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An Evolutionary Game Approach to the Issues of Migration, Nationalism, Assimilation and Enclaves

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Abstract

I use evolutionary game theory to address the relation between nationalism and immigration, studying how two different populations in a country, one composed of national citizens and the other of immigrants, evolve over time. Both populations depart from some polymorphic initial state. A national citizen may behave either nationalistically or may welcome immigrants. Immigrants may have an interest in learning the host country language or not. I also account for the presence of enclaves, which make the immigrants' own population effects important. The results show that six types of evolutionary equilibria are possible, although they never co-exist in the state space. A low cost of learning the host country language leads to complete assimilation of immigrants over time. Enclaves make assimilation a less competitive strategy. A high cost of learning may lead to peaceful multiculturalism or to political instability depending on the ability of policy makers to prevent nationalistic attitudes.

Keywords: Evolutionary games, replicator dynamics, migration, nationalism, enclaves.

JEL classification: C62; C72; C73; F22; F52.

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1 Introduction

In this paper, I analyze the long run effect on the behaviour of individuals resulting from a continuous relationship between nationals and immigrants, with a focus on the case in which both nationalism and cultural barriers co-exist. Nationalism and immigration have been very closely related issues in Europe for the last decade. Nationalist parties have been growing in many countries across the European Union (EU) with some of them already having obtained seats in the European Parliament.[1] In the UK, the British National Party (BNP) has increasingly been present in the media and has a strong anti-immigration policy. The main issue behind the BNP's immigration policy is the fact that about 3.7 million immigrants have entered the UK since 1997, the vast majority from non-EU countries. The party also states that 20% of the current UK residents were either born overseas or from foreign parents. According to the BNP, this strong immigration inflow is the main contributing factor to the loss of what is referred to as British indigenous identity over the past decades and is creating a population of "white Britons second-class citizens".

To address the relation between nationalism and immigration, I study how two different populations in a country, one composed of national citizens and the other of immigrants, evolve over time given both depart from some polymorphic[2] initial state and assuming that among the national citizens some individuals behave in a nationalistic (chauvinist) way, while others simply welcome immigrants. The latter population has individuals who have an interest in becoming assimilated. They make an effort to achieve this by learning the language of the host country (used in this paper as a proxy for cultural assimilation), while other immigrants simply do not have this interest and do not learn at all. Hence, the main objective is to understand how the presence of different levels of nationalism and assimilation, together with the existence of immigrants' enclaves, affects the equilibrium states of both populations. Depending on the interaction among these factors, we may end up with monomorphic or polymorphic populations. I assume two cases: a first one in which immigrants live dispersed across the country, making interactions within their own population insignificant. Later, I extend this model to allow for the case in which a significant number of immigrants live in an enclave and interactions among them are more likely to occur.

The use of language assimilation as a proxy for the acquisition of the host country's culture by immigrants is justified by the fact that language and religion have been the two most important cultural infrastructures serving as bases for national differentiation and modern demands of autonomy in Europe [Shafir (1995)].

The importance of investigating the influence of enclaves in the process of immigrants' assimi-

[1] In the 2009 European Parliament (EP) election, the French Front National won 3 seats and the British National Party got 2 seats. Other European nationalist political parties include the Italian Fiamma Tricolore (1 seat in the 2004 EP election), the Spanish Democracia Nacional, the Portuguese Partido Nacional Renovador, among others.

[2] Polymorphism is simply heterogeneity. From the literature on Biology, polymorphism is related to the existence of more than one phenotype, which are the organism's observable properties or behaviours. One example with respect to behaviour would be the hawk and the dove behaviours in the classical hawk-dove game and an example with respect to phisical property would be the spotted and the melanistic (also known as black) jaguars.

lation follows from the empirical literature on migration. Lazear (1999) found a negative correlation between immigrants' fluency in the host country language and concentration in his study for the US. The same kind of behaviour was found by Chiswick and Miller (2001) using data from the 1991 Canadian census. In Toronto, where a concentration of Portuguese speakers exists, after 15 years living in Canada, 35% of the latter are able to speak English or French and make use of the host country language even at home. The same figure rises to 42% for the case of immigrants living in Canadian areas where zero percent of the population speak the language of their country of origin.

