# Optimising woody-weed control

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Woody weeds pose significant threats to the 12.3 billion dollar Australian grazing industry. These weeds reduce stocking rate, increase mustering effort, and impede cattle access to waterways. Two major concerns of woody-weed management are the high cost of weed management with respect to grazing gross margins, and episodic seedling recruitments due to climatic conditions. This case study uses a Stochastic Dynamic Programming (SDP) model to determine the optimal weed management decisions for chinee apple (*Ziziphus mauritiana*) in northern Australian rangelands to maximise grazing profits. Weed management techniques investigated include: no-control, burning, poisoning, and mechanical removal (blade ploughing). The model provides clear weed management thresholds and decision rules, with respect to weed-free gross margins and weed management costs.

Key words: woody weeds, weed control, chinee apple, rangelands, grazing, stochastic dynamic programming

## Introduction

To date it is unknown if woody-weed management in the Australian rangelands is financially viable and, if so, what the best strategies are for given weed species, grazing gross-margins, weed management efficacies and cost structures. This research aims to develop an economic framework to establish optimal control decisions of woody-weeds for Australian rangeland graziers. It is unlikely that any one method of control will result in an optimal solution (Monjardino *et al.*, 2005) and this modelling framework enables us to explore a range of integrated weed management (IWM) strategies.

Historically, many bioeconomic models have sought to combine economic and ecological modelling disciplines, but lacked biological realism as they over-simplify population dynamics (Deacon *et al.*, 1998). Economic optimisation models for weed control decisions have tended to be for annual weeds in cropping systems (Jones *et al.*, 2006; Pannell *et al.*, 2004; Jones & Medd, 2000; Pandey & Medd, 1990; Pannell, 1990; Taylor & Burt, 1984), often assuming the whole weed population is at the same stage of development, i.e. same life-cycle stage. Densities are often measured in the number of seeds or plants within an area. However, optimal management strategies for long-lived perennial weeds need to consider the size of individual plants, their seed production, effects on pasture production, and the efficacies of different management strategies against different life-cycle stages. The model developed here accommodates this complex suite of biophysical and economic parameters and we apply it to chinee apple (*Ziziphus mauritiana*) in northern Australian rangeland upland zones.

This study is based on modelling an average hectare within the Australian rangelands and assumes: (1) seeds are evenly distributed in each hectare and population density for the area is homogenous, (2) weeds do not impede on the production of neighbouring areas, (3) there are no economies of scale in weed management, (4) all prices are constant over time, and (5) an area can be managed independently of its neighbours - in reality neighbouring areas are often in a similar state, and are co-managed.

## Stochastic population model

A stage projection matrix model is used to estimate future weed populations and the effect of control. The temporal transition of the weed population can be represented as:

$$\mathrm{x}_{t+1} = (\mathrm{H}_t \mathrm{x}_t) \circ \Upsilon_t - (\varphi_{u_t} \circ \mathrm{x}_t) \quad (1)$$

$\Upsilon_t$ (a Hadamard product) is the stochastic recruitment of seedlings based on Charters Towers' rainfall data (values are selected by Monte-Carlo sampling), $u_t$ is the control decision, represented by an element from a vector $\mathrm{U}$ of possible management actions, and the mortality rate for each life-cycle stage in response to the control decision is represented by an efficacy vector $\varphi$ from a matrix $\Phi$ for different management actions. $\mathrm{H}_t$ is a density-dependent stage projection matrix with dimensions n x n; where n is the number of life cycle stages. $\mathrm{x}_t$ is the population vector for the number of individuals in each life stage, at time *t*. The three main life cycle stages of woody weeds are seeds, juveniles and adults. Seeds are broken into sub states, new seeds (NS) and seed bank (SB)). As are the juvenile (J), and adults (A) into sub stages $(J_1, J_2, ..., J_m)$ and $(A_1, A_2, ..., A_q)$, based on the time required to reach maturity and plant longevity (Figure 1). For a detailed description of how $\mathrm{H}_t$ is derived over time see Zull *et al.*, (2008).

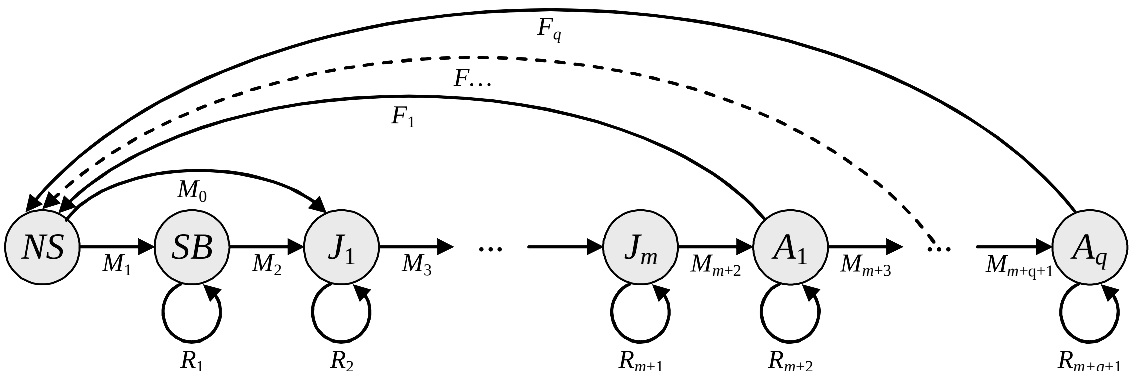


*Figure 1. Life cycle diagram for woody weeds*

Weed damage to pasture production is modelled using Cousens' (1985) rectangular-hyperbola function, as follows:

