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A Theory of Factor Allocation and Plant Size

Thomas J. Holmes*

Federal Reserve Bank of Minneapolis,
University of Minnesota,
and National Bureau of Economic Research

Matthew F. Mitchell*

University of Iowa
and Federal Reserve Bank of Minneapolis

ABSTRACT

In this paper we develop a theory of how factors interact at the plant level. The theory has implications for (1) the micro foundations for capital-skill complementarity, (2) the relationship between factor allocation and plant size, and (3) the effects of trade and growth on the skill premium. The theory is consistent with certain facts about factor allocation and factor price changes in the 19th and 20th centuries.

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1 Introduction

In this paper we develop a theory of how factors interact at the plant level. The theory has implications for (1) the relationship between factor allocation and plant size, (2) the micro foundations for capital-skill complementarity, and (3) the effects of trade and growth on the skill premium. The theory is consistent with certain facts about factor allocation and factor price changes in the 19th and 20th centuries.

The main idea in our theory is that there is an analogy between the way capital relates to unskilled labor and the way unskilled labor relates to skilled labor. Capital can do relatively simple *mechanical* tasks that unskilled labor would otherwise do, but only if high setup costs are incurred. Analogously, unskilled labor can do complex *brain* tasks skilled labor would otherwise do, but only if high setup costs are incurred.

To explain, consider a simple mechanical task such as emptying the trash or moving a box from point A to point B. Unskilled labor has *general* ability to undertake such simple tasks. An unskilled worker hired just five minutes ago could first empty the trash and then move a box with virtually no training. It may be possible to obtain a machine to take out the trash, but this would in general require extensive setup costs, for example, construction of a conveyer belt that would have to be designed to fit a particular space. Moreover, we expect that a different machine would have to be obtained to move the box. Machines tend to be specific in tasks they can be used for, at least as compared to the general ability of the human body to undertake simple mechanical tasks. This continues to be true even for the now available computer-controlled machinery that is much more flexible than equipment from earlier years.

Next consider a complex task that might ordinarily be assigned to a skilled worker. It may be possible to *routinize* this task so that an unskilled worker can do it, but only by incurring setup costs. For example, suppose a software company needs employees to staff a helpline. The company could hire skilled computer experts who have general knowledge of computer problems. Alternatively, the company could invest in routinizing the tasks, training relatively unskilled workers to answer a narrow set of specific questions and developing a system for routing calls. There is a large literature on *de-skilling* through Taylorist principles, and this phenomenon was thought to be particularly important in the early 20th century as unskilled workers on assembly lines began to replace skilled artisans.[1]

To incorporate these ideas, we develop a model with the following features. To produce output at any plant, a variety of tasks needs to be performed. The firm must decide which inputs (capital, unskilled labor, or skilled labor) should do which tasks. Tasks vary in *complexity,* and more complicated tasks require more *setup costs*. Skilled workers, with their high level of general-purpose knowledge, have low setup costs. The setup costs of unskilled workers are higher, and the setup costs of capital are higher still. Thus capital can be thought of as an extreme form of unskilled labor.

In the optimal assignment of tasks, there is a partition. Skilled workers are assigned complex tasks that would require extensive setup costs if undertaken by unskilled workers or capital. Capital is assigned the relatively easy-to-master tasks such as those that involve the movement of objects. Unskilled labor is assigned the in-between tasks. Thus on one margin capital substitutes for unskilled labor, while on another margin unskilled labor substitutes for skilled labor.

Our first set of results concerns the relationship between factor allocation and plant size. As we discuss further in Section 2, in today's economy, employees of larger plants tend to be more skilled than employees of smaller plants—a positive size-skill relationship. Larger plants also are also more capital intensive—a positive size-capital relationship. These facts have led researchers to develop theories of why large plants might have higher quality workers. (See Oi and Idson (1999) for a survey.)

A century ago, the pattern was reversed. In the late 19th century and early 20th century, the size-skill relationship was negative, as we document in Section 2. The substitution of unskilled labor for skilled labor was a key characteristic of the mass production techniques developed by industrialists such as Henry Ford. Large mass production factories were loaded with unskilled workers, while small "craft" shops employed skilled artisans.

Historians know well that the size-skill relationship was negative in 1900. And labor economists know well that the size-skill relationship is positive today. But no previous

[1] See, for example, Brown and Philips (1986). There is a literature in sociology that emphasizes the de-skilling. See, in particular, Braverman (1974).

analysis has tried to address both facts at the same time like we do here. In our theory, the relationship can go either way. Larger plants tend to substitute capital for unskilled labor and unskilled labor for skilled labor, because the larger scale makes it more worthwhile to pay fixed costs to lower marginal costs. Thus the net effect of plant size on the skilled labor share is ambiguous. We are able to derive a simple condition determining the direction of the net effect. We also examine how the size-capital relationship is determined, and we connect this to the size-skill relationship: a positive size-skill relationship implies a positive size-capital relationship. This is the pattern we observe in the latter part of the 20th century.

Our second set of results concerns capital-skill complementarity. Previous empirical work, including Griliches (1969) and Krusell et al. (2000), has found that capital and skill are complements. In this literature the production process is a "black box." Constant elasticity of substitution production functions are assumed to hold, and elasticities of substitution are estimated. This paper provides micro foundations of the production process in which capital-skill complementarity is derived. We find that capital and unskilled labor tend to be similar in a sense that will be clarified in the paper.

Our third set of results concern the effects of market expansion and productivity growth on factor prices. These forces allow plants to expand in size to exploit economies of scale. Chandler (1990) shows how lower transportation costs enabled firms in the late 19th century to expand market areas and increase plant sizes. Section 2 shows that increases in output per plant have been the broad trend in the 20th century as well. In our model, increases in plant size driven by expansion of markets and growth have general equilibrium effects on factor prices. Increased scale makes it easier to substitute capital for unskilled labor (because capital has higher setup costs), and this tends to raise the skill premium. But higher scale also makes it easier to substitute higher-setup-cost unskilled labor for skilled labor, and this tends to reduce the skill premium. The net effect on the skill premium is ambiguous in general.

Our main result connects the cross-section relationship between plant size and factor mix with changes over time in the skill premium. It is intuitive that there should be a connection. If larger plants employ relatively more skilled workers and macroeconomic changes lead to an increase in average plant size, we might expect the relative demand for skilled labor to go up and the skill premium to increase. While this intuition is part of the story, it is incomplete, and our analysis clarifies the precise connection between these two issues. We show that if the size-skill relationship is negative, then the skill premium necessarily trends down. But the skill premium also trends down even if the size-skill relationship is flat. The skill premium trends up only if the size-skill relationship is sufficiently positive. The model has a bias toward a falling skill premium because expansion of scale diminishes the importance of fixed cost, which is skilled's advantage over unskilled.

Our results are consistent with the historical pattern. Goldin and Katz (2000) have documented that the time pattern of the skill premium over the 20th century is roughly a U-shape. Since the size-skill relationship was negative in the late 19th century, our theory predicts the skill premium should have been falling, consistent with what happened. Since the skill premium has risen in recent decades, the theory predicts that this size-skill relationship should have been positive in recent decades, consistent with what happened.

Our analysis of changes over time in the skill premium is closely related to previous work. Goldin and Katz (1998), Caselli (1999), Mobius (2000), and Mitchell (2001) all have models where changes in technology lead first to a reduction and then to an increase in the skill premium. What distinguishes our work from this set of papers is our attempt to connect changes in the skill premium to cross-section relationships between plants of varying size. Furthermore, we show how the observed U-shaped pattern of the skill premium can be generated in a model even when there is no technological change. Expansion of markets and capital deepening, forces that raise plant size, are sufficient to obtain this result.

We note that an expansion of markets in our model is the same thing as an increase in trade. The channel through which increased trade affects the skill premium in our model is very different from the channel in the standard model, which is based on Hecksher-Ohlin arguments. In our paper, we interpret trade as simply the merging of multiple, identical countries, so it is simply a scaling up of market size. As a result, trade has no effect on the skill premium through the conventional channel. Here, trade allows plants to enjoy scale economies, and as plants expand relative factor demands are affected. This is consistent with the plant level evidence from Berman, Bound, and Griliches (1994), who find that the increase in demand for skilled labor is within industries and not due to a reallocation across industries, as in Hecksher-Ohlin. Our analysis of the effect of trade between similar countries is similar in spirit to that of Acemoglu (2003), who also identifies a channel (in his case endogenous technological change) through which increases in market size affect the skill premium.

2 Supporting Evidence

This section provides supporting evidence for assertions made in the introduction.

2.1 The Size-Skill Relationship

Suppose for now that an individual's pay can be used as a proxy for his or her skill. It is a well-known and robust fact that in today's economy larger plants have higher-paid workers (Brown and Medoff (1989)). It is not as well appreciated that the size-pay relationship has changed over time. Using micro data from the Census of Manufactures over the 1963-1986 period, Davis and Haltiwanger (1991) show there was a sharp upward trend in the relationship over this period. Idson (2001) also reports recent increases. Atack, Bateman, and Margo (2000) analyze Census micro data from the late 19th century and report a fundamentally different relationship between size and pay. In a simple linear regression of log wage on log size, they find a *negative* relationship. In a regression with a quadratic term, they find an inverted U-shaped relationship that is first increasing and then decreasing.

These results are illustrated in Table 1. We use employment to measure plant size because the Census Bureau has published data in this format in a consistent way over a long period, enabling us to examine the long-run trend. To construct the table, we first calculated average pay for each plant-size category by dividing total payroll in the category by total employment. For example, in the 1997 census, average pay calculated this way for plants with 2,500 or more employees equaled \$52,100. We then normalized by dividing through by average pay in the entire manufacturing sector. The mean in 1997 was \$33,900, so the normalized wage in 1997 in the 2,500+ size category is 1.54=52.1/33.9, which is the figure reported in the table. Thus average pay in the largest size category is 54 percent higher than the average wage, a substantial premium.

Going from left to right in the table, we move forward in time. Observe that for the largest two size classes, the premium increases monotonically with time. Thus the table replicates Davis and Haltiwanger's (1991) previous findings that apply for the time period beginning with the 1960s. But note there is also a substantial increase from 1947 to 1967 of 1.13 to 1.26. We conclude that the upward trend in the size-pay premium began well before the 1960s.

