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Binomial menu auctions in government formation

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Abstract

In a menu auction, players submit bids for all choices the auctioneer A can make, and A then makes the choice that maximizes the sum of bids. In a binomial menu auction (BMA), players submit acceptance sets (indicating which choices they would support), and A chooses the option that maximizes his utility subject to acceptance of the respective players. Monetary transfers may be implicit, but players may also bid by offering "favors" and the like. BMAs provide a unified representation of both monetary and non-monetary bidding, which I apply to model government formation. First, I analyze general BMAs, characterize the solution under complete information and establish outcome uniqueness (for both, sealed bid and Dutch formats). Second, in case monetary transfers are possible, BMAs are shown to implement VCG mechanisms. Finally, in case transfers are impossible, BMAs extend the model of proto-coalition bargaining and are specifically applied to government formation.

JEL–Codes: C72, C78, D44

Keywords: menu auction, demand commitment, proto-coalition bargaining, VCG mechanism

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1 Introduction

A well-established branch of bargaining analyses rests on the demand commitment paradigm. In these analyses, the players first state “utility demands,” and subsequently a decision maker chooses the option (allocation of posts or funds, perhaps a political platform) that maximizes his utility subject to satisfying the utility demands. This idea reaches back to the Nash demand game (Nash, 1950), and starting with Winter (1994a,b), it has experienced a revival in coalitional bargaining. For example, Morelli (1999) and Montero and Vidal-Puga (2007, 2010) analyze which bargaining protocols induce equitable equilibrium results in majority bargaining, Caruana et al. (2007) analyze a demand game where players may revise demands, Vidal-Puga (2004) and Breitmoser (2009) study the equilibrium outcomes if the identity of the coalition formateur is held constant throughout the game. The latter class of games is particularly suitable to model government formation, as government formateurs are often appointed by third parties such as presidents, and therefore have an exogenous identity throughout the game.

The present paper theoretically analyses a model that merges the defining features of demand commitment bargaining with those of proto-coalition bargaining (Diermeier and Merlo, 2000; Baron and Diermeier, 2001). The latter appears to be the most promising current approach toward empirical analyses of government formation. In proto-coalition bargaining, a coalition “formateur” appoints a proto-coalition and subsequently, if all of its members agree, they enter multilateral negotiations to allocate cabinet posts (the model of these negotiations typically follows Merlo and Wilson, 1995, 1998). If at least one of them disagrees, a care-taker government assumes office. This model can account for empirical phenomena such as minority and surplus governments and its structure can be extended straightforwardly to allow for stochastic deviations from strict best responses (by adding logistic errors). The resulting structural models have been estimated based on real-world data and are used in counterfactual policy experiments (Diermeier et al., 2002, 2003, 2007) to evaluate institutional and constitutional design.

The combined model proposed here is shown to maintain the desirable features of proto-coalition bargaining—outcome uniqueness and generally characterized solution—and it is straightforwardly extensible to a structural model by including logistic errors. That is, the increased complexity that follows from considering pre-commitments prior to the formateur's choice does not obstruct the tractability of the proto-coalition model, in both theoretical and empirical analyses, but it allows for more precise descriptions of government formation.

This is particularly interesting, as it seems to be consensual that strategic pre-commitments affect government formation.[1] It has to be noted, however, that pre-commitments are usually not made in the form of utility demands in this context. More typically, and more generally, parties pre-commit by stating "acceptance sets" indicating which options are acceptable and which are not. For example, parties may pre-commit to negotiate only with a specific other party, or not to negotiate with say left/right wing parties, or to negotiate only on the condition that person X is not on the cabinet. In response to these pre-commitments, the formateur then chooses the option maximizing his utility. Such "acceptance set bidding" is analyzed in this study.

Implicitly, the formateur auctions off inclusion in the government coalition. Due to the relation to "menu auctions" (Bernheim and Whinston, 1986), I refer to the model as a "binomial menu auction" (BMA). In a menu auction, the bidders bid independent amounts for all the choices that the auctioneer can make (and the auctioneer maximizes the sum of bids), and in BMAs, they signal acceptance with respect to all of the choices (and the auctioneer maximizes utility). The BMA is a general model of acceptance set bidding, but related approaches have been developed for public goods provision (Bag and Winter, 1999), network formation (Mutuswami and Winter, 2002), and legislative coalition formation (Montero and Vidal-Puga, 2007).

The present paper complements a companion paper (Breitmoser, 2010), where I focus on an alternative extension to the standard proto-coalition model, namely on the case that the formateur can revise his proposal to the proto-coalition after rejections (with infinite time horizon). Here, I maintain the standard assumption that the formateur can make a single proposal. This one-round game is shown to be outcome

[1] Examples abound, not least in Germany with its frequent government formations at both the federal and the regional level. For example, the liberal democrats (FDP) often pre-commit to coalesce only with the conservatives (CDU) and the CDU usually pre-commits not to coalesce with the left wing "Die Linke."

equivalent to the T-round game if $T < \infty$, however, which implies that the present study and Breitmoser (2010) are fairly exact complements (note that I also consider pre-commitments in the infinite-horizon analysis reported in Breitmoser, 2010).

Section 3 establishes generic outcome uniqueness of BMAs in a general model of complete information (assuming trembling-hand perfection). Interestingly, the feature that players state acceptance sets actually proves to be key in establishing uniqueness. The outcome is generally not unique if players state utility demands (see Section 5), and thus the notion of acceptance set bidding is not only more intuitive in modeling government formation, but also more conclusive in its predictions. Similarly, if we would model government formation as a menu auction (not "binomial," that is), we would have to assume that transfer payments can be made (or committed to) prior to the formateur's choice, and still it does not induce generic uniqueness even if we restrict bids to be compatible with utility demands (which is called "truthful bidding" in Bernheim and Whinston, 1986).

Sections 4 and 5 apply the general results of Section 3 to the two arguably most salient special cases, namely to the cases that monetary transfers between formateur (auctioneer) and bidders are perfectly possible (Section 4) and entirely impossible (Section 5). In the case with transfers, the BMA is a generalized auction and relates closely to VCG mechanisms (following Vickrey, 1961, Clarke, 1971, Groves, 1973, see e.g. Krishna, 2002). The BMA outcome is shown to be socially efficient, individually desirable (bidders are not worse off than they were if the auction had not taken place), and the implied payment profile is bounded below by the VCG payments.

In the case without transfers, the BMA directly extends the proto-coalition model. The general results apply straightforwardly, and I show by example that the BMA is not individually desirable in the above sense, i.e. the "winning" bidders may commit to accept options where they are worse off than under the care-taker government. In turn, such pre-commitment is reasonable if it prevents alternative, subjectively worse government coalitions and illustrates why pre-commitments are relevant in the first place: they help to prevent or enable specific choices of the formateur. Yet, the formateur does not generally benefit from the possibility that bidders can pre-commit, and in the two examples discussed below, he is actually worse off than in the original proto-coalition model. In general, the predictions differ, and empirical research is required to investigate which kinds of pre-commitments actually matter in government formation.

Alternative cases where players articulate specific demands (e.g. that they are allocated a particular cabinet post or that person X is not appointed as prime minister), can be analyzed similarly. This is discussed briefly in the concluding Section 6.

2 Definitions

Notation The auctioneer is denoted as A and the set of bidders is $N = \{1, \ldots, n\}$, with $n < \infty$ and typical elements $i, j \in N$. Also define $N_A := N \cup \{A\}$. The choice set of A is finite, non-empty, and denoted as $\mathcal{R}$. For each $r \in \mathcal{R}$, the set of bidders whose agreement is required to implement r is denoted as $N(r) \subset N$. For example, in the context of government formation, $N(r)$ would be the government coalition, and in a single-object auction, $N(r)$ would be the singleton set containing the winning (and paying) bidder.

The valuations are denoted as $v_i : \mathcal{R} \to \mathbb{R}$, $i \in N_A$. They are non-degenerate in the sense that all players can rank any option, for which their agreement is required, in relation to any other option. That is, for all $i \in N$,

$$\forall r, r' \in \mathcal{R}: \quad i \in N(r) \ \wedge \ r \neq r' \quad \Rightarrow \quad v_i(r) \neq v_i(r'), \tag{1}$$

and correspondingly $v_A(r) \neq v_A(r')$ for all $r \neq r'$. In turn, bidders may be indifferent between options for which their agreement is not required. In government formation, valuations that are generic in this sense follow immediately from the assumption that the choice set is the set of government coalitions (the standard assumption in proto-coalition models, see Section 5). In games such as single-object Vickrey auctions, on the contrary, genericity does not seem so immediate, but in equilibrium, the winning bidder pays either the valuation v of the first loser or $v + \varepsilon$ (with ε being the smallest currency unit), and this depends on how the first loser and A act in cases of indifference. By assuming generic values, I implicitly assume that these cases of indifference are resolved, to be able to focus on the strategic analysis of BMAs.

