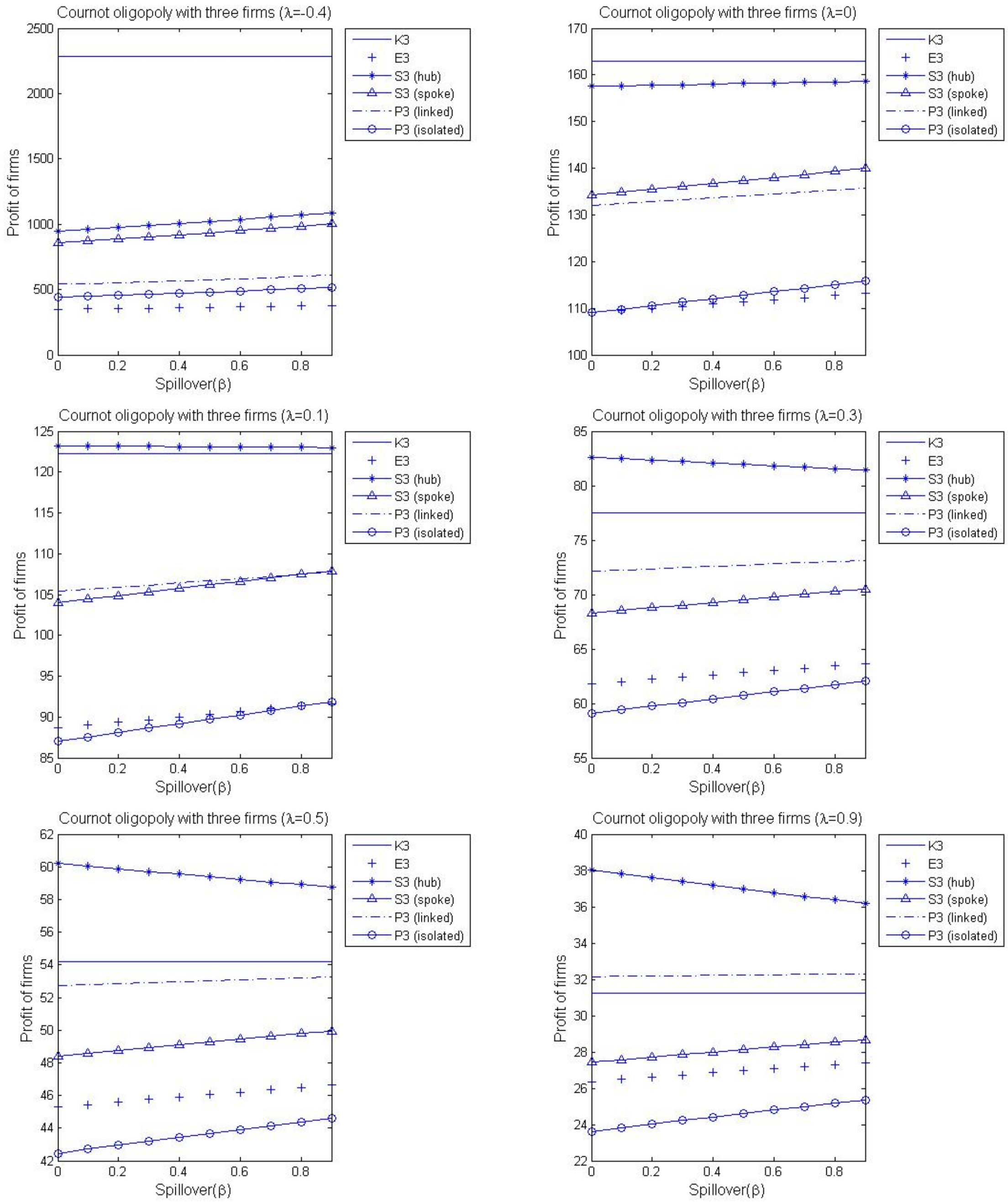

Figure 9: Profit of firms in the four networks for substitution degree $\lambda = -0.4, 0, 0.1, 0.3, 0.5, 0.9$, respectively.

