Ellsberg Paradox and Second-order Preference Theories on Ambiguity: Some New Experimental Evidence

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Abstract

We study the two-color problem by Ellsberg (1961) with the modification that the decision maker draws twice with replacement and a different color wins in each draw. The 50-50 risky urn turns out to have the highest risk conceivable among all prospects including the ambiguous one, while all feasible color distributions are mean-preserving spreads to one another. We show that the well-known second-order sophisticated theories like MEU, CEU, and REU as well as Savage's first-order theory of SEU share the same predictions in our design, for any first-order risk attitude. Yet, we observe that substantial numbers of subjects violate the theory predictions even in this simple design.

KEYWORDS: Ellsberg paradox, Ambiguity, Second-order risk, Second-order preference theory, Experiment

JEL classification: C91, D81

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1. Introduction

The Ellsberg Paradox refers to the outcome from Ellsberg's (1961) thought experiments, that missing information about objective probabilities can affect people's decision making in a way that is inconsistent with Savage's (1954) subjective expected utility theory (SEU). Facing two urns simultaneously in Ellsberg's two-color problem, one with 50 red and 50 black balls (the risky urn) and the other with 100 balls in an unknown combination of red and black balls (the ambiguous urn), most people prefer to bet on the risky urn whichever the winning color is. This phenomenon is often called ambiguity aversion. Many subsequent experimental studies confirm Ellsberg's finding as for example surveyed in Camerer and Weber (1992).

To rationalize the Ellsberg-type ambiguity aversion by extending the SEU theory, Gilboa and Schmeidler (1989) develop the maxmin expected utility (MEU) theory where the decision maker (DM) has a set of prior beliefs associated with the ambiguous prospect and assign the minimal SEU utility based on this set as their MEU utility. Given MEU, it is not a paradox anymore.[1]

This seminal work sets off a large literature on what Ergin and Gul (2009) called "second-order probabilistically sophisticated" preferences. Abandoning Savage's axiom of "reduction of compound lotteries", various theories on preferences on second or higher order priors have been developed that are also capable of rationalizing the original Ellsberg paradox. Among these, the smooth ambiguity or Recursive Expected Utility (REU) model by Klibanoff, Marinacci, and Mukerji (2005) has the most operational form by assuming one utility function for each belief order in a space of two-stage lotteries.[2] Recent applications of REU on studies of asset pricing

[1] Schmeidler (1989) develops the related Choquet Expected Utility (CEU) theory based on axioms. The MEU/CEU theory has been further generalized to the general multiple-prior approach. For further literature, see Casadesus-Masanell, Klibanoff, and Ozdenoren (2000), Eichberger and Kelsey (2009).

[2] For further two-stage approaches to ambiguity aversion, see Chew, Karni and Safra (1987), Nau (2006), Segal (1987, 1990), and Seo (2009) among others.

show that it indeed enables an internally consistent calibration of ambiguity attitudes that fits the data and explains issues such as the equity premium puzzle.[3]

Given the success in the applied fields, more experimental investigations about the behavioral foundation of the two-stage preference theories are needed. Recently, Halevy (2007) has an ingenious experimental design to test the predictions of various two-stage preference theories for consistency, where subjects are asked to reveal their certainty-equivalent evaluations for gambles from the original Ellsberg urns to urns with objective second-order priors that have the same mean as the 50-50 urn. He manages to sort each of his subjects into one of the existing (second-order) preference theories surveyed in his paper.

However, all previous experiments on ambiguity aversion we are aware of, including Halevy (2007) as well as variations like "bundling" by Halevy and Feltcamp (2005), share the feature that the ambiguous prospect can be associated with a first-order lottery that is of either lower mean or higher variance than the original risky prospect. Thus, they still leave enough slack for, say, a pessimistic belief of the maxmin first-order lottery to step in as rationalization for the paradox. One extreme test for robustness of the second-order preference theories' validity is to design an experiment that eliminates this room for maneuvering

This is exactly the purpose of the present paper. We construct a decision problem that is a simple modification of Ellsberg's two-color problem. The risky urn contains 5 red and 5 white balls, while the ambiguous urn contains 10 balls of an unknown combination of the two colors. Instead of drawing only once in the urn of the decision maker's choice, we ask them to draw twice with replacement. Crucial is the rule that a *different* color wins the same amount of 50 Yuan in each different draw. In our experiment, red drawn first and white drawn second will ensure a payoff of 100 Yuan.