The auctioneer A has an outside option, i.e. there exists $r \in \mathcal{R}$ such that $N(r) = \emptyset$ (e.g. that A resigns as government formateur or that he calls off the auction). The outside option is $r_{\min} \in \arg\min_r v_A(r)$, i.e. the option that A values the least. This is assumed without loss of generality, as options that A actually values less than his outside option can be eliminated from $\mathcal{R}$ without affecting the equilibrium outcome (as A would not choose them in any case).

To abbreviate notation, write $r \succ_i r'$ if $v_i(r) > v_i(r')$ ($i \in N_A$), and $r \succ_C r'$ if $r \succ_i r'$ for all $i \in C$ ($C \subseteq N_A$). Finally, for all $R \subseteq \mathcal{R}$ define $\min R$ and $\max R$ such that

$$\min R \in \arg\min_{r \in R} v_A(r) \qquad\qquad \max R \in \arg\max_{r \in R} v_A(r). \tag{2}$$

Sealed bid BMA First, the bidders $i \in N$ simultaneously submit acceptance sets, and second A chooses $r \in \mathcal{R}$ subject to acceptance by all $i \in N(r)$. For all $i \in N$, define $\mathcal{R}_i = \{r \in \mathcal{R} \mid i \in N(r)\}$. A pure strategy of $i \in N$ is $s_i : \mathcal{R}_i \to \{0,1\}$, where $s_i(r) = 1$ indicates that i accepts r and $s_i(r) = 0$ indicates that i does not. The choice $r^*(s)$ of A and the payoff $\pi_i(s)$ associated with $s = (s_i)_{i \in N}$ is

$$\pi_i(s) = v_i(r^*(s)) \qquad \text{where } r^*(s) = \max\big\{r \in \mathcal{R} \mid \forall i \in N(r) : s_i(r) = 1\big\}. \tag{3}$$

Randomized strategies are defined as behavior strategies $\sigma_i : \mathcal{R}_i \to [0,1]$. That is, the decisions with respect to any pair $r \neq r'$ are stochastically independent (players accept/reject options by sequentially filling in a form). Expected payoffs $\pi_i(\sigma)$ are defined correspondingly.

Dutch BMA The game proceeds in rounds, and in each round, A proposes an option $r \in \mathcal{R}$, and in response the players $i \in N(r)$ simultaneously vote on r. If all accept, then r is implemented, and otherwise a new round begins. A proposes the options in decreasing order (under $\succ_A$) and may skip options.[2] Let H_A denote the set of histories after which A has to propose an option and for all $h \in H_A$ let $\mathcal{R}(h)$ denote the set of proposals that A can make after history h. Pure strategies of A are denoted as

[2]This assumption generalizes single-object Dutch auctions, where A proposes prices in decreasing order. In more general contexts, sticking to decreasing order is not always optimal. Relaxing the order would lead to a generalized Dutch auction, however, while the conventional one is to be analyzed here.

$\sigma_A : H_A \to \mathcal{R}$ s.t. $\sigma_A(h) \in \mathcal{R}(h)$. Further, if H_i is the set of histories after which $i \in N$ has to vote and $r(h)$ denotes the standing proposal to be voted on, the pure strategies of $i \in N$ are $\sigma_i : H_i \to \{0,1\}$, with $\sigma_i(h) = 1$ indicating acceptance of $r(h)$.

The remainder specifies the (fairly standard) payoff functions in Dutch auctions. It can be skipped if it is not of explicit interest. Given a strategy profile $\sigma = (\sigma_i)_{i \in N_A}$ and a history $h \in H_A$ after which A is to make a proposal, the augmented history containing A's proposal and the votes made in response is denoted as

$$h^a = \big(h, \sigma_A(h), \big\{\sigma_i\big(h, \sigma_A(h)\big)\big\}_{i \in N(\sigma_A(h))}\big). \tag{4}$$

Using $\delta \in (0,1)$ as the discount factor, the payoff of $i \in N_A$ is (defined recursively)

$$\pi_i(\sigma|h) = \begin{cases} v_i\big(\sigma_A(h)\big), & \text{if } \sigma_i\big((h, \sigma_A(h))\big) = 1 \text{ for all } i \in N(\sigma_A(h)), \\ \delta \pi_i(\sigma|h^a), & \text{otherwise.} \end{cases} \tag{5}$$

The payoffs are well-defined, as A is eventually forced to propose the outside option (i.e. to end the auction), following which the condition that all requisite players accept is empty and thereby satisfied trivially. The extension to behavior strategies and the definition of expected payoffs are standard.

3 Characterization of sealed-bid and Dutch BMAs

The equilibrium outcome of the Dutch auction is determined by backward induction. The key to uniqueness is trembling-hand perfection, which enforces *sincere bidding*, i.e. that bidders give approval to some $r \in \mathcal{R}$ if and only if they prefer r to the continuation payoff. In conjunction with non-degenerateness sincere bidding allows us to backward induce the outcome uniquely.[3] In contrast, under subgame perfection (i.e. without trembling hands), players do not generally bid sincerely. For example, they may reject any proposal $r \in \mathcal{R}$ where $|N(r)| > 1$, and this holds even if all of them prefer r to the respective continuation outcome. In such cases, rejecting r is weakly dominated, but unilateral deviations to "accept" do not pay off without perfection.

[3] To be precise, sincere bidding additionally requires that current actions do not affect future tremble probabilities (i.e. that the tremble probabilities in a given state do not depend on the history of play leading to this state). This fairly intuitive assumption will be made as well.

To characterize the equilibrium outcome, let $R_{-\max} := R \setminus \{\max R\}$ for all $R \subseteq \mathcal{R}$, with $\max R$ as the payoff maximizing $r \in R$ in the eyes of A, see (2). Define $f : \mathcal{P}(\mathcal{R}) \to \mathcal{R}$ such that $f(R) \in R$ for all singleton sets $R \subseteq \mathcal{R}$ and

$$f(R) = \begin{cases} \max R, & \text{if } \max R \succ_{N(\max R)} f\big(R_{-\max}\big), \\ f\big(R_{-\max}\big), & \text{otherwise,} \end{cases} \tag{6}$$

for all non-singleton sets $R \subseteq \mathcal{R}$. The interpretation of f is as follows: if at any stage in the Dutch auction A's proposal set is $R \subseteq \mathcal{R}$, then the perfect equilibrium outcome is $f(R)$. Intuitively, if all players in $N(\max R)$ prefer $\max R$ over the outcome that results after eliminating $\max R$ from the possibility set, then they would accept $\max R$ when proposed (and hence A would propose it), and otherwise they would reject it, $\max R$ is strategically irrelevant, and $f\big(R_{-\max}\big)$ must result.

Proposition 3.1 (Dutch BMA)**.** *There exists* $\underline{\delta} < 1$ *such that for all* $\delta \in \big(\underline{\delta}, 1\big)$*, the perfect equilibrium outcome of the Dutch auction is* $f(\mathcal{R})$*.*

Proof. Define $R_1 = \{\min \mathcal{R}\}$ and for all $i > 1$, $R_k = R_{k-1} \cup \min(\mathcal{R} \setminus R_{k-1})$. Hence, for k high enough, $R_k = R_{k-1} = \mathcal{R}$. I claim that the following applies in the unique perfect equilibrium for all $k \geq 1$: after all histories $h \in H_A$ such that $\mathcal{R}(h) = R_k$, $f(R_k)$ results along the equilibrium path.[4] This claim is satisfied for $k = 1$, since $\min \mathcal{R}$ necessarily results if $\mathcal{R}(h) = \{\min \mathcal{R}\}$. The remainder shows that it holds for k if it holds for all $k' < k$. For contradiction, assume the opposite: some $r \neq f(R_k)$ results in a perfect equilibrium. First, consider the case $r = \max R_k$. Since $r \neq f(R_k)$, this implies $f(R_k) \neq \max R_k$. Define $r' := f(R_k)$. Under the induction assumption, if A proposes $\max R_k$ and the bidders reject it, then r' results. In addition, $r' \equiv f(R_k) \neq \max R_k$ implies $\max R_k \not\succ_{N(r)} r'$, i.e. there exists $i \in N(r)$ such that $\max R_k \prec_i r'$ (generically). Hence, under perfection (i.e. sincere voting, given $\delta \approx 1$) this $i \in N(r)$ does not accept $r = \max R_k$, which yields the contradiction. Second, consider the case $r \prec_A \max R_k$ and $f(R_k) = \max R_k$. An inversion of the previous argument yields the contradiction. Third, consider $r \prec_A \max R_k$, $f(R_k) \prec_A \max R_k$, and $f(R_k) \succ_A r$. Now, A can deviate profitably toward proposing $r' := f(R_k)$ in the considered subgame, following which r' results by the induction assumption. Finally, consider $r \prec_A \max R_k$,

[4]Recall that $\mathcal{R}(h)$ denotes the set of proposals that A can propose after history h.