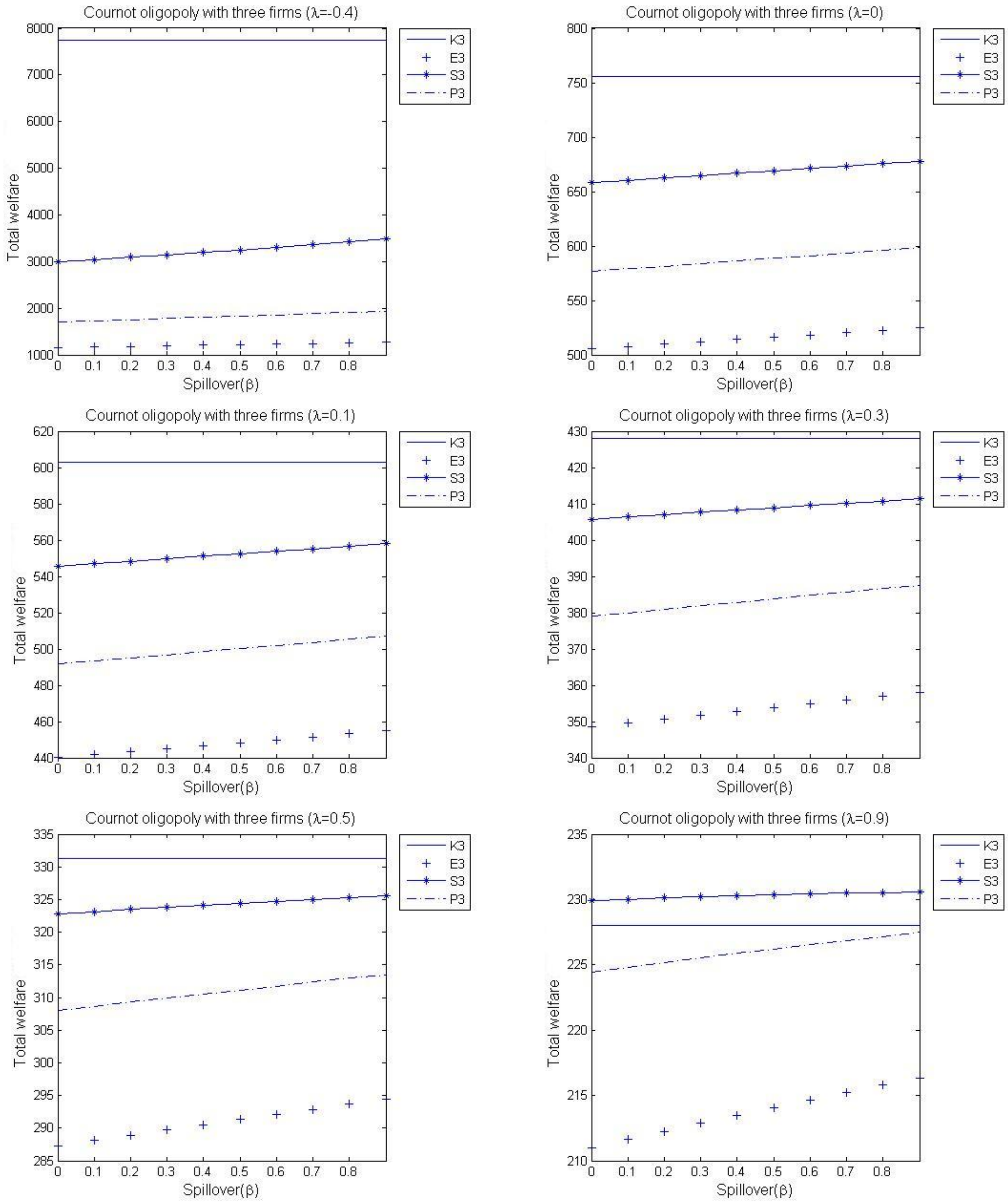

Figure 10: Total welfare of firms in the four networks for substitution degree $\lambda = -0.4, 0, 0.1, 0.3, 0.5, 0.9$, respectively.

also find that in this range of λ that the spillover parameter β has a positive effect on profits. For other values of $\lambda > 0.1$ the hub firm, within the star network S_3, has profits which are the highest. This implies that for substitute goods firms prefer to have central position in the R&D network since then the center (hub) firm can benefit of accessing all information and gain a higher profit. However, within this range of λ we find for the state network S_3 that as the spillover parameter β increases, the profits of the hub firm will decrease while the profits for the spoke firm will increase. For all other networks and for all values of $\lambda > 0.1$ we find that profits increase as $\beta > 0$.

The profits shown for the firms in the complete network K_3 and the star network S_3 suggest that K_3 may be unique stable network. For any R&D network to be stable it requires that the following two conditions be satisfied:

1. For any two firms $ij \in G$, $\pi_i(G) \geq \pi_i(G - ij)$ and $\pi_j(G) \geq \pi_j(G - ij)$
2. For any two firms $ij \notin G$, if $\pi_i(G) < \pi_i(G + ij)$ then $\pi_j(G) > \pi_j(G + ij)$

Here we use G to denote a generic R&D network, where $G - ij$ is the network resulting from the network G after deleting the link ij and $G + ij$ is the network resulting from the network G after adding the link ij.

From this definition, we can say that the network G is stable if no firm can obtain higher profit from deleting one of their links and no other two firms benefit from adding a link between them with one benefiting strictly. We can see that the second condition of the stability of networks is satisfied at once, since we can not add another link to the network K_3. Therefore, let us see the first condition by removing the link between firms i and j.

We find that $\pi_i(K_3) \geq \pi_i(K_3 - ij)$ and $\pi_j(K_3) \geq \pi_j(K_3 - ij)$ for all i and j. This is clear since the profits of firms increase with growing the collaborative links and this means the complete network is stable network. To see the uniqueness, let $G \neq K_3$ to be a stable network. Then, G is different from the complete network by at least one link between firm i and firm j i.e ij. By adding this link, we have $G + ij = K_3$ and then, we find that $\pi_i(K_3) > \pi_i(G)$ and $\pi_j(K_3) > \pi_j(G)$. This yields that the network G is not stable and the complete network K_3 is the unique stable network.

In terms of network efficiency, Figure 10 shows that the size total welfare for each of the four R&D networks depends on the λ. For complementary and independent goods ($\lambda \leq 0$), the total welfare grows as the number of cooperative links between firms increases, making the complete network K_3 the unique efficient network. For substitute goods, this is true if the substitution degree is not high ($\lambda < 0.9$). For $\lambda \geq 0.9$, the star network S_3 becomes the most efficient network.

4.2 Six firms in the market

In previous part with three firms, it is simple to consider all different networks. With six firms, it is not possible to do this. Therefore, we consider some significant networks that could represent the structure of the market in R&D. These are shown in Figure 11.

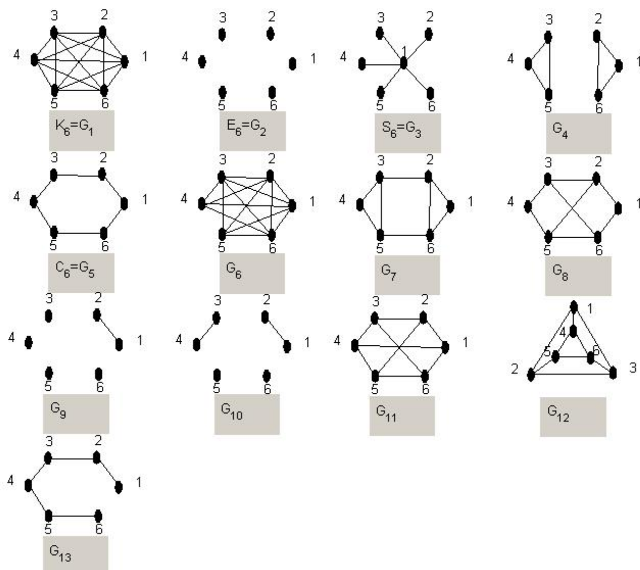

Figure 11: Some possible distinct networks that could generated from six firms.

In the following, we summarize the most important results for the case of differentiated Cournot oligopoly with six firms:

Efforts of firms in R&D:

Impact of the standard of the competition on the effort of firms:

The effort of firms in R&D in the complete network declines with increasing the strength of the competition among firms. This is true for other networks in particular the expenditure of firms on R&D is higher (lower) for less (more) competitive market if the spillover is high.

Impact of the collaborative links on the effort of firms:

1. If the quantities are strategic complements:
   (a) The effort of firms increases with respect to the cooperative links or we can say the effort of firms increases with degree of firms.
   (b) Firms in more dense network invest the highest in R&D whereas firms in networks with no cooperative links spend the lowest.

