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## Liquidity and Insurance for the Unemployed\*

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### ABSTRACT

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We study the optimal design of unemployment insurance for workers sampling job opportunities over time. We focus on the optimal timing of benefits and the desirability of allowing workers to freely access a riskless asset. When workers have constant absolute risk aversion preferences, it is optimal to use a very simple policy: a constant benefit during unemployment, a constant tax during employment that does not depend on the duration of the spell, and free access to savings using a riskless asset. Away from this benchmark, for constant relative risk aversion preferences, the welfare gains of more elaborate policies are minuscule. Our results highlight two largely distinct roles for policy toward the unemployed: (a) ensuring workers have sufficient liquidity to smooth their consumption; and (b) providing unemployment benefits that serve as insurance against the uncertain duration of unemployment spells.

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# 1 Introduction

There is wide variation in the duration of unemployment benefits across OECD countries (Figure 1). In Italy, the United Kingdom, and the United States, benefits last for six months. In Germany benefits lapse after one year and in France after five years. In Belgium they last forever. Which country has the right policy?

A standard argument for terminating benefits after a few quarters is that extending the duration of benefits lengthens the duration of jobless spells (Katz and Meyer, 1990). But benefits also provide insurance and help workers maintain smooth consumption while unemployed (Gruber, 1997). Determining which policy is best requires a dynamic model of optimal unemployment insurance. Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) develop such models and show that benefits should optimally decline during a jobless spell. Many economists have interpreted these results as broadly supportive of the Italian, British, and American version of unemployment insurance. But they hinge on some subtle assumptions, notably a restriction that the unemployed can neither borrow nor save and so consume their benefits in each period. This means that unemployment benefits play a dual role: they insure workers against uncertainty in the prospect of finding a job, and they provide workers with the ability to smooth consumption while unemployed.

In this paper, we reexamine the optimal timing of benefits, distinguishing the two roles by allowing the worker to borrow and save. Our main conclusion is that when workers have sufficient liquidity, in either assets or capacity to borrow, a constant benefit schedule of unlimited duration is optimal or nearly optimal.<sup>1</sup> The constant benefit schedule insures against unemployment risk, while workers' ability to dissave or borrow allows them to avoid temporary drops in consumption.

Our results suggest two conceptually distinct roles for policy toward the unemployed. First, ensuring that workers have sufficient liquidity to smooth their consumption; and second, providing constant unemployment benefits that serve as insurance against the uncertain duration of unemployment spells. This dichotomy is consistent with the spirit of Feldstein and Altman's (1998) recent policy proposal for unemployment insurance savings accounts (see also Feldstein, 2005).

We represent the unemployed worker's situation using McCall's (1970) model of sequential

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<sup>1</sup>This does not necessarily imply that the Belgian policy is optimal. Policies differ along other dimensions, notably in the maximum yearly benefit; see OECD (2002), Table 2.2. This paper focuses on the optimal duration of benefits. In ongoing work for a separate paper, we examine the determinants of the optimal level.

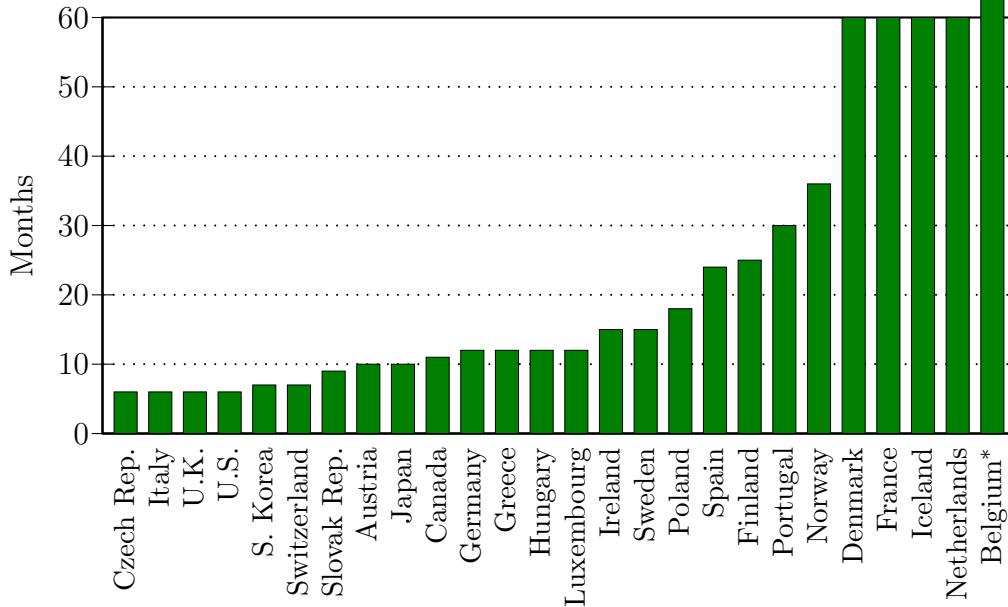


Figure 1: Duration of unemployment benefits in select OECD countries. Source: OECD Benefits and Wages 2002, Table 2.2. \*Duration is unlimited in Belgium.

job search. Each period, a risk-averse, infinitely lived unemployed worker gets a wage offer from a known distribution. If she accepts the offer, she keeps the job at a constant wage forever. If she rejects it, she searches again the following period.

Our main purpose is to compare two unemployment insurance policies. We begin by considering a simple insurance policy, *constant benefits*, where the worker receives a constant benefit while she is unemployed and pays a constant tax once she is employed. The worker can borrow and lend using a riskless bond. We show that the worker adopts a constant reservation wage although her assets and consumption decline during a jobless spell. The reservation wage is increasing in both the unemployment benefit and the employment tax, a form of moral hazard. An insurance agency sets the level of benefits and taxes to minimize the cost of providing the worker with a given level of utility.

We then consider *optimal unemployment insurance*. An insurance agency dictates a duration-dependent consumption level for the unemployed, funded by an employment tax that depends on the length of the jobless spell. The worker has no access to capital markets and so must consume her after-tax income in each period. Absent direct monitoring of wage offers or randomization schemes, this is the best insurance system possible. The path of unemployment consumption and employment taxes determines the worker's reservation wage, which the insurance agency cannot directly control. It sets this path to minimize the

cost of providing the worker with a given level of utility.

Our main result is that with constant absolute risk aversion (CARA) preferences and no lower bound on consumption, constant benefits and optimal unemployment insurance are equivalent. That is, the cost of providing the worker with a given level of utility is the same, her reservation wage is the same, and the path of her consumption is the same under both insurance systems. In both cases consumption falls by a constant amount each period that the worker is unemployed, both during and after the unemployment spell. When the worker can borrow and save, this is consistent with a constant benefit and tax.

Our result that the optimal unemployment insurance can do no better than constant benefits with borrowing and savings contrasts with a large literature on the need for savings constraints in dynamic moral hazard models.<sup>2</sup> Rogerson (1985) considers an environment in which a risk-averse worker must make a hidden effort decision that affects her risk-neutral employer’s profits. He proves that optimal insurance is characterized by an inverse Euler equation,

$$\frac{1}{u'(c_t)} = \mathbb{E}_t \left( \frac{1}{\beta(1+r)u'(c_{t+1})} \right),$$

where  $\mathbb{E}_t$  is the expectation operator conditional on information available at date  $t$ . In particular, since the function  $1/x$  is convex,

$$u'(c_t) < \mathbb{E}_t \beta(1+r)u'(c_{t+1}).$$

An individual facing this path of consumption would consume less today and more tomorrow and hence is “savings-constrained” by optimal insurance. In contrast, in our model a worker confronted with the optimal unemployment insurance policy satisfies this Euler condition with equality.

We also explore optimal insurance with constant relative risk aversion (CRRA) preferences. We do this for two reasons. First, CRRA preferences are theoretically more appealing than CARA preferences. We want to explore the robustness of our findings to this assumption. And second, this introduces a nonnegativity constraint on consumption that limits a worker’s debt to the amount that she can repay even in the worst possible state of the world, Aiyagari’s (1994) natural borrowing limit. We highlight an interesting interaction between

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<sup>2</sup>A recent example is Golosov, Kocherlakota, and Tsyvinski (2003), who emphasize that capital taxation may discourage saving. Allen (1985) and Cole and Kocherlakota (2001) provide a particularly striking example of the cost of unobserved savings in a dynamic economy with asymmetric information. They prove that if a worker privately observes her income and has access to a hidden saving technology, then no insurance is possible.

a worker's ability to borrow and optimal insurance.

The perfect equivalence between optimal unemployment insurance and constant benefits breaks down with CRRA preferences, but we find that our results with CARA provide an important benchmark. As in the CARA case, optimal unemployment insurance dictates a declining path of consumption for unemployed workers and an increasing tax upon reemployment. However, the implicit subsidy to unemployment, the amount that a worker's expected lifetime transfer from the insurance agency rises if she stays unemployed for an additional period, *increases* very slowly during a jobless spell.

By its very definition, constant benefits are always at least as costly as optimal insurance. But if the worker has enough liquidity so as to have a minimal chance of approaching any lower bound on assets, the additional cost of constant benefits is minuscule, less than  $10^{-7}$  weeks (or about 0.01 seconds) of income in our leading example. The difference between the optimal time-varying and time-invariant subsidy is also very small.

If the worker is near her debt limit, the difference between constant benefits and optimal unemployment insurance is larger. This is because benefits are forced to play the dual role of providing insurance and smoothing consumption. However, using benefits to create liquidity in this indirect way is likely to be less efficient than measures designed to address the liquidity problem directly.

The general message that emerges from our model is that unemployment insurance policy should be simple—a constant benefit and tax, combined with measures to ensure that workers have the liquidity to maintain their consumption level during a jobless spell. Our intuition for these results is the following. With CARA utility the fall in assets and consumption that occurs during an unemployment spell does not affect attitudes toward risk; as a consequence, the optimal unemployment subsidy is constant. With CRRA utility, the worker becomes more risk averse as consumption falls; this explains why the optimal subsidy increases over time. However, this wealth effect is small during a typical, or even relatively prolonged, unemployment spell provided the worker is able to smooth her consumption.

Before proceeding, we note that our use of a sequential search model departs from [Hopenhayn and Nicolini \(1997\)](#) and many others, which assumes that there is only a job search effort decision.<sup>3</sup> There are three reasons for this modeling choice. First, our model produces stark results on optimal policy in a straightforward way, which we believe is in-

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<sup>3</sup>[Shavell and Weiss \(1979\)](#) allow for both hidden search effort and hidden wage draws. See also exercise 21.3 in [Ljungqvist and Sargent \(2004\)](#). However, both of these models assume that employed workers cannot be taxed, and neither examines optimal benefits when workers have access to liquidity.

trinsically useful. On the other hand, the sequential search model is not critical for these results. Indeed, the paper most closely related to ours is [Werning \(2002\)](#), which introduces hidden borrowing and savings into the [Hopenhayn and Nicolini \(1997\)](#) search effort model, and some of his results are analogous to ones we report here. For example, he proves that constant benefits and taxes are optimal under CARA preferences if the cost of search is monetary. Despite this, and in contrast to our results here, in [Werning \(2002\)](#) constant benefits are not equivalent to optimal unemployment insurance, even with CARA utility, since it is always desirable to exclude the worker from the asset market.

