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Step-by-Step Migration to Efficient Agglomerations*

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ABSTRACT

Recent literature suggests that historical accidents can trap economies in inefficient equilibria. In a prototype model in the literature, there are two locations, the productive South and the unproductive North. By accident of history, the industry starts in the North. Because of agglomeration economies, the industry may reside in the North forever—an inefficient outcome. This paper modifies the standard model by assuming there is a continuum of locations between the North and the South. Productivity gradually increases as one moves South. There is a unique long-run equilibrium in this economy where all agents locate at the most productive locations.

*The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

Recently, the possibility of multiple equilibria under increasing returns has received much attention. Work by Farrell and Saloner (1985), Arthur (1989), and Krugman (1991a) has emphasized that in such situations equilibrium outcomes may be determined by accidents of history. Moreover, the equilibrium selected by the historical accident may turn out to be inefficient. In this paper, I make a slight modification in the existing prototype model and show that the economy always migrates to the efficient location pattern.

The underlying ideas in this literature can be illustrated with a simple example. Suppose there are two possible locations for production in an industry, the North and the South. Suppose that the South is better suited for production in this industry than the North, so that in an efficient allocation the entire industry is in the South. By accident of history, the industry starts in the North. If agglomeration economies are important, the industry may be trapped in the North forever—an inefficient outcome. This can happen if the value of the agglomeration benefits in the North to an individual producer exceeds the benefit of the natural advantage of the South.

This paper considers two modifications to this standard model. The first modification is that instead of there being two discrete locations, North and South, there is a continuum of locations along a North-South axis. The natural suitability for production gradually gets better as one moves South until some southernmost endpoint is reached. The second modification is that space is a scarce resource in the model. It is not feasible to concentrate all production at a single point. Rather, production must be spread out over an interval of locations.

The main result of this paper is that there is a unique long-run equilibrium in this economy in which all production occurs in the most efficient locations. If by historical accident the industry is initially at the north end of the location spectrum, the industry will eventually migrate to the more suitable territory at the south end of the location spectrum. In some cases the migration is abrupt, with all new entrants in the initial period jumping immediately to the most efficient locations at the south end of the location spectrum. In other cases the migration is gradual, with each new cohort of entrants shifting the center of the industry south by a small step. I call this latter type of case a *step-by-step* migration.

To obtain my result, it is not enough to simply assume a continuum of locations. It is also necessary to make space a scarce resource. If I did not add this second modification onto the model, there might exist a continuum of equilibria in the economy with each point x in the location space being a possible equilibrium production site. To see why, suppose a particular agent takes as given that all the other agents in the economy will locate at a point x. If this particular agent were to deviate from the location x, it would lead to a first-order loss in the particular agent's welfare, if transportation costs are big compared to the rate at which the natural advantage increases as one moves south. Hence, there could be an equilibrium where all agents locate at any inefficient point x.

This logic breaks down when space is a scarce resource. Suppose that the industry were initially located towards the north end of the location spectrum. By assumption, the industry cannot be concentrated at single point, so suppose the industry is spread out over an interval $[\underline{x}, \overline{x}]$. First consider the case where the natural suitability for production does not vary by location. In this case, the most desirable location in the economy would be at the industry center $c \equiv \frac{\underline{x}+\overline{x}}{2}$ because this central location would minimize transportation costs. Because the center would be the best location in this case, the cost of a small deviation from the center would only be a second-order loss. Now suppose that suitability for production increases as one moves south. Here, there is a first-order gain to deviating from the center and making a slight movement in the direction of the South. This first-order gain outweighs any second-order cost from an increase in transportation costs. Therefore, if the industry is initially located on some interval $[\underline{x}, \overline{x}]$, the new agents entering the economy will prefer to locate at a point that is south of the center of this existing industry. This force tends to shift the center of the industry further south in each period. It prevents the industry from getting stuck at an inefficient set of locations in the North.

While the industry always occupies the efficient locations in the long run, the equilibrium transition path is not necessarily efficient. For certain parameters, a social planner might specify that the industry immediately jump to the best locations at the south end of the location spectrum. While there always exists an equilibrium that decentralizes the planner's solution that the industry jump, in some cases there also exists a second, inefficient, equilibrium transition path where the industry migrates in a gradual, step-by-step, fashion. Thus, there may exist an equilibrium path where the transition is slower than what a planner would do. The reverse is also true. There may exist an equilibrium where the migration is too fast—the industry takes a jump along the equilibrium path, but the planner would move the industry in a step-by-step fashion.

The theme of this paper is that market forces have a way of preventing economies from getting trapped forever in inefficient allocations. A variety of recent papers have made similar points. Liebowitz and Margolis (1995) present a number of arguments that discount the importance of lock-in. Rauch (1993) argues that if an economy were initially in an inefficient location, agents (land developers) would emerge to coordinate the migration to the efficient location. This is a Coasian argument for why we should expect efficiency. My paper is fundamentally different from the Rauch paper because the efficient location is obtained in the long run without the help of any analog of land developers to coordinate the migration. My paper is closely related to a previous paper, Holmes (1996), which considers an environment with two discrete locations, North and South, but a continuum of different product qualities. In this previous paper, low quality products migrate to the South first, and they are followed by successively higher quality products. In the previous paper, it is possible for part of the industry to remain stuck in the North if the natural advantage of the South is small enough. The current paper is different in that the unique long-run equilibrium is for all production to be in the South, for any positive natural advantage of the South, no matter how small.

This paper is also related to the literature on vintage capital models. At the beginning of each period there exists a capital stock from past investments that is tied down to a particular set of locations. The question faced in this economy is where to build the new factories. There is a benefit to building the new plants as far south as possible because land becomes more productive as one moves south. The bigger the step size south in each period, the larger rate of growth in welfare measures from one period to the next. But this dynamic gain of a larger step size comes at a static cost in any given period. The further south the new plants are, the further the new plants are away from the existing plants. This is problematic because separating the new plants from the old plants precludes the agglomeration benefits that would emerge if old and new were near each other. The trade-off in costs and benefits from increasing the step size is analogous to the trade-off found in vintage capital models such as those in Chari and Hopenhayn (1991), Parente (1994), and Jovanovic and Nyarko (1996). In these models, the benefits of faster adoption of new technologies must be weighed against the cost of increasing the rate of obsolescence of past investments. Jovanovic and Nyarko (1996) ask the question of whether or not it would ever be optimal to stop adopting new technologies in light of this trade-off. They find that it may be optimal to stop at a technology level below that maximum level. In my related but different structure, it is never optimal, nor is it ever an equilibrium for the economy to get stuck in an allocation where productivity is less than its maximum possible level.

2. The Static Model

This section describes and analyzes a static version of the model. I begin with the static model because it is easier to explain that the dynamic model and much of the intuition for what happens in the dynamic case follows from what happens in the static case. I extend the model to the dynamic case in the next section.

Agglomeration benefits emerge in the model because producers value access to a large variety of local specialist suppliers. To model this, I follow the recent literature, e.g., Abdel-Rahman (1988) and Krugman (1991b), in applying the structure of the Spence (1976) and Dixit-Stiglitz (1977) formulations to a geographic context.

A. Description of the Model

The set of locations in the economy is the interval $[0, \xi]$.[1] Locations are indexed by $x \in [0, \xi]$. The higher is x, the further south is the location.

There are two kinds of agents in the economy, *suppliers* and *assemblers*. Suppliers employ an outside good called *dollars* to manufacture intermediate inputs. Assemblers use these intermediate inputs to manufacture the single final good. The price of the single final good is normalized to one dollar.

There is a continuum of suppliers indexed by s on the interval $[0, \sigma]$, where $\sigma < \xi$. Each supplier chooses a location x to set up a factory. Let $\ell^s(s) \in [0, \xi]$ be the location choice of supplier s. Each point x in the location space can hold at most a single supplier. Formally, in a feasible set of location decisions, $\ell^s(s') \neq \ell^s(s'')$, if $s' \neq s''$. For example, one feasible allocation is for suppliers to locate on the interval $[0, \sigma]$; i.e., $\ell^s(s) = s$. Note that my earlier assumption that $\sigma < \xi$ implies that there is more than enough room in the location space $[0, \xi]$ to fit the entire set of suppliers.

The locations vary in the marginal cost to produce intermediate inputs. The marginal cost at location x is $e^{-\theta x}$ dollars. Thus at location $x = 0$ marginal cost is unity, and the marginal cost decreases with x at the rate of θ per unit distance.

Suppliers differ in the variety of input that they supply, in addition to differing in their location. For example, in the automobile industry, suppliers differ in their product, e.g., windshield wipers or seat belts, in addition to differing in the address of their plants. There is a continuum of input varieties indexed by $y \in [0, \infty)$. Let $y(s)$ denote the variety choice by supplier s. Each

[1] Krugman and Venables (1995) also consider an environment in which the location space is a continuum. The issues they consider are very different from the issues considered here.

supplier s also chooses a price $p(s)$ denominated in dollars.

There is a continuum of assemblers in the economy indexed by a on the interval $[0, \alpha]$. Each assembler constructs units of a composite intermediate input from the output of the suppliers. Suppose a particular assembler employs an amount $h(y)$ of each specialized input y. The production of the composite intermediate input m is

$$(1) \qquad m = \left[\int_0^\infty h(y)^{\frac{\lambda-1}{\lambda}} dy \right]^{\frac{\lambda}{\lambda-1}},$$

where λ is the constant elasticity of substitution. This production function is standard in the literature (e.g., Abdel-Rahman (1988), Krugman (1991b)). As is standard in the literature, assume that $\lambda > 1$. Define the markup parameter μ to be $\mu \equiv \frac{\lambda}{\lambda-1}$. The bigger is μ, the stronger is the preference for variety.

Assemblers convert the m units of the composite intermediate input to q units of the final good with the production function

$$(2) \qquad q = m^{\frac{\phi-1}{\phi}}.$$

The parameter ϕ is the elasticity of supply with respect to changes in the price of the composite intermediate input. Assume $\phi > 1$.

Assemblers incur a transportation cost when acquiring intermediate inputs. Suppose an assembler locates at x^a and purchases an intermediate input produced at x. Of the amount shipped, a fraction $e^{-\tau|x^a - x|}$ survives the trip, and a fraction $1 - e^{-\tau|x^a - x|}$ is dissipated as a transportation cost. The bigger is the transportation cost parameter τ, the greater is the output lost in transit. This is an *iceberg* transportation cost as in Krugman (1991b).

I consider two alternative cases regarding feasible distributions of assemblers.

Case 1. The distribution of assemblers can be any arbitrary distribution, including a mass point at a single location. Formally, the location choice function $\ell^a(a)$ can be any function mapping from the set of assemblers $[0, \alpha]$ to the set of locations $[0, \xi]$.

Case 2. There is room at each location x for at most a single assembler and a single supplier. Formally, the location choice function $\ell^a(a)$ must be such that $\ell^a(a') \neq \ell^a(a'')$, if $a' \neq a''$.

In Case 1, space is not a scarce resource for assemblers. In this case, the analysis is very tractable, enabling me to obtain general results. Case 2 is a more plausible case, since this case makes space a scarce resource for assemblers just as it is for suppliers. But the analysis of this case is fairly complex, and my results for this case are more limited than my results for Case 1.

B. Equilibrium in the Static Model

An allocation in this economy is a set of functions $\{\ell^s(\cdot), y(\cdot), p(\cdot), \ell^a(\cdot), q(\cdot), h(\cdot,\cdot)\}$ where $\ell^s(s)$, $y(s)$, and $p(s)$ specify for each supplier s the location choice, the variety choice, and the price choice and $\ell^a(a)$, $q(a)$, $h(s,a)$ specify for each assembler a the location choice, the quantity of final-good output, and the demand for specialized inputs from supplier s. A feasible allocation is one where the location choices of the agents are feasible and the output levels are feasible given the input levels.

All agents in the economy make their decisions simultaneously. When supplier s chooses its location $\ell^s(s)$ and its price $p(s)$, it takes as given the location decisions of the other suppliers $\ell^s(\cdot)$, the location decisions of the assemblers $\ell^a(\cdot)$, and the output levels of the other suppliers $h(\cdot,\cdot)$. Analogously, when assemblers make their location decisions, they take as given the location decisions of all the other agents in the economy and the prices of the various specialized-intermediate inputs. An equilibrium in this economy is a feasible allocation in which each agent maximizes profits given the set of choices available to the agent.

To determine the equilibrium, note first that each supplier will obviously pick a different variety y to produce, and given the symmetry of (1), it is irrelevant which one each selects. Following standard arguments, it is profit-maximizing for each supplier to set price equal to a constant markup μ over cost. A supplier at location x has a marginal cost of $e^{-\theta x}$ dollars. Hence, the price of a good produced at location x, before transportation costs, is

$$(3) \qquad p(x) = \mu e^{-\theta x}.$$

Suppose an assembler is located at x^a and purchases a specialized input from a supplier at location x. A fraction $e^{-\tau|x^a - x|}$ remains after the transportation costs, so in order to receive one delivered unit, the assembler has to purchase $e^{\tau|x^a - x|}$ units. Hence, the delivered price of one unit of good produced at x delivered to x^a is

$$(4) \qquad p^d(x, x^a) = \mu e^{-\theta x + \tau|x^a - x|}.$$

It is useful to introduce some additional notation. Take as given that the location choices of suppliers are given by some function $\ell^s(\cdot)$, and take as given that suppliers price according to the rule (3). Taking the locations and prices as given, let $v(x)$ be the minimum cost of constructing one unit of the composite intermediate input at location x. It is clear that the problem of picking the location that maximizes the assembler's profit is equivalent to the problem of picking the location with the lowest cost $v(x)$ of the composite intermediate. Analysis of the problem of minimizing the cost of constructing one unit of the composite yields the following lemma:

Lemma 1. Take as given that suppliers locate on the interval $[c^s - \frac{\sigma}{2}, c^s + \frac{\sigma}{2}]$ with center c^s and that suppliers price according to (3). The cost of the composite intermediate $v(x)$ is u-shaped as a function of x, and the minimum is attained at a point c^{a*} strictly further south than the supplier network center c^s; i.e., $c^{a*} > c^s$. Furthermore, $v(c^s + y) < v(c^s - y)$ for all $y > 0$; i.e., the composite cost at a location y units south of the center is less than the composite cost at a location y units north of the center.