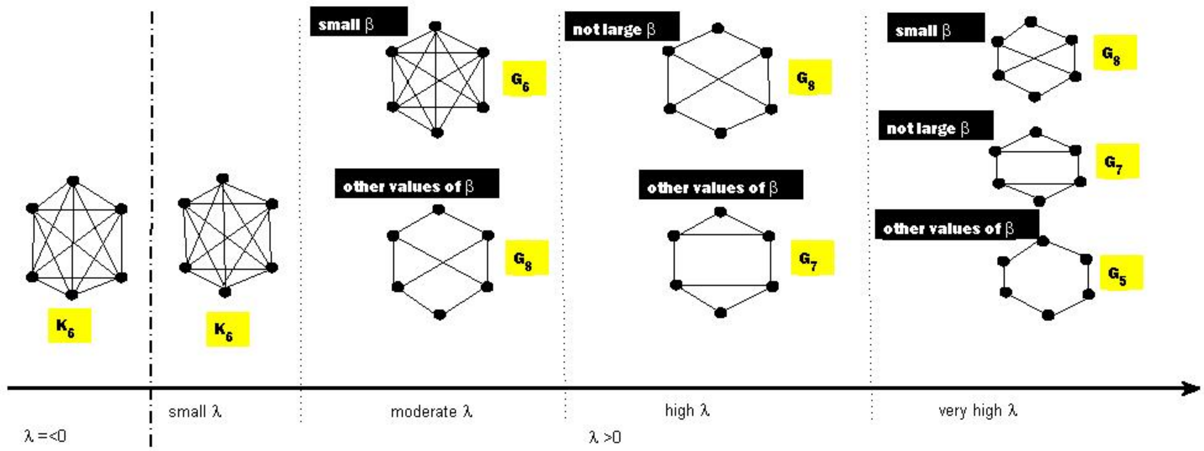

Figure 12: Efficient networks among the list of studied R&D networks for six firms under Cournot competition.

2. If the quantities are strategic substitutes:
   (a) The effort of firms declines with growing the cooperative links.
   (b) Firms in more dense network invest the lowest in R&D whereas firms in networks with no collaborative links have the highest R&D effort.

The impact of collaborative firms on outside firms:

1. The R&D effort of neighbors of cooperative firms goes up with growing the collaborative links if the quantities are strategic complements.
2. If quantities are strategic substitutes, the effort of neighbors of cooperative firms declines with growing the collaborative links.

Results in pertaining to regular networks:

Firms in the regular networks such as in the complete network G_1 and cycle network G_5 lead to symmetric equilibria among firms, so output, prices, R&D effort and profits are identical for all firms at the sub game perfect equilibrium. This is because regular networks have identical degree for all nodes. As regular networks symmetric this implies for both Goyal and Moraga-Gonzales (2001) and our paper, the R&D effort will be identical across all firms with a given R&D network at the equilibrium.

Stability of networks:

The more dense network i.e the complete network (K_6) is the unique stable network for all differentiated goods.

Efficiency of networks: (See Figure 12)

1. If the quantities are strategic complements, the dense network i.e the complete network is the efficient network.
2. If the quantities are strategic substitutes in Cournot competition, the efficient network differs with respect to values of the substitution degree and the spillover as a following:
   (a) If the substitution degree is small, the dense network is the efficient network
   (b) If the substitution degree is moderate (eg. $\lambda = 0.5$) and the spillover is small ($\beta \leq 0.1$), then G_6 is efficient. For other values of the spillover network G_8 is the efficient.
   (c) If the substitution degree is high (eg. $\lambda = 0.7$) and spillover is not large ($\beta \leq 0.5$), then network G_8 is efficient. For other values of the spillover, network G_7 is efficient.
   (d) If the substitution degree is very high (eg. $\lambda = 0.9$) and the spillover is small ($\beta \leq 0.35$), then G_8 is efficient. For intermediate spillover levels ($0.35 < \beta < 0.7$) G_7 is efficient and for high spillovers G_5 network is efficient.

5 Bertrand competition with product differentiation

In this section, we discuss Bertrand competition. We begin by examining the case where the size of the market is three firms and then we assume that there are six firms. The R&D networks for the three and six firm models we are as given in Figures 6 and 11, respectively.

5.1 Three firms in the market

Firstly, the inverse demand function for each of the three firms i, j and k is given by

$$p_i = a - q_i - \lambda q_j - \lambda q_k, \quad i,j,k = 1,2,3 \tag{31}$$

This then yields the demand function for firm i

$$q_i = \frac{(1-\lambda)a - (1+\lambda)p_i + \lambda(p_j + p_k)}{(1-\lambda)(2\lambda+1)}, \quad i,j,k = 1,2,3. \tag{32}$$

By substituting q_i into the profit function for firm i, this yields the profit function for firm i:

$$\pi_i = (p_i - c_i)\left[\frac{(1-\lambda)a - (1+\lambda)p_i + \lambda(p_j + p_k)}{(1-\lambda)(2\lambda+1)}\right] - \gamma x_i^2, \quad i,j,k = 1,2,3. \tag{33}$$

To find the reaction function of the price for firms i, we calculate $\frac{\partial \pi_i}{\partial p_i} = 0$ to get

$$p_i = \frac{(1-\lambda)a + \lambda(p_j + p_k) + (1+\lambda)c_i}{2(1+\lambda)}, \quad i, j, k = 1, 2, 3 \tag{34}$$

Note that, the second order condition of maximization yields $-2(1-\lambda)(2\lambda+1)/(1+\lambda) < 0$. This condition is satisfied for substitute and independent goods ($\lambda > 0$ and $\lambda = 0$ respectively) and for complement goods, this condition is satisfied if $\lambda > -0.5$. From this, we let the substitution parameter $\lambda \in [-0.4, 1)$.

Analogously, we can find that for firms j and k and by substitution among the best response functions, we have the equilibrium price for firm i

$$p_i^* = \frac{(1-\lambda)(3\lambda+2)a + (1+\lambda)(\lambda+2)c_i + \lambda(\lambda+1)(c_j + c_k)}{2(3\lambda+2)}, \quad i, j, k = 1, 2, 3. \tag{35}$$

Upon substituting p_i^* into demand function, q_i, and profit function, π_i, we arrive at the equilibrium quantities and profits for firms $i = 1, 2, 3$:

$$q_i^* = \frac{(1+\lambda)}{(1-\lambda)(2\lambda+1)} \left[\frac{(1-\lambda)(3\lambda+2)a + (\lambda^2 - 3\lambda - 2)c_i + \lambda(\lambda+1)(c_j + c_k)}{2(3\lambda+2)} \right] \tag{36}$$

and

$$\pi_i^* = \frac{(1+\lambda)}{(1-\lambda)(2\lambda+1)} \left[\frac{(1-\lambda)(3\lambda+2)a + (\lambda^2 - 3\lambda - 2)c_i + \lambda(\lambda+1)(c_j + c_k)}{2(3\lambda+2)} \right]^2 - \gamma x_i^2 \tag{37}$$

We now want to find the sub game perfect equilibrium R&D effort and quantities for each of the firms in the complete (K_3), empty ($\overline{K_3}$), star (S_3) and partial (P_3) R&D networks:

1. **Complete network:** The equilibrium R&D effort and output for each firm in the complete network are given as follows:

$$x_k^* = \frac{(1-\lambda^2)(a - \bar{c})}{4(2\lambda+1)\gamma - 3(1-\lambda^2)} \tag{38}$$

   and

$$q_k^* = \frac{2\gamma(a - \bar{c})(1+\lambda)}{4(2\lambda+1)\gamma - 3(1-\lambda^2)}. \tag{39}$$

2. **Empty network:** The equilibrium R&D effort and output for each firm is given by

$$x_e^* = \frac{1}{A} \left[9(a - \bar{c})(\lambda+1)(18 + (27 - 2\beta)\lambda - (2\beta+9)\lambda^2) \right] \tag{40}$$

   and

$$q_e^* = \frac{1}{A} \left[162\gamma(3\lambda+2)(a - \bar{c})(\lambda+1) \right], \tag{41}$$

   where $A = 648\gamma - 36\beta - 162 + \lambda^2(1944\gamma + 8\beta^2 - 162) + \lambda(2268\gamma + 4\beta^2 - 72\beta - 405) + \lambda^3(4\beta^2 + 36\beta + 81)$.