$$D_t = \frac{\psi \mathrm{x}_t}{1 + \frac{\psi \mathrm{x}_t}{\tau}} \quad (2)$$

$D_t$ is the proportion of pasture-production lost ha$^{-1}$ at time *t* due to weeds, $\psi$ is a damage vector index for the average amount of pasture production lost per weed in each life stage (as weed density approaches zero), and $\tau$ is the maximum proportion of pasture-production loss as weed density approaches infinity. Damage to pasture production can be reduced through control *u*. The financial return (benefit) in any time period is:

$$B_t = \pi_{wf}(1 - D_t\{\mathrm{x}_t, u_t\}) - C_{u,t} \quad (3)$$

where $\pi_{wf}$ is the weed-free grazing gross margin ha$^{-1}$, and $C_t$ is the cost of control *u* in time *t*.

A key assumption of Optimal Control (OC) is that the equation of motion is continuously differentiable with respect to the state and control variables. Many woody-weed control practises have dichotomous application rates (i.e. burning or mechanical removal of plants do not have degrees of application it is or is not applied) hence not continuously differentiable, and OC can not be used. An alternative to OC is Dynamic programming (DP) which is a computationally effective method for solving maximisation problems and does not rely on differentiation (Kennedy, 1988). DP offers the benefit of identifying thresholds i.e., apply a given control ($u$) if an infestation is greater than a given level, based on the temporal effects of the population dynamics and efficacy of control measures; thus providing biological thresholds for management actions. The DP groups all possible population structures into 'states' and divides the planning horizon into 'stages', and uses backward recursion to seek optimal decisions. Solutions were obtained for a planning horizon ($T$) of 25 years, with a discount rate of 5 per cent, in MATLAB (Mathworks, 2007).

## Control measures

There are four different control methods considered in this study. Although, not mutually exclusive in the field they are seldom used in the same year, but can be applied in consecutive years. The variable costs for weed management are density-dependent and increase with increasing density as more effort and materials are required to control very dense infestations. Fixed costs for each controlled hectare, are unrelated to weed density, but do vary from treatment to treatment. These may include searching for weeds, setting up, transport, etc. Costs are listed in Table 1. The Efficacies of the four control options are provided in Table 2, representing the proportion of individuals removed from the different life-cycles stages, after natural or climatic mortality.

*Table 1: Control options* $u_i$ *and cost per hectare*

| $i$ | Control method ($u_i$) | Fixed costs ha$^{-1}$ | Variable costs ha$^{-1}$ | Total Cost ha$^{-1}$ (full density ) |
|---|---|---|---|---|
| 1. | No control | $0 | $0 | $0 |
| 2. | Burning | $15 | $0 | $15 |
| 3. | Chemical (poisoning) | $37.50 | $112.5 | $150 |
| 4. | Mechanical (blade ploughing) | $50 | $50 | $100 |

*Table 2: Efficacies of different chinee apple management options in upland zones. Values represent the proportion of the population that is removed in each life-cycle stage.*

| | | Control method (U) and efficacy | | | |
|---|---|---|---|---|---|
| | | No control (1) | Burning (2) | Chemical (3) | Mechanical (4) |
| Life-cycle stage | New seeds (NS) | 0 | 0.9 | 0 | 0 |
| | Seedbank (SB) | 0 | 0.9 | 0 | 0 |
| | Seedlings (J1) | 0 | 0.4 | 0.8 | 0.05 |
| | Small juveniles (J2) | 0 | 0.2 | 0.9 | 0.05 |
| | Medium juveniles (J3) | 0 | 0.01 | 0.9 | 0.5 |
| | Large juveniles (J4) | 0 | 0.01 | 0.9 | 0.5 |
| | Small adults (A1) | 0 | 0.01 | 0.95 | 0.95 |
| | Medium adults (A2) | 0 | 0.01 | 0.95 | 0.95 |
| | Large adults (A3) | 0 | 0.01 | 0.95 | 0.95 |
| | Largest adults (A4) | 0 | 0.01 | 0.95 | 0.95 |