The proof is in the appendix. To see the intuition, suppose an assembler were to locate at the center of the supplier network c^s. The price of intermediate inputs to the south of the center is less than the price of intermediate inputs to the north of the center. This follows because price is a constant markup over cost, and marginal cost falls as one moves south. Hence, an assembler located at c^s would tend to substitute away from inputs produced north of c^s towards inputs produced south of c^s. Since more than half of the total inputs purchased are obtained from suppliers south of c^s, such an assembler can lower its transportation cost bill by shifting its location south of c^s. It is worth noting that a key assumption underlying this result is that suppliers are spread out on the interval $[c^s - \frac{\sigma}{2}, c^s + \frac{\sigma}{2}]$. If instead, it were feasible for the mass of suppliers to concentrate at a single point c^s, the optimal assembly location would be at the center c^s.

Lemma 1 implies the following about the optimal location decisions of assemblers given that suppliers are located on the interval $[c^s - \frac{\sigma}{2}, c^s + \frac{\sigma}{2}]$. In Case 1, they all concentrate at the single point c^{a*} that minimizes $v(x)$. In Case 2 where assemblers are forced to spread out, the lemma implies that either they all concentrate on the interval $[\xi - \alpha, \xi]$ at the southernmost end of the location space or they concentrate on an interval $[c^a - \frac{\alpha}{2}, c^a + \frac{\alpha}{2}]$ with a center that is strictly further south than the center of the supplier network; i.e., $c^a > c^s$. In either Case 1 or Case 2, the result is the same that the assembly center is strictly further south than the supplier center.[2]

Now consider the location choices of specialized-input suppliers. Each supplier takes as given the location choices of the other suppliers $\ell^s(\cdot)$, the location choices of the assemblers $\ell^a(\cdot)$, and the production levels $h(\cdot, \cdot)$ of the other suppliers. Let $\pi^s(x)$ be supplier profit at location x, taking as given the choices of all the other agents.

[2] This statement assumes that $\alpha < \sigma$. In Case 2, if $\alpha > \sigma$ and assemblers and suppliers are both at the south end of the location space, then the center of assembly will be further north than the center of the suppliers.

At this point it is convenient to provide a separate treatment of the two alternative cases.

Case 1

Consider Case 1 where it is feasible to concentrate all assemblers at a single point c^a. The following lemma provides a formula for how supplier profit $\pi^s(x)$ depends upon the location of the supplier.

Lemma 2. Suppose that Case 1 applies and that all assemblers are concentrated at the point c^a. The profit of a supplier located at x takes the form

$$(5) \qquad \pi^s(x) = k e^{\frac{1}{\mu-1}[\theta x - \tau|c^a - x|]},$$

where k is a positive constant that is independent of x. This function is single-peaked and has the property that $\pi^s(c^a + y) > \pi^s(c^a - y)$ for all $y > 0$.

According to Lemma 2, the profit at a location y units south of the assembly center c^a is strictly greater than the profit at a location y units north of the center. The intuition for this is straightforward. The transportation cost to c^a is the same from both points, but the marginal cost of production is lower at the more southern point $c^a + y$ than the more northern point $c^a - y$. Given the shape of the profit function specified in Lemma 2, the optimal location choices of the suppliers will be an interval $[c^{s\prime} - \frac{\sigma}{2}, c^{s\prime} + \frac{\sigma}{2}]$ with some center $c^{s\prime}$ south of the assembly center c^a. This is illustrated in Figure 1. There the thick black line segment is the set of points of measure σ with the highest profit. The center of the set of profit-maximizing points is strictly further south than the assembly center c^a.

It is important for this result that it is not possible to concentrate all the suppliers at the same point. If $\theta < \tau$, then the profit-maximizing supplier location is at the assembly center c^a, as in Figure 1. Hence, if it were feasible for all suppliers to concentrate at a single point and if $\theta < \tau$, any location point could be an equilibrium location point. Given that the mass of assembly activity is at an arbitrary point c^a, all suppliers would locate there, and this would make it optimal for assemblers to locate at c^a. In contrast, if suppliers are forced to spread out as assumed here, even though the maximum supplier profit for $\theta < \tau$ is at c^a, because the profit function $\pi^s(\cdot)$ is asymmetric around c^a, the center of the set of profit-maximizing supplier locations is strictly greater than c^a.

I can now state the main result for this section.

Proposition 1. Assume that Case 1 applies. There exists a unique equilibrium. In the equilibrium allocation, suppliers locate on the interval $[\xi - \sigma, \xi]$ with center $c^{s*} \equiv \xi - \frac{\sigma}{2}$ at the southernmost end of the location space. Assemblers locate at a point c^{a*} satisfying $c^{a*} > \xi - \frac{\sigma}{2} \equiv c^{s*}$.

This proposition says that in the unique equilibrium of this economy, suppliers occupy the efficient locations at the south end. To get an idea of how the proof goes, suppose to the contrary that suppliers locate on an interval $[\overline{x} - \sigma, \overline{x}]$ that is not at the southernmost end, i.e., where $\overline{x} < \xi$. Lemma 1 implies assemblers will occupy a point c^a that is south of the center $c^s = \overline{x} - \frac{\sigma}{2}$ of the supplier interval. Lemma 2 then implies that the profit $\pi(\overline{x})$ at the south end of the supplier network is bigger than the profit $\pi(\overline{x} - \sigma)$ at the north end of the network. This implies that there exist unoccupied locations just south of the supplier network that yield greater profit than some locations within the supplier network. This is inconsistent with equilibrium.

The above argument eliminates from being possible equilibrium allocations where suppliers occupy an interval $[\overline{x} - \sigma, \overline{x}]$ with $\overline{x} < \xi$. The proof of Proposition 1 in the appendix shows that a location pattern where suppliers are not distributed along an interval cannot be an equilibrium. The only possibility left is the case where suppliers occupy the interval $[\xi - \sigma, \xi]$ at the south end of the location spectrum. It is immediate from Lemmas 1 and 2 that this location pattern is an equilibrium location pattern.

Case 2

Now consider Case 2 where assemblers are forced to spread out. Suppose that assemblers occupy an interval $[c^a - \frac{\alpha}{2}, c^a + \frac{\alpha}{2}]$. The profit of a supplier at x, taking as given the choices of all the other agents, equals

$$(6) \qquad \pi^s(x) = \int_{c^a - \frac{\alpha}{2}}^{c^a + \frac{\alpha}{2}} \tilde{\pi}(x, x^a) dx^a,$$

where $\tilde{\pi}(x, x^a)$ is the profit a supplier at x earns on sales to an assembler at x^a.

The fact that there is an interval of assembler locations makes the analysis here more complicated than the analysis for Case 1. Here there are trade-offs in the comparison of supplier profit between a location at the assembler center c^a and an alternative site x' north of c^a. A disadvantage of the x' location is that profits from sales to assemblers south of c^a are lower for a supplier at the x' location than for a supplier at the c^a location; i.e., $\tilde{\pi}(x', x^a) < \tilde{\pi}(c^a, x^a)$, for $x^a \geq c^a$. But if $\theta < \tau$, location x' yields higher profits on assemblers located north of x'; i.e., $\tilde{\pi}(x', x^a) > \tilde{\pi}(c^a, x^a)$, $x^a \geq x'$. These trade-offs make it difficult to compare supplier profit at alternate locations. Nevertheless, it is possible to prove some analytical results, including

Lemma 3. Assume that Case 2 applies and that $\alpha \leq \sigma$. Assume that suppliers and assemblers are distributed on the intervals $[c^s - \frac{\sigma}{2}, c^s + \frac{\sigma}{2}]$ and $[c^a - \frac{\alpha}{2}, c^a + \frac{\alpha}{2}]$ and that $c^s \geq c^a$. Under these assumptions, $\pi^s(c^s + \frac{\sigma}{2}) > \pi^s(c^s - \frac{\sigma}{2})$.

This result is analogous to the result of Lemma 2 for Case 1. The result says that if the assembler center is at least as far south as the supplier center, then profit at the southern end of the supplier interval is strictly greater than the profit at the northern end of the supplier interval. Intuition suggests that this result should follow from the fact that costs fall as one moves south. But I should note the assumption that assembler demand for the composite is elastic, $\phi > 1$, also plays a role in the result. An elastic demand ensures that total spending on intermediate inputs is greater for assemblers located near the south end of the supplier network (where locally produced inputs are relatively cheap) than for assemblers located near the north end of the supplier network (where locally produced inputs are relatively expensive).[3]

Lemma 3 along with Lemma 1 implies

Proposition 2. Assume that Case 2 applies and that $\alpha \leq \sigma$. An allocation in which suppliers locate on an interval $[\overline{x} - \sigma, \overline{x}]$ for $\overline{x} < \xi$ cannot be an equilibrium allocation.

The proof of Proposition 2 follows the same arguments given in the discussion of Case 1. Suppose suppliers occupied an interval $[\overline{x} - \sigma, \overline{x}]$ not at the south end; i.e., $\overline{x} < \xi$. Assemblers would locate at an interval with a center further south than the supplier center. (Note the assumption that $\alpha \leq \sigma$ guarantees that this is feasible). Lemma 3 implies that the profit at unoccupied locations just south of the supplier interval is greater than the profit at the north end of the supplier interval. This is inconsistent with equilibrium.

From Proposition 2, we know that if an interval of supplier locations $[\overline{x} - \sigma, \overline{x}]$ is not at the south end, it *cannot* be an equilibrium. The next issue is whether or not an interval of supplier locations at the south end *can* be an equilibrium. My results on this issue are arrived at in two

[3]Supplier profits are a constant fraction $\frac{\mu-1}{\mu}$ of revenues. Consider an alternative version of the model where assemblers have an inelastic demand for one unit of the composite intermediate. An assembler located at the north end of the supplier interval will pay a higher price for this one composite unit than an assembler located at the south end. Since supplier profits are a constant portion $\frac{\mu-1}{\mu}$ of revenues, aggregate supplier profit derived from assemblers at the north end will be greater than aggregate supplier profit derived from assemblers at the south end. If the transportation cost parameter τ is large enough, most of the aggregate supplier profit from assemblers at the north end will go to suppliers at the north end. This makes the north end of the supplier interval attractive to suppliers when τ is large.

different ways, analytical methods and numerical computations. For the case in which the elasticity of assembler composite demand ϕ is relatively big, I have obtained the following analytical result:

Proposition 3. Assume that Case 2 applies, that $\alpha \leq \sigma$, and that $\phi \geq \lambda$. There exists an equilibrium where suppliers occupy the interval $[\xi - \sigma, \xi]$ at the south end of the location spectrum.

The proposition assumes that the elasticity of composite demand is bigger than the elasticity of demand for individual specialized inputs. The remaining case where $\phi \in (1, \lambda)$ is difficult to analyze because the profit function $\pi(x)$ is not always single-peaked.[4] (This makes things difficult because I have to show that the profit at $\pi(\xi - \sigma)$ is less than the profit in the interior of the interval $(\xi - \sigma, \xi)$.) Hence, for this case I have been forced to look at numerical examples. In the examples I considered, an equilibrium with suppliers bunched at the south end always existed, even for extreme values of the parameters. On the basis of this numerical analysis, I conjecture that the statement in Proposition 3 also applies for the case of $\phi \in (1, \lambda)$.

The assumption that $\alpha \leq \sigma$ is important for Proposition 3. Suppose α is much bigger than σ and that suppliers occupy the locations $[\xi - \sigma, \xi]$ at the south end of the location spectrum. Since α is much bigger than σ, the supplier network would be far from the center $\xi - \frac{\alpha}{2}$ of the assembler locations $[\xi - \alpha, \xi]$. If the transportation cost parameter τ is big enough, it is clear that this allocation would be neither an efficient allocation nor an equilibrium allocation. Rather, it would be efficient to shift suppliers closer to the center of the assembler locations and to spread the suppliers out as well, and these forces will be reflected in any equilibrium allocation.

3. The Dynamic Model

Now consider an overlapping generations version of the model. In each period t, a measure σ of suppliers enters the economy. Suppliers live for two periods. When young, a supplier s makes a location decision that is fixed over the two periods of the supplier's life. Let $\ell_t^s(s)$ denote the location decision of supplier s born in period t. Assume that each location x can be occupied by at most a single old supplier and a single young supplier. Under this assumption the two generations can overlap in space as well as time.[5]

[4] Suppose that $\alpha = \sigma$ and that ϕ is close to one. If τ is big enough, there may be humps in $\pi(x)$ close to both of the endpoints in the supplier interval.

[5] An alternative approach is to assume that each location can be occupied by at most one supplier, young or old. This alternative assumption is perhaps more realistic. However, the results under this alternative assumption are qualitatively the same as my results. In addition, this alternative assumption comes at the cost of making the analysis somewhat awkward.

There is a measure α of assemblers in each period. To keep the analysis simple, assume that Case 1 applies so that assemblers can concentrate at a single point. Also, to keep the analysis simple, assume that assemblers can alter their location every period.

In the beginning of period t, the state variable in the economy is the set of location decisions $\ell^s_{t-1}(\cdot)$ made by the suppliers that entered in the previous period. Let $t = 1$ be the initial period. Let the locations $\ell^s_0(\cdot)$ of the old suppliers alive at the beginning of period $t = 1$ be the initial state.

Assume agents in the economy act to maximized discounted profits and use a discount factor of $\delta \in [0, 1)$. It is straightforward to extend the definition of equilibrium in the static case to equilibrium in this dynamic case.

A *stationary equilibrium* in this economy is an initial state $\ell^s_0(\cdot)$ and a set of choices by each agent in each period such that each agent is maximizing profit given the choices of the other agents and such that all new entering cohorts of suppliers select the same locations as the old suppliers in the initial state. The arguments used in the proof of Proposition 1 apply to

Proposition 4. There exists a unique stationary equilibrium in which each generation of suppliers is located on the interval $[\xi - \sigma, \xi]$ with center $c^{s*} \equiv \xi - \frac{\sigma}{2}$. Assemblers locate at a point c^{a*} satisfying $c^{a*} > \xi - \frac{\sigma}{2} \equiv c^{s*}$.