3. **Star network:** The optimal effort and output for the hub firm are given by

$$x_h^* = \frac{((\lambda^2-1)(a-\overline{c})(\lambda^3(\beta^2+4\beta-288\gamma-32)+\lambda^2(2\beta^2-8\beta-96\gamma)}{\lambda B_1+\lambda^2 B_2+\lambda^3 B_3+\lambda^4 B_4+\lambda^5 B_5+B_6} + \frac{\lambda(64-20\beta+256\gamma+\beta^2)+(128\gamma-8\beta+32)))}{\lambda B_1+\lambda^2 B_2+\lambda^3 B_3+\lambda^4 B_4+\lambda^5 B_5+B_6} \tag{42}$$

and

$$q_h^* = \frac{(2\gamma(a-\overline{c})(\lambda+1)(\lambda^3(\beta^2+4\beta-288\gamma-32)+\lambda^2(2\beta^2-8\beta-96\gamma)}{\lambda B_1+\lambda^2 B_2+\lambda^3 B_3+\lambda^4 B_4+\lambda^5 B_5+B_6} + \frac{\lambda(20\beta-\beta^2-256\gamma-64)+(8\beta-128\gamma-32)))}{\lambda B_1+\lambda^2 B_2+\lambda^3 B_3+\lambda^4 B_4+\lambda^5 B_5+B_6} \tag{43}$$

The optimal effort and output for each spoke firms is given by

$$x_s^* = \frac{(8\gamma(a-\overline{c})(3\lambda-2\lambda^2-3\lambda^3+2)((\beta-8)\lambda+(\beta+8)\lambda^2-8))}{\lambda B_1+\lambda^2 B_2+\lambda^3 B_3+\lambda^4 B_4+\lambda^5 B_5+B_6} \tag{44}$$

and

$$q_s^* = \frac{(64\gamma^2(3\lambda+2)^2(a-\overline{c})(\lambda-1)(\lambda+1))}{\lambda B_1+\lambda^2 B_2+\lambda^3 B_3+\lambda^4 B_4+\lambda^5 B_5+B_6} \tag{45}$$

where $B_1 = \beta^2-4\beta^2\gamma+80\beta\gamma-2048\gamma^2-20\beta+512\gamma+64$, $B_2 = 2\beta^2-12\beta^2\gamma-1664\gamma^2-480\gamma-32$, $B_3 = 24\beta-8\beta^2\gamma-80\beta\gamma+1920\gamma^2-1184\gamma-96$, $B_4 = 4\beta^2\gamma-2\beta^2+32\beta\gamma+8\beta+2304\gamma^2+96\gamma$, $B_5 = 4\beta^2\gamma-\beta^2+64\beta\gamma-4\beta+544\gamma+32$, and $B_6 = 32\beta\gamma-8\beta-512\gamma^2+256\gamma+32$

4. **Partial network:** The optimal effort and output for the linked firm is given by

$$x_\ell^* = \frac{3(\lambda+1)(a-\overline{c})(\lambda(12-\beta)-\lambda^2(\beta-2)+12)(\lambda(90-21\beta-288\gamma+\beta^2)}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7} + \frac{\lambda^2(36-18\beta+108\gamma+2\beta^2)+\lambda^3(\beta^2-3\beta+324\gamma-18)+(36-144\gamma-6\beta)}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7} \tag{46}$$

and

$$q_\ell^* = \frac{36\gamma(3\lambda+2)(a-\overline{c})(\lambda+1)(\lambda^3(324\gamma-3\beta+\beta^2-18)+\lambda^2(2\beta^2\lambda^2-18\beta+36+108\gamma)}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7} + \frac{\lambda(\beta^2-21\beta-288\gamma+90)+(36-144\gamma-6\beta))}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7}. \tag{47}$$

The optimal effort and output for the isolated firms is given by

$$x_u^* = \frac{6(\lambda+1)(a-\overline{c})(\lambda(9-\beta)-\lambda^2(\beta+3)+6)(\lambda^3(\beta^2+6\beta+324\gamma-72)}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7} + \frac{\lambda^2(2\beta^2-12\beta+108\gamma)+\lambda(\beta^2-30\beta-288\gamma+144)+(72-144\gamma-12\beta))}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7} \tag{48}$$

and

$$q_u^* = \frac{(36\gamma(3\lambda+2)(a-\overline{c})(\lambda+1)(\lambda^3(\beta^2+6\beta+324\gamma-72)}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7}$$
$$+\frac{\lambda^2(2\beta^2-12\beta+108\gamma)+\lambda(\beta^2-30\beta-288\gamma+144)+(72-144\gamma-12\beta)))}{\lambda R_1+\lambda^2 R_2+\lambda^3 R_3+\lambda^4 R_4+\lambda^5 R_5+\lambda^6 R_6+R_7} \quad (49)$$

where $R_1 = 324\beta^2 - 18\beta^3 - 3456\beta\gamma + 648\beta - 114048\gamma^2 + 75168\gamma - 11664$, $R_2 = \beta^4 - 75\beta^3 + 216\beta^2\gamma + 396\beta^2 - 16416\beta\gamma + 2700\beta - 191808\gamma^2 + 110160\gamma - 15552$,$R_3 = 4\beta^4 - 102\beta^3 + 972\beta^2\gamma - 108\beta^2 + 16200\gamma - 26352\beta\gamma - 23328\gamma^2 + 3672\beta - 1296$, $R_4 = 6\beta^4 - 36\beta^3 + 1620\beta^2\gamma - 468\beta^2 - 13392\beta\gamma + 1296\beta + 209952\gamma^2 - 65448\gamma + 9072$, $R_5 = 4\beta^4 + 24\beta^3 + 1188\beta^2\gamma - 216\beta^2 + 3888\beta\gamma + 139968\gamma^2 - 864\beta - 13608\gamma + 2592$, $R_6 = \beta^4 + 15\beta^3 + 324\beta^2\gamma + 3888\beta\gamma + 17496\gamma - 540\beta - 1296$, and $R_7 = 72\beta^2 - 20736\gamma^2 + 15552\gamma - 2592$.