In the framework I adopt, the country always keeps shares of nationals and immigrants over time. What changes over time (over generations) is their attitude toward nationalistic ideas and learning, respectively. Hence, I am interested in analyzing whether immigrants become assimilated or not, without being concerned if they get fully incorporated into the national collective at all. Using the terminology of Shafir (1995), incorporation into the host country is composed by two stages: assimilation, which means cultural incorporation through language learning and marrying local spouses; and integration, meaning immigrants are structurally incorporated into local economics, political and other organizations. Only when the latter is achieved, an immigrant can be regarded as fully incorporated. I assume immigrants are able to either achieve cultural incorporation or no incorporation at all. This is the main difference between learner and non-learner immigrants.

In his work, Shafir (1995) classifies three main immigrant-host relation frameworks: assimilation, which is based on the expectation that immigrants are pushed out of agrarian origin regions and pulled by modern host regions where, although they struggle to deal with a new culture and generally live in bad conditions, they make an effort to achieve incorporation over time. At the other extreme, segmentation views immigration just as part of the capitalist process in which host countries seek for complementary labour force and immigrants are directed toward less economically advantageous sectors, creating then permanent barriers for their social assimilation and integration. The third framework is that of multiculturalism, which lies in between the former ones. Multiculturalism in fact can have two distinct characteristics. On the one hand, as applied in countries such as Sweden, UK and Australia, immigrants have the right and support to keep their language and culture if they desire, under the expectation that at least their descendents will eventually become nationals or incorporated individuals. On the other hand, multiculturalism can be as perverse as segmentation, such as in post-independent Latvia and Estonia, where there exists a tendency to treat Russian and Slavic nationals as a permanent minority. With respect to nationalism, this can be a corporate one, which is opposed to immigrants and their integration or a hegemonic one, which offers a window for the integration of immigrants. Although distinct, both emerge from fears of "denationalization" of the host country culture. It is important to stress that nationalistic movements have a kind of erratic evolution with many countries having shifted from corporate nationalism to a hegemonic one and vice-versa over time. Other countries, especially in Western Europe, present what Shafir calls primordial nationalism, which emerges due to the unbalanced modernization of all sectors of a country's society. The dissatisfaction of the sectors that benefit less from this modernization process is then translated into hostility towards the immigrants who are viewed as the symbol of this unbalacement.

In the model I present in section 2, nationalism is more related to the corporate or primordial types.[3] I take into account that some sort of nationalistic behaviour exists among some nationals, and depending on the attitude of policy makers towards its prevention or not, this nationalism may spread over time or even disappear, also depending on the interaction between nationals and immigrants. I show that the result of such interaction leads to all possible three frameworks of immigrant-host relations. The one that emerges depends, among other factors, on the effort immigrants have to make to overcome the cultural barrier in the host country, while at the same time they may have to deal with nationalistic behaviour.

With respect to the labour relations between hosts and immigrants, I adopt a simple assumption in which nationals work in essential services, while immigrants work in non-essential activities. My terminology essential services refers to well established sectors of the country's economy or public services which in many countries are restricted to national or naturalized immigrant employees. I do not assume any further kind of labour market structure in the model or wage differentials. This is justified not only for the sake of simplicity but also due to the mixed evidence presented in the empirical labour economics literature. Borjas (1994) concluded that the literature so far did not provide enough understanding about how immigrants' inflows affect the natives in the host country labour market. In his paper, he presents several examples in which both natives' wages and employment were not affected by a significant inflow of immigrants. One case in which there was an immigration shock, occurred in France in 1962 after Algeria's independence and the work by Hunt (1992) is often cited in the literature. The massive inflow of 900 thousand people to France led to an increase of 1.9% in the French population and 1.6% in its labour force. This repatriation movement resulted in an increase in unemployment of 0.3p.p. with little impact on the affected localities. An even more challenging case of massive immigration took place a decade later in Portugal. According to Carrington and Lima (1996), in 1975, one year after the Armed Forces Movement (MFA) took the power in Portugal, thus ending the Salazar-Caetano regime period, there was a massive inflow of 600 thousand people from the former colonies to Portugal, especially from Angola and Mozambique. Even in a country that by that time had a population of 9 million, the authors ended concluding that overall there was no large adverse effect of immigration on the Portuguese labour market outcomes. One possible problem with the above results, according to Borjas (1994), is that in general labour markets are assumed as closed entities once immigration takes place. In reality, when a strong inflow of immigrants occurs, natives may also emigrate and internal migration may also slow down towards the areas receiving immigrants. As a counter example, Borjas, Freeman and Katz (1992) tried to