Table 1 summarizes all possible first-order lotteries given this payoff rule, with π_i coding for the lottery with *i* red balls and 10-*i* white balls. There are exactly 11 of

[3] See Chen, Ju, and Miao (2009), Hansen (2007), Hansen and Sargent (2008), Ju and Miao (2009).

them. Each column lists the distribution of monetary outcome, its mean and its variance. For example, the urn with 4 red and 6 white balls, π_4, gives us the probabilities of .24, .52, and .24 to earn the prize of 0, 50, and 100 Yuan, respectively; with a mean of 50 Yuan and a variance of 34.64. Obviously, our modified Ellsberg risky prospect, π_5, has the highest variance of 35.36, while all color compositions yield the same mean payoff.

Table 1: Complete List of Potential First-order Lotteries

	π_0	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}
Red	0	1	2	3	4	5	6	7	8	9	10
White	10	9	8	7	6	5	4	3	2	1	0
p(0)	0	.09	.16	.21	.24	.25	.24	.21	.16	.09	0
p(50)	1	.82	.68	.58	.52	.50	.52	.58	.68	.82	1
p(100)	0	.09	.16	.21	.24	.25	.24	.21	.16	.09	0
mean	50	50	50	50	50	50	50	50	50	50	50
std	0	21.21	28.28	32.40	34.64	35.36	34.64	32.40	28.28	21.21	0

Our experiment consists of three decision problems, sequentially presented to the subjects. We first elicit subjects' risk attitude regarding simple lotteries using a modified BDM mechanism. We then let them make a decision between the risky and ambiguous urn in the modified Ellsberg problem, followed by a decision between the risky urn and one with uniformly distributed compound lottery. Both the first-order theory of SEU and the two-stage ones of MEU/CEU and REU turn out to have the same prediction that (first-order) risk-averse subjects should prefer the ambiguous and the uniform compound urns to the risky urn in our design!

We ran this experiment in two different treatments, one in the classroom with 12 out of 150 participants randomly chosen for monetary payment and another under lab conditions with all 75 subjects paid according to their decisions. Among the 137 (72) subjects in the random-pay (pay-all) treatment with consistent revelation of risk attitude, 53 (25), 42 (25) and 42 (22) turn out to be risk averse, neutral, and seeking respectively. Among the risk-averse subjects, 53% (44%) prefer the risky urn over the ambiguous one, while 45% (36%) prefer it over the urn with objective second-order risk, in separate decision problems. Among the risk-seeking subjects, 55% (36%) prefer the ambiguous urn, while 57% (50%) prefer the urn with objective second-order risk, respectively. Both violate the theory predictions. With 99% confidence, at least 58% (40%) of non-risk-neutral subjects have violated the theory predictions at least once over the two main decisions.

These findings cast serious doubt on rational preference theories as the exclusive explanation for decisions under uncertainty involving ambiguity and compound lotteries. It appears that many subjects never activated the arithmetic mode of explicit risk assessment, in face of ambiguity. This seems consistent with arguments from psychologically motivated experiments on ambiguity that probabilities involved are not the only factors that help shape decisions.

Heath and Tversky (1991), for instance, attribute the ambiguity preference to the competence which the subjects felt towards the source of the ambiguity. Fox and Tversky (1995) consider the Ellsberg phenomenon an inherently comparative effect and state that it does not arise in an independent or separate evaluation of uncertain prospects (the comparative ignorance hypothesis).[4] Psychological studies in general identify multiple processes (some more effortful and analytic, others automatic, associative, and often emotion-based) being in play for decisions under risk or uncertainty (Weber and Johnson, 2008).

Hsu, Bhatt, Adolphs, Tranel and Camerer (2005) find that the level of ambiguity in choices correlates positively with activation in the areas relating the integration of emotional and cognitive input (the orbitofrontal cortex, OFC) and reaction to emotional information (the amygdala). Their data show that the amygdala and OFC reacted rapidly, yet the dorsal striatum (activation in this area is correlated with the

[4] See Chow and Sarin (2002) and Stecher, Shields and Dickhaut (2010) for more discussions on factors inducing ambiguity aversion.

expected value of actual choices and reward prediction) reacted slowly. The control group of patients with OFC lesions in fact did not distinguish between degrees of uncertainty (ambiguity and risk) and behaved consistently with SEU theory. This evidence suggests that, when facing ambiguity, the amygdala and OFC activate first and deal with missing information independent of its risk implication. In another study, Huettel et al. (2006) show, that activation within the lateral prefrontal cortex was predicted by ambiguity preference, while activation of the posterior parietal cortex was predicted by risk preference.[5]

In the next section, we discuss our experimental design and derive the associated theoretical predictions. Section 3 discusses the data. Section 4 concludes the paper.

2. Experimental Design, Procedure, and Theoretical Predictions

Subjects face three simple decision problems one after another. Problem 1 is meant to test their (first-order) risk attitude, in a modified BDM procedure (Becker, DeGroot, and Marschak, 1964). On a list of 20 cases of sure payoffs that ranges from 5 to 100 Yuan in steps of 5 Yuan, subjects have to choose either the sure payoff or the risky one, Choice B for every case. Choice B is the risky urn with 5 red and 5 white balls. Its value is decided by the following rule. A person draws twice with replacement. If the first draw is red and the second is white, he gets 100 Yuan; if the two draws are of the same color, he gets 50 Yuan; but if the two colors are in the order of white first and red second, he gets 0.