Suppose that the initial state is such that the old suppliers are located on the interval $[0, \sigma]$ at the north end of the location spectrum. Think of this as being due to some historical accident. Will the economy converge to the steady state? Will it do so in finite time? How does an equilibrium transition path compare with the path of a social planner? This rest of this section addresses these questions. But before I tackle these questions, I need to digress and make a distinction between *fragmented* and *unfragmented* allocations.

A. A Focus on Unfragmented Allocations

An *unfragmented* allocation is an allocation where, in each period, the entire measure α of assemblers is concentrated at a single point c^a_t and the entire entering cohort of suppliers locates on an interval $[c^s_t - \frac{\sigma}{2}, c^s_t + \frac{\sigma}{2}]$. A *fragmented* allocation is where at least one of these two conditions is not satisfied in some period.

In the family of models considered in this paper, a fragmented equilibrium often exists, and such an equilibrium is often inefficient (though it should be noted that these equilibria are usually unstable). For example, in a model with two discrete locations, North and South, there might be an equilibrium where half the assemblers locate in the North and half locate in the South. This might be an equilibrium since, given that the agglomeration benefits are the same at the two locations, assemblers might be indifferent between the two locations. Such an equilibrium might be inefficient because all might be better off if the entire industry were concentrated in one location allowing for a greater level of agglomeration benefits.

For this model, I know from Proposition 1 that, for the static case, there does not exist a fragmented equilibrium. I know from Proposition 4 that, for the dynamic model, the allocation in the unique stationary equilibrium is unfragmented. Outside of the stationary case, I do not know whether or not there exists an equilibrium in the dynamic model in which there is any fragmentation along the equilibrium path. I do know that I can make small changes in the model which allow the possibility of a fragmented equilibrium.[6] The existence of old suppliers in fixed locations that do not necessarily maximize current profit is crucial for this construction. (In the static model all locations have to maximize current profit, and a fragmented equilibrium is impossible.)

In this section, I avoid the difficulties inherent in analyzing fragmented equilibria by restricting attention to unfragmented equilibria. This allows me to focus on the main question posed in the introduction: namely, suppose an industry is initially concentrated in an inefficient northern location; does the industry eventually migrate to the more efficient southern location? By ignoring fragmented equilibria I am unable to address a different question: namely, suppose that assembly operations are fragmented in an inefficient way across space; what market forces operate to ensure the industry is eventually concentrated in an efficient way? At this point I can only say that in the static version of the model an inefficient fragmented equilibrium is impossible. The dynamic case allowing the possibility of fragmentation along the equilibrium path is left for future research.

B. Does the Industry Migrate South?

This subsection asks whether or not the industry eventually migrates to the southern end of the location spectrum, given that the initial old suppliers are located at the northern end of the location spectrum. To state my result, I will define a critical time period $\hat{t}$ by

$$\hat{t} \equiv \frac{2\tau(\xi - \sigma)}{\theta\sigma}.$$

[6] For example, suppose that in the initial period the old suppliers occupy the unit interval $[0,1]$. I can construct an example where the new suppliers have a measure $\sigma = 0.9$ in which there exists an equilibrium with fragmented assemblers in the initial period. In this example equilibrium, the new suppliers locate on the interval $[1.7, 2.6]$ in the initial period. A small fraction of the assemblers locate at a point close to the center of the old-supplier network, and the remaining assemblers locate at a point near the center of the new-supplier network.

For technical reasons it is convenient to assume there is a finite horizon T in the economy, where T is large and $T > \hat{t}$. My result is

Proposition 5. Suppose that in the initial state old suppliers occupy the interval $[0, \sigma]$. In any unfragmented equilibrium, there exists a critical time period t', where $t' \leq \hat{t}$, such that in any period $t \geq t'$, all new suppliers locate on the interval $[\xi - \sigma, \xi]$ at the south end of the location space. Before period t', new suppliers locate on the interval $[c_t^s - \frac{\sigma}{2}, c_t^s + \frac{\sigma}{2}]$ with center c_t^s, and this center shifts south by at least $\frac{\theta\sigma}{2\tau}$ in each period; i.e., $c_t^s > c_{t-1}^s + \frac{\theta\sigma}{2\tau}$, for $t \leq t'$.

The result says that in any unfragmented equilibrium, the industry migrates south in a monotone fashion. In each period the set of new suppliers shifts south at least by an amount $\frac{\theta\sigma}{2\tau}$. With a migration of this rate, the industry necessarily bangs up against the southern endpoint of the location spectrum by period $\hat{t}$.

Proposition 4 does not say anything about whether or not an unfragmented equilibrium exists. It is straightforward to show that an unfragmented equilibrium exists for the case of $\delta = 0$. I present an existence result for the case of $\delta > 0$ in Proposition 9.

C. The Social Planner's Solution

The previous subsection answers the question of whether or not the industry migrates south. (It does). The next question is whether or not the transition path is efficient. As a first step in answering this question, this subsection considers the problem of a social planner maximizing discounted expected profit.

To make the analysis as simple as possible, I assume that each assembler has an inelastic demand for one unit of the composite and uses this to produce one unit of the final good. All the claims in Propositions 1, 4, and 5 apply for this alternative assumption, and it simplifies the analysis considerably here. Under this assumption, the only efficiency issue that needs to be considered is the location decisions of the agents. Given a set of location decisions of the agents, the intermediate goods are constructed in an efficient way in the equilibrium allocation. Even though there is a wedge of $\mu - 1$ between the price and the marginal cost of each input, the wedge is the same for each good, resulting in an efficient mix of each input.

Continuing to make things as simple as possible, assume that the discount factor $\delta = 0$. Take as given that in the initial state the old suppliers are on the interval $[0, \sigma]$. The assumption that $\delta = 0$ implies that the problem of maximizing total surplus here reduces to solving for the assembler locations and new-supplier locations that minimize the average total cost in the period of delivering one composite intermediate to each assembler. It is straightforward to show that in any solution to this problem, all assemblers are concentrated in the same location c^a. If $\theta < \tau$, it is straightforward to show that when assemblers locate at c^a, the locations for the new suppliers that minimize average total cost are the locations on the interval $[\underline{x}, \overline{x}]$, where

$$(7) \qquad \underline{x} = c^a - \frac{\sigma}{2} + \frac{\theta\sigma}{2\tau} \text{ and } \overline{x} = c^a + \frac{\sigma}{2} + \frac{\theta\sigma}{2\tau}$$

in the interior case defined by

$$\textit{Interior Case: } c^a + \frac{\sigma}{2} + \frac{\theta\sigma}{2\tau} < \xi,$$

and

$$(8) \qquad \underline{x} = \xi - \sigma \text{ and } \overline{x} = \underline{x} + \sigma$$

in the corner case defined by

$$\textit{Corner Case: } c^a + \frac{\sigma}{2} + \frac{\theta\sigma}{2\tau} \geq \xi.$$

In the interior case (7), this solution sets the marginal cost of supplying location c^a at the north end $\underline{x}$ of the interval equal to the marginal cost at the south end $\overline{x}$, where marginal cost includes transportation cost as well as production cost.

Figure 2 plots average total cost (ATC) as a function of the assembler location c^a, taking as given that the new suppliers are located on the interval given by (7), i.e., that the new-supplier locations minimize average total cost given the assembler location c^a. In this numerical example, $\sigma = 1$. The ATC function has a number of properties. At $c^a = 0.5$, the center of the old suppliers, this function is strictly decreasing. The function continues to decrease until it reaches a local minimum at 0.7 which is within the network of old suppliers. Beyond 0.7, the function first increases and then eventually decreases by so much that for large enough c^a, the average total cost is less than at the local minimum at 0.7.

The shape of the ATC function here has the following explanation. It is necessarily decreasing at 0.5, the center point of the old-supplier network. This follows because even if the new suppliers did not exist, it would minimize transportation costs on purchases from old suppliers alone to shift the assembly location south of the old-supplier network center (recall Lemma 1). As c^a is increased beyond the old-supplier center 0.5, eventually for high enough c^a there is a trade-off from increasing c^a. Raising c^a puts assemblers at a further distance from the old suppliers, which raises transportation costs. However, raising c^a shifts the new suppliers further south, lowering their cost. The local minimum at 0.7 is a point where these two offsetting effects balance. For a range of c^a beyond this point, the first effect of an increase in c^a on increasing transportation costs from old suppliers dominates the second effect of lowering the costs of the new suppliers. Eventually, however, this reverses. For large enough c^a, assemblers are buying so little from old suppliers that the effect is negligible, and the second effect dominates. In the limit, as c^a goes to infinity, average total cost goes to zero as the costs of the new suppliers goes to zero. Hence, for large enough c^a, average total cost is lower than at the local minimum for small c^a.

If the cost decline parameter θ is large enough, the shape of the ATC function is different from that in Figure 2, for it will be strictly decreasing over its entire range. To make the point I want to make here, it is sufficient to limit attention to the case where the ATC function takes the shape in Figure 2. Formally, assume that θ is small enough so that

$$(9) \qquad \min_{c^a \in [\frac{\sigma}{2}, \sigma]} ATC(c^a) < ATC(\sigma)$$

holds.[7]

The discussion of average total cost so far has ignored the fact there is an endpoint ξ of the location space. This leads to minor modifications. Suppose, for example, that $\xi = 2$. The parameters for this particular example imply that the cost-minimizing way to distribute the new suppliers is for the center of the new suppliers c^s_{new} to be equal to $c^a + 0.1$. So given that $\xi = 2$, the endpoint constraint is not binding if $c^a < 1.4$. So for $c^a < 1.4$, the ATC when we take account of the constraint is the same as without the constraint. For $c^a \geq 1.4$, the constraint is binding, so all new suppliers are located at $[1, 2]$, for all c^a in this range. The imposition of this endpoint constraint shifts up the ATC for c^a in the range $[1.4, 2]$, as illustrated with the dotted lines in Figure 2. The figure also illustrates the analogous case of $\xi = 3$.

It is clear from inspection of Figure 2 that there are two possible forms for the solution to the social planner's problem. The first form is optimal if ξ is small. In this case the optimum is to set the assembly center at the local optimum in the interior of the old-supplier network, i.e., at 0.7 in the example. The new suppliers are located nearby according to the rule (7). Call this first possibility the *step-by-step strategy*. In the example in the figure, the step-by-step strategy is optimal, for

[7] Note that in the limiting case of $\theta = 0$, the condition necessarily holds because the minimum of $ATC(c^a)$ is attained at $c^a = \frac{\sigma}{2}$. Since there is no cost advantage as one moves south, the optimum is obtained by placing assemblers in the center of the interval of old suppliers and by exactly overlapping the interval of new suppliers with the interval of old suppliers.

instance, if $\xi = 2$. The second form the solution might take arises if ξ is large (e.g., $\xi = 3$ in the example). In this case the optimal strategy is to immediately jump towards the southernmost end of the interval by placing new suppliers on the interval $[\xi - \sigma, \xi]$. Call this second possibility the *jump strategy.* Note that if in the initial period, the step-by-step strategy is optimal, then the step-by-step strategy is optimal in all further periods since the relative return from the jump strategy falls. (The distance between the center of the old suppliers and the south end of the location space falls each period.) This discussion suggests the intuition for the following proposition, which is proved in the appendix.

Proposition 6. Assume that θ is small enough that (9) holds. Assume that $\delta = 0$. There exists a critical level $\hat{\xi}$ of ξ where $\hat{\xi} > \sigma$ with the following properties. If $\xi < \hat{\xi}$, the unique solution to the social planner's problem specifies that the supplier center shift by a constant step $z^* < \sigma$ in each period, $c_t^s = c_{t-1}^s + z^*$, until the endpoint is reached where all suppliers are located at the south end $[\xi - \sigma, \xi]$. If $\xi > \hat{\xi}$, the unique solution specifies that in the initial period the new suppliers jump to the south end $[\xi - \sigma, \xi]$ of the location spectrum and that all future generations of suppliers also locate there.

Note that in the proposition if $\xi < \hat{\xi}$ so that the step-by-step strategy is optimal, the step size z^* is less than σ. Hence the old and new generations overlap geographically. There is a continuous movement south; no locations are missed. If $\xi > \hat{\xi}$, there is a discontinuous jump to the South.

D. Is the Equilibrium Path Efficient?

Now that I have characterized the social planner's solution, I can relate it to the equilibrium outcome.

Proposition 7. Assume that $\delta = 0$. Then the social planner's solution can be decentralized as an equilibrium allocation.

Proposition 7 presents the good news that any socially efficient allocation can be decentralized as an equilibrium allocation. It is a straightforward result. Taking c^a as given, the social planner picks the locations for the new suppliers that minimize average total cost, but these are the same locations that maximize profit for the suppliers.

Define a *step-by-step equilibrium* to be the analog of the step-by-step form of the planner's solution. Specifically, in a step-by-step equilibrium, the center of the new-supplier interval shifts over by a constant amount $z^* < \sigma$ in each period, i.e., $c_t^s = c_{t-1}^s + z^*$, and the location of assemblers shifts over by a constant amount z^* in each period, i.e., $c_t^a = c_{t-1}^a + z^*$. These shifts take place until the interval of new suppliers runs into the boundary at the southern endpoint, at which point the shifts stop. The next proposition presents the bad news.

Proposition 8. Assume that $\delta = 0$ and that θ is small enough so that (9) holds. Then for any ξ, there exists a step-by-step equilibrium with a step size $z^* < \sigma$. Furthermore, there exists a $\xi' < \hat{\xi}$, such that if $\xi \in (\xi', \xi]$, there also exists a jump equilibrium where all new suppliers locate at the southern end $[\xi - \sigma, \xi]$ beginning in the initial period.

Under the assumptions of Proposition 8, there exists a step-by-step equilibrium for any level of ξ. It is straightforward to see why. The step-by-step approach is a local optimum for the social planner. Given that all other new suppliers locate in the step-by-step formation, it is globally optimal for an individual supplier to join the formation. However, for large ξ, it is not globally optimal for the social planner to take the step-by-step approach. This illustrates that the migration south in the market allocation can be too slow; it is efficient to jump, but the market takes baby steps. The second part of the result shows that the reverse is also true. For a range of ξ just below $\hat{\xi}$, the globally optimal solution for the social planner is the step-by-step approach. However, for ξ in this range there exists a jump equilibrium. Hence, it is possible for the market to move too fast.