$f(R_k) \prec_A \max R_k$, and $r \succ_A f(R_k)$. Here, $r' := f(R_k)$ implies $r \not\succ_{N(r)} r'$, i.e. there exists $i \in N(r)$ such that $r \prec_i r'$. For $\delta \approx 1$, this $i \in N(r)$ rejects r when it is proposed by A (under perfection), and since r must be proposed along the path of play to become the outcome, this yields the contradiction. By induction, the proof is completed for all k. □

The next proposition establishes outcome equivalence with sealed-bid BMAs. Let me start with two examples, however, to illustrate the underlying intuition and in particular to illustrate the obstacles in establishing outcome uniqueness of sealed-bid BMAs. The first example is a single-object auction with two bidders (B and C) under complete information. Let B's valuation of the object be 60 and the one of C be 40. The subgame-perfect equilibrium (SPE) outcomes of the Dutch auction are 40 and 41 (if $\varepsilon = 1$ is the smallest currency unit), depending on how C acts in cases of indifference, but in the sealed bid auction, the range of SPE outcomes is 40–60. Trembling-hand perfection refines both sets of equilibrium outcomes toward the singleton $\{40\}$ in this case. This example suggests that perfection possibly induces uniqueness in general, but as the next one shows, it is not generally sufficient. For, we also have to relax the assumption that strategies be "monotone."

Definition 3.2 (Monotone strategy)**.** Let $\sigma_i : \mathcal{R}_i \to \{0,1\}$ denote a sealed bid. It is monotone if there exists $d_i \in \mathbb{R}$ such that $\sigma_i(r) = 1 \Leftrightarrow v_i(r) \geq d_i$ for all $r \in \mathcal{R}_i$.

Bids in single-object auctions are monotone in this sense, as are truthful bids in menu auctions and demands in demand bargaining games. Now, consider the allocation of 100 dollars (or, cabinet posts) between A, B, and C in a Nash demand game. That is, bidders B and C state demand commitments $d_i \in \{0,1,\ldots,100\}$, and in response, auctioneer A chooses an allocation. All allocations $\mathbf{x} \in \mathbb{N}^3$ such that $x_A + x_B + x_C = 100$ are feasible, and unanimity is required (i.e. both demands must be met by the chosen allocation, if possible). In this game, all demand commitments satisfying $d_B + d_C = 100$ are compatible with perfection, i.e. perfection does not yield uniqueness. Nor does genericity. For example, if players have quasi-lexicographic preferences over the set of allocations $\mathbf{X}$, with the own payoff x_i being the primary criterion, then the multiplicity of equilibria is sustained without modification.

We will see, however, that additionally relaxing monotonicity induces uniqueness. This may be counter-intuitive at first glance, as relaxing monotonicity vastly increases the strategy space. In the Nash demand game, for example, all outcomes $\mathbf{x} \in \mathbf{X}$ can then result under subgame perfection, including the Pareto inefficient ones.[5] Furthermore, for any list of sealed bids σ_{-i} of the opponents of any player $i \in N$, one of i's best responses is monotone, i.e. relaxing monotonicity does not allow players to formulate better responses to their opponents.

However, it allows the players to formulate more robust responses, i.e. responses that are better under full support, and unraveling the implications of this effect yields outcome uniqueness. Perhaps surprisingly, the resulting equilibrium analysis turns out to mimic that of the Dutch auction, which leads to the outcome equivalence.

Proposition 3.3 (Sealed-bid BMA)**.** *The perfect equilibrium outcome of the sealed-bid BMA is $f(\mathcal{R})$.*

Remark 3.4. *Proposition 3.3 implies that the equilibrium outcome is in the core, i.e. there is no option $r \in \mathcal{R}$ that A and all players in $N(r)$ prefer to it. Hence, it is also Pareto efficient.*

Remark 3.5. *The BMA outcome $f(\mathcal{R})$ results equivalently in any T-round BMA, with $T < \infty$, where the BMA is repeated up to T rounds until A proposes an option $r \in \mathcal{R}$ that all $i \in N(r)$ accepted in the respective round (for a proof, see appendix).*

Proof of Proposition 3.3. The proof makes use of $f : \mathcal{P}(\mathcal{R}) \to \mathcal{R}$ as defined in Eq. (6). The following pure strategy profile is shown to be the unique perfect equilibrium.

1. For all $r \in \mathcal{R}$ and all $i \in N(r)$, i submits acceptance with respect to r if and only if $r \succsim_i r'$ for $r' = f\big(\{r'' | r \succsim_A r''\}\big)$.
2. A chooses the most favored option $r \in \mathcal{R}$ for which all requisite bidders submitted acceptance.

It is straightforward to verify that this strategy profile results in $f(\mathcal{R})$ along the path of play. Point 2 is trivial. Point 1 is proven by induction. It is true for $r = \min \mathcal{R}$,

[5] For any $\mathbf{x} \in \mathbf{X}$, an SPE inducing $\mathbf{x}$ is as follows: both B and C submit acceptance with respect to $\mathbf{x}$, they do not accept any $\mathbf{x}' \neq \mathbf{x}$, and thus A will have to choose $\mathbf{x}$.

since $N(\min \mathcal{R}) = \emptyset$ by assumption (hence Point 1 is empty with respect to $\min \mathcal{R}$). The following shows that Point 1 is satisfied for $r \in \mathcal{R}$ if it is satisfied for all $r' \in \mathcal{R}$ such that $r \succ_A r'$. Assume the opposite and let τ denote a perfect equilibrium (satisfying the induction assumption) where some $i \in N(r)$ acts differently than prescribed with respect to r. Fix this player $i \in N(r)$. By definition, τ is the limit point of ε-equilibria τ^{ε} as the profile of perturbations ε approaches zero. Fix ε close to zero, fix the corresponding ε-equilibrium τ^{ε}, and define the following terms: ε' is the probability that no $r'' \succ_A r$ can be implemented by A (under τ^{ε}), ε'' is the probability that all $j \in N(r) : j \neq i$ submit acceptance with respect to r, and v_i' is i's expected payoff of submitting rejection with respect to r conditional on the assumption that no $r'' \succ_A r$ can be implemented by A. Under the induction assumption, $\varepsilon \approx 0$ implies $v_i' \approx v_i(r')$ for $r' = f\big(\{r'' | r \succ_A r''\}\big)$, i.e. r' is the "next-best" outcome conditional on no $r'' \succsim_A r$ being implementable. Thus, i's decision with respect to r is relevant with probability $\varepsilon' \cdot \varepsilon'' > 0$, and for $\varepsilon \approx 0$ he is best off submitting acceptance if and only if $r \succ_i r'$ using $r' = f\big(\{r'' | r \succ_A r''\}\big)$. Generically, this is equivalent to the claim and thus contradicts the above assumption. By induction, Point 1 is established for all $r \in \mathcal{R}$. □

Briefly, let us look at the relation to English auctions. Assume for simplicity that jump bids are not possible. That is, A proposes all options $r \in \mathcal{R}$ in increasing order, the respective players $i \in N(r)$ state whether they agree to r getting implemented, and if all agree, then r becomes the "standing high bid." The result of the auction, i.e. the option that will be implemented, is the option that constitutes the standing high bid when all options had been proposed.