Problem 2 is our main test for predictions regarding ambiguity aversion. In this problem, subjects have to decide between Choice B as described above and Choice C. Choice C is the ambiguous urn containing 10 balls in the combination of unknown colors of red and white. It could be any number between 0 red balls (and 10 white balls) to 10 red balls (and 0 white balls). The drawing and payoff rules are exactly the

[5] Hsu, in private communication, pointed out that the difference in implicated brain parts between Hsu et al. (2005) and Huettel et al. (2006) might be due to the design difference that learning might have occurred over repeatedly facing the same task in the latter. In this sense, our design is closer to Hsu et al.

same as with Choice B. Note, however, the novel feature from our double-drawing lottery design of different color winning each round ensures that the mean of the lottery is always 50 Yuan, independent of the color composition in the urn. Consequently, all compound lotteries can be ordered as mean-preserving spreads to one another, with Choice B being associated with the highest possible variance. Table 1 summarizes the statistical characteristics of potential first-order lotteries in our design.

Problem 3 is a test on preference over objective compound lotteries, where subjects are to choose between (the simple risk) Choice B and (the second-order risk) Choice D. Again, the drawing and winning rules are the same, but the number of red and white balls under Choice D is determined as follows: one ticket is drawn from a bag containing 11 tickets with the numbers 0 to 10 written on them. The number on the drawn ticket will determine the number of red balls in the urn. For example, if the number 3 is drawn, then there will be 3 red balls and 7 black balls in the urn.

Procedure

We ran two treatments that differ only in the number of subjects chosen for real payment after making decisions as described above. In the random-pay treatment (RP) conducted in classrooms, subjects were informed right after the instruction about Problem 1, that only three in the classroom (12/150 in total) would be randomly chosen to have their decisions implemented and get paid. In the pay-all treatment (PA), participants were recruited in the traditional manner for lab experiments. All subjects got paid following the implementation of their decisions after the session.[6]

A total of 225 subjects participated. The RP treatment, conducted in October 2009, had four experimental sessions with a total of 150 students who voluntarily stayed for about a half hour after classes at Shanghai University of Finance and Economics. In three sessions, subjects made decisions in the sequence of Problems 1, 2 and 3, while in the fourth session the sequence was Problems 1, 3 and 2. The PA treatment was run

[6] Due to time constraints, subjects had the option to implement their decisions either one by one right after the session or later at the experimenter's office with an impersonal ID card we handed out.

with 75 students in two sessions, in November 2010. Subjects made decisions in the sequence of Problem 1, 2 and 3. All participants were first-year college students of various majors ranging from economics and management to science and language.

Our instructions were done with a PowerPoint presentation. After the explanation of each problem, students were instructed to make their decisions right away and to hand them in before they got instructions for the next problem. Details of the instructions can be found in the appendix.

To increase credibility, we demonstrated drawings with the urn to be used later in Choices B and D during instructions. Choice C urn was secretly prepared before the session and placed on the counter for all to see until the end. After all decision sheets were collected, random drawings determined subjects for real payment in each RP session. Then, subjects were called upon to have their decisions implemented one by one. They got paid in cash according to the realization of their decisions and were dispatched, unless they opted not to wait and would rather do so in the experimenter's office later instead. Average payoff for all 87 subjects with real payment in the two treatments was 62.2 Yuan.[7]

Theoretical Predictions

Given the structure of our two-draw design, only three outcomes $X = \{0, 50, 100\}$ are possible from the gambles B, C and D. Let $S = \{\pi_i, i = 0, ..., 10\}$ denote the feasible set of first-order lotteries under our design, where π_i refers to the lottery with *i* red and (10-*i*) white balls in the urn. Given the rule that red wins the first draw and white the second, for each π_i, the probability is $q_i = i(10-i)/100$ for either of outcomes of 0 and 100 Yuan, and 1-2q_i for the outcome of 50 Yuan. Due to our symmetrical design, {*i*-red, (10-*i*)-white} and {(10-*i*)-red, *i*-white} urns induce stochastically

[7] Note that 1 USD = ca. 6.8 Yuan. Regular student jobs paid about 7 Yuan per hour and average first jobs for fresh graduates paid below 20 Yuan per hour. As a side note, students indeed regularly requested to inspect the content of the ambiguous urn C after the decision implementation.

equivalent prospects, in all aspects relevant for decision under risk. Mean for π_i is the same 50 for all *i*. But the variance increases from *i*= 0 to *i*=5 and then symmetrically decreases from *i*=5 to *i*=10. In other words, a more color-balanced urn represents a mean-preserving spread gamble to a less balanced urn. Table 1 summarizes the risk characteristics of all 11 elements in *S*.

