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An Experimental Study of Learning and Limited Information in Games

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ABSTRACT

We report on experiments that tested the predictions of competing theories of learning in games. Experimental subjects played a version of the three-person matching-pennies game. The unique mixed-strategy Nash equilibrium of this game is locally unstable under naive Bayesian learning. Sophisticated Bayesian learning predicts that expectations will converge to Nash equilibrium if players observe the entire history of play. Neither theory requires payoffs to be common knowledge. We develop maximum-likelihood tests for the independence conditions implied by the mixed-strategy Nash equilibrium. We find that perfect monitoring was sufficient and complete payoff information was unnecessary for average play to be consistent with the equilibrium (as is predicted by sophisticated Bayesian learning). When subjects had imperfect monitoring and incomplete payoff information, average play was inconsistent with the equilibrium.

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1. INTRODUCTION

Since the concept of equilibrium in noncooperative games was proposed in the 1950s, theorists have modeled learning dynamics in games. Their work has addressed several interesting questions: What sorts of dynamic processes converge to equilibrium? How much information and coordination do these processes require? How large is the class of environments in which particular dynamic processes converge to equilibrium? Recent work has identified several environments in which particular learning processes cause players to converge, or fail to converge, to a Nash equilibrium.

Nash equilibrium is the primary solution concept in noncooperative game theory, even though the theory does not typically specify how players arrive at equilibrium. Limits on players' abilities to compute the equilibrium and predict their opponents' strategies may cause players to deviate from the Nash equilibrium prediction. Therefore, specifying the process by which players arrive at equilibrium is extremely important.[1]

We know from earlier laboratory experiments that players (or *subjects*) do not instantaneously arrive at the Nash equilibrium, even when it is unique; often, subjects with extensive experience fail to converge to equilibrium.[2] While these findings suggest that subjects might be learning about a Nash equilibrium, researchers have only recently specified precisely how or what these subjects might be learning. Because experiments based on competing theories of learning that make different predictions are rare, separating situation-specific behavior from learning dynamics is difficult. At best, existing research has identified some qualitative features of learning, or *adaptive behavior*, by subjects in specific experiments.[3]

In this paper, we present a series of experiments that test the predictions of competing theories of learning in games. We compare the predictive power of different economic theories under controlled conditions with human subjects motivated by money. We identify distinct learning processes proposed by theorists with particular informational assumptions and, where competing theories differ in their predictions, manipulate the information provided to players in laboratory experiments. In particular, we consider a situation in which the equilibrium strategy is considered difficult to learn: a mixed-strategy Nash equilibrium.[4]

Our results suggest that the amount of information available to players significantly affected whether play was consistent with the unique symmetric Nash equilibrium. When players could imperfectly observe the entire history of play (had *imperfect monitoring*) and had incomplete information about the payoffs of other players, average play was inconsistent with the equilibrium. In contrast, perfect monitoring of other players' choices was sufficient for average play to come close to a Nash equilibrium in many respects, even when players only had access to incomplete information about others' payoffs. Similar conclusions followed when players had perfect monitoring of other players' actions and the payoffs of all players were common knowledge. Our results show that heterogeneity in players' strategies persisted when players imperfectly observed the history of play and had incomplete information about the payoffs of other players. Heterogeneity in players' strategies disappeared with experience when players could perfectly monitor other players' choices and had incomplete information about other players' payoffs. No heterogeneity in strategies occurred among even inexperienced players when players had both perfect monitoring and complete payoff information.

Our results provide qualitative support for certain theories of learning in games. In particular, we find some support for theories which assert that complete information may not be necessary as long as players have perfect monitoring. Our results also suggest that perfect monitoring is sufficient, and in its absence, players may use adaptive behavior that does not converge, even though they might learn some features of the equilibrium strategy over time. While theory suggests that players may have difficulty learning equilibria, if they are provided with enough information and experience, the task of arriving at a Nash equilibrium may be simplified, even for a mixed-strategy equilibrium that incorporates subtle randomizations.

2. THEORETICAL MODELS AND EXPERIMENTS

Much theoretical work as well as some experimental tests now focus on learning in games. Explicit models have recently been developed in which partially informed or myopic agents interact repeatedly. No one—except possibly Boylan and El-Gamal (1993)—has tested these models, and our interest lies in the testable restrictions that the models impose on learning behavior in the laboratory. We test whether the experimental data are more or less consistent with the distributions predicted by particular learning models.

Jordan (1991) provides sufficient conditions for players following a Bayesian strategy process to converge to equilibrium. He specifies that players are uncertain about the characteristics of other players and that all players have perfect monitoring. As play proceeds, all players update their beliefs about the characteristics of other players. As they attempt to rationalize what characteristics of other players could be consistent with the optimality of those players' observed decisions, players gradually learn the game, and ultimately, their expectations converge to a Nash equilibrium. Similar results are derived by Kalai and Lehrer (1993). They present a closely related model of rational learning. However, both models require that players process a large amount of information every period. We refer to these models as *sophisticated Bayesian learning*. If we think that a more limited information-processing capacity is reasonable, we must investigate alternative theories of adaptive behavior.

A class of models that explicitly requires less information to be processed by players, called *naive Bayesian learning*, is presented by Fudenberg and Kreps (1993) and by Jordan (1993). The basic premise of naive Bayesian learning theory is that players regard other players not as strategists but rather as statistical processes against which a series of optimal responses must be chosen. Under naive Bayesian learning, players make decisions that are *myopically optimal:* they are the best possible responses to one-shot interactions, given players' beliefs. Over time, players revise their beliefs about these hypothesized processes, and so the time series of decisions changes. The issue is whether this form of adaptive behavior converges to equilibrium.

In fact, adaptive behavior may not converge, as Shapley (1964) observes. Jordan (1993) shows that even in simple situations, naive Bayesian learning generates a dynamic learning process that leads to cycles. While it is simple and intuitively appealing, naive Bayesian learning often fails to converge in games.[5] Fudenberg and Kreps (1993) point out that for mixed-strategy equilibria, players have difficulty learning other players' strategies. This finding suggests that games with mixed-strategy equilibria are an important starting point for studying how and if subjects learn to play equilibria.

Some prior experimental evidence exists on the form of learning in games. Boylan and El-Gamal (1993) analyze other researchers' data and conclude that fictitious play is a better predictor than a Cournot best response for a variety of experiments on normal form games. Van Huyck et al. (1991) examine the power of deterministic dynamic models in predicting the convergence of bargaining behavior in a game between two populations of players.

Two recent papers that examine the stochastic process of learning are closely related to our work. Mookherjee and Sopher (1993) conduct an experiment using the zero-sum game of two-person matching-pennies, in which even naive Bayesian learning is predicted to converge to equilibrium. They find that the distribution of play is close to a Nash equilibrium at the aggregate level and that knowledge about other players' payoffs has a significant effect on a player's strategy. But no simple learning model is found to explain the nature of the choices made. Bloomfield (1995) reports an experiment for a 2×2 game with a unique mixed-strategy equilibrium but no equilibria in pure strategies. Bloomfield finds stronger evidence of adaptive behavior when players know only the relative performance of their own strategies than when the performance of both players' strategies is common knowledge.

In this paper, we examine environments where simple rules of learning, such as naive Bayesian learning, lead to cycles. If the rule that best describes behavior is indeed naive Bayesian learning, then that rule imposes restrictions on the data. The distribution of players' choices should then be different from what it would be in a Nash equilibrium. Players' strategies converging to the Nash equilibrium would be a refutation of naive Bayesian learning. But failing to converge over a finite horizon is inconclusive—a skeptic could argue that an experiment in which convergence does not occur is too short. The average length of our experiments was 75 periods, which we consider long enough to greatly reduce the finite-horizon problem.

2.1. *The Three-Person Matching-Pennies Game*

In all the experiments in this study, subjects repeatedly played a version of the three-person matching-pennies game (see Figure 1), which was first described in Jordan (1993). While theoretical modelers usually study more complex environments, they could make much more convincing arguments if they could show that cognitive or experiential limitations which might inhibit convergence are present in the simple concrete situations used in their experiments. The simplicity of the three-person matching-pennies game makes it ideally suited to this task. Since the game is symmetric in decisions and players, the unique mixed-strategy Nash equilibrium is also symmetric. This structure reduces the computational burden on subjects, however, and could conceivably bias the experiment toward finding the equilibrium.[6]

The three-person matching-pennies game has three players (1, 2, and 3), each of whom can choose either heads (message A) or tails (message B). Each player's payoff depends on a comparison of the choice of the player with that of the player's neighbor, where 1's neighbor is 2, 2's neighbor is 3, and 3's neighbor is 1. The goal of each player is to avoid matching the player's neighbor. If 1 and 2 both choose heads (or tails), then 1 earns nothing; but if one of the players chooses heads and the other chooses tails, then 1 earns a penny. Similarly, 2's payoff is determined by comparing the decisions of 2 and 3; 3's, by comparing the decisions of 3 and 1. This game has a unique equilibrium in which all three players choose heads with probability 1/2 and tails with probability 1/2.

The game is formally described as follows. It has three players, and each player has a set of pure strategies S = {heads, tails} with $s_i \in S$, i = 1, 2, 3, denoting the choice of player i. Player i's payoff function, p_i: $S^3 \rightarrow \mathbb{R}$, is defined for each i as

$$(1) \qquad \pi_1(s_1,s_2,s_3) = \begin{cases} 0, & \text{if } s_1 = s_2 \\ 1, & \text{if } s_1 \neq s_2 \end{cases}$$

$$(2) \qquad \pi_2(s_1,s_2,s_3) = \begin{cases} 0, & \text{if } s_2 = s_3 \\ 1, & \text{if } s_2 \neq s_3 \end{cases}$$

$$(3) \qquad \pi_3(s_1,s_2,s_3) = \begin{cases} 0, & \text{if } s_3 = s_1 \\ 1, & \text{if } s_3 \neq s_1 \end{cases}.$$

The unique Nash equilibrium for this game involves each player randomizing between heads and tails with equal probability. Let p denote the probability with which heads is played. Then the unique Nash equilibrium is the point (1/2,1/2,1/2) in the unit cube in $\mathbb{R}^3$. If we partition $\{\text{heads, tails}\}^3$ into eight octants, the unique and symmetric mixed-strategy equilibrium leads to a uniform distribution of actions over the eight octants. In terms of empirical predictions, the Nash equilibrium implies that the marginal distribution of actions of each player will be uniform between heads and tails during each period; that is, $p_i = 1/2$ for all i = 1, 2, 3. Let p_i^t be player i's mixed strategy at time t. Then p_i^t and $p_j^{t'}$ are independent for all $i \neq j$, and for all t, t′, where $t \neq t'$. In addition, for all i, p_i^t is independent of the history of actions until period t.

By contrast, the Nash equilibrium is locally unstable under any naive Bayesian learning dynamic. As a consequence, for almost every initial set of beliefs, naive Bayesian learning approaches a limit cycle in which play is distributed over six octants, excluding the two in which all three players choose heads or tails. The details of this cycle are discussed in Jordan (1993).

2.2. *The Modified Three-Person Matching-Pennies Game*

Studying learning dynamics in the laboratory requires experiments with repeated interactions. However, repeating a one-shot game increases the set of equilibria. In particular, cooperative equilibria might be supported by threats to punish deviations by individual players. This possibility has been advanced as an explanation for the results of prisoners' dilemma experiments (see Kreps et al. (1982)).

This confounding effect should be minimized by designing a game in which such individual punishments are difficult to execute. Therefore, we modify the three-person matching-pennies game by replacing each player in the original game with a population of three players (see Figure 2).

The modified game is formally described as follows. It has nine players, and each player has a set of pure strategies S = {heads, tails} with $s_i \in S$, i = 1, 2, ..., 9, denoting the choice of player i. Players are assigned to populations, and each population consists of three players. Thus, if the players are serially numbered 1, 2, ..., 9, the first three (1,2,3) are assigned to population 1, the next three (4,5,6) to population 2, and the last three (7,8,9) to population 3.

Let $\sigma = (s_1, s_2, \ldots, s_9)$ be a vector of the players' choices in a particular period. Let $\chi(\cdot)$ be an indicator function that assumes the value one when its argument is true and zero when its argument is false. For any given period, payoffs to player i are as described here:[7]

$$\text{(4)} \qquad \pi_i(\sigma) \equiv \begin{cases} 10 \sum_{k=4}^{6} \chi(s_i \neq s_k), & \text{if } i = 1, 2, 3 \\ 10 \sum_{k=7}^{9} \chi(s_i \neq s_k), & \text{if } i = 4, 5, 6 \\ 10 \sum_{k=1}^{3} \chi(s_i \neq s_k), & \text{if } i = 7, 8, 9 \end{cases}$$

One Nash equilibrium for this game involves each player randomizing uniformly between heads and tails. A continuum of equilibria also exists, and all of these equilibria generate the same distribution of $\{\text{heads, tails}\}^3$ for each population. Let p_i represent the probability with which player i selects heads, and let s_i denote the choice of player i. A Nash equilibrium for this game is a vector $\{p_i\}_{i=1}^{9}$ such that each triple $\{s_{3i-2}, s_{3i-1}, s_{3i}\}$ is distributed as binomial (3,1/2), that is, with probability parameter 1/2 for i = 1, 2, 3. The unique symmetric Nash equilibrium involves p_i = 1/2 for all i. An example of an asymmetric equilibrium is $p_i = 1/2$ for all i = 1, 2, ..., 7; p_8 = 1; and $p_9 = 0$. Another asymmetric equilibrium is $p_i = 1$, $p_2 = 1/2$, $p_3 = 0$, $p_4 = p_5 = p_6$ = 1/2, $p_7 = 1$, $p_8 = 1/2$, and $p_9 = 0$. In the modified game, the nonconvergence of naive Bayesian learning that we described for the three-player game is preserved. Because of symmetry, one possible outcome of naive Bayesian learning is the limit cycle.