The Census Bureau did not publish payroll by establishment size before 1947, so it is not possible to use Census tabulations to extend the table before that year. However, if we go back to 1880, we can use the 5 percent sample of the Census micro data collected by Atack and Bateman (1999). These data were obtained from raw manuscript data that are publicly available.[2] These are the data used by Atack, Bateman, and Margo (2000) discussed above. With these sample data, it is possible to estimate payroll and employment by size class and extend the table to 1880, only we need to change the size groupings.[3] Plants in 1880 were dramatically smaller than today or in 1947, as the median in 1880 had only three employees. This fact, combined with the fact that we have only a 5 percent sample, means there are very few observations in some of the cells in the original groupings. To deal with this fact, we have aggregated the larger size groupings and disaggregated the smallest grouping. With these groupings, we have over 200 observations in each cell.[4]

The results in Table 2 illustrate the inverted U-shaped pattern reported by Atack, Bateman, and Margo (2000). Pay rises with size for very small plants, but then flattens out over the range from 20 to 100 to 110 percent of the average wage. Pay then falls to 5 percent below the average pay, a drop of 15 percent. It is worth noting that if we were to look at the larger plants for which we have relatively few observations, the drop is even larger. For the 50 plants with more than 250 employees, the average pay is 14 percent below the average.

[2] The raw Census data become publicly available 72 years after they are collected.

[3] Our aim here is to construct the variables for 1880 to make them as consistent as possible with the way the comparable variables are constructed for the later years. The mean is employment weighted, just as it is in the years. The mean also uses the sampling weights. (Atack and Bateman oversample small states.) We use average annual employment in the denominator. Atack and Bateman make a correction for the number of months the plant was operated. If we make this correction, it does not qualitatively affect the results.

[4] The cell counts are 11,750, 770, 263, and 209.

For the 15 plants with more than 500 employees, the average pay is 20 percent below the average.

Our interest here is in the size-skill relationship. Pay may depend upon other factors besides skill, so now we consider other measures of skill. Brown and Medoff (1989) and Troske (1999) both find that adding controls for worker quality such as education reduces the coefficient on establishment size in a wage regression. This indicates plant size is correlated with observable worker quality measures. Abowd and Kramarz (1999) use matched worker-firm data and show that plant size is positively correlated with measures of worker quality. All of these studies use data from the 1970s or later.

In empirical work, it is common to classify production workers as unskilled workers and nonproduction workers as skilled workers. Table 3 presents nonproduction worker share by plant size, normalized by average nonproduction worker share. In 1997, the share for plants in the 2,500+ category was .405 while the average share was .282, so the normalized share is $1.44 = .405/.282$. In 1997 the share in the largest size class was substantially larger than the average, but otherwise the relationship is relatively flat. In 1987, 1977, and 1967, the relationship is steep in the second highest class as well as the top class. There is a clear pattern that this relationship has steepened over time.

In the 1880 data, there is no classification by production/nonproduction worker status. Employment is divided up by men, women, and children. Since women mainly worked as production workers during this time period and since this is certainly true about children, we expect the women/children share of employment to be positively correlated with the production worker share. Table 2 shows that this share increases sharply with plant size, with the largest plant having almost half of its workers being women and children.

Unlike the censuses before and after it, the 1890 census collected information that was directly related to skills. Workers were classified into five categories based on the type of work that they did. Three of these categories can be regarded as *skilled* work. These include officers, clerks, and skilled workers. The other two categories, unskilled laborers and pieceworkers, can be classified as unskilled. Unfortunately, micro data from the 1890 Census is unavailable. However, state level data are published for nine important industries.[5] For

[5] The tabulations are in Table 8 of U.S. Census (1895). The industries are Agricultural Implements, Boots

each state we calculated the percentage of male workers in the state that were skilled workers as well as the average employment size for plants in the state. Figure 1 plots these data points for the carriage and wagon industry as well as a regression line. There is a strong negative relationship. The R-squared of the regression is .54. An analogous pattern occurs in the other industries. In eight out of nine industries, the regression line is negative, and in the one case where it is not (the paper industry) the slope is not statistically significant. If we aggregate the data in a regression in logs with state and industry fixed effects, the estimated elasticity of skill share with respect to average size is $-.16$ with a standard error of .02. If we use average sales instead of average employment as a measure of size, the elasticity falls somewhat to $-.11$ (standard error .02), but still continues to be quite high, especially considering the large observed variations in plant size. Given the clean definition of skill used here, we regard this as our strongest evidence that the size-skill relation was negative in the late 19th century.

2.2 The Size-Capital Relationship

We turn now to the issue of capital intensity. It is a widely held view that large plants are more capital-intensive than small plants. Table 4 provides evidence on this relationship. The Census tabulations by establishment size do not report information about the *stock* of capital, but they do report information about the flow (that is, new investment). We constructed Table 4 by first dividing new capital expenditures by total employment to obtain a measure of capital intensity and then normalizing this by the average capital intensity in manufacturing. In 1997 this measure rises sharply with size. When we look at the other years, it is notable that this series is not as smooth as the series in Table 1 and Table 3. Nonetheless, there is a clear pattern that the size-capital relationship has steepened in recent years.

Table 2 illustrates the size-capital relationship using the micro data from the Atack-Bateman sample. Here capital is measured by book value, and the normalization is as before. The largest size class has the lowest ratio of capital to workers.

and Shoes, Carriages and Wagons, Dairy, Flour, Leather, Paper, Meatpacking, and Slaughtering. These nine industries accounted for 10 percent of 1890 employment.

2.3 Increasing Plant Size

Chandler (1990) shows how expansion of markets in the late 19th century significantly increased plant sizes. Here we present evidence that plant sizes have increased through the 20th century.

Table 5 presents real value added per establishment for manufacturing plants in the United States in millions of 2000 dollars for various years.[6] In 1954 the mean manufacturing plant had a value added of $2 million. By 1997, this had increased to $5.3 million, in real terms. There is "upsizing" in the number of widgets that are coming out of the factory door. As one might expect, if we look at particular industries, we can find cases where the pattern is flat or even declining. But the overall pattern for most industries is an increase in output per plant. Evidently, "effective" inputs have been increasing at the average manufacturing plant, and this is the concept of "size" that is relevant for our analysis.

While our theory is perhaps most applicable to the manufacturing sector, it is potentially applicable to other sectors of the economy as well. Table 6 presents mean establishment employment size for the broad sectors of the economy in the postwar period. The table illustrates the decline in mean manufacturing employment. Over the 1953 to 1997 period, mean employment size in manufacturing fell 22 percent, and there was a fall in mining and transportation of similar magnitude. But consider the retail and service sectors. These are huge sectors, together accounting for over half of total employment in 1997. Mean employment size in these two sectors grew at extremely high rates, 88 and 156 percent, respectively, and this trend continues in the more recent data. When we also take into account the increases in productivity in these sectors, we can safely conclude that increases in the quantity of effective inputs allocated at the establishment level have been the typical case throughout the economy.

[6] The value added and establishment count data are from the Census of Manufacturers. The price deflator are the producer price index. We use industry-level deflators for all years except 1923, for which we use the broader industrial commodity deflator.

2.4 The Skill Premium

It is now well known that the skill premium has increased in recent decades (Katz and Murphy (1992)). There is also a consensus among economic historians that the skill premium fell during the late 19th century. Figure 2 shows the time series for the skill premium, measured as the return to one year of college, over the 20th century. The data are from Goldin and Katz (2000).[7]

We report the return to college because of the availability of a century-long series for it, but the primary features of the series are consistent with other measures of the return to skill and pay dispersion. For instance, the 90-10 or 80-20 wage ratio, looked at from the perspective of studies on various parts of the century, are similar: a fall in premium in the first half of the century, followed by an increase in the last quarter of the century. The return to high school, also reported in Goldin and Katz (2000) for the century, moves in a very similar pattern.

3 The Model

A fundamental component of the model is the existence of setup costs. Given the scale economies, the firms in our model have market power. In particular, firms sell differentiated products. Some firms have more desirable products than other firms, introducing variation in size across firms.

All firms face the same production technology. Each firm does a continuum of tasks that are ranked by the degree of complexity. More complex tasks require more setup. There are three inputs—capital, unskilled labor, and skilled labor—that are in fixed supply to the economy. These inputs vary in the setup cost required to undertake any particular task.

[7] The premium is calculated for young men, by comparing the wages of those completing exactly 12 years of schooling to those completing exactly 16, and dividing by 4.

3.1 Preferences and Technology

A representative household consumes a continuum of differentiated products indexed by $u \in [0, m]$. The differentiated goods are aggregated to a composite good through a CES production function with elasticity of substitution σ:

$$Q = \left[\int_0^m \theta(u) q(u)^{\frac{\sigma-1}{\sigma}} du \right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

Observe that the differentiated goods vary in the weight $\theta(u) > 0$ that they enter the CES function. Assume that higher u goods have higher weight, $\theta'(u) \geq 0$. With this specification, a consumer will buy more of the higher u good than a lower u good if the two goods have the same price. Normalize the scaling so that $\theta(0) = 1$. Assume $\sigma < 1$, so firms face inelastic demand. We make this assumption for convenience, because it simplifies the pricing formulas (with inelastic demand firms limit price).[8]

The technology for producing each differentiated product is the same. There is a continuum of tasks indexed by z on the unit interval $z \in [0, 1]$. Let $x(z)$ denote the level of activity of task z. The quantity of differentiated product produced given $x(\cdot)$ is CES with elasticity of substitution ω,

$$\begin{aligned} q &= \left(\int_0^1 x(z)^{\frac{\omega-1}{\omega}} dz \right)^{\frac{\omega}{\omega-1}}, \text{ if } \omega \neq 1 & (2) \\ &= \exp\left(\int_0^1 \ln x(z) dz \right), \text{ if } \omega = 1. & (3) \end{aligned}$$

There are three factors of production indexed by $j \in \{1, 2, 3\}$ in increasing order of skill. Capital is $j = 1$, unskilled labor is $j = 2$, and skilled labor is $j = 3$. The total endowment of factor j in the economy is $\overline{X}_j$.

Undertaking each task entails a variable cost component and a fixed cost component. The variable cost is constant. Assume all three input types are equally efficient at the variable cost component in that one unit of the input is needed to undertake one unit of the task. Let $x_j(z)$ denote the use of factor j at task z. Then

$$x(z) = \sum_{j=1}^{3} x_j(z)$$

[8] We have also worked out the case where $\sigma > 1$ and obtained similar results.

is the total amount of task undertaken.