According to the subjective expected utility (SEU) theory by Savage (1954), decision makers assign subjective (first-order) probability $\pi_C \in \overline{S}$ where $\overline{S}$ denotes the convex hull of *S* and evaluate Choice C with π_C together with a utility function *u* defined on the outcome space *X*. Note, a compound second-order lottery is a probability distribution $\mu \in \Delta(S)$ with support in the space of first-order lotteries *S*. Thus, Choice D can be formally identified as $\mu_D = (1/11, \pi_i)_{i=0}^{10} \in \Delta(S)$. As SEU satisfies the axiom of reduction of compound lotteries (ROCL) in case of objective compound lotteries, the DM evaluates Choice D with the first-order probability $\pi_D = \sum_{i=0}^{10} \pi_i / 11 \in \overline{S}$. Since mean of π_D is 50 and $\operatorname{var}\pi_D = \sum_i \operatorname{var}\pi_i / 11 < \operatorname{var}\pi_5$, π_5 is a mean-preserving spread to π_D. Consequently, a risk-averse DM is to prefer D to B, while risk-seeking ones are to prefer B to D, in Problem 3. Similarly, as *any* feasible subjective probability in $\overline{S}$ turns out to be a strict mean-preserving contraction to π_5 unless it degenerately puts all weight on the latter, a risk-averse (-seeking) DM is also to prefer C to B (B to C), in Problem 2. This exactly illustrates the fundamental difference to Ellsberg's design where a subjective probability can be associated with the ambiguity prospect that may yield higher mean or lower variance than the benchmark risky prospect.

The theories of Maxmin Expected Utility (MEU) by Gilboa and Schmeidler (1989) and of Choquet Expected Utility (CEU) by Schmeidler (1989) attempt to rationalize the Ellsberg paradox by generalizing SEU into a set-valued theory, based on axioms. In particular, the DM could change to another "pessimistic" belief of which first-order probability $\pi \in \overline{S}$ to use for risk assessment, depending on different presentation of the ambiguous case. However, due to design, the most pessimistic such evaluation of Choice C for a risk-averse DM is always π_5. Consequently, MEU/CEU predicts that the risk-averse DM prefers C to B, just like SEU does. Moreover, since MEU/CEU merely extends SEU under ambiguity, its prediction for Problem 3 is the same as SEU's, too.

Under the model of Recursive Expected Utility (REU) for decisions under second-order uncertainty by KMM (2005) that has found wide use in applied fields, the preference of a rational DM under risk or uncertainty is characterized by two different expected utility functions. For any first-order lottery $\pi \in S$, the DM may use a von-Neumann-Morgenstern utility index u to calculate its certainty equivalent $C_\pi = \sum_x u(x)\pi(x)$. Then, for each second-order lottery $\mu \in \Delta(S)$, the DM may use another utility index $v(\cdot)$ to calculate the ultimate expected utility as

$$U(\mu) = \sum_{\pi \in \operatorname{supp} \mu} V(c_\pi)\mu(\pi)$$

For our problem, the certainty equivalent value of first-order gamble π_i is calculated as $C_i = q_i[u(0) + u(100)] + (1-2q_i)u(50)$. Under the assumption of concavity and monotonicity of $u(\cdot)$, which also implies first-order risk aversion for the DM, and given the ranking of the variance in *S*, we derive the following ranking of associated certainty equivalents.

$C_0 > C_1 > C_2 > C_3 > C_4 > C_5$ and $C_i = C_{10-i}$ for i = 0, 1, 2, 3 or 4.

If $v(\cdot)$ is concave and monotone, and the decision maker evaluates a bet on the bundle from the ambiguous urn using an arbitrary prior $\mu \in \Delta(S)$, expected value of Choice C is $U(\mu) = \sum_{i=0}^{10} v(C_i) \cdot \mu(\pi_i)$. Since $v(C_i) > v(C_5)$ for all $i \neq 5$, we have

$$U(\mu) = \sum_{i=0}^{10} v(C_i) \cdot \mu(\pi_i) > v(C_5) = U(B)$$

For Choice D, μ is the special case of the uniform distribution on *S*. As a result, a REU decision maker who has risk averse utility functions $v(\cdot)$ and $u(\cdot)$ will rank the three choices as follows, *U*(D)>*U*(B) and *U*(C)>*U*(B).[8] In summary, we have our following theoretical prediction to test.