Each player in the modified game is matched with three neighbors (rather than one) in each period. This game reveals how well a given, unchanging set of players learns to play Nash equilibria. A consequence of our design is that the same players repeatedly encounter one another every period. This design could potentially lead to *repeated-game effects.*[8]

We think that repeated-game effects are not an issue in the modified game because the unique perfect equilibrium for that game is a repetition of the mixed-strategy Nash equilibrium every period. The unique equilibrium rules out trigger strategies, such as those proposed by Green and Porter (1984) or Benoit and Krishna (1985). Repeated-game effects could still be caused by incomplete information, however (see Ledyard (1986) and McKelvey and Palfrey (1992)).

Since we want to test whether players learn to play the Nash equilibrium in a game with incomplete information, we must minimize the likelihood of repeated-game effects, which could invalidate our statistical inference. This is why we have designed the modified game using populations. With such a design, repeated-game effects are less likely to be present and trigger strategies are more difficult to administer. In particular, punishing three players or inferring their types is less likely to be the driving force behind the experiment.

3. EXPERIMENTAL DESIGN

In the first of our three experimental environments, *perfect monitoring and complete payoff information* (PM-CP), subjects have complete information about the game. They observe the entire history of the game and know exactly how each player's payoffs are determined. This is a baseline environment in which convergence is a desirable property if a Nash equilibrium is to have some predictive content. In this baseline environment, both sophisticated and naive Bayesian learning are feasible.

In the second environment, *perfect monitoring and incomplete payoff information* (PM-IP), subjects have less information about the game. At the end of every period, they observe the actions of every player. Thus both sophisticated and naive Bayesian learning are feasible here too. A player does not know the incentives of the other players, but if the player's cognitive limitations are not too severe, he or she could learn what is needed to converge. Even if the experiment is too short, it yields a testable implication: if cycles occur, then naive Bayesian learning theory describes behavior better than sophisticated Bayesian learning theory.

In the third environment, *imperfect monitoring and incomplete payoff information* (IM-IP), subjects receive the least amount of information about the game. They know that their payoffs depend on a comparison of their actions with those of their three neighbors, but they lack sufficient information to infer the incentives of their neighbors. They also do not observe the actions of the other five players. At the end of each period, however, they do observe the choices of their three neighbors. The goal of experiments in IM-IP is to see whether subjects can learn the process that generates their neighbors' choices and respond to it. This environment is close to that posited by Fudenberg and Kreps (1993) and Jordan (1993). Naive Bayesian learning theory predicts that players will not converge and that players' choices over time will follow a particular cyclical pattern. In IM-IP, players do not have enough information for sophisticated Bayesian learning because this type of learning requires that players should be able to use the entire history of the game to form their strategies.

We conducted a total of 18 experiments:[9] five sessions in each of the PM-CP, PM-IP, and IM-IP environments with inexperienced subjects and three sessions repeated in the IM-IP environment using subjects who had been in an experiment involving IM-IP environments before.

Each session consisted of nine subjects recruited by telephone from the student population at the University of Minnesota between January 11 and May 15, 1993. Subjects were predominantly undergraduates, and all had previously participated in at least one unrelated individual-choice experiment. Subjects who participated were paid $5 plus earnings; subjects who were bumped due to overbooking were paid $8. Upon arrival at the laboratory, each subject drew a chip without replacement from a cup; this chip indicated seat placement in the experiment. Each subject was surrounded by partitions designed to maintain privacy. A no-talking rule was strictly enforced.

Experiments were conducted over a computerized network. Subjects were paid their earnings in cash over the duration of the experiment, which ranged from 45 minutes to an hour and a half.

Once subjects were seated, the instructions were read aloud by an experimenter. These instructions are reproduced in Appendixes A, B, and C. After the instructions were read and any questions were answered, the first period started. In each period, each subject selected either message A or message B, called here *heads* or *tails*. Once all nine subjects had confirmed their selections, each subject was informed about his or her payoffs and given relevant information (which varied according to the environment) about the choices of other players. The computer waited until all subjects had made their choices so that subjects could not reveal their identity by making early or late decisions. Experiments were run for n periods, where n was distributed uniformly between 70 and 79 periods.[10]

4. ECONOMETRIC METHODS

Our goal was to test whether the subjects in each experiment played the unique mixed-strategy equilibrium of selecting heads with probability 1/2, where the probability of heads was independent of all players' past actions.

One simple way to test this proposition is simply to look at the fraction of the times that heads was played in each experiment and test whether that fraction was equal to 1/2. But this test ignores the possibility of *strategic interdependence*—that a player's actions in one period could depend on the player's own past play or that of the player's opponents. In this section, we develop new tests of strategic interdependence.

Suppose we consider a matching-pennies game which has $m = 1, 2, \ldots, M$ populations of $n = 1, 2, \ldots, N$ players each. The payoff for a member of a population m in a given period is determined by comparing the person's play in that period to that of the members of population m + 1, using the rules described in Section 3, except that members of population M are compared to members of population 1.

Suppose, further, that each player bases his or her current play on, at most, the previous period's play by all players whose actions the player can observe. In this case, we will denote the conditional probability that player n of population m will play heads in the current period as $P_{m_n}(i_{1_1}, i_{1_2}, \ldots, i_{M_N})$, where i_{m_n} is an indicator variable that takes a value of one if player n of population m played heads in the previous period and zero if player n played tails.

Since the conditional probability that player n of population m will play heads in the current period could be different for each combination of plays that this player could observe in the previous period, potentially 2^{MN} different conditional probabilities exist of playing heads currently, one for each past observable state. In the three-person, three-population matching-pennies game, up to 512 different conditional probabilities exist that player n of population m will play heads. Further, the conditional probability of playing heads in the current period may differ for each player.

If we observed the conditional probabilities of playing heads for each player for each of the possible states in the previous period, we could compute the state-transition matrix from one period to the next as a function of those conditional probabilities.

Suppose that we are interested in the element of the $2^{MN} \times 2^{MN}$ state-transition matrix that indicates the conditional probability that all players will play heads in this period, given that they all played heads in the previous period. We could denote that probability as $P(1,1,\ldots,1 \mid 1,1,\ldots,1)$.

The key to computing this conditional probability as a function of the conditional probabilities that the individual players will play heads is to note that the current play of each player is conditionally independent of the current play of all other players. Thus

$$(5) \qquad P(1,1,\ldots,1 \mid 1,1,\ldots,1) = \prod_{m=1}^{M} \prod_{n=1}^{N} P_{m_n}(1,1,\ldots,1).$$

Similarly, the conditional probability of all players playing heads in the current period, conditioned on any arbitrary state of play in the previous period, is simply

(6) $$P(1,1,\ldots,1 \mid i_{1_1},i_{1_2},\ldots,i_{M_N}) = \prod_{m=1}^{M}\prod_{n=1}^{N} P_{m_n}(i_{1_1},i_{1_2},\ldots,i_{M_N}).$$

Equation (6) completely describes the last row of the state-transition matrix as a function of MN $\times$ 2^{MN} conditional probabilities for the individual players.

We can compute the entire state-transition matrix by noting that for player n of population m, the conditional probability of playing tails, given any arbitrary state of play in the previous period, is $1 - P_{m_n}(i_{1_1},i_{1_2},\ldots,i_{M_N})$. If j_{m_n} is an indicator variable that takes the value of one if member n of population m plays heads in the current period and zero if member n plays tails, then we can compute any element of the state-transition matrix

(7) $$P(j_{1_1},j_{1_2},\ldots,j_{M_N} \mid i_{1_1},i_{1_2},\ldots,i_{M_N}) = \prod_{\substack{m \\ \forall J_{m_n}=1}}\prod_{n} P_{m_n}(i_{1_1},i_{1_2},\ldots,i_{M_N}) \prod_{\substack{m \\ \forall J_{m_n}=0}}\prod_{n} (1 - P_{m_n}(i_{1_1},i_{1_2},\ldots,i_{M_N})).$$

Note that all the elements in the $2^{MN} \times 2^{MN}$ transition matrix are functions of the MN $\times$ 2^{MN} conditional probabilities used to compute the last row of the state-transition matrix.

4.1. *Restrictions*

Although this parameterization greatly reduces the number of free elements in the state-transition matrix, the number of free parameters is still far too large to estimate if the conditional probabilities are unknown. In the three-person, three-population matching-pennies game, the unrestricted state-transition matrix would have 262,144 parameters. Even the restricted version, described above, would still have 4,608 parameters. We must, therefore, use additional restrictions to reduce the number of free parameters in the state-transition matrix. We propose the following additional restrictions.

RESTRICTION 1: *Irrelevance of Within-Population Permutations. Permutations in the previous period's play within populations do not affect a player's conditional probability of playing heads in the current period. Only the total number of heads played in the previous period by a given population can affect the current conditional probability.*

Suppose we denote the total number of heads played by population m in the previous period as $r_m = \Sigma_{n=1}^{N} i_{m_n}$. Then this restriction means that $P_{m_n}(r_1,r_2,\ldots,r_M) = P_{m_n}(i_{1_1},i_{1_2},\ldots,i_{M_N})$.

RESTRICTION 2: *Within-Population Symmetry. Each player within a population has the same conditional probability.*

This restriction, when combined with Restriction 1, is exactly the same as de Finetti's (1990) concept of partial-column exchangeability.[11] Note that these two restrictions imply that

(8) $$P_m(r_1,r_2,\ldots,r_M) \equiv P_{m_n}(r_1,r_2,\ldots,r_M) = P_{m_k}(r_1,r_2,\ldots,r_M), \quad \text{for all } k, n \ni 1 \leq k, n \leq N.$$

RESTRICTION 3: *Circular Symmetry Across Populations. Since the payoff for a member of population m is based on a comparison of that member's play with the play of all members of population m + 1 and the payoff for a member of population M is based on a comparison of that member's play with the play of all members of population* 1, *it seems reasonable to assume that the conditional probability of playing heads in the current period for a member of population m,* $P_m(r_1,r_2,\ldots,r_M)$ *is the same as the conditional probability of playing heads in the current period for a member of population* $m + \ell = P_{m+\ell}(s_1,s_2,\ldots,s_M)$ *if and only if*

(9) $$s_m = r_{m+\ell}, \quad \text{for all } m,\ 1 \leq m \leq M - \ell,$$

(10) $s_m = r_{m-M+\ell}$, *for all m,* $m - \ell < m \leq M$.

So, for example, in the three-person, three-population matching-pennies game,

(11) $P_1(1,1,0) = P_2(1,0,1) = P_3(0,1,1)$.

Using Restrictions 1 and 2, we can limit the number of unique conditional probabilities that a member of population m will play heads in the current period to $(N+1)^M$, or the number of unique values of heads that a person can play in a given period raised to the power of the number of populations. Restriction 3 implies that the conditional probabilities for all other populations are just permutations of the conditional probabilities for population 1. Thus Restrictions 1, 2, and 3 reduce the number of free parameters in the three-person, three-population matching-pennies game from 4,608 to 64.

Although we have greatly reduced the number of free parameters in the state-transition matrix, so far we have implicitly allowed for considerable flexibility. First, we have allowed the players of each population to condition on the play of the players of *all* other populations. Second, we have assumed that members of each population have the *same* information about the past play of *every* population. We must impose additional restrictions that are consistent with the payoff and information structure of the three-person matching-pennies game, as described in Section 3.

RESTRICTION 4: *Irrelevance of Unmatched Population. Since the payoff for a member of each population is based on a comparison of his or her play with that of the next population, at most, the player will condition current play on the past play of his or her own population and that of the next population.*

In addition, the number of distinguishable states for each of these two populations may be different. Suppose that a player of population 1 can distinguish among a states for his or her own population and b states for the next population. Then the player can distinguish, at most, a $\times$ b different states in the play of the two populations during the previous period. This would restrict the total number of unique conditional probabilities to a $\times$ b. In the case of full information, as with the three-person, three-population matching-pennies game, a = b = 4, so 16 unique conditional probabilities exist.

Thus our most general unrestricted model will have only 16 free parameters, instead of 262,144. We will now discuss how we conduct our statistical inference and how we test whether the players in our experiments condition on the past play of their own population or the next population in determining the probability that they will play heads in the current period.

4.2. *Maximum Likelihood Estimation*

Suppose that in each period S possible states exist that describe the combination of play by all players. (In the three-person, three-population matching-pennies game, S = 512.) We have shown that the S^2 elements of the state-transition matrix can be described as a function of a $\times$ b free parameters. (In the full-information, three-person, three-population matching-pennies game, we need only 16 parameters.)