The three factors differ in setup cost. Assume that type 3 has zero setup cost. Let $\phi_2(z)$ be the setup cost for type 2 and $\phi_2(z) + \phi_1(z)$ be the setup cost of type 1, where $\phi_j(z) > 0$ for $z > 0$. Thus setting up factor 1 requires all the fixed costs needed to set up factor 2 plus additional fixed costs. Assume $\phi_j(z)$ is continuously differentiable and that for $j \in \{1, 2\}$ and $z > 0$,

$$\phi_j'(z) > 0. \tag{4}$$

Thus higher z goods require more setup. Higher z tasks are more complex.

The idea that capital has high fixed costs is not controversial; indeed, it is common to consider fixed costs for capital but none for labor. The fixed cost of capital has a natural interpretation in this model. In order to be able to do a particular task, it is essential that capital be designed for the task. A screwdriver is well designed for use on screws, but is not very effective on nails. The high fixed cost reflects the specificity of capital.

While capital typically has high fixed costs, we recognize that some capital is very scalable. A small plant might use a few computers and a large plant many, and the cost of this hardware might be proportional to output. However, many types of capital are very specialized. Banks use similar computers but have specialized software, despite the seemingly similar tasks that various banks undertake. Car plants use specialized equipment that is produced specifically for the setup of the plant.

Less standard is the treatment of fixed costs across different skill levels of labor. What we have in mind is that skills give workers general knowledge that allows them to move between tasks easily. Unskilled workers must be taught to do each task, at relatively high costs. Skilled workers can figure out how to do tasks without much difficulty. Another way to interpret the assumption is that it implies that unskilled workers are relatively efficient, compared to skilled workers, when they have very specialized jobs. The routinization of jobs on the assembly line allowed unskilled workers to be engaged where skilled workers had been necessary. The narrow scope of each job meant that even unskilled workers could pick up the necessary understanding to do the job properly.

Our analysis will depend heavily on an elasticity concept. Define the setup cost elasticity for j to be

$$\eta_j(z) \equiv \frac{\phi_j'(z)(1-z)}{\phi_j(z)}.$$

This elasticity relates the percentage change in setup costs to the percentage change in the $1-z$ tasks above z. If the setup cost takes the following functional form,

$$\phi_j(z) = \alpha_j(1-z)^{-\theta_j}, \tag{5}$$

then setup cost elasticity is constant, $\eta_j(z) = \theta_j$. We will be interested in cases of both constant and nonconstant setup cost elasticity.

3.2 The Cost Minimization Problem

Before defining equilibrium, it is useful to study the cost minimization problem of a firm. Let the numeraire be the composite good, and let (w_1, w_2, w_3) be the vector of input prices. Let w_j denote the price of a unit of factor j in terms of the numeraire. Since factors have identical productivity in the variable component of each task, but higher j factors have uniformly lower setup costs, it must be the case that, in any equilibrium, $w_1 < w_2 < w_3$.

Consider the cost minimization problem of a firm producing q units of output. The firm must choose how much of each task z to undertake and which factor to employ at this task. (Because of setup costs, each task is assigned to only one factor). Since the factors are equally productive at the variable component but higher j have higher pay, higher j are more costly in the variable component. But higher j have lower fixed costs since $\phi_j(z) > 0$, so there is a trade-off. Since total setup increases in z, $\phi_j'(z) \geq 0$, it is immediate that the optimal assignment of tasks will consist of a pair of cutoff rules (z_1, z_2) such that factor 1 is assigned $z < z_1$, factor 2 is assigned $z \in (z_1, z_2)$, and factor 3 is assigned $z > z_2$. Within each range, the intensity is constant. Let x_j denote the intensity of factor j, in the range where it is used.

It is useful to decompose the cost minimization problem into two parts. The first part takes as given that the firm uses cutoffs (z_1, z_2) and determines the optimal mix across tasks. Recall that aside from the setup cost, the production function for the differentiated product is the constant returns CES form (2). Fixing (z_1, z_2) and given constant returns conditional on these cutoffs, the cost minimizing input mix does not depend upon q. Let $\tilde{x}_j$ be the cost minimizing level at which to operate those tasks assigned to factor j to produce a *single* unit of output. The demand $\tilde{x}_j$ is implicitly a function of the pay and the cutoffs. The per unit input demands satisfy the following problem:

$$\tilde{c}(z_1, z_2) \equiv \min_{\{(x_1,x_2,x_3) \text{ such that } q=1\}} \left[z_1 x_1 w_1 + (z_2 - z_1)\, x_2 w_2 + (1 - z_2)\, x_3 w_3\right]. \tag{6}$$

The solution for this CES case is standard. The ratio of task intensities satisfies

$$\frac{\tilde{x}_j}{\tilde{x}_k} = \left(\frac{w_j}{w_k}\right)^{-\omega} \tag{7}$$

and the minimized cost is

$$\begin{aligned} \tilde{c}(z_1, z_2) &= \left(z_1 w_1^{1-\omega} + (z_2 - z_1)\, w_2^{1-\omega} + (1 - z_2)\, w_3^{1-\omega}\right)^{\frac{1}{1-\omega}}, \text{ if } \omega \neq 1 \\ &= w_1^{z_1} w_2^{z_2 - z_1} w_3^{1-z_2}, \text{ if } \omega = 1. \end{aligned} \tag{8}$$

Note that the minimized cost per unit is written as a function of the cutoffs but it also depends implicitly on the pay as well. Since $w_1 < w_2 < w_3$, it is immediate that

$$\frac{\partial \tilde{c}(z_1, z_2)}{\partial z_j} < 0$$

for $j \in \{1, 2\}$. Increasing the cutoff z_j replaces input $j+1$ with input j, which is less costly and equally productive in the variable component of the task. Thus increasing the cutoff lowers the variable cost per unit.

The second part of the cost minimization problem is to choose the cutoffs z. Given a choice of cutoffs, the total expenditure on setup costs across all tasks is

$$f(z_1, z_2) = \left[\int_0^{z_1} \phi_1(z) dz + \int_0^{z_2} \phi_2(z) dz\right].$$

Observe that the firm pays $\phi_2(z)$ on all tasks done by either 1 or 2. In addition, it must pay $\phi_1(z)$ on all tasks done by 1. Given output level q, the firm chooses z_1 and z_2 to minimize the sum of variable costs plus setup costs:

$$c(q) = \min_{z_1, z_2} q\tilde{c}(z_1, z_2) + f(z_1, z_2). \tag{9}$$

This is a strictly convex problem since $\tilde{c}(z_1, z_2)$ is convex and $f(z_1, z_2)$ is strictly convex under assumption (4). The first-order condition for the choice of z_j is

$$q\frac{\partial \tilde{c}(z_1, z_2)}{\partial z_j} + \phi_j(z_j) = 0. \tag{10}$$

The first term is the reduction in variable cost from increasing z_j, weighted by output q. The second term $\phi_j(z_j)$ is the marginal increment in total fixed cost f from shifting task z_j away from factor $j+1$ to factor j.

For later use, we rewrite the first-order necessary condition for the choice of cutoff z_j. In the Cobb-Douglas case ($\omega = 0$), the cost function can be written as

$$\tilde{c} = w_1^{z_1} w_2^{z_2 - z_1} w_3^{1-z_2} = e^{z_1 \ln w_1 + (z_2 - z_1)\ln w_2 + (1-z_2)\ln w_3}.$$

So the slope is

$$\frac{\partial \tilde{c}}{\partial z_j} = \tilde{c}\left[\ln w_j - \ln w_{j+1}\right]. \tag{11}$$

In the Cobb-Douglas case, total expenditure on each task is the same. With a unit measure of tasks, total expenditure overall must equal expenditure on any individual task. In particular, it equals that on a task assigned to factor 3,

$$q\tilde{c} = w_3 x_3. \tag{12}$$

Substituting (11) and (12) into the first-order condition (10) for the choice of z_j yields

$$w_3 x_3 \left[\ln w_j - \ln w_{j+1}\right] + \phi_j(z_j) = 0. \tag{13}$$

Using an analogous derivation for the general ω case, we can rewrite the first-order necessary condition as[9]

$$\frac{1}{1-\omega} w_3^{\omega} x_3 \left(w_j^{1-\omega} - w_{j+1}^{1-\omega}\right) + \phi_j(z_j) = 0. \tag{14}$$

3.3 Equilibrium

Before discussing equilibrium, we first note a consequence of our assumption that for factor 3 (skilled labor), there is no setup cost for any task. If one unit of skilled labor were equally divided across all the tasks, one unit of output would result. With no setup costs, producing output in this way is constant returns to scale, and the cost per unit of output is w_3. We assume that this constant returns avenue for obtaining output is freely available to consumers.

[9] This derivation, contained in the Appendix, uses the marginal rate of substitution condition to write x_1 and x_2 as functions of w_3 and x_3.

Now consider the behavior of producers. There is a single producer of each differentiated good u. A producer cannot charge consumers a price greater than w_3 since consumers would use the constant returns alternative just discussed to obtain the product at a cost of w_3 per unit. By the assumption that $\sigma < 1$, demand is inelastic for prices below w_3. Hence, it is immediate that producers will set a limit price up to the consumer's reservation price of w_3; that is, $p(u) = w_3$ for each differentiated product.

We exploit this structure of a constant limit price across all products to simplify our definition of an equilibrium. For simplicity of the definition, define $n_j(u)$ to be the number of tasks that the producer of product u assigns factor j:

$$\begin{aligned} n_1(u) &\equiv z_1(u) \\ n_2(u) &\equiv z_2(u) - z_1(u) \\ n_3(u) &\equiv 1 - z_2(u). \end{aligned}$$

Definition 1 *An equilibrium is a list of functions* $(p(u), q(u), z_j(u),\ x_j(u))$ *and factor prices* w_j *such that the following conditions hold.*

(1) *Limit Pricing*: $p(u) = w_3$.

(2) *Marginal Rate of Substitution Condition*: $q(u) = \theta(u)^{\sigma} q(0)$.