E. An Unbounded Location Space

This subsection considers an alternative version of the model where the location space is unbounded; i.e., the location space is $[0, \infty)$ instead of $[0, \xi]$. Eliminating the endpoint eliminates the horizon effect in the analysis. New suppliers can shift over by a constant amount z in each period without the economy ever running into a bound. Considering this alternative version of the model facilitates the analysis of the case where the discount factor δ is positive. By considering this case, I am able to derive a number of comparative statics results.

For this subsection, I return to the case where assemblers have the production function given by (2) and the elasticity of assembly supply is ϕ. (The results also apply in the alternative case where supply is inelastic.)

Consider a planner's problem of selecting an assembly point $c_1^a = c^a$ in period 1 and a constant step size z, so that the assembly center point is $c_t^a = c^a + (t-1)z$ in period t and new suppliers shift over by z in each period. Suppose the objective of the planner is to maximize the discounted sum of total assembler and total supplier profit. It is straightforward to show that if the minimum cost to construct a delivered unit of the composite input in period t is v_t, then the sum of assembler profit and supplier profit in period t is

$$(10) \quad w_t = kv_t^{-(\phi-1)},$$

for some constant $k > 0$. Along the step-by-step path, the composite cost falls at the rate

$$(11) \quad v_{t+1} = e^{-\theta z} v_t.$$

Using (10) and (11), we know that discounted social welfare is

$$W(z, c^a) = \frac{1}{1 - \delta e^{(\phi-1)\theta z}} kv_1(z, c^a)^{-(\phi-1)},$$

where $v_1(z, c^a)$ is the minimum composite cost in period 1 as a function of z and c^a. It is clear that there does not exist a solution to the social planner's problem. By making the step size z arbitrarily large, the planner can make the average composite cost v_t arbitrarily close to zero and make discounted profits arbitrarily large.

Consider next a constrained planner's problem

$$(12) \quad \max_{z \leq \sigma,\ c^a} W(z, c^a).$$

In this problem, the planner is constrained to keep the step size no bigger than σ. This requires that the generations of suppliers overlap. This might be a reasonable constraint to impose if we think about the points x on the location space as technologies. In this interpretation, the assumption amounts to a constraint that no technologies be skipped. There is a quality ladder, and society has to go through each of the rungs on the ladder as in Grossman and Helpman (1991).

The next proposition states the result of this section.

Proposition 9. Take as given the model parameters μ, τ, σ, δ, and ϕ. There exists a $\hat{\theta} > 0$, such that if $\theta < \hat{\theta}$, there exists a unique step-by-step equilibrium with a constant step $z^* < \sigma$. There also exists a unique optimum to the constrained social planner's problem (12), and the planner's constrained optimum coincides with the equilibrium. For θ in the range $(0, \hat{\theta})$ the equilibrium step size z^* has the following comparative statics properties. It *decreases* with μ and τ. It *increases* with θ, δ, and ϕ.

According to the proposition, if the rate θ at which cost falls is not too big, there exists a unique step-by-step equilibrium. The equilibrium step size coincides with the planner's optimal step, given that the planner is constrained from making a jump and must pursue a step-by-step strategy instead. The model has intuitive comparative statics properties. The more important the agglomeration economies, through either the preference-for-variety parameter μ or transportation costs τ, the slower the migration. The more cost falls as one moves south, the more important the future; and the greater the responsiveness of assembly supply to the delivered composite price, the faster the migration.

4. Concluding Remarks

A large recent literature emphasizes that historical accidents can trap economies in inefficient agglomerations. In this paper, I modify a standard model in the literature by first assuming that space is a continuum and second assuming that it is infeasible for the entire economy to agglomerate at a single point. Under these two reasonable assumptions, I show that the economy can always find its way out of the trap.

I think more research with this kind of model is warranted because studying the forces highlighted in this model may help us understand trends in important industries like the automobile industry and the computer industry.

Consider, for example, the U.S. automobile industry. Access to local suppliers in this industry has become increasingly important due to the adoption of just-in-time production methods (see Rubenstein (1992)). For a number of years most of this industry was concentrated in Michigan. But the new assembly plants built since the 1980s have gradually shifted the center of this industry in the direction of the South. The first Japanese transplant assembly plant was the Honda plant in southern Ohio. By picking an Ohio location, Honda obtained access to the network of suppliers that existed in Michigan and northern Ohio and spurred the entry of new suppliers in southern Ohio. Subsequent Japanese plants were built in Tennessee and Kentucky. The most recent new automobile plant to be announced is the Mercedes plant in Alabama. Alabama would have been an unlikely choice for an automobile plant 15 years ago. The choice is not so unthinkable now given the supplier network that has emerged in Tennessee. The automobile industry appears to be experiencing a step-by-step migration that parallels, to some extent, the migration in the model economy.

Next consider the computer industry. There certainly has been much discussion about network externalities in the adoption of standards for operating systems for computers. It is possible to reinterpret the location space in my model as a space of computer standards. Think of a specialized supplier in the model as a specialized piece of software. A piece of software is *near* a standard if is designed to work under the standard. Most existing software has been written in 16-bit code for use with DOS and Microsoft Windows 3.1. Microsoft NT is a superior operating system because it is completely 32-bit. However, the old 16-bit software does not always work well on NT. Rather than have us jump from DOS to NT, the social planner (a.k.a. Bill Gates) has engineered a step-by-step migration between these two standards with the introduction of Windows 95. Windows 95 was explicitly written to run the old 16-bit software well and also serve as a platform for 32-bit software. But it is something less than a full 32-bit operating system. As more 32-bit software is written, we will become less dependent upon the 16-bit software, and eventually we can all take the next step to a fully 32-bit operating system like NT.

One of the most controversial issues in the computer industry is the dominance of Microsoft.[8] Some people think that Apple's Macintosh standard is inherently superior to the Microsoft standard. In their view, Microsoft's dominance of the market is a bad equilibrium arising from network externalities in the widespread adoption of a standard. The model that I have presented here in its current form is not suitable for analysis of this issue. There is no analog in the model of two large players battling it out. But I think some of the ideas of this paper may be relevant here. Those who worry that we will be stuck forever with an inefficient Microsoft technology make a mistake in characterizing the market as having two discrete alternative standards, Macintosh versus Microsoft. In reality, there is a continuum of standards made up of convex combinations of these two cases. Microsoft has gradually changed its standards in a step-by-step fashion, from DOS to Windows 3.1 to Windows 95, to emulate the good things about the Macintosh.

[8]See, for example, Taylor (1993) and Gleick (1995) for articles with the the titles "The Microsoft monopoly: How do you restrain an 800-pound gorilla?" and "What to do with the Microsoft monster."

Appendix

Lemma 1. Take as given that suppliers locate on the interval $[c^s - \frac{\sigma}{2}, c^s + \frac{\sigma}{2}]$ with center c^s and that suppliers price according to (3). The cost of the composite intermediate $v(x)$ is u-shaped as a function of x, and the minimum is attained at a point c^{a*} strictly greater than the supplier network center c^s. Furthermore, $v(c^s + y) < v(c^s - y)$ for all $y > 0$; i.e., the composite cost at a location y units south of the center is less than the composite cost at a location y units north of the center.

Proof. It is obvious that $v(x)$ is strictly decreasing for $x < c^s - \frac{\sigma}{2}$ and strictly increasing for $x > c^s + \frac{\sigma}{2}$. So consider a location point x^a in the interior of the supply network $(c^s - \frac{\sigma}{2}, c^s + \frac{\sigma}{2})$. Let $h(x, x^a)$ denote the optimal purchases from a supplier located at x for an assembler at x^a. The assembler sets the MRS between an input produced at its location x^a and an input produced at some other point x equal to the ratio of prices,

$$MRS = \frac{h\left(x^a, x^a\right)^{\frac{1-\mu}{\mu}}}{h(x, x^a)^{\frac{1-\mu}{\mu}}} = \frac{\mu e^{-\theta x^a}}{\mu e^{-\theta x + \tau |x^a - x|}} = e^{-\theta(x^a - x) - \tau |x^a - x|}.$$

This implies that

$$\text{(13)} \quad h\left(x, x^a\right) = h\left(x^a, x^a\right) e^{\frac{\mu}{\mu-1}\left(-\theta(x^a - x) - \tau |x^a - x|\right)}.$$

Let $\underline{x} = c^s - \frac{\sigma}{2}$ and $\overline{x} = c^s + \frac{\sigma}{2}$ be the endpoints of the interval containing the supplier network. The average total cost of the composite to an assembler at x^a is

$$\begin{aligned}
\text{(14)} \quad v(x^a) &= \frac{\int_{\underline{x}}^{\overline{x}} p^d(x, x^a) h\left(x, x^a\right) dx}{\left[\int_{\underline{x}}^{\overline{x}} h\left(x, x^a\right)^{\frac{1}{\mu}} dx\right]^{\mu}} \\
&= \frac{\int_{\underline{x}}^{\overline{x}} \mu e^{-\theta x + \tau |x^a - x|} h\left(x^a, x^a\right) e^{\frac{\mu}{\mu-1}\left(-\theta(x^a - x) - \tau |x^a - x|\right)} dx}{\left[\int_{\underline{x}}^{\overline{x}} h\left(x^a, x^a\right)^{\frac{1}{\mu}} e^{\frac{\mu}{\mu-1}\left(-\theta(x^a - x) - \tau |x^a - x|\right)\frac{1}{\mu}} dx\right]^{\mu}} \\
&= \mu \frac{\int_{\underline{x}}^{\overline{x}} e^{\frac{1}{\mu-1}\left(\theta x - \tau |x^a - x|\right)} dx}{\left[\int_{\underline{x}}^{\overline{x}} e^{\frac{1}{\mu-1}\left(\theta x - \tau |x^a - x|\right)} dx\right]^{\mu}} \\
&= \mu \left[\int_{\underline{x}}^{\overline{x}} e^{\frac{1}{\mu-1}\left(\theta x - \tau |x^a - x|\right)} dx\right]^{-(\mu-1)} \\
\text{(15)} \quad &= \mu \left[u(x(^a))\right]^{-(\mu-1)},
\end{aligned}$$

where the function $u(x^a)$ is defined by

$$\text{(16)} \quad u(x^a) \equiv \int_{\underline{x}}^{\overline{x}} e^{\frac{1}{\mu-1}\left(\theta x - \tau |x^a - x|\right)} dx.$$

Assemblers will select their locations to minimize $v(x^a)$, which, since $\mu > 1$, is equivalent to maximizing $u(x^a)$. We can write $u(x^a)$ as

$$(17) \quad u(x^a) \equiv \int_{\underline{x}}^{x^a} e^{\frac{1}{\mu-1}(\theta x - \tau(x^a - x))} dx + \int_{x^a}^{\overline{x}} e^{\frac{1}{\mu-1}(\theta x - \tau(x - x^a))} dx.$$

The slope of $u(x^a)$ is

$$\begin{aligned}(18) \quad \frac{du}{dx^a} &= \frac{\tau}{\mu-1}\left[-\int_{\underline{x}}^{x^a} e^{\frac{1}{\mu-1}(\theta x - \tau(x^a - x))} dx + \int_{x^a}^{\overline{x}} e^{\frac{1}{\mu-1}(\theta x - \tau(x - x^a))} dx\right] \\ &= \frac{\tau}{\mu-1}\left[\begin{array}{c} -\frac{\mu-1}{\tau+\theta}\left[e^{\frac{1}{\mu-1}\theta x^a} - e^{\frac{1}{\mu-1}(\theta\underline{x} - \tau(x^a - \underline{x}))}\right] \\ -\frac{\mu-1}{\tau-\theta}\left[e^{(\theta\overline{x} - \tau(\overline{x} - x^a))} - e^{\frac{1}{\mu-1}\theta x^a}\right] \end{array}\right] \\ &= \tau e^{\frac{1}{\mu-1}\theta x^a} F(x^a, \underline{x}, \overline{x}),\end{aligned}$$

where

$$(19) \quad F(x^a, \underline{x}, \overline{x}) \equiv -\frac{1}{\tau+\theta}\left[1 - e^{-\frac{1}{\mu-1}(\tau+\theta)(x^a - \underline{x})}\right] + \frac{1}{\tau-\theta}\left[1 - e^{-\frac{1}{\mu-1}(\tau-\theta)(\overline{x} - x^a)}\right].$$

Straightforward differentiation shows that F is strictly decreasing in x^a. It is also straightforward to show that $F(\underline{x} + \frac{\sigma}{2}, \underline{x}, \overline{x}) > 0$, for $\theta > 0$. These two facts and the fact that $\frac{du}{dx^a} = \tau e^{\frac{1}{\mu-1}\theta x^a} F$ imply that there is a unique x^{a*} maximizing $u(x^a)$ that satisfies $x^{a*} > \underline{x} + \frac{\sigma}{2} = c^s$. This proves that $u(x^a)$ is single-peaked and that the maximum is south of the center c^s, which in turn implies that $v(x^a)$ is u-shaped with a minimum south of the center c^s. The result that $v(c^s + y) < v(c^s - y)$ for $y > 0$ follows from straightforward but tedious calculations using the formula (17).■

Lemma 2. Suppose that Case 1 applies and that all assemblers are concentrated at the point c^a. The profit of a supplier located at x takes the form

$$(20) \quad \pi^s(x) = k e^{\frac{1}{\mu-1}[\theta x - \tau|c^a - x|]},$$

where k is a positive constant that is independent of x. This function is single-peaked and has the property that $\pi^s(c^a + y) > \pi^s(c^a - y)$ for all $y > 0$.

