Volatility Spillovers in Agricultural Commodity Markets: An Application Involving Implied Volatilities from Options Markets

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Abstract

This article provides a new approach to analyze the issue of volatility spillovers. In particular, we investigate relationships and transmissions between implied volatilities in corn and soybean markets – two of the most important agricultural commodity markets in the United States. Using weekly average data from 2001 to 2010, we estimate a VAR model with Fourier seasonal components as exogenous variables. Results from this model indicate that volatility spillovers exist from the corn market to the soybean market, but there is no volatility spillover from the soybean market to the corn market. Impulse response functions from this model show that a standard positive shock in the implied volatility of corn has a positive impact on responses of the implied volatility of soybeans. However, responses of the implied volatility of corn to a shock in the soybean market are not significant. To examine the time invariance property of this model, we conduct three bootstrap versions of Chow tests (sample-split, break-point, and Chow forecast). All of these tests suggest significant structural break points in several time periods. To improve the accuracy of our model, we develop a threshold VAR model with four regimes that depend on previous levels of volatilities. Results from the threshold VAR model indicate that when both volatilities are relatively low, volatility spills over from the corn market to the soybean market, but when the implied volatility of soybeans is relatively high, volatility spillover effects reveal an opposite direction. Finally, using futures prices, we estimate a BEKK-GARCH model, which is commonly used to investigate volatility spillover effects. Results from the BEKK model show that volatility spillovers exist between the two markets, which is different from what we have found using implied volatilities.

1. Introduction

A volume of research has been performed on patterns of volatility across different commodities, times, and locations. Most of these studies that have investigated the problem of volatility spillovers in commodity markets have applied (multivariate) GARCH models using historical "backward-looking" price data. In this study, we introduce a new approach to analyze the issue of volatility spillovers. In particular, we examine relationships and transmissions between implied volatilities that are derived from options prices.

An implied volatility is calculated by applying an option pricing formula, the most common of which is the Black-Scholes formula. The advantage of using this kind of volatility instead of alternatives that are based on historical or lagged data is that the implied volatility is a forward-looking and market-based measure of price variability and uncertainty, thereby interpreting the market's collective expectation of the future volatility of the price of the underlying asset.

Research on volatility spillovers in agricultural commodity markets has become an important issue for market participants whose production and marketing decisions are often impacted by uncertainty and risks in commodity markets. To date, only a few studies have addressed this topic. Volatility spillovers exist among agricultural commodity markets because most such commodities share common market information, are typically imperfect substitutes in demand, and compete in the usage of some common inputs, such as land and labor. Changes in the volatility of one market will often trigger reactions in other markets. Our intent is to model such interactions. An understanding of the overall market behavior and the transmission of risks and shocks across interrelated markets requires an understanding of these relationships and in particular the mechanism for transmission among different markets. The dynamics of these linkages is also an important indicator of overall market behavior and performance.

In this study, we investigate the volatility spillover effects between corn and soybean markets using implied volatilities derived from nearby options contracts in these two markets. We select these two markets because of their important role in U.S.

agricultural commodity markets. In Section 4.2, we apply a VAR model with Fourier seasonal components as exogenous variables, and impulse response functions to analyze the volatility spillover effects between these two markets. In Section 4.3, we perform three bootstrap versions of Chow tests to test for structural changes in the VAR model, and find significant structural break points around the year 2003, 2006 and 2008. The Energy Independence and Security Act of 2007 may be the cause of those break points around 2006. To improve the performance of our model, in Section 4.4, we estimate a threshold VAR with four regimes that depend on previous levels of volatilities to overcome the structural change problem. Finally, in Section 4.5, we estimate a BEKK-GARCH model, a commonly used model to investigate volatility spillover effects, and compare results with what we have found in Section 4.2.

2. Previous Research

The time-varying volatility, as a measure of risk, has attracted considerable attention since the 1980s due to its importance in analyzing price data and the development of econometric models. Understanding the behavior of volatility is crucial for making market decisions such as hedging strategies and asset location decisions. The time-varying volatility is usually modeled by Engle's (ARCH) models or Bollerslev's generalized autoregressive conditional heteroscedasticity (GARCH) models.

In agricultural economics, researchers started to realize the importance of price volatility and to study the source of price volatility three decades ago. For example, among those early studies, Anderson (1985) investigated determinants of futures price volatilities in eight major agricultural commodity markets. And Streeter and Tomek (1992) performed an integrated study about the futures price volatility in the soybean market. ARCH and GARCH models have proven useful in many studies on agricultural commodity price risks (or volatilities). For instance, Aradhyula and Holt (1990) argued that the application of the GARCH model improved the forecasting accuracy of measuring changes in price risk over time.

As the development of econometric tools for resolving the problem of heteroscedasticity, models have been extended to the multivariate dimension. This new type of multivariate model triggered the popularity of another research topic – volatility spillovers. By definition, examining volatility spillover effects answers the question that how price volatility of one commodity is affected by previous values of price volatilities of other commodities. Understanding the transmission mechanism of price risks between markets is especially important for market participants, producers, researchers, and policy makers. For example, when making policy changes in the market of one commodity, policy makers need to consider how its price volatility spills over to price volatilities of its substitutes through market channels.

Although the topic of volatility spillovers has been extensively discussed in studies of financial markets, very few studies have been done in agricultural commodity markets. Among these few studies, Natcher and Weaver (1999) discussed the transmission of price volatilities in beef markets. Apergis and Rezitis (2003) investigated volatility spillover effects across agricultural input prices, output prices and retail food prices in Greece. Buguk, Hudson, and Hanson (2003) tested volatility spillovers for prices in the supply chain, and found strong evidence of price volatility spillovers from feeding material (corn, soybeans, menhaden) to catfish feed and farm- and wholesale-level catfish prices.

Another type of volatility, the "implied volatility", can also be used to investigate the price variability (or risk). Differing from the historical volatility, an implied volatility is a forward-looking measure of the price variability, and it is calculated from an option pricing formula, such as the Black-Scholes model and the Cox-Ross-Rubinstein binomial model. Given the values of the option price, interest rate, and time to expiration, the option pricing formula relates the option price to the volatility of the underlying asset. To calculate the implied volatility, we need to enter the prices of options premiums into an option pricing model and then solve for the volatility. This type of volatility is the market's estimate of how volatile the underlying futures prices will be from the present until the option's expiration. The question of whether the implied volatility is a good forecast of future volatility was discussed by many researchers during the 1970s and 1980s. Some of them (e.g., Latane and Rendleman1976; Chiras and Manaster 1978; Beckers 1981) suggested that the implied volatility performed better than the historical volatility. Although some researchers found conflicting results, most studies still supported the conclusion that the implied volatility could forecast the future volatility effectively.

Although the implied volatility is widely considered to be a good way to measure the future volatility, very little research on implied volatilities has been done in agriculture. McNew and Espinosa (1994) found that USDA reports have strong impacts on implied volatilities in corn and soybean markets by demonstrating a strong relationship between USDA crop reports and implied volatilities. To examine the importance of implied volatility in agricultural markets, Giot (2003) compared the incremental information content of lagged implied volatility to results from GARCH models. He found that past squared returns only marginally improve the information content provided by the lagged implied volatility, and VaR (Value at Risk) models that rely on lagged implied volatility perform as well as those derived from the GARCH models. A more recent study refers to the work of Isengildina-Massa, Irwin, Good and Gomez (2008). They found that WASDE reports lead to a statistically significant reduction of implied volatility in corn and soybean markets.

3. Methodology

3.1. VAR model

Considering the advantage of implied volatility and the importance of volatility spillovers in agricultural commodity markets, we develop a new method to examine the issue of volatility spillovers. In particular, we estimate a vector autoregressive (VAR) model with Fourier seasonal components as the exogenous variables, using implied volatilities of corn and soybeans.