Hypothesis *According to the theories of SEU, MEU/CEU, and REU, risk-averse individuals (CE<50 in Problem 1) are to choose C over B in Problem 2 and D over B in Problem 3, while risk-seeking individuals (CE>50 in Problem 1) are to choose B over C or D in both Problem 2 and 3.*[9]

Note that any decision in Problems 2 and 3 by a Problem-1 risk-neutral individual is trivially consistent with the theory prediction. And the theories predict that people with non-neutral risk attitudes should have a strict preference among the two choices in both Problems 2 and 3, which makes it redundant to provide the option of indifference between the two choices in Problems 2 and 3 in the design.

3. Experimental Results

Problem 1 elicits individuals' risk attitude. The certainty equivalent value (thereafter CE) of the risky lottery (Choice B) in our experiment is defined as the lowest value, at which one starts to prefer sure payoff to the lottery. The majority of subjects revealed monotone behavior of switching from B to A with increasing sure payoffs. Only 13 out of 150 subjects (8.6%) in the random-pay treatment and 3 out of 75 subjects (4%) in the pay-all treatment switched back from A to B,[10] which are

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[8] In fact, *v* only needs to be monotone for this result to be correct, due to the special structure of our gamble design. Note, if $\mu \in \Delta(\overline{S})$ is allowed, then Jensen's inequality is needed to prove the claim, which requires *v* to be convex.

[9] In fact, the same can be proved for the so-called Recursive Non-Expected Utility (RNEU) by Segal (1987, 1990). For the sake of brevity, we elect to not discuss its tedious proof here.

[10] Note that the t-test yields p-value of 0.1004 indicating no significant difference. The 90% confidence

deemed anomaly and excluded from our data analysis. Subsequent analysis only uses the remaining samples of 137 subjects in RP and of 72 subjects in PA, respectively.

The average certainty equivalent value for the RP treatment (PA treatment) is 46.3 (49.65) with standard deviation of 16.1 (11.11). We have 38.68% (34.72%) of the subjects with CE<50, 30.66% (34.72%) with CE=50 and 30.66% (30.56%) with CE>50, which correspond to the attitudes of risk aversion, risk neutrality and risk seeking respectively.[11] Wilcoxon signed-rank test reveals no significant difference between RP and PA regarding subject risk attitudes (p=0.2867). Figure B1 in the appendix shows the distributions of subjects' CE values.

As a methodological note, most experiments on the Ellsberg paradox use the standard BDM mechanism in which the subject is asked to state a minimum certainty-equivalent selling price to give up the lottery he has been endowed with.[12] This auction procedure provides a formal incentive for the subject to truthfully reveal their CE of the lottery. However, in its original form it appears hard for some subjects to comprehend. In a pilot study where subjects were to make binary decisions first and to reveal a BDM price for each of their choices second, 26 out of 89 subjects (29.2%) displayed inconsistent evaluations.[13] Thus, we choose to use a modified version of the BDM mechanism to elicit subjects' first-order risk attitude. First, instead of asking subjects to reveal a single selling price, we ask them to make 20 simple binary decisions, where a randomizing device[14] determines which of them is realized.[15] In

interval of the binomial proportion test is [.074, .151] for 13/150 in RP, and [.011, .100] for 3/75 in PA. Thus, neither sample rejects with 10% significance the hypothesis that q ∊ [.074, .100] be the expected ratio of inconsistency.

[11] For comparison, Halevy (2007) uses the standard BDM mechanism. In a sample of 105 subjects, 31.7%, 30.5%, and 38.5% of people are risk averse, neutral, and seeking, respectively.

[12] See summary in Stecher, Shields and Dickhaut (2010) for example.

[13] More specifically, aside from Problem 2 and 3 binary decisions as in this paper, subjects in the pilot faced another choice between urn B and an urn with equal likelihood of either 3 or 7 red balls. After the binary decision is made, the subject has to announce their selling price for their *preferred* prospect.

[14] To quote Harrison and Rustrom (2008), "For the instrument to elicit truthful responses, the experimenter must ensure that the subject realizes that the choice of a buying price does not depend on the stated selling price. If there is reason to suspect that subjects do not understand this independence, the use of physical randomizing devices (e.g., die or bingo cages) may mitigate such strategic thinking." And the 29.2% inconsistency rate encountered in the mentioned pilot using the original

addition, the binary decision in our modified BDM is similar in shape to the subsequent parts of the experiments, which facilitates the comparison to ambiguity attitudes.[16]