## Curse of dimensionality

Individual woody weeds can exist in one of many life-cycle stages for many annual cycles. Additionally, individuals in different life-cycle stages will have different effects on pasture production. Even the efficacy of different management strategies is dependent on plants' life-cycle stage. This means that the state of the weed population must be described by the state of its life-cycle stages. However, this will result in a large number of possible combinations of states. For example, if there are ten life stages and each stage can have ten states, then the population can be one of a possible $10^{10}$ states. Presuming there are four control variables, and a 25 year timeline, DP requires $10^{22}$ iterations (Kennedy, 1988). In reality land managers do not need such detailed information on population structures to develop or implement fine scale management strategies. Decisions are more likely to be based on the total number of seeds, and damage from juveniles and adults. If the lifecycle is reduced to 3 main stages the total number of states declines to $10^3$ and the numerical problem only requires $10^8$ iterations. Put another way, if it takes four hours to solve the reduced problem, it will take about 45 billion years to solve the unreduced problem, highlighting the curse-of-dimensionality' (Bellman, 1957). Any chinee apple infestation is assumed to be in one of 5280 population states ($Z$), derived from 11 seeds, 24 juvenile, and 20 adult states. The number of seeds in seed life cycle stages, range between 0 and 256,000, and damages to pasture production from juvenile and adult states range between zero and 3.83 and 43.65 per cent.

Reducing the DP model from ten life cycles to three is complex, as vital ecological information may be lost. A transition probability function (TPF) is therefore used to capture the transition from one state to others, based on the full population dynamics of the plant and method of control, whilst decreasing the number of state variables. Let the reduced state variable be denoted by $z$, a function of $\mathrm{x}$. To derive $z$ requires two steps. First a '*sample set*' of various possible states over time ($\mathrm{x}_t$) is derived using the stage projection matrix model Eq.(1), capturing the population dynamics of new and recovering infestations, from different management scenarios. Then $\mathrm{x}_t$ is condensed into $z_t$ states. This requires a summation of the seeds and the area occupied by individual juveniles and adults. The next step is to map the relationships between the 'state variables' $\mathrm{x}_i$ and $z_i$, through the truncation of $\mathrm{x}_t$ and $z_t$. Values are stored in matrices X and Z, which will be used as lookup tables whilst solving the DP solution.

## Stochastic dynamic programming framework

The decision rule is now based on the current state and the probabilities of going into other states. The state of the infestation $z_t\{\mathrm{x}_t\}$ is known before selecting a control value ($u_t$), resulting in known current benefit $B\{z_t\{\mathrm{x}_t\},u_t\}$. However, with stochastic influences the future states of the weed population are unknown, $z_{t+1}\{\mathrm{x}_{t+1}\} = f\{\mathrm{x}_t, u_t, \Upsilon_{t+1}\}$, as are future rewards $R_{t+1} = \pi_{wf}\left(1 - D_{t+1}\{z_t\{x_t\}, u_t, \Upsilon_{t+1}\}\right) - C_{u,t+1}$. The expectation operator of episodic recruitment $\Upsilon_t$ has known probabilities that are assumed to be independently and identically distributed (*iid*). The equation of motion is replaced with a three-dimensional TPF whose element $P_{ijd}$ represents the transition probability from state *i* to state *j*, given decision *d* if control was applied ($u_d$). Let the Markovian probability matrix $P(u_d) \in \Re^n$ denote the state transition probabilities when policy $u_d$ is followed. The recursive equation with stochastic recruitment is:

$$V_t\{z_t\{x_t\}\} = \underset{u_t}{Max}\left[ B_t\{z_t\{x_t\},u_t\} + \beta \sum_{j=1}^{n} p_{ij}\{u_d\} V_{t+1}\{z_j\{x_j\}\} \right]; \qquad t = T,...,1 \qquad (4)$$

subject to

$$P_{ij}\{u_d\} = pr\{z_{t+1} = z_j \mid z_t\{x_t\} = z_i\{x_i\},\ u_t = u_d\} \qquad (5)$$

$\beta$ is the discount factor $(1+r)^{-1}$ for discount rate *r*. $V_t$ is the optimal return in the current period. The solution is solved backwards, from *t* = *T* to *t* = 1. The recursive Eq. (4) provides the optimal decision policy $U^*\{X\}$ for any given state.

# Applying control rules U*

As the problem is autonomous, based on Markov chain processes where the future is independent of the past, an suite of decision rules $U^*\{Z\{X\}\}$ can be obtained (Odom et al., 2003). This ‘package’ of control decisions can be used to manage any infestation based on its current state. $u^*$ is a function of the populations state (*z*) which in turn is a function of number and size of juvenile and adult plants as well as the number of seeds.

Figure 2 investigates how $U^*$ changes with respect to changes in the number of seeds, and the damages from juvenile and adult plants. For example, in Figure 2 (b), assuming there are few seeds and low levels of damage from adult plants (S(1) & A(1)), U* suggests No-control until the level of damage from juveniles to pasture production is > 1.6 per cent; after this point chemical control is used. The model was run with normal control costs and $\pi_{wf}$ = $20.