After testing for stationarity for implied volatilities of corn and soybeans, if they both appear to be stationary in levels, a vector autoregressive model of order p with exogenous variables (VAR(p)) can be conducted. The VAR(p) model is shown as follows:

$$y_t = a + \sum_{i=1}^{p} \mathrm{B}_i y_{t-1} + Dx_t + u_t \text{ , (3.1),}$$

where $y_t = (c_t, s_t)'$, c_t is the implied volatility of corn, s_t is the implied volatility of soybeans, $a = (a_1, a_2)'$ is a 2×1 vector of intercept terms, matrix

$$\mathrm{B}_i = \begin{bmatrix} b_{11,i} & b_{12,i} \\ b_{21,i} & b_{22,i} \end{bmatrix}$$

is the coefficient matrix, x_t is a $k\times1$ vector of exogenous variables, D is a $2\times k$ matrix of parameters, and $u_t = (u_{1t}, u_{2t})'$ is a 2-dimensional white noise, that is, given the information at $t-1$, $E(u_t) = 0$, $E(u_t u_t') = \Sigma_u$, and $E(u_t u_s') = 0$ if $s \neq t$.

In this study, we apply Fourier seasonal components as exogenous variables to depict the periodicity of implied volatilities. The Fourier seasonal component is defined as

$$f_i = \sum_{j=1}^{J} \left\{ \alpha_j \cos\left(\frac{2\pi j}{52} w\right) + \beta_j \sin\left(\frac{2\pi j}{52} w\right) \right\},$$

where w represents the number of week in the year. Thus, the VAR(p) model of our study can be written as

$$\begin{bmatrix} c_t \\ s_t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \sum_{i=1}^{p} \begin{bmatrix} b_{11,i} & b_{12,i} \\ b_{21,i} & b_{22,i} \end{bmatrix} \begin{bmatrix} c_{t-i} \\ s_{t-i} \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{J} \left\{ \alpha_j \cos\left(\frac{2\pi j}{52} w\right) + \beta_j \sin\left(\frac{2\pi j}{52} w\right) \right\} \\ \sum_{j=1}^{J} \left\{ \gamma_j \cos\left(\frac{2\pi j}{52} w\right) + \theta_j \sin\left(\frac{2\pi j}{52} w\right) \right\} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}.$$

To test volatility spillovers effects, we only need to test the significance of $b_{12,i}$'s and $b_{21,i}$'s. For example, if the null hypothesis is no volatility spillover from the soybean market to the corn market, then we should test the significance of $b_{12,i}$'s.

In this study, we also use impulse responses to measure volatility spillover effects. With stationary time series variables, an impulse response function is generally applied to discuss responses of a variable to a shock. A VAR(p) model with stationary variables can be rewritten as a vector moving average model with infinite order (VMA($+\infty$))

$$y_t = c + \varepsilon_t + \sum_{i=1}^{\infty} \Psi_i \varepsilon_{t-i}, \quad (3.2)$$

where Ψ_i's are 2×2 matrices, and ε_t is a 2-dimensional white noise, that is, given the information at $t-1$, $E(\varepsilon_t)=0$, $E(\varepsilon_t \varepsilon_t{}')=\Sigma_\varepsilon$, and $E(\varepsilon_t \varepsilon_s{}')=0$ if $s \neq t$. The response of the i^{th} variable to one standard positive shock (one unit change) in the j^{th} variable h periods before is estimated by the ij^{th} element in Ψ_h (coefficient matrix at lag h).

3.2. Bootstrapped Chow Tests

In the second step of this study, we conduct structural change tests to check the time invariance property of the model. The most commonly used structural change test in time series analysis is the Chow test. When the structural break point is unknown, although the structural change test can be conducted repeatedly for a range of potential structural break points, the outcomes of these repeated tests are not independent, and thus the p-values from the series of tests may be misleading (Andrews, 1993). Some research has been done to resolve this problem (see, for example, Andrews and Ploberger 1994; and Hansen 1997), by making corrections to the p-values or critical values. Furthermore, Candelon and Lutkepohl (2001) developed the bootstrap versions of Chow tests to improve the accuracy for testing in common sample sizes. Lutkepohl (2004) also extended the bootstrap versions of Chow tests to multivariate models.

In this study, we conduct three bootstrap versions of Chow tests to examine the time invariance property of our model. In particular, they are sample-split, break-point, and forecast bootstrapped Chow tests. Assuming that a structural break has happened at time t, the sample-split and break-point tests compare parameter estimates obtained from the model using data before t with those from the same model but using data after t. The sample-split test assumes that the variance-covariance matrix is invariant for the two subsamples, while the break-point test also checks the constancy of the variance-covariance matrix. The Chow forecast test checks whether forecasts from the model for the first subsample are compatible with observations in the second subsample. For more details, please refer to Appendix A.

3.3. Threshold Model

From the results of bootstrapped Chow tests, we found significant structural break points in the VAR model (see Section 4.3). To improve the accuracy of our model, we estimate a threshold VAR model with four regimes:

High volatility of corn – High volatility of soybeans,

High volatility of corn – Low volatility of soybeans,

Low volatility of corn – High volatility of soybeans,

Low volatility of corn – Low volatility of soybeans.

The regimes are defined by the previous levels of implied volatilities of corn and soybeans. The reason we use the levels of implied volatility to define the regimes is because we believe that the dynamic transmissions of volatilities may behave very differently from a low volatility regime to a high volatility regime, according to the properties of the data. The threshold VAR model we propose is

$$y_t = \begin{cases} a_1 + \sum_{i=1}^{p} B_{1i} y_{t-1} + D_1 x_t + u_t & \text{if } c_{t-1} \geq C_1 \text{ and } s_{t-1} \geq C_2 \\ a_2 + \sum_{i=1}^{p} B_{2i} y_{t-1} + D_2 x_t + u_t & \text{if } c_{t-1} \geq C_1 \text{ and } s_{t-1} < C_2 \\ a_3 + \sum_{i=1}^{p} B_{3i} y_{t-1} + D_3 x_t + u_t & \text{if } c_{t-1} < C_1 \text{ and } s_{t-1} \geq C_2 \\ a_4 + \sum_{i=1}^{p} B_{4i} y_{t-1} + D_4 x_t + u_t & \text{if } c_{t-1} < C_1 \text{ and } s_{t-1} < C_2 \end{cases}$$

C_1 and C_2 define the thresholds of this model, and they are chosen by the maximum likelihood method.

3.4. BEKK-GARCH Model

A traditional approach to testing volatility spillovers is to estimate a GARCH model and to test the significance of the parameter estimates. A popular type of multivariate GARCH models used to examine volatility spillover effects is the BEKK model, which ensures the positive semi-definite property of the variance-covariance matrix. The purpose of estimating a BEKK-GARCH model using futures price returns is to compare our results from the VAR model with those from the traditionally used method. A BEKK model with two time series is shown as follows:

$$\begin{bmatrix} r_{ct} \\ r_{st} \end{bmatrix} = \begin{bmatrix} \eta_{01} \\ \eta_{02} \end{bmatrix} + \sum_{i=1}^{p} \begin{bmatrix} \eta_{11,i} & \eta_{12,i} \\ \eta_{21,i} & \eta_{22,i} \end{bmatrix} \begin{bmatrix} r_{c,t-i} \\ r_{s,t-i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{ct} \\ \varepsilon_{st} \end{bmatrix}, \quad (3.3)$$

$$\begin{bmatrix} \varepsilon_{ct} \\ \varepsilon_{st} \end{bmatrix} \Bigg| \Psi_{t-1} \sim N(0, H_t),$$