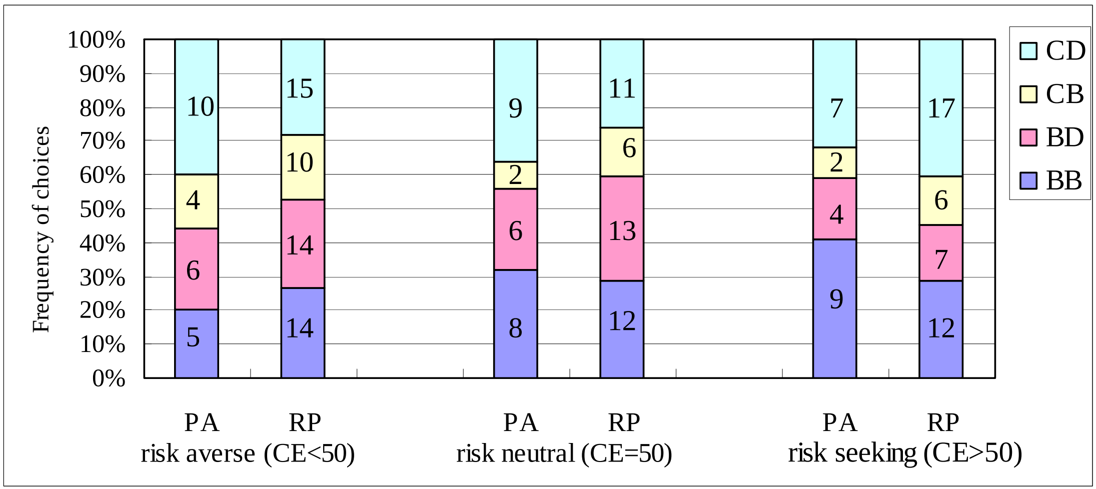

Figure 1: Distribution of Choice Profiles

Figure 1 summarizes the distribution of individual behavior in both Problem 2 and 3 decisions, separately grouped for different risk attitudes. Each subject's behavior is characterized by one of the four types of choice combination BB, BD, CB and CD, which they made in Problem 2 and Problem 3 respectively. Simple regression reveals that orders of Problem 2 and 3 have no significant effect on decisions, so that we can pool data from all sessions within each treatment.[17] Moreover, the chi-square goodness-of-fit test yields p-values of 0.7739 (0.3449), 0.4298 (0.2035) and 0.062 (0.1529), for the risk-averse, -neutral and -seeking samples in RP (PA), which reject the null hypotheses of pure behavior randomization in any case.

Table 2 summarizes all relevant cases of violations against the theoretical Hypothesis, along with the lower bounds of their 99% and 95% binomial proportion

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BDM fittingly echoes this reasoning.

[15] Sapienza, Zingales and Maestripieri (2009) use a similarly modified BDM method, which they consider an adaptation from the mechanism used in Holt and Laury (2002).

[16] Weber and Johnson (2008) argue that, when measuring levels of risk taking with the objective of predicting risk taking in other situations, it is important to use a decision task that is as similar as possible to the situation for which behavior is being predicted.

[17] See Table B1 in Appendix B.

confidence intervals.

Table 2: Violations to Hypothesis and Confidence Intervals

	Sample size	Problem 2	Problem 3	Problems 2&3
Random-pay risk averse	53	52.83% [28] (.3466/.3864)	45.28% [24] (.2786/.3156)	71.7% [38] (.5339/.5765)
Random-pay risk seeking	42	54.76% [23] (.3423/.3867)	57.14% [24] (.3644/4096)	71.43% [30] (.5062/.5542)
Random-pay combined	95	53.68% [51] (.4008/.4315)	50.53% [48] (.3705/.4007)	71.58% [68] (.5825/.6140)
Pay-all risk averse	25	44% [11] (.1974/.2440)	36% [9] (.1399/.1797)	60% [15] (.3298/.3867)
Pay-all risk seeking	22	40.91% [9] (.1618/.2071)	50% [11] (.2293/.2822)	59.09% [13] (.3046/.3635)
Pay-all combined	47	42.55% [20] (.2451/.2826)	42.55% [20] (.2451/.2826)	59.57% [28] (.3989/.4427)

Note: Numbers in parentheses (-/-) refer to lower bounds of 99% and 95% confidence intervals. Numbers in [-] refer to size of violation observations. Under risk aversion, violation refers to choices of B in Problem 2 or 3, and to non-CD choice profiles under Problems 2&3. Under risk seeking, violation refers to choices of C in Problem 2 or D in Problem 3, and to non-BB under Problems 2&3.

We consistently observe at least 40% violations in all but one situation (36%). The Pay-all treatment seems to induce slightly lower numbers of violations than the Random-pay treatment does, across the board. Yet, the Wilcoxon signed-rank test reveals no significant difference between the treatments, with p=0.4696 (p=0.4416), p=0.7789 (p=0.8200), and p=0.2963 (p=0.5886) for risk-averse, -neutral and –seeking subjects in Problem 2 (Problem 3), respectively.