Second, the sequential search model is empirically relevant. Starting with the work of [Feldstein and Poterba \(1984\)](#), a number of authors have documented that an increase in unemployment benefits raises workers' reservation wage and consequently reduces the rate at which they find jobs. The sequential search model is a natural one for thinking about this fact. Third, the sequential search model is the backbone of most research on equilibrium unemployment. At the heart of the [Lucas and Prescott \(1974\)](#) equilibrium search model and of versions of the [Pissarides \(1985\)](#) matching model with heterogeneous firms are individual sequential search problems. More recently, [Ljungqvist and Sargent \(1998\)](#) examine a large economy in which each individual engages in sequential job search from an exogenous wage distribution.

This paper proceeds as follows. [Section 2](#) describes the model's environment and the two policies we consider. [Section 3](#) then establishes the equivalence between the two systems under CARA preferences. [Section 4](#) quantitatively evaluates optimal unemployment insurance and optimal constant benefits with CRRA preferences, highlighting the relationship between unemployment insurance and liquidity. [Section 5](#) concludes.

## 2 Two Policies for the Unemployed

We begin by describing the common physical environment of the model. We then discuss the two policies we consider, *constant benefits* and *optimal unemployment insurance*.

### 2.1 The Unemployed Worker

There is a single risk-averse worker who maximizes the expected present value of utility from consumption,

$$\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where  $\beta < 1$  represents the discount factor and  $u(c)$  is the increasing, concave period utility function.

At the start of each period, a worker can be employed at a wage  $w$  or unemployed. A worker employed at  $w$  produces  $w$  units of the consumption good in each period; she never leaves her job. An unemployed worker receives a single independent wage draw from the cumulative distribution function  $F$ .<sup>4</sup> Let  $\underline{w} \geq 0$  denote the lower bound of the wage distribution. The worker observes the wage and decides whether to accept or reject it. If she accepts  $w$ , she is employed and produces  $w$  units of the consumption good in the current and all future periods. If she rejects  $w$ , she produces nothing and is unemployed at the start of the next period. In either case, the worker decides how much to consume at the end of the period, after observing the wage draw. The worker cannot recall past wage offers.

We assume that an unemployment agency only observes whether the worker is employed or unemployed. In particular, it does not observe the worker's wage, even after she decides to take a job.<sup>5</sup> The objective of the unemployment insurance agency is to minimize the cost of providing the worker with a given level of utility. We assume costs are discounted at rate  $r = \beta^{-1} - 1$ .

## 2.2 Policy I: Constant Benefits

The policy we call *constant benefits* is defined by a constant unemployment benefit  $\bar{b}$ , a constant employment tax  $\bar{\tau}$ , and perfect access to a riskless asset with net return  $r = \beta^{-1} - 1$ , subject to a no-Ponzi-game condition.<sup>6</sup>

Since the worker's problem is stationary, we present it recursively. Start by considering a worker who is employed at wage  $w$  and has assets  $a$  with budget constraint  $a' = (1 + r)a + w - \bar{\tau} - c$ . Since  $\beta(1 + r) = 1$ , she consumes her after tax-income plus the interest on her assets  $c_e(a, w) = ra + w - \bar{\tau}$ , so that assets are kept constant,  $a' = a$ . This means that her

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<sup>4</sup>We assume that  $F$  is continuous and has finite expectation and that there is some chance of drawing a positive wage, so  $F(w) < 1$  for some  $w > 0$ .

<sup>5</sup>If the wage were observable, an unemployment insurance agency could tax employed workers 100 percent and redistribute the proceeds as a lump-sum transfer. Workers would be indifferent about taking a job and hence would follow any instructions on which wages to accept or reject. This makes it feasible to obtain the first best, complete insurance with the maximum possible income. Private information is a simple way to prevent the first best, but other modeling assumptions could also make the first best unattainable, e.g., moral hazard among employed workers.

<sup>6</sup>That is, debt must grow slower than the interest rate,  $\lim_{t \rightarrow \infty} (1 + r)^{-t} a_t \geq 0$ , with probability one, where  $a_t$  denotes asset holdings. Together with the sequence of intertemporal budget constraints, this is equivalent to imposing a present-value lifetime budget constraint, with probability one.

lifetime utility is

$$V_e(a, w; \bar{\tau}) = \frac{u(ra + w - \bar{\tau})}{1 - \beta}. \quad (1)$$

Next consider an unemployed worker with assets  $a$  and let  $V_u(a; \bar{b}, \bar{\tau})$  denote her expected lifetime utility, given policy parameters  $\bar{b}$  and  $\bar{\tau}$ . This must satisfy the Bellman equation

$$V_u(a; \bar{b}, \bar{\tau}) = \int_w^\infty \max \left\{ \max_c (u(c) + \beta V_u(a'; \bar{b}, \bar{\tau})), \frac{u(ra + w - \bar{\tau})}{1 - \beta} \right\} dF(w), \quad (2)$$

where  $a' = (1 + r)a + \bar{b} - c$ . An unemployed worker chooses whether to accept a job or not. If she does take the job, her utility is given by  $V_e(a, w; \bar{\tau})$  in equation (1). Otherwise, she collects unemployment benefits  $\bar{b}$ , consumes  $c$  and saves  $a'$  in the current period, and remains unemployed into the next period, obtaining expected continuation utility  $V_u(a'; \bar{b}, \bar{\tau})$ .

The solution to the Bellman equation defines the worker's unemployment consumption  $c_u(a)$ , reservation wage  $\bar{w}(a)$ , and next period's assets  $a'(a)$ , conditional on this period's assets. Given these objects, the cost of the unemployment insurance system is defined recursively by

$$S(a; \bar{b}, \bar{\tau}) = \left( \bar{b} + \frac{S(a'(a); \bar{b}, \bar{\tau})}{1 + r} \right) F(\bar{w}(a)) - \frac{(1 + r)\bar{\tau}(1 - F(\bar{w}(a)))}{r}. \quad (3)$$

A worker with assets  $a$  fails to find a job with probability  $F(\bar{w}(a))$ . In this event, the cost of the unemployment insurance system is the unemployment benefit  $\bar{b}$  plus the discounted continuation cost  $S(a')$ . If she finds a job, the present value of her tax payments is  $\frac{(1+r)\bar{\tau}}{r}$ .

An unemployment insurance agency chooses  $\bar{b}$  and  $\bar{\tau}$  to maximize the worker's utility given some available resources and an initial asset level. Equivalently, we consider the dual problem of minimizing the total resource cost of delivering a certain utility for the worker. The optimal constant benefit policy solves  $C^c(v_0, a) \equiv \min_{\bar{b}, \bar{\tau}} S(a; \bar{b}, \bar{\tau}) + (1 + r)a$  subject to  $V_u(a; \bar{b}, \bar{\tau}) = v_0$ .

Since there are no *ad hoc* constraints on borrowing, a standard Ricardian equivalence argument implies that  $V(a; \bar{b}, \bar{\tau}) = V(a + x; \bar{b} - rx, \bar{\tau} + rx)$ . The same is true with total cost, so it follows that  $C^c(v_0, a)$  is independent of  $a$ . Abusing notation we write  $C^c(v_0)$ .



## 2.3 Policy II: Optimal Unemployment Insurance

Under optimal unemployment insurance, a worker who is unemployed in period  $t$  consumes  $b_t$ , while a worker who finds a job in period  $t$  pays a tax  $\tau_t$ , depending on when she finds a job, for the remainder of her life. One can conceive of more complicated insurance policies where the agency asks the worker to report her wage draws, advises her on whether to take the job, and makes payments conditional on the worker's entire history of reports. That is, one can model unemployment insurance as a revelation mechanism in a principal-agent problem. We prove in [Appendix A](#) that the policy we consider here does as well as any deterministic mechanism as long as absolute risk aversion is non-increasing.

Given  $\{b_t\}$  and  $\{\tau_t\}$ , consider a worker who chooses a sequence of reservation wages  $\{\bar{w}_t\}$ . Her lifetime utility is

$$U(\{\bar{w}_t, b_t, \tau_t\}) = \sum_{t=0}^{\infty} \beta^t \left( \prod_{s=0}^{t-1} F(\bar{w}_s) \right) \left( u(b_t)F(\bar{w}_t) + \int_{\bar{w}_t}^{\infty} \frac{u(w - \tau_t)}{1 - \beta} dF(w) \right) \quad (4)$$

The worker is unemployed at the start of period  $t$  with probability  $\prod_{s=0}^{t-1} F(\bar{w}_s)$ . If she draws a wage below  $\bar{w}_t$ , she rejects it and her period utility is  $u(b_t)$ . If she draws a wage above  $\bar{w}_t$ , she takes the job and gets utility  $u(w - \tau_t)$  each period, forever.

Now consider an unemployment insurance agency that sets the sequence of unemployment consumption and employment taxes  $\{b_t, \tau_t\}$  to minimize the cost of providing the worker with utility  $v_0$ :

$$C^*(v_0) \equiv \min_{\{\bar{w}_t, b_t, \tau_t\}} \sum_{t=0}^{\infty} (1+r)^{-t} \left( \prod_{s=0}^{t-1} F(\bar{w}_s) \right) \left( b_t F(\bar{w}_t) - \frac{1+r}{r} \tau_t (1 - F(\bar{w}_t)) \right), \quad (5)$$

subject to two constraints. First, the worker's utility must equal  $v_0$  if she uses the recommended reservation wage sequence,  $v_0 = U(\{\bar{w}_t, b_t, \tau_t\})$ . And second, she must do at least as well using the recommended reservation wage sequence as any other sequence  $\{\hat{w}_t\}$ ,  $U(\{\bar{w}_t, b_t, \tau_t\}) \geq U(\{\hat{w}_t, b_t, \tau_t\})$ . That is, the agency recognizes that the worker will choose her reservation wage sequence  $\{\bar{w}_t\}$  to maximize her utility given  $\{b_t, \tau_t\}$ . The solution to this problem describes optimal unemployment benefits.

It is useful to express this problem recursively. The cost function defined above must

solve the Bellman equation

$$C^*(v) = \min_{\bar{w}, b, v', \tau} \left( \left( b + \frac{C^*(v')}{1+r} \right) F(\bar{w}) - \frac{1+r}{r} \tau (1 - F(\bar{w})) \right) \quad (6)$$

subject to

$$v = (u(b) + \beta v') F(\bar{w}) + \int_{\bar{w}}^{\infty} \frac{u(w - \tau)}{1 - \beta} dF(w) \quad (7)$$

$$u(b) + \beta v' = \frac{u(\bar{w} - \tau)}{1 - \beta}. \quad (8)$$

Moreover, the optimal sequence  $\{\bar{w}_t, b_t, \tau_t\}$  must be generated by the Bellman equation's policy functions.