In the following, we give a numerical example of three firms where we assume that $a = 120$ and $\overline{c} = 100$, then, we plot the R&D effort, the profit of firms and the total welfare. For this we found that $\gamma = 4$ is sufficient to obtain non-negative results for all variables. As in differentiated Cournot modeled examined earlier, we focus on three basic aspects related to the effort of firms: R&D effort, profits and welfare.

Firstly we note Figure 13 show that there is a inverse in the relationship between the strength of competition between firms (as characterized by the substitutability parameter λ) and the R&D effort of individual firms. In the complete network, we observe that the optimal R&D effort for each firm declines as the degree of substitution, λ, increases. We also note that this result also holds for the other three R&D and is true for all values of the R&D spillover β. Hence, we can say that the R&D effort of firms depends on the degree of the competition among them, with R&D effort decreasing as products become perfect substitutes. For complementary goods when the substitution degree diverges from zero, the R&D efforts of firms in the complete R&D network is highest. This is also true for other R&D networks whenever the R&D spillovers are high. However, when goods are highly substitutable firms spend the lowest amount on R&D. This tells us that more R&D effort is expected to be between firms that produce complementary products and this also coincides with our findings with respect to Cournot competition.

Figure 14 shows the effect of investment of firms on each other and spillovers. In general the R&D effort of each firm has positive effects on other firms' R&D efforts if firms selling either complementary or independent goods. In this case the R&D effort of each firm encourages other firms to spend more on R&D (i.e $\partial x_i/\partial x_j > 0$ for all firms i and j). In this case, the R&D effort decision of each firm can be regarded called a strategic complement. Also, for complementary and independent goods, an increase in the spillover β will raise the incentive of firms to invest more in R&D (i.e $\partial x_i/\partial \beta > 0$ for each firm i). If firms are producing products that are substitutes, in the complete network firms encourage each other to invest more in R&D. However this relationship changes in networks that are only partially connected, where higher degrees of substitution degree lead to a negative relationship between R&D effort applied by inside and outside firms (i.e $\partial x_s/\partial x_s < 0$ and $\partial x_\ell/\partial x_u < 0$). We also find that there is a significant impact of the spillover parameter β on incentive of firms in R&D, with higher β leading to lower R&D efforts (i.e $\partial x_i/\partial \beta < 0$).

Figure 14 also shows the influence of collaborative links on the R&D efforts of firms. For complementary and independent goods and for highly differentiated substituted goods, the R&D efforts of firms increase as the number of collaborative links in the network increase. However, for substitute goods which are not highly differentiated ($\lambda > 0.1$), we find that the R&D effort of firms declines with the number of cooperative links. Moreover, the existence a direct R&D collaborative link between any two firms will also impact on the R&D efforts of third party firms. We find that when any two firms create a direct link (i.e., form a RJV), then the R&D effort of the third firm will increases if firms sell complement and independent goods. However if firms sell goods that are substitutes, then the R&D effort of the third party firm firms declines if the other two firms decide to form a direct collaborative link.

In terms of the profit of firms (see Figure 15) we find that the profit of firms goes up as the size of the RJV increases. This implies that the complete network, K_3, will dominate all other R&D networks in term of stability. We also observe that since firms prefer to make collaborative links between each other when selling complementary and independent goods. However, when goods are substitutes, they prefer to become the center of the network. In this way they can access all information and gain higher profit. However, the structure of cooperation in the star network, S_3 is not stable because the spoke firms will always want to create a link between themselves and other firms in order to raise their profits. This causes the hub firm's profits to decline. We also find that the size of firm level profits is influenced by the size of the R&D spillover β. For complementary and independent goods, we find that profits increase as the spillover increases. For substitute goods this is true, except for the profit of the hub firm in the star network S_3. which declines as spillovers increase.

We observe in Figure 16 that total welfare increases as the number the collaborative links in the R&D network increases and this is true for all goods, except for when the substitution degree is high. The idea that can be extracted from here is that, except when good are perfect substitutes, the complete network K_3 is the efficient network. However, when the substitution degree is high the star network S_3 is the most efficient. Moreover we find that when the degree of substitution is close to one, the partial network is the most efficient. In addition, the size of the R&D spillovers β impacts on the size of the total welfare generated by each R&D network. For all differentiated goods that are not highly substitute, total welfare increases with increases in the spillovers β. Whereas, if the substitution degree is high, the total welfare in the star network declines.