Let us denote the conditional probability of moving from state i in period $t - 1$ to state j in period t as P_{ij}. We know that for given values of the parameters β, P_{ij} is a function of those parameters. We also know that $\Sigma_{j=1}^{S} P_{ij} = 1$.

The likelihood of observing a given transition pair, given the parameters β, is

$$(12) \qquad \mathscr{L}(I,J \mid \beta) = \prod_{i=1}^{S} \prod_{j=1}^{S} P_{ij}^{d_{ij}}$$

where d_{ij} is an indicator variable that takes on a value of one if and only if (i,j) = (I,J) and zero otherwise. If we observe a vector of N transition pairs, X, the log-likelihood of the sample is

$$(13) \qquad \ell(X|\beta) = \sum_{i=1}^{S} \sum_{j=1}^{S} C_{ij} \ln P_{ij}$$

where C_{ij} is the count of the number of transitions from i to j in the sample. All estimates we present in Section 5 are the result of maximizing the log-likelihood function (13).

4.3. *Tests of Strategic Interdependence*

We propose three likelihood ratio tests to determine the extent of strategic interaction in our experimental data. All of these tests assume that players use the same strategy throughout the experiment. These tests employ successively stronger restrictions to determine whether players use the unique mixed-strategy equilibrium in each case. For ease of exposition, we restrict our discussion to the cases of experiments in PM-CP and PM-IP, where the players have full information about the past play of all other players. All of these tests assume Restrictions 1–4 discussed above.

4.3.1. *Mixed-Strategy Equilibrium*

The simplest test of mixed-strategy play is to restrict the model further than Restrictions 1–4 by assuming that each player plays heads with the same probability regardless of the past play of his or her population or the next population. This leaves only one parameter to estimate: the unconditional probability of playing heads. We can test whether each player plays the mixed-strategy equilibrium by comparing the maximized value of the likelihood function in the one-parameter model with the value of the likelihood function obtained by restricting the probability of playing heads to 1/2. Two times the log-difference of the likelihood values should be distributed asymptotically as a χ^2_1 random variable if each player plays the mixed-strategy equilibrium.

4.3.2. *Past Play of the Player's Own Population*

If players are able to unconditionally randomize their play, we can test the next level of randomization: Do players adequately randomize on their own population's past play? Suppose we allow different probabilities that a player will play heads in the current period for each of the values of the number of heads played by that player's population in the last period: 0, 1, 2, or 3. Based on this assumption and Restrictions 1–4, we have four free parameters in our model. If each player plays the mixed-strategy equilibrium, no statistically significant difference should exist between the maximized value of the likelihood for this four-parameter model and the value of the likelihood if the probability of playing heads is 1/2. Two times the log-difference of these likelihood values should be distributed asymptotically as a χ^2_4 random variable if each player's strategy is conditionally independent of the past play of his or her own population.

4.3.3. *Past Play of Both the Player's Own and the Next Population*

If players are able to unconditionally randomize and randomize with respect to their own population, we can conduct our final test: Do players simultaneously randomize with respect to the past play of both their own population and the next population?

In the three-person, three-population matching-pennies game, when a player has full information about the past play of all players, the unrestricted model described in Section 3 allows for 16 different probabilities that a player will play heads in the current period—one for each combination of the total number of heads played in the previous period by the player's population and the next population. Of course, in the mixed-strategy equilibrium, all of these probabilities should be the same and each should be 1/2.

If we maximize the likelihood function of this 16-parameter model, we can form a likelihood ratio test by comparing the maximized value of the likelihood function for this model with the value of the likelihood function, assuming that the probability of playing heads is 1/2. Two times the log difference of the likelihood values should be distributed asymptotically as a χ^2_{16} random variable if each player's strategy is conditionally independent of the past play of his or her own population and the next population.

5. EMPIRICAL RESULTS

In Figures 3, 4, and 5, we graph a typical nine-subject session for each of the three environments. A solid square indicates that a subject chose message B (tails); a space, that he or she chose message A (heads). Thus a solid bar indicates a run of message B's of some length, and a gap indicates a run of message A's. One conclusion which follows immediately from the raw choice data is that the naive Bayesian prediction of run lengths increasing (asymptotically, according to the Fibonacci ratio) is not supported.

Our failure to find the pattern of runs predicted by the naive Bayesian model indicates that we should make a closer examination of the mixed-strategy prediction, i.e., $p = 1/2$. To do this, we aggregate our individual data in two ways. First, we present the empirical distribution of runs in individual choices, and second, we look at average period earnings.

We define a run of length n to be n-consecutive choices of heads or n-consecutive choices of tails, followed by a switch to the opposite choice, where these choices are all made by the same individual. Within each environment, we compare the frequency of actual runs across subjects, sessions, and periods. Figures 6, 7, and 8 present these frequencies for all periods, the first half of each session, and the second half of each session. We compare the actual frequency of runs to the expected frequency distribution under the maintained assumption that subjects are playing a mixed strategy with $p = 1/2$. Note that this frequency distribution is multinomial with probability weights of $(p,p^2,p^3,\ldots)$.

In PM-CP, we find that the actual distribution of runs is close to the expected distribution. In PM-IP and IM-IP, however, the actual distributions differ from the expected distributions in systematic and similar ways. These differences, which are stronger in the second half of the data, mainly consist of a much lower frequency of runs of length one and a much higher frequency of runs of length three. Thus the pattern of runs that would be observed if the mixed-strategy equilibrium were being played are not present in the data.

Comparing average earnings per period, we find further evidence for systematic differences between our baseline environment, PM-CP, and our environments PM-IP and IM-IP. In Figure 9, we plot average earnings for all subjects and all sessions within a given environment. We also indicate the 95 percent confidence interval for these means. Note that if subjects are playing the mixed-strategy equilibrium, then expected period earnings are 15 cents, while the naive Bayesian limit cycle results in a higher expected period earnings of 20 cents. The baseline PM-CP results in significantly lower earnings than in the other environments. And the payoff in both of those environments significantly differs from either the 15-cent prediction or the 20-cent prediction.

Tables I–III show the results of the econometric tests described in Section 4 for the three sets of experiments. These tables show how differences in the information available to players affect their ability to play the unique symmetric mixed-strategy equilibrium and whether players learn, in the sense that they are better able to play the mixed-strategy equilibrium in the second half of the experiment than in the first.

In Table I, which shows the results for the first set of experiments, the first row displays the tests for unconditional randomization. With perfect monitoring and complete payoff information, players in this set of experiments unconditionally randomized in each half of the experiment and for the experiment as a whole.

The second row of Table I displays these tests for randomization with respect to the past play of a player's own population. Players in this set of experiments randomized with respect to the past play of their own population in both the first and the second half of each experiment. But, for the experiments as a whole, randomization is rejected at the 5 percent level. This rejection for the entire sample can occur, even though randomization was not rejected for each half of the experiment, because this test has more power—the same number of parameters are estimated with more observations. Therefore, we can detect some deviation from complete randomization in a sample of over 3,300 observations.

The third row of Table I displays the tests for simultaneous randomization with respect to the past play of a player's own population and the next population. The results shown here seem anomalous. The first column of the third row shows that in the first half of each experiment, players adequately randomized. But the second column shows that we can reject at the 5 percent level the hypothesis that they adequately randomized in the second half of each experiment. And the third column shows that they failed to adequately randomize over each experiment, as a whole. These results suggest that players forgot, rather than learned, how to randomize. The results also suggest that for the experiment as a whole, players' strategies depended on the past play of the next population, which is a violation of mixed-strategy equilibrium.

In Table II, which shows the results for the second set of experiments, the first row displays the tests for unconditional randomization. With perfect monitoring and incomplete payoff information, players in this set of experiments unconditionally randomized in each half of the experiment and for the experiment as a whole.

The second row of Table II displays the tests for randomization with respect to the past play of a player's population. Players in this set of experiments randomized in this respect in each half of the experiment and for the experiment as a whole.

The third row of Table II displays the tests for simultaneous randomization with respect to a player's own population and the next population. Players in this set of experiments simultaneously randomized in each half of the experiment. But we can reject at the 5 percent level the hypothesis that players adequately randomized over the entire experiment.

The most interesting result from this set of experiments is that no statistically significant difference existed in players' abilities to perform any particular randomization between the first and the second half of each experiment. Players appeared to learn quickly enough so that they adequately randomized in the first half of each experiment, on average, even though they had incomplete information about payoffs. In fact, the players in this set of experiments did a better job of randomization than the players in the first set of experiments, who had both perfect observability and complete information about payoffs.

Table III shows the results for the third set of experiments. The first column of the first row shows that in the first half of each experiment, players could not unconditionally randomize their play with imperfect monitoring and incomplete information; that is, we can reject the hypothesis of unconditional randomization at the 5 percent level. But the second column of that row shows that players learned to unconditionally randomize by the second half of the experiment. This is evidence of adaptive behavior.

However, even in the second half of the experiment, players were not able to randomize along other dimensions. The second and third rows of the second column show that we can reject at the 5 percent level the hypothesis that players successfully randomized with respect to their own past play or that they simultaneously randomized with respect to their own past play and the past play of the next population.[12]

6. TESTS FOR HOMOGENEITY

Our analysis thus far has assumed that all players have *homogeneous strategies;* that is, the probability that a player will play heads conditional on the past play of the player's own population and the next population is the same for all players. This assumption may be extremely unrealistic at the start of an experiment for environments with either imperfect monitoring or incomplete payoff information.

A test of whether players attain the mixed-strategy Nash equilibrium that controls for potential heterogeneity would be stronger than the tests we have used thus far. If players learned successfully about the structure of the game, they would eventually have homogeneous strategies: every player would play the unique mixed-strategy equilibrium. In this section, we propose a test of homogeneous strategies. We test not only whether players, on average, attain the mixed-strategy equilibrium, but also whether they all attain it.

Let us consider one way to model persistent individual heterogeneity in choice probabilities: a probit random-effects model. (This model was first considered by Heckman (1981).) Suppose each player acts as if his or her probability of playing heads in a given period is affected by three components: (i) a constant α, which is the same for all players; (ii) a player-specific draw of a mean-zero random variable μ_{m_n}, which is constant for a player across time but uncorrelated across players; and (iii) a period-specific mean-zero random variable $\epsilon_{m_n,t}$, which is independent across players and time periods. For a given period t, player m_n will choose heads if

$$(14) \qquad \alpha + \mu_{m_n} + \epsilon_{m_n,t} > 0$$

and will choose tails otherwise. If, in addition to the previous assumptions about μ_{m_n} and $\epsilon_{m_n,t}$, we assume that μ_{m_n} and $\epsilon_{m_n,t}$ are normally distributed, we can estimate (14) using a probit random-effects model. Of course, if the players are playing the mixed-strategy Nash equilibrium, the estimated value of α should also be zero. In addition, if players have homogeneous strategies, then $\text{var}(\mu) = \text{var}(\mu_{m_n})$ for all m, n should be zero. We can thus perform a joint test of homogeneity and mixed-strategy Nash play by testing whether $\alpha = 0$ and $\text{var}(\mu) = 0$.

The results of this test are displayed in Table IV for each of the three sets of experiments. We perform the test using data from each set of experiments as a whole and from the first and second halves of the experiments, to see whether any type of learning occurs. All estimates presented use Keane's (1994) panel-data method-of-simulated-moments estimator, which has been shown in a Monte Carlo study to be superior to other probit random-effects estimators. The results in the first row of Table IV show that when players had perfect monitoring and complete payoff information, the test fails to reject the null hypothesis of a mixed-strategy Nash equilibrium play and homogeneous strategies for the experiments as a whole or for each half separately. This result suggests that in this environment *all* players immediately recognized the mixed-strategy equilibrium and used it in all periods; no learning was necessary.

The results in the second row of Table IV show that when players had perfect monitoring but incomplete payoff information, the test fails to reject the null hypothesis of mixed-strategy Nash equilibrium play and homogeneous strategies for the combined sample. However, the test results for the two halves of the sample are very different: the hypothesis of mixed-strategy play and homogeneity is rejected for the first half of the experiments, but not for the second half.

Since we already know from Table II that the hypothesis of mixed-strategy play alone is not rejected in either half of the PM-IP experiments, the rejection of the joint hypothesis for the first half of the PM-IP environment must be due to heterogeneity in strategies across players. Such heterogeneity is perfectly rational at the beginning of these experiments since the players did not know each others' payoffs.

However, as players acquired more information about the likely characteristics of other players in the experiment by observing the choices of the other players, the players' strategies appear to converge to a homogeneous strategy. One way to interpret the lack of rejection of the joint hypothesis of mixed-strategy play and homogeneity in the second half of the PM-IP experiments is that by the second half of the experiment, *all* players were playing the mixed-strategy equilibrium. This behavior is consistent with sophisticated Bayesian learning.