An unemployed worker starts the period with some promised utility  $v$ . The agency chooses consumption for the unemployed  $b$ , the tax  $\tau$  it will collect on workers who become employed in the current period, the worker's continuation utility if she remains unemployed  $v'$ , and the reservation wage  $\bar{w}$  in order to minimize its cost. If the worker gets an offer below the reservation wage, then the cost is the unemployment consumption  $b$  plus the discounted cost of delivering continuation utility  $v'$  in the next period. If instead the worker finds a job above the reservation wage, then the agency's costs are reduced by the present value of taxes. Equation (7) imposes that the policy must deliver utility  $v$  to the worker. Finally, equation (8) is the incentive constraint, which incorporates the fact that the worker sets her reservation wage at the point of indifference between accepting and rejecting the wage.

### 3 Equivalence for a Benchmark: CARA Utility

There are two disadvantages to constant benefits relative to optimal unemployment insurance. First, there is a restriction on the time path of unemployment benefits and taxes, so  $b_t$  and  $\tau_t$  are constant. Second, the planner does not directly control the worker's consumption and so is constrained by her savings choices. This can be thought of as an additional dimension of moral hazard. In general, constant benefits are more costly than optimal unemployment insurance:  $C^c(v) \geq C^*(v)$ ; however, in this section we prove analytically that constant benefits achieve the same outcome as optimal unemployment insurance for the case with CARA preferences,  $u(c) = -\exp(-\rho c)$  with  $c \in \mathbb{R}$ . A key feature is that there is no limit on the amount of debt that workers can accrue and all workers have the same attitude

towards lotteries over future wages, which makes the model particularly tractable. We later show that these results provide a good benchmark for other preference specifications.

For the results in this section, it is convenient to define

$$\text{CE}(\bar{w}) \equiv u^{-1} \left( \int_{\underline{w}}^{\infty} u(\max\{\bar{w}, w\}) dF(w) \right), \quad (9)$$

the certainty equivalent for a worker of a lottery offering the maximum of  $\bar{w}$  and  $w \sim F$ . The CARA utility function has a convenient property that we exploit throughout this section,  $u(c_1 + c_2) = -u(c_1)u(c_2)$  for any  $c_1$  and  $c_2$ .

### 3.1 Constant Benefits

We characterize constant benefits in two steps. First, we characterize individual behavior given unemployment benefits  $\bar{b}$ , employment taxes  $\bar{\tau}$ , and assets  $a$ . Then we discuss how to choose these parameters optimally. It is convenient to define the net benefit or unemployment subsidy by  $\bar{B} \equiv \bar{b} + \bar{\tau}$ .

The first step follows from solving the Bellman equation (2).

**Proposition 1** *Assume CARA preferences. The reservation wage, consumption and utility of the unemployed satisfy*

$$(1 + r)\bar{w} = \text{CE}(\bar{w}) + r\bar{B}. \quad (10)$$

$$c_u(a) = ra + \bar{w} - \bar{\tau} \quad (11)$$

$$V_u(a) = \frac{u(ra - \bar{\tau} + \text{CE}(\bar{w}))}{1 - \beta} \quad (12)$$

**Proof.** In Appendix B. ■

Equation (10) generalizes a standard equation for a risk-neutral worker's reservation wage, e.g., equation (6.3.3) in [Ljungqvist and Sargent \(2004\)](#), to an environment with risk aversion and savings. It can be reexpressed as the condition that a worker is indifferent between accepting her net wage  $\bar{w} - \bar{\tau}$  today and rejecting it, getting her unemployment benefit today, and then earning the certainty equivalent  $\text{CE}(\bar{w}) - \bar{\tau}$  thereafter:

$$\frac{\bar{w} - \bar{\tau}}{1 - \beta} = \bar{b} + \beta \frac{\text{CE}(\bar{w}) - \bar{\tau}}{1 - \beta}.$$

Equation (10) indicates that the reservation wage  $\bar{w}$  is increasing in the net unemployment

subsidy  $\bar{B}$ . This is the essence of the moral hazard problem in our model—the more one tries to protect the worker against unemployment by raising unemployment benefits and funding the benefits by an employment tax, the more selective she becomes. The equation also shows that a worker’s assets  $a$  do not affect her reservation wage, so it is constant during a spell of unemployment.

Consumption in equation (11) has a permanent income form with a constant precautionary savings component. Assets fall by  $\bar{w} - \bar{B} > 0$  as long as there is some chance of getting a wage in excess of the unemployment subsidy,  $F(\bar{B}) < 1$ .<sup>7</sup> Consumption falls by  $CE(\bar{w}) - \bar{w}$  each period that the worker remains unemployed. Unemployed workers face uncertainty: a wage draw above  $CE(\bar{w})$  is good news leading to an increase in consumption, while a wage draw below  $CE(\bar{w})$  is bad news leading to a decline in consumption.

The next step is to minimize the cost of providing the worker with utility  $v_0$ . Using the result that the reservation wage is constant, equation (3) becomes

$$S(a; \bar{b}, \bar{\tau}) = \frac{1+r}{r} \left( \frac{r\bar{b}F(\bar{w}) - (1+r)\bar{\tau}(1-F(\bar{w}))}{1+r-F(\bar{w})} \right), \quad (13)$$

which is independent of  $a$ . Optimal constant benefit policy minimizes  $S(a; \bar{b}, \bar{\tau}) + (1+r)a$  subject to (10) and (12).

**Proposition 2** *Assume CARA preferences. Then the optimal constant benefits policy is independent of  $v_0$ . The reservation wage satisfies  $\bar{w}^* \in \arg \max_{\bar{w}} \Phi(\bar{w})$ , where*

$$\Phi(\bar{w}) \equiv \frac{CE(\bar{w}) - \bar{w}F(\bar{w})}{1+r-F(\bar{w})}, \quad (14)$$

and  $\bar{b}$  and  $\bar{\tau}$  are then determined by equations (10) and (12). The minimum cost is

$$C^c(v_0) = \frac{1+r}{r} \left( u^{-1}((1-\beta)v_0) - (1+r)\Phi(\bar{w}^*) \right), \quad (15)$$

independent of  $a$ .

**Proof.** Use equations (10) and (12) to solve for  $\bar{b}$  and  $\bar{\tau}$  as functions of  $\bar{w}$  and  $v = V_u(a; \bar{b}, \bar{\tau})$ . Substituting into the cost (13) delivers the desired result. ■

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<sup>7</sup>Substitute equation (11) into the unemployed worker’s budget constraint to get  $a' = a + \bar{B} - \bar{w}$ . If  $\bar{w} \leq \bar{B}$ , condition (10) implies  $\bar{w} \geq CE(\bar{w})$ . But the definition of the certainty equivalent (9) implies this is possible only if  $F(\bar{w}) = 1$ , a contradiction.

Our next result characterizes the worker's allocation given optimal policy. We take current unemployment utility  $v$  as a state variable, express the allocation as a function of  $v$ , and describe the evolution of  $v$ .

**Proposition 3** *Assume CARA preferences. Let  $v$  denote the utility promised to the unemployed at the beginning of a period. Then if the agent remains unemployed, she consumes*

$$c_u(v) = \bar{w}^* - \text{CE}(\bar{w}^*) + u^{-1}((1 - \beta)v) \quad (16)$$

and her utility evolves to

$$v'(v) = -u(\bar{w}^* - \text{CE}(\bar{w}^*))v. \quad (17)$$

If she accepts a job at wage  $w$ , she forever after consumes

$$c_e(v, w) = w - \text{CE}(\bar{w}^*) + u^{-1}((1 - \beta)v). \quad (18)$$

**Proof.** This follows directly by changing variables from  $a$  to  $v = V_u(a; \bar{b}, \bar{\tau})$  using equations (11)–(12),  $c_e(a, w) = ra + w - \bar{\tau}$ , and the budget constraint  $a' = (1 + r)a + b - c$ .

■

This proposition will be useful when comparing constant benefits with optimal unemployment insurance, which we turn to now.

## 3.2 Optimal Unemployment Insurance

We characterize optimal unemployment insurance using the Bellman equation (6)–(8). To do so, it is convenient to first deduce the shape of the cost function directly from the sequence problem.

**Lemma 1** *Assume CARA preferences. The cost function satisfies*

$$C^*(v_0) = \frac{1+r}{r} u^{-1}((1 - \beta)v_0) + C^*\left(\frac{u(0)}{1 - \beta}\right). \quad (19)$$

Moreover, let  $\{\bar{w}_t^*, b_t^*, \tau_t^*\}$  denote the optimum for initial promised utility  $u(0)/(1 - \beta)$ . Then  $\{\bar{w}_t^*, b_t^* + x, \tau_t^* - x\}$  with  $x \equiv u^{-1}((1 - \beta)v_0)$  is optimal for any other initial promise  $v_0$ .

**Proof.** Let  $\hat{b}_t \equiv b_t + x$  and  $\hat{\tau}_t \equiv \tau_t - x$  for all  $t$ . Use equation (4) and CARA preferences to show that adding a constant  $x$  to unemployment consumption in each period and subtracting

the same constant  $x$  from the employment tax simply multiplies lifetime utility by the positive constant  $-u(x)$ , that is  $U(\{\bar{w}_t, b_t, \tau_t\}) = -u(x)U(\{\bar{w}, \hat{b}, \hat{\tau}\})$ . The result follows immediately. ■

The optimal path for consumption shifts in parallel with promised utility, while the path for the reservation wage is unchanged. The cost function reflects these two features. Indeed, since promised utility is a state variable for the problem, the lemma implies that the optimal reservation wage path will be constant. These results are implications of the absence of wealth effects with CARA preferences.

To solve the agency's problem further, we substitute the cost function from (19) into (6) and use the incentive constraint (8) to eliminate the employment tax  $\tau$ . The Bellman equation at  $v = u(0)/(1 - \beta)$  then becomes

$$C^*\left(\frac{u(0)}{1 - \beta}\right) = \min_{\bar{w}, b, v'} \left( \left( b + \frac{u^{-1}((1 - \beta)v')}{r} + \frac{1}{1 + r} C^*\left(\frac{u(0)}{1 - \beta}\right) \right) F(\bar{w}) - \frac{1 + r}{r} \left( \bar{w} - u^{-1}((1 - \beta)(u(b) + \beta v')) \right) (1 - F(\bar{w})) \right) \quad (20)$$

$$\text{subject to } \frac{u(0)}{1 - \beta} = -(u(b) + \beta v')u(\text{CE}(\bar{w}) - \bar{w}). \quad (21)$$

The solution to this cost minimization problem must solve the subproblem of minimizing the cost  $b + u^{-1}((1 - \beta)v')/r$  of providing a given level of utility  $u(b) + \beta v'$  to those remaining unemployed. The first order condition for this problem yields

$$(1 - \beta)v' = u(b) \quad (22)$$

or equivalently  $u(b) + \beta v' = u(b)/(1 - \beta)$ . The promise keeping constraint (21) is then equivalent to  $b = \bar{w} - \text{CE}(\bar{w})$ . Substitute these conditions into the Bellman equation to eliminate  $b$  and  $v'$ , and solve for  $C^*(u(0)/(1 - \beta))$  to obtain

$$C^*\left(\frac{u(0)}{1 - \beta}\right) = \frac{(1 + r)^2}{r} \min_{\bar{w}} \frac{F(\bar{w})\bar{w} - \text{CE}(\bar{w})}{1 + r - F(\bar{w})} = -\frac{(1 + r)^2}{r} \max_{\bar{w}} \Phi(\bar{w}), \quad (23)$$

where  $\Phi(\bar{w})$  is defined by equation (14).