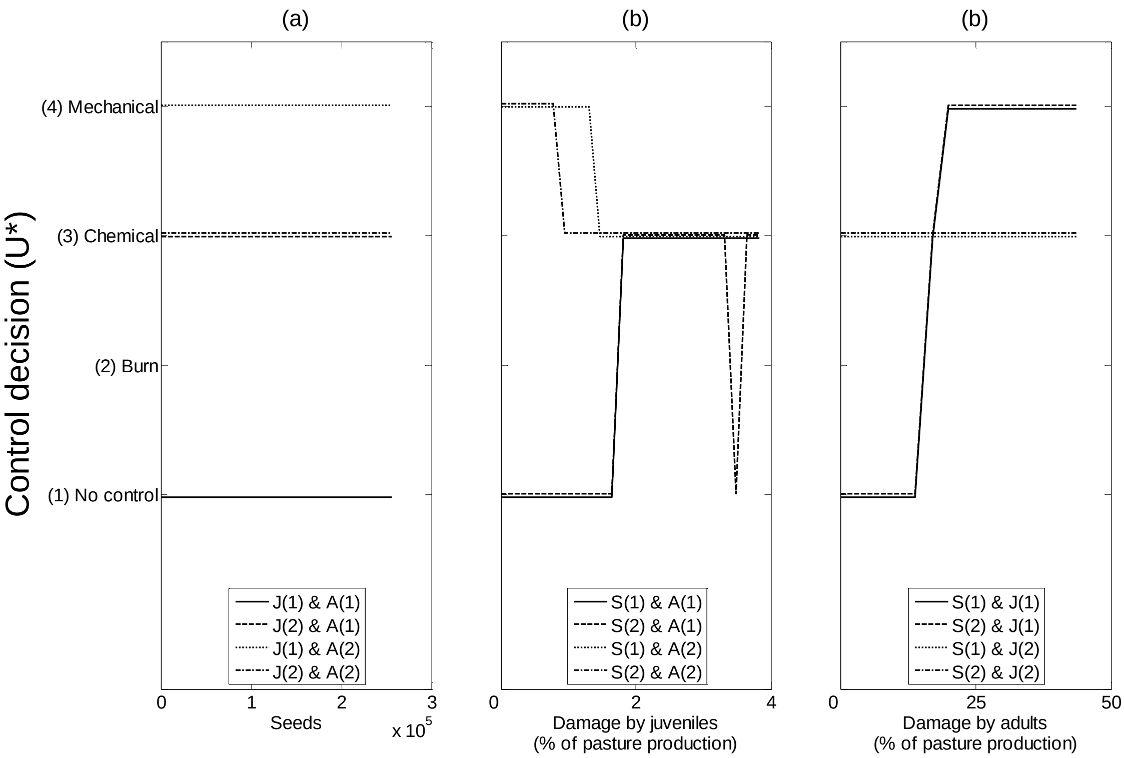


*Figure 2: Recommended control decision* $(U^*)$ *for changes in the quantity of (a) seed, (b) juvenile and (c) adult damages, whilst keeping other state parameters fixed. S(1) = low, S(2) = high seedbank; J(1) = low, J(2) = high juvenile damage; and A(1) = low, and A(2) = high adult damage.*

As there are three primary measures for the state of the infestation to consider (seeds, juvenile and adults damage), all three have been individually varied whilst keeping the others fixed (at either low or high values). Figure 2 (a) indicates that control decisions are not based on the number of seeds present. Although the number of seeds will affect how the population state will change it has little affect on control decisions ( U* ). Changes in both the level of damage to grazing pasture production by juveniles and adults will affect the control decision implemented, Figure 2(b) & (c). The figures for total weed density thresholds are not presented due to their likeness to damage thresholds.

As the control decision appears to be independent of the quantity of seeds present, a control decision table can be constructed based on the level of damage from both juvenile and adult plants (Figure 3). Note that the maximum level of damage to pasture production from juvenile plants (4%) is far less than that from adults (44%).

Figure 3 illustrates a number of thresholds between treatment types, being between No-control and Burning, No-control and Chemical control, and between Chemical and Mechanical control. Burning was only chosen as an option when damages from both juvenile and adults plants is low.

| Proportion of damage per ha by juveniles \ Proportion of damage per ha by adults | 0 | 0.017 | 0.063 | 0.102 | 0.138 | 0.170 | 0.199 | 0.226 | 0.251 | 0.274 | 0.296 | 0.316 | 0.335 | 0.369 | 0.385 | 0.400 | 0.414 | 0.427 | 0.437 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 1 | 1 | 1 | 1 | 1 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0.0006 | 1 | 1 | 1 | 1 | 1 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0.0026 | 1 | 1 | 1 | 1 | 1 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0.0044 | 1 | 1 | 2 | 1 | 1 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0.0061 | 1 | 1 | 2 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0.0078 | 2 | 2 | 2 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 0.0095 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 3 |
| 0.0113 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 3 |
| 0.0130 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 0.0147 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 0.0165 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 0.0181 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 0.0200 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 0.0208 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 0.0231 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | 3 | 3 |
| 0.0248 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | 3 | 3 |
| 0.0265 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | 3 | 3 |
| 0.0282 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | 3 | 3 |
| 0.0296 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 0.0314 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

| | |
|---|---|
| 1 | No control |
| 2 | Burn |
| 3 | Chemical |
| 4 | Mechanical |