In standard contexts, bidders bid in English auctions simply for one reason: their bid might be the standing high bid even in the end (which has positive probability if they are incompletely informed or if we assume trembling-hand perfection). Hence, they bid if and only if they prefer the auction to end with their bid rather than with the current standing high bid. This strategy is myopic in the sense that it neglects the long-term implications of one's bid, but in standard auctions, being myopic is sufficient. In turn, if we assume that players are myopic, equivalence between English auctions and Dutch/sealed bid auctions can be established also for BMAs.

Definition 3.6. A myopic bidder in an English auction accepts option $r \in \mathcal{R}$ if and only if he prefers r to the standing high bid.

Lemma 3.7. *In the English auction, the equilibrium outcome if bidders are myopic is $f(\mathcal{R})$.*

Proof. The proof is very similar to the induction in Dutch auctions and skipped. $\square$

It is easy to see that a general equivalence between English and Dutch auctions cannot be established for farsighted bidders. This is not a characteristic of BMAs, but an implication of the completeness of information and the possible interdependence of valuations in our model: As players can backward induce who would outbid their bids, there is a strategic reason to deviate from myopic behavior.

4 Relation to VCG mechanisms in "transfer games"

This section consider auctions where monetary transfers from the bidders $i \in N$ to the auctioneer A are possible. As for this case, BMAs are shown to relate to VCG mechanisms in that their outcome is socially efficient, individually desirable, and induces the VCG payments at the lower bound. This holds, although a BMA constitutes a first-price auction, i.e. winners have to pay their bids. In contrast, in "menu auctions" as analyzed by Bernheim and Whinston (1986)—with bids rather than acceptance sets—comparable results have been established only for the significantly stronger assumptions of either truthful bidding or coalition proofness, while merely perfection is required in BMAs. Recall that truthful bidding in the sense of Bernheim and Whinston (1986) essentially restricts each bidder's strategy set to the choice of a utility demand d_i (as in Def. 3.2) instead of a menu of bids. In relation to their results, the present section shows that relaxing the requirement of monotonicity as in BMAs does not obstruct the desirable properties of menu auctions in "transfer games."

We establish these results for a general, abstract auction called "game of economic influence." Here, A auctions off a decision of himself to the bidders paying the most. Further discussion of such generalized auctions can be found in Bernheim and Whinston (1986) and more recently Nisan (2007). This framework extends multi-object auctions (which are briefly discussed at the end of this section) by allowing (amongst others) that valuations depend on the overall allocation of objects, as analyzed for example by Jehiel et al. (1996, 1999) and Jehiel and Moldovanu (2000, 2001), and thus it also applies to procurement auctions, bilateral trade, and contributions to public goods and public projects (see e.g. Nisan, 2007, p. 220ff, for a more comprehensive discussion of the applications). The model is particularly interesting for the present study, as it also provides a general representation of government formation in case transfers are possible.

The auctioneer's choice set is denoted as $\mathcal{A}$, which might be the set of possible government coalitions, and the profile of payments of the bidders to the auctioneer is $\mathbf{x} \in X \subset \mathbb{R}^N$, with

$$\mathbf{X} := \{\mathbf{x} \in \mathbb{R}^N \mid \forall i \in N \exists n_i \in \mathbb{N} : \; x_i = n_i \varepsilon\}. \tag{7}$$

The smallest currency unit is denoted as $\varepsilon > 0$, and bids as well as valuations are multiples of ε. In the following definition, I assume that budgets are essentially unlimited.

Definition 4.1 (Game of economic influence)**.** Define a non-empty choice set $\mathcal{A}$, non-negative valuations $u_i : \mathcal{A} \to \mathbb{R}$ for all $i \in N_A$, and budgets $\bar{x}_i > \max_{i' \in N} \max_{a \in \mathcal{A}} u_{i'}(a)$ for all $i \in N$. The auctioneer's choice set is

$$\mathcal{R} = \{(a, \mathbf{x}) \in \mathcal{A} \times \mathbf{X} \mid \forall i \in N \exists n_i \in \mathbb{N} : \; x_i = n_i \varepsilon \leq \bar{x}_i\},$$

and for all $(a, \mathbf{x}) \in \mathcal{R}$, define $N(a, \mathbf{x}) = \{i \in N \mid x_i > 0\}$ and the valuations

$$\forall i \neq A : \lfloor v_i(a, \mathbf{x}) \rfloor = (u_i(a) - x_i)/\varepsilon \quad \text{and} \quad \lfloor v_A(a, \mathbf{x}) \rfloor = u_A(a) + \sum_{i \in N} x_i/\varepsilon.$$

A few comments on the definition of the valuation functions may be helpful. Primarily, the valuations are stated in terms of the smallest currency unit ε (without loss, one can assume $\varepsilon = 1$, e.g. 1 cent), and *rounded down to the nearest integer* they equate with utility less expenses. The definition allows for perturbations after the decimal point in order to ensure that non-degenerateness can be satisfied. Formally, the perturbations encapsulate how the players decide in case they are otherwise indifferent. The following introduces a restriction on how indifference is resolved which allows me to avoid the plethora of case distinctions necessary otherwise. Variations of this assumption affect the equilibrium outcome merely on the order of ε, however, i.e.

the assumption becomes irrelevant as ε tends to zero. The restriction states that players prefer aggressive bids (i.e. in favor of the auctioneer) in case they are otherwise indifferent, i.e. for all $i \in N$,

$$\lfloor v_i(a,\mathbf{x}) \rfloor = \lfloor v_i(a',\mathbf{x}') \rfloor \quad \wedge \quad v_A(a,\mathbf{x}) > v_A(a',\mathbf{x}') \quad \Rightarrow \quad v_i(a,\mathbf{x}) > v_i(a',\mathbf{x}'). \quad (8)$$

Additionally, I assume $u_i(\emptyset) = 0$ for all $i \in N$, i.e. the utilities are normalized with respect to the outside option and the outside option is "bad" in the sense that no other option $a \in \mathcal{A}$ is worse in the eyes of any $i \in N$ (as u_i is non-negative). The "badness" assumption is a standard assumption in auction analyses, and it is made (and used) precisely for this reason, but note that it is less typical in government formation. For example, a party may prefer the care-taker government over government coalitions not including this party.

Define the set of socially efficient choices as

$$\mathcal{A}^{\text{eff}} := \arg\max_{a \in \mathcal{A}} \sum_{i \in N_A} u_i(a) \quad (9)$$

and assume for simplicity that it is a singleton. In case $u_A = 0$, i.e. if the auctioneer is indifferent with respect to the decision he makes, the VCG mechanism based on the "Clarke pivot rule" (see e.g. Nisan, 2007) results in the efficient allocation $a \in \mathcal{A}^{\text{eff}}$ and the payments

$$x_i^* = \max_{a \in \mathcal{A}} \sum_{j \in N \setminus \{i\}} u_j(a) - \sum_{j \in N \setminus \{i\}} u_j(a^*) \quad \forall i \in N. \quad (10)$$

If we additionally allow for $u_A \neq 0$, we obtain the generalized VCG payments

$$x_i^* = \max_{a \in \mathcal{A}} \sum_{j \in N_A \setminus \{i\}} u_j(a) - \sum_{j \in N_A \setminus \{i\}} u_j(a^*) \quad (11)$$

for all $i \in N$. The generalized payment vector $\mathbf{x}^*$ in case $u_A \neq 0$ is attained in VCG mechanisms if we include a dummy bidder equipped with A's preferences.

Proposition 4.2 (Games of economic influence)**.** *The BMA outcome $(a,\mathbf{x})$ satisfies social efficiency ($a \in \mathcal{A}^{\mathit{eff}}$), individual desirability ($u_i(a) \geq x_i$ for all $i \in N$), and $\mathbf{x} \geq \mathbf{x}^*$ with $\mathbf{x}^*$ as the VCG payments.*

Proof. The BMA outcome is characterized as a sequence $\{(a^k, \mathbf{x}^k)\}_{k\geq 1}$ of iteratively dominant options. I write that $(a, \mathbf{x})$ "dominates" $(a', \mathbf{x}')$ if $v_i(a, \mathbf{x}) > v_i(a', \mathbf{x}')$ for all $i \in \{A\} \cup N(a, \mathbf{x})$.