The optimal reservation wage  $\bar{w}^*$  is independent of promised utility and hence constant over time. Substituting equation (23) into equation (19) proves that the cost to the agency of providing a worker with utility  $v$  is identical to the cost with constant benefits  $C^c(v)$  in equation (15).

Once we have found the optimal reservation wage  $\bar{w}^*$ , the associated unemployment consumption, employment tax and continuation utility fall out using equations (7), (8), and (22) along with Lemma 1. The next proposition summarizes the main result of this section.

**Proposition 4** *Assume CARA preferences. Under optimal unemployment insurance, the reservation wage is constant over time and maximizes  $\Phi(\bar{w})$ , given by (14). If an agent has expected utility  $v$  and remains unemployed, she consumes  $c_u(v)$  (equation 16) and has continuation utility  $v'(v)$  (equation 17). If she accepts a job at wage  $w$ , she consumes  $c_e(v, w)$  (equation 18) forever. This is the same allocation as under an optimal constant benefit and the cost is the same,  $C^c(v_0) = C^*(v_0)$ .*

Thus, when the worker can borrow and lend at the same rate as the agency, a very simple policy attains the same allocation as optimal unemployment insurance. Of course, Ricardian equivalence implies that the timing of transfers is not pinned down, only the net subsidy to unemployment. If a worker takes a job in period  $t$ , she must pay taxes equal to  $\frac{(1+r)\tau_t}{r}$  in present value terms. If she remains unemployed for one more period, she receives a benefit  $b_t$  and then pays taxes  $\frac{\tau_{t+1}}{r}$  in present value terms. The sum of these is the unemployment subsidy, a measure of insurance:

$$B_t \equiv b_t + \frac{(1+r)\tau_t}{r} - \frac{\tau_{t+1}}{r} \quad (24)$$

Using  $b_t = c^u(v_t)$  and  $\tau_t = w - c^e(v_t, w)$  with equations (16)–(18), we find that  $B_t = ((1+r)\bar{w}^* - CE(\bar{w}))/r$ . The unemployment subsidy is constant and the same as  $\bar{B}$  in the problem with constant benefits, given by equation (10).

At the other end of the spectrum from Ricardian equivalence, imagine a worker who can neither borrow nor save and so lives hand-to-mouth consuming current income. In this extreme case, benefits and taxes are uniquely pinned down, as in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). One interpretation of this extreme case is that it calls for decreasing benefits and increasing taxes. However, it is equivalent to think of the insurance agency simultaneously lending to the worker and providing her with a constant unemployment subsidy. Conceptually, even in this case, it remains useful to distinguish between these two components of policy.

## 4 Liquidity and Wealth Effects: CRRA Utility

The sharp closed form results obtained so far were derived under an assumption of CARA preferences and in particular allowed consumption to be negative. We now turn to workers with constant relative risk aversion (CRRA) preferences with nonnegative constraint on consumption. Let  $\sigma > 0$  denote the coefficient of relative risk aversion. Then the period utility function is  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$  for  $\sigma \neq 1$ , with  $u(c) = \log(c)$  corresponding to risk aversion of one.

We again consider our two alternative policies: *optimal unemployment insurance* and *constant benefits*. The equivalence between these two policies breaks down with CRRA preferences. Nevertheless, we find little welfare gain in moving from constant benefits to optimal unemployment insurance. Moreover, we find that the optimal policy and allocations obtained analytically with CARA provide an excellent approximation for our CRRA specifications. For both reasons, we conclude that the CARA case is indeed a very useful benchmark.

### 4.1 Optimal Unemployment Insurance

Optimal unemployment insurance solves the Bellman equation (6)–(8). The form of the utility function is different with CRRA preferences than with CARA preferences, and so the analytical expression for the cost function in equation (19) no longer holds. We therefore use numerical simulations to examine the economy.

To proceed we need to make choices for the discount factor  $\beta = (1+r)^{-1}$ , the coefficient of relative risk aversion  $\sigma$ , and the wage distribution  $F(w)$ . As in [Hopenhayn and Nicolini \(1997\)](#), we view a period as representing a week and set  $\beta = 0.999$ , equivalent to an annual discount factor of 0.949. We fix the coefficient of relative risk aversion at  $\sigma = 1.5$  but later consider the robustness of our results to a higher value,  $\sigma = 6$ .<sup>8</sup>

We adopt a Frechet wage distribution,  $F(w) = \exp(-zw^{-\theta})$  with support  $(0, \infty)$ , and parameters  $z, \theta > 0$ .<sup>9</sup> With CRRA the parameter  $z$  acts as an uninteresting scaling factor,

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<sup>8</sup>[Hopenhayn and Nicolini \(1997\)](#) use the much lower value of  $\sigma = 1/2$  in their baseline calibration. They argue that over short horizons, a high intertemporal elasticity of substitution may be appropriate. In our view, this remark resonates introspectively, but is at the same time misleading since it confounds attitudes regarding consumption and net income paths. In their model, consumption and net income were equivalent; but our model allows saving and borrowing, and as a result a worker displays an infinite elasticity of substitution with respect to the timing of transfers.

<sup>9</sup>A Frechet distribution has some desirable properties in this environment. First, it displays positive skewness. Second, suppose a worker receives  $n$  wage draws within a period from a Frechet distribution with parameters  $(\hat{z}, \theta)$ , and must decide whether to accept the maximum of these draws. The distribution of the maximum wage draw is also Frechet with the same  $\theta$  and  $z = n\hat{z}$ . Thus there is no loss of generality in our



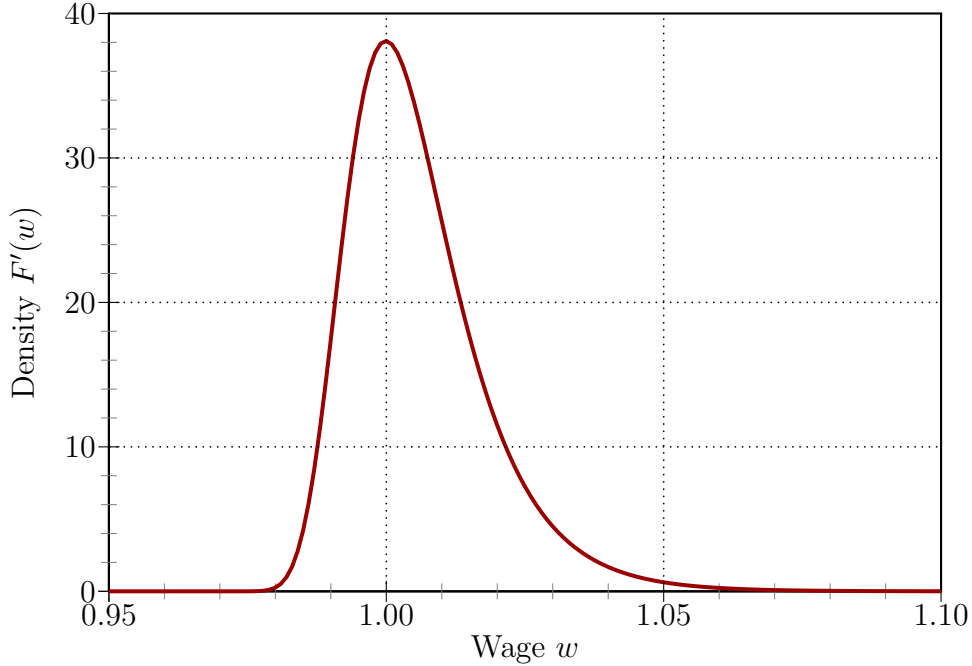


Figure 2: Wage density:  $F'(w) = \theta w^{-\theta-1} e^{-w^{-\theta}}$ ,  $\theta = 103.56$ .

so we normalize by setting  $z = 1$ . The mean log wage draw is then  $\frac{\gamma}{\theta}$ , where  $\gamma \approx 0.577$  is Euler's constant, and the standard deviation of log wages is  $\frac{\pi}{\sqrt{6\theta}} \approx \frac{1.28}{\theta}$ .

Following [Hopenhayn and Nicolini \(1997\)](#), we set  $\theta$  so that the mean duration of an unemployment spell is about ten weeks, consistent with evidence in [Meyer \(1990\)](#) on a weekly job finding probability of 10 percent for the United States. This requires setting  $\theta = 103.56$ , giving a standard deviation of log wages of about 1.2 percent.<sup>10</sup> [Figure 2](#) plots the density function  $F'(w)$ . We also consider the robustness of our results to changes in the wage distribution, in particular to a substantial decrease in  $\theta$ , which increases the dispersion in wages, raising the option value of job search and the expected duration of unemployment.

It will be useful to have a way of comparing the cost or policy functions obtained from our CRRA specification with those obtained from the CARA case. To this end, note that a worker with a constant coefficient of relative risk aversion  $\sigma$  who consumes  $c$  has local coefficient of absolute risk aversion equal to  $\sigma/c$ . This suggests comparing the cost or policy functions obtained for a CRRA worker with the approximation provided by those of a fictitious CARA worker with coefficient of absolute risk aversion  $\rho = \sigma/u^{-1}((1 - \beta)v)$ , where

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assumption that the worker gets one wage draw per period.

<sup>10</sup>Specifically, we chose  $\theta$  so that a risk-neutral worker without unemployment insurance would have exactly  $F(\bar{w}) = 0.90$ , making use of equation (10).

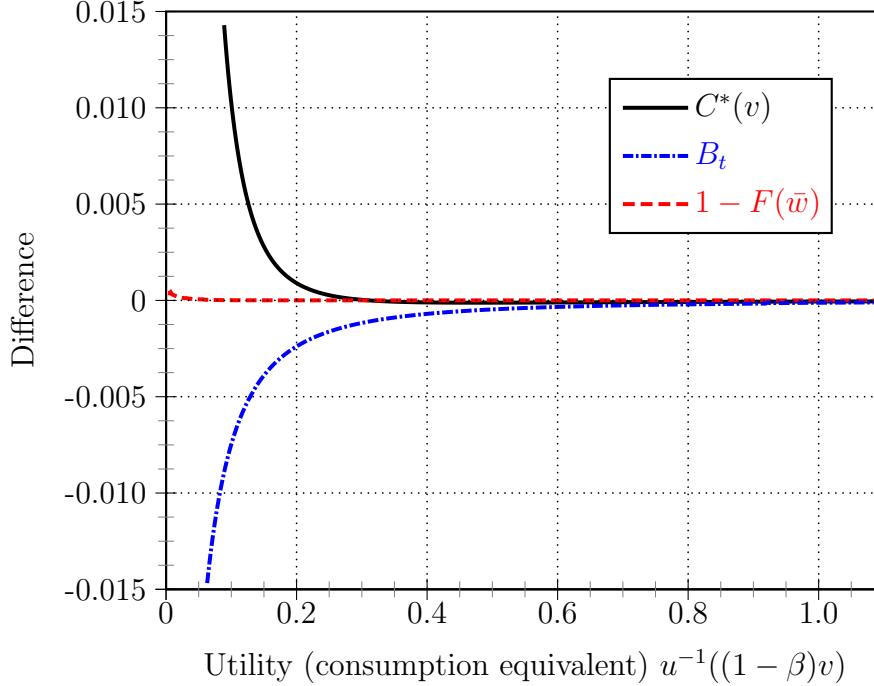


Figure 3: Difference between actual values and CARA approximations.

we take the consumption equivalent utility as a proxy for consumption. The approximations can be computed analytically using our results from [Section 3](#).