[3] In my model xenophobia is the feeling of hating the immigrants which affects the nationalists due to their fear of denationalization of the host country culture. Their reaction to this hate/fear are their nationalistic attitudes against the immigrants, translated into hostility in my model through a tentative of boycotting them. According to the Oxford and the Cobuild English dictionaries, xenophobia "is a fear of people from other countries, or a strong dislike of them." In other words, xenophobia is the harm derived from having to face immigrants while nationalism is the reaction against immigrants.

analyze the macro impact of immigration and found that, between 1980 and 1988, a significant decline in the wages of high school dropouts was a result of the inflow of a large number of low skilled immigrants.

The main contribution of my paper is to employ evolutionary game theory (EGT) to study migration and its relation with nationalism, assimilation and the existence of enclaves. To my best knowledge such a framework has not been developed yet. EGT has been used to study many other problems such as the evolution of crime over time [Cressman, Morrison and Wen (1998)], the banking system and corporate governance in the particular case of post-socialist Lithuania [Marmefelt (2004)] and the effect of economic agents' behaviour on the long-run performance of the economy [Carrera (2009)]. In the latter, depending on the initial conditions, a country ends up either in a low-level (poverty trap) or high-level equilibrium. While Cressman, Morrison and Wen (1998) employ the replicator dynamics with no own population effects to model the dynamic adjustment process, Marmefelt (2004) includes own population effects in the replicator equations. Many other examples of EGT models as well as different dynamic adjustment processes can be found and a paper with a very interesting discussion on EGT is Friedman (1998).

In order to model the dynamic adjustment process of the behaviour of individuals over time in my model, I employ the replicator dynamics, initially without own population effects. Later, I extend the model to include them for the study of the effects of immigrants' enclaves. In evolutionary games employing replicator dynamics, agents from a single or multiple populations are randomly paired to play a stage game. These pairwise contests repeat over time, where the time intervals are assumed to be infinitesimal.[4] Each agent is programmed to play a given pure strategy, depending on his type.[5] A strategy in my model is a behaviour, which can be to learn or not (to behave nationalistic or not) for the case of the population of immigrants (nationals). The payoff an individual receives for adopting a given behaviour depends not only on the stage game payoffs but also on the opponent's population state, that is, the proportions of individuals adopting the n possible opponent's behaviours. Strategies leading to payoffs above the own population average payoff increase their proportion of adoption among the own population individuals over time. Strategies that do worse than average, become extinct over time. Hence, replicator dynamics deals with the role of selection, similarly to natural selection. Successful behaviours tend to proliferate over time among the population while unsuccessful behaviours become extinct.[6]

My results show that when immigrants live dispersed among the host country population, an

[4]See Maynard Smith (1982) for a description of animal contests in a biological context of an evolutionary game. The so-called Hawk-Dove game assumes a single population game in which two animals can behave in one of two ways during a pairwise contest: either adopt the Hawk or Dove behaviour. The expected number of offspring of each animal depends on both the adopted behaviour and on the distribution of behaviours among the population's individuals. Payoffs are dynamical over time.

[5]In every stage game, an individual plays a pure strategy against a completely mixed strategy which is the population state, i.e., the population shares of individuals who are programmed to play each pure strategy available.