$$\begin{aligned} H_t &= \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} \\ \sigma_{21,t} & \sigma_{2,t}^2 \end{bmatrix} \\ &= \begin{bmatrix} \varpi_1 & 0 \\ \varpi_2 & \varpi_3 \end{bmatrix} \begin{bmatrix} \varpi_1 & \varpi_2 \\ 0 & \varpi_3 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{21} \\ \alpha_{12} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} & \varepsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \quad (3.4) \\ &\quad + \begin{bmatrix} \beta_{11} & \beta_{21} \\ \beta_{12} & \beta_{22} \end{bmatrix} \begin{bmatrix} \sigma_{1,t}^2 & \sigma_{12,t} \\ \sigma_{21,t} & \sigma_{2,t}^2 \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \end{aligned}$$

where r_{ct} and r_{st} are corn and soybean futures returns (defined as $r_{ct} = \ln(p_{ct}) - \ln(p_{c,t-1})$ and $r_{st} = \ln(p_{st}) - \ln(p_{s,t-1})$, where p_{ct} and p_{st} are the nearest futures prices of corn and soybeans), ψ_{t-1} is the known information at time t, and H_t is the time-varying variance-covariance matrix. After several steps of derivation, $\sigma_{1,t}^2$ and $\sigma_{2,t}^2$ can be rewritten as

$$\begin{aligned} \sigma_{1,t}^2 &= \varpi_1^2 + \alpha_{11}^2 \varepsilon_{1,t-1}^2 + 2\alpha_{11}\alpha_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{21}^2 \varepsilon_{2,t-1}^2 + \beta_{11}^2 \sigma_{1,t-1}^2 + 2\beta_{11}\beta_{21}\sigma_{21,t-1} + \beta_{21}^2 \sigma_{2,t-1}^2 \\ \sigma_{12,t} &= \varpi_1 \varpi_2 + \alpha_{11}\alpha_{12}\varepsilon_{1,t-1}^2 + \alpha_{11}\alpha_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{12}\alpha_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{22}\alpha_{21}\varepsilon_{2,t-1}^2 \\ &\quad + \beta_{11}\beta_{12}\sigma_{1,t-1}^2 + \beta_{12}\beta_{21}\sigma_{21,t-1} + \beta_{11}\beta_{22}\sigma_{12,t-1} + \beta_{22}\beta_{21}\sigma_{2,t-1}^2 \\ \sigma_{2,t}^2 &= (\varpi_2^2 + \varpi_3^2) + \alpha_{12}^2 \varepsilon_{1,t-1}^2 + 2\alpha_{22}\alpha_{12}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + \alpha_{22}^2 \varepsilon_{2,t-1}^2 + \beta_{12}^2 \sigma_{1,t-1}^2 + 2\beta_{22}\beta_{12}\sigma_{12,t-1} + \beta_{22}^2 \sigma_{2,t-1}^2 \end{aligned}$$

From the equations above, testing for return spillover effects from one market to the other is equivalent to testing the significance of $\eta_{12,i}$ and $\eta_{21,i}$ in equation (3.3). To test volatility spillover effects from soybeans to corn, we need to perform the hypothesis test:

$$H_0 : \alpha_{21} = \beta_{21} = 0 \quad \text{(No volatility Spillover from soybeans to corn)}$$

$$H_a : \text{at least one of them not } 0 .$$

And to test volatility spillover effects from corn to soybeans, we need to perform the hypothesis test:

$$H_0 : \alpha_{12} = \beta_{12} = 0 \quad \text{(No volatility Spillover from corn to soybeans)}$$

$$H_a : \text{at least one of them not } 0$$

4. Results

4.1. Data

The data we use in this study consist of weekly average implied volatilities derived from nearby option contracts in corn and soybean markets from 1/5/2001 to 11/1/2010. These implied volatilities are calculated from the Black option pricing model, using the mean of the two nearest-the-money calls and the two nearest-the-money puts.

Figure 1 shows the time series plots for the weekly average prices of the nearest corn and soybean futures contracts. It illustrates that the futures markets of corn and soybeans have undergone dramatic changes since 2007. Specifically, both price levels have increased significantly since 2007. The cause of these dramatic changes is the significant structural shocks from the Energy Independence and Security Act of 2007. This Energy Act sets a modified standard that starts at 9.0 billion gallons of renewable fuel in 2008 and rises to 36 billion gallons by 2022. 21 billion gallons of the latter total is required to be obtained from ethanol and other advanced biofuels. This modified standard has increased the demand for corn which is the major source for ethanol. On the other hand, the price of soybeans is highly correlated with the price of corn, because corn and soybeans are basically grown in the same region and compete for the same land. Changes in the demand for corn will probably incur changes in production decisions of soybeans, and therefore affect the price of soybeans. In particular, as higher corn prices bid away acreage toward corn, soybean prices will rise.

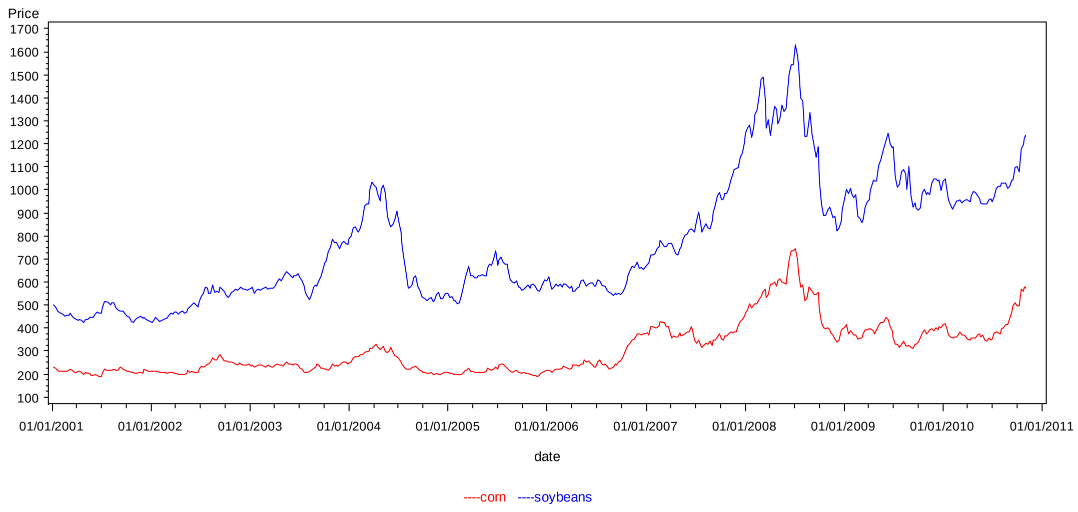

Figure 1. Prices for the Nearest Futures Contracts of Corn and Soybeans (Weekly Average)

Figure 2 and 3 show the time series plots of weekly average implied volatility for corn and soybean markets. For the corn market, the implied volatility displays a strong periodical pattern before 2007. The maximum implied volatilities appear to occur approximately in June and July, immediately prior to the harvest season of corn. This is a period of time when new information regarding the upcoming crop is being processed by the market. The minimum implied volatilities appear in winter, following harvest. For the soybean market, though the seasonal pattern of the implied volatility is not as significant as in the corn market, implied volatility still displays regular patterns before 2006. The minimum implied volatilities generally appear to occur in winter.

Changes in futures prices will also result in changes in price volatilities. Implied volatilities in these two markets have changed remarkably since 2007. For example, the implied volatility of corn remained at a relatively high level after 2007. The average implied volatility increased by approximately 47%, compared with the average over the period from 2001 to 2006. For the soybean market, implied volatility increased to a relatively high level from the beginning of 2008, and then it started to decline from the middle of 2009. Descriptive statistics of these implied volatilities are reported in Table 1.