Note the boundaries [y, z] of any (100-x)% confidence interval can be interpreted as implying that a hypothetical parameter within [y, z] cannot be statistically rejected to the significance level of x%, while it can be rejected outside of [y, z] to the level of x%. Consequently, we conclude that our data cannot reject at the 5% significance level that at least 38.64% (24.4%) risk-averse and 38.67% (20.71%) risk-seeking subjects would regularly violate the theoretical prediction in Problem 2 under the RP (PA) treatment. The situation does not change much if subjects face Problem 3 where the ambiguous choice C of Problem 2 is replaced by choice D with objective second-order risk. With 95% confidence, we expect at least 31.56% (17.97%) of risk-averse and 40.96% (28.22%) of risk-seeking people to behave in violation of the prediction of second-order preference theories under risk, in treatment RP (PA). Moreover, among the revealed non-risk-neutral people, we expect with 95% confidence that at least 61.4% (44.27%) would violate the theory predictions at least once after facing both Problems 2 and 3 in RP (PA).

In summary, our data reject the theoretical predictions of the Hypothesis and suggest that a substantial share of people regularly behave in a way that is inconsistent with all preference theories.

4. Concluding Discussion

Ambiguity aversion has often been rationalized with second-order probabilistic sophistication approaches of rational preference theory. Our results cast serious doubt on their universal applicability. Even in fairly simple decision problems as our Problems 2 and 3, there is a substantial share of individuals whose behavior cannot be explained with any existing (second-order) economic preference theory.

Recent neuroimaging studies like Hsu et al. (2005) and Huettel et al. (2006) compare brain activation of people who choose between ambiguous vs. risky options and suggest that these two types of decision making follow different brain mechanisms and processing paths. Our findings suggest that this dichotomy in brain activity may be triggered for a sizeable share of people even when ambiguity is completely dissociated from higher risk as in Problem 2 here.

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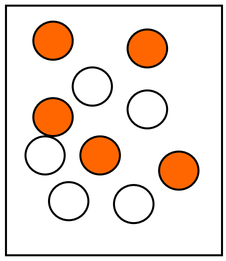
Appendix A: Instructions (Slides translated from Chinese original)

[Slide 1] **Problem 1**: Making a choice between option A and urn B

- Urn B contains 5 red balls and 5 white balls.



B

- **Payoff rule for urn B:** Two balls are to be drawn from urn B with replacement.

  You get 50 Yuan if the first ball drawn is red and nothing if it is white. Conversely, you get 50 Yuan if the second ball drawn is a white and nothing if it is red. You get paid the sum of money earned in the two draws.

[Slide 2] Decision sheet for Problem 1

Make a choice by checking either option A or urn B in each row			
Situation	**Payoff of Option A**	**Option A**	**Urn B**
1	5 Yuan		
2	10 Yuan		
3	15 Yuan		
…	…		
9	45 Yuan		
10	50 Yuan		
…	…		
19	95 Yuan		
20	100 Yuan		

[Slide 3] Rules for session ending implementation of decisions

- [**used in the RP treatment**] Because of the time constraint, at the end of the experiment, we will randomly choose 3 students for real monetary payment. Every selected student will draw one of the three cards numbered 1, 2 3, which represent three decision problems in today's experiment. We will pay you depending on the realization of your decision in that problem.

  [**used in the PA treatment**] In the end of the experiment, you will be paid based on the realization of your choices. You draw one of the three cards numbered 1, 2 3, which represent three decision problems in today's experiment. We will pay you depending on the realization of your decision in that problem.

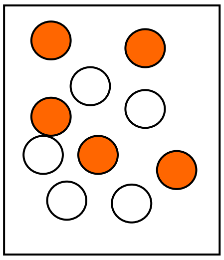
- For example, one draws a card of number 1 and would realize his payoffs from Problem 1. He is asked to draw one of the twenty cards representing 20 situations of option A, and he draws number 1. If his choice in situation 1 is "urn B", then we will let him draw balls from urn B to realize his payoffs. If his choice in situation 1 is A, then we will pay him 5 Yuan immediately.

[Slide 4] **Problem 2**: Make a choice between urn B and urn C

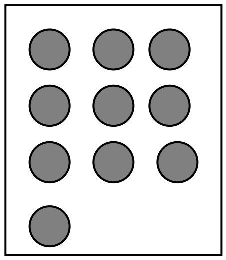
- Urn C contains a mixture of 10 red and white balls. The number of red and white balls is unknown; it could be any number between 0 red balls (and 10 white balls) to 10 red balls (and 0 white balls).

- **Payoff rule for urn C: same as Payoff rule for urn B**.

[Slide 5] Decision sheet for Problem 2



B



C

Question: If you are asked to make a choice between urn B and urn C, which urn will you choose?