Proof. Let $\tilde{\pi}(x, x^a)$ be the profit a supplier located at x obtains from sales to an assembler located at x^a. I first show that this profit takes the following form:

$$(21) \quad \tilde{\pi}(x, x^a) = e^{\frac{1}{\mu-1}[\theta x - \tau|x^a - x|]}(\mu - 1)e^{-\frac{\mu}{\mu-1}\theta x^a} h(x^a, x^a),$$

where $h(x^a, x^a)$ is what an assembler located at x^a would purchase from a supplier at x^a if such a supplier existed. To see that (21) must hold, note that for every one unit that is delivered, $e^{\tau|x^a - x|}$ needs to be shipped because of the transportation cost. Recall that the marginal cost at x is $e^{-\theta x}$ and that the price is a markup μ over this. Since an assembler at x^a purchases $h(x, x^a)$ units from a supplier at x, the supplier's profit from these sales is

$$\tilde{\pi}(x, x^a) = (\mu - 1)e^{-\theta x} e^{\tau|x^a - x|} h(x, x^a),$$

which reduces to (21) when the formula (13) for $h(x, x^a)$ is substituted in. Suppose, then, as stipulated in the lemma, that the entire measure α of assemblers is concentrated at a point c^a. The profit at x is

$$\begin{aligned} \pi^s(x) &= \alpha\pi(x, c^a) \\ &= ke^{\frac{1}{\mu-1}[\theta x - \tau|c^a - x|]}, \end{aligned}$$

for k defined by

$$k \equiv \alpha(\mu - 1)e^{-\frac{\mu}{\mu-1}\theta x^a} h(x^a, x^a).\blacksquare$$

Proposition 1. Assume that Case 1 applies. There exists a unique equilibrium. In the equilibrium allocation, suppliers locate on the interval $[\xi - \sigma, \xi]$ with center $c^{s*} \equiv \xi - \frac{\sigma}{2}$ at the southernmost end of the location space. Assemblers locate at a point c^{a*} satisfying $c^{a*} > \xi - \frac{\sigma}{2} \equiv c^{s*}$.

Proof. Following the arguments given in the text, the only thing I need to show here is that there cannot exist an equilibrium with suppliers located on some subset L of the location space which is different from $[\xi - \sigma, \xi]$. Suppose this is not true and there exists an equilibrium with supplier locations $L \neq [\xi - \sigma, \xi]$. Define $\underline{x}^s$ to be the infimum over the set of equilibrium locations L. Since $L \neq [\xi - \sigma, \xi]$, $\underline{x}^s < \xi - \sigma$.

Analogous to the proof of Lemma 1, here assemblers will choose locations that maximize $u(x^a)$ defined by

$$(22) \quad u(x^a) \equiv \int_L e^{\frac{1}{\mu-1}(\theta x - \tau|x^a - x|)} dx.$$

Let u^* be the maximum of $u(x^a)$ over the unit interval. Let $\underline{x}^a$ be the minimum x^a such that $u(x^a) = u^*$. (By the continuity of $u(x^a)$, such a minimum must exist.)

The first step is to prove that $\underline{x}^s < \underline{x}^a$. Suppose not, that $\underline{x}^a \leq \underline{x}^s$. This says that all suppliers are south of $\underline{x}^a$. It is straightforward to show that $u(x^a)$ is strictly increasing in x^a at $\underline{x}^a$. This contradicts the definition of $\underline{x}^a$ of being in the set of maximizers of $u(x^a)$. Hence, $\underline{x}^s < \underline{x}^a$.

I next claim that $(\underline{x}^s, \underline{x}^a] \subseteq L$; i.e., there must be suppliers in the interval of points between $\underline{x}^s$ and $\underline{x}^a$. Let $\pi(x)$ be the profit to a supplier at location x. Since all assemblers are at $\underline{x}^a$ or south of $\underline{x}^a$, it is immediate that $\pi(x)$ is strictly increasing in x on the interval $(\underline{x}^s, \underline{x}^a]$. Since the profit at all points $(\underline{x}^s, \underline{x}^a]$ is strictly greater than at $\underline{x}^s$ and since $\underline{x}^s$ is the infimum of where suppliers locate, from the definition of equilibrium all points $(\underline{x}^s, \underline{x}^a]$ must be occupied by suppliers.

I next claim there must exist a nonempty set of locations south of $\underline{x}^a$ that are not in L. This follows since $\underline{x}^s < \xi - \sigma$, since $(\underline{x}^s, \underline{x}^a] \subseteq L$, and since the measure of L is σ. Let x' be the infimum of the set of points $x > \underline{x}^a$ that are not in L.

I next claim that

$$\underline{x}^a > \underline{x}^s + \frac{1}{2}\left(x' - \underline{x}^s\right) \tag{23}$$

must hold. To see this, note that by the definition of $\underline{x}^s$ and x' the interval $(\underline{x}^s, x')$ is completely occupied by suppliers (i.e., $(\underline{x}^s, x') \subseteq L$), and furthermore, there are no suppliers below $\underline{x}^s$. Consider first what assembler location preferences would be if I were to truncate the supplier distribution and throw out all suppliers located above x'. Let $\tilde{L} = (\underline{x}^s, x')$ be this truncated distribution. Let $u(x^a, \tilde{L})$ be the assembler utility of location x^a given this set of supplier locations $\tilde{L}$. Follow the same proof as in Lemma 1: there is a unique maximum $\tilde{x}^a$ of $u(\tilde{x}^a, \tilde{L})$ satisfying $\tilde{x}^a > \underline{x}^s + \frac{1}{2}\left(x' - \underline{x}^s\right)$. Consider two points x_1 and x_2 such that $x_1 < x_2 \leq x'$. Using the formula (22) for the location utility function gives

$$\begin{aligned}
u(x_2, \tilde{L}) - u(x_1, \tilde{L}) &= \int_{\tilde{L}} \left[e^{\frac{1}{\mu-1}(\theta x - \tau |x_2 - x|)} - e^{\frac{1}{\mu-1}(\theta x - \tau |x_1 - x|)} \right] dx \\
&\leq \int_{L} \left[e^{\frac{1}{\mu-1}(\theta x - \tau |x_2 - x|)} - e^{\frac{1}{\mu-1}(\theta x - \tau |x_1 - x|)} \right] dx \\
&= u(x_2, \tilde{L}) - u(x_1, \tilde{L}).
\end{aligned}$$

The inequality holds because for any x in L that is not in $\tilde{L}$, the term in brackets is positive (for any such x, $x \geq x_2 > x_1$). This says that if an assembler prefers x_2 to an x_1 when $x_1 < x_2 \leq x'$ holds given truncated supplier distribution $\tilde{L}$, then the consumer prefers x_2 to x_1 at the actual supplier distribution L. Since the maximum $\tilde{x}^a$ of $u(x^a, \tilde{L})$ satisfies $\tilde{x}^a > \underline{x}^a + \frac{1}{2}\left(x' - \underline{x}^a\right)$, it cannot be true that a maximum of $u(x^a, L)$ satisfies $x^a \leq \underline{x}^a + \frac{1}{2}\left(x' - \underline{x}^a\right)$. This implies that (23) holds as claimed.

I next show that $\pi^s(x') > \pi^s(\underline{x}^s)$. Inequality (23) implies that at any location x^a where there is an assembler, the location is closer to x' than to $\underline{x}^s$. Since $x' > \underline{x}^a$ also holds, Lemma 2 then implies that the profit $\pi(x', x^a)$ to a supplier at x' from sales to x^a is greater than the profit $\pi(\underline{x}^a, x^a)$ to a supplier at $\underline{x}^s$ from sales to x^a. Since this comparison is true for all x^a where consumers locate, $\pi^s(x') > \pi^s(\underline{x}^s)$ must hold.

Recall that x' is the infimum of locations above $\underline{x}^a$ that are not in L while $\underline{x}^s$ is the infimum of locations in L. The fact that $\pi^s(x') > \pi^s(\underline{x}^s)$ and the continuity of π^s imply that there exists a point in L with strictly less profit than a point outside of L. This contradicts the definition of equilibrium and completes the proof of the proposition.■

Lemma 3. Assume that Case 2 applies and that $\alpha \leq \sigma$. Assume that suppliers and assemblers are distributed on the intervals $[c^s - \frac{\sigma}{2}, c^s + \frac{\sigma}{2}]$ and $[c^a - \frac{\alpha}{2}, c^a + \frac{\alpha}{2}]$ and that $c^s \geq c^a$. Under these assumptions, $\pi^s(c^s + \frac{\sigma}{2}) > \pi^s(c^s - \frac{\sigma}{2})$.

Proof. The proof has two parts. Part 1 treats the case of $\phi \geq \lambda$. Part 2 treats the case of $\phi \in (1, \lambda)$.

Part 1: The Case of $\phi \geq \lambda$

For this case I prove a slightly more general result that I later use in the proof of Proposition 3. The more general result is that for $\phi \geq \lambda$ and $\alpha \leq \sigma$ and any $y > 0$, $\pi^s(c^s + y) > \pi^s(c^s - y)$. The proof for this claim has six steps.

Step 1

This step shows that the following is true:

$$(24) \quad h(x^a, x^a) = k_1 e^{\frac{\mu}{\mu-1}\theta x^a} \left[u(x^a)\right]^{\phi(\mu-1)-\mu}.$$

To prove this, note first that

$$(25) \quad h(x^a, x^a) = m(x^a) g(x^a, x^a),$$

where $m(x^a)$ is the demand for delivered units of the composite for an assembler located at x^a and $g(x, x^a)$ is the amount of the specialized input produced at x in the cost-minimizing bundle of inputs at location x^a used to construct one unit of the composite. Recall that the assembler production function is $q = m^{\frac{\phi-1}{\phi}}$. I first show that

$$\begin{aligned}(26) \quad m(x^a) &= \left[\frac{\phi}{\phi-1}\right]^{-\phi} v(x^a)^{-\phi} \\ &= \left[\frac{\phi}{\phi-1}\right]^{-\phi} \left[\mu u(x^a)^{-(\mu-1)}\right]^{-\phi}.\end{aligned}$$

The first equality follows from the FONC for the profit-maximizing choice of m. The second inequality follows from the definition (16) of $u(x^a)$.

I next obtain a formula for $g(x^a, x^a)$. Analogous to (13), cost minimization implies that

$$(27) \quad g\left(x, x^a\right) = g\left(x^a, x^a\right) e^{\frac{\mu}{\mu-1}\left(-\theta(x^a - x) - \tau|x^a - x|\right)}.$$

By definition, the combination of the input levels $g(x, x^a)$ at x^a must construct one unit of the composite. From the production function (1),

$$(28) \quad \begin{aligned} 1 &= \left[\int_{\underline{x}}^{\overline{x}} g(x, x^a)^{\frac{1}{\mu}}\right]^{\mu} \\ &= g(x^a, x^a) e^{-\frac{\mu}{\mu-1}\theta x^a} \left[\int_{\underline{x}}^{\overline{x}} e^{\frac{1}{\mu-1}(\theta x - \tau|x^a - x|)}\right]^{\mu} \\ &= g(x^a, x^a) e^{-\frac{\mu}{\mu-1}\theta x^a} u(c^a)^{\mu}, \end{aligned}$$

where the third equality follows from the definition (16) of $u(x^a)$. Solving (28) for $g(x^a, x^a)$ and substituting this and (26) into (25) proves that (24) holds for k_1 defined by

$$k_1 \equiv \left[\frac{\phi}{\phi - 1}\right]^{-\phi} \mu^{-\phi}.$$

Step 2

This step shows that

$$(29) \quad \tilde{\pi}(x, x^a) = k_2 e^{\frac{1}{\mu-1}[\theta x - \tau|x^a - x|]} \left[u(x^a)\right]^{\phi(\mu-1)-\mu}.$$

This follows from substituting (24) into (21).

Step 3

Define new parameters $\omega \equiv \phi(\mu - 1) - \mu$ and $\beta = \frac{1}{\mu-1}$. The assumption that $\phi \geq \lambda$ is equivalent to the assumption that $\omega \geq 0$. Assume that $y \in (0, \frac{\alpha}{2}]$ and that $c^a = c^s$. This step shows that

$$(30) \quad \int_{c^a - \frac{\alpha}{2}}^{c^a + \frac{\alpha}{2}} e^{-\beta|x^a - c^a - y|} u(x^a)^{\omega} dx^a - \int_{c^a - \frac{\alpha}{2}}^{c^a + \frac{\alpha}{2}} e^{-\beta|x^a - c^a + y|} u(x^a)^{\omega} dx^a \geq 0.$$

To show this, I first rewrite the first term on the LHS of (30):

$$(31) \quad \begin{aligned} &\textstyle\int_{c^a - \frac{\alpha}{2}}^{c^a + \frac{\alpha}{2}} e^{-\beta|x^a - c^a - y|} u(x^a)^{\omega} dx^a \\ &= \textstyle\int_{c^a - \frac{\alpha}{2}}^{c^a} e^{-\beta|x^a - c^a - y|} u(x^a)^{\omega} dx + \int_{c^a}^{c^a + \frac{\alpha}{2}} e^{-\beta|x^a - c^a - y|} u(x^a)^{\omega} dx^a \\ &= \textstyle\int_{c^a - \frac{\alpha}{2}}^{c^a} e^{-\beta|x^a - c^a - y|} u(x^a)^{\omega} dx + \int_{c^a - \frac{\alpha}{2}}^{c^a} e^{-\beta|\tilde{x}^a - c^a + y|} u(2c^a - \tilde{x}^a)^{\omega} d\tilde{x}^a. \end{aligned}$$

The second term of the second equality follows from making the substitution $\tilde{x}^a = 2c^a - x^a$ for the variable of integration. Analogously, the second term on the LHS of (30) can be written as

$$\begin{aligned}
&\int_{c^a-\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\beta|x^a-c^a+y|}u(x^a)^\omega dx^a \\
&= \int_{c^a-\frac{\alpha}{2}}^{c^a} e^{-\beta|x^a-c^a+y|}u(x^a)^\omega dx + \int_{c^a-\frac{\alpha}{2}}^{c^a} e^{-\beta|\tilde{x}^a-c^a-y|}u(2c^a-\tilde{x}^a)^\omega d\tilde{x}^a.
\end{aligned} \tag{32}$$

Using (31) and (32) we can rewrite the difference (30) as

$$\begin{aligned}
&\int_{c^a-\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\beta|x^a-c^a-y|}u(x^a)^\omega dx^a - \int_{c^a-\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\beta|x^a-c^a+y|}u(x^a)^\omega dx^a \\
&= \int_{c^a-\frac{\alpha}{2}}^{c^a} \left[e^{-\beta|x^a-c^a-y|} - e^{-\beta|x^a-c^a+y|}\right] u(x^a)^\omega dx \\
&\quad + \int_{c^a-\frac{\alpha}{2}}^{c^a} \left[e^{-\beta|\tilde{x}^a-c^a+y|} - e^{-\beta|\tilde{x}^a-c^a-y|}\right] u(2c^a-\tilde{x}^a)^\omega d\tilde{x}^a \\
&= \int_{c^a-\frac{\alpha}{2}}^{c^a} \left[e^{-\beta|x^a-c^a+y|} - e^{-\beta|x^a-c^a-y|}\right] \left[u(2c^a-x^a)^\omega - u(x^a)^\omega\right] dx^a.
\end{aligned}$$

The first bracketed term in the integral is positive since $y \in (0, \frac{\alpha}{2}]$. The second bracketed term is also positive. This follows since Lemma 1 and $c^a = c^s$ imply that $u(2c^a - x^a) - u(x^a) > 0$. The fact that $\omega \geq 0$ then implies that $u(2c^a - x^a)^\omega - u(x^a)^\omega \geq 0$. This proves that (30) holds.