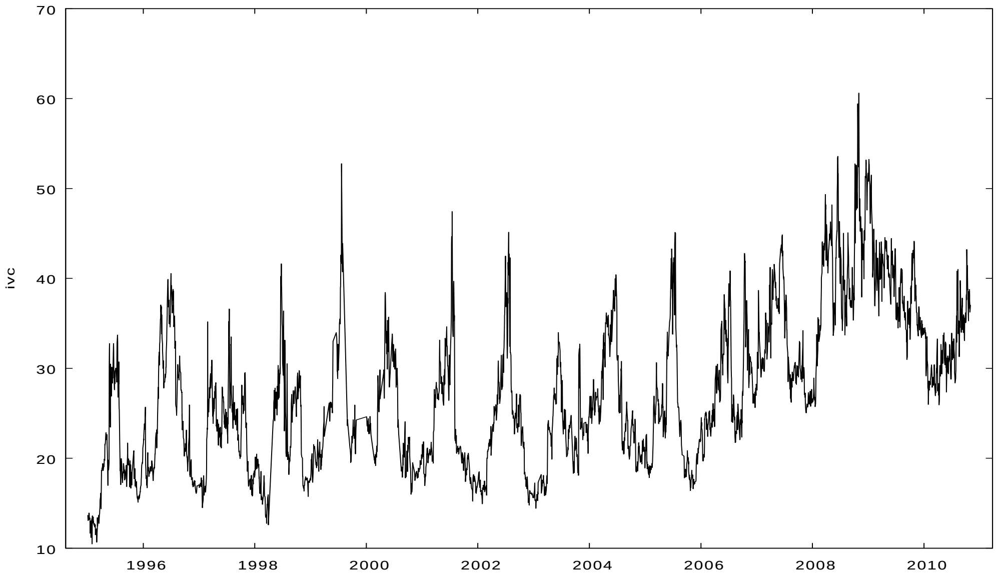

Figure 2. Weekly Average Implied volatility in the corn market

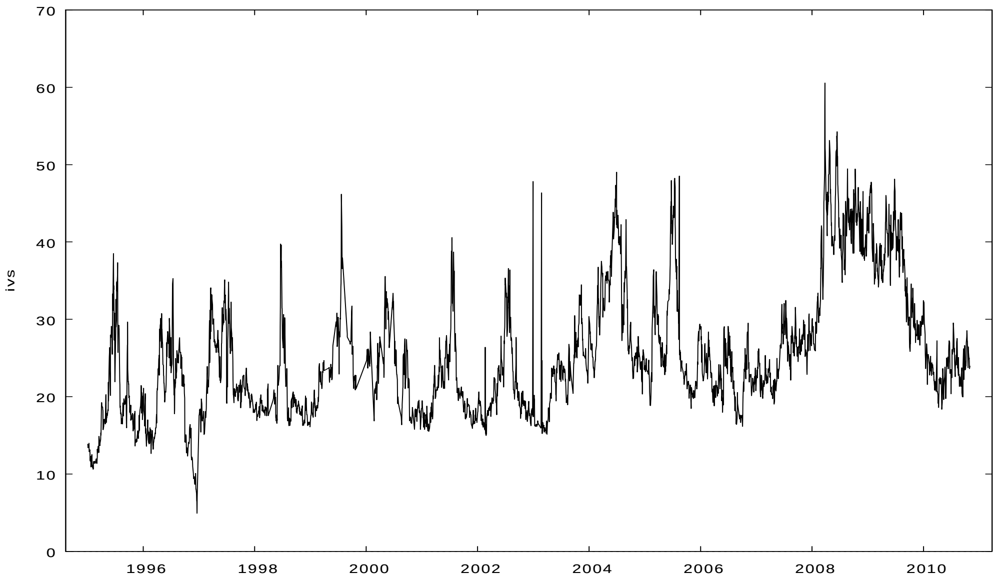

Figure 3. Weekly Average Implied volatility in the soybean market

Table 1. Summary Statistics

	c_t	s_t
Observations	513	513
Mean	29.451	27.333
Standard Deviation	8.497	14.903
Minimum	15.07	13.71
Maximum	55.64	54.19
Skewness	0.414	0.882
Kurtosis	-0.523	-0.045

In the analysis of time series data, generally, the first step is to test for stationarity (or unit-root) for the data because non-stationary data under the ordinary least squares framework tend to result in biased estimates. We conduct both augmented Dickey-Fuller (ADF) and Phillips-Perron tests to evaluate the stationarity for the two endogenous variables in the model. Table 2 shows the ADF and Phillips-Perron test statistics, which indicates that both variables are stationary at a 5% significance level. Thus, no cointegration test needs to be conducted.

Table 2. ADF and Phillips-Perron Tests Results

	ADF Test		Phillips-Perron	
Variable	Single Mean	Trend	Single Mean	Trend
c_t	-3.84***	-4.88***	-3.52***	-4.43***
s_t	-3.83***	-4.04***	-3.37***	-3.91**

Note: (1) ***, and ** refer to the rejection of the null hypothesis of a unit root at 1% and 5%.
(2) No unit root in levels is found at 1% significance level.

4.2. VAR model with Fourier Seasonal Components

The order of the VAR model and the order of the Fourier seasonal components are decided by the Schwartz Bayesian Criterion (SBC), a commonly used criterion for determining the order of a VAR model. By the minimum value of SBC, the order of the VAR model and the order of the Fourier seasonal components are both one. That is, two trigonometric exogenous variables, $\cos(2\pi w/52)$ and $\sin(2\pi w/52)$, are included as exogenous variables in the VAR model. Thus, the VAR(1) model becomes

$$\begin{bmatrix} c_t \\ s_t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} c_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha & \beta \\ \gamma & \theta \end{bmatrix} \begin{bmatrix} \cos\left(\frac{2\pi}{52} w\right) \\ \sin\left(\frac{2\pi}{52} w\right) \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}. \quad (4.1)$$

Estimates of coefficients in this VAR(1) model are shown in Table 3. The results of the estimates indicate that volatility spillovers exist from the corn market to the soybean market at a significance level of 5%. However, there is no significant volatility spillover from the soybean market to the corn market.

Tabel 3. Estimates of the VAR(1) model

	c_t equation	s_t equation
constant	1.534	1.018
	(0.435)***	(0.437)**
c_{t-1}	0.960	0.051
	(0.021)***	(0.021)**
s_{t-1}	-0.012	0.908
	(0.021)	(0.021)***
$\cos(2\pi w/52)$	-0.023	-0.072
	(0.165)	(0.166)
$\sin(2\pi w/52)$	0.466	0.326
	(0.156)***	(0.157)**

Note: (1) Standard errors are in parentheses.
(2) ***, **, and * denote significance level of 1%, 5% and 10%.

Impulse response functions illustrate the effect of a positive shock (a one unit change) in a variable on the future values of the other variables and itself. In this study, we used the simple impulse response functions to examine effects of a shock in an implied volatility. Unlike an orthogonal impulse response function, a simple impulse response function (equation (3.2)) is not affected by the ordering of the variables.

Figure 4 and 5 show the impulse responses of the VAR(1) model up to 40 weeks after a positive shock (a one unit change in level) in one variable. A shock in the implied volatility of corn has a positive and significant impact on the implied volatility of soybeans. The significance persists for approximately 33 weeks at a 5% significance level. The responses increase for about 13 weeks, and then start to decline. However, a shock in the implied volatility of soybeans has no significant impact on the implied volatility of corn.