Table IV also shows that players had more difficulty in uniformly playing the mixed-strategy equilibrium when they had imperfect monitoring and incomplete payoff information. Note that the joint hypothesis of mixed-strategy play and homogeneity is completely rejected for the IM-ID environment, which did not happen for either of the other two environments. The joint hypothesis is also rejected at the 1 percent level for the first half of each experiment and at the 10 percent level for the second half of each experiment. In fact, the joint hypothesis is almost rejected at the 5 percent level for the second half of each experiment. This poor performance of the joint hypothesis suggests that when players are provided with not only incomplete payoff information, but also imperfect monitoring, they have difficulty in uniformly playing the mixed-strategy equilibrium.

7. CONCLUSIONS

Our analysis of experimental results from the three-person matching-pennies game suggests that the type of information available to subjects significantly affected whether play was consistent with the unique symmetric mixed-strategy Nash equilibrium. The information also had a significant effect on whether the choices of subjects in a particular population were conditionally independent of the past play of subjects in their own population or subjects in the neighboring population.

For the imperfect monitoring and incomplete payoff information environment (IM-IP), we reject the hypothesis that players randomized unconditionally in the first half of the experiment, but we fail to reject unconditional randomization in the second half. This finding suggests that players learned one implication of a mixed-strategy Nash equilibrium. However, we reject the hypothesis that players' choices were independent of the past play of the neighboring population and that of their own population. This observation is consistent with learning theories which suggest that with imperfect monitoring and incomplete payoff information, players may use adaptive behavior that does not converge to a Nash equilibrium.

For the perfect monitoring and incomplete payoff information environment (PM-IP), for each half of the experiment, we fail to reject the hypothesis that players randomized unconditionally or randomized with respect to the past play of their own population or that of the neighboring population. This finding suggests that players' behavior satisfies several key features of a mixed-strategy Nash equilibrium. Further, the first and second halves of the experiment are qualitatively similar. This observation is consistent with theories of sophisticated Bayesian learning, which suggest that perfect monitoring may suffice for convergence to a Nash equilibrium, even in the presence of incomplete payoff information. However, we reject the hypothesis that players' choices were independent of the past play of the neighboring population and that of their own population for the entire experiment.

For the perfect monitoring and complete payoff information environment (PM-CP), which served as our baseline environment, for each half of the experiment, we fail to reject unconditional randomization or randomization with respect to the past play of a player's own population. This finding is also consistent with some of the predictions of sophisticated Bayesian theories of learning. However, we find that players were able to simultaneously randomize with respect to the past play of their own population and that of the neighboring population in the first half, but were unable to do so in the second half. This failure suggests that play in the second half of this experiment, despite perfect monitoring and common knowledge of payoffs, was less consistent with a Nash equilibrium in the second half of the experiment than in the first.

Our tests for individual heterogeneity suggest that in the first half of PM-IP and IM-IP, players were heterogeneous in their choices. However, for both environments, we cannot reject the joint hypothesis of homogeneity and Nash equilibrium play in the second half of each experiment. We interpret this as evidence of players learning one aspect of the equilibrium strategy, namely, unconditional randomization with probability 1/2. For PM-CP, we conclude that all players immediately recognized the equilibrium strategy and did not require time to learn it. Even though players learned homogeneous unconditional Nash equilibrium play in the second half, we can reject the joint hypothesis over the entire experiment for IM-IP.

These results provide qualitative support for certain theories of learning in games. In particular, the experiments provide insights into the issue of how much information subjects need to arrive at a Nash equilibrium. Results from the PM-IP environment provide support for theories of sophisticated Bayesian learning since these results suggest that perfect monitoring is sufficient. Results from the IM-IP environment further suggest that perfect monitoring is necessary, and without it, players may use adaptive behavior that does not converge. Results from the PM-CP and PM-IP environments suggest that the problem of learning mixed-strategy equilibria may be considerably simplified, if not solved, by providing historical information on other players' choices and repeating the game several times.

Our conclusions are limited, however, since we have not explicitly analyzed the dynamics of the process of convergence to a Nash equilibrium and how significantly players' choices deviate from the best response. Developing a deeper understanding of the dynamics of learning seems to be a promising area for further research.

In summary, by studying a game which is known to be locally unstable with respect to all naive Bayesian learning dynamics, we can examine the degree of sophistication in the learning dynamics of cash-motivated players. By analyzing our data, we can develop specific maximum-likelihood tests for the independence conditions implied by mixed-strategy behavior. Our results suggest that the amount of information available to players significantly affects their ability to achieve the independence conditions predicted by the mixed-strategy Nash equilibrium. Furthermore, we find support for theories of sophisticated Bayesian learning which assert that complete information may not be necessary as long as players have perfect monitoring.

Footnotes

[1]This issue is discussed by Roth and Erev (1993) and Davis and Holt (1993).

[2]Roth and Erev cite the ultimatum game in Roth et al. (1991).

[3]One example of an experiment showing qualitative features of learning is found in the work of Van Huyck et al. (1991), who test for evolutionarily stable strategies.

[4]Arguments about why a mixed-strategy equilibrium may be difficult to learn are presented in Fudenberg and Kreps (1993) and Jordan (1993).

[5]It is well known that fictitious play, a variant of naive Bayesian learning, converges in zero-sum games (shown by Brown (1951) and Robinson (1951)) and in 2×2 two-person nonzero-sum games (Miyasawa (1961)) but not in a simple 3×3 two-person game (Shapley (1964)).

[6]Richard Boylan has informed us of an unpublished experiment in which a skew-symmetric mixed-strategy Nash equilibrium was not attained.

[7]Payoffs are in terms of dimes rather than pennies—hence the prefix 10.

[8]While the repetition could potentially introduce repeated-game effects into the dynamics of the experiment, an alternative less vulnerable to this problem would involve repeated interactions among a changing set of players. We think that players might have more difficulty learning in this environment than in a stationary environment.

[9]To avoid leakage of information within the subject pool, all experiments were run sequentially: first, the five inexperienced IM-IP experiments; second, the three experienced IM-IP experiments; third, the five PM-IP experiments; and fourth, the five PM-CP experiments.

[10]This was done to prevent subjects from predicting the last period.

[11]See de Finetti (1990). Exchangeability is partial because observations are exchangeable only within a population. Exchangeability does not extend to rows because, in our unrestricted model, we are allowing current play to potentially depend on past play.

[12]Note that we test for randomization with respect to the past play of a player, not that of the player's population. This difference arises because the actions of the other members of a player's team cannot be monitored. Imperfect monitoring also affects the number of degrees of freedom in these tests because states that were distinguishable in the previous set of experiments are indistinguishable in this set of experiments.

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TABLE I[a]

PERFECT MONITORING AND

COMPLETE PAYOFF INFORMATION

Test	df	First half	Second half	Full experiment
1	1	1.040	0.890	0.010
		(0.308)	(0.345)	(0.920)
2	4	7.640	4.630	9.850
		(0.106)	(0.327)	(0.043)
3	16	19.200	41.100	69.000
		(0.256)	(0.001)	(0.000)
		N = 1,647	N = 1,665	N = 3,312

[a]Significance levels in parentheses.

TABLE II[a]

PERFECT MONITORING AND

INCOMPLETE PAYOFF INFORMATION

Test	df	First half	Second half	Full experiment
1	1	0.100	1.000	0.880
		(0.752)	(0.317)	(0.348)
2	4	5.000	6.100	8.540
		(0.287)	(0.192)	(0.074)
3	16	14.340	25.640	29.960
		(0.573)	(0.059)	(0.019)
		N = 1,611	N = 1,638	N = 3,249

[a]Significance levels in parentheses.

TABLE III[a]

IMPERFECT MONITORING AND INCOMPLETE PAYOFF INFORMATION

Test	df	First half	Second half	Full experiment
1	1	5.150 (0.023)	0.950 (0.330)	5.220 (0.022)
2	2	5.830 (0.054)	9.590 (0.008)	12.360 (0.002)
3	8	51.870 (0.000)	82.990 (0.000)	128.860 (0.000)
		N = 2,610	N = 2,664	N = 5,274

[a]Significance levels in parentheses.

TABLE IV

JOINT TESTS OF HOMOGENEITY AND MIXED-STRATEGY NASH EQUILIBRIUM PLAY

		Test Statistics		
Environment	df	First half	Second half	Full experiment
PM-CP	2	0.007 (0.999)	0.008 (0.999)	0.086 (0.958)
		N = 1,647	N = 1,655	N = 3,312
PM-IP	2	6.302* (0.043)	0.516 (0.773)	2.961 (0.228)
		N = 1,611	N = 1,638	N = 3,249
IM-IP	2	23.374* (0.000)	5.899 (0.052)	12.977* (0.002)
		N = 2,610	N = 2,664	N = 5,274

*Indicates significance at the 5 percent level.

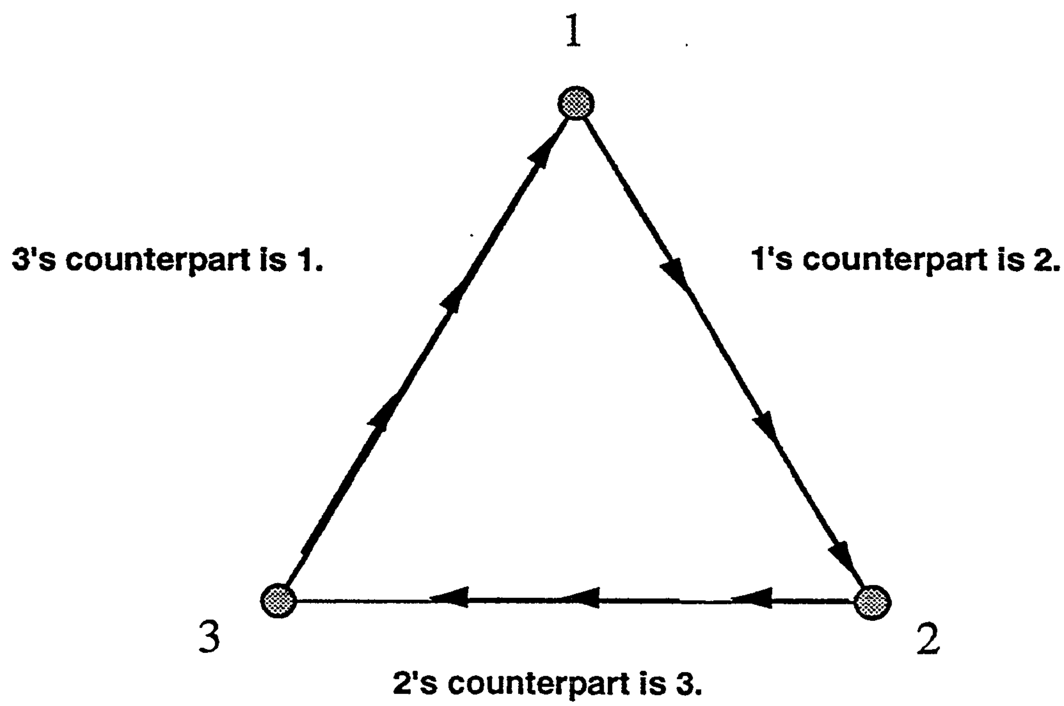


FIGURE 1. — The three-person matching-pennies game.

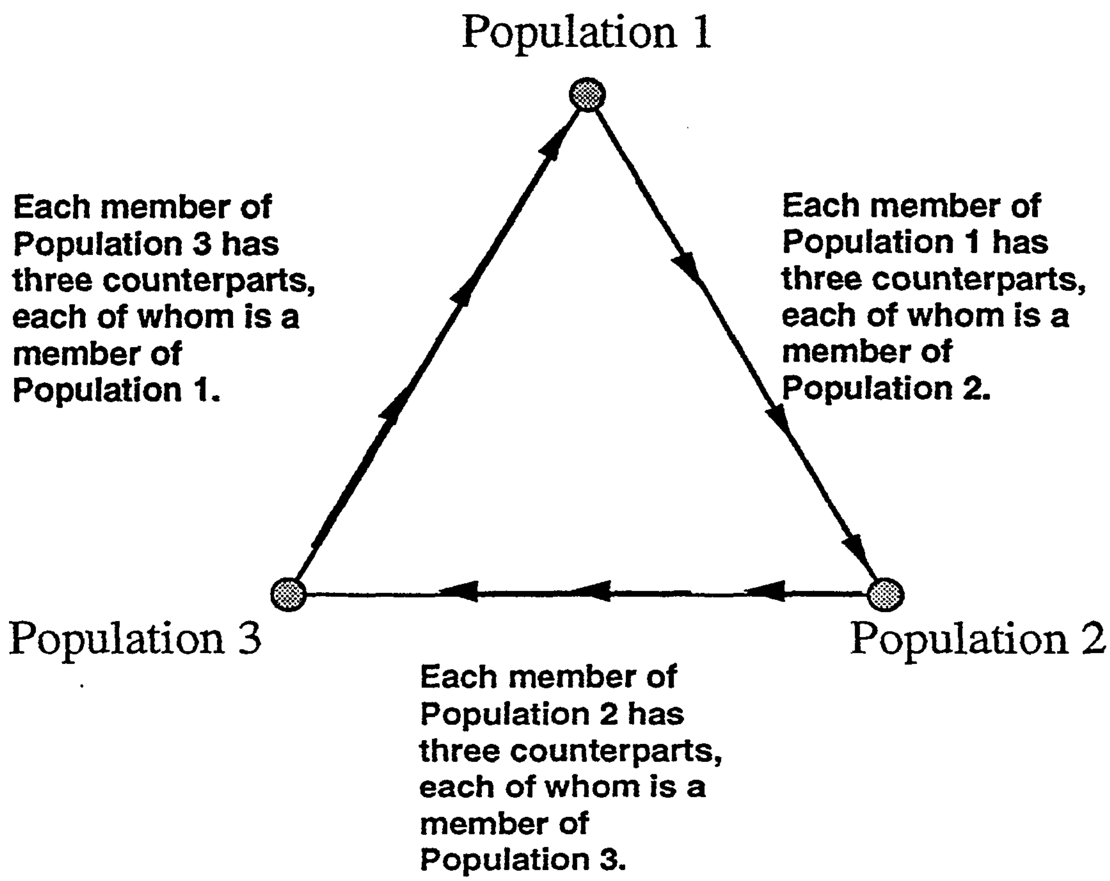


FIGURE 2. — The population version of the three-person matching-pennies game.