*Figure 3: Control decision* ($U^*$) *with respect to changes in juvenile and adult damages, whilst ignoring the quantity of seeds (set at 190,000 seeds ha*$^{-1}$*).*

To see how the decision rule (U*) could affect a woody-weed population see Figure 4. figures (a), (b) and (c) indicate the damage, control, and NPVS over the same time period. The management decision rule (U*) was applied to a theoretical fully developed chinee apple population in an upland zone over a 100-year time period. This provides an indication of temporal weed damage, management decisions, and financial benefits. Eq. (1) was used to generate $x_t$ over time.

Results indicate that controlling chinee apple can increase profit margins, Figure 4(c). although it took 45 years to break even when control strategies were implemented. The solid line represents how the damage to pasture production is affected by climatic condition, when control is not administered. Damage to pasture production is decreased from about 45 per cent down to around 5 percent when control decision U* is administered, given normal weed management costs and $\pi_{wf}$ = $20.

The model also indicates that it accounts for climatic condition. The solid line represents the damage if control was not implemented, which in this simulation decreased over the first 40 year, and then increased. This has also been reflected in the controlled population where the infestation is initially managed and the not controlled for another 30 years. However, when the level of damage from the untreated population is high, indicating favourable climatic for chinee apple, so to is the frequency of control.

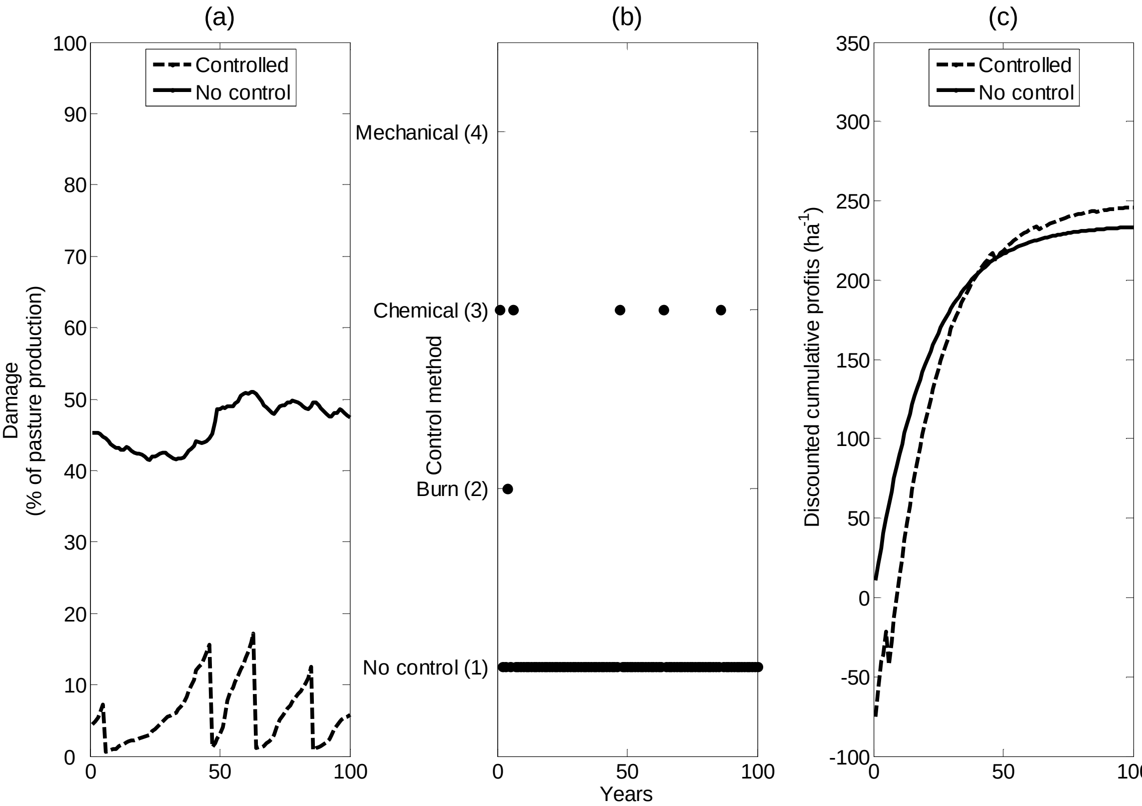


*Figure 4: Simulation is based on normal weed management costs and $\pi_{wf}$ = $20. (a) Shows the damage over time for both controlled and non-controlled infestations. (b) The types of control options implemented. (c) Discounted cumulative profits for controlled and non-controlled infestations.*