Individual desirability is established by showing $u_i(a^k) \geq x_i^k$ for all i and k. (i) $u_i(a^k) \geq x_i^k$ applies for all i and k where $i \notin N(a^k, \mathbf{x}^k)$. For, $u_i(a) \geq 0$ applies for all a and $x_i = 0$ follows from $i \notin N(a^k, \mathbf{x}^k)$. (ii) $u_i(a^k) \geq x_i^k$ for all i and $k = 1$. For, $(a^1, \mathbf{x}^1)$ is A's outside option by assumption, i.e. $N(a^1, \mathbf{x}^1) = \emptyset$, which implies $x_i^1 = 0$ for all i, and by $u_i \geq 0$, the claim follows. (iii) If $u_i(a^k) \geq x_i^k$ for all i, then $u_i(a^{k+1}) \geq x_i^{k+1}$ for all i. By (i), this applies for all $i \notin N(a^{k+1}, \mathbf{x}^{k+1})$, for all $i \in N(a^{k+1}, \mathbf{x}^{k+1})$, it follows from the iterated dominance $u_i(a^{k+1}) - x_i^{k+1} \geq u_i(a^k) - x_i^k$ and the induction assumption $u_i(a^k) - x_i^k \geq 0$ for all i. Completing the induction yields $u_i(a^k) \geq x_i^k$ for all i and k.

Social efficiency Assume the opposite, i.e. $a \notin \mathcal{A}^{\text{eff}}$. Again, let $\mathbf{x}$ denote the payment vector in equilibrium. Using $x_i \leq u_i(a)$ for all i, the assumed social inefficiency of a implies that there exists $a' \in \mathcal{A}^{\text{eff}}$ such that $\sum_i x_i < \sum_i u_i(a')$. Given (8), this implies that there exists $\mathbf{x}'$ such that $(a', \mathbf{x}')$ dominates $(a, \mathbf{x})$, which in turn contradicts the initial assumption that $(a, \mathbf{x})$ is the equilibrium outcome $f(\mathcal{R})$.

VCG payments: Assume the opposite, i.e. some $(a, \mathbf{x})$ results where a is socially efficient and $\mathbf{x} \ngeq \mathbf{x}^*$. Fix $i \in N$ such that $x_i < x_i^*$ and $a' \in \arg\max_{a'' \in \mathcal{A}} \sum_{j \in N_A \setminus \{i\}} u_i(a)$. By definition of $\mathbf{x}^*$ and $x_i < x_i^*$,

$$x_i + u_A(a) + \sum_{j \neq i} \left[u_j(a) - x_j\right] < u_A(a') + \sum_{j \neq i} \left[u_j(a') - x_j\right],$$

and using $\mathbf{x}' \in \mathbb{R}^N$ such that $x_i' = 0$ and $x_j' = \max\{0, u_j(a') - u_j(a) + x_j\}$ for all $j \neq i$,

$$u_A(a') + \sum_{i \in N} x_j' > u_A(a) + \sum_{i \in N} x_j \quad \text{and} \quad u_j(a') - x_j' \geq u_j(a) - x_j \quad \forall j \neq i : x_j' > 0$$

follows. Given (8), $(a', \mathbf{x}')$ dominates $(a, \mathbf{x})$, which contradicts the assumption that $(a, \mathbf{x})$ is the BMA outcome. □

That is, the general lower bounds for the individual payments are the VCG payments. The individual upper bounds correspond with the individual valuations of the efficient allocation (by individual desirability). The lower bound for the (aggregate) revenue is the sum of the VCG payments, but perhaps surprisingly, the general upper bound for the revenue is $\max_{a \neq a^*} \sum_{i \in N} u_i(a)$ (plus epsilon). In single-object auctions, the upper bound of the revenue equates with the second-highest valuation over all bidders (plus epsilon), but in multi-object auctions, it can be relatively close to the social welfare. To see how the auctioneer can extract almost all of the consumer surplus, recall that by Proposition 3.3, $f(\mathcal{R})$ is the final element $(a^K, \mathbf{x}^K)$ of a sequence $\{(a^k, \mathbf{x}^k)\}_{k \geq 1}^K$ where, for all $k > 1$, $(a^k, \mathbf{x}^k)$ "dominates" $(a^{k-1}, \mathbf{x}^{k-1})$. Then, if A proposes the options $\{(a^k, \mathbf{x}^k)\}_{k \geq 1}^K$ in increasing order (as in an English auction), myopic bidders accept all of them and eventually also $(a^K, \mathbf{x}^K)$. (If bidders are farsighted, then the auctioneer can use the Dutch format.) Given this, the proof that the revenue of $\max_{a \neq a^*} \sum_{i \in N} u_i(a)$ is the general upper bound reduces to showing that A's preferences may be such that the resulting sequence $\{(a^k, \mathbf{x}^k)\}_{k \geq 1}^K$ induces a revenue at the upper bound $\max_{a \neq a^*} \sum_{i \in N} u_i(a)$ without violating $\lfloor v_A(a, \mathbf{x}) \rfloor = u_A(a) + \sum_{i \in N} x_i / \varepsilon$.[6]

Briefly, let me also illustrate the relation to standard (multi-object) auctions. The set of objects to be auctioned off is denoted as O, it is non-empty and finite, and the set of possible allocations of the objects to the players is $\mathbf{A} = (N \cup \{A\})^O$. For example, for any $o \in O$ and $\mathbf{a} \in \mathbf{A}$, $a_o = i$ indicates that $i \in N$ is allocated object o, while $a_o = A$ indicates that A keeps object o. The utility that i derives from being allocated $O' \subseteq O$ is denoted as $u_i(O')$, but to abbreviate notation I write $u_i(\mathbf{a}) := u_i(\{o | a_o = i\})$ (preferences are not interdependent, however). To define the VCG payments in this case, it is conventional to first define the welfare over the subset of players $N' \subseteq N$ if allocations are restricted to the subset $\mathbf{A}' \subseteq \mathbf{A}$ as

$$W(N', \mathbf{A}') = \sum_{i \in N'} u_i(\mathbf{a}^*) \quad \text{with} \quad \mathbf{a}^* \in \arg\max_{\mathbf{a} \in \mathbf{A}'} \sum_{i \in N} u_i(\mathbf{a}). \tag{12}$$

Now, using $N_{-i} = N_A \setminus \{i\}$ and $\mathbf{A}_{-i} = (N_{-i})^O$ as the set of allocations over players in N_{-i}, the VCG payment of $i \in N$ is the "externality" that he imposes on his opponents:

$$x_i^* = W(N_{-i}, \mathbf{A}_{-i}) - W(N_{-i}, \mathbf{A}). \tag{13}$$

[6]The construction of such preferences is in general straightforward. Note only that options where all players pay more than the VCG prices are not dominated, (formally: if $a \in \mathcal{A}^{\text{eff}}$ and $\mathbf{x} \in \mathbf{X}$ such that $u_i(a) > 0 \Rightarrow x_i > x_i^*$ for all $i \in N$, then $(a, \mathbf{x})$ is undominated), which implies that the resulting sequence $\{(a^k, \mathbf{x}^k)\}_{k \geq 1}^K$ must circumvent such options.

Since this is a special of the general model considered above, Proposition 4.2 implies that the BMA outcome is socially efficient and induces at least the VCG payments Eq. (13) also in this case. For example, it implements the Vickrey auction if there is a single object.

5 Coalition formation without transfers

Aside from being a model of auctions with transfers, BMAs also allow us to model auctions without (monetary) transfers. In government formation, for example, bidders compete by offering participation in coalitions the formateur finds preferable. In other non-monetary auctions, bidders may offer favors, support in a committee, and the like. The following focuses on the case of government formation.

Assume that elections had been held, and A, the player recognized as coalition "formateur," is a third party without any seats in the parliament (the latter assumption can be altered easily). The parliament has 100 seats and the number of seats of any $i \in N$ is denoted as $w_i \in \mathbb{N}$, with $\sum_{i \in N} w_i = 100$. A's task is to propose a coalition $C \subseteq N$ that controls the majority of seats, i.e. such that $\sum_{i \in C} w_i > 50$. Hence, the choice set is $\mathcal{R} = \left\{ C \subseteq N | \sum_{i \in C} w_i > 50 \right\} \cup \{\emptyset\}$, with "$\emptyset$" as the outside option.

After having been chosen by A, the coalition C enters multilateral negotiations to allocate cabinet posts and to fix a political platform. Without restricting the protocol of these negotiations, we can assume that its outcome is anticipated correctly in equilibrium, and hence that all players have well-defined preferences over the set of majority coalitions $C \subseteq N$ that may enter negotiations. Let $u_i : \mathcal{R} \to \mathbb{R}$ denote the corresponding utility function for all $i \in N_A$.