We begin by discussing our results for the minimum cost of providing a worker with a given level of utility. Recall that with CARA utility, this cost is linear when utility is measured in consumption equivalent units with a slope of  $\frac{1+x}{r}$ ; see equation (19). For our CRRA specification, we find that the cost is nearly linear with almost the same slope. For this reason, we do not graph the cost function. Instead, we compare the cost obtained from the CRRA specification with an approximation provided by the CARA exercise. The solid black line in [Figure 3](#) shows that the difference between  $C^*(v)$  and this CARA approximation is small, less than 0.0001 in absolute value when utility exceeds a certainty equivalent of 0.3.

Turning to the optimal allocation and policy, the left panel in [Figure 4](#) shows that, as a function of the worker's promised utility, unemployment consumption  $b$  is increasing (solid brown line) while employment taxes  $\tau$  are decreasing (dashed orange line). The right panel shows how  $b$  and  $\tau$  evolve over an unemployment spell starting with initial promised value  $v_0 = u(1.1)/(1 - \beta)$ . Although initially unemployment consumption is high and the employment tax is slightly negative, after a sufficiently long unemployment spell the employment tax rises to a high level and unemployment consumption falls to nearly zero. Putting these together, a worker's expected utility  $v_t$  declines over time. This line is not graphed because

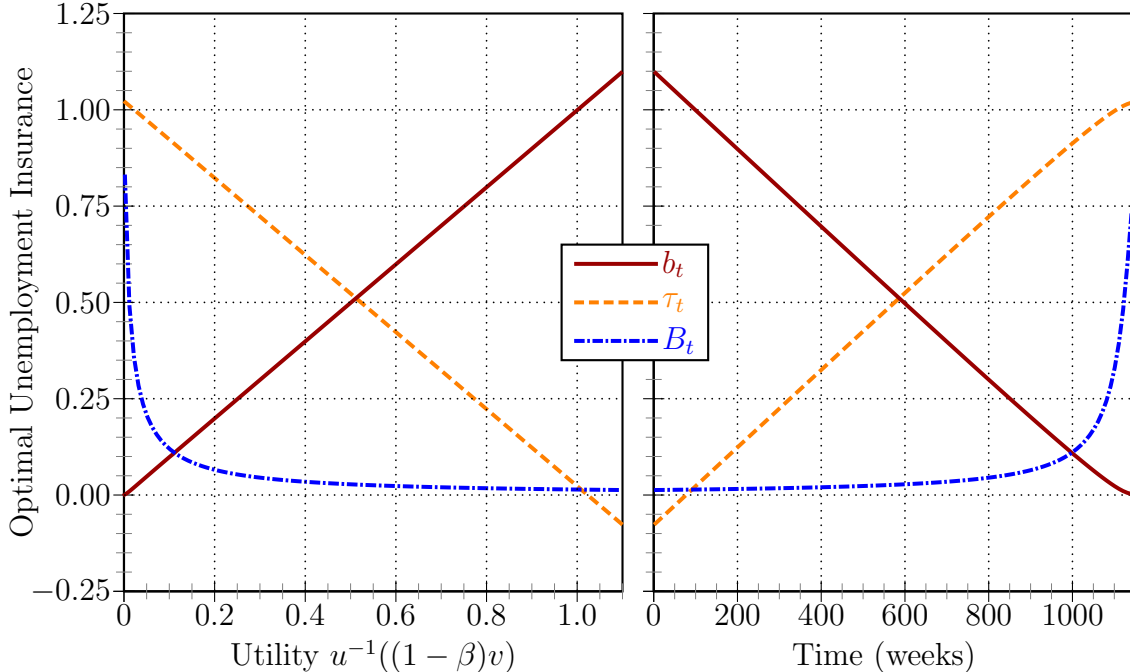


Figure 4: Optimal unemployment consumption  $b_t$ , employment taxes  $\tau_t$ , and unemployment subsidy  $B_t = b_t + \frac{(1+r)\tau_t}{r} - \frac{\tau_{t+1}}{r}$ .

it is only slightly higher than, and would be scarcely distinguishable from, unemployment consumption  $b_t$ .

We also look at the subsidy to unemployment, the additional resources that a worker gets by remaining unemployed for one more period, as previously defined in equation (24). The dash-dot blue line in Figure 4 shows that this unemployment subsidy is small when utility is high at the start of an unemployment spell and then increases gradually as promised utility falls and the spell continues. The dash-dot blue line in Figure 3 illustrates the high accuracy for  $B_t$  of the CARA approximation with  $\rho = \sigma/u^{-1}((1-\beta)v)$ .

The unemployment subsidy  $B_t$  paints a very different picture of optimal unemployment insurance than do unemployment consumption  $b_t$  or employment taxes  $\tau_t$  in isolation. The picture for  $b_t$  and  $\tau_t$  in Hopenhayn and Nicolini (1997) is qualitatively similar. Werning (2002) computes the net subsidy to unemployment from their allocation and finds that it is nearly constant, starting quite low and rising very slowly. This distinction between unemployment consumption and subsidies is crucial in understanding the difference between the results of this paper on the one hand, and Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) on the other.

Finally, Figure 5 shows the probability that an unemployed worker accepts a job,  $1 -$

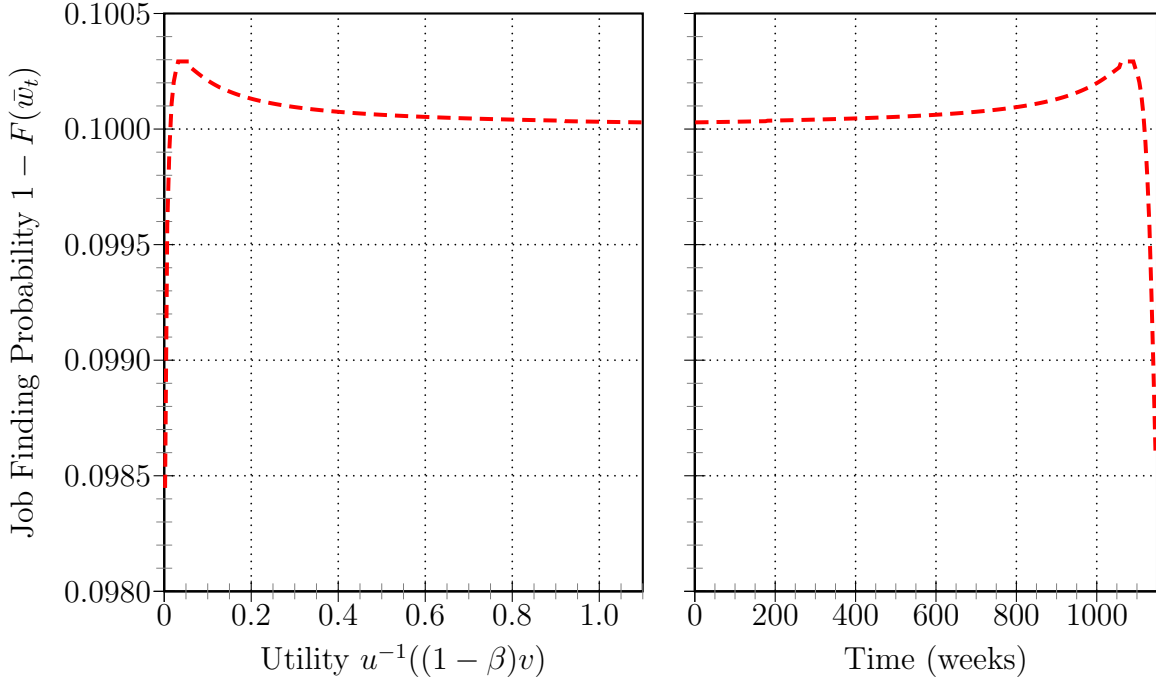


Figure 5: Optimal Job Finding Probability  $1 - F(\bar{w}_t)$ .

$F(\bar{w}_t)$ , under optimal unemployment insurance. This starts just above 10 percent per week when promised utility is high, then initially rises before falling when promised utility is very low. This non-monotonicity illustrates two opposing forces at play as the worker gets poorer: the increase in absolute risk aversion and the increase in unemployment subsidies. The first effect encourages the worker to accept more jobs, while the second effect, which is partly an endogenous response to the first, encourages her to become more selective. The dashed red line in [Figure 3](#) shows the high accuracy of the CARA approximation. It follows that the non-monotonicity of the job finding rate with respect to promised utility  $v$  found in our CRRA specification reflects a non-monotonicity with respect to risk aversion  $\rho$  in the CARA case. The next subsection explores this notion further by studying the constant benefits policy with CRRA utility.

Before closing, it is important to emphasize that in this CRRA specification, the probability that a worker remains unemployed for 100 weeks or more is remote, on the order of  $10^{-5}$ . Over the relevant time period, the unemployment subsidy and the job finding probability are virtually constant. In this sense, our results with CARA provide an excellent benchmark for the CRRA specification.

## 4.2 Constant Benefits

The Bellman equation (2) describes the problem of an unemployed worker with assets  $a$  facing a constant benefit  $\bar{b}$  and a constant employment tax  $\bar{\tau}$ . In addition, since consumption is nonnegative, assets  $a'$  cannot fall below some, possibly negative, level  $\underline{a}$ , Aiyagari's (1994) natural borrowing limit. A worker can borrow as long as she can pay the interest on her debt following any sequence of wage draws. The natural borrowing limit is

$$\underline{a} = -\frac{\max\{\bar{b}, \underline{w} - \bar{\tau}\}}{r} = -\frac{\max\{\bar{B}, \underline{w}\} - \bar{\tau}}{r}.$$

Thus, the details of the natural borrowing limit depend on whether the smallest possible wage,  $\underline{w}$ , is bigger or smaller than the net unemployment benefit,  $\bar{B} \equiv \bar{b} + \bar{\tau}$ .