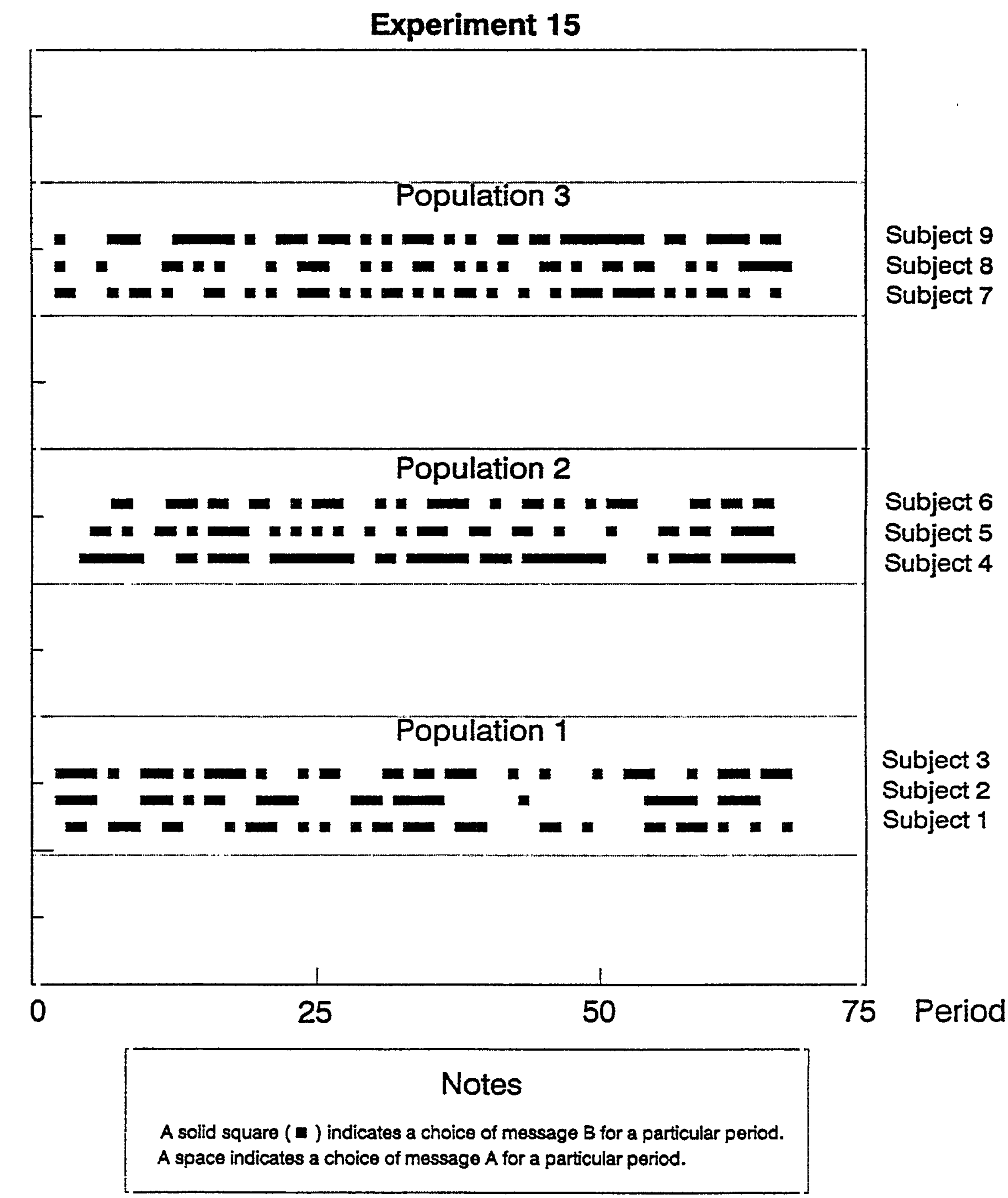
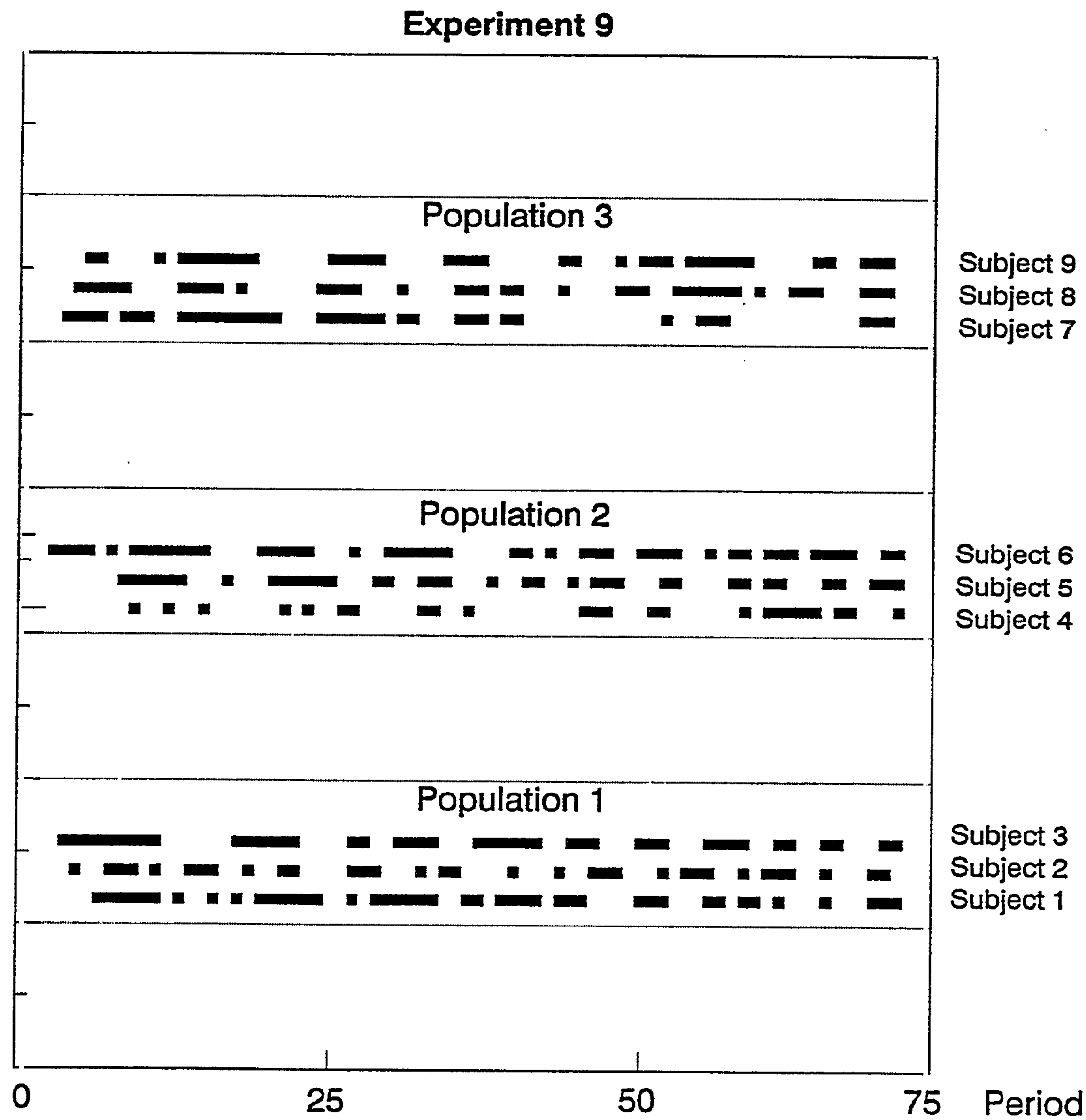


FIGURE 3. - Individual choices of message B under perfect monitoring and complete payoff information.



Notes

A solid square (■) indicates a choice of message B for a particular period.
A space indicates a choice of message A for a particular period.

FIGURE 4. - Individual choices of message B under perfect monitoring and incomplete payoff information.

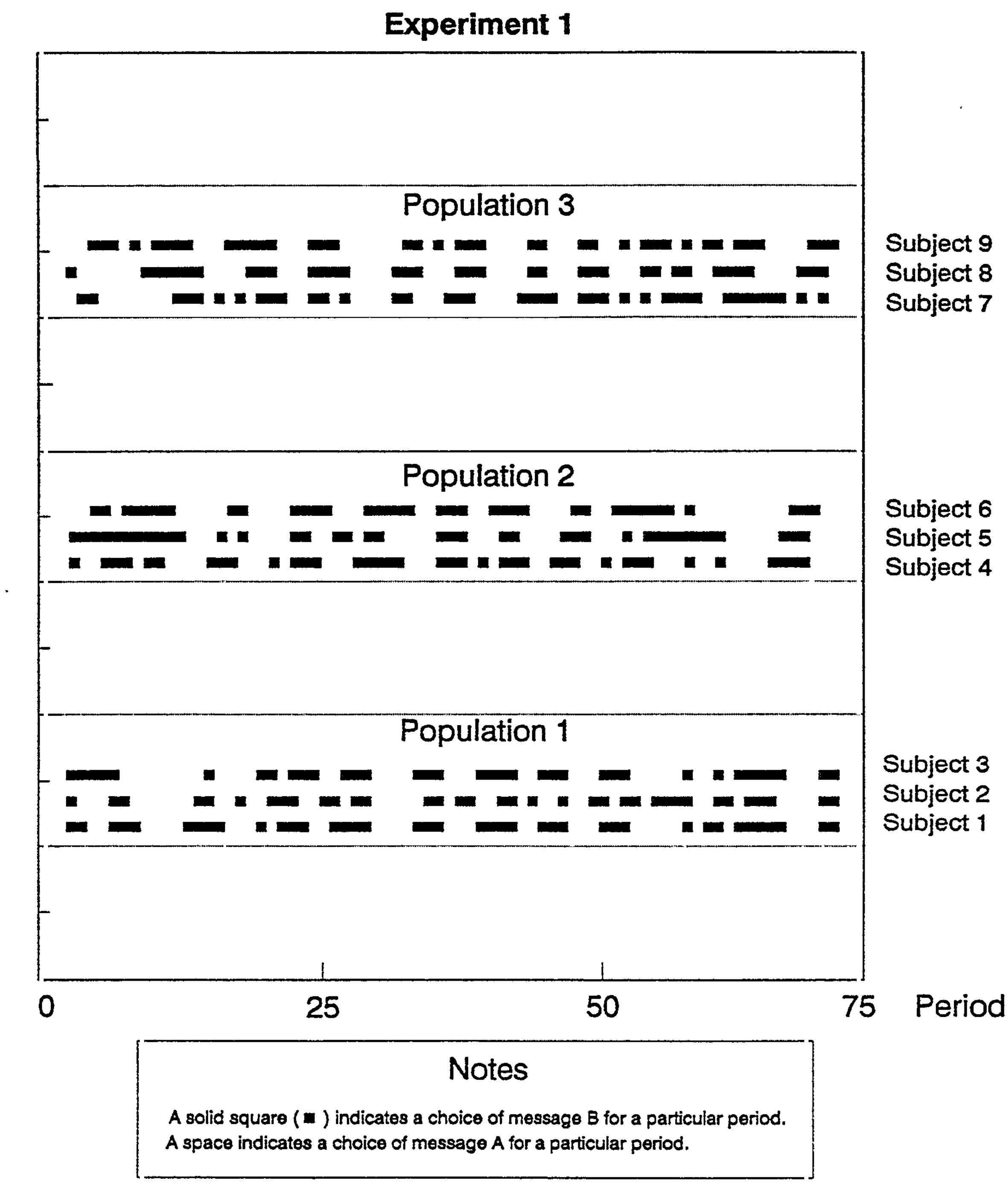


FIGURE 5. - Individual choices of message B under imperfect monitoring and incomplete payoff information.