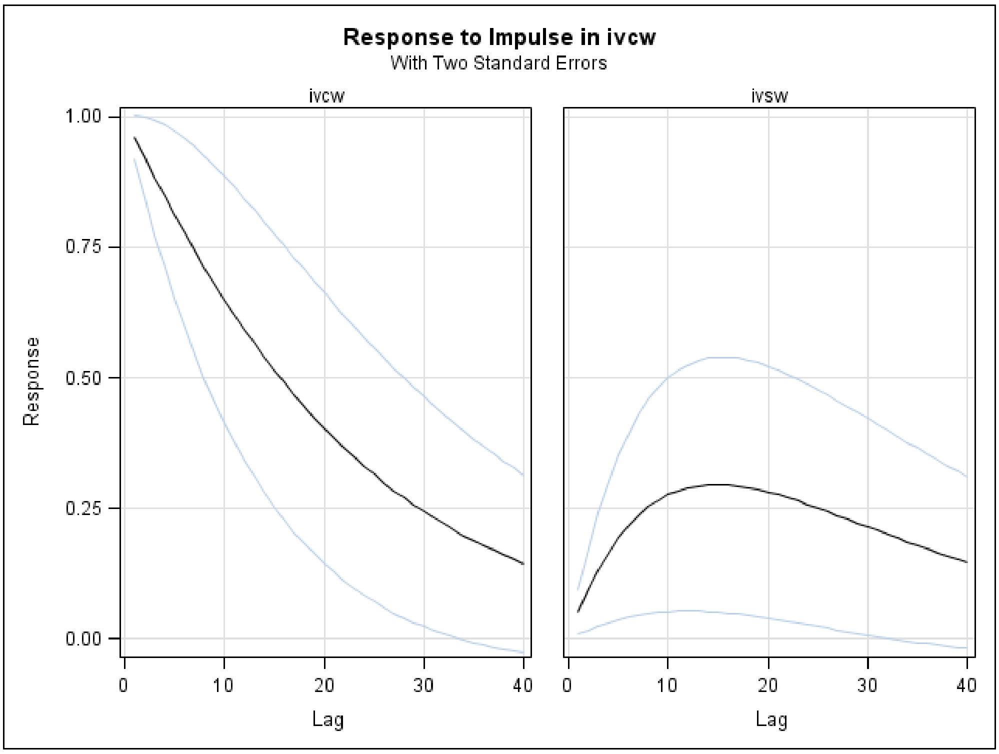

Figure 4. Responses to impulse in the implied volatility of corn

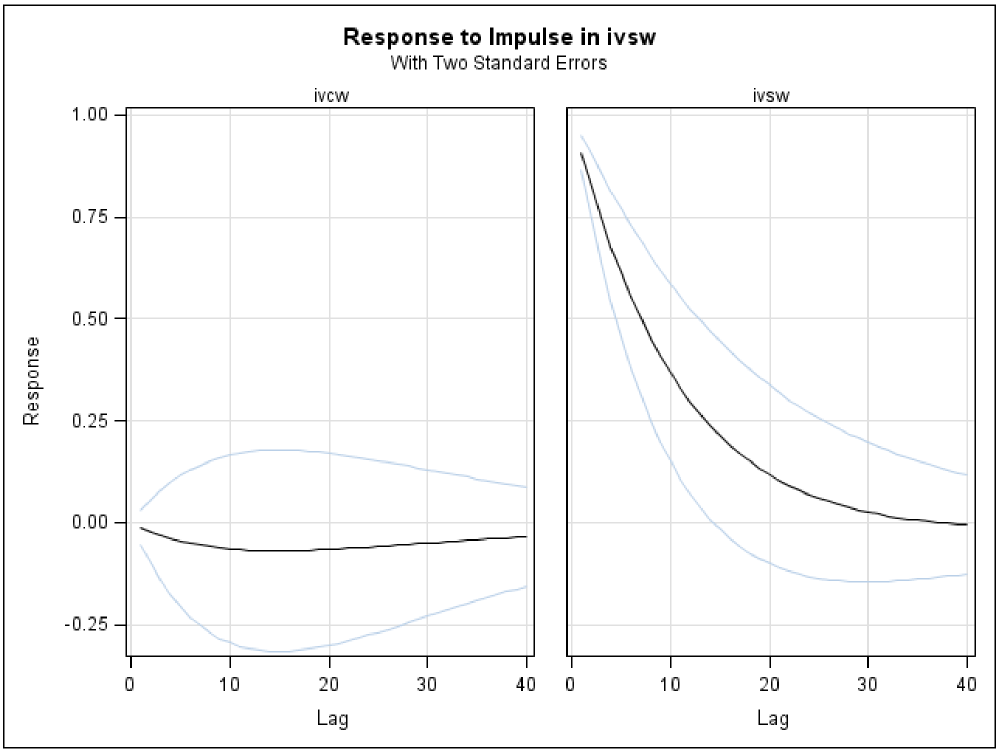

Figure 5. Responses to impulse in the implied volatility of soybeans

4.3. Structural Change Tests

From Figure 2 and 3, we observed dramatic changes in implied volatilities in corn and soybean markets since 2007. To check the time invariance property of our model, three bootstrap versions of Chow tests (break-point, sample-split and forecast) are conducted. Figure 5 shows the p-values for these three types of Chow tests when structural change points are unknown. Small p-values suggest structural changes at a given time. For example, significant structural changes can be observed around 2003, the second quarter of 2006, and the first quarter of 2008.

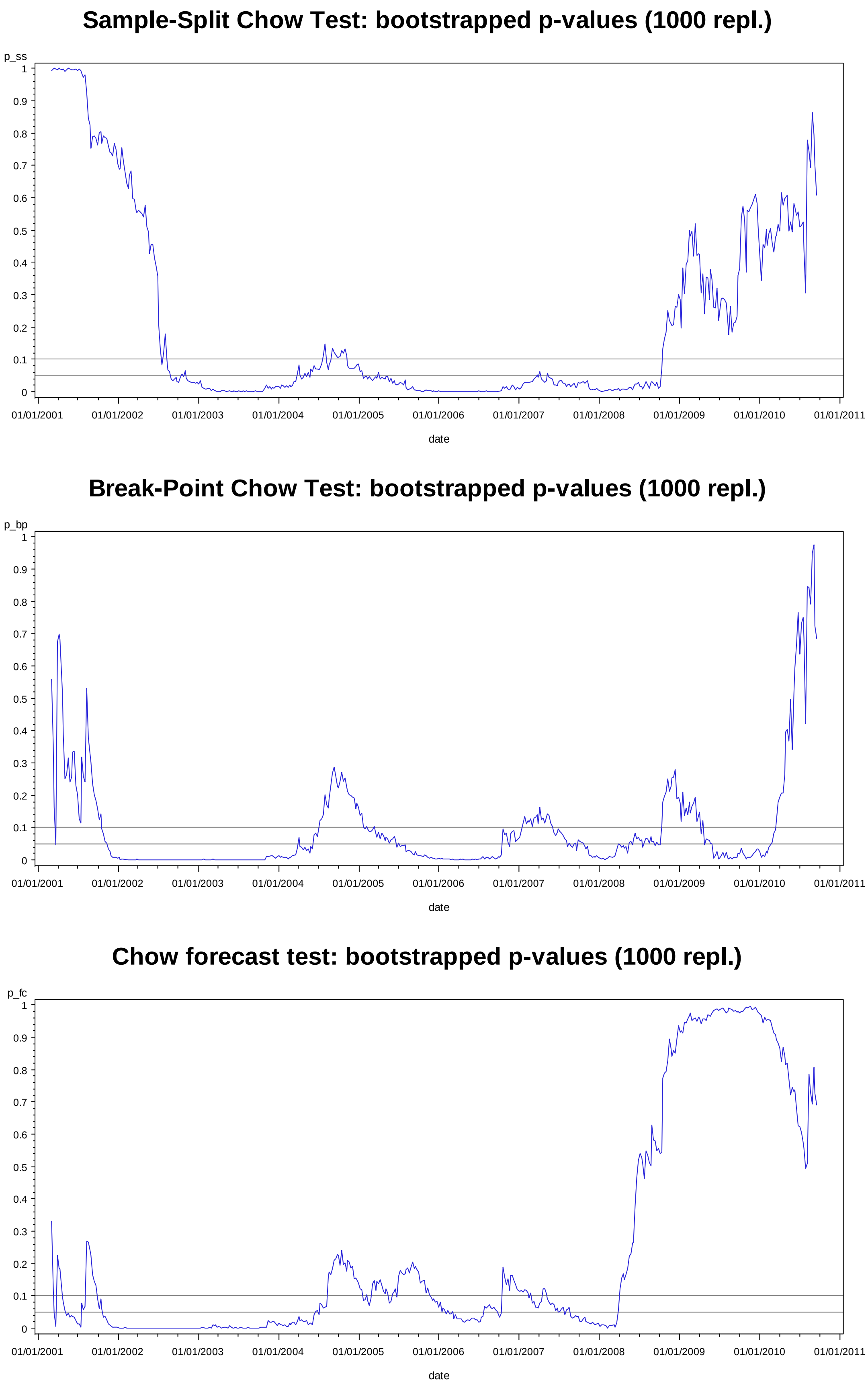

Figure 6. Bootstrapped Versions of Chow Tests

After the structural change tests, we separate the data set into two subsamples to compare results from the subsamples. The first subsample contains observations before 2007, and the second contains observations from 2006. The year 2006 is covered in both subsamples. The reason of separating the data set in this way is because dramatic changes can be observed after 2007 from Figure 1 to 3, and significant structural changes can be found during the year 2006 (especially in the second quarter) from the three bootstrap versions of Chow Tests. Also, we want to observe the changes aroused by the shock of the Energy Act of 2007.