In this case, where transfers from $i \in N$ to A, to influence A's decision, are impossible, we essentially arrive at the proto-coalition model of government formation. Here, the coalition C chosen by A is the "proto-coalition," and following A's choice, all $i \in C$ vote on entering negotiations within this proto-coalition (anticipating the negotiation outcome). The proto-coalition forms if all $i \in C$ accept, and otherwise a care-taker government assumes office (the outside option "$\emptyset$"). In contrast, BMAs allow us to model that players can pre-commit with respect to their coalition choices.[7]

Definition 5.1 (NTU government formation)**.** Given $u : \mathcal{P}(N) \to \mathbb{R}^{N\cup\{A\}}$, define $\mathcal{R} = \left\{C \subseteq N | \sum_{i\in C} w_i > 50\right\} \cup \{\emptyset\}$, and for all $C \in \mathcal{R}$ define both $N(C) = C$ and $v_i(C) = u_i(C)$, $i \in N \cup \{A\}$.

By Proposition 3.3, the BMA outcome of "NTU government formation" is generically unique, while uniqueness does not generally result if players state utility demands. This can be seen immediately if we assume that the outside option "$\emptyset$" is prohibitively bad, i.e. if all parties prefer any coalition over the care-taker government. In this case, the set of perfect equilibrium outcomes if players state utility demands d_i rather than acceptance sets is the set of all choices that are undominated in $\mathcal{R}$, i.e. generically the set

$$U(\mathcal{R}) := \left\{C \in \mathcal{R} \mid \nexists C' \in \mathcal{R} \forall i \in C' \cup \{A\} : \ v_i(C') > v_i(C)\right\}. \tag{14}$$

The proof is straightforward and therefore skipped (further discussion of utility demands in this particular context can be found in Bolle and Breitmoser, 2008). Interestingly, this same set of undominated options (called "quasi core") results in the infinite-horizon game with acceptance sets bidding (Breitmoser, 2010). That is, in a game with either infinite time horizon or utility demands, any undominated option may result, while a specific undominated option is isolated in the BMA, where players state acceptance sets with finite time horizon (see Remark 3.5, the extension from $T = 1$ to $T < \infty$ is possible).

Two examples are provided in Table 1. In these (hypothetical) games, A faces a five-party parliament, i.e. $N = \{1,2,3,4,5\}$, with seat shares $\mathbf{w} = (20,28,11,33,8)$. The parties are ordered from far left to far right, and their ideal political platforms are $\overline{\mathbf{p}} = (0.2,0.4,0.5,0.6,0.8)$. Assume that if the proto-coalition $C \subseteq N$, with $C \neq \emptyset$, forms, the negotiations imply that the government platform $p(C)$ is the weighted mean of the individual platforms, and that the cabinet posts are allocated propor-

[7] An alternative generalization of proto-coalition bargaining is considered in Breitmoser (2010), which analyzes various *infinite-horizon models* of proto-coalition bargaining (i.e. A can revise his coalition choice after rejections), some of which also allow for pre-commitments.

tionally to the voting weights (as in "Gamson's law," see Gamson, 1961).

$$p(C) = \sum_{i\in C} w_i \overline{p}_i / \sum_{i\in C} w_i \qquad\qquad x_i(C) = \begin{cases} w_i/\sum_{i\in C} w_i, & \text{if } i \in C \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

The valuations are, for all all $i \in N$, linearly increasing in the own number of cabinet posts and linearly decreasing in the distance between government platform and individual platform,

$$v_i(C) = (1-\alpha)x_i(C) - \alpha \cdot \left|\overline{p}_i - p(C)\right|,$$

and correspondingly, for formateur A (who is not allocated cabinet posts), it is

$$v_A(C) = 1 - \alpha \cdot \left|\overline{p}_A - p(C)\right|.$$

The formateur's ideal platforms are $\overline{p}_A = 0.3$ and $\overline{p}_A = 0.4$ in Tables 1a and 1b, respectively.

In Tables 1a and 1b, each line refers to a possible choice of A (a majority coalition), and his options are ordered according to his preferences (in decreasing order). Note that both examples satisfy genericity of valuations. Hence, the equilibrium analysis indeed follows directly from Proposition 3.3. The individual acceptance decisions are listed in the right-most column. "1" indicates acceptance, "0" indicates rejection, and "–" indicates that the respective player is not in the corresponding proto-coalition. The lines set in bold-face type refer to proto-coalitions that are accepted by all members, and the highest of them is chosen by A in equilibrium.

In case $\overline{p}_A = 0.3$, A appoints the proto-coalition $C = (1,2,5)$, and in case $\overline{p}_A = 0.4$, A appoints $(1,2,3)$. Either choice conflicts with A's preferences, as A actually prefers $(1,2,3)$ if his ideal platform is $\overline{p}_A = 0.3$ and he prefers $(1,2,5)$ if $\overline{p}_A = 0.4$. These conflicts are a consequence of the strategic interaction between A and the players $i \in N$ in BMAs. A cannot implement his respectively favored coalition because at least one of the players pre-commits not to accept it in equilibrium. In the equilibrium of the original model of proto-coalition bargaining, where parties cannot pre-commit credibly, A simply chooses the proto-coalition maximizing his utility subject to the participation constraint. In the above example, A's choices would be $(1,2,3)$ in case $\overline{p}_A = 0.3$ and $(1,2,4)$ in case $\overline{p}_A = 0.4$. In both cases, A would therefore be better off

Figure 1: Two examples of NTU government formation (as defined in the text)

(a) *A*'s ideal platform is 0.3

Coalition	Valuations						Accept
	A	1	2	3	4	5	
1,2,3	0.975	0.094	0.213	0.019	−0.125	−0.225	1, 0, 1, −, −
1,2,5	**0.957**	**0.086**	**0.243**	**−0.057**	**−0.107**	**−0.136**	**1**, **1**, −, −, **1**
1,2,3,5	0.948	0.047	0.207	0.034	−0.098	−0.138	0, 1, 1, −, 1
1,2,4	0.934	0.007	0.157	−0.034	0.12	−0.184	0, 1, −, 0, −
1,2,3,4	0.93	−0.011	0.132	0.03	0.099	−0.18	0, 1, 1, 0, −
1,4	**0.925**	**0.064**	**−0.025**	**−0.025**	**0.236**	**−0.175**	**1**, −, −, **1**, −
1,3,4	0.921	0.027	−0.029	0.065	0.187	−0.171	1, −, 1, 0, −
1,2,4,5	0.917	−0.02	0.125	−0.017	0.118	−0.122	1, 0, −, 0, 1
1,2,3,4,5	0.916	−0.035	0.105	0.04	0.1	−0.125	1, 0, 1, 0, 1
1,4,5	0.902	0.016	−0.048	−0.002	0.218	−0.087	1, −, −, 0, 1
1,3,4,5	0.902	−0.009	−0.048	0.074	0.177	−0.097	1, −, 1, 0, 1
2,3,4	0.897	−0.153	0.141	0.073	0.183	−0.147	−, 0, 1, 0, −
2,4	**0.896**	**−0.154**	**0.175**	**−0.004**	**0.225**	**−0.146**	−, **1**, −, **1**, −
2,3,4,5	0.882	−0.168	0.107	0.051	0.174	−0.082	−, 1, 1, 1, 0
2,4,5	0.879	−0.171	0.132	−0.021	0.21	−0.071	−, 1, −, 1, 0
3,4,5	0.845	−0.205	−0.105	0.051	0.312	−0.018	−, −, 1, 1, 0
∅	0	0	0	0	0	0	