Step 4

This step shows that for $y \in (0, \frac{\alpha}{2}]$ and $c^s = c^a$, $\pi(c^s + y) > \pi(c^s - y)$ as claimed in the lemma. This follows because

$$\begin{aligned}
&\pi(c^s + y) - \pi(c^s - y) \\
&= k_2 e^{\frac{1}{\mu-1}\theta(c^a+y)} \int_{c^a-\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\frac{1}{\mu-1}|x^a-c^a-y|}u(x^a)^\omega dx^a \\
&\quad -k_2 e^{\frac{1}{\mu-1}\theta(c^a-y)} \int_{c^a-\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\beta|x^a-c^a+y|}u(x^a)^\omega dx^a \\
&> 0.
\end{aligned}$$

The equality follows from using the formula for each $\tilde{\pi}(x, x^a)$ from (29). The inequality follows from (30) and the fact that $k_2 e^{\frac{1}{\mu-1}\theta(c^a+y)} > k_2 e^{\frac{1}{\mu-1}\theta(c^a-y)}$.

Step 5

This step considers the case of $y > \frac{\alpha}{2}$, continuing the assumption that $c^a = c^s$. For such y, the formula for $\tilde{\pi}(x, x^a)$ from (29) implies that

$$\pi(c^a + y) = e^{\frac{1}{\mu-1}(\theta-\tau)(y-\frac{\alpha}{2})}\pi(c^a + \frac{\alpha}{2})$$

and

$$\pi(c^a - y) = e^{\frac{1}{\mu-1}(-\theta-\tau)(y-\frac{\alpha}{2})}\pi(c^a - \frac{\alpha}{2}).$$

These relations and the fact that $\pi(c^a + \frac{\alpha}{2}) > \pi(c^a - \frac{\alpha}{2})$ holds from Step 4 then imply that $\pi(c^a + y) > \pi(c^a - y)$. Hence, the steps up to this point have shown that $\pi(c^s + y) > \pi(c^s - y)$, for $c^s = c^a$ for any $y > 0$.

Step 6

This step shows that the result holds for $c^a > c^s$. Note first that if $\tau \leq \theta$, then the profit $\pi(c^s + y)$ is obviously greater than $\pi(c^s - y)$ because a producer at $c^s + y$ has lower delivered cost than a producer at $c^s - y$ for all locations in the economy. So assume that $\tau > \theta$.

Let $\hat{\pi}(x, \underline{x}^a, \overline{x}^a)$ be profit at location x when the assemblers locate on the interval $[\underline{x}^a, \overline{x}^a]$. I need to show that for $c^a > c^s$,

$$\hat{\pi}(c^s + y, c^a - \frac{\alpha}{2}, c^a + \frac{\alpha}{2}) - \hat{\pi}(c^s - y, c^a - \frac{\alpha}{2}, c^a + \frac{\alpha}{2}) > 0. \tag{33}$$

Suppose first that $c^a - \frac{\alpha}{2} \geq c^s - \frac{\theta}{\tau}y$ or $c^a \geq c^s + \frac{\alpha}{2} - \frac{\theta}{\tau}y$. It is straightforward to calculate that the cost (including production and transportation cost) to a supplier at $c^s - y$ to deliver a unit of input to the northernmost assembler at $c^a - \frac{\alpha}{2}$ is at least as great as the cost to a supplier at $c^s + y$. It follows immediately that total profits must be higher at $c^s + y$ than at $c^s - y$.

Now consider the case where $c^a \in (c^s, c^s + \frac{\alpha}{2} - \frac{\theta}{\tau}y)$. I need to show that (33) holds. From Step 5, I know that (33) holds for $c^a = c^s$. So it is sufficient to show that the LHS of (33) increases in c^a for c^a in the specified range. The slope of the LHS of (33) is

$$\begin{aligned} \frac{d\hat{\pi}(c^s + y)}{dc^a} - \frac{d\hat{\pi}(c^s - y)}{dc^a} &= \frac{\partial\hat{\pi}(c^s + y)}{\partial\underline{x}^a} + \frac{\partial\hat{\pi}(c^s + y)}{\partial\overline{x}^a} - \frac{\partial\hat{\pi}(c^s - y)}{\partial\underline{x}^a} - \frac{\partial\hat{\pi}(c^s - y)}{\partial\overline{x}^a} \\ &= [\tilde{\pi}(c^s + y, \overline{x}^a) - \tilde{\pi}(c^s - y, \overline{x}^a)] \\ &\quad + [\tilde{\pi}(c^s - y, \underline{x}^a) - \tilde{\pi}(c^s + y, \underline{x}^a)]. \end{aligned} \tag{34}$$

For c^a in the specified range, $c^s - y < c^s + y < \overline{x}^a$ and $\underline{x}^a < c^s - \frac{\theta}{\tau}y$. These imply that both bracketed terms in (34) are strictly positive, so that the LHS of (33) is strictly increasing. Put in another way, a supplier at $c^s + y$ makes more profit on the southernmost assembler $\overline{x}^a$ than the $c^s - y$ supplier does and less profit on the northernmost assembler $\underline{x}^a$. So if the distribution of assemblers is shifted south, then the net increase in profit for the $c^s + y$ supplier is bigger than the net increase in profit for the $c^s - y$ supplier.

This concludes the proof for part 1 of the lemma.

Part 2: The Case of $\phi \in (1, \lambda)$

If I can show that the result holds for $c^a = c^s$, then the case of $c^a > c^s$ follows from the argument in Step 6 above. So assume that $c^a = c^s$. I need to show that $\pi(c^a + \frac{\alpha}{2}) > \pi(c^a - \frac{\alpha}{2})$. I can write $\pi(c^a + \frac{\alpha}{2})$ as

$$\pi(c^a + \frac{\alpha}{2}) = \int_0^{\frac{\alpha}{2}} \left[\tilde{\pi}(c^a + \frac{\alpha}{2}, c^a + z) + \tilde{\pi}(c^a + \frac{\alpha}{2}, c^a - z)\right] dz,$$

and I can write $\pi(c^a - \frac{\alpha}{2})$ in an analogous way. Hence, to show that $\pi(c^a + \frac{\alpha}{2}) > \pi(c^a - \frac{\alpha}{2})$, it is sufficient to show that

$$\text{(35)} \quad \tilde{\pi}(c^a + \frac{\alpha}{2}, c^a + z) > \tilde{\pi}(c^a - \frac{\alpha}{2}, c^a - z)$$

for all $z \in [-\frac{\alpha}{2}, \frac{\alpha}{2}]$.

Using (29), to show (35) I must show that

$$\begin{aligned} & k_2 e^{\frac{1}{\mu-1}\left[\theta(c^a+\frac{\alpha}{2})-\tau(\frac{\alpha}{2}-z)\right]} \left[u(c^a+z)\right]^{\phi(\mu-1)-\mu} \\ & > k_2 e^{\frac{1}{\mu-1}\left[\theta(c^a-\frac{\alpha}{2})-\tau(\frac{\alpha}{2}-z)\right]} \left[u(c^a-z)\right]^{\phi(\mu-1)-\mu}, \end{aligned}$$

which holds if and only if

$$\text{(36)} \quad e^{\frac{1}{\mu-1}\theta\alpha} > \left[\frac{u(c^a-z)}{u(c^a+z)}\right]^{\phi(\mu-1)-\mu}.$$

Note that since $\phi < \lambda$, $\phi(\mu-1) - \mu < 0$ (recall that $\lambda \equiv \frac{\mu}{\mu-1}$). If $z \leq 0$, then from Lemma 1, $u(c^a - z) > u(c^a + z)$. The fact that $\phi(\mu-1) - \mu < 0$ then implies that the RHS of (36) is less than one. The RHS is strictly greater than one, so the inequality holds if $z \leq 0$. It remains to consider the case where $z > 0$. In this case, the term in brackets on the RHS is less than one. Since $\phi(\mu-1) - \mu < 0$, the inequality (36) holds if and only if

$$\text{(37)} \quad e^{\frac{1}{\mu-\phi(\mu-1)}\frac{1}{\mu-1}\theta\alpha} > \frac{u(c^a+z)}{u(c^a-z)}.$$

Since $\phi > 1$, since the LHS is increasing in ϕ, and since the RHS is independent of ϕ, it is sufficient to show that the inequality holds for $\phi = 1$ or that

$$\text{(38)} \quad e^{\frac{1}{\mu-1}\theta\alpha} > \frac{u(c^a+z)}{u(c^a-z)}.$$

The next step is to obtain a formula for $u(x^a)$. For $x^a \in (\underline{x}, \overline{x})$ (the set of points containing the suppliers),

$$\begin{aligned} \text{(39)} \quad u(x^a) &\equiv \int_{\underline{x}}^{x^a} e^{\frac{1}{\mu-1}(\theta x - \tau(x^a - x))} dx + \int_{x^a}^{\overline{x}} e^{\frac{1}{\mu-1}(\theta x - \tau(x - x^a))} dx \\ \text{(40)} \quad &= \frac{\mu-1}{\tau+\theta}\left(e^{\frac{1}{\mu-1}\theta x^a} - e^{\frac{1}{\mu-1}(\theta\underline{x} - \tau(x^a - \underline{x}))}\right) - \frac{\mu-1}{\tau-\theta}\left(e^{\frac{1}{\mu-1}(\theta\overline{x} - \tau(\overline{x} - x^a))} - e^{\frac{1}{\mu-1}\theta x^a}\right) \\ \text{(41)} \quad &= e^{\frac{1}{\mu-1}\theta x^a}(\mu-1)G(x^a, \underline{x}, \overline{x}), \end{aligned}$$

where $G(x^a, \underline{x}, \overline{x})$ is defined by

$$G(x^a, \underline{x}, \overline{x}) \equiv \left(\frac{1}{\tau+\theta}\left(1 - e^{-\frac{1}{\mu-1}(\tau+\theta)(x^a - \underline{x})}\right) + \frac{1}{\tau-\theta}\left(1 - e^{-\frac{1}{\mu-1}(\tau-\theta)(\overline{x} - x^a)}\right)\right).$$

Using (39), we know that the inequality holds if and only if

$$e^{\frac{1}{\mu-1}\theta\alpha} > e^{\frac{1}{\mu-1}\theta 2z}\frac{G(c^a+z, c^a-\frac{\sigma}{2}, c^a+\frac{\sigma}{2})}{G(c^a-z, c^a-\frac{\sigma}{2}, c^a+\frac{\sigma}{2})}. \tag{42}$$

Since $z \leq \frac{\alpha}{2}$, to prove (42) it is sufficient to show that

$$\begin{aligned} 0 &< G(c^a-z, c^a-\frac{\sigma}{2}, c^a+\frac{\sigma}{2}) - G(c^a+z, c^a-\frac{\sigma}{2}, c^a+\frac{\sigma}{2}). \\ &= \frac{1}{\tau+\theta}\left(-e^{-\frac{1}{\mu-1}(\tau+\theta)(\frac{\sigma}{2}-z)} + e^{-\frac{1}{\mu-1}(\tau+\theta)(\frac{\sigma}{2}+z)}\right) \\ &\quad + \frac{1}{\tau-\theta}\left(-e^{-\frac{1}{\mu-1}(\tau-\theta)(\frac{\sigma}{2}+z)} + e^{-\frac{1}{\mu-1}(\tau-\theta)(\frac{\sigma}{2}-z)}\right), \end{aligned} \tag{43}$$

where the equality uses the definition of G. It is straightforward to show that (43) holds by showing first that at $z = 0$ the RHS of (43) is zero and that it is strictly increasing in z for $z \leq \frac{\sigma}{2}$.■

Proposition 3. Assume that Case 2 applies, that $\alpha \leq \sigma$, and that $\phi \geq \lambda$. There exists an equilibrium where suppliers occupy the interval $[\xi-\sigma, \xi]$ at the south end of the location spectrum.

Proof. Take as given that suppliers occupy the interval $[\xi-\sigma, \xi]$. Lemma 1 and the fact that $\alpha \leq \sigma$ imply that assemblers will occupy an interval $[c^a-\frac{\alpha}{2}, c^a+\frac{\alpha}{2}]$ with a center south of the supplier center, $c^a \geq \xi - \frac{\sigma}{2}$.

Suppose it is optimal for suppliers to locate at $[\xi-\sigma, \xi]$, if assemblers are in an interval $[c^a-\frac{\alpha}{2}, c^a+\frac{\alpha}{2}]$ with a center at the supplier center, $c^a = c^s = \xi - \frac{\sigma}{2}$. Then by arguments similar to those used in Step 6 of the proof of Lemma 3, it continues to be optimal for suppliers to locate there for any assembly center south of the supplier center, $c^a \geq c^s$. Hence, it is sufficient to show that $[\xi-\sigma, \xi]$ is optimal for suppliers, when $c^a = c^s$. This is what I will show.

It is clear that $\pi^s(x)$ is increasing for $x < \xi - \sigma$. So to prove that $[\xi-\sigma, \xi]$ are the profit-maximizing locations, it is sufficient to show that

$$\pi^s(x) > \pi^s(\xi-\sigma) \tag{44}$$

for all $x > \xi - \sigma$. I will show that $\pi^s(x)$ is strictly increasing for $x \in (\xi-\sigma, c^s)$. This, along with the result from Part 1 of the proof of Lemma 3 that $\pi^s(c^s+y) > \pi^s(c^s-y)$ when $c^s = c^a$ and $y > 0$, will imply that (44) holds.