[6]Replicator dynamics deals poorly with mutation given that if one strategy is not represented in the system of ordinary differential equations at time zero, no individual in the population will ever adopt it in the future [Gintis (2000)].

equilibrium with complete cultural assimilation of the former group occurs only if the effort to learn the host country culture is low. Although this pattern showed to be independent of whether the host country adopts measures to prevent nationalism or not, there is indeed a difference in the pattern of evolution depending on the ability of policy makers to prevent nationalistic attitudes. For the case when the cost of learning is neither low nor high, while the adoption of policies to prevent nationalism leads to evolution to a monomorphic state of both populations, free nationalism leads both immigrants and nationals to evolve keeping a permanent polymorphic state. In the former, immigrants do not learn and nationalists disappear over time while, in the latter, both populations will oscillate over time with respect to the shares of each of their types. Also, when the cost of learning the host country language is high, while official prevention of nationalism leads to peaceful multiculturalism, free nationalism leads to an evolutionary equilibrium related to political instability in the long run.

The introduction of immigrants' enclaves in the model leads to two main effects: in line with the empirical literature on migration, the existence of enclaves in the host country makes assimilation a less competitive strategy. Depending on the size of the enclave and the extra utility it adds to non-learners, an evolutionary equilibrium with full assimilation may not exist. The second impact of the existence of enclaves is that, when nationalism is prevented, depending on the cost of learning, immigrants may evolve to a polyphormic state in equilibrium, while nationals become a monomorphic population. On the other hand, when there are no measures to prevent nationalism, both populations may evolve to an evolutionary equilibrium in which they continue to be polymorphic as at the initial conditions, just with different shares of each of their two types of individuals. In any possible case I studied, there are never multiple evolutionary equilibria. If this was the case, the evolution of the two populations toward their equilibrium would be dependent on the initial conditions of the problem. Given in my model there is always one single equilibrium, the initial conditions only dictate the lenght of time it will take both populations to evolve to the equilibrium.

The remainder of the paper is organized as follows: in section 2, I set up and solve the model for the two different cases with respect to the existence of enclaves, section 3 presents a numerical simulation, section 4 gives directions for further research and section 5 concludes.

2 Model

2.1 Countries without immigrants' enclaves

I consider a country in which there are two very large distinct populations: national citizens and immigrants. The latter group is composed of individuals who come from the same origin country. Nationals work in essential services such as utilities services, hospitals, transport and general public services. Immigrants work performing other activities. I do not make any assumption with respect to rationality in the sense that individuals do not need to be rational at all. Over time, individuals meet in downtown in a pairwise meeting composed of one randomly selected individual of each population, that is, an immigrant wishes to make use of an essential service and, hence, has to meet with a national who is an employee there. During such a meeting, an individual from the population of national citizens may behave either in a nationalistic way (pure strategy N - nationalist) or welcoming anyone (strategy W - welcome). The immigrant may be an individual fluent in the host country language (strategy L - learner) or may not have learned it (strategy H - non-learner).

Learner-type immigrants are able to have full access to any service, independently of the type of employee they meet, and are able to interact with the place where they live when they travel from their houses to downtown (in the sense of being able to read the newspapers, signs, advertisements and have a full understanding of what is going on in the streets). They obtain an utility $f_1(P)$ and $f_2(K)$ from the service and the interaction, respectively. On the other hand, they incur an effort (or a cost) of learning the language $f_3(c)$,[7] which depends on the language distance between the origin and host countries official idioms ξ, with $\frac{\partial f_3(c)}{\partial \xi} > 0$. Every individual is equally gifted with respect to language learning, so this effort to learn is the same for every immigrant. Non-learner-type immigrants do not incur this cost but also do not obtain any utility from interaction in the streets and, whenever meeting with a nationalist-type employee, they are not able to have full access to the public service obtaining an utility $\alpha f_1(P)$, $\alpha \in [0,1)$. The parameter α, controlled by the government, measures the ability nationalists have to boycott immigrants when dealing with them. In the extreme case of complete boycott, non-learners are fully prevented from getting any service, $\alpha = 0$. Such boycotts are very likely when there is no legal mechanism in the host country to avoid them or, even worse, when such boycotts are officialized by the host country constitution. Independently of the nationalists' degree of xenophobia, policies to reduce or eliminate this kind of barrier in the access to the services may exist, such as the availability of forms in different languages as well as information about the citizens' rights at the public services offices. Both types of immigrants face a disutility $f_4(\Delta)$ due to feeling discriminated whenever they meet with a nationalist-type. In this paper, discrimination is a result of feeling rejected by someone, which can arise independently of being able to access the services or not. Therefore, the possible payoffs for an immigrant, resulting from the pairwise meeting between national and immigrant individuals, measured in terms of utility, are:[8]

$$u^m(N,L) = f(P,K) - f(c,\Delta)$$
$$u^m(N,H) = \alpha f_1(P) - f_4(\Delta)$$

[7] More precisely, $f_3(c)$ is the cost of learning per pairwise meeting.