In the first case,  $\underline{w} > \bar{B}$ , and the natural borrowing limit is  $-\frac{\underline{w} - \bar{\tau}}{r}$ , since a worker with assets above this level could always have positive consumption and pay the interest on her debt by taking the next job offer. This implies that a change in the left tail of the wage distribution can substantially affect a worker's debt limit, and potentially her behavior, even if she is extremely unlikely ever to accept a wage from this part of the distribution.<sup>11</sup>

If  $\underline{w} \leq \bar{B}$ , the natural borrowing limit is determined by a worker's ability to use her unemployment income to pay the interest on her debt, so  $\underline{a} = -\frac{\bar{b}}{r}$ . An increase in the net unemployment benefit, obtained by an increase in  $\bar{b}$  and a budget balance change in  $\bar{\tau}$ , then has two distinct effects. It transfers income to states in which the worker does not find a job (insurance) and it allows the worker to go further into debt while she is unemployed (liquidity). The liquidity effect is absent from the model with CARA preferences because there is no borrowing limit.

Appendix C discusses an efficient method of solving the worker's Bellman equation (2) for  $V_u(a; \bar{b}; \bar{\tau})$  and equation (3)  $S(a; \bar{b}, \bar{\tau})$ . It is then simple to choose  $\bar{b}$  and  $\bar{\tau}$  to minimize the total resource cost  $(1+r)a + S(a; \bar{b}, \bar{\tau})$  of providing a worker with utility  $v = V_u(a; \bar{b}, \bar{\tau})$ . We parameterize the economy as before:  $\beta = 0.999$ ,  $\sigma = 1.5$ ,  $F(w) = \exp(-w^{-\theta})$ , and  $\theta = 103.56$ . Thus, we begin with a specification where  $\underline{w} = 0 \leq \bar{B}$  so that we are in the case where benefits affect the borrowing constraint. In the next subsection we turn to the other case.

To start, we examine the cost of providing the worker with a given level of utility. Theoretically, this is higher than the cost under optimal unemployment insurance. Rather than

<sup>11</sup>With CARA preferences, Proposition 1 shows that the distribution of wages below the reservation wage is irrelevant for equilibrium behavior. This is because there is no borrowing limit with CARA preferences.

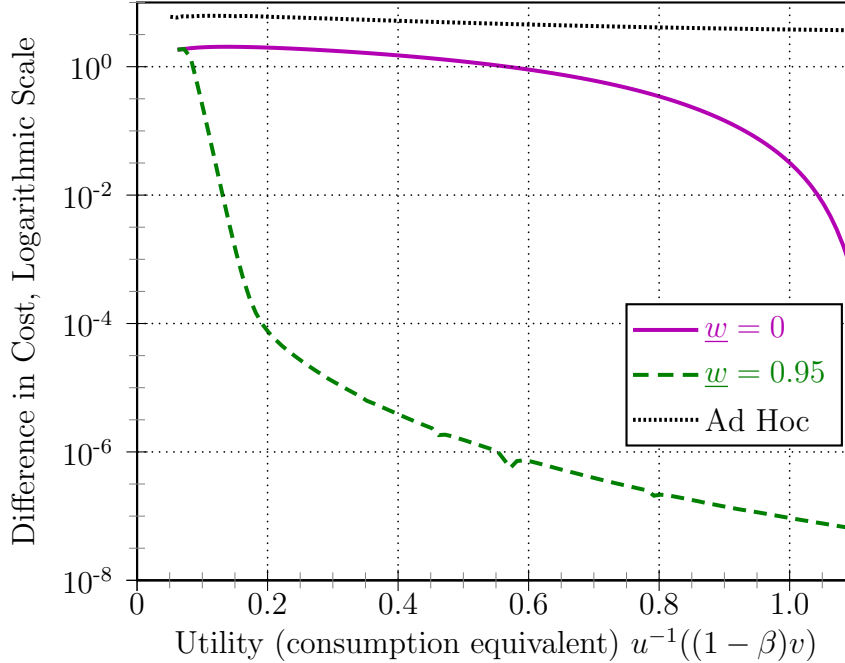


Figure 6: Additional cost of constant benefits.

showing the cost directly, the solid purple line in Figure 6 plots the additional cost of constant benefits on a logarithmic scale. At small values of utility, the cost of constant benefits is reasonably large, equal to a few weeks of consumption. But at high levels of utility, near those corresponding to a net resource cost of zero, around  $v = \frac{u(1.03)}{1-\beta}$ , the additional cost of using a constant benefits system is very small, about 0.01 weeks of consumption.

On the other hand, Figure 7 shows that the optimal constant unemployment subsidy  $\bar{B}$  for a worker who starts an unemployment spell with a given level of utility (solid purple line) is typically much higher than the optimal time-varying unemployment subsidy  $\bar{B}_t$  for a worker with the same level of utility (dash-dot blue line). Workers with lower utility demand higher benefits for two reasons. First, they have higher absolute risk aversion, and value insurance more. Second, relaxing the borrowing constraint is more important to them because they are closer to it.

### 4.3 Liquidity

The goal of this section is to isolate the insurance role of unemployment benefits. To do this, first observe that under the Frechet wage distribution  $F(w) = \exp(-w^{-103.56})$ , the probability that a single wage draw is less than 0.95 is minuscule, approximately  $10^{-88}$ .

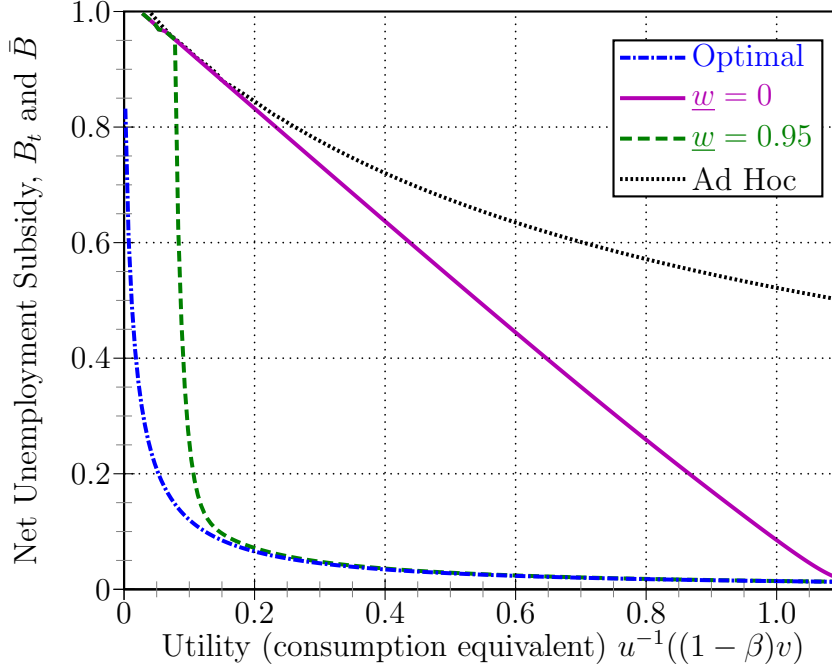


Figure 7: Net Unemployment Subsidy  $B_t = b_t + \frac{(1+r)\tau_t}{r} - \frac{\tau_{t+1}}{r}$  and  $\bar{B} = \bar{b} + \bar{\tau}$ .

Consider an economy very similar to this one but with the wage distribution  $\tilde{F}(w) = F(w)$  if  $w \geq \underline{w} \equiv 0.95$  and  $\tilde{F}(w) = 0$  otherwise, i.e., with a (small) mass point at 0.95. It turns out that the optimal reservation wage always exceeds 0.95 and this change has no effect on optimal unemployment insurance policy. But if in the original economy we had  $\bar{B} < .95$ , then the natural borrowing limit was  $-\bar{b}/r$  while in the modified economy it is  $-(\underline{w} - \bar{\tau})/r$ . Thus, this slight change in the wage distribution may lead to a significant increase in the borrowing limit, i.e., in the availability of liquidity.

The green dashed lines in Figure 6 and Figure 7 show how this matters. When  $\underline{w} > \bar{B}$ , unemployment subsidies no longer play a role in increasing a worker's liquidity. The unemployment subsidy turns into a pure insurance mechanism and is much lower than with  $\underline{w} = 0$ . In fact, the optimal subsidy is only slightly higher than the optimal time-varying unemployment subsidy at the same level of utility (dash-dot blue line), at least when utility is high. For example, at  $v = \frac{u(0.5)}{1-\beta}$ , the optimal time-varying subsidy is 0.0278, rising to 0.0283 during a ten-week unemployment spell. The optimal constant subsidy is 0.0288 if  $\underline{w} = 0.95$  but 0.540 if  $\underline{w} = 0$ . There is little need for time-varying unemployment subsidies when  $\underline{w}$  is high, and hence little additional cost of providing utility through a constant subsidy (Figure 6).

A slight modification in policy has an effect that is similar to this change in technology.

Suppose that after the worker draws a wage, she has the option of exiting the labor market and collecting  $\tilde{w} - \bar{\tau}$  thereafter. She accepts this option if her reservation wage  $\bar{w}$  falls below  $\tilde{w}$ .<sup>12</sup> When this happens, the cost is  $\frac{(1+r)(\tilde{w}-\bar{\tau})F(\tilde{w})}{r}$ , since  $\tilde{w}$  must be paid in all future periods.

Like the lower bound on the wage distribution, this option can substantially affect a worker's debt limit and her behavior even if she is extremely unlikely ever to exercise it. The only difference is that the policy involves a cost to the planner, while the alternative distribution does not. However, since the odds of getting a single wage draw below 0.95 are negligible, the odds of the worker ever accepting  $\tilde{w}$  and hence the cost of the policy are infinitesimal. This means that the cost and optimal unemployment subsidy under constant benefits are indistinguishable with  $\tilde{w} = 0.95$  or with  $\underline{w} = 0.95$ .

In summary, when the distribution of wages is such that workers have liquidity problems, optimal unemployment insurance is well-mimicked by a two-part policy: a subsidy to unemployment, which insures workers against the failure to find a good job; and measures to ensure that workers are able to smooth their consumption over unusually long sequences of bad wage draws. Insuring workers against the small probability of a very bad shock provides liquidity. Together the two policies mimic optimal unemployment insurance, which involves a nearly constant unemployment subsidy for a long period of time, followed by a sharp increase in the subsidy when workers are sufficiently poor (Figure 4).

An open question is how to interpret the finding that raising the lower bound on the wage distribution from 0 to 0.95 can have a significant effect on the constant benefit policy even if  $F(0.95) \approx 10^{-88}$ . In our view, it is a shortcoming of exogenous incomplete markets models that vanishingly small probability events can significantly affect borrowing. On the other hand, we view the simplicity and transparency of the exogenous incomplete market model as a virtue.

#### 4.4 Ad Hoc Borrowing Constraints

Although our analysis focuses on the natural borrowing limit, it is useful to note that policy can easily circumvent any tighter *ad hoc* limit. To be concrete, suppose borrowing is prohibited but that the natural borrowing limit is negative,  $\underline{a} < 0$ . Consider giving the worker a lump-sum transfer  $-(1+r)\underline{a}$  at the start of the initial period, while lowering her unemployment benefit to  $\bar{b} + r\underline{a}$  and raising her employment tax to  $\bar{\tau} - r\underline{a}$ . This simply

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<sup>12</sup>In our numerical examples, a worker only accepts  $\tilde{w}$  if doing so is the only way she can pay the interest on her debt.



changes the timing of payments, and is equivalent to providing the worker with a risk-free loan, but is an ideal instrument for circumventing any ad hoc borrowing constraint.