Using the formula (29) for $\tilde{\pi}(x, x^a)$ and integrating over the interval of assembler locations yields the following formula for profit $\pi^s(x)$:

$$\pi^s(x) = k_2 e^{\frac{1}{\mu-1}\theta x}\int_{c^a-\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\frac{1}{\mu-1}\tau|x^a-x|}\left[u(x^a)\right]^{\phi(\mu-1)-\mu} dx^a.$$

Letting $\beta \equiv \frac{1}{\mu-1}$ and $\omega \equiv \phi(\mu-1)-\mu$, we can rewrite this as

(45) $\quad \pi^s(x) = k_2 e^{\frac{1}{\mu-1}\theta x} H(x),$

for $H(x)$ defined by

$$H(x) \equiv \int_{c^a-\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\beta|y-x|} u(y)^\omega dy.$$

From inspection of (45), to show that $\pi^s(x)$ is increasing over the range $x \in (\xi-\sigma, c^a)$, it is sufficient to show that $H(x)$ is weakly increasing over this range. For $x \in (\xi-\sigma, c^a-\frac{\alpha}{2}]$, $H(x)$ is obviously increasing. So consider $x \in (c^a-\frac{\alpha}{2}, c^a)$. For x in this range,

$$H(x) = \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-y)} u(y)^\omega dy + \int_{x}^{c^a+\frac{\alpha}{2}} e^{-\beta(y-x)} u(y)^\omega dy.$$

The slope for $x \in (c^a-\frac{\alpha}{2}, c^a)$ is

$$\begin{aligned}
H'(x) &= -\beta \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-x^a)} u(x^a)^\omega dx^a + \beta \int_{x}^{c^a+\frac{\alpha}{2}} e^{-\beta(x^a-x)} u(x^a)^\omega dx^a \\
&= -\beta \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-x^a)} u(x^a)^\omega dx^a + \beta \int_{x}^{2x-c^a+\frac{\alpha}{2}} e^{-\beta(x^a-x)} u(x^a)^\omega dx^a \\
&\quad +\beta \int_{2x-c^a+\frac{\alpha}{2}}^{c^a+\frac{\alpha}{2}} e^{-\beta(x^a-x)} u(x^a)^\omega dx^a \\
&> -\beta \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-x^a)} u(x^a)^\omega dx^a + \beta \int_{x}^{2x-c^a+\frac{\alpha}{2}} e^{-\beta(x^a-x)} u(x^a)^\omega dx^a \\
&= -\beta \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-x^a)} u(x^a)^\omega dx^a + \beta \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(2x-x^a-x)} u(2x-x^a)^\omega dx^a \\
&= -\beta \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-x^a)} u(x^a)^\omega dx^a + \beta \int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-x^a)} u(2x-x^a)^\omega dx^a \\
&> \beta \left[\int_{c^a-\frac{\alpha}{2}}^{x} e^{-\beta(x-x^a)} \left(u(2x-x^a)^\omega - u(x^a)^\omega \right) dx^a \right].
\end{aligned}$$

By Lemma 1, $u(2x-x^a) > u(x^a)$, for $x \in (c^a-\frac{\alpha}{2}, c^a)$ and $x^a \in (c^a-\frac{\alpha}{2}, x)$. Since $\omega \geq 0$, this implies that $u(2x-x^a)^\omega - u(x^a)^\omega \geq 0$. Hence, $H'(x) > 0$ for x in this range as claimed. This completes the proof of Part 2.■

Proposition 5. Suppose that in the initial state, old suppliers occupy the interval $[0,\sigma]$. In any unfragmented equilibrium, there exists a critical time period t' where $t' \leq \hat{t}$, such that in any period $t \geq t'$, all new suppliers locate on the interval $[\xi-\sigma, \xi]$ at the south end of the location space. Before period t', new suppliers locate on the interval $[c_t^s - \frac{\sigma}{2}, c_t^s + \frac{\sigma}{2}]$ with center c_t^s, and this center shifts south by at least $\frac{\theta\sigma}{2\tau}$ in each period, i.e., $c_t^s > c_{t-1}^s + \frac{\theta\sigma}{2\tau}$, for $t \leq t'$.

Proof. I will show that the following is true. Take as given that at period t, the old suppliers are located on $[c^s_{t-1} - \frac{\sigma}{2}, c^s_{t-1} + \frac{\sigma}{2}]$. In any equilibrium, the interval of new entrants must be located at least $\frac{\theta\sigma}{2\tau}$ units further south than the old suppliers if this is feasible, and otherwise the new entrants are located at $[\xi - \sigma, \xi]$. Furthermore, the profit in period t of an old supplier at $c^s_{t-1} + \frac{\sigma}{2}$ is strictly greater than the profit of an old supplier at $c^s_{t-1} - \frac{\sigma}{2}$.

Note that the claim is obvious if $\tau \leq \theta$, so assume that $\tau > \theta$.

I first show that this claim is true for $t = T$. Suppose that assemblers locate at c^a_T in period T. It is straightforward to show that Lemma 2 applies in this case, so the profit function for period T profits has the form

$$(46) \quad \pi^s_T(x) = ke^{\frac{1}{\mu-1}\left[\theta x - \tau\left|c^a_T - x\right|\right]}.$$

This implies that new entrants in period T will locate at $[c^a_T - \frac{\sigma}{2} + \frac{\sigma\theta}{2\tau}, c^a_T + \frac{\sigma}{2} + \frac{\sigma\theta}{2\tau}]$ if this interval is contained within the location space, and otherwise they will locate at $[\xi - \sigma, \xi]$. Hence, if new suppliers do not locate at $[\xi - \sigma, \xi]$, assemblers locate at a point c^a_T that is $\frac{\sigma\theta}{2\tau}$ units north of the center of the new-supplier interval. It must be the case that $c^a_T > c^s_{T-1}$. If $c^a_T \leq c^s_{T-1}$, assemblers would be locating north of the old-supplier center c^s_{T-1} as well as the new-supplier center. A straightforward extension of Lemma 1 would show that this contracts optimal assembler location behavior. The claim that the profit of an old supplier at $c^s_{t-1} + \frac{\sigma}{2}$ is strictly greater than the profit of an old supplier at $c^s_{t-1} - \frac{\sigma}{2}$ follows from the fact that $c^a_T > c^s_{T-1}$.

Now suppose that the claim is true for some $t + 1$. I will show the result is true for t. By induction the result will be true for all t. Since the claim is true for $t + 1$, I know that for suppliers that enter in period t, the profit in $t + 1$ is higher for the supplier that located at $c^s_t + \frac{\sigma}{2}$ than for the supplier at $c^s_t - \frac{\sigma}{2}$. I will use this fact shortly. Now note that if the new suppliers locate at $[\xi - \sigma, \xi]$, the result is true, so suppose that new suppliers locate at some interval with center $c^s_t < \xi - \frac{\sigma}{2}$. Let c^a_t denote the location of assemblers in period t. I show that $c^a_t + \frac{\sigma\theta}{2\tau} \leq c^s_t$ must hold. Suppose it did not. The formula for profit in period t is proportional to (46). Hence, if $c^a_t + \frac{\sigma\theta}{2\tau} > c^s_t$, a new supplier at $c^s_t + \frac{\sigma}{2}$ would make strictly greater profit in period t than a supplier at $c^s_t - \frac{\sigma}{2}$. As mentioned above, the profit in period $t + 1$ is also higher, so total discounted profits would be higher at $c^s_t + \frac{\sigma}{2}$ than at $c^s_t - \frac{\sigma}{2}$. This is inconsistent with equilibrium if $c^s_t + \frac{\sigma}{2} < \xi$. Hence, $c^a_t + \frac{\sigma\theta}{2\tau} \leq c^s_t$ must hold. By the same argument used in the $t = T$ case, $c^a_t \geq c^s_{t-1}$ must hold. This proves the claim.

Proposition 6. Assume that θ is small enough that (9) holds. Assume that $\delta = 0$. There exists a critical level $\hat{\xi}$ of ξ where $\hat{\xi} > \sigma$ with the following properties. If $\xi < \hat{\xi}$, the unique solution to the social planner's problem specifies that the supplier center shift by a constant step $z^* < \sigma$ in each period, $c_t^s = c_{t-1}^s + z^*$, until the endpoint is reached where all suppliers are located at the south end $[\xi - \sigma, \xi]$. If $\xi > \hat{\xi}$, the unique solution specifies that in the initial period the new suppliers jump to the south end $[\xi - \sigma, \xi]$ of the location spectrum and that all future generations of suppliers also locate there.

Proof. The social planner's problem is reduced to picking a c_t^a in each period and new-supplier locations according to (7) and (8) to maximize assembler profit in a period. (Note that supplier profit is proportional to assembler profit.)

Consider the initial period where the old suppliers are at $[0, \sigma]$. Following earlier arguments, we know that the location c_1^a that maximizes assembler profit in period 1 is the location that maximizes

$$(47) \quad U_1(c^a) = u(c^a, 0, \sigma) + u(c^a, \underline{x}^s(c^a), \overline{x}^s(c^a)),$$

where $\underline{x}^s(c^a)$ and $\overline{x}^s(c^a)$ are defined by (7) and (8) and $u(c^a, \underline{x}, \overline{x})$ is defined by (16). It is straightforward to show that for $c^a \geq \sigma$

$$(48) \quad u(c^a, 0, \sigma) = e^{-\frac{\tau}{\mu-1}(c^a - \sigma)} u(\sigma, 0, \sigma),$$

while for $c^a \in [\sigma, \xi - \frac{\sigma}{2} + \frac{\theta\sigma}{2\tau}]$

$$(49) \quad u(c^a, \underline{x}^s(c^a), \overline{x}^s(c^a)) = e^{\frac{\theta}{\mu-1}(c^a - \sigma)} u(\sigma, \underline{x}^s(\sigma), \overline{x}^s(\sigma)).$$

Inspection of (48) and (49) reveals that $U_1(c^a)$ is strictly convex for $c^a \in [\sigma, \xi - \frac{\sigma}{2} + \frac{\theta\sigma}{2\tau}]$. Hence, in the social planner's solution the optimum must be either $c^a < \sigma$, or $c^a > \xi - \frac{\sigma}{2} + \frac{\theta\sigma}{2\tau}$. The latter case corresponds to putting the assemblers in $[\xi - \sigma, \xi]$.

It is obvious that if ξ is large enough, the planner will put the new suppliers at $[\xi - \sigma, \xi]$. It is also clear that if this is optimal for some ξ', it is optimal for all $\xi > \xi'$. Hence, there must exist a $\hat{\xi} > \sigma$ such that if $\xi > \hat{\xi}$, then the optimal choice in the initial period is to put the suppliers at $[\xi - \sigma, \xi]$, and if $\xi < \hat{\xi}$ it is optimal to set $c^a < \sigma$. This proves that the result holds for the initial period. The case for later periods follows from the argument given in the body of this paper.

Proposition 9

This subsection contains the proof of Proposition 9.

In period 1, the initial period, the current generation of old suppliers is located at $[0, \sigma]$. The new suppliers take a step z and locate at $[z, z+\sigma]$. Assemblers locate at a point c^a. In a step-by-step equilibrium, the generation of new suppliers that arrives in period 2 takes the same size step as the new suppliers in period 1; i.e., they locate in the interval $[2z, 2z+\sigma]$. Furthermore, assemblers in period 2 take the same size step and locate at $c^a + z$. Analogously, in period 3, suppliers locate at $[3z, 3z+\sigma]$, and so forth.

If a step z is consistent with optimal behavior for the agents in period 1, then the step z is consistent with optimal behavior for the agents in later periods, since the objective functions for the later agents are the same as the objective functions for the agents in period 1, except for a multiplicative constant that depends upon θ and z. Therefore, to determine the conditions for a step-by-step equilibrium, it is sufficient to look at the behavior of the agents in period 1. In particular, it must be profit-maximizing for the new suppliers in period 1 to locate at $[z, z+\sigma]$, and it must be profit-maximizing for assemblers to locate at c^a in period 1.

The new suppliers in period 1 care about their profits in periods 1 and 2. In order to write an expression for profit, let me first define h_t^a as the demand of an assembler in period t for units of specialized inputs from the supplier sharing the same location c_t^a as the assembler in period t. Using the formula for the single-period profit (20), we know that the discounted profit of a supplier locating at x in period 1 when assemblers are located at c^a in period 1 and $c^a + z$ in period 2 is

$$(50) \quad \begin{aligned} \pi(x) \;=\; & e^{\frac{1}{\mu-1}[\theta x - \tau|c^a - x|]}(\mu-1)e^{-\frac{\mu}{\mu-1}\theta c^a}h_1^a \\ & +\delta e^{\frac{1}{\mu-1}[\theta x - \tau|c^a + z - x|]}(\mu-1)e^{-\frac{\mu}{\mu-1}\theta(c^a+z)}h_2^a. \end{aligned}$$

I can simplify this by noting that demand in period 2 bears the following relation to the demand in period 1:

$$(51) \quad h_2^a = e^{\phi\theta z}h_1^a.$$

To see this, note that since the entire economy shifts south by the amount z between period 1 and period 2, the minimum delivered cost at the assembly center to construct one unit of the composite intermediate falls to a fraction $e^{-\theta z}$ of its previous level; i.e.,

$$(52) \quad v_2(c_2^a) = e^{-\theta z}v_1(c_2^a).$$

The relation (51) then follows because the assembler's elasticity of demand for units of the composite intermediate is ϕ. Using (51) and dividing through by common factors gives that profit at x is proportional to

$$(53) \quad \pi(x) \propto e^{\frac{1}{\mu-1}[\theta x - \tau|c^a - x|]} + \delta e^{[\phi - \frac{\mu}{\mu-1}]\theta z}e^{\frac{1}{\mu-1}[\theta x - \tau|c^a + z - x|]}.$$