(b) *A*'s ideal platform is 0.4

Coalition	Valuations						Accept
	A	1	2	3	4	5	
1,2,3,5	0.998	0.047	0.207	0.034	−0.098	−0.138	0, 0, 1, −, 1
1,2,5	0.993	0.086	0.243	−0.057	−0.107	−0.136	0, 1, −, −, 1
1,2,4	0.984	0.007	0.157	−0.034	0.12	−0.184	0, 0, −, 1, −
1,2,3,4	0.98	−0.011	0.132	0.03	0.099	−0.18	0, 0, 1, 1, −
1,4	0.975	0.064	−0.025	−0.025	0.236	−0.175	0, −, −, 1, −
1,2,3	**0.975**	**0.094**	**0.213**	**0.019**	**−0.125**	**−0.225**	**1**, **1**, **1**, −, −
1,3,4	0.971	0.027	−0.029	0.065	0.187	−0.171	1, −, 1, 0, −
1,2,4,5	0.967	−0.02	0.125	−0.017	0.118	−0.122	1, 0, −, 0, 1
1,2,3,4,5	0.966	−0.035	0.105	0.04	0.1	−0.125	1, 0, 1, 0, 1
1,4,5	0.952	0.016	−0.048	−0.002	0.218	−0.087	1, −, −, 0, 1
1,3,4,5	0.952	−0.009	−0.048	0.074	0.177	−0.097	1, −, 1, 0, 1
2,3,4	0.947	−0.153	0.141	0.073	0.183	−0.147	−, 0, 1, 0, −
2,4	**0.946**	**−0.154**	**0.175**	**−0.004**	**0.225**	**−0.146**	−, **1**, −, **1**, −
2,3,4,5	0.932	−0.168	0.107	0.051	0.174	−0.082	−, 1, 1, 1, 0
2,4,5	0.929	−0.171	0.132	−0.021	0.21	−0.071	−, 1, −, 1, 0
3,4,5	0.895	−0.205	−0.105	0.051	0.312	−0.018	−, −, 1, 1, 0
∅	0	0	0	0	0	0	

if the parties would not be able to pre-commit credibly, while at least one of the parties would be worse off. In general, the parties benefit from the possibility to pre-commit individually, which explains why it can be observed in practice.

Also, note how the BMA rationalizes strategic delegation in government formation, and in fact the appointment of formateurs in the first place. The formateur is usually appointed by the president, and as can be seen in both examples above, the president may gain by appointing a formateur with preferences different from his own. This is generally not optimal in the standard proto-coalition model, i.e. when parties are assumed to be unable pre-commit credibly.

Finally, the coalition $C = (1,2,5)$ chosen in case $\overline{p}_A = 0.3$ is unconnected and includes both of the extreme parties. The prediction of such outcomes (which are observed empirically) obviously depends on the form of the utility function. For example, if utility is quadratic (rather than linear) in the distance between government platform and individual platform, such outcomes would be predicted comparably rarely. Then again, the functional form of utility in government formation is to be determined by empirical research, and to estimate utility functions, structural modeling is required. Arguably, due to their simple two-stage move structure, BMAs provide a convenient basis for structural modeling (following e.g. Turocy, 2005, 2010).

6 Conclusion

In this paper, I analyzed a general model of auctions under complete information that allows for both monetary and non-monetary bids. The auction is based on the notion of acceptance set bidding, i.e. players indicate which choices of the auctioneer they would accept, and the auctioneer then maximizes utility (subject to acceptance). The equilibrium outcome has been shown to be generically unique and it has been characterized by a simple, but general program. The outcome was shown to be equivalent in sealed-bid and Dutch formats of the BMA. In auctions with monetary transfers, BMAs relate closely to VCG mechanisms (see Section 4), and in games without transfers, they relate closely to proto-coalition bargaining (see Section 5). The BMA itself is more general, however, as it allows players to bid by offering arbitrary transfers to influence the decision maker, and hence it provides a general framework for analyses of economic influence.

As for government formation, on which I focused in this study, an interesting next step would be the application of the model in empirical analyses. Following the structural approach developed by Diermeier et al. (2002, 2003, 2007), where the formateur's decision is modeled using a logit choice function, the BMA can be analyzed using the agent logit equilibrium (McKelvey and Palfrey, 1998), which is well-established and technically straightforward (in particular, if one follows Turocy, 2010). This structural approach would allow researchers to estimate functional forms and parameterization of utility in government formation, possibly also to distinguish credible and non-credible pre-commitments, and finally to discriminate between the two extensive forms with finite time horizon (as discussed here) and infinite time horizon (as discussed in Breitmoser, 2010). The BMAs is sufficiently general to analyze these questions, and based on the results of such empirical work, it might be possible to analyze refined theoretical models.

References

Bag, P. and Winter, E. (1999). Simple Subscription Mechanisms for Excludable Public Goods. *Journal of Economic Theory*, 87(1):72–94.

Baron, D. and Diermeier, D. (2001). Elections, governments, and parliaments in proportional representation systems. *The Quarterly Journal of Economics*, 116(3):933–967.

Bernheim, B. and Whinston, M. (1986). Menu auctions, resource allocation, and economic influence. *Quarterly Journal of Economics*, 101(1):1–31.

Bolle, F. and Breitmoser, Y. (2008). Coalition formation, agenda selection, and power. In Braham, M. and Steffen, F., editors, *Power, Freedom, and Voting*, chapter 10, pages 185–208. Springer.

Breitmoser, Y. (2009). Demand commitments in majority bargaining or how formateurs get their way. *International Journal of Game Theory*, 38(2):183–191.

Breitmoser, Y. (2010). Proto-coalition bargaining and the core. MPRA working paper.

Caruana, G., Einav, L., and Quint, D. (2007). Multilateral bargaining with concession costs. *Journal of Economic Theory*, 132(1):147–166.

Clarke, E. (1971). Multipart pricing of public goods. *Public Choice*, 11(1):17–33.

Diermeier, D., Eraslan, H., and Merlo, A. (2002). Coalition governments and comparative constitutional design. *European Economic Review*, 46(4-5):893–907.

Diermeier, D., Eraslan, H., and Merlo, A. (2003). A structural model of government formation. *Econometrica*, 71(1):27–70.

Diermeier, D., Eraslan, H., and Merlo, A. (2007). Bicameralism and government formation. *Quarterly Journal of Political Science*, 2:227–252.

Diermeier, D. and Merlo, A. (2000). Government turnover in parliamentary democracies. *Journal of Economic Theory*, 94(1):46–79.

Gamson, W. (1961). A theory of coalition formation. *American Sociological Review*, pages 373–382.

Groves, T. (1973). Incentives in teams. *Econometrica*, 41(4):617–631.

Jehiel, P. and Moldovanu, B. (2000). Auctions with downstream interaction among buyers. *RAND Journal of Economics*, pages 768–791.

Jehiel, P. and Moldovanu, B. (2001). Efficient design with interdependent valuations. *Econometrica*, 69(5):1237–1259.

Jehiel, P., Moldovanu, B., and Stacchetti, E. (1996). How (not) to sell nuclear weapons. *American Economic Review*, 86(4):814–829.

Jehiel, P., Moldovanu, B., and Stacchetti, E. (1999). Multidimensional mechanism design for auctions with externalities. *Journal of Economic Theory*, 85(2):258–293.

Krishna, V. (2002). *Auction theory*. Academic press.

McKelvey, R. and Palfrey, T. (1998). Quantal response equilibria for extensive form games. *Experimental Economics*, 1(1):9–41.

Merlo, A. and Wilson, C. (1995). A stochastic model of sequential bargaining with complete information. *Econometrica*, 63(2):371–399.

Merlo, A. and Wilson, C. (1998). Efficient delays in a stochastic model of bargaining. *Economic Theory*, 11(1):39–55.

Montero, M. and Vidal-Puga, J. (2010). Demand bargaining and proportional payoffs in majority games. *Games and Economic Behavior*.

Montero, M. and Vidal-Puga, J. J. (2007). Demand commitment in legislative bargaining. *American Political Science Review*, 101(4):847–50.

Morelli, M. (1999). Demand competition and policy compromise in legislative bargaining. *The American Political Science Review*, 93(4):809–820.

Mutuswami, S. and Winter, E. (2002). Subscription Mechanisms for Network Formation. *Journal of Economic Theory*, 106(2):242–264.

Nash, J. (1950). The bargaining problem. *Econometrica*, 18(2):155–162.

Nisan, N. (2007). Introduction to mechanism design (for computer scientists). In Nisan, N., Roughgarden, T., Tardos, E., and Vazirani, V. V., editors, *Algorithmic game theory*, pages 209–242. Cambridge University Press.

Turocy, T. (2005). A dynamic homotopy interpretation of the logistic quantal response equilibrium correspondence. *Games and Economic Behavior*, 51(2):243–263.

Turocy, T. (2010). Computing sequential equilibria using agent quantal response equilibria. *Economic Theory*, 42(1):255–269.

Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance*, 16(1):8–37.