The estimates of the VAR models for the two subsamples are shown in Table 4. Results from the first subsample suggest that volatility spillovers exist from the corn market to the soybean market at a significance level of 10%. However, no spillover effect can be observed for the second subsample. These results are quite different from what we found in Section 4.2 for the complete data set. Figure 7 to 10 show the impulse responses up to 40 weeks after a shock in one variable for each subsample. These impulse responses are also quite different from those in Section 4.2. For the first subsample (Figure 7 and 8), a shock in the implied volatility of corn has a positive effect on the implied volatility of soybeans (the two blue lines indicate the interval at 5% significance level), while a shock in the implied volatility of soybeans has a negative effect on the implied volatility of corn. For the second subsample (Figure 10), a shock in the implied volatility of soybeans has a positive effect on the implied volatility of corn.

Table 4. Estimates of VAR(1) for two subsamples

	2001-2006		2006-2010	
	c_t equation	s_t equation	c_t equation	s_t equation
constant	3.588	1.427	3.130	0.835
	(0.756)***	(0.772)*	(0.978)***	(0.945)
c_{t-1}	0.888	0.063	0.874	0.028
	(0.036)***	(0.037)*	(0.042)***	(0.040)
s_{t-1}	-0.031	0.878	0.045	0.940
	(0.029)	(0.029)***	(0.032)	(0.031)***
$\cos(2\pi w/52)$	-0.583	-0.096	0.182	-0.014
	(0.255)**	(0.260)	(0.253)	(0.245)
$\sin(2\pi w/52)$	0.757	0.409	0.185	0.230
	(0.194)***	(0.198)**	(0.249)	(0.240)

Note: Standard errors are in parentheses.

***, **, and * denote significance level of 1%, 5% and 10%.

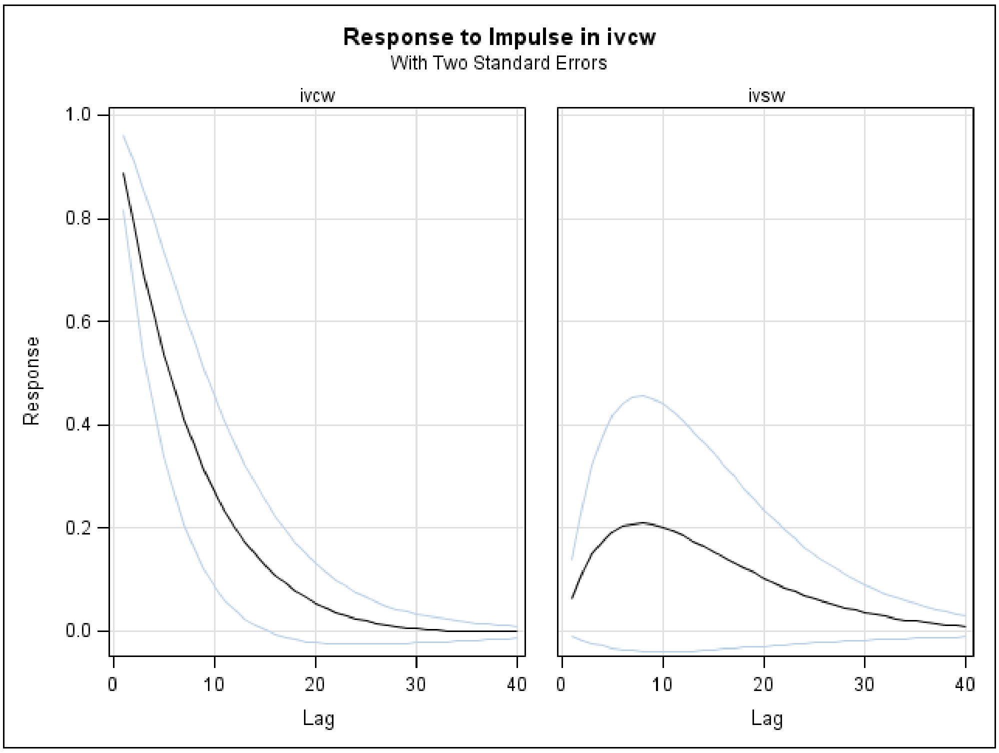

Figure 7. Responses to impulse in the implied volatility of corn (2001-2006)

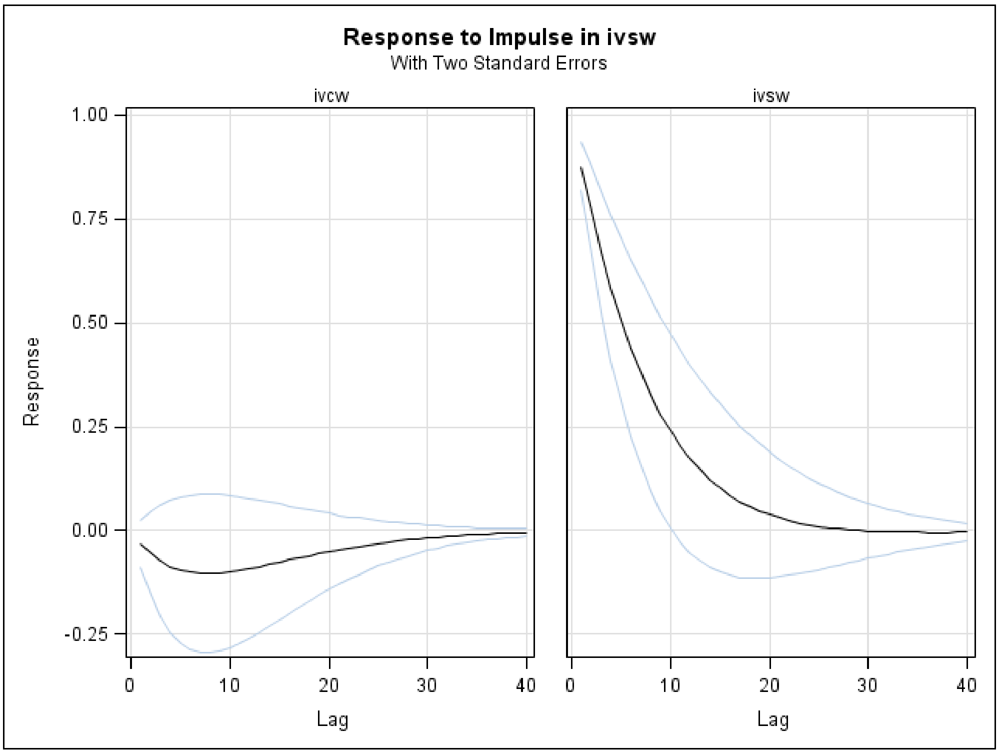

Figure 8. Responses to impulse in the implied volatility of soybeans (2001-2006)