Since new suppliers in period 1 locate on the interval $[z, z+\sigma]$, a necessary condition for equilibrium is that profit be equal at the endpoint z and $z+\sigma$ of the interval; i.e.,

$$(54) \quad \pi(z) - \pi(z+\sigma) = 0.$$

Before I rewrite this expression, let me assume that

$$(55) \quad z < \frac{\sigma}{2} \leq c^a \leq \sigma.$$

I will show later that for small θ this is a necessary condition for equilibrium. This assumption implies that a supplier at the endpoint z of the new-supplier interval $[z, z+\sigma]$ is further north than the assembly centers in both periods and that the supplier at the other endpoint $z+\sigma$ is further south than the assembly centers in both periods. Knowing this enables me to order the differences within the absolute value expressions in (53) so that I can take out the absolute value symbols. Substituting (53) into (54) and dividing by common factors yields the following condition for equilibrium in the choices of the supplier locations as a function of the step z and the center c^a:

$$S(z, c^a) = 0,$$

where the function $S(z, c^a)$ is defined by

$$\begin{aligned}(56) \quad S(z, c^a) \quad \equiv \quad & e^{-\frac{1}{\mu-1}\tau(c^a - z)} + \delta e^{[\phi - \frac{\mu}{\mu-1}]\theta z} e^{-\frac{1}{\mu-1}\tau c^a} \\ & - e^{\frac{1}{\mu-1}[\theta\sigma - \tau(z+\sigma-c^a)]} - \delta e^{[\phi-\frac{\mu}{\mu-1}]\theta z} e^{\frac{1}{\mu-1}[\theta\sigma - \tau(\sigma - c^a)]}.\end{aligned}$$

I now turn to the condition for optimal assembler behavior. Recall the earlier analysis of average total cost (14) in the static model with a single interval of suppliers. It is straightforward to generalize this formula to the case where there is a set of suppliers $[0, \sigma]$ and a second set at $[g, g+\sigma]$. Following the earlier analysis, the location that minimizes ATC is the location that maximizes

$$(57) \quad U(x^a) = u(x^a, 0, \sigma) + u(x^a, z, z+\sigma),$$

where $u(x^a, \underline{x}, \overline{x})$ is defined by (16). The profit-maximizing location c^a solves the first-order condition

$$\begin{aligned}(58) \quad U'(c^a) \quad &= \quad \frac{du(c^a, 0, \sigma)}{dx^a} + \frac{du(c^a, z, z+\sigma)}{dx^a} \\ &= \quad \tau e^{\frac{1}{\mu-1}\theta c^a} \left[F(c^a, 0, \sigma) + F(c^a, z, z+\sigma)\right] = 0,\end{aligned}$$

which uses the formula (18) for the slope. Taking (58) and substituting in the formula (19) for $F(x^a, \underline{x}, \overline{x})$ and dividing by common factors yields the following condition for equilibrium in the assembly sector:

$$A(z, c^a) = 0,$$

where the function $A(z, c^a)$ is defined by

$$\begin{aligned}(59)\quad A(z,c^a) &= 2\left(\frac{\tau+\theta}{\tau-\theta}-1\right)+e^{-\frac{1}{\mu-1}(\tau+\theta)(c^a-z)}+e^{-\frac{1}{\mu-1}(\tau+\theta)c^a}\\ &\quad -\frac{\tau+\theta}{\tau-\theta}e^{-\frac{1}{\mu-1}(\tau-\theta)(z+\sigma-c^a)}-\frac{\tau+\theta}{\tau-\theta}e^{-\frac{1}{\mu-1}(\tau-\theta)(\sigma-c^a)}.\end{aligned}$$

Consider the polar case of $\theta^\circ = 0$. Let $z^\circ = 0$ and $c^{a\circ} = \frac{\sigma}{2}$. It is straightforward to calculate that

$$\begin{aligned}A(z^\circ, c^{a\circ}, \theta^\circ) &= 0\\ S(z^\circ, c^{a\circ}, \theta^\circ) &= 0.\end{aligned}$$

Hence, the necessary conditions for an equilibrium are satisfied. It is straightforward to show that the sufficient conditions for an equilibrium are also satisfied, e.g., that $c^{a\circ} = \frac{\sigma}{2}$ is the unique globally optimal choice of assemblers given that the step size is $z^\circ = 0$. Now consider what happens for small positive θ. Straightforward differentiation of S and A shows that the following are true when evaluated at the point $(z^\circ, c^{a\circ}, \theta^\circ) = (0, \frac{\sigma}{2}, 0)$:

$$\begin{aligned}&\frac{\partial A}{\partial z} = \frac{\partial S}{\partial z} > 0\\ (60)\quad &\frac{\partial A}{\partial c} < \frac{\partial S}{\partial c} < 0\\ &\frac{\partial A}{\partial \theta} > 0 \text{ and } \frac{\partial S}{\partial \theta} < 0.\end{aligned}$$

A standard application of the implicit function theorem shows that for small θ there exist unique functions $z^*(\theta)$ and $c^{a*}(\theta)$ satisfying the necessary conditions

$$\begin{aligned}A(z^*(\theta), c^{a*}(\theta), \theta) &= 0\\ S(z^*(\theta), c^{a*}(\theta), \theta) &= 0\end{aligned}$$

for an equilibrium and satisfying

$$\frac{dz^*(\theta)}{d\theta} > 0 \text{ and } \frac{dc^{a*}(\theta)}{d\theta} > 0.$$

It is straightforward to show that for small θ, $z^*(\theta)$ and $c^{a*}(\theta)$ satisfy the sufficient condition for an equilibrium. I claim that for small θ this is the unique equilibrium. Suppose not. Then for each n, there exists a $\theta_n < \frac{1}{n}$, with at least one step-by-step equilibrium (z_n, c_n^a) besides $(z^*(\theta), c^{a*}(\theta))$. By the implicit function theorem this other equilibrium must be outside a ball around the limit point $(z^\circ, c^{a\circ}) = (0, \frac{\sigma}{2})$. By the definition of a step-by-step equilibrium, $z_n \in [0, \sigma]$, and this implies that $c_n^a \in [0, 2\sigma]$. Since the sequence $\{z_n, c_n^a\}$ is bounded, there exists a convergent subsequence. Let (z_∞, c_∞^a) be the limit of this convergent sequence. Since every element of the subsequence is bounded away from $(z^\circ, c^{a\circ}) = (0, \frac{\sigma}{2})$, the limit is bounded away from $(z^\circ, c^{a\circ}) = (0, \frac{\sigma}{2})$; i.e., $z_\infty > 0$ and $c_\infty^a > \frac{\sigma}{2}$. Since θ_n converges to zero, (z_∞, c_∞^a) must be an equilibrium in the case where $\theta = 0$. But straightforward analysis shows that for $\delta < 1$, there can be no such alternative equilibrium in the $\theta = 0$ case. This shows that the equilibrium $(z^*(\theta), c^{a*}(\theta))$ is the unique equilibrium for small θ.

The comparative statics claims follow from the following. It is straightforward to show that for $\theta > 0$ and any (z, c^a) satisfying $0 < z < \frac{\sigma}{2} \leq c^a \leq \sigma$, the following hold:

$$\frac{\partial A(z,c^a,\theta,\delta)}{\partial \delta} = 0$$
$$\frac{\partial A(z,c^a,\theta,\phi)}{\partial \phi} = 0$$
$$\frac{\partial S(z,c^a,\theta,\delta)}{\partial \delta} < 0$$
$$\frac{\partial S(z,c^a,\theta,\phi)}{\partial \phi} < 0.$$

These relations, along with (60), imply that z^* strictly increases in δ and ϕ for small θ.

With the help of MAPLE (a computer program for symbolic manipulation), it can be shown that the following functions have the following signs when evaluated at the limit point $(z^\circ, c^{a\circ}, \theta^\circ) = (0, \frac{\sigma}{2}, 0)$:

$$\begin{aligned} -\frac{\partial^2 A}{\partial\theta\partial\tau}\frac{\partial S}{\partial c} + \frac{\partial^2 S}{\partial\theta\partial\tau}\frac{\partial A}{\partial c} &< 0 \\ -\frac{\partial^2 A}{\partial\theta\partial\mu}\frac{\partial S}{\partial c} + \frac{\partial^2 S}{\partial\theta\partial\mu}\frac{\partial A}{\partial c} &< 0. \end{aligned}$$

Straightforward arguments using L'Hospital's rule show that these imply that z^* strictly decreases in τ and μ for small θ. This completes the proof of the comparative statics results stated in Proposition 9.

It remains to show that for small enough θ, there is a unique solution to the constrained social planner's problem and that this solution coincides with the equilibrium allocation. Let w_t be the sum of assembler and supplier profit in period t. It is straightforward to show (see the lemma below) that

$$(61) \quad w_t = k v_t^{-(\phi-1)}$$

for some $k > 0$. Consider the problem of a social planner picking a step size z and an assembly center c^a in period 1 (implying an assembly center $c^a + (t-1)z$ in period t) to maximize

$$W(z, c^a) = w_1 + \delta w_2 + \delta^2 w_3 + \ldots.$$

Let $\tilde{W}(z)$ be defined as follows:

$$\tilde{W}(z, \theta) \equiv \max_c W(z, c^a, \theta).$$

Consider the following constrained social planner's problem:

$$(62) \quad \max_{z \leq \sigma} \tilde{W}(z, \theta).$$

It is straightforward to show that in the limiting case of $\theta = 0$, the unique solution to this problem is at $z = 0$. Furthermore, it can be shown that

$$\frac{\partial \tilde{W}(0,0)}{\partial z} = 0$$

and

$$\frac{\partial^2 \tilde{W}(0,0)}{\partial z^2} < 0.$$

By the implicit function theorem, by continuity, and by the fact that $z = 0$ is the unique solution for $\theta = 0$, for small θ there exists a unique $z^*_{planner}(\theta)$ solving (62).

The final thing to prove in the proposition is that $z^*_{planner}(\theta) = z^*_{equilibrium}(\theta)$. Note that for a given z, the social planner's choice of c must be the cost-minimizing level, i.e., the choice of c must satisfy $A(z, c) = 0$. So I need to show that $S(z, c) = 0$. To show this is the case, take as given that the planner is setting the assembly centers at the levels $c^a_t = c^* + (t-c)z^*$ in each period and that beginning in period $t = 2$, the planner will locate the new suppliers entering in period t on the interval $[tz^*, \sigma + tz^*]$; i.e., $z_t = z^*$, for $t \geq 2$. If z^* and c^* are a solution to the constrained social planner's problem, it must be optimal to set $z_1 = z^*$ in period 1, if we take this other stuff as given. Discounted profit as a function of the z_1 in period 1 is

$$\begin{aligned}
(63) \quad W(z_1) \quad = \quad & k^{-(\mu-1)} \left[u(c^*, 0, \sigma) + u(c^*, z_1, z_1 + \sigma)\right]^{(\mu-1)(\phi-1)} \\
& + \delta k^{-(\mu-1)} \left[u(c^* + z^*, z_1, z_1 + \sigma) + u(c^* + z^*, 2z^*, 2z^* + \sigma)\right]^{(\mu-1)(\phi-1)} \\
& + \text{discounted profit in period 3 and later.}
\end{aligned}$$

Note that a deviation in z_1 in period 1 away from z^* affects discounted profit in period 1 and 2 but does not affect discounted profit in periods 3 or later. The formula (63) follows from (57), (14), and (61).

In order that $z_1 = z^*$ solve the planner's problem, it must solve the first-order necessary condition

$$W'(z^*) = 0.$$

Straightforward calculations show that this condition is equivalent to $S(z^*, c^*) = 0$. This completes the proof.■

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Figure 1
Supplier Profit as a Function of Supplier Location x
Given Assemblers are at c^a

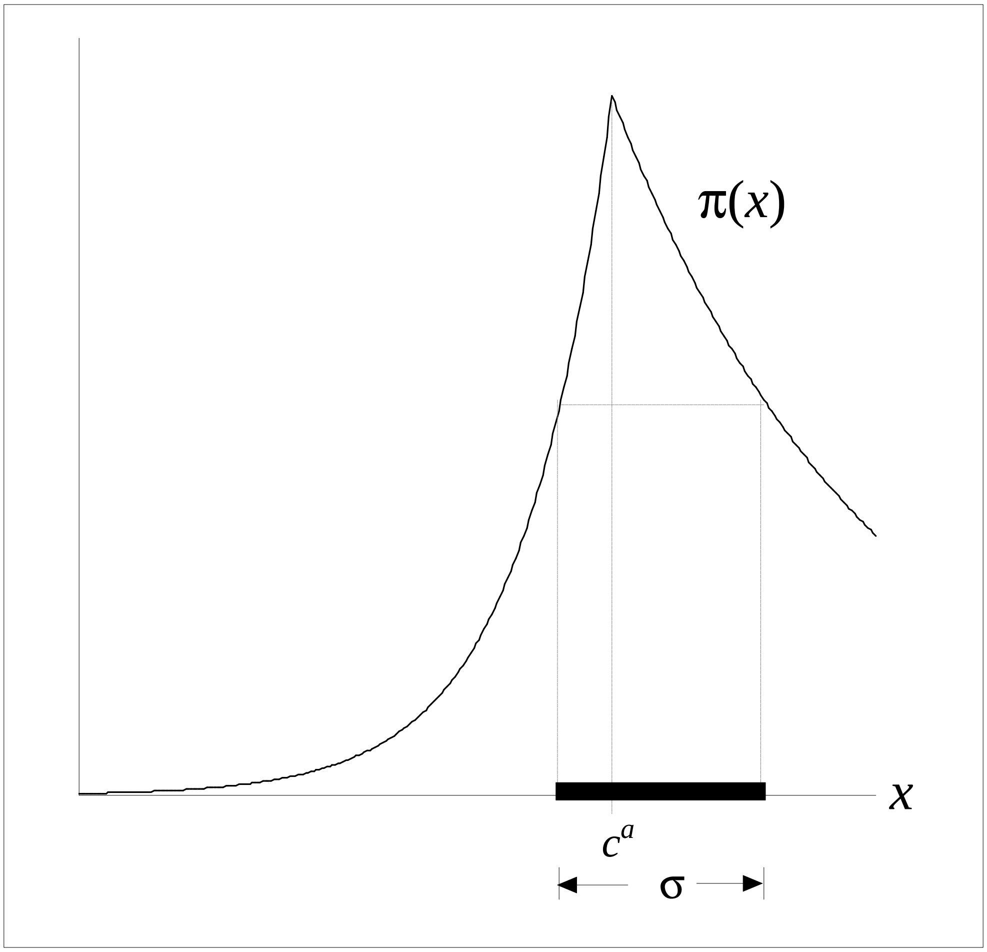

Figure 2
Average Total Cost for Social Planner