Results in Figure 4 are based on a set of randomly chosen climatic events. Therefore, every simulation run will result in different temporal weed damage, management decisions, and financial benefits. Future climatic events and their impacts are unknown. Therefore current decisions must be based on the probability of future benefits and costs. To investigate the probable benefits of using U* compared to ignoring the infestation; 400 simulations where run. The NPV of weed management is the difference in NPVs from managing the infestation and ignoring it. On average U* will result in weed management having a NPV of \$10.61 $ha^{-1}$, with NPVs ranging between \$-94.72 and \$22.87 $ha^{-1}$ (Figure 5). The lower and upper $10^{th}$ percentiles where \$0.17 and \$14.91 $ha^{-1}$, with U* resulting in a positive NPV 90 per cent of the time.

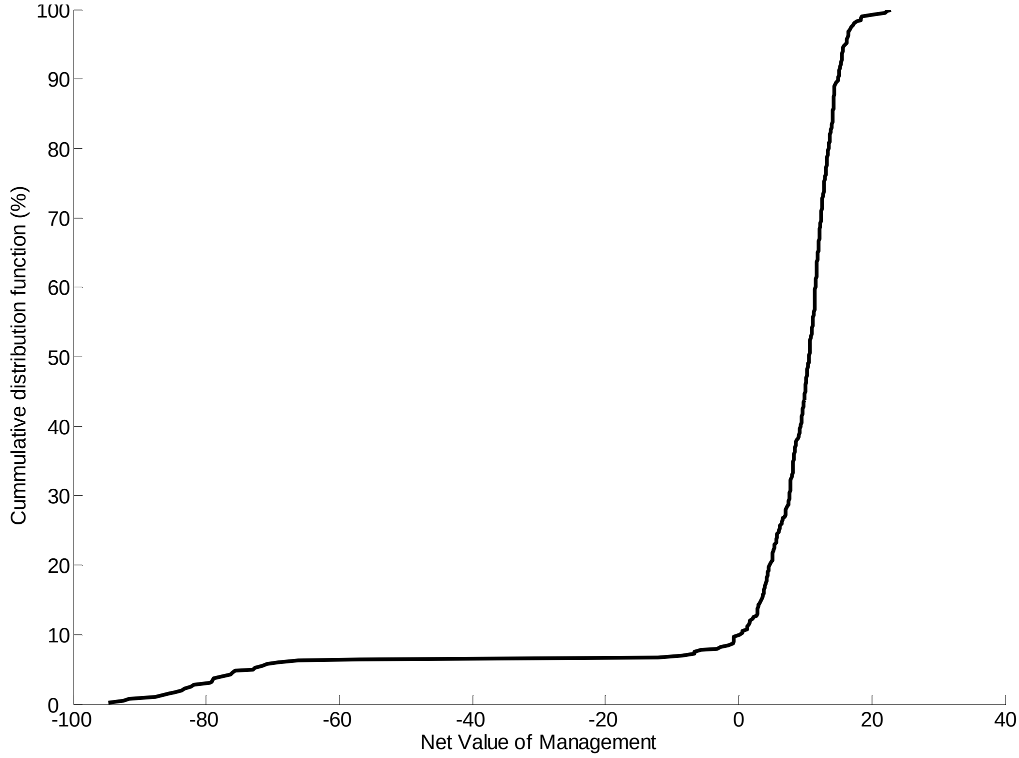


*Figure 5: Cumulative distribution function of net present values (NPV) of using control decision* U* *, based on normal weed management costs and* $\pi_{wf}$ *= \$20.*

Distribution of NPVs is due to episodic recruitment of chinee apple, as a result of climatic conditions. The lower ten per cent of Figure 5 with negative NPVs is due to adverse climatic conditions after a chosen method of weed management was implemented, i.e. higher rainfall and levels of recruitment.

# Evaluating the model with changing control costs and gross margins

The model thus far has resulted in higher NPVs for grazers if control decision U* is administered for normal weed management cost and $\pi_{wf}$ =\$20. The question now remains how will expected NPVs change with respect to changes in weed management costs and/or weed-free gross margins. Additionally, the dimensionally of the SDP model needs to be tested to ensure that U* retains the correct set of control decisions for different combinations of weed management costs and grazing gross margins.

To investigate the effects of changing weed management cost and $\pi_{wf}$, both parameters where changed simultaneously and the SDP was solved. U* was applied against the full stochastic population model (Eq.(1)) for 400 iterations, for each combination of parameters. Weed management costs are expressed as a percentage deduction of current costs. The expected (average) total NPVs are presented in Figure 6. The black area represents the expected NPVs when ignoring the infestation, and the grey area is when U* is administrated. As $\pi_{wf}$ increases there is a clear bifurcation between the expected NPVs from ignoring the infestation and controlling using U*. When $\pi_{wf}$ is low the expected NPVs from the U* mostly coincides with ignoring the infestation, as one of the control options of U* is to "No Control".