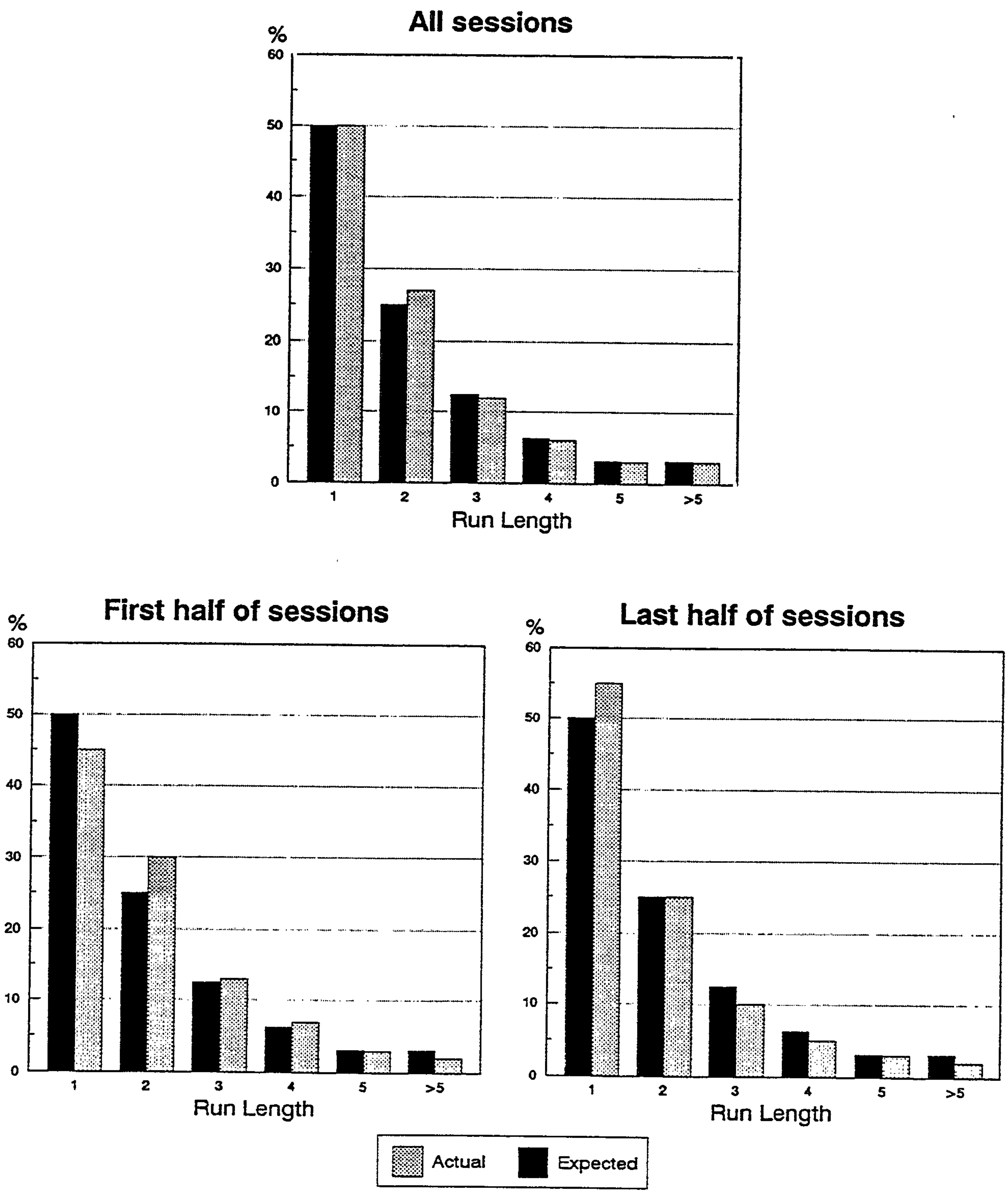


FIGURE 6. - Frequency of runs under perfect monitoring and complete payoff information.

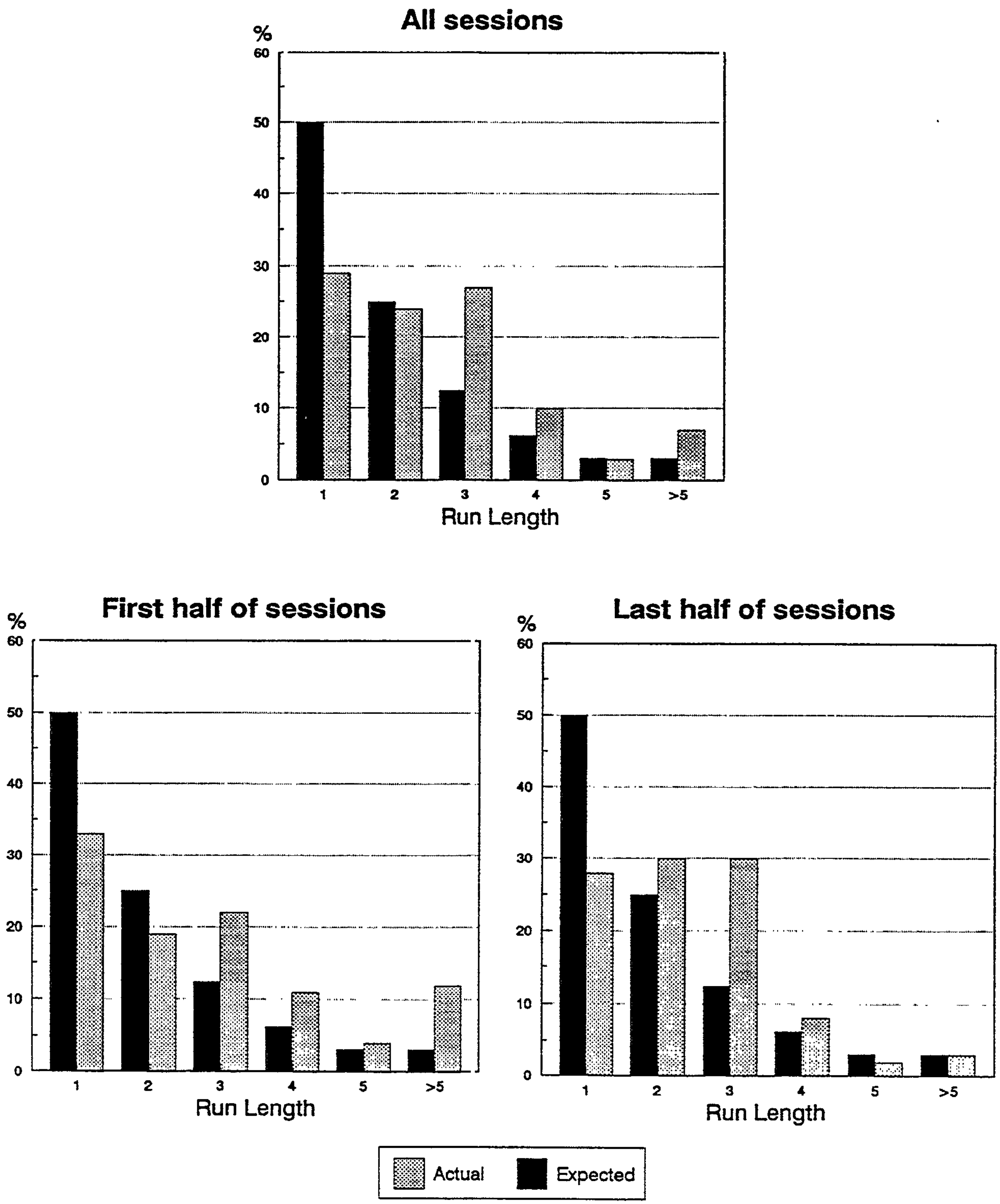


FIGURE 7. - Frequency of runs under perfect monitoring and incomplete payoff information.

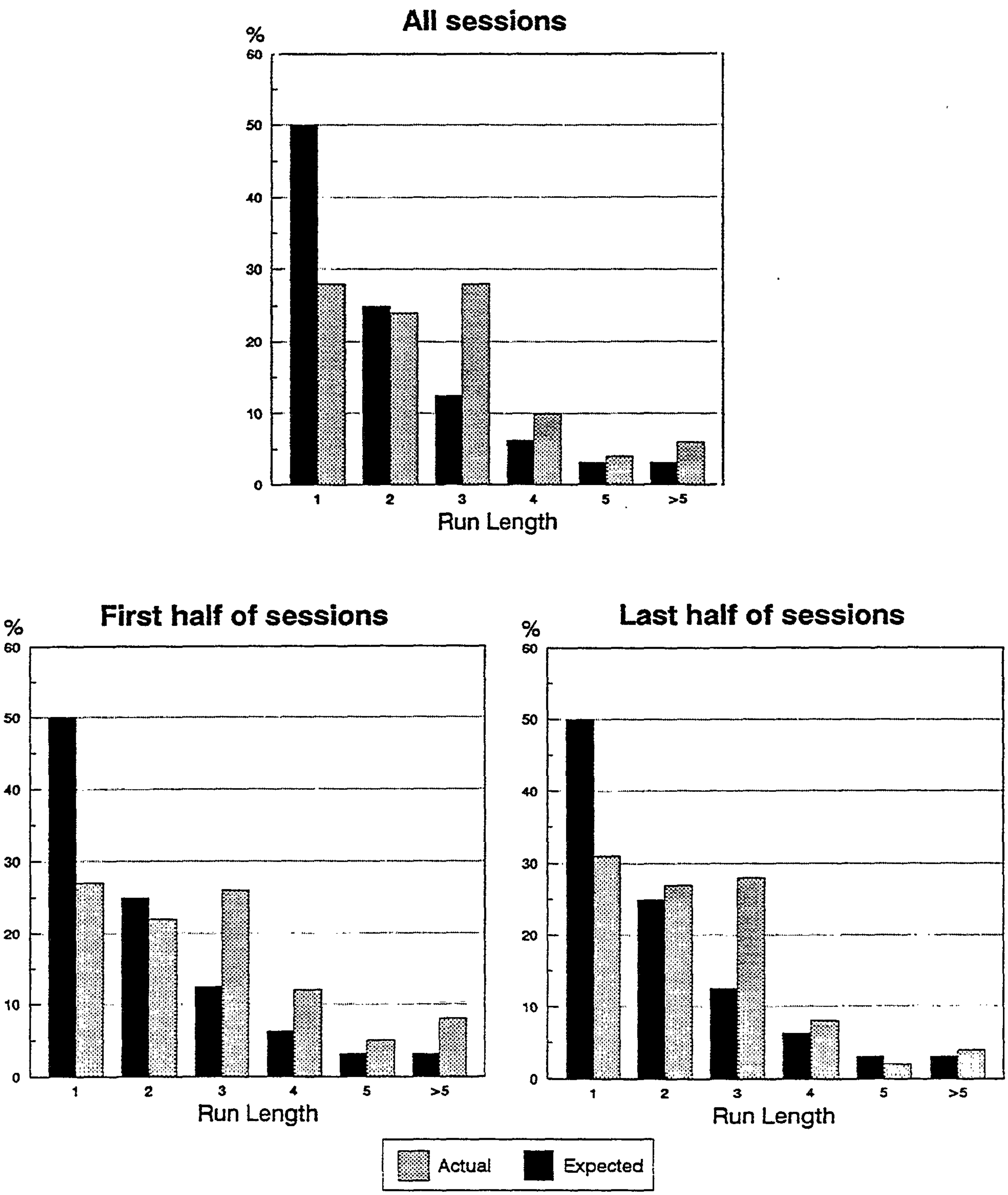


FIGURE 8. - Frequency of runs under imperfect monitoring and incomplete payoff information.