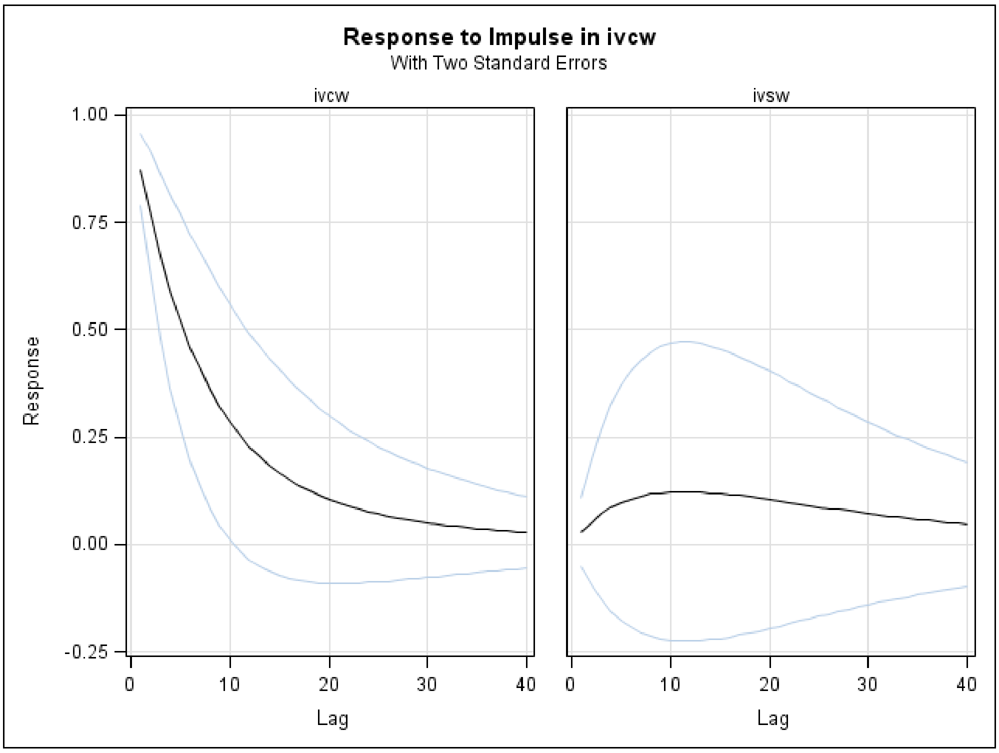

Figure 9. Responses to impulse in the implied volatility of soybeans (2006-2010)

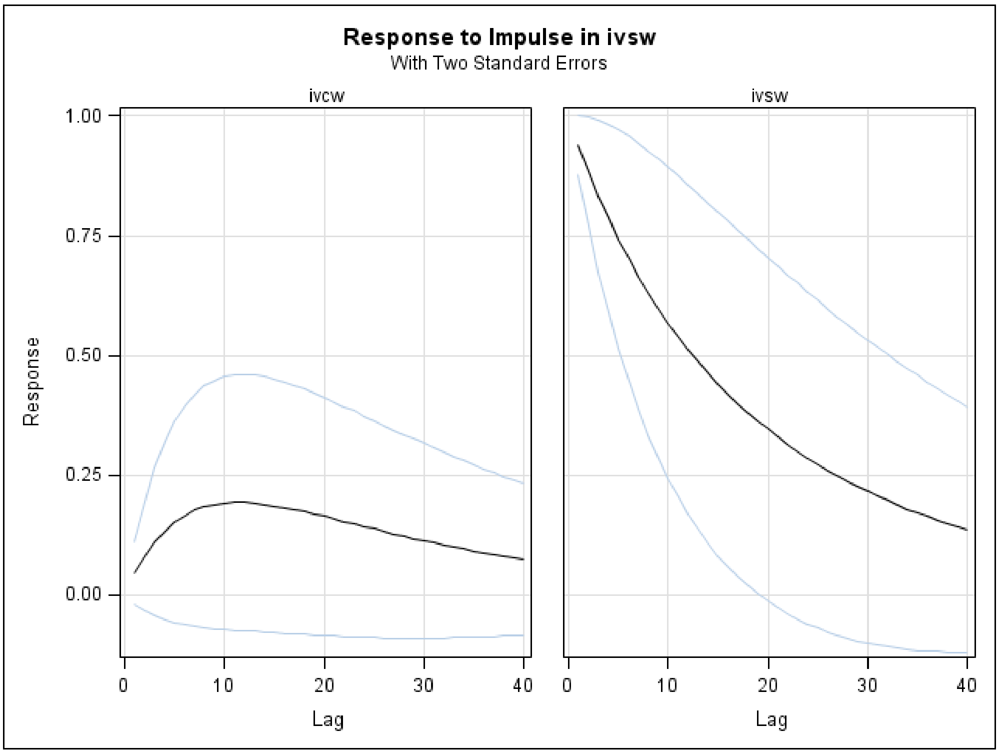

Figure 10. Responses to impulse in the implied volatility of soybeans (2006-2010)

4.4. Threshold Model

Results in Section 4.3 indicate that significant structural changes may exist in our model. And transmissions of volatilities between the corn and soybeans markets may behave differently from a low volatility condition to a high volatility condition. Therefore, we develop a threshold model with four thresholds (see Section 3.3), and results of this model are shown in Table 5. The optimal values of C_1 and C_2 are 35.355 and 31.926, which are obtained from the maximum likelihood method. These two values are both in the middle of the ranges of these two implied volatilities.

Table 5. Estimates of the Threshold VAR model

	$c_{t-1} \geq 35.355$ and $s_{t-1} \geq 31.926$ ($n_1 = 98$)		$c_{t-1} \geq 35.355$ and $s_{t-1} < 31.926$ ($n_2 = 37$)	
	c_t equation	s_t equation	c_t equation	s_t equation
constant	13.533	12.609	13.064	2.441
	(2.917)***	(2.978)***	(7.507)*	(7.665)
c_{t-1}	0.546	-0.104	0.657	-0.045
	(0.083)***	(0.084)	(0.203)***	(0.207)
s_{t-1}	0.140	0.796	-0.031	0.982
	(0.071)*	(0.073)***	(0.157)	(0.161)***
$\cos(2\pi w/52)$	2.377	0.368	-0.308	-0.277
	(0.477)***	(0.487)	(0.771)	(0.788)
$\sin(2\pi w/52)$	0.181	0.696	0.396	0.129
	(0.362)	(0.369)*	(0.813)	(0.830)

	$c_{t-1} < 35.355$ and $s_{t-1} \geq 31.926$ ($n_3 = 30$)		$c_{t-1} < 35.355$ and $s_{t-1} < 31.926$ ($n_4 = 348$)	
	c_t equation	s_t equation	c_t equation	s_t equation
constant	22.206	25.489	1.428	1.912
	(7.284)***	(7.437)***	(0.833)*	(0.850)
c_{t-1}	0.761	-0.090	0.998	0.057
	(0.121)***	(0.123)	(0.033)***	(0.033)*
s_{t-1}	-0.467	0.295	-0.047	0.859
	(0.169)***	(0.173)*	(0.041)	(0.042)***
$\cos(2\pi w/52)$	-3.312	-3.848	-0.119	0.054
	(1.109)***	(1.132)***	(0.206)	(0.211)
$\sin(2\pi w/52)$	1.706	0.564	0.391	0.207
	(0.636)***	(0.649)	(0.189)**	(0.193)

Note: n_i is the number of observations in Regime i.

From the results of Table 5, coefficients of b_{12} (based on equation (4.1)) are significant in the two regimes where $s_{t-1} \geq 31.926$. The coefficient of b_{21} is significant only in the regime where $c_{t-1} < 35.355$ and $s_{t-1} < 31.926$. Therefore, we can conclude that when the volatility of soybeans is high (Regime 1 and 3), volatility spillovers exist from the soybean market to the corn market. In Regime 1 (high volatility of corn), the volatility of soybeans has a positive spillover effect on the volatility of corn; while in Regime 3 (low volatility of corn), this effect becomes negative. And when both volatilities are low, volatility spillovers exist from the corn market to the soybean market.