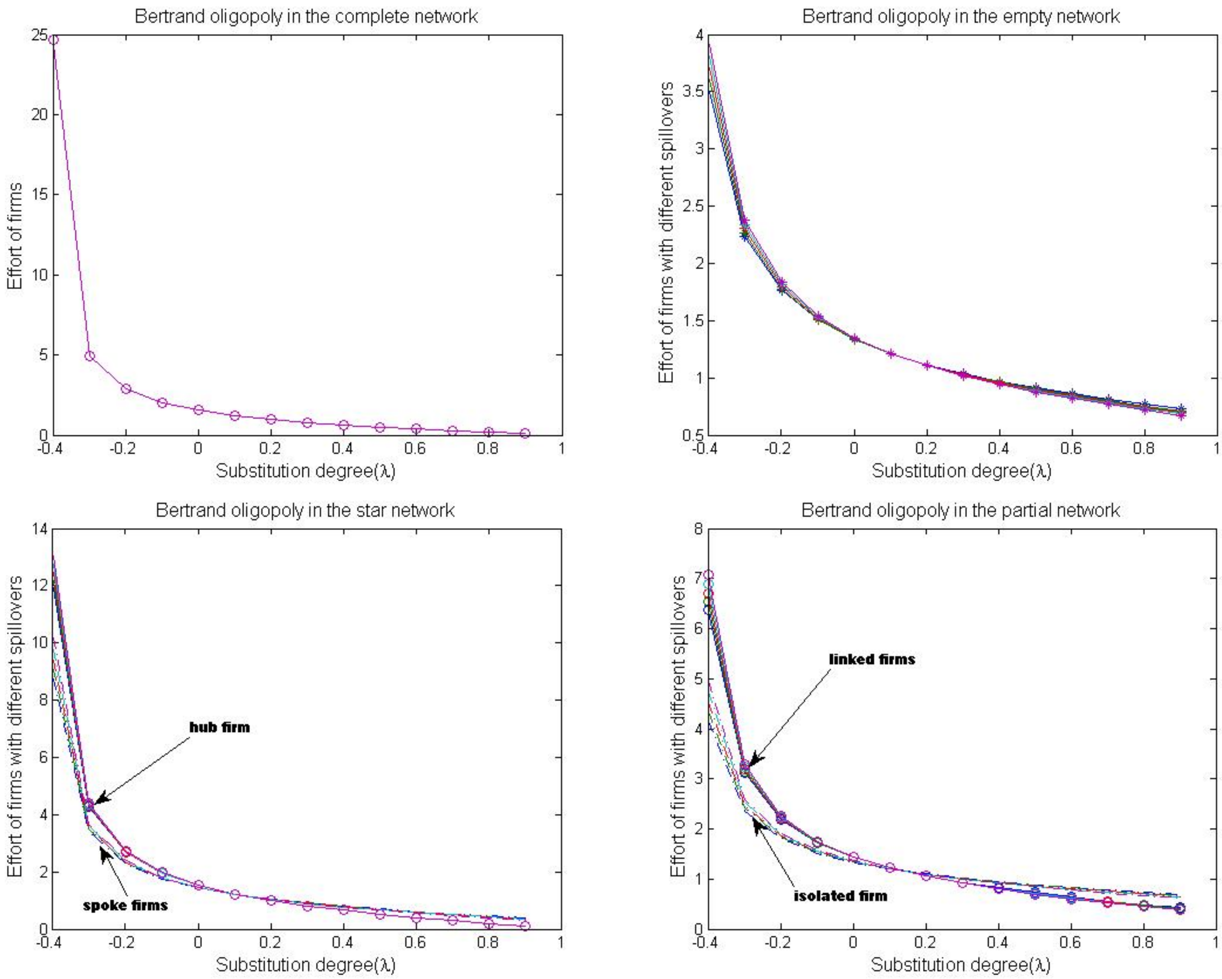

Figure 13: R&D effort of firms in the four networks under Bertrand competition declines with respect to the product differentiation.

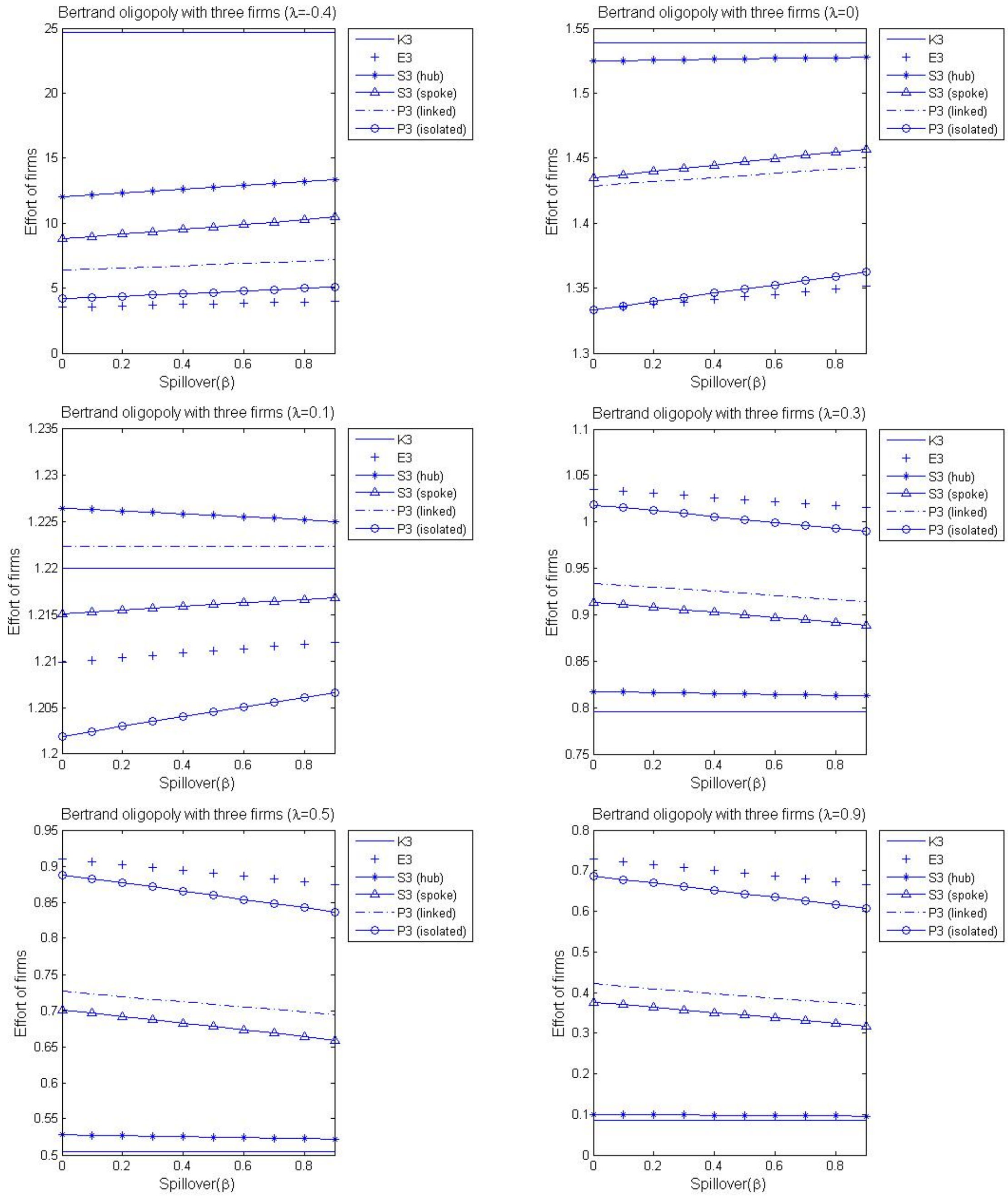

Figure 14: The R&D effort of firms in the four networks for $\lambda = -0.4, 0, 0.1, 0.3, 0.5, 0.9$ respectively.