(3) *Cost Minimization:*

$$(z_1(u), z_2(u)) = \arg\min_{z_1,z_2} q(u)\tilde{c}(z_1, z_2, w_1, w_2, w_3) + f(z_1, z_2)$$
$$x_j(u) = q(u)\tilde{x}_j(z_1(u), z_2(u), w_1, w_2, w_3).$$

(4) *Differentiated Goods Market Clearing:*

$$\left(n_2(u)\left(x_1(u)\right)^{\frac{\omega-1}{\omega}} + n_2(u)\left(x_2(u)\right)^{\frac{\omega-1}{\omega}} + n_3(u)\left(x_3(u)\right)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}} = q(u).$$

(5) *Factor Market Clearing:*

$$\textstyle\int_0^m n_j(u)x_j(u)du = \bar{X}_j, \text{ for each } j.$$

(6) *Household's Budget Constraint:*

$$Q - \int_0^m f(z_1(u), z_2(u))du = \bar{X}_1 w_1 + \bar{X}_2 w_2 + \bar{X}_3 w_3 + \Pi.$$

Condition (2) follows from consumer utility maximization. It is the marginal rate of substitution condition between good u and good 0. (Recall $\theta(0) = 1$ and that prices $p(u)$ are constant for all u.) Since it is optimal for firms to set the limit price, the analysis of the firm's problem reduces to minimizing the cost of producing the quantity demanded at the limit price. This is condition (3). Conditions (4), (5), and (6) are market clearing conditions. The left-hand side of the household's budget constraint (6) reflects the fact that households consume all of the final good output (Q) except that used in the production of fixed costs. Profits from the differentiated goods firms are denoted Π.

It is easy to derive an explicit formula for the differentiated product price p (and also w_3 since $p = w_3$). Let $\tilde{q}(u)$ be the cost minimizing quantity of differentiated good u required to produce a *single* unit of the composite. From the marginal rate of substitution condition, we have $\tilde{q}(u) = \theta(u)^{\sigma}\tilde{q}(0)$. From the composite production function,

$$\begin{aligned} 1 &= \left[\int_0^m \theta(u)\tilde{q}(u)^{\frac{\sigma-1}{\sigma}}du\right]^{\frac{\sigma}{\sigma-1}} \\ &= \tilde{q}(0)\left[\int_0^m \theta(u)^{\sigma}du\right]^{\frac{\sigma}{\sigma-1}}. \end{aligned}$$

Thus,

$$\tilde{q}(u) = \theta(u)^{\sigma}\left[\int_0^m \theta(u)^{\sigma}du\right]^{-\frac{\sigma}{\sigma-1}}.$$

Since the composite is the numeraire, the total price of this cost minimizing bundle must equal one,

$$1 = p\int_0^m \tilde{q}(u)du = p\left[\int_0^m \theta(u)^{\sigma}du\right]^{-\frac{1}{\sigma-1}}.$$

We can then solve for the equilibrium differentiated product price p,

$$p = \left[\int_0^m \theta(u)^{\sigma}du\right]^{\frac{1}{\sigma-1}}. \tag{15}$$

We next present our result for the existence of an equilibrium. Here we restrict attention to the case where there are a finite number of different product types.

Proposition 1 *Suppose the set $\{\theta(u), u \in [0, m]\}$ is finite. An equilibrium exists.*

Proof. See the Appendix. ■

4 Factor Allocation and Plant Size

In this section we consider the relationship between plant size and factor allocation. In this economy, more desirable goods (higher u) have higher q. All firms in the economy face the same wages and have access to the same technology. So in order to study how factor allocation depends upon plant size, we need to study how the cost-minimizing factor demands vary with q.

Our first step is to determine how the cutoffs z_1 and z_2 vary with plant size. The two cutoffs solve the two first-order necessary conditions,

$$\begin{aligned} q\frac{\partial \tilde{c}(z_1, z_2)}{\partial z_1} + \phi_1(z_1) &= 0 \\ q\frac{\partial \tilde{c}(z_1, z_2)}{\partial z_2} + \phi_2(z_2) &= 0. \end{aligned} \tag{16}$$

Our result is the following proposition.

Proposition 2 *In the solution to the cost minimization problem (9), z_1 and z_2 both increase in q.*

Proof. Observe from (11) for $\omega = 1$ and from (29) in the Appendix for $\omega \neq 1$ that

$$\frac{\partial \tilde{c}}{\partial z_2}\frac{\partial^2 \tilde{c}}{\partial z_1 \partial z_2} - \frac{\partial \tilde{c}}{\partial z_1}\frac{\partial^2 \tilde{c}}{\partial z_2 \partial z_2} = 0. \tag{17}$$

Totally differentiating the two first-order conditions (16) for z_1 and z_2, using Cramer's rule, and (17) yields

$$\frac{\partial z_1}{\partial q} = -\frac{1}{|H|}\frac{\partial \tilde{c}}{\partial z_1}\phi_2' = \frac{1}{|H|}\frac{1}{q}\phi_1\phi_2' > 0, \tag{18}$$

where H is the Hessian, $|H| > 0$, and where the first-order necessary condition is used to substitute in ϕ_1/q. Analogously,

$$\frac{\partial z_2}{\partial q} = \frac{1}{|H|}\frac{1}{q}\phi_2\phi_1' > 0. \tag{19}$$

■

An increase in the target quantity q places more weight on the benefit of cost reduction for increasing the cutoffs. So it is intuitive that a higher q would increase the optimal cutoffs.

With greater economies of scale, a larger firm substitutes capital for unskilled labor and unskilled labor for skilled labor.

Define the *size-skill* relation as the ratio of the demand for skilled and unskilled workers as a function of plant size:

$$\begin{aligned} s(q) &\equiv \frac{X_3(q)}{X_2(q)} = \frac{(1 - z_2(q))\, x_3(q)}{(z_2(q) - z_1(q))\, x_2(q)} \\ &= \frac{1 - z_2(q)}{z_2(q) - z_1(q)} \left(\frac{w_3}{w_2}\right)^{-\omega}. \end{aligned}$$

Observe that the ratio of intensities $\frac{x_3}{x_2}$ is the same for firms of different sizes because this depends upon the ratio of factor prices through (7), the same for all firms. Thus to understand how the skill ratio varies with q, we need only look at the behavior of the ratio of numbers of tasks assigned.

It is immediate that there are two conflicting forces at work. Higher q raises z_1, so capital replaces unskilled workers. It raises z_2, so unskilled workers replace skilled. The net effect on the size-skill relationship is ambiguous. It turns out to depend upon a simple comparison of the elasticity of setup costs, as demonstrated in the following proposition.

Proposition 3 *The slope $s'(q)$ of the size-skill relationship is positive, zero, or negative as $\eta_2(q)$ is greater than, equal to, or less than $\eta_1(q)$.*

Proof. The slope $s\prime(q)$ has the sign of

$$\begin{aligned} &-\frac{dz_2}{dq}(z_2 - z_1) - \left(\frac{dz_2}{dq} - \frac{dz_1}{dq}\right)(1 - z_2) \\ =\ &\frac{dz_1}{dq}(1 - z_2) - \frac{dz_2}{dq}(1 - z_1). \end{aligned}$$

Substituting in (18) and (19) and multiplying by $q|H|$, the slope $s\prime(q)$ has the sign of

$$\phi_1\phi_2'(1 - z_2) - \phi_2\phi_1'(1 - z_1)$$

or, dividing by $\phi_1\phi_2$,

$$\eta_2 - \eta_1.$$

■

It is intuitive that the relative setup cost elasticities should play a crucial role. If η_1 is small relative to η_2, it is relatively cheaper to shift the z_1 margin (capital replacing unskilled labor) than the z_2 margin (unskilled labor replacing skilled) so the size-skill relationship increases. Note that our model does not have an unambiguous prediction for the size-skill relationship, which gives it a chance to replicate the experience of the changes from the late 19th century to the late 20th century.

Next define the *size-capital relationship* as

$$\begin{aligned} k(q) &\equiv \frac{X_1}{X_2 + X_3} = \frac{x_1 z_1}{x_2 (z_2 - z_1) + x_3(1 - z_2)} \\ &= \frac{z_1}{\left(\frac{w_2}{w_1}\right)^{-\omega} (z_2 - z_1) + \left(\frac{w_3}{w_1}\right)^{-\omega} (1 - z_2)}, \end{aligned}$$

where X_j, x_j, and z_j are all implicitly functions of q. The relationship specifies how the ratio of capital to total labor (unskilled plus skilled) varies with plant size. Again there are two offsetting effects. The measure of tasks z_1 assigned to capital increases with plant size, and this tends to increase the ratio k. But the increase in the z_2 cutoff tends to decrease the ratio k. Unskilled labor is cheap compared to skilled labor, $w_2 < w_3$, so tasks assigned to unskilled labor are operated at a relatively high intensity, $x_2 > x_2$. As z_2 is increased and tasks are transferred from skilled to unskilled labor, this factor increases the denominator in k.

If it is difficult to substitute unskilled for skilled (η_2 large), then z_2 won't increase much, and we expect the second offsetting effect to be small. This suggests the possibility of another result that compares η_2 and η_1. Our result is the following proposition.

Proposition 4 *The size-capital relationship $k(q)$ strictly increases if $\eta_2(q) \geq \eta_1(q)$.*

Proof. See the Appendix. ■

Note this is not an if and only if result like before. A positive size-skill relationship $\eta_2 > \eta_1$ implies a positive size-capital relationship, but a negative size-skill relationship does not imply a negative size-capital relationship. In fact, in the limiting case of Leontief, $x_3 = x_2$, the offsetting intensity effect drops out and the size-capital relationship is always positive. Note that in the late 20th century, both the size-capital and the size-skill relationships were strongly positive; the model connects these facts.

5 Factor Prices

This section examines the impact of changes in the stock of endowments on factor prices. We use the analysis to examine two issues. First, we consider the issue of capital-skill complementarity. Second, we examine the impact of market expansion and productivity growth on the skill premium.

To make the general equilibrium analysis as simple as possible, we examine the limiting case where all plants are the same size. By continuity, our results apply when plant sizes differ, but a sufficiently large amount of the probability weight is concentrated near a particular plant size type. We also assume that the total measure of products is $m = 1$, so that $p = 1$ from formula (15) and therefore $w_3 = 1$.

To simplify the presentation of this section, we assume $\omega \geq 1$. The $\omega < 1$ analysis is the same except that case requires us to take into account the possibility of a corner solution where all of factor 1 or 2 is completely disposed of. With $\omega \geq 1$, things are simpler since all factor prices will be strictly positive and there is no disposal.

5.1 The Demand for Capital

This subsection presents some preliminary results that we use in the next two subsections. It provides the details of how an equilibrium is constructed for the one-type case. We simplify the general equilibrium analysis down to a simple equation that has a natural interpretation as demand equating supply for capital.

Let z_1 and z_2 denote the cutoffs of the representative firm. We will construct an equilibrium by beginning with an arbitrary z_2 and derive the equilibrium demand for capital $\hat{D}_1(z_2)$ that is consistent with this level of z_2. We then compare demand to the exogenous supply $\overline{X}_1$. An equilibrium is where demand equals supply, $\hat{D}_1(z_2) = \overline{X}_1$. This demand-equals-supply condition will prove useful for the analysis of general equilibrium effects, since there is a close relationship between the skill premium and the equilibrium z_2.