4.5. BEKK-GARCH Model

The data we use in this section are weekly average prices of the nearest corn and soybean futures contracts. Table 6 reports descriptive statistics of the returns. And Table 7 shows the results for the bivariate BEKK-GARCH model (equation (3.3) and (3.4)). These results indicate that parameters η_{21}, α_{12}, α_{21}, and β_{12} are statistically significant, which means return spillovers exist from the corn market to the soybean market, and volatility spillover effects exist between these two markets. Results from the BEKK model are quite different from what we have obtained from Section 4.2.

Table 6. Summary Statistics for returns of corn and soybeans

	r_{ct}	r_{st}
Mean	0.00181	0.00176
Standard Deviation	0.03631	0.03299
Minimum	-0.15119	-0.13300
Maximum	0.13453	0.09685
Skewness	-0.17141	-0.75245
Kurtosis	1.49920	1.79848

Table 7. Estimates of the BEKK(1,1) Model

Parameter	Estimate	Standard Error	P-Value
η_{01}	0.00152	0.00158	0.3381
η_{02}	0.00129	0.00141	0.3603
η_{11}	0.20622	0.05365	0.0001
η_{12}	-0.02658	0.05906	0.6529
η_{21}	0.12264	0.04792	0.0108
η_{22}	0.16089	0.05274	0.0024
ϖ_{1}	-0.03531	0.00160	0.0000
ϖ_{2}	-0.01589	0.00227	0.0000
ϖ_{3}	0.01164	0.00076	0.0000
α_{11}	0.39387	0.10890	0.0003
α_{12}	1.49556	0.15134	0.0000
α_{21}	0.37142	0.12850	0.0039
α_{22}	0.52223	0.16165	0.0012
β_{11}	0.19242	0.16190	0.2346
β_{12}	-0.24579	0.10134	0.0153
β_{21}	-0.01450	0.09898	0.8835
β_{22}	0.42265	0.00863	0.0000

5. Conclusion and Discussion

In this study, we investigate the relationships and transmissions between implied volatilities in two major agricultural commodity markets – corn and soybeans, by applying a VAR model, impulse response functions, bootstrap versions of structural change tests, and a threshold VAR model. In the first step, the VAR model suggests that volatility spillovers exist from the corn market to the soybean market, but there is no volatility spillover from the soybean market to the corn market.

From the results of three bootstrap versions of Chow tests, we can conclude that there may be several structural break points in our model. Then, taking into account the dramatic changes for both implied volatilities, we develop a threshold VAR model with four regimes depending on the levels of previous volatilities. Results from the threshold VAR model suggest that volatility spillovers from the corn market to the soybean market only exists when both volatilities are at relatively low levels. When the soybean market is in a high volatility situation, volatility may spill over from the soybean market to the corn market.

Finally, we estimate a bivariate BEKK-GARCH model to examine the volatility spillover effects using futures returns. Results from this model provide evidence of double-directional volatility spillover effects between the two markets, which is different from what we have found in Section 4.2. In other words, conclusions from the historical backward futures data may be different from those based on the forward-looking measure of volatilities.

Appendix A: Bootstrap Versions of Chow Test

Suppose that the structural break occurred at time T_B for a K-dimensional VAR(p) model with M exogeneous variables:

$$y_t = a + \sum_{i=1}^{p} \mathrm{B}_i y_{t-1} + Dx_t + u_t .$$

Then the model under consideration is estimated from the full sample T observations and the first T_1 and last T_2 observations, where $T_1 = T_B - 1$ and $T_2 = T - T_B + 1$. The residuals from the full sample and the two subsamples are denoted as $\hat{u}_t$, $\hat{u}_t^{(1)}$, and $\hat{u}_t^{(2)}$, respectively. $\hat{u}_t$, $\hat{u}_t^{(1)}$, and $\hat{u}_t^{(2)}$ are $K \times 1$ vectors. Define

$$\hat{\Sigma}_{12} = \frac{\sum_1^{T_1} \hat{u}_t \hat{u}_t' + \sum_{T-T_2+1}^{T} \hat{u}_t \hat{u}_t'}{T_1 + T_2}$$

$$\hat{\Sigma}_{(12)} = \frac{\sum_1^{T_1} \hat{u}_t \hat{u}_t'}{T_1} + \frac{\sum_{T-T_2+1}^{T} \hat{u}_t \hat{u}_t'}{T_2}$$

$$\hat{\Sigma}_{(1)} = \frac{\sum_1^{T_1} \hat{u}_t^{(1)} \hat{u}_t^{(1)\prime}}{T_1}$$

$$\hat{\Sigma}_{(2)} = \frac{\sum_1^{T-T_2+1} \hat{u}_t^{(2)} \hat{u}_t^{(2)\prime}}{T_2},$$

The sample-split test statistics is given by

$$\lambda_{SS} = \left(T_1 + T_2\right)\left\{\log\left|\hat{\Sigma}_{12}\right| - \log\left|\frac{T_1\hat{\Sigma}_{(1)} + T_2\hat{\Sigma}_{(2)}}{T_1 + T_2}\right|\right\}.$$

The break-point test statistics is given by

$$\lambda_{BP} = \left(T_1 + T_2\right)\log\left|\hat{\Sigma}_{(12)}\right| - T_1 \log\left|\hat{\Sigma}_{(1)}\right| - T_2 \log\left|\hat{\Sigma}_{(2)}\right| .$$

λ_{SS} and λ_{BP} have approximate χ^2-distributions. The degrees of freedom (DoF) are the difference between the sum of the number of free coefficients in the first and last subsamples and the number of free coefficient in the full sample model. For λ_{SS}, the DoF is $pK^2 + K + KM$, and for λ_{BP}, the DoF is $pK^2 + K + KM + K\left(K+1\right)/2$.

The Chow forecast test statistic is given by

$$\lambda_{CF} = \frac{1-\left(1-R_r^2\right)^{1/s}}{\left(1-R_r^2\right)^{1/s}} \cdot \frac{Ns-q}{Kk^*} \approx F\left(Kk^*, Ns-q\right),$$

where $k^* = T - T_1$ is the number of forecast periods considered by the test,

$$R_r^2 = 1 - \left(\frac{T_1}{T}\right)^K \left|\hat{\Sigma}_{(1)}\right| \left(\left|\hat{\Sigma}_{(1)}\right|\right)^{-1},$$

$$s = \left(\frac{K^2 k^{*2} - 4}{K^2 + k^{*2} - 5}\right)^{1/2}, \quad q = \frac{Kk^*}{2} - 1,$$

$$N = T - k_1 - k^* - \left(K - k^* + 1\right)/2,$$

and k_1 is the number of regressors in the restricted time-invariant model.

Bootstrap versions of Chow tests are obtained by estimating the model of interest, denoting the residuals as $\hat{u}_t$, computing centered residuals $\hat{u}_1 - \bar{u}$, $\hat{u}_2 - \bar{u}$, …, $\hat{u}_T - \bar{u}$, where $\bar{u} = \sum_{t=1}^{T} \hat{u}_t / T$, and generating bootstrap residuals $u_1^*, u_2^*, \ldots, u_T^*$ by randomly drawing with replacement from the centered residuals. These quantities are then used to compute the bootstrap time series recursively starting from given presample values $y_{-p+1}, \ldots, y_0$. The model of interest is then reestimated with and without stability restriction and a bootstrap version of the statistic of interest, say λ_{SS}^*, λ_{BP}^*, or λ_{CF}^*, is computed. Repeating these steps a large number of times, a critical value is then obtained as the relevant percentage point, say λ_{crit}^*, from the empirical distribution of the bootstrap test statistic and the stability hypothesis is rejected if $\lambda > \lambda_{crit}^*$. Alternatively, the P-value of the test can be estimated as the fraction of times that the values of the bootstrap statistics exceed λ.

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