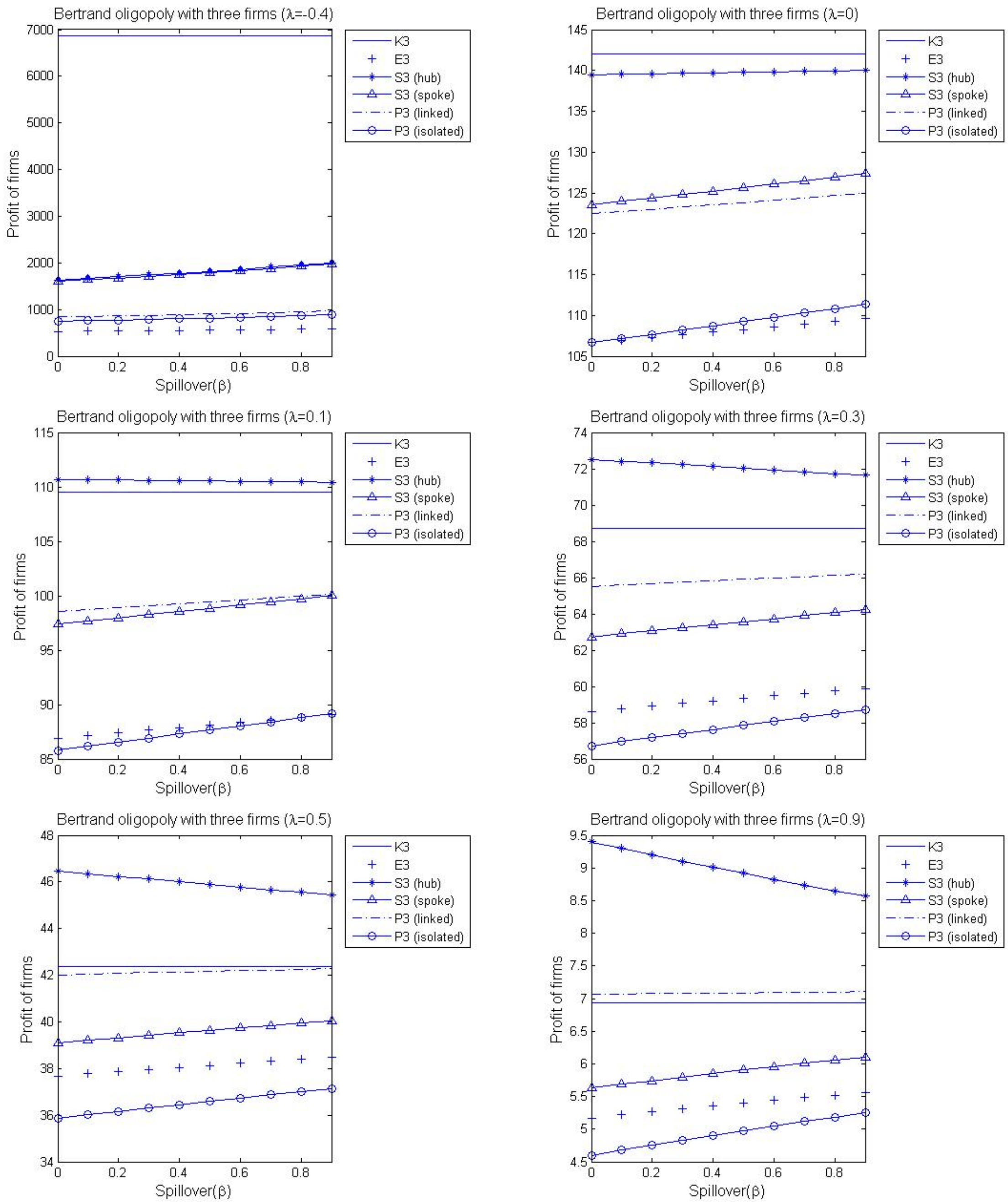

Figure 15: Profit of firms in the four networks for $\lambda = -0.4, 0, 0.1, 0.3, 0.5, 0.9$ respectively.

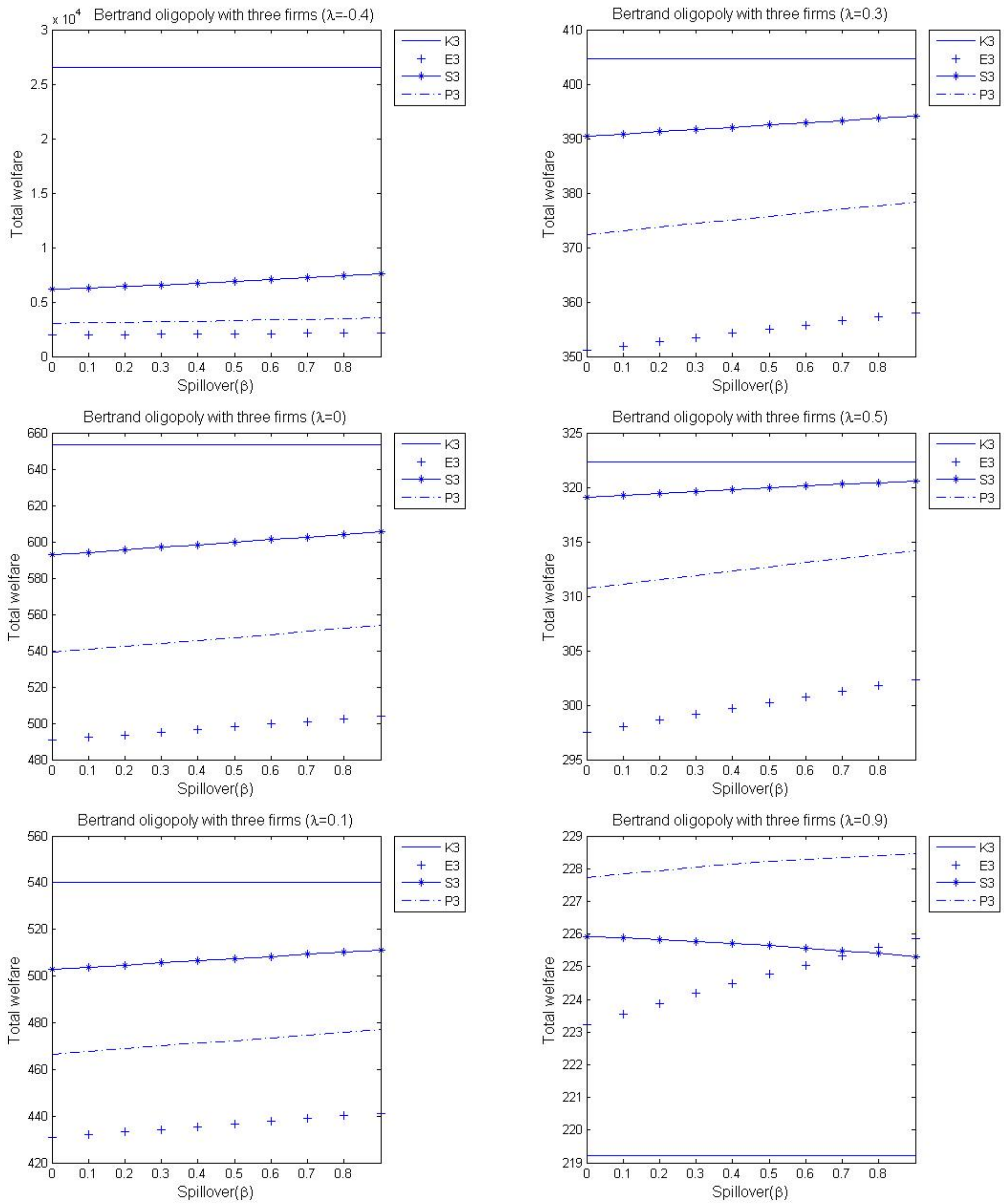

Figure 16: Total welfare in the four networks for $\lambda = -0.4, 0, 0.1, 0.3, 0.5, 0.9$ respectively.