Vidal-Puga, J. (2004). Bargaining with commitments. *International Journal of Game Theory*, 33(1):129–144.

Winter, E. (1994a). The demand commitment bargaining and snowballing cooperation. *Economic Theory*, 4(2):255–273.

Winter, E. (1994b). Non-cooperative bargaining in natural monopolies. *Journal of Economic Theory*, 64:202–220.

Binomial menu auctions: Supplementary proofs

Existence of a monotone best response (see page 10)

Lemma A.1. *In sealed-bid BMAs, for all $i \in N$ and any list of sealed bids σ_{-i} of i's opponents, one of i's best responses is monotone.*

Proof. For contradiction assume the opposite, i.e. that all best responses are non-monotone, let σ_i' denote a best response, and define r' as the outcome that A will implement in response to (σ_i', σ_{-i}). I claim that the monotone strategy σ_i based on $d_i := v_i(r')$ must also be a best response to σ_{-i}. To see this, note that A's choice in response to either (σ_i', σ_{-i}) or (σ_i, σ_{-i}) satisfies (under subgame perfection)

$$r' \in \underset{r \in R(\sigma_i', \sigma_{-i})}{\arg\max}\; v_A(r) \qquad\qquad r'' \in \underset{r \in R(\sigma_i, \sigma_{-i})}{\arg\max}\; v_A(r) \tag{16}$$

using $R(\sigma_i', \sigma_{-i}) = \{r \in \mathcal{R} | \sigma_i(r) = 1 \forall i \in N(r)\}$. By construction, $v_i(r) < d_i$ for all $r \in R(\sigma_i', \sigma_{-i}) \setminus R(\sigma_i, \sigma_{-i})$ and $v_i(r) > d_i$ for all $r \in R(\sigma_i, \sigma_{-i}) \setminus R(\sigma_i', \sigma_{-i})$. This implies $v_i(r') \leq v_i(r'')$. Since σ_i' is a best response by assumption, $v_i(r') \geq v_i(r'')$, and thus $v_i(r') = v_i(r'')$, i.e. both σ_i' and σ_i are best responses (the contradiction). $\square$

T-round BMA (see Remark 3.5)

Definition A.2 (T-round BMA)**.** The game proceeds for up to T rounds. In each round $t \leq T$, all players $i \in N$ submit acceptance sets $s_i : \mathcal{R}_i \to \{0,1\}$ and A responds by proposing some $r \in \mathcal{R}$. The game ends with outcome r if $s_i(r) = 1$ for all $i \in N(r)$, it ends with the outcome $\min \mathcal{R}$ if there exists $i \in N(r)$ such that $s_i(r) = 0$ and round $t = T$ is reached, and a new round begins otherwise.

In the following, I assume that players do not discount future payoffs, but the result holds equivalently for discount factors sufficiently close to 1.

Lemma A.3. *In any perfect equilibrium of the T-round BMA, for any $T < \infty$, $f(\mathcal{R})$ results along the path of play.*

Proof. By Proposition 3.3, the lemma holds true for round $t = T$. The following shows that it holds true for round $t < T$ if it holds true for round $t' = t+1$. Assume that a perfect equilibrium exists violating the claim and let $r \neq f(\mathcal{R})$ denote the respective equilibrium outcome in round t. By the induction assumption, $r^* = f(\mathcal{R})$ is the continuation outcome. Since $r \neq r^*$, either $r \succ_A r^*$ or $r^* \succ_A r$ applies. If $r^* \succ_A r$, then A is better off deviating unilaterally from proposing r toward proposing r^* in round t. For, it necessarily implies that $r^* \succ_A r$ results. Alternatively, if $r \succ_A r^*$, define $R \subset \mathcal{R}$ as the set of options $r' \succ_A r^*$ that all $i \in N(r')$ signaled to accept in round t. Define $r' = \min R$ as the least preferable of these options in the eyes of A. By definition of $r^* \equiv f(\mathcal{R})$, there exists $i \in N(r)$ such that $r^* \succ_i r'$ (r^* is undominated), which implies that this player $i \in N$ is better off deviating unilaterally from pre-committing to accept r' toward rejecting it. Under full support, i's decision with respect to r' is relevant with positive probability, and if it relevant, i is better off rejecting r', as this implies that r^* results (either in round t or in $t+1$, as it is the continuation outcome by assumption). By induction, the lemma therefore holds for all t. $\square$

On the claim made in Footnote 6

Lemma A.4. *If $a \in \mathcal{A}^{eff}$ and $\mathbf{x} \in \mathbf{X}$ such that $u_i(a) > 0 \Rightarrow x_i > x_i^*$ for all $i \in N$, then $(a, \mathbf{x})$ is undominated.*

Proof. Assume the opposite, i.e. that some $(a', \mathbf{x}') \in \mathcal{R}$ dominates $(a, \mathbf{x})$. First, consider the case $a' = a$. Dominance requires $\mathbf{x}' \neq \mathbf{x}$, but A prefers $\mathbf{x}$ if $\sum_{i \in N} x_i' < \sum_{i \in N} x_i$, and some $i \in N$ with $x_i' > 0$ prefers $\mathbf{x}$ otherwise. Second, in case $a' \neq a$, $\sum_{i \in N_A} u_i(a') < \sum_{i \in N_A} u_i(a)$ follows by efficiency of a. Since dominance requires $u_A(a') + \sum_{i \in N} x_i' \geq u_A(a) + \sum_{i \in N} x_i$, this implies

$$\sum_{i \in N} \left[u_i(a') - x_i' \right] < \sum_{i \in N} \left[u_i(a) - x_i \right]. \tag{17}$$

Hence, there exist $i \in N$ such that $u_i(a') - x_i' < u_i(a) - x_i$. Define $N' \subset N$ as the set of all $i \in N$ such that $u_i(a') - x_i' < u_i(a) - x_i$, which by the previous observation is non-empty. The initially assumed dominance can be satisfied only if $i \notin N(a', \mathbf{x}')$ for all $i \in N'$. For all $i \in N'$, this implies $x_i' = 0 \Rightarrow u_i(a') - x_i' \geq 0 \Rightarrow u_i(a) - x_i > 0 \Rightarrow$

$u_i(a) > 0$, and hence $x_i > x_i^* \geq 0$ by assumption. Thus

$$\sum_{j\in N\setminus N'} x'_j = \sum_{j\in N} x'_j \geq \sum_{j\in N} x_j > \sum_{i\in N'} x_i^* + \sum_{j\in N\setminus N'} x_j$$

where, using $\sum_{j\in N_A\setminus\{i\}} u_j(a') \leq \max_{a''\in\mathcal{A}} \sum_{j\in N_A\setminus\{i\}} u_j(a'')$ for all $i \in N'$,

$$\begin{aligned}
\sum_{i\in N'} x_i^* &= \sum_{i\in N'} \left(\max_{a''\in\mathcal{A}} \textstyle\sum_{j\in N_A\setminus\{i\}} u_j(a'') - \sum_{j\in N_A\setminus\{i\}} u_j(a)\right) \\
&\geq \sum_{i\in N'} \sum_{j\in N_A\setminus\{i\}} \left[u_j(a') - u_j(a)\right] \\
&\geq \sum_{j\in N_A\setminus N'} \left[u_j(a') - u_j(a)\right] + \left(|N'| - 1\right) \sum_{j\in N_A} \left[u_j(a') - u_j(a)\right] \\
&\geq \sum_{j\in N_A\setminus N'} \left[u_j(a') - u_j(a)\right]
\end{aligned}$$

and combined this yields

$$\sum_{j\in N_A\setminus N'} u_j(a') - \sum_{j\in N\setminus N'} x'_j < \sum_{j\in N_A\setminus N'} u_j(a) - \sum_{j\in N\setminus N'} x_j.$$

By dominance, i.e. by $(a', \mathbf{x}') \succ_A (a, \mathbf{x})$,

$$u_A(a') + \sum_{i\in N\setminus N'} x'_i = u_A(a') + \sum_{i\in N} x'_i \geq u_A(a) + \sum_{i\in N} x_i > u_A(a) + \sum_{i\in N\setminus N'} x_i,$$

and hence there must exist $j \in N\setminus N'$ such that $u_j(a') - x'_j < u_j(a) - x_j$, which contradicts the initial assumption N' contains all $i \in N$ such that $u_i(a') - x'_i < u_i(a) - x_i$. $\square$