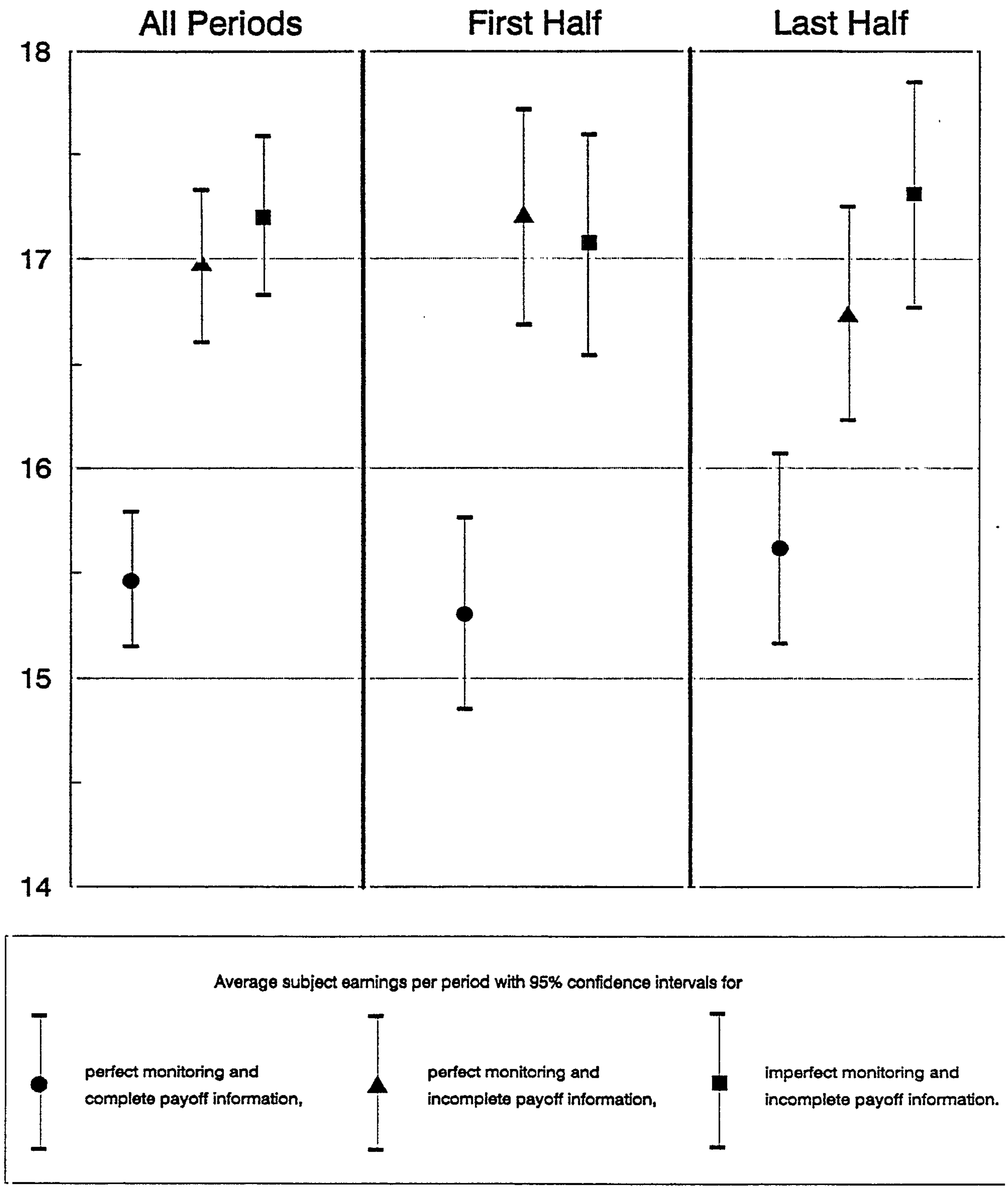


FIGURE 9. - Comparison of average earnings across environments.

Appendix A: Instructions for Environment IM-IP

Instructions

This is an experiment in decision making. Various research foundations have provided funds for this research. The instructions are simple and if you follow them carefully, you can earn a considerable amount of money which will be paid to you in cash. Before we proceed to describe the experiment, we would like to lay down some simple rules.

(i) Please do not talk during the experiment. Communication of any form without the permission of the experimenter is not allowed. The partitions are there to provide privacy.

(ii) If you have any questions at any time during the experiment , please raise your hand. An experimenter or a monitor will answer your questions.

(iii) To repeat, this is an experiment in decision-making. You will have to make your own choices in this experiment. We cannot recommend particular decisions to you. But we can clarify questions about procedures.

The experiment will consist of a series of many separate decision making periods. In each period you will be paired with three other persons. We will call these persons your counterparts. The identity of your counterparts will not change over time.

Payoffs

In each period, your payoffs will be based on your decision and the decisions of your three counterparts. Each period you can choose either A or B. Your decision will be compared with the decision of each of your counterparts. If you make the same decision as your counterpart, you will earn zero: this will happen if both of you choose A or both of you choose B. If you choose A and your counterpart chooses B, or you choose B and your counterpart chooses A, then you will earn 10 cents. These payoffs in cents are listed in Table 1 where the row is your decision, and the column is the decision by your counterpart. Thus if you choose A and your counterpart chooses B, your earnings are 10 cents.

Table 1
Your earnings (in cents)
Your counterpart's decision

Your decision	A	B
A	0	10
B	10	0

At the beginning of the experiment you will see the following screen.

```
Period: 1                    Payoffs: 0 for MATCH, 10 for NOT MATCH
------------------------------------------HISTORY----------------------------------------------
            Your            Counterparts'     Earnings
            Choice          Choices           Given                    Period
Period      (A/B)           # A's   # B's     # A's + # B's    =       Earnings




------------------------------------------MESSAGES---------------------------------------------
       Please input your choice (A/B) ?
```

At this stage you should enter your decision, either A or B, by pressing the appropriate key on the keyboard.

The computer will ask you to confirm your decision. For instance if you chose B the screen will look like this:

```
Period:  1                              Payoffs: 0  for MATCH, 10 for NOT MATCH
----------------------------------------HISTORY----------------------------------------
                    Your                Counterparts'   Earnings
                    Choice              Choices         Given                   Period
Period              (A/B)               # A's   # B's   # A's + # B's    =      Earnings




----------------------------------------MESSAGES----------------------------------------
          Your CHOICE is B.  Is this Correct ?
          Press y to accept this number or press n to try again.
```

You should confirm your decision by pressing y or press n if you wish to change it.

Once all participants have made decisions, the computer will calculate earnings and report them to everyone. These will appear on a screen that will resemble the figure below. We will now explain the numbers in the figure to make sure you understand how you will earn money during this experiment.

```
Period:  1                              Payoffs: 0  for MATCH, 10 for NOT MATCH
----------------------------------------HISTORY----------------------------------------
                    Your                Counterparts'   Earnings
                    Choice              Choices         Given                   Period
Period              (A/B)               # A's   # B's   # A's + # B's    =      Earnings
1                   B                   2       1       20       0              20




----------------------------------------MESSAGES----------------------------------------
Your choice was B.
2 counterparts chose A and 1 chose B
resulting in a payoff of 20 cents.     Press SPACE-BAR to continue
```

In the setting described above, you chose B. Of the three counterparts you were matched with, 2 chose A, and 1 chose B. So you MATCHED 1 (earnings zero per match) , and DID NOT MATCH with 2 (earnings 10 cents per match), which adds up to period earnings of 20 cents.

At the end of the period, please press the SPACE-BAR on your keyboard to continue to the next period. The same procedure will be repeated for several periods. At the end of the experiment, we will pay you the total earnings (sum of your period earnings) in cash.

Here's what the screen might look like after 5 periods.

Period: 5 Payoffs: 0 for MATCH, 10 for NOT MATCH

--------HISTORY--------

Period	Your Choice (A/B)	Counterparts' Choices # A's	# B's	Earnings Given # A's +	# B's	=	Period Earnings
1	B	2	1	20	0		20
2	A	1	2	0	20		20
3	A	2	1	0	10		10
4	B	0	3	0	0		0
5	B	2	1	20	0		20

--------MESSAGES--------

Your choice was B.
2 counterparts chose A and 1 chose B
resulting in a payoff of 20 cents. Press SPACE-BAR to continue

To ensure that you understand the instructions answer the following questions. An experimenter will check your answers when you are finished.

(a) Please write **T** if you believe the following statement is true or **F** if false.

T **F** You will be matched with the same three counterparts every period.

(b) In period 5, you chose A. Two of your counterparts chose B, and one chose A. Your earnings for this period were (circle one)

zero **10 c** **20 c** **30 c** **none of these**

(c) In period 11, you chose B. Two of your counterparts chose A, and one chose B. Your earnings for this period were (circle one)

zero **10 c** **20 c** **30 c** **none of these**

(d) In period 18, you chose A. All three counterparts chose B. Your earnings for this period were (circle one)

zero **10 c** **20 c** **30 c** **none of these**

(e) In period 25, you chose B. All three counterparts chose B. Your earnings for this period were (circle one)

zero **10 c** **20 c** **30 c** **none of these**

Are there any questions?

Appendix B: Instructions for Environment PM-IP

Instructions

This is an experiment in decision making. Various research foundations have provided funds for this research. The instructions are simple and if you follow them carefully, you can earn a considerable amount of money which will be paid to you in cash. Before we proceed to describe the experiment, we would like to lay down some simple rules.

(i) Please do not talk during the experiment. Communication of any form without the permission of the experimenter is not allowed. The partitions are there to provide privacy.

(ii) If you have any questions at any time during the experiment , please raise your hand. An experimenter or a monitor will answer your questions.

(iii) To repeat, this is an experiment in decision-making. You will have to make your own choices in this experiment. We cannot recommend particular decisions to you. But we can clarify questions about procedures.

The experiment will consist of a series of many separate decision making periods. In each period you will be paired with three other persons. We will call these persons your counterparts. The identity of your counterparts will not change over time.

Payoffs

In each period, your payoffs will be based on your decision and the decisions of your three counterparts. Each period you can choose either A or B. Your decision will be compared with the decision of each of your counterparts. If you make the same decision as your counterpart, you will earn zero: this will happen if both of you choose A or both of you choose B. If you choose A and your counterpart chooses B, or you choose B and your counterpart chooses A, then you will earn 10 cents. These payoffs in cents are listed in Table 1 where the row is your decision, and the column is the decision by your counterpart. Thus if you choose A and your counterpart chooses B, your earnings are 10 cents.

Table 1
Your earnings (in cents)
Your counterpart's decision

Your decision	A	B
A	0	10
B	10	0

At the beginning of the experiment you will see the following screen.

```
Period: 1                    Payoffs: 0 for MATCH, 10 for NOT MATCH
-----------------------------------------HISTORY---------------------------------------------
        Your     Counterparts' Total      Earnings                          Other Decision
        Choice    Choices                 Given            Period           Makers' Choices
Period  (A/B)    S   T   U   #As #Bs As + Bs  =            Earnings         V   W   X   Y   Z




------------------------------------------MESSAGES---------------------------------------------
        Please input your choice (A/B) ?
```

At this stage you should enter your decision, either A or B, by pressing the appropriate key on the keyboard.

The computer will ask you to confirm your decision. For instance if you chose B the screen will look like this:

Period: 1							Payoffs: 0 for MATCH, 10 for NOT MATCH								
-------------------HISTORY-------------------															
	Your Choice	Counterparts' Choices			Total		Earnings Given		Period	Other Decision Makers' Choices					
Period	(A/B)	S	T	U	#As	#Bs	As +	Bs =	Earnings	V	W	X	Y	Z	

-------------------MESSAGES-------------------

Your CHOICE is B. Is this Correct ?
Press y to accept this number or press n to try again.

You should confirm your decision by pressing y or press n if you wish to change it.

Once all participants have made decisions, the computer will calculate earnings and report them to everyone. These will appear on a screen that will resemble the figure below. We will now explain the numbers in the figure to make sure you understand how you will earn money during this experiment.

Period: 1							Payoffs: 0 for MATCH, 10 for NOT MATCH							
-------------------HISTORY-------------------														
	Your Choice	Counterparts' Choices			Total		Earnings Given		Period	Other Decision Makers' Choices				
Period	(A/B)	S	T	U	#As	#Bs	As +	Bs =	Earnings	V	W	X	Y	Z
1	B	A	A	B	2	1	20	0	20	B	A	B	A	B

-------------------MESSAGES-------------------

Your choice was B.
2 counterparts chose A and 1 chose B
resulting in a payoff of 20 cents. Press SPACE-BAR to continue

In the setting described above, you chose B. Of the three counterparts you were matched with, 2 chose A, and 1 chose B. So you MATCHED 1 (earnings zero per match) , and DID NOT MATCH with 2 (earnings 10 cents per match), which adds up to period earnings of 20 cents. On your screen, S, T and U are the columns for your three counterparts. So S is your first counterpart, T your second and U your third. Their choices are shown separately, just before the earnings calculation. There are five other people in the room, listed as V,W, X, Y and Z on your screen. Their choices will also be reported to you at the end of each period of the experiment. Thus V is the first of these five people, W the second, and so on.

At the end of the period, please press the SPACE-BAR on your keyboard to continue to the next period. The same procedure will be repeated for several periods. At the end of the experiment, we will pay you the total earnings (sum of your period earnings) in cash.
Here's what the screen might look like after 5 periods.

Period: 5 Payoffs: 0 for MATCH, 10 for NOT MATCH

--HISTORY--

Period	Your Choice (A/B)	Counterparts' Total Choices S	T	U	#As	#Bs	Earnings Given As +	Bs =	Period Earnings	Other Decision Makers' Choices V	W	X	Y	Z
1	B	A	A	B	2	1	20	0	20	B	A	B	A	B
2	A	A	A	A	3	0	0	0	0	A	A	B	A	B
3	A	B	B	B	0	3	0	30	30	A	B	B	A	A
4	A	A	B	A	2	1	0	10	10	B	A	B	B	B
5	B	B	A	A	2	1	20	0	20	B	A	B	A	B

--MESSAGES--

Your choice was B.
2 counterparts chose A and 1 chose B
resulting in a payoff of 20 cents. Press SPACE-BAR to continue

So if you look down the column for S, you will see the decisions made by person S during the experiment for the last 5 periods on this screen. To repeat, your three counterparts are code-named S,T and U; you can see the decision for your first counterpart in column S, for your second in column T, for your third in column U; though not necessarily in that order. The same person's decision is in each column every period. So for example, S chose A, A, B, A and B in the first five periods.

To ensure that you understand the instructions answer the following questions. An experimenter will check your answers when you are finished.

(a) Please write **T** if you believe the following statement is true or **F** if false.
T **F** You will be matched with the same three counterparts every period.