[8] The shape of the utility functions does not affect the dynamics of the system. What really matters is that all the utility functions are assumed as non-negative, increasing and starting from the origin $(0,0)$. They can be either concave or convex and what affects the selection mechanism in the replicator dynamics equations are the net payoffs in the stage game matrix because different values and signs in the elements of the payoff matrix lead to different coefficients in the ordinary differential equations and, hence, different possible evolutionary equilibria and patterns of stability.

$$u^m(W,L) = f(P,K) - f_3(c)$$
$$u^m(W,H) = f_1(P)$$

where $f_i(\cdot) \in \Re_+$; $f_i(0) = 0$; $f_i'(\cdot) > 0$; $f_i''(\cdot) < 0 \vee f_i''(\cdot) > 0$ and $f(x,y) = f_i(x) + f_j(y)$. From the above payoffs, $u^m(W,L) > u^m(N,L)$ and $u^m(W,H) > u^m(N,H)$ always hold, that is, following the logic, immigrants always prefer to meet a welcome-type national independently of their type.

Nationalists, on the other hand, face a trade-off due to the combination of the degree of xenophobia X they feel and their capacity to boycott immigrants. While their sentiment of xenophobia brings them a disutility $f_5(X)$ caused by being forced to interact with immigrants at their workplace, understood as a "hate for aliens", on the other hand they also obtain some pleasure when they are able to restrict the immigrants' access to essential services. This pleasure gives them utility $(1-\alpha)f_1(P)$, which is exactly the utility level they are able to extract from non-learners when the latter have partial access to the services they request. Overall, nationalists have positive utility levels whenever the amount of harm they are able to inflict on immigrants more than offsets the burden of having to face them.[9] Welcome-type nationals are indifferent between dealing with learners or non-learners and they do not earn or loose any utility from interacting with immigrants. Hence, the possible payoffs for a national are:[10]

$$u^n(N,L) = -f_5(X)$$
$$u^n(N,H) = (1-\alpha)f_1(P) - f_5(X)$$
$$u^n(W,L) = 0$$
$$u^n(W,H) = 0$$

[9] The level of utility nationalists obtain from harming the non-learner-type immigrants, $(1-\alpha)f_1(P)$, depends on two aspects: the first aspect is the utility immigrants get from fully using the public services, $f_1(P)$. The higher this utility is, the greater the pleasure a nationalist obtains if she is able to boycott a non-learner making the public service she wishes to use unavailable. The second aspect is the parameter α. This parameter depends on exogenous policies set by the government to try to enforce the accessibility of the public services to everyone. The higher this parameter is, the greater the accessibility of non-learners to the public services is, making the boycott carried by the nationalists less successfull and, hence, bringing the latter less utility from harming the immigrants.

[10] In reality, the nationals also obtain some utility from the use of the public services. Assuming their utility is given by a function $g(P)$, which would be the same for both nationalist and welcome-type nationals, a national individual would get this utility in a pairwise meeting with another national, i.e., a meeting between a national wishing to use the public service and another national who worked there. Hence, this process would be an own-population effect for the population of nationals and their payoff matrix would be given by:

$$\begin{array}{cc} & \begin{array}{cc} N & W \end{array} \\ \begin{array}{c} N \\ W \end{array} & \begin{pmatrix} g(P),0 & g(P),0 \\ g(P),0 & g(P),0 \end{pmatrix} \end{array}$$

where the row player is the one seeking for the service and the column one is the employee providing it. In the replicator dynamics, this kind of own-population effect would have no impact on the selection mechanism over time given that the payoff from any strategy (nationalist or welcome) would equal the nationals' population average payoff, i.e., $u(e^1,x) - u(x,x) = u(e^2,x) - u(x,x) = g(P)[p + (1-p)] - g(P)[p^2 + 2p(1-p) + (1-p)^2] = 0$. Therefore, I do not take into account the own-population effect of the nationals in the model, derived from their own use of the public services, but one should have in mind that the existence of the public services in the host country brings positive utility not only for the immigrants but also for the nationals.