Given a cutoff z_2 and given that in any equilibrium factor 3 is equally distributed among the unit measure of firms, the intensity of tasks undertaken by factor 3 at each firm must then be

$$\hat{x}_3(z_2) = \frac{\overline{X}_3}{(1-z_2)},$$

which we write as an explicit function of z_2. Substituting this into the first-order necessary condition (14) for z_2, noting that $w_3 = 1$, and rearranging yields w_2 as an explicit function of z_2,

$$\hat{w}_2(z_2) = \left[1 + (\omega - 1)\,\frac{1-z_2}{\overline{X}_3}\phi_2(z_2)\right]^{-\frac{1}{\omega-1}}. \tag{20}$$

Observe our assumption that $\omega \geq 1$ implies that $0 < \hat{w}_2(z_2) < 1$ for all z_2.

Next we back out the z_1 that is implied by the marginal technical rate of substitution condition. Cost minimization implies

$$\frac{x_3}{x_2} = w_2^{\omega}.$$

But given that a measure $1 - z_2$ of tasks are assigned to factor 3 and $z_2 - z_1$ to factor 2, the implied levels of x_3 and x_2 yield

$$\frac{\overline{X}_3}{\overline{X}_2}\,\frac{z_2 - z_1}{1 - z_2} = w_2^{\omega}. \tag{21}$$

Solving this leads to an expression for z_1 as a function of z_2,

$$\hat{z}_1(z_2) = z_2 - (1 - z_2)\,\frac{\overline{X}_2}{\overline{X}_3}\hat{w}_2^{\omega}(z_2). \tag{22}$$

Observe this is negative at $z_2 = 0$. It is also immediate that

$$\lim_{z_2 \to 1} \hat{z}_1(z_2) = 1.$$

So define z_2° by

$$z_2^{\circ} = \max\left\{z_2\text{: such that } \hat{z}_1(z_2) = 0\right\}.$$

Observe that $\hat{z}_1(z_2) > 0$, for all $z_2 > z_2^{\circ}$. This is the range of z_2 that we will consider.

The next step is to determine the level of w_1 that is implied by the first-order condition in the choice of z_1. This equals

$$\hat{w}_1(z_2) = \left[w_2^{1-\omega} + (\omega - 1)\,\frac{1-z_2}{\bar{X}_3}\phi_1(\hat{z}_1(z_2))\right]^{-\frac{1}{\omega-1}}, \tag{23}$$

which satisfies $0 < \hat{w}_1(z_2) < \hat{w}_2(z_2)$, given $\omega \geq 1$.

Having determined all this, we can calculate the intensity used by factor 1,

$$\hat{x}_1(z_2) = \hat{x}_3(z_2)\hat{w}_1(z_2)^{-\omega}.$$

Putting this all together leads to what we call the *demand for capital* given z_2,

$$\begin{aligned} \hat{D}_1(z_2) &= \hat{z}_1\hat{x}_1 \\ &= \hat{z}_1\hat{x}_3\hat{w}_1^{-\omega}. \end{aligned} \tag{24}$$

At this point we make two observations. First, by definition of the cutoff z_2°, $\hat{D}_1(z_2^\circ) = 0$. Second, since z_1 goes to 1 and w_1 is bounded above by 1, and since $\hat{x}_3$ goes to infinity, the limit of demand is infinite near z_2 equal to one,

$$\lim_{z_2 \to 1} \hat{D}_1(z_2) = \infty.$$

These two observations and continuity of $\hat{D}_1(z_2)$ imply an equilibrium exists where demand equals supply,

$$\hat{D}_1(z_2) = \overline{X}_1.$$

If demand $\hat{D}_1(z_2)$ is everywhere upward sloping in z_2, the equilibrium is unique. It is intuitive that it should be upward sloping. An increase in z_2 means fewer tasks are assigned to factor 3, increasing work for the other two factors. For the case of constant setup cost elasticity given by equation (5), we can show that if $\eta_1 \geq 1$ and $\eta_2 \geq 1$, then demand is strictly monotonic. Figure 3 illustrates this for the parameters $\eta_1 = \eta_2 = 1$ and $\bar{X}_2 = \bar{X}_3 = 1$.[10] The demand curve as well as two supply curves are illustrated. Observe that if the supply of capital $\bar{X}_1 = 1$, then the equilibrium $z_2 = .35$. Skilled labor undertakes a fraction $1 - z_2 = .65$ of all the tasks, but accounts for only $\frac{1}{3} = \frac{\bar{X}_3}{\bar{X}_1 + \bar{X}_2 + \bar{X}_3}$ of the labor stock. The other two factors are concentrated in disproportionately fewer tasks to keep setup costs low.

While an upward sloping relationship appears to be the typical case, for extreme parameters it is possible to construct examples where a portion of the demand relationship is nonmonotonic. Figure 4 is an example with $\eta_1 = \eta_1 = .01$ and $\alpha_1 = \alpha_2 = 30$ where this occurs. In such a case, depending on where the supply line cuts, there may be multiple equilibria. If there are multiple equilibria, there will be at least one *irregular* equilibrium

[10] The multiplicative constant for setup cost is $\alpha_1 = \alpha_2 = 1$.

where demand cuts supply from above as well *regular* equilibria where demand cuts supply from below. For our comparative statics analysis, we will strict attention only to regular equilibria:

Regularity Condition. Restrict the set of equilibria to include only those z_2^e that satisfy

$$\frac{d\hat{D}_1(z_2^e)}{dz_2} > 0,$$

in addition to $\hat{D}_1(z_2^e) = \bar{X}_3$.

This is analogous to a stability condition. Note a regular equilibrium will always exist. In our comparative statics analysis, in the neighborhood of regular equilibrium z_2^e, we will assume a continuous equilibrium selection around z_2^e. Our comparative statics results are meant to be interpreted as local results.

We consider comparative statics with three model parameters, $\bar{X}_1$, $\bar{X}_2$, and a parameter λ that scales up all the inputs in a proportionate way,

$$\bar{X}_j = \lambda \xi_j,$$

for a vector of constants (ξ_1, ξ_2, ξ_3). We have in mind two interpretations of the scaling parameter λ. First, an increase in trade possibilities is equivalent to an increase in λ. If two separate identical economies are merged by the expansion of trade, it is identical to a doubling of λ. Second, an increase in total factor productivity is equivalent to an increase in λ.

The next result is critical for the subsequent analysis.

Proposition 5 *Let z_2^e be the cutoff in a regular equilibrium. It strictly increases in (i) the capital stock $\bar{X}_1$, (ii) the stock of unskilled labor $\bar{X}_2$, or (iii) an increase in the scaling λ of all three factors (that is, trade or productivity growth).*

Proof. (i) Since demand $\hat{D}_1(z_2)$ is upward sloping in a regular equilibrium, it is immediate that z_2 increases in $\bar{X}_1$. (ii) Fixing z_2, the function $\hat{z}_1(z_2, \overline{X}_2)$ in (22) strictly decreases in $\overline{X}_2$. This implies that $\hat{w}_1(z_2, \overline{X}_2)$ from (23) strictly increases in $\overline{X}_2$ using $\phi_1'(z) > 0$. Hence $\hat{D}_1(z_2, \overline{X}_2)$ strictly decreases in $\overline{X}_2$. This shift down in $\hat{D}_1$ increases the equilibrium z_2. (iii) Define adjusted demand in a way that removes the scaling,

$$\widehat{AD}_1(z_2, \lambda) = \frac{\hat{D}_1(z_2^e, \lambda)}{\lambda} = \hat{z}_1 \frac{1}{\hat{w}_1} \frac{\hat{x}_3}{\lambda} = \hat{z}_1 \frac{1}{\hat{w}_1} \frac{\xi_j}{1 - z_2}.$$

Analogously, adjusted supply is

$$\widehat{AS}_1(z_2, \lambda) = \frac{\overline{X}_1}{\lambda} = \xi_1.$$

In equilibrium adjusted demand equals adjusted supply. Since adjusted supply is a constant, it is sufficient to show that $\widehat{AD}_1(z_2, \lambda)$ decreases in λ. Observe first that fixing z_2, $\hat{w}_2(z_2, \lambda)$ in (20) strictly increases in λ. From (22), $\hat{z}_1(z_2, \lambda)$ strictly decreases in λ. Since $\phi_1'(z_1) > 0$ by assumption, all results imply that $\hat{w}_1(z_2, \lambda)$ in (23) strictly increases in λ. Together, these results imply that $\widehat{AD}_1(z_2, \lambda)$ is strictly decreasing in λ. ■

It is intuitive that as the stock of either capital $\bar{X}_1$ or unskilled labor $\bar{X}_2$ increases, z_2 increases so that the number of tasks $1 - z_2$ done by skilled labor decreases. Indeed, the effect of an increase in capital is immediate in the supply and demand curve in Figure 3. If all factors increase proportionately, the number of tasks $1 - z_2$ done by skilled labor also decreases. Skilled labor is allocated a disproportionate number of tasks to save on fix costs. As the scale of the economy increases, this disadvantage of capital and unskilled relative to skilled labor decreases.

5.2 Capital-Skill Complementarity

We define capital and skill as complements if an increase in $\overline{X}_1$ lowers the equilibrium value of w_2. Recall that $w_3 = 1$, so a decrease in w_2 corresponds to a decrease in the relative pay of unskilled labor or an increase in the skill premium.

From Proposition 5, an increase in capital $\bar{X}_1$ raises the cutoff z_2. From inspection of formula (20), it is immediate that $\hat{w}_2(z_2)$ increases in z_2 if and only if $(1 - z_2)\,\phi_2(z_2)$ increases in z_2. In turn, this occurs if and only if the setup elasticity exceeds unity, $\eta_2(z_2) > 1$. We conclude the following.

Proposition 6 *Capital and skill are complements if and only if* $\eta_2(z_2) > 1$.

Recall from Proposition 5 that z_2 increases in $\bar{X}_2$ as well as $\bar{X}_1$. Moreover, like $\bar{X}_1$, $\bar{X}_2$ does not directly enter the formula $\hat{w}_2(z_2)$. Hence, the direction of the effect of an increase in $\bar{X}_2$ on w_2 is the same as the direction of the effect of an increase in $\bar{X}_1$. We conclude the following.