(b) In period 5, you chose A. Two of your counterparts chose B, and one chose A. Your earnings for this period were (circle one)
zero **10 c** **20 c** **30 c** **none of these**

(c) In period 11, you chose B. Two of your counterparts chose A, and one chose B. Your earnings for this period were (circle one)
zero **10 c** **20 c** **30 c** **none of these**

(d) In period 18, you chose A. All three counterparts chose B. Your earnings for this period were (circle one)
zero **10 c** **20 c** **30 c** **none of these**

(e) In period 25, you chose B. All three counterparts chose B. Your earnings for this period were (circle one)
zero **10 c** **20 c** **30 c** **none of these**

(f) Please write **T** if you believe the following statement is true or **F** if false.
T **F** Decisions reported under a particular column such as S, T or U [for counterparts] or V,W, X, Y or Z [for other decision makers] are those of the same person every period.

(g) Please match the two items reported below:
1. Three counterparts' choices (i) V, W, X, Y or Z
2. Other five decision-makers' choices (ii) S, T, U

Are there any questions?

Appendix C: Instructions for Environment PM-CP

<u>Instructions</u>

This is an experiment in decision making. Various research foundations have provided funds for this research. The instructions are simple and if you follow them carefully, you can earn a considerable amount of money which will be paid to you in cash. Before we proceed to describe the experiment, we would like to lay down some simple rules.

(i) Please do not talk during the experiment. Communication of any form without the permission of the experimenter is not allowed. The partitions are there to provide privacy.

(ii) If you have any questions at any time during the experiment, please raise your hand. An experimenter or a monitor will answer your questions.

(iii) To repeat, this is an experiment in decision-making. You will have to make your own choices in this experiment. We cannot recommend particular decisions to you. But we can clarify questions about procedures.

The experiment will consist of a series of many separate decision making periods. In each period you will be paired with three other persons. We will call these persons your counterparts. The identity of your counterparts will not change over time.

<u>Payoffs</u>

In each period, your payoffs will be based on your decision and the decisions of your three counterparts. Each period you can choose either A or B. Your decision will be compared with the decision of each of your counterparts. If you make the same decision as your counterpart, you will earn zero: this will happen if both of you choose A or both of you choose B. If you choose A and your counterpart chooses B, or you choose B and your counterpart chooses A, then you will earn 10 cents. These payoffs in cents are listed in Table 1 where the row is your decision, and the column is the decision by your counterpart. Thus if you choose A and your counterpart chooses B, your earnings are 10 cents.

Table 1

Your earnings (in <u>cents</u>)

Your counterpart's decision

Your decision	A	B
A	0	10
B	10	0

At the beginning of the experiment you will see the following screen.

```
Period: 1                    Payoffs: 0 for MATCH, 10 for NOT MATCH
-----------------------------------------HISTORY----------------------------------------------
                                                                   Your        Other
        Your    Counterparts' Total    Earnings      Period      Group's     Group's
        Choice     Choices             Given         Earnings   Choices     Choices
Period  (A/B)   S   T   U  #As #Bs     As + Bs  =                V   W       X  Y  Z




-----------------------------------------MESSAGES---------------------------------------------
        Please input your choice (A/B) ?
```

At this stage you should enter your decision, either A or B, by pressing the appropriate key on the keyboard.

The computer will ask you to confirm your decision. For instance if you chose B the screen will look like this:

Period: 1 Payoffs: 0 for MATCH, 10 for NOT MATCH

---HISTORY---

	Your Choice	Counterparts' Choices			Total		Earnings Given		Period Earnings	Your Group's Choices		Other Group's Choices		
Period	(A/B)	S	T	U	#As	#Bs	As +	Bs =		V	W	X	Y	Z

---MESSAGES---

Your CHOICE is B. Is this Correct ?
Press y to accept this number or press n to try again.

You should confirm your decision by pressing y or press n if you wish to change it.

Once all participants have made decisions, the computer will calculate earnings and report them to everyone. These will appear on a screen that will resemble the figure below. We will now explain the numbers in the figure to make sure you understand how you will earn money during this experiment.

Period: 1 Payoffs: 0 for MATCH, 10 for NOT MATCH

---HISTORY---

	Your Choice	Counterparts' Choices			Total		Earnings Given		Period Earnings	Your Group's Choices		Other Group's Choices		
Period	(A/B)	S	T	U	#As	#Bs	As +	Bs	=	V	W	X	Y	Z
1	B	A	A	B	2	1	20	0	20	B	A	B	A	B

---MESSAGES---

Your choice was B.
2 counterparts chose A and 1 chose B
resulting in a payoff of 20 cents. Press SPACE-BAR to continue

In the setting described above, you chose B. Of the three counterparts you were matched with, 2 chose A, and 1 chose B. So you MATCHED 1 (earnings zero per match), and DID NOT MATCH with 2 (earnings 10 cents per match), which adds up to period earnings of 20 cents. On your screen, S, T and U are the columns for your three counterparts. So S is your first counterpart, T your second and U your third. Their choices are shown separately, just before the earnings calculation.

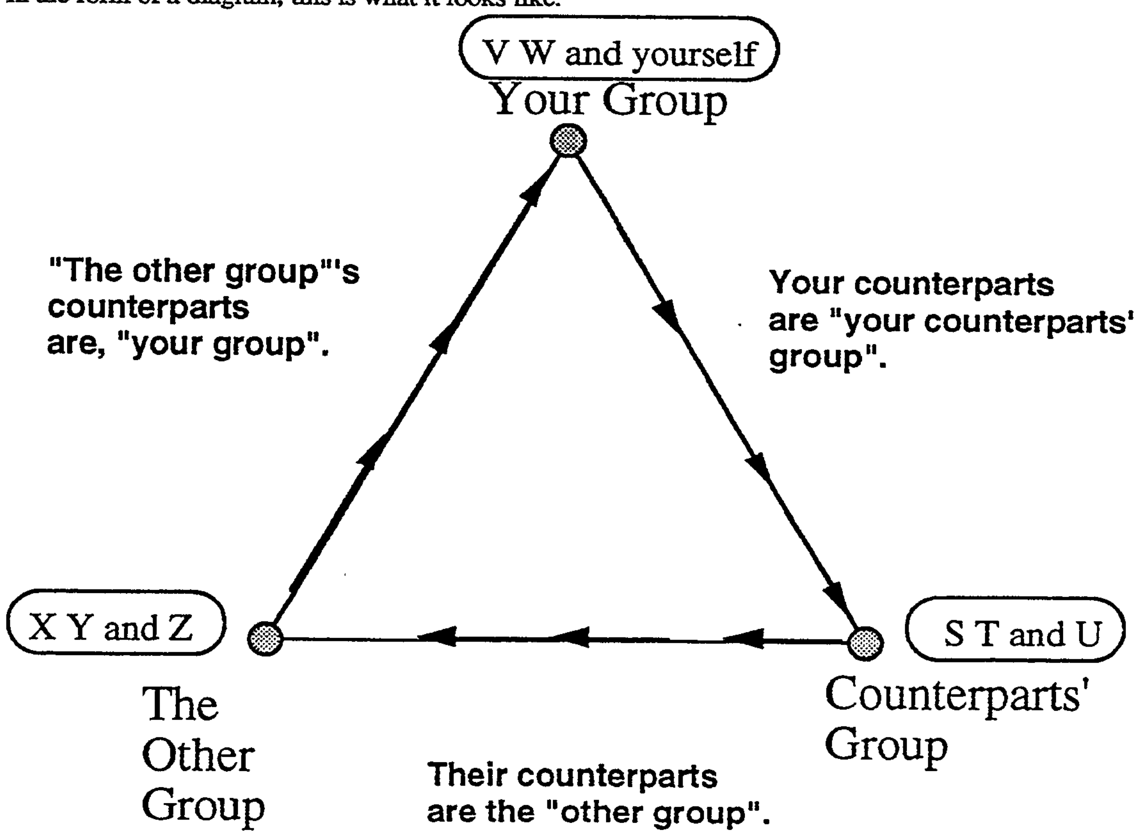
In this experiment, there are nine subjects divided into three groups. As far as you are concerned, these are "your group", "your counterparts' group" and the "other group." Each group has three members. Once people have been assigned to groups at the beginning of the experiment, there are no further changes, so members of these three groups are the same throughout the experiment. "Your counterparts' group" consists of S,T and U. Their choices appear under the columns S, T and U respectively. In "your group" there are two other members, denoted by V and W, whose choices appear under the columns V and W respectively. In the "other group" there are three other members, denoted by X, Y and Z: their choices appear under the columns X, Y and Z respectively. To repeat, "your group" will always consist of the same three people V, W and yourself; "your counterparts' group" will always consist of the same three people S, T and U and the "other group" will always consist of the same three people X, Y and Z.

For all the people in your group, i.e. yourself, V and W, your counterparts will always be "your counterparts' group" consisting of S,T and U. So each person in your group has three counterparts, namely S, T and U.

Now, let's describe what happens to "your counterparts' group" consisting of S,T and U. Their counterparts will always be the "other group" consisting of X, Y and Z. So each person in "your counterparts' group" has three counterparts, namely X, Y and Z. Thus S has three counterparts, namely X, Y and Z. The same is true for T and also for U.

Now, let's describe what happens to the "other group" consisting of X, Y and Z. Their counterparts will always be "your group" consisting of V, W and yourself. So each person in the "other group" has three counterparts, namely V, W and yourself. Thus X has three counterparts, namely X, Y and Z. The same is true for Y and also for Z.

In the form of a diagram, this is what it looks like:



Follow the arrows to see which group is the counterpart of which other group. **To summarize, everyone in your group has the same three counterparts, in what we call your counterparts' group. Everyone of them has the same three counterparts, in what we call the other group. Everyone in the other group has the same three counterparts, and they are your group.**

Remember any person's payoffs depend upon a comparison of their choices with their counterparts' choices. So if you choose B and your counterparts S,T and U respectively choose A, A and B, you earn 20 cents. But S's payoffs depend on a comparison of their choices with the other group's choices i.e. comparing the choices of S with those of X,Y and Z. [The same is true of T and U.] And X's choices are compared with those of you and the two other people in your group V and W to determine X's payoffs.

At the end of the period, please press the SPACE-BAR on your keyboard to continue to the next period. The same procedure will be repeated for several periods. At the end of the experiment, we will pay you the total earnings (sum of your period earnings) in cash.

Here's what the screen might look like after 5 periods.

Period: 5 Payoffs: 0 for MATCH, 10 for NOT MATCH

---HISTORY---

Period	Your Choice (A/B)	Counterparts' Choices S	T	U	Total #As	#Bs	Earnings Given As +	Bs	= Period Earnings	Your Group's Choices V	W	Other Group's Choices X	Y	Z
1	B	A	A	B	2	1	20	0	20	B	A	B	A	B
2	A	A	A	A	3	0	0	0	0	A	A	B	A	B
3	A	B	B	B	0	3	0	30	30	A	B	B	A	A
4	A	A	B	A	2	1	0	10	10	B	A	B	B	B
5	B	B	A	A	2	1	20	0	20	B	A	B	A	B

---MESSAGES---

Your choice was B.
2 counterparts chose A and 1 chose B
resulting in a payoff of 20 cents. Press SPACE-BAR to continue

So if you look down the column for S, you will see the decisions made by person S during the experiment for the last 5 periods on this screen. To repeat, your three counterparts are code-named S,T and U; you can see the decision for your first counterpart in column S, for your second in column T, for your third in column U; though not necessarily in that order. The same person's decision is in each column every period. So for example, S chose A, A, B, A and B in the first five periods.

To ensure that you understand the instructions answer the following questions. An experimenter will check your answers when you are finished.

(a) Please write **T** if you believe the following statement is true or **F** if false.
T **F** You will be matched with the same three counterparts every period.

(b) In period 5, you chose A. Two of your counterparts chose B, and one chose A. Your earnings for this period were (circle one)
zero **10 c** **20 c** **30 c** **none of these**

(c) In period 11, you chose B. Two of your counterparts chose A, and one chose B. Your earnings for this period were (circle one)
zero **10 c** **20 c** **30 c** **none of these**

(d) In period 18, you chose A. All three counterparts chose B. Your earnings for this period were (circle one)

zero **10 c** **20 c** **30 c** **none of these**

(e) In period 25, you chose B. All three counterparts chose B. Your earnings for this period were (circle one)

zero **10 c** **20 c** **30 c** **none of these**

(f) Please write **T** if you believe the following statement is true or **F** if false.

T **F** Decisions reported under a particular column such as S, T or U [for counterparts] or V,W, X, Y or Z [for other decision makers] are those of the same person every period.

(g) Please match the two items reported below:

1. Three counterparts' choices (i) V, W, X, Y or Z
2. Other five decision-makers' choices (ii) S, T, U

(h) For period 1, the choices were as follows

Period	Your Choice (A/B)	Counterparts' Choices S	T	U	Total #As	#Bs	Earnings Given As +	Bs	Period Earnings =	Your Group's Choices V	W	Other Group's Choices X	Y	Z
1	B	A	A	B	2	1	20	0	20	B	A	B	A	B

* How much did S earn? (circle one)

zero **10 c** **20 c** **30 c** **none of these**

* How much did X earn? (circle one)

zero **10 c** **20 c** **30 c** **none of these**

Are there any questions?