The dynamics governing the model is the standard replicator dynamics [Weibull (1997)], also known as Malthusian dynamics [Friedman (1998)]. Since local shifts in the payoff functions do not affect the standard replicator dynamics, I make a transformation in the payoff matrices of immigrants (M) and nationals (N) following Friedman (1998), in which all elements of the main diagonal become zero. For an alternative transformation in which the off-main diagonal elements become zero instead, see Weibull (1997). Hence, using $i_1 = i_{12} - i_{22}$ and $i_2 = i_{21} - i_{11}$, $(i = n, m)$:

$$M = \begin{pmatrix} 0 & m_1 \\ m_2 & 0 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix}$$

In order to make this set up clear, I should emphasize that m_{12} is the payoff a learner gets when facing a welcome-type national, while n_{12} is the payoff a nationalist obtains when confronted with a non-learner-type immigrant, such that merging both matrices M and N above, we end up with:

$$\begin{array}{cc} & \begin{array}{cc} L & H \end{array} \\ \begin{array}{c} N \\ W \end{array} & \begin{pmatrix} 0,0 & n_1, m_2 \\ n_2, m_1 & 0,0 \end{pmatrix} \end{array}$$

To make the notation easier, from now on I will simply call $f(\cdot)$ any utility function $f_i(\cdot)$. Assuming p is the proportion of nationalists among the population of nationals while q is the proportion of learners among the population of immigrants, assuming x (y) is equivalent of having a player in the population of nationals (immigrants) playing a strategy in the interior of the corresponding simplex Δ^i; $i = n (i = m)$, from matrices M and N, the population states $s^n = (p; 1-p)$ and $s^m = (q; 1-q)$ are governed over time, respectively, by:

$$\dot{p} = (u(e^1, y) - u(x, y))p = (n_1(1-q) - n_1(1-q)p - n_2(1-p)q)p$$

$$\Rightarrow \dot{p} = (n_1(1-q) - n_2 q)p(1-p) \tag{1}$$

and

$$\dot{q} = (u(x, e^1) - u(x, y))q = (m_1(1-p) - m_1(1-p)q - m_2(1-q)p)q$$

$$\Rightarrow \dot{q} = (m_1(1-p) - m_2 p)q(1-q) \tag{2}$$

The game overall state space is therefore the square $\theta = s^n \times s^m = [0,1]^2 \subset \Re^2$ and, for the sake of simplicity, I will represent a given state as (p,q). Substituting n_1, n_2, m_1 and m_2 in eqs. (1) and (2) for the payoffs:[11]

$$\dot{p} = (((1-\alpha)f(P) - f(X))(1-q) - f(X)q)p(1-p) \tag{3}$$

and

$$\dot{q} = ((f(K) - f(c))(1-p) - (f(c) - (1-\alpha)f(P) - f(K))p)q(1-q) \tag{4}$$

From eqs. (3-4), we may have the following cases:

$$(1-\alpha)f(P) > f(X) \Rightarrow n_1 > 0; n_2 > 0 \tag{5}$$

$$(1-\alpha)f(P) < f(X) \Rightarrow n_1 < 0; n_2 > 0 \tag{6}$$

[11]In appendix I, I present a detailed derivation of both equations (1) and (2). According to Friedman (1998), evolutionary games are a natural approach in Biology given the fitness of a biological trait or behaviour depends both on the prevalence of that behaviour in the current population and on the prevalence of alternative behaviours. He gives an example of an aggressive animal having a higher fitness when that behaviour is rare among the population. In such case, the chance of facing a conteraggression would be rare. Hence, while the behaviour was relatively rare, the genetic transmission of the aggressive behaviour in the population over time would be very successful. Friedman states that, although genetic transmission may sometimes face constraints when applied in economic problems, models adopting EGT are useful whenever the outcome of an agent interacting with other agents depends on the others' behaviour and on the agent's behaviour. In my model each population has only two possible behaviours. Looking at equation (1) for the case of nationals, one can see that whenever $u(e^1,y) < u(x,y)$, i.e., whenever the adoption of the nationalistic behaviour does worse than average when dealing with a polymorphic population of immigrants (hence contesting against a mixed strategy y), the proportion of nationalists decreases over time. Even if one assumes that individuals are "wired" to always play the same pure strategy over life as in many games in Biology, the natural selection mechanism behind the replicator dynamics here can be understood in the sense that, at early stages of human life, a recently born individual has not her personnality fully defined yet. At the same time, she is exposed to the behaviours which are present in the environment (home, school, etc...). Hence in the early stage of life, during the learning process, one will be exposed to nationalistic and welcome type influences until making her final decision with respect to her personnality. If to behave nationalistically leads more often to a lower payoff than to welcome immigrants, it is more likely through natural selection that over time, the proportion of nationalists will decrease because less young nationals will adopt ('be wired to') a nationalistic personnality. Once the personnality is defined, one does not change it anymore over life. Also in countries with a very large population of welcome types, to show a nationalistic trait would be more discouraged. With respect to the immigrants, the reasoning behind equation (2) is the same, i.e., whenever $u(x,e^1) < u(x,y)$, the proportion of learners decreases over time. With respect to immigrants, immigration decision depends on information people receive about a place. If to be a non-learner pays-off, in the long run more people with a non-learner behaviour will be attracted by the host country than people with a learner-type behaviour. Or alternatively, in the long run more new immigrants will tend to keep their own culture after their arrival instead of learning the host country culture.

$$f(c) \in [0; f(K) - \varepsilon] \Rightarrow m_1 > 0; m_2 < 0 \tag{7}$$

$$f(c) \in [f(K) + \varepsilon; f(K) + (1 - \alpha) f(P) - \varepsilon] \Rightarrow m_1 < 0; m_2 < 0 \tag{8}$$

$$f(c) \in [f(K) + (1 - \alpha) f(P) + \varepsilon; +\infty) \Rightarrow m_1 < 0; m_2 > 0 \tag{9}$$

where $\varepsilon \to 0$. Equation (5) refers to a country where nationalist behaviour is not prevented by the government (α is low) and individuals supporting movements against immigrants are able to create barriers making the life of non-assimilated immigrants in the host country difficult given that they are not able to benefit fully from the use of essential services. Someone may argue that it could also be the case that xenophobia was very low and, actually, every immigrant could access any service (that is, X very close to zero and α high, even close to 1) but this would be a less interesting case and, given the paper focus on nationalism, I shall assume xenophobia by nationalists does exist and is relevant, in line with the former possibility. Equation (6) refers to the opposite situation, in which xenophobia exists and is strong among the proportion of the nationals adopting nationalistic behaviour but there is some institutional policy giving enough protection for the access of any kind of individual to any service (α is high). This latter scenario is similar to the cases of Catalonia and Basque country in Spain with respect to the inflow of Castilians. Also recent examples are those of Latvia and Estonia during the USSR era, in which such countries had a welfare level above the other former soviet republics, attracting immigrants from Russia, Ukraine and Belarus. Such immigration was not well received creating a strong level of nationalism in these countries but at the same time, both immigration and the access of immigrants to institutions and the labour market could not be prevented by nationalistic attitudes given these regions played a limited role in Moscow's central political institutions [see Shafir (1995)].

With respect to the effort of learning, $f(c)$ may fall in one of 3 intervals which I will call low, intermediate and high cost of learning. Eq. (7) refers to a situation in which both host and origin countries share the same or a very closely related language (eg: Argentina and Brazil). In this case, the cost of language acquisition is low. The other extreme is given in eq. (9), in which immigrants have a very different cultural background when compared to the nationals of the host country and language acquisition is very costly. I now analyze the six possible combinations when immigrants do not live in enclaves. In all cases, I always depart from an initial condition according to which both populations of nationals and immigrants living in the coutry are polymorphic.