Optimal unemployment insurance, taken literally as a policy geared towards a hand-to-mouth consumer, is also a loan. It pays out the duration-dependent sequence  $b_t$  and collects taxes  $\tau_t$  (as in [Hopenhayn and Nicolini, 1997](#)). [Figure 4](#) shows that the net subsidy  $B_t$  may be much lower than consumption while unemployed, reflecting the accumulating employment tax liability over the jobless spell. Thus, an important component of the agency's gross transfers are not net present value transfers; the agency pays out early on and collects later, much as a loan.

For completeness, we also consider briefly the case where, for some unspecified and arbitrary reason, the insurance agency does not circumvent the ad hoc borrowing constraint. This may significantly affect both the cost and level of optimal constant benefits. To take an extreme case, suppose a worker has no assets and no ability to borrow, so she must consume her benefit each period she is unemployed. We compute the optimal constant benefit and taxes for this case. The black dotted line in [Figure 6](#) shows that this raises the cost of providing the worker with a particular level of utility by about three to six weeks of income, a substantial amount given that unemployment spells last for only ten weeks. The need to provide both insurance and consumption smoothing makes the optimal unemployment subsidy much higher, in excess of 0.5 over the usual range of utility ([Figure 7](#)).

## 4.5 Robustness

This section asks the extent to which our results depend on the wage distribution, in particular on the assumption that a worker finds a job in ten weeks on average. There are a few reasons to explore this assumption. First, our results indicate that constant unemployment benefits and constant employment taxes do almost as well as a fully optimal unemployment insurance policy. It could be that this result would go away if unemployment spells tended to last longer and therefore presented a bigger risk to individuals. Second, in many countries, notably much of Europe, unemployment duration is substantially longer, although this is at least in part a response to unemployment benefits that are high compared to workers' income prospects ([Ljungqvist and Sargent, 1998](#); [Blanchard and Wolfers, 2000](#)). And third, workers typically experience multiple spells of unemployment before locating a long-term job ([Hall, 1995](#)). Although modeling this explicitly would go beyond the scope of this paper, raising unemployment duration may capture some aspect of this longer job search process.

To explore this possibility, we choose  $\theta = 21.084$  so that the weekly job finding probability is about  $1 - F(\bar{w}) = 0.020$ , one-fifth of the earlier level.<sup>13</sup> This raises the unconditional standard deviation of wages, by a factor of five, to 0.027, which increases the option value of job search. We revisit our main conclusions under this alternative parameterization:

- *Under optimal unemployment insurance, the subsidy  $B_t$  rises slowly.* Suppose we start a worker with utility equal to a constant consumption of 1.2. The optimal subsidy is 0.080 and rises to 0.145 during the first 10.75 years of unemployment, during which time her utility falls in half, to a consumption equivalent of 0.6.
- *The optimal job finding rate changes slowly.* In the same experiment, it rises from 2.005 percent per week to 2.010 percent per week during the first 10.75 years of unemployment.
- *The CARA case provides a good approximation.* CARA would suggest an initial unemployment subsidy of 0.084, rising to 0.156 when absolute risk aversion doubles. The approximate and exact job finding probabilities are indistinguishable.
- If the lowest wage is high, here  $\underline{w} = 1.03$ , the optimal constant subsidy is similar to the optimal time-varying unemployment subsidy, 0.087 at the start of the unemployment spell and 0.175 once the worker's utility has fallen to 0.6. Moreover, the cost of constant benefits is small, approximately 0.0004 at the start and 0.017 for a worker with utility 0.6.
- If lowest wage is zero, the optimal constant unemployment subsidy is higher, 0.240 at the start of the unemployment spell and 0.698 for a worker with utility of 0.6. The cost of constant benefits is also higher, 0.745 and 5.56 at these two utility levels. This last number still only represents about a 1 percent increase in the cost of the unemployment insurance system.

We have also examined the robustness of our results to higher risk aversion by setting  $\sigma = 6$ . Optimal unemployment benefits are higher than the benchmark with  $\sigma = 1.5$ , as the CARA approximations would also suggest. For example, for a worker with utility equal to  $u(1)/(1 - \beta)$ , the optimal unemployment subsidy rises by a factor of four from 1.4 percent to 5.5 percent. Otherwise this change in preferences has little effect on our results.

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<sup>13</sup>Specifically, we set  $\theta$  to ensure that a risk-neutral worker without unemployment insurance would have  $1 - F(\bar{w}) = 0.020$ .

## 5 Conclusion

This paper characterizes optimal unemployment insurance in the [McCall \(1970\)](#) sequential search model. Our main result is that with CARA preferences, constant benefits coupled with free access to borrowing and lending of a riskless asset is optimal. In particular, it is inefficient to distort the worker’s savings behavior. With CRRA preferences, the exact optimality of constant benefits breaks down. We find that the optimal unemployment subsidy *rises* very slowly over time. However, we find little loss to a constant unemployment subsidy if workers are given free access to enough liquidity. This quantitative result is robust to the key parameters of the model.

There are important advantages to simple policies with free access to markets that our model does not capture. Free choice of savings decisions may be intrinsically valuable for philosophical reasons ([Friedman, 1962](#); [Feldstein, 2005](#)). Such policies may also be valuable on practical grounds because they are likely to be more robust to the numerous real-world considerations that are not included in our model.

This paper has not focused on the optimal level of unemployment subsidies, but rather on their optimal timing and on the desirability of allowing workers free access to the asset market. In the examples in this paper, the optimal unemployment subsidy turns out to be low unless the worker’s utility is also quite low. They are not, however, out of line with results in [Gruber \(1997\)](#), who computed benefits somewhere between 0 and 10 percent of wages to be optimal for the United States. Still, it is possible to construct examples where the optimal unemployment subsidy is much higher.<sup>14</sup> In ongoing separate work, we focus on the determinants of the level of benefits.

We have deliberately written a stark model of job search in order to keep the analysis relatively simple and focus on the forces that we believe are most important. Nevertheless, the model lends itself to a number of extensions, some of which we mention here. First, to keep our analysis comparable to [Shavell and Weiss \(1979\)](#) and [Hopenhayn and Nicolini \(1997\)](#), we have assumed that all jobs last forever. Relaxing this assumption permits an examination of how optimal unemployment subsidies depend on a worker’s entire labor market experience.<sup>15</sup> Second, we have focused on deterministic unemployment insurance

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<sup>14</sup>Suppose the wage draw can take on two values,  $w_1 < w_2$ . Moreover, suppose the first-best features the worker only accepting the high wage offer. Then setting the unemployment subsidy to  $w_2$  has the desired effect and moreover fully insures the unemployed; it attains the first-best and is thus optimal.

<sup>15</sup>[Wang and Williamson \(1996\)](#) and [Zhao \(2000\)](#) have explored optimal unemployment insurance, without borrowing and saving, in an economy with repeated spells of unemployment.

mechanisms. There are situations in which an employment lottery can reduce the cost of optimal unemployment insurance even if workers have CARA or CRRA preferences.<sup>16</sup> Future research should explore the potential gains from using employment lotteries and their interpretation. Third, we have assumed that wage draws are independent over time. By introducing some serial correlation, this model could potentially capture the idea that some people are much more likely to obtain a high wage job quickly, while others learn early on that a high wage is an unlikely event. Our results suggest that for these and other extensions, it will be important to evaluate the relative efficiency of simple benefit policies coupled with free access to the asset market and to distinguish between insuring workers against uncertainty in the duration of a jobless spell and ensuring their ability to smooth consumption while unemployed.

## Appendix

### A General Mechanisms

This section uses the revelation principle to set up the most general deterministic mechanism that an unemployment insurance agency might contemplate given the assumed asymmetry of information. We allow the worker to make reports on the privately observed wage and we allow taxes to vary during an employment spell. We show that neither of these capabilities is useful: the planner does just as well by offering unemployment benefits that depend on the duration of unemployment, and setting employment taxes that depend on the duration of the previous unemployment spell, not on employment tenure.

#### A.1 The Recursive Mechanism

For notational convenience, we present the general mechanism directly in its recursive form—this can be justified along the lines of [Spear and Srivastava \(1987\)](#). Our general mechanism involves the following steps:

1. The unemployed worker starts the period with some promise for expected lifetime utility  $v$ .

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<sup>16</sup>We are grateful to Daron Acemoglu for pointing out this possibility.

2. The worker then receives a wage offer  $w$  from the distribution  $F(w)$  and makes a report  $\hat{w}$  to the planner.
3. If the worker reports  $\hat{w} < \bar{w}$ , she rejects the job, receives unemployment benefit  $b(\hat{w})$ , and is promised continuation utility  $v'(\hat{w})$ , starting the next period in step 1, described above, with this value.
4. If the worker reports  $\hat{w} \geq \bar{w}$ , she accepts the job and pays a tax  $\tau(\hat{w}, n)$  in each subsequent period  $n = 1, 2, \dots$

## A.2 The Planner's Problem

The full planner's problem may be expressed recursively as follows:

$$C(v) = \min_{\bar{w}, \{b\}, \{v'\}, \{\tau\}} \int_{\underline{w}}^{\bar{w}} \left( b(w) + \frac{C(v'(w))}{1+r} \right) dF(w) + \int_{\bar{w}}^{\infty} \left( \sum_{n=0}^{\infty} (1+r)^{-n} \tau(w, n) \right) dF(w)$$

subject to the promise keeping constraint

$$v = \int_{\underline{w}}^{\bar{w}} (u(b(w)) + \beta v'(w)) dF(w) + \int_{\bar{w}}^{\infty} \left( \sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n)) \right) dF(w)$$

and a set of truth telling constraints for all  $w, \hat{w}$ :

$$\sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n)) \geq \sum_{n=0}^{\infty} \beta^n u(w - \tau(\hat{w}, n)) \quad w, \hat{w} \geq \bar{w} \quad (25)$$

$$\sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n)) \geq u(b(\hat{w})) + \beta v'(\hat{w}) \quad w \geq \bar{w} > \hat{w} \quad (26)$$

$$u(b(w)) + \beta v'(w) \geq \sum_{n=0}^{\infty} \beta^n u(w - \tau(\hat{w}, n)) \quad \hat{w} \geq \bar{w} > w \quad (27)$$

$$u(b(w)) + \beta v'(w) \geq u(b(\hat{w})) + \beta v'(\hat{w}) \quad \bar{w} > w, \hat{w} \quad (28)$$

We proceed to simplify the planner's problem.