Proposition 7 *Capital and skill are complements if and only if w_2 decreases in the supply of unskilled labor $\bar{X}_2$.*

It is possible in this model for an increase in the supply of unskilled labor to *raise* the price of unskilled labor. This happens because greater supply allows for scale economies that can attenuate unskilled labor's disadvantage of high fixed cost relative to skilled labor. In such a case, capital and skilled labor are substitutes. But if an increase in the supply of unskilled labor lowers its own price, capital and skilled labor are complements. To the extent this intuitively plausible condition is the empirically relevant one, our theory is consistent with the findings of capital-skill complementarity in the literature (for example, Griliches (1969), Krusell et al. (2000)).

5.3 Market Expansion and Growth

We now turn to the effect on the skill premium of scaling up all factors proportionately by increasing the parameter λ. The effect of increased trade on the skill premium depends in large measure on the shape of the setup cost functions. When λ increases, there are more resources available to produce the fixed set of products. As a result, plants grow, which tends to make firms want to substitute capital for unskilled labor and unskilled labor for skilled labor. The change in the demand for unskilled labor (relative to skilled labor) depends on how costly it is, in terms of the ϕ functions, to substitute at the two margins z_1 and z_2. Our earlier analysis showed that the size-skill relationship provided information about the relative shapes of ϕ_1 and ϕ_2. Our next result ties this together.

Proposition 8 *Suppose at given market size λ° the size-skill relationship is nonpositive. Then in market size λ around λ° strictly increases w_2, so the skill premium falls.*

Proof. We will show that if w_2 does not strictly increase in λ around λ°, then $\eta_1(z_1^\circ) < \eta_2(z_2^\circ)$ must hold. Proposition 3 then implies that the size-skill relationship must be strictly positive, which is sufficient to prove the claim.

We begin with some preliminaries. First, we show that z_1 must strictly increase. Recall formula (22) for z_1:

$$z_1 = z_2 - (1 - z_2)\,\frac{\xi_2}{\xi_3} w_2^{\omega}. \tag{25}$$

Differentiating with respect to λ yields

$$\frac{dz_1}{d\lambda} = \frac{dz_2}{d\lambda}(1 + \frac{\xi_2}{\xi_3} w_2^\omega) - \frac{\xi_2}{\xi_3}(1 - z_2)\omega w_2^{\omega - 1}\frac{dw_2}{d\lambda} > 0.$$

This is strictly positive since $\frac{dz_2}{d\lambda} > 0$ from Proposition 5 and since $\frac{dw_2}{d\lambda} \leq 0$ by hypothesis. Since both z_1 and z_2 strictly increase, it is then immediate from a formula analogous to (21) for w_1^ω that w_1 strictly increases.

Next observe that

$$\begin{aligned} \frac{\frac{dz_1}{d\lambda}}{\frac{dz_2}{d\lambda}} &\geq \left(1 + \frac{\overline{X}_2}{\overline{X}_3} w_2^\omega\right) \\ &= 1 + \frac{\overline{X}_2}{\overline{X}_3}\frac{\overline{X}_3}{\overline{X}_2}\frac{z_2 - z_1}{1 - z_2} = \frac{1 - z_1}{1 - z_2}. \end{aligned} \tag{26}$$

The weak inequality follows from differentiating (25) and using the fact that w_2 is weakly decreasing by hypothesis. The second line substitutes in w_2^ω from (21).

We now turn to the main step of the proof. From (14), the two first-order necessary conditions for the choice of z_1 and z_2 can be written

$$\begin{aligned} \frac{1}{1-\omega} x_3 \left(w_2^{1-\omega} - w_1^{1-\omega}\right) &= \phi_1(z_1) \\ \frac{1}{1-\omega} x_3 \left(1 - w_2^{1-\omega}\right) &= \phi_2(z_2). \end{aligned}$$

Dividing yields

$$\frac{\left(1 - w_2^{1-\omega}\right)}{\left(w_2^{1-\omega} - w_1^{1-\omega}\right)} = \frac{\phi_2(z_2)}{\phi_1(z_1)}.$$

Since w_2 weakly decreases while w_1 strictly increases, the left-hand side must strictly increase. Differentiating the right-hand side and multiplying through by a positive factor, we have

$$0 < \phi_2'(z_2)\phi_1(z_1)\frac{dz_2}{d\lambda} - \phi_1'(z_1)\phi_2(z_2)\frac{dz_1}{d\lambda}.$$

A rearrangement including multiplying through by $(1 - z_2)$ yields

$$\begin{aligned} 0 &< (1 - z_2)\frac{\phi_2'(z_2)}{\phi_2(z_2)} - (1 - z_1)\frac{\phi_1'(z_1)}{\phi_1(z_1)}\frac{(1 - z_2)}{(1 - z_1)}\frac{\frac{dz_1}{d\lambda}}{\frac{dz_2}{d\lambda}} \\ &\leq \eta_2(z_2) - \eta_1(z_1), \end{aligned}$$

where the weak inequality uses (26). ■

This result provides a link between the size-skill relationship and the effect of market expansion and growth on the skill premium. In particular, a negative or zero size-skill relationship implies that expansion of markets reduces the skill premium. Chandler (1990) argues that in the late 19th century, the expansion of markets brought about by advances in transportation led firms to increase plant size and adopt mass production technologies. Goldin and Katz (1998) argue that the advent of mass production led to reductions in the skill premium. Our theory connects these two separate points, using the size-skill relationship to draw the connection. The negative size-skill relationship of the late 19th century was a signal that expansion of markets would *necessarily* have a negative impact on the skill premium.

For some parameters of our model, an expansion of markets can increase the skill premium, as we discuss in the numerical example below. When that happens, our result above tells us that the size-skill relationship must be strictly positive. The process of market expansion has continued throughout the 20th century. The positive size-skill relationship that has become increasingly steep implies that the effect of an expansion of markets today on the skill premium *may* be positive.

6 An Example

This section presents a particular example of our model that exhibits the broad trends that we discussed in the introduction. The example is meant to capture the following scenario. In the late 19th century, mechanical tasks such as those involved in the manufacture of a vehicle were undertaken by both unskilled workers and craftsmen who were highly trained and who were considered skilled workers. As market size expanded, it was relatively easy to substitute unskilled workers for skilled workers at these mechanical tasks. Over time, unskilled workers pushed skilled workers out of mechanical tasks altogether; skilled workers shifted entirely to working on nonmechanical tasks like management. While in come cases it may be possible to substitute unskilled for skilled workers in these “brain tasks” (like the computer helpline discussed in the introduction), in general this substitution is quite difficult. As a result, unskilled labor today is faring relatively poorly on its two fronts. In its rear front where it competes with capital, it is losing out as capital displaces its mechanical tasks. But in its forward front where it competes with skilled labor, it is unable to displace skilled labor at management-like tasks.

In this example, we keep the technology the same, including the ϕ_j functions. We also hold fixed the ratio of skilled workers to unskilled workers. We recognize that there have been major technological changes over the course of a century that would affect the ϕ_1 and ϕ_2 functions and that these changes would affect the skill premium. We also recognize that there have been changes in the ratio of skilled to unskilled workers through schooling and that these changes would affect the skill premium. Our intent here is to abstract from these well-known forces so that we can focus on the potential role of market expansion and capital deepening in accounting for the observed patterns.

The particular assumptions we make about how the economy changes over time are as follows:

$$\begin{aligned} X_1(t) &= \xi_1 e^{\gamma t + \kappa t} \\ X_2(t) &= \xi_2 e^{\gamma t} \\ X_3(t) &= \xi_3 e^{\gamma t} \end{aligned}$$

for $\gamma > 0$ and $\kappa > 0$. Here, skilled and unskilled labor grow at the same rate γ so the ratio stays fixed. Capital grows at a higher rate that exceeds labor by an amount κ. We view the γ parameter as capturing the force of market expansion; the κ parameter captures capital deepening.

The specification of the setup cost functions is crucial for determining the dynamics. If we were to choose the constant elasticity specification (5) for both ϕ_1 and ϕ_2, Proposition 3 would then imply the sign of the size-skill relationship would be constant over time. We would have no hope of capturing the switch in sign, from negative to positive, that has occurred in the data. This leads us to adopt the following nonconstant elasticity specification of ϕ_2,

$$\phi_2(z) = \alpha_2 (1 - z - \beta_2)^{-\theta_2},$$

for $\beta_2 > 0$. Here the elasticity is

$$\eta_2(z) = \theta_2 \left[\frac{1 - z}{1 - z - \beta_2} \right], \tag{27}$$

which increases in z and goes to infinity as z goes to $1-\beta$. This captures the idea that as we move up in the hierarchy of tasks, it becomes increasingly more difficult to substitute skilled for skilled labor. The set of tasks $[1-\beta, 1]$ at the top of the complexity scale can be thought of as management tasks that only skilled workers are capable of doing.

For simplicity, we assume that ϕ_1 takes the constant elasticity form (5). Having the elasticity of ϕ_2 change relative to that of ϕ_1 is enough to drive the patterns we will exhibit in the model.

As t goes to infinity, the quantity of each factor gets arbitrarily large, since $\gamma > 0$. Moreover, capital's share of the productive units gets arbitrarily large, since $\kappa > 0$. Hence, $z_2(t)$ must approach $1-\beta$. As we go back in time and t goes to minus infinity, it is clear that $z_2(t)$ must go to zero, since setup cost becomes prohibitive. Assume

$$\eta_2(0) = \frac{\theta_2}{1-\beta_2} < \min\{\theta_1, 1\}.$$

Then if we go back far enough in time, $\eta_2(t) < \eta_1 = \theta_1$. From Proposition 3, the size-skill relationship is negative. From Proposition 8, the effect of the market expansion (γ) on the skill premium is negative. In addition, since $\eta_2(0) < 1$, the effect of an increase in capital on the skill premium is negative, from Proposition 6. Thus far enough back in time the skill premium is falling.

The setup elasticity η_2 for factor 2 given by (27) strictly increases over time and goes to infinity. There is a critical time period $\hat{t}$ where $\eta_2 = \eta_1$. Before this point the size-skill relationship is positive; after this point it is negative. There is also a point where η_2 exceeds unity. From Proposition 6, capital and skill are complements after this point in time. In simulations of numerical examples we have found that far enough into the future the skill premium increases in market size. Unskilled labor bangs into the constraint that it cannot cut management jobs. But expansion of markets enables capital to cut into tasks done by unskilled workers.