5.2 Results for differentiated Bertrand oligopoly with six firms

Here we summarize our results for asymmetric R&D networks containing six firms for Bertrand competition. As the reader will note most of the results are quite similar to the three firm model and are therefore summarized. The most different results relate to the efficiency of different R&D networks. This is due to the larger number of firms in this market giving rise to greater variety of collaborative structures in this market.

R&D Effort:

Impact of the degree of substitutability on the R&D effort of firms

In the complete network, firms in very weak competitive market spend the highest amount on R&D where the expenditure becomes the lowest in strong competitive market. This is true for other networks as well as the effort of firms is the highest (lowest) for less (more) competitive market if the value of the spillover is high.

Impact of the collaborative links on the R&D effort of firms

1. If prices are strategic substitutes:
   (a) Then the R&D effort of firms increases with respect to the cooperative links or we can say the effort of firms increases with degree of firms.
   (b) Then for more dense R&D network, firms spend the higher amount on R&D
2. If prices are strategic complements:
   (a) For independent goods, we have the same result when the prices are strategic substitutes.
   (b) for substitute goods:
      i. If the substitution degree is not close to zero ($\lambda > 0.1$), then effort of firms declines with growing the cooperative links.
      ii. Firms in the dense network invest the lowest amount in R&D if the substitution degree is not small.

Impact of two collaborative firms on effort of outside firms:

1. If prices are strategic complements, then the R&D effort of neighboring firms (i.e. outsiders - those not participating directly in the RJV) increases as the number collaborative links in the network grow.
2. If prices are strategic complements
   (a) When goods are either independent, R&D effort of neighboring firms increases as collaborative links in the network grow.

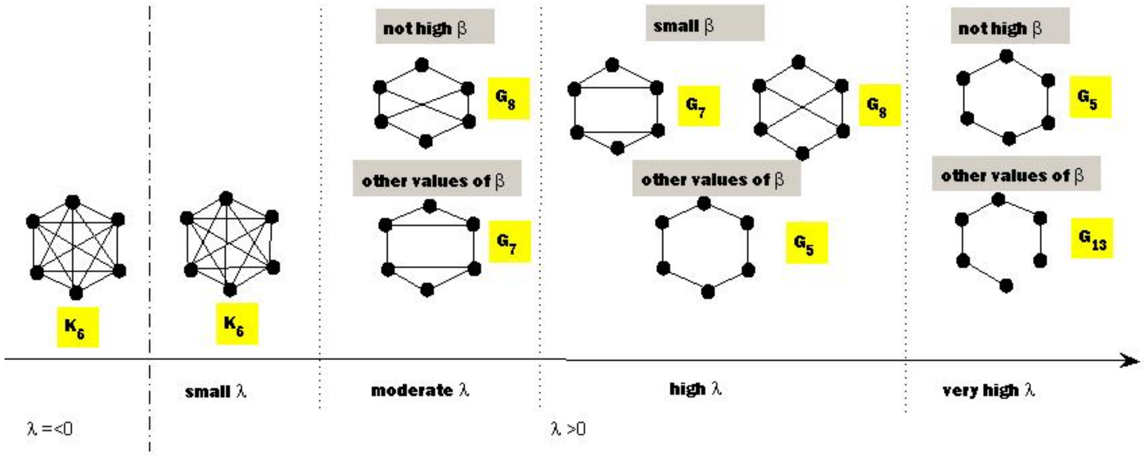

Figure 17: Efficient networks among the list of studied R&D networks for six firms under Bertrand competition.

(b) For substitute goods, the effort of neighboring firms declines as collaborative links in the R&D network increase.

Results for regular networks:

In regular networks, at the sub-game perfect equilibrium all firms have the same output for all economic variables.

Stability of networks:

The dense network i.e the complete network, K_6, is the stable unique network for all substitution degree

Efficiency of networks: (See Figure 17)

1. If prices are strategic substitutes, then the complete network K_6 is the efficient network
2. When prices are strategic complements:
   (a) If the substitution degree is small ($\lambda < 0.2$), then the complete network K_6 is the efficient network.
   (b) If the substitution degree is moderate (eg. $\lambda = 0.5$)
      i. When the R&D spillover is not high, the G_8 network is the efficient.
      ii. For other values of the R&D spillover, the cycle network G_7 is the efficient network.
   (c) If the substitution degree is high, but not close to one (eg. $\lambda = 0.7$)

i. When the R&D spillover is small, the networks G_7 and G_8 are the most efficient.

ii. for other values of the spillover the G_5 is the efficient network.

(d) If the substitution degree is very high (or close to one eg. $\lambda = 0.9$)

i. Then G_5 is the efficient network if R&D spillover is not high.

ii. For other values of the R&D spillover, the network G_{13} is the efficient network.

6 Conclusion

In this section, we conclude our work by writing summary of our results. In studying the profits of firms and total welfare, we found the following. For both strategic substitutes and complements, we observed that firms prefer to make collaborative links with all firms in an industry. This means that the complete network is the unique stable network. This result is consistent with Goyal and Moraga-Gonzalez (2001). For total welfare, we found results differ with the degree of the competition. If firms are not in a strongly competitive market, the complete network is the unique efficient network whereas in strongly competitive market, the star network becomes the efficient. This result differs with Goyal and Moraga-Gonzalez (2001), since the complete and star networks never efficient whereas for the other two networks their relative degree of efficiency depends on the value of the spillover.

Our results are different, depending on strength of the competition among firms. Under Cournot competition with three firms in the market, we found that there is more collaboration between firms that sell complementary products. Also, firms spend the highest value on R&D when they form the complete network if they are in weak competitive market. However, firms in the complete network expend the lowest amount on R&D effort if the competition among firms increases, i.e. firms' products become closer substitutes. By increasing the size of the market to be six firms we found about similar results for effort and profit (stability) of firms. However, in term of the efficiency of networks, the complete network is the efficient network if firms are in weakly competitive market, but by increasing the degree of the differentiation, the efficient network becomes a less connected network. This tell us that the conflict between stability and efficiency does not occur in weakly competitive market.

Under Bertrand competition with three firms, we found similar results to Cournot with respect to R&D effort and the profits of firms. However, in term of the efficiency of the R&D network, we found that the complete network is efficient if firms are not in more competitive market where then the star and the partial network become efficient depending on the value of the degree differentiation. For six firms in the market, we found similar results with Cournot with six firms in terms of the effort and profit of firms. However, in terms of the efficiency of the R&D network, we found they with increasing the competition among firms, the efficient networks becomes the less connected network.

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