U* resulted in equal to or higher expected NPVs than ignoring the infestation except for a small section, see Figure 6, where $10 \le \pi_{wf} \le 15$. As weed-free grazing gross margins are set to be positive ($\pi_{wf}$ >$0), and the damage from weeds is never greater than one ($0 \le \psi \le \tau \le 1$), an unmanaged (ignored) infestation will always have a positive NPV. Likewise, stochastic climatic conditions will affect woody-weed population dynamics; however, it can never occupy more than 100 per cent of the area, and therefore the premise remains that the NPVs $\ge \$0$. Control decision U* should have the same or a higher NPV than ignoring the infestation. The lower NPVs from U* may be due to the truncation of the states being too coarse and the predicted transition between population states within the TPF is over estimated. For example, in reality the infestation may require two years to go from truncated State-A to State-B; however the TPF may have estimated that it takes one only year. One solution is to increase the number of states that is, the infestation is truncated into a state between these two states in the first year and then moves into State-B in the following year. However this will result in the curse-of-dimensionality (Bellman, 1957). Additionally, it may not be of any real benefit out in the field. The control decision U* only results in lower NPVs when the benefits of control are marginal, i.e. when the NPVs from U* are similar to ignoring the infestation. An alternative is to accept U* only when it has the same or greater NPV than for ignoring the infestation.

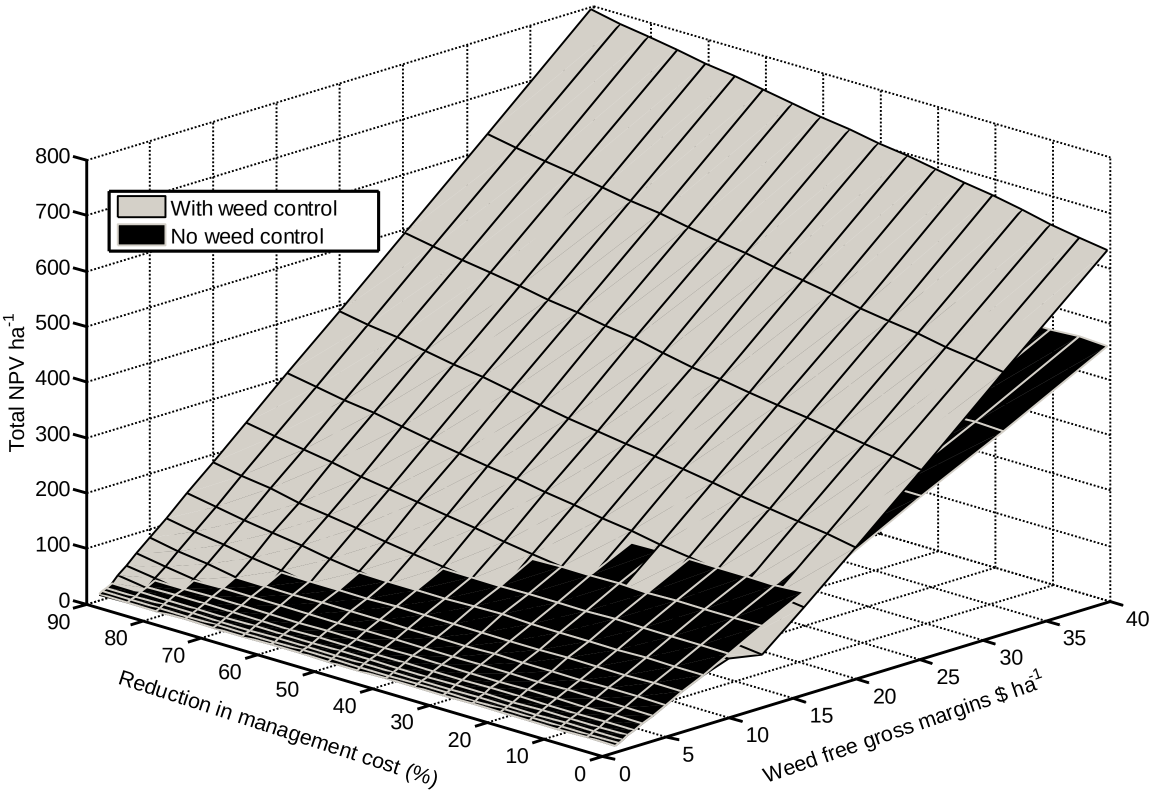


*Figure 6: Expected total NPVs from grazing with and without using U* with respect to changing weed management costs and $\pi_{wf}$.*

Now the expected benefits of woody weed management will always have NPVs $\geq$ 0. To determine the set of control decisions U* has one additional step. First, U* is defined; second, if expected NPV is positive it is accepted, if not it is rejected and the infestation is ignored. Based on this procedure a single threshold frontier between ignoring and managing the chinee apple infestation has been established, for given weed management costs and $\pi_{wf}$ (Figure 7).