☐Urn B ☐Urn C

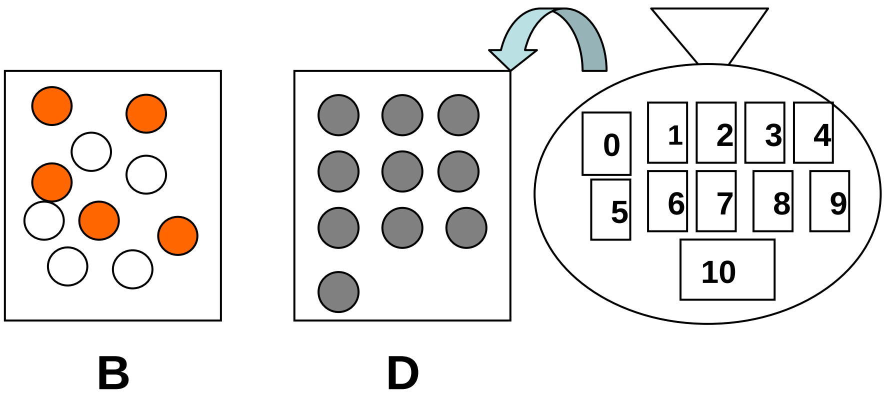
[Slide 6] **Problem 3**: Make a choice between urn B and urn D

- Urn D contains a mixture of 10 red and white balls. The number of red and white balls is determined as follows: one ticket is drawn from a bag containing 11 tickets with the numbers 0 to 10 written on them. The number written on the drawn ticket will determine the number of red balls in the urn. For example, if the number 3 is drawn, then there will be 3 red balls and 7 black balls in the urn.

- **Payoff rule for urn D: same as Payoff rule for urn B**.

[Slide 7] Decision sheet for Problem 3

Draw the number of red balls in urn D



Question: If you are asked to make a choice between urn B and urn D, which urn will you choose?

☐Urn B ☐Urn D

Gender ☐Male ☐Female

Appendix B: Further Data

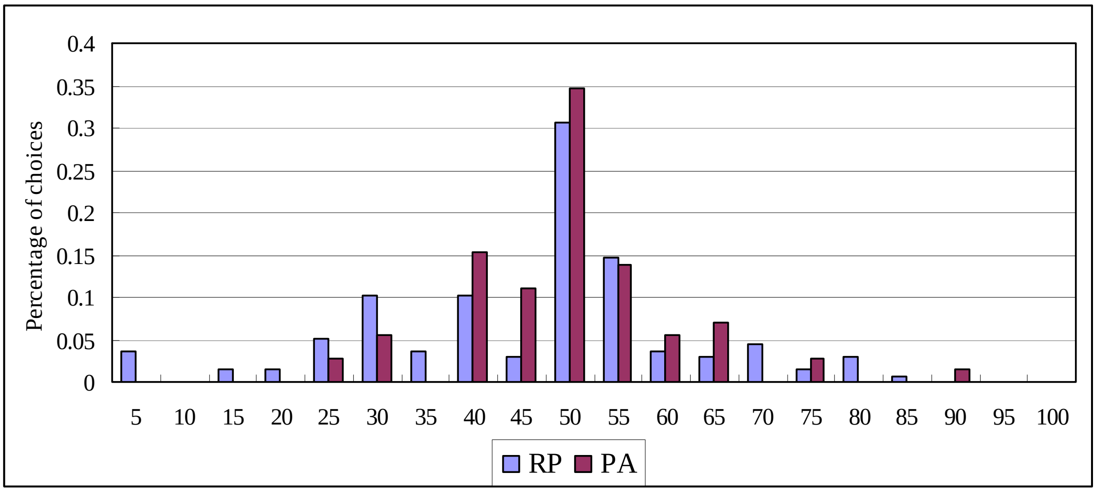


Figure B1: Distribution of Certainty Equivalent Value in Problem 1

On the order effect of Games 2 and 3 in random-pay treatment

Whether the placement of Problem 2 and Problem 3 affects decisions is a justified concern. To control for this, in the random-pay treatment, we collected 100 observations in the sequence of Problems 1, 2 and 3 and 37 observations in the sequence of Problems 1, 3 and 2, for reserved order between Problems 2 and 3.

Logistic regressions reported in Table B1 confirm that no order effect is found, as the coefficients of the variable Order prove to be insignificant for both Problem 2 and 3 decisions. Interestingly, Problem 1 CE level also has no effect on Problem 2 and 3 behavior. There is, however, a significant gender effect in the Problem 3 decision, where females tend to choose B less frequently than D.

Table B1: Binary logit model of Choice B in Problem 2 and Problem 3

	Choice B in Problem 2	Choice B in Problem 3
Order	-0.08 (0.387)	0.184 (0.398)
CE	-0.018 (0.055)	-0.051 (0.057)
Female	0.042 (0.353)	-0.708* (0.360)
Constant	0.304 (0.660)	0.502 (0.677)
Observation	135	135
R square	0.0009	0.0253

* Significant at 10% level