**Lemma 2** (a) Suppose an optimum has the schedules  $b(w)$  and  $v'(w)$ , then the mechanism that replaces these with a constant schedule  $b = b(\hat{w})$  and  $v' = v'(\hat{w})$  for any  $\hat{w}$  (with a slight abuse of notation) is also optimal. (b) The incentive constraints (25)–(28) can be replaced

with the single equality condition,

$$u(b) + \beta v' = \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n)), \quad (29)$$

and constraint (25).

**Proof.** (a) Condition (28) implies that  $u(b(w)) + \beta v'(w) = \max_{w \leq \bar{w}} (u(b(w)) + \beta v'(w)) \equiv x$ , independent of  $w$ . From the planner's objective function we see that given  $x$  any  $(b(w), v'(w)) \in \arg \max_{b, v'} \{b + C(v')\}$  subject to  $u(b) + \beta v' = x$  is optimal. Consequently, one can select a solution that is independent of  $w$ .

(b) For constant  $b$  and  $v'$ , the constraint (28) is trivially satisfied. Since the right-hand side of constraint (27) is increasing in  $w$ , it is equivalent to

$$u(b) + \beta v' \geq \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\hat{w}, n))$$

for all  $\hat{w} \geq \bar{w}$ . Constraint (25) implies  $\hat{w} = \bar{w}$  maximizes the right-hand side of this inequality, so it reduces to

$$u(b) + \beta v' \geq \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n)). \quad (30)$$

Next, note that inequality (26) is now equivalent for all  $w \geq \bar{w}$

$$\sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n)) \geq u(b) + \beta v'.$$

If  $w > \bar{w}$ , then

$$\sum_{n=0}^{\infty} \beta^n u(w - \tau(w, n)) \geq \sum_{n=0}^{\infty} \beta^n u(w - \tau(\bar{w}, n)) > \sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n)),$$

where the first inequality uses (25) and the second uses monotonicity of the utility function. Therefore the preceding inequality is tightest when  $w = \bar{w}$ , so inequality (26) is equivalent to

$$\sum_{n=0}^{\infty} \beta^n u(\bar{w} - \tau(\bar{w}, n)) \geq u(b) + \beta v'. \quad (31)$$

Inequalities (30) and (31) hold if and only if equation (29) holds, completing the proof. ■

### A.3 Constant Absolute Risk Aversion

So far we have not made any assumptions about the period utility function  $u$  except that it is increasing. This section examines the implications of having constant absolute risk aversion preferences.

**Lemma 3** *With CARA utility, an optimum must feature the tax on the employed  $\tau(w, n)$  being independent of  $w$  and  $n$ .*

**Proof.** With exponential utility the  $w$  on both sides of (25) cancels, implying that the remaining term  $\sum_{n=0}^{\infty} \beta^n u(-\tau(w, n))$  must be some value independent of  $w$ . Let  $x$  denote this value. It follows that an optimum must solve the subproblem

$$\begin{aligned} \min_{\{\tau\}} & \left( - \sum_{n=0}^{\infty} \beta^n \tau(w, n) \right) \\ \text{subject to } x &= \sum_{n=0}^{\infty} \beta^n u(-\tau(w, n)). \end{aligned}$$

The first order condition for this problem reveals that an  $\tau(w, n)$  must be independent of  $(w, n)$ . ■

Lemmas 2 and 3 allow us to rewrite the planning problem as in (6)–(8). Private information prevents “employment insurance,” so the tax rate  $\tau$  is independent of the wage. With CARA preferences and jobs that last forever, the wage effectively acts as a permanent multiplicative taste shock. This ensures that all employed workers have the same preferences over transfer schemes, which makes it impossible to separate workers according to their actual wages. Since workers have concave utility, introducing variability in taxes is not efficient.

With non-CARA utility, workers with different wages rank tax schedules differently. In some cases, it may be possible to exploit these differences in rankings to separate workers according to their wage; see Prescott and Townsend (1984) for an example. If workers have decreasing absolute risk aversion (DARA), including CRRA preferences, those earning lower wages are more reluctant to accept intertemporal variability in taxes. One can therefore induce these workers to reveal their wage by giving them a choice between a time-varying employment tax with a low discounted cost and a constant tax with a high cost. High wage workers would opt for the time-varying schedule. This does not, however, reduce the planner’s cost of providing an unemployed worker with a given level of utility, since it transfers income from low wage to high wage workers. It is therefore not optimal.

## B Proof of Proposition 1

The worker's sequence problem implies that the value function must have the form  $V_u(a) = u(ra - \bar{\tau} + k_1)/(1 - \beta)$ , for some constant  $k_1$ . We determine this constant, and the rest of the solution along with it.

The maximization with respect to consumption in equation (2) delivers

$$c_u(a) = ra + (1 + r)^{-1}(r\bar{b} + k_1 - \bar{\tau}) \quad (32)$$

Substituting this back into the value function (2) gives

$$V_u(a) = \frac{1}{1 - \beta} \int_{\underline{w}}^{\infty} \max \left\{ u \left( ra + \frac{r\bar{b} + k_1 - \bar{\tau}}{1 + r} \right), u(ra + w - \bar{\tau}) \right\} dF(w). \quad (33)$$

This implies that the worker accepts all wages  $w$  exceeding a reservation wage  $\bar{w}$  defined by  $\bar{w} = (r\bar{B} + k_1)/(1 + r)$ , where again  $\bar{B} \equiv \bar{b} + \bar{\tau}$ . Use this and the identity  $u(c_1 + c_2) = -u(c_1)u(c_2)$  to write (33) as

$$V_u(a; \bar{b}, \bar{\tau}) = -\frac{u(ra - \bar{\tau})}{1 - \beta} \int_{\underline{w}}^{\infty} u(\max\{\bar{w}, w\}) dF(w) = \frac{u(ra - \bar{\tau} + \text{CE}(\bar{w}))}{1 - \beta}, \quad (34)$$

establishing equation (12). Substituting  $k_1 = \text{CE}(\bar{w})$  into equation (32), and  $\bar{w} = (r\bar{B} + k_1)/(1 + r)$  delivers equations (10) and (11).

## C Computing the Worker's Value Function

We are interested in solving equation (2). Ricardian equivalence implies that we can set the employment tax  $\bar{\tau}$  to zero without loss in generality. First assume  $\bar{B} \geq \underline{w}$ . It is numerically impossible to work with the natural borrowing limit  $\underline{a} = -\frac{\bar{B}}{r}$ , and so instead we impose an ad hoc borrowing constraint,  $\underline{a} > -\frac{\bar{B}}{r}$ . We consider values of  $\underline{a}$  arbitrarily close to  $-\frac{\bar{B}}{r}$  to test the sensitivity of the results to the exact borrowing limit. The results we report in the paper are not sensitive to this choice.

Take a worker with assets  $a$  very slightly greater than  $a_0 \equiv \underline{a}$ . A worker at this point will consume enough to reach the borrowing limit. This implies

$$V_u(a) = \max_{\bar{w}} \left( \left( u((1 + r)a + \bar{B} - a_0) + \beta V_u(a_0) \right) F(\bar{w}) + \int_{\bar{w}}^{\infty} \frac{u(ra + w)}{1 - \beta} dF(w) \right).$$



We can evaluate this directly at  $a = a_0$ :

$$V_u(a_0) = \max_{\bar{w}_0} \frac{u(ra_0 + \bar{B})F(\bar{w}_0) + \frac{1}{1-\beta} \int_{\bar{w}_0}^{\infty} u(ra_0 + w)dF(w)}{1 - \beta F(\bar{w}_0)}.$$

In addition, we can differentiate with respect to  $a$  using the envelope condition and evaluate at  $\underline{a}$  to get

$$V'_u(a_0) = (1+r) \left( u'(ra_0 + \bar{B})F(\bar{w}_0) + \int_{\bar{w}_0}^{\infty} u'(ra_0 + w)dF(w) \right),$$

where we use  $1 - \beta = \frac{r}{1+r}$  to simplify the expression.

Next, for any  $n \geq 1$ , we define recursively  $a_n$ ,  $V_u(a_n)$ , and  $V'_u(a_n)$ . Let  $a_n$  be the highest level of  $a$  such that a worker with assets  $a$  this period wants to have assets  $a_{n-1}$  next period. Equation (2) implies

$$u'((1+r)a_n + \bar{B} - a_{n-1}) = \beta V'_u(a_{n-1}),$$

which uniquely defines  $a_n$  since  $u'$  is decreasing and  $V'_u(a_{n-1})$  is known. It follows that the reservation wage solves

$$u((1+r)a_n + \bar{B} - a_{n-1}) + \beta V_u(a_{n-1}) = \frac{u(ra_n + \bar{w}_n)}{1 - \beta},$$

the value function solves

$$V_u(a_n) = \left( u((1+r)a_n + \bar{B} - a_{n-1}) + \beta V_u(a_{n-1}) \right) F(\bar{w}_n) + \int_{\bar{w}_n}^{\infty} \frac{u(ra_n + w)}{1 - \beta} dF(w),$$

and, using the envelope theorem again, its derivative solves

$$V'_u(a_n) = (1+r) \left( u'((1+r)a_n + \bar{B} - a_{n-1})F(\bar{w}_n) + \int_{\bar{w}_n}^{\infty} u'(ra_n + w)dF(w) \right).$$

For each level of assets  $a_n$ , we can also compute the expected discounted cost of the unemployment insurance system. At  $a_0$ ,

$$s_0 = \left( \bar{B} + \frac{s_0}{1+r} \right) F(\bar{w}_0) = \frac{(1+r)\bar{B}F(\bar{w}_0)}{1+r - F(\bar{w})},$$

while thereafter

$$s_n = \left( \bar{B} + \frac{s_{n-1}}{1+r} \right) F(\bar{w}_{n-1}).$$

The cost of providing the worker with utility  $V_u(a_n)$  is  $(1+r)a_n + s_n$ . We interpolate this to compute the cost at arbitrary utility levels and choose unemployment benefits  $\bar{B}$  to minimize cost.

Next consider  $\underline{w} > \bar{B}$ , so accepting any job is the worker's best option. A worker's assets cannot fall below  $-\frac{\underline{w}}{r}$ . In fact, if a worker who ends one period with assets  $a < -\frac{\underline{w}+r\bar{B}}{r(1+r)}$  remains unemployed, her assets the following period are no higher than  $a' = (1+r)a + \bar{B} < -\frac{\underline{w}}{r}$ , a violation of the borrowing constraint. Thus the natural borrowing limit is  $a_0 = \underline{a} = -\frac{\underline{w}+r\bar{B}}{r(1+r)}$ . Below this point a worker must accept any job.  $V_u(a_0)$  and  $V'_u(a_0)$  are slightly changed by this constraint:

$$V_u(a_0) = \frac{\int_{\underline{w}}^{\infty} u(ra_0 + w)dF(w)}{1 - \beta},$$

and

$$V'_u(a_0) = (1+r) \int_{\underline{w}}^{\infty} u'(ra_0 + w)dF(w).$$

The cost of a worker at the borrowing limit is zero, since she accepts any job. Given these initial conditions,  $a_n$ ,  $V_u(a_n)$ ,  $V'_u(a_n)$ , and  $s_n$  are defined using the same inductive formulae as before.

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