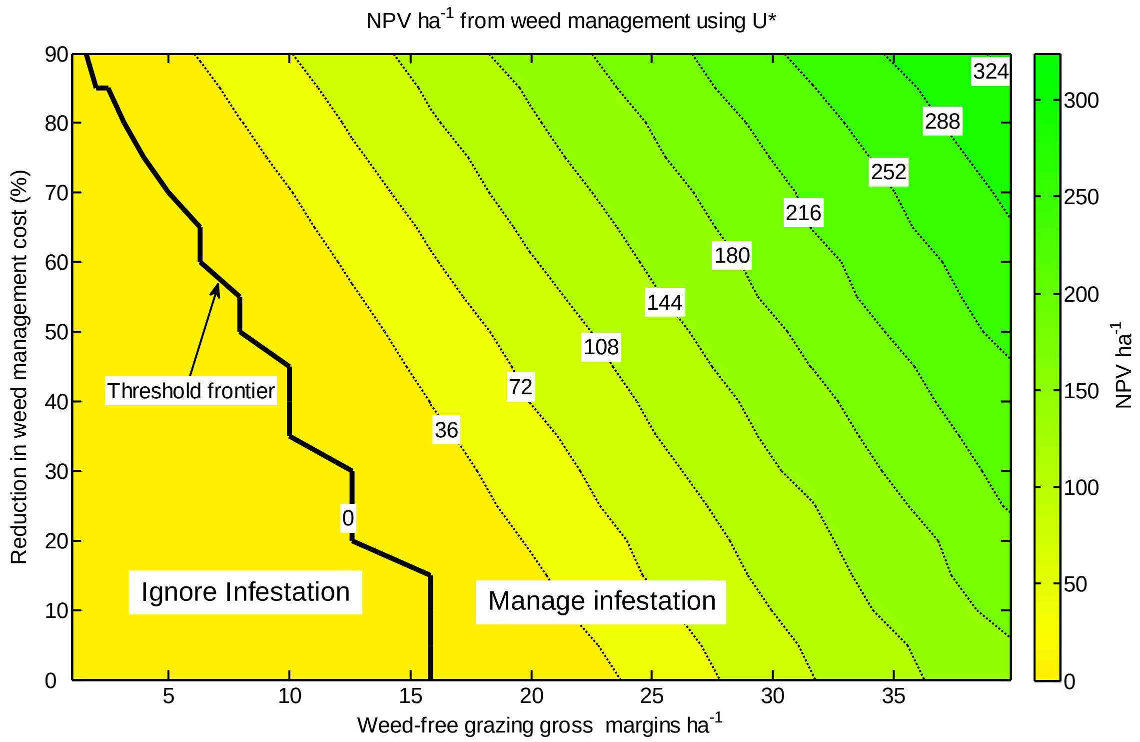


*Figure 7: Expected NPVs from using* $U^*\{X\}$. *The lines and their values represent expected NPVs from using U* .*

## Discussion

An economic SDP framework has been developed to help determine optimal control decisions for woody-weed management in the Australian rangelands. The framework captures the stochasticity of the system and complex population dynamics, whilst significantly reducing the 'curse of dimensionality', commonly encountered in DP models. This was achieved through the truncation of possible populations, which must weigh up cost of losing valuable population dynamics information.

This framework provides a contingent based management tool, i.e. given all the known information, if the weed population is in state x, then administer management decisions $u^*\{x\}$. This decision does not consider whether management has or will be undertaken in the future. It is Markovian in the true sense, management decisions are made independently at each stage in time (Nemhauser, 1966). As the weed population changes so too does the optimal control strategy. Therefore, there will be a number of control thresholds, for each type of weed control (Pannell, 1990), see Figure 4. In this research, the term "threshold frontier" (Figure 7) has been used to indicate the point after which an infestation is managed – below this point it is ignored. Beyond the threshold frontier the infestation will be controlled, but not actively treated every year.

Some of the simulations represented here have higher weed-free grazing gross margins ($\pi_{wf}$) than those currently achievable from grazing. Current $\pi_{wf}$ values are estimated to be around \$4.50, \$6.50, and \$13.21 $ha^{-1}$, for poor, average and good land conditions (MacLeod, 2007, pers. Comm.,). The premise for including higher gross margins was to show the relationship between different control variables, population dynamics of the infestation and climatic effects. Moreover, if only low, yet realistic, gross margins were used many of the results would suggest “No Control”. Under current weed management costs, $\pi_{wf}$ must be > \$15 to justify control if chinee apple. If control costs are halved the threshold would be $\pi_{wf}$ >10.

The results from this study indicate a minimum level of weed control and a maximum acceptable level of woody-weed density and damage. Including ecological and public costs is likely to further increase the intensity of weed management and decrease optimal weed densities. Additionally, potential spatial spread into non-infested areas was not included, nor was their potential damage and control costs. This indicates that these results give a minimum level of control for graziers.

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