Figure 5 plots the evolution over time of factor allocations, the skill premium (defined by $(w_3 - w_2)/w_2$), where the elasticity $\varepsilon_s \equiv \frac{s'(q)q}{s(q)}$, and the size-capital elasticity ε_k is defined in the analogous way. The numerical example satisfies the restrictions above. The monotonic increase over time in the size-skill relationship and the U-shape of the skill premium is a robust pattern across various parameter values satisfying these restrictions. Note that at the bottom of the U where the skill premium begins to rise, the size-skill relationship is strictly positive (as must be the case from Proposition 8). This pattern is consistent with the U.S. experience, as the size-skill relationship was small, but strictly positive at midcentury. (See Tables 1 and 3.)

In general, the size-capital relationship can increase or decrease over time for parameters satisfying the above restrictions. In the example illustrated, the size-capital relationship increases. This example matches the pattern in the U.S. data.

7 Conclusion

We have introduced a model of task assignment at the plant level. Increased scale of production makes it worthwhile to substitute capital for unskilled labor and unskilled labor for skilled labor. The model delivers several empirically relevant results. It connects the positive size-skill relationship and positive size-capital relationship of the late 20th century to each other, it connects capital-skill complementarity to the positive size-skill relationship, and it connects the effect of market expansion on the skill premium to the size-skill relationship. The model is flexible enough to account for the ends of two centuries while at the same time connecting time series movements to observable cross-section implications. Indeed, as we showed in the last section, the model is even capable of delivering the qualitative trends without any changes in technology, but simply through changes in the stock of inputs.

We do not discount the importance of changes in technology; surely, for instance, the advent of computer controlled machines has made capital more flexible, lowering its fixed cost. But our model has made implicit statements about that, as well: to the extent that capital has become more flexible, so that it can substitute more readily for unskilled workers, there will be an increased chance that the size-skill relationship will be positive, implying a positive size-capital relationship; capital and skills will be complements, and increased trade may no longer be the force for a falling skill premium that it was before. All of this is consistent with the recent evidence. Taking these ideas of technological change to the data in a more quantitative way is a future direction where the model could be put to use.

8 Appendix: Proofs

Calculations for Section 3

Recall that the unit cost function for the general ω case can be written as

$$\tilde{c} = \left(z_1 w_1^{1-\omega} + (z_2 - z_1)\, w_2^{1-\omega} + (1 - z_2)\, w_3^{1-\omega}\right)^{\frac{1}{1-\omega}}. \tag{28}$$

Now

$$\frac{\partial \tilde{c}}{\partial z_j} = \frac{1}{1-\omega} \tilde{c}^{\omega} \left(w_2^{1-\omega} - w_3^{1-\omega}\right). \tag{29}$$

From the marginal rate of substitution condition for cost minimization, we have

$$w_j = w_3 \left(\frac{x_3}{x_j}\right)^{\frac{1}{\omega}}. \tag{30}$$

Substituting (30) for w_1 and w_2 into (28) and taking it to the ω power yields

$$\begin{aligned} \tilde{c}^{\omega} &= \left(z_1 \left(w_3 \left(\frac{x_3}{x_1}\right)^{\frac{1}{\omega}}\right)^{1-\omega} + (z_2 - z_1) \left(w_3 \left(\frac{x_3}{x_2}\right)^{\frac{1}{\omega}}\right)^{1-\omega} + (1 - z_2)\, w_3^{1-\omega}\right)^{\frac{\omega}{1-\omega}} \\ &= w_3^{\omega} x_3 \left[z_1 x_1^{\frac{\omega-1}{\omega}} + (z_2 - z_1)\, x_2^{\frac{\omega-1}{\omega}} + (1 - z_2)\, x_3^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{1-\omega}} = w_3^{\omega} x_3 q^{-1}. \end{aligned}$$

Substituting this into the first-order necessary condition then yields the expression reported in the text

$$\begin{aligned} 0 &= q \frac{\partial \tilde{c}(z_1, z_2)}{\partial z_j} + \phi_j(z_j) \\ &= \frac{1}{1-\omega} w_3^{\omega} x_3 \left(w_j^{1-\omega} - w_{j+1}^{1-\omega}\right) + \phi_j(z_j). \end{aligned} \tag{31}$$

Proof of Proposition 1. Suppose there is a finite set of product types indexed by $i \in \{1, 2, ..., I\}$, and let θ^i be the weight of product i and m^i the mass of products of this type. Let z_j^i denote the j-cutoff of product j. Let $z = (z_1^1, z_2^1, z_1^2, z_2^2, ..., z_1^I, z_2^I)$ be the vector of all cutoffs. Define the space of feasible cutoffs $Z \subset R^{2I}$ to be any z such that $0 \le z_j^i \le z_2^j \le 1$ for $j \in \{1, 2\}, i \in \{1, 2, ..., I\}$. Clearly, Z is compact.

Start with an arbitrary $\hat{z} \in Z$. Define $\hat{n}_j^i$ to be the associated measures of tasks performed by factor j at firm i, $\hat{n}_1^i = \hat{z}_1^i$, $\hat{n}_2^i = \hat{z}_2^i - \hat{z}_1^i$, $\hat{n}_3^i = 1 - z_2^i$. For $\hat{z}$ such that $\hat{n}_j^i > 0$ for all i and j, calculate the unique vectors $\left(\lambda^1, \lambda^2, ..., \lambda^I\right)$, $(q^1, q^2, ..., q^I)$ and $(x_1^1,\, x_2^1,\, x_3^1)$ that solve

$$
\begin{aligned}
x_j^i &= \lambda^i x_j^1 && (32)\\
\sum_{i=1}^{I} m^i \hat{n}_j^i x_j^i &= \bar{X}_j,\; j \in \{1,2,3\} && (33)\\
\left(\hat{n}_2^i \left(x_1^i\right)^{\frac{\omega-1}{\omega}} + \hat{n}_2^i \left(x_2^i\right)^{\frac{\omega-1}{\omega}} + \hat{n}_3^i \left(x_3^i\right)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}} &= q^i\\
\left(\theta^i\right)^{\sigma} q^1 &= q^i,
\end{aligned}
$$

where $\lambda^1 = 1$. Observe that in any equilibrium, given the CES production function, the ratio of intensities across a firm of type i and a firm of type 1 is a constant ratio across factors. The first condition above imposes this constant ratio. The construction determines the input vectors that satisfy this constant ratio, as well as conditions (1), (2), (4), and (5) in the definition of an equilibrium.

Denote the intensity levels by $x_j^i(\hat{z})$, a continuous function by the implicit function theorem. Extend this to all of Z by taking the limit of the continuous $x_j^j(\hat{z})$. Compute $w_1(\hat{z})$ and $w_2(\hat{z})$ according to $w_j(\hat{z}) = w_3\left(\frac{x_3^1(\hat{z})}{x_j^1(\hat{z})}\right)^{\frac{1}{\omega}}$ for $w_3 = p$ where p is defined by (15).

Next, solve the cost-minimization problem for type i,

$$z^i(w_1, w_2) = \arg\min_{0 \le z_1 \le z_2 \le 1} q^i \tilde{c}(z_1, z_2; w_1, w_2) + f(z_1, z_2),$$

where $\tilde{c}$ is the unit cost function defined in (6). Since the problem is strictly convex and continuous, the solutions $z^i(w_1, w_2)$ are continuous functions.

Define

$$z^*(\hat{z}) \equiv z^1(w_1(\hat{z}), w_2(\hat{z})) \times z^2(w_1(\hat{z}), w_2(\hat{z})) \times .... \times z^I(w_1(\hat{z}), w_2(\hat{z})).$$

Since this is a continuous function on a compact set, there is a fixed point $\hat{z} \in z^*(\hat{z})$. Construct $(q^i, p^i, w_j,\, z_j^i,\, x_j^i)$ according to (32), $p^i = p$ from (15), and $x_j^i = \lambda^i x_j^1$. By construction, this satisfies conditions (1) through (5) of equilibrium whenever $\hat{n}_j^i > 0$ for all i and j. Notice that, if $\hat{n}_j^i = 0$, any x_j^i will maintain market clearing (since it is always multiplied by zero), so the fixed point is an equilibrium in those cases as well. The household's budget constraint (6) holds by Walras law. ■

Proof of Proposition 4. Differentiating $k(q)$ and rearranging shows that $k'(q) > 0$ if and only if

$$\frac{x_2}{x_1}\left(\frac{dz_1}{dq}z_2 - \frac{dz_2}{dq}z_1\right) + \frac{x_3}{x_1}\left(\frac{dz_1}{dq}(1-z_2) + \frac{dz_2}{dq}z_1\right) > 0. \tag{34}$$

We will show that $\eta_2 \geq \eta_1$ implies that the first term is positive, which will complete the proof since the second term is positive. Now

$$\begin{aligned} 0 &\leq \phi_1\phi_2'\frac{(1-z_2)}{(1-z_1)} - \phi_2\phi_1' \\ &< \phi_1\phi_2'\frac{z_2}{z_1} - \phi_2\phi_1'. \end{aligned}$$

The weak inequality follows from $\eta_2 \geq \eta_1$. The strict inequality follows from $z_2 > z_1$. Hence,

$$\phi_1\phi_2'z_2 - \phi_2\phi_1'z_1 > 0.$$

Using the formulas for $\frac{dz_1}{dq}$ and $\frac{dz_2}{dq}$, it follows that the first term of (34) is strictly positive.

■

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Table 1
Payroll per Employee by Establishment Size and Year
(Normalized relative to average across all establishment sizes)

Employment Size Category	1947	1954	1967	1977	1987	1997
1-99	.91	.86	.88	.85	.82	.85
100-249	.96	.94	.89	.87	.89	.92
250-499	.98	.96	.91	.89	.92	.95
500-999	1.01	1.02	.98	.99	1.02	1.02
1,000-2499	1.05	1.09	1.09	1.14	1.20	1.20
2,500+	1.13	1.19	1.26	1.39	1.48	1.54

Source: Authors' calculations from tabulations by establishment size in U.S. Census (1950, 1957, 1971, 1981, 1993, 2001).

Table 2
Plant Characteristics by Plant Size
1880 Census of Manufactures
Atack-Bateman Sample

Employment Size Category	Pay per Employee (normalized)	Women/Children Share of Workforce	Capital Intensity (normalized)
1-19	.98	.09	1.07
20-49	1.10	.19	1.30
50-99	1.10	.25	1.27
100+	.95	.40	.74

Source: Authors' calculations with the Atack-Bateman sample data discussed in Atack and Bateman (1999).

