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Abstract. Following a recent paper by Fragnelli and Marina (2009) concerned with the susceptibility of the Steinhaus-Knaster procedure for discrete fair division problems to insincere representations of the agents' preferences, the present short note shows that modified procedures proposed in the literature present a reduced degree of vulnerability to strategic manipulations.
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In a recent contribution Fragnelli and Marina (2009) analyze the effects of strategic bidding under the sealed bids procedure for discrete fair division problems originally proposed by Steinhaus (1946) and Knaster (1946), taking into account the possibility of information exchange among the agents. As a result of their analysis the Authors conclude that the procedure "is a manipulable mechanism ... each agent has the possibility of a declaration for an object that increases his payoff w.r.t. the true valuation ... two agents may improve their payoffs with coordinated false declarations ... completely risk averse agents [may obtain] a total gain that does not depend on the declarations of the other agents".
The purpose of the present short note is to propose a minor extension of the above investigation showing that an appropriate modification of the mechanism of distribution of the revenue after the auction ensures not only envy free allocations in case of sincere bidding – a property not satisfied by the original method – but also a reduced degree of vulnerability to strategic manipulations.

To this end, let us denote by N the number of agents and by q_i the fraction of the set of items to be divided that agent i is entitled to, $i = 1, 2,..., N$; moreover, assuming, for sake of simplicity and without loss of generality, that there is just one (indivisible) item to be assigned, let us denote by b_i the sealed bid of agent i, where $b_1 \geq b_2 \geq ... \geq b_N$ (possibly after a renumbering of the agents) and recall that in the original procedure as a result of the auction step the item is assigned to the highest bidder (at random, if there is a tie for the greatest bid) who pays

$$p_1 = (1 - q_1)b_1 - q_1(b_1 - m)$$

and each agent $j, j > 1$, receives the compensatory cash amount

$$p_j = q_j b_j + q_j(b_1 - m)$$

where $m = q_1 b_1 + ... + q_N b_N$. Under sincere bidding the resulting allocation is proportional and efficient but, in general, not envy free.
The class of modified procedures to be considered (see Chisholm 2000; see also Willson 2003, Corradi and Corradi 2005) is based on a *k*-double auction (see Cramton,

Gibbons and Klemperer 1987): the item is assigned to the highest bidder; in the subsequent cash transfer distribution the winning bidder pays $p_1 = (1 - q_1)X$, where X is a convex combination of b_2 and b_1, $X = (1 - k)\, b_1 + kb_2$, $0 \le k \le 1$, and each agent j, $j > 1$, receives the compensatory cash amount $p_j = q_j X$; in particular,

a_1) $X = b_1$ (as in a standard first-bid auction),

a_2) $X = b_2$ (as in a Vickrey second-bid auction).
The resulting division is efficient and envy free.
Now let us examine possible effects of insincere bidding.

a_1) In this case the payoff of each agent, except the winning bidder, is independent of his own bid, so he has no incentive at all to misrepresent his valuation in order to obtain any gain. As for the winning bidder, only in case he acquires reliable information about the bids of all his opponents he can safely increase his payoff by insincerely submitting a bid slightly above the second highest bid, if the highest bid is unique; if there is a tie for the greatest bid, e.g. $b_1 = b_2 > b_3$, then agent 1 and agent 2 could collude, agreeing to insincerely submit a bid slightly above the second highest bid, say $b_3 + \varepsilon$, with $\varepsilon < b_1 - b_3$, and consequently sharing the total gain

$$(1 - q_1 - q_2)(b_1 - b_3 - \varepsilon) .$$

(We note, however, that in this scenario each agent could find it in his interest to defect from the agreement since by raising his bid slightly above the agreed price he can rule out the possibility that the item be assigned to the rival.)

a_2) In this case the price that the winning bidder pays is determined by the competitors' bids alone and does not depend on any action the bidder himself can undertake. If any agent j has reliable information about any other agent h's bid, with $b_h > b_j$, then he has some incentive to insincerely bid slightly less than b_h : indeed, if b_h turns out to be the highest bid, then such a manipulation has the effect of maximizing the second price and consequently the payoff of all agents except the winning bidder, otherwise it has no effect. It may be pointed out that while in general, under the assumption that the strategies of bidders are limited to the submission of bids, each bidder has a well defined best bid (his true valuation) regardless of how he believes his rivals will bid, in the present context, in which each agent is simultaneously a seller (*pro-quota*) and a potential buyer (via his bid) the incentives encouraging strategic bidding do not disappear.

It is clear that for different values of X, with $b_2 < X < b_1$, the resulting envy free methods mix aspects of both first-price and second-price auctions and, with reference to strategic manipulations, they undergo the limitations of both.

Note. According to a suggestion by Chisholm, cit., the auction stage of the envy-free methods could be conducted in formats other than a sealed bid auction, e.g. by holding an English open outcry auction. Now, different auction formats are known to lead to different results in terms of viability and profitability of collusion, see Graham and Marshall (1987), Marshall and Marx (2007), Marshall and Marx (2008). In any case, in the present context using the envy free methods neither individual nor collusive manipulations by completely risk averse agents allow of safe gains regardless of the other agents' bids. □

To summarize, we can conclude that, in comparison with the original Steinhaus-Knaster procedure, the modified procedures not only yield envy free allocations under sincere bidding, but also are less vulnerable to manipulative conducts. In particular, from the foregoing discussion it follows that in any case no room is left to safe profitable collusions among completely risk averse agents: moreover, method a_1 fails to induce sincere revelations of the agents' valuations only in the extreme (somewhat unrealistic) case in which the winning bidder has exact knowledge of the bids of all other agents, and even in this case only the winner turns out to be tempted to adopt an exploitative strategy: as noted by Willson, cit., p. 259, in real life situations this gives a definite advantage over methods where in principle all agents have incentives to misrepresent their valuations.

Addendum. The unique *equitable* variant of the Steinhaus-Knaster procedure – named "Adjusted Knaster" by Raith (2000) (see also Chisholm, cit., Corradi and Corradi 2002, Sanchez 2002) – presents the same vulnerability to cheating as the original procedure. Indeed, the winning bidder pays $p_1 = (1 - q_1 b_1 / m) b_1$ for the item assigned and each agent j, $j > 1$, receives the compensatory amount $p_j = (q_j b_j / m) b_1$: it is readily seen that p_j is a strictly increasing function of b_j, p_k is a strictly decreasing function of b_j, $k \neq j$, so that the main points raised by Fragnelli and Marina still hold. In particular, each agent maintains a double incentive to bid insincerely, to increase his payoff and to protect himself from losing money as a consequence of other agents' strategic bidding.

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