Table 3
Nonproduction Worker Share by Establishment Size and Year
(Normalized relative to average across all establishment sizes)

Employment Size Category	1947	1954	1967	1977	1987	1997
1-99	1.03	.95	.86	.92	.92	.96
100-249	.94	.94	.93	.92	.90	.95
250-499	.92	.94	.90	.92	.89	.93
500-999	.95	.99	.96	1.01	.93	.94
1,000-2499	1.05	1.04	1.09	1.10	1.12	1.03
2,500+	1.07	1.14	1.27	1.21	1.42	1.44

Source: Authors' calculations from tabulations by establishment size in U.S. Census (1950, 1957, 1971, 1981, 1993, 2001).

Table 4

Normalized Capital Intensity by Establishment Size and Year
(Normalized relative to average across all establishment sizes)

Employment Size Category	1954	1967	1977	1987	1997
1-99	.75	.91	.77	.63	.61
100-249	.92	.84	.91	.84	.88
250-499	.91	.87	.96	.93	1.03
500-999	1.11	1.09	.97	1.20	1.20
1,000-2499	1.39	1.16	1.41	1.50	1.65
2,500+	1.11	1.17	1.24	1.55	1.54

Source: Authors' calculations from tabulations by establishment size in U.S. Census (1957, 1971, 1981, 1993, 2001).

Table 5
Value Added per Estabslishment
Manufacturing Plants
(Millions of 2000 dollars)

Industry	1923	1954	1967	1977	1987	1997
All Manufacturing	1.0	2.0	4.1	3.5	4.3	5.3
Food Processing	0.3	1.2	2.7	3.8	7.3	7.5
Textiles and Apparel	0.5	0.6	1.4	1.7	2.3	2.9
Chemicals	1.4	3.8	8.8	10.7	14.2	17.5
Rubber	4.7	4.5	3.2	3.0	3.7	4.9
Leather	1.0	1.7	2.8	2.7	2.5	2.9
Non-metallic Mineral Products	1.0	1.8	2.4	2.5	2.7	3.2
Metals and Metal Products	2.3	3.0	4.3	3.7	3.4	2.9
Transportation	3.6	11.2	13.6	14.1	16.7	17.9

Source: Authors' calculations from tabulations by establishment size in U.S. Census (1926, 1957, 1971, 1981, 1993, 2001).

Table 6
Employment per Establishment
County Business Patterns

	Employees per Establishment 1953	Employees per Establishment 1977	Employees per Establishment 1997	1997 Employment Share	Growth in Employees per Est. 1953-97
TOTAL	11.8	14.9	15.3	1.00	30
Agricultural Services, Forestry, and Fishing	4.7	5.4	6.2	0.01	31
Mining	29.8	29.9	21.9	0.01	–27
Construction	9.0	8.1	8.3	0.05	–9
Manufacturing	60.6	59.9	47.4	0.18	–22
Transportation and Public Utilities	27.3	24.2	20.8	0.06	–24
Wholesale Trade	11.9	12.2	12.8	0.06	8
Retail Trade	7.4	10.6	13.8	0.21	88
Finance, Insurance, and Real Estate	8.5	11.1	10.9	0.07	29
Services	5.7	11.4	14.7	0.35	156

Source: Authors' calculations from U.S. Census (1955, 1977, 1999).

Figure 1
Skilled Worker Share and Average Size in the Carriage and Wagon Industry
by State
1890 Census

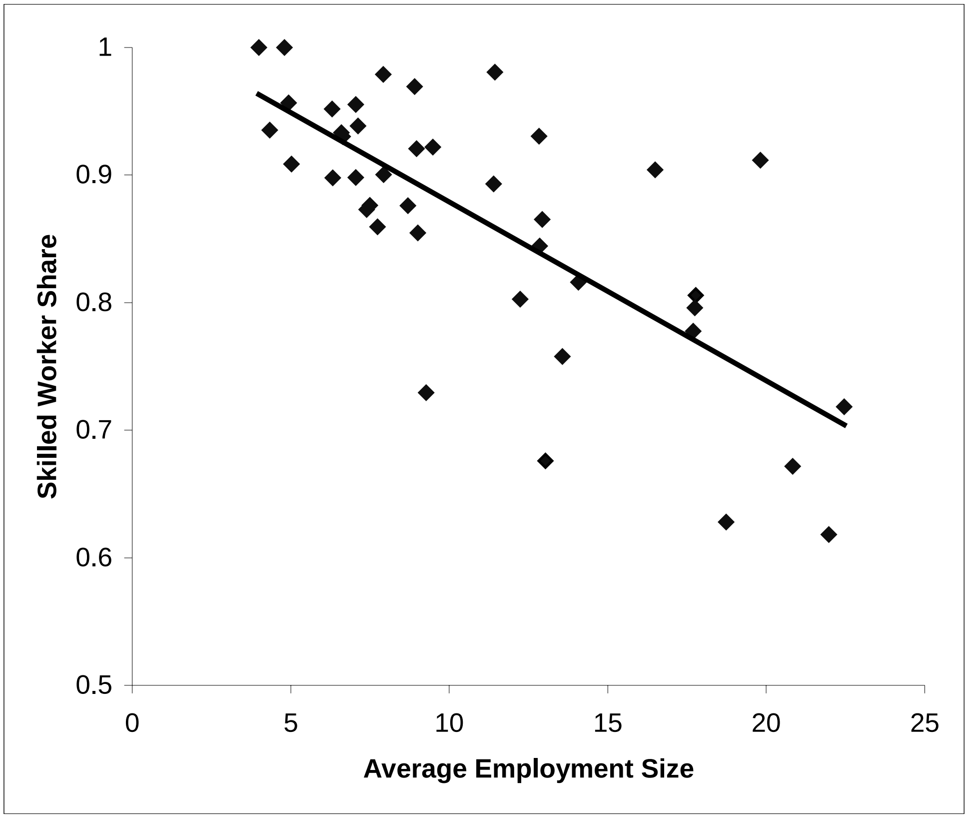
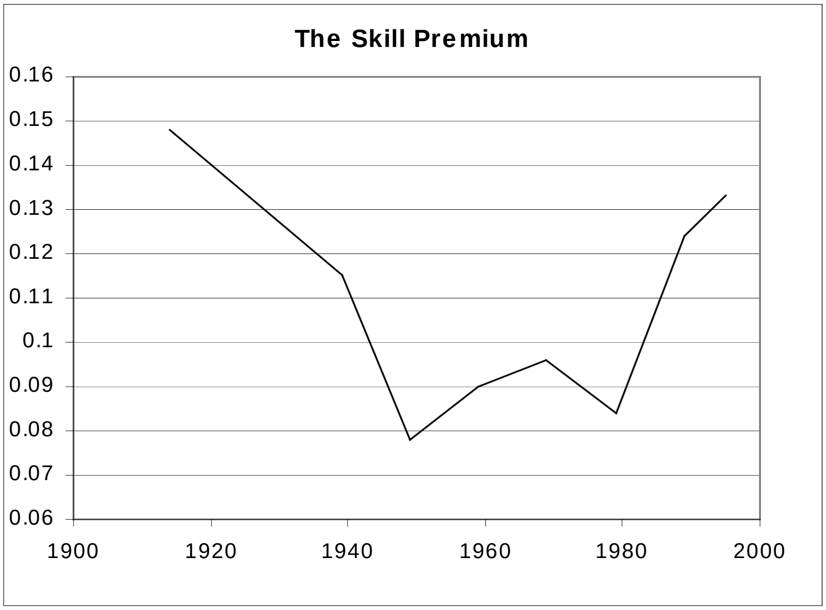


Figure 2
The Skill Premium (Return to College) in the 20th Century



Source: Goldin and Katz (2000).

Figure 3
Supply and Demand for Capital

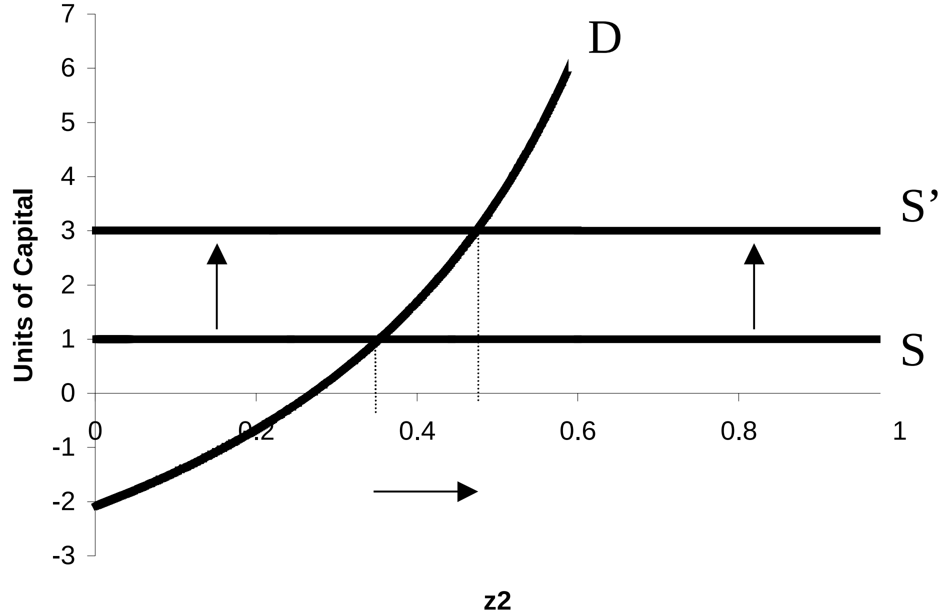


Figure 4
Example of Nonmonotone Demand

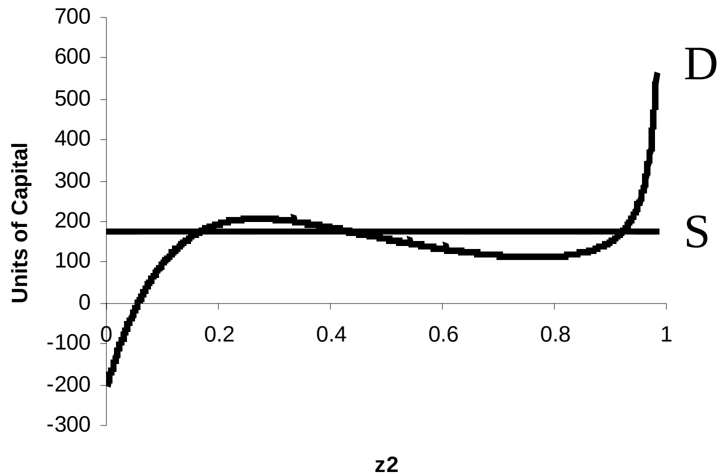


Figure 5
Factor Allocation and Prices Over Time
A Numerical Example