Proposition 1: The game state space θ contains at most five stationary states, θ^0, all them may be Nash equilibria, θ^{NE}, but only four of them may be evolutionary stable states (ESS), θ^{ESS}, and evolutionary equilibria (EE), θ^{EE}.

Proof: Equations 3 and 4 lead to the following fixed points: $(0,0)$; $(1,0)$; $(0,1)$; $(1,1)$ and $(\overline{p} = \frac{f(c)-f(k)}{(1-\alpha)f(P)}, \overline{q} = \frac{(1-\alpha)f(P)-f(X)}{(1-\alpha)f(P)})$, the latter when $f(c) \in [f(K)+\varepsilon; f(K)+(1-\alpha)f(P)-\varepsilon]$ and $(1-\alpha)f(P) > f(X)$. The former four ones are Nash equilibria when at least one of the populations has a dominant strategy. The latter is a Nash equilibrium in mixed strategies, which is a Liapunov stable state under replicator dynamics with no own population effects but not assymptotically stable (and, hence, not EE). The rest of the proof comes from the fact that every strict Nash equilibrium is evolutionary stable and when the dynamics of the population states is governed by replicator dynamics, every ESS is also an EE$_\bullet$

2.1.1 Host countries with weak or no anti-nationalist laws

In this section I analyze all possibilities when immigrants move to a country in which nationalistic movements do exist and the central government does not implement enough measures to prevent it.

Proposition 2: When the cost of learning the host country language is low and there is no prevention against nationalism, over time both populations become monomorphic with all nationals welcoming immigrants and the latter becoming completely assimilated.

Proof: In this case, $n_i > 0$; $m_1 > 0$; $m_2 < 0$; hence, $\dot{q} > 0$ and in the long run non-learner-types disappear from the immigrants' population.[12] To learn is a dominant strategy for the population of immigrants. The best response for the population of nationals is to become welcome-types and nationalism also disappears in the long run. Hence, over time, non-learners and nationalists decrease their share in their respective populations until becoming extinct and in the limit both populations become monomorphic. $\theta^{NE} = \theta^{ESS} = \theta^{EE} = (0,1)_\bullet$

This is the context in host countries where language distance is very small. Countries which languages are close tend to share the same norms and values, which is a facilitator or even an incentive to attract immigrants who end up learning the language once they start to live in the host society. On the side of the nationals, nationalistic behaviour does not pay over time given the proximity between the cultures of both origin and host countries, which makes it possible, independently of the level of xenophobia and nationalistic boycott against immigrants, for any individual to have access to the services because they all become learners. In other words, given assimilation takes place over time, calls for nationalistic behaviour among the nationals population lose strength in the long run and nationalistic attitudes become extinct.

Proposition 3: In countries with very high costs of learning the language and where no prevention against nationalism exists, populations of nationals and immigrants evolve to a monomorphic state in which immigrants are not assimilated and nationals behave nationalistically. In this case, multiculturalism is an equilibrium which may de-stabilyze the country politically in the long run.

Proof: We have the opposite from proposition 2, with $m_1 < 0$; $m_2 > 0$; $\dot{q} < 0$. The dominant

[12] In appendix I, I present an alternative analytical proof of propositions 2-5, based on the linearization of the non-linear system of ordinary differential equations composed of equations (1) and (2) at the stationary points.

strategy for the population of immigrants now is not to learn the language while nationals best respond raising nationalistic behaviour. $\theta^{NE} = \theta^{ESS} = \theta^{EE} = (1,0)$, which is located at the southeast corner of the square $[0,1]^2_{\bullet}$

In fact, such kind of multiculturalism is not the one implemented in countries such as the UK, Sweden, Canada and Australia, which tend to welcome immigrants. This is more likely a kind of forced multiculturalism implemented on the nationals (indigenous population) by an outside power, similar to the metropolitan policies in the former colonies during the past. The incapacity to achieve self determination in Africa and Asia, together with the association of the indigenous culture to second class citizens, such that Europeans did not want to learn it, led to strong nationalistic movements which over time evolved to guerrila, civil war and a massive outflow of the European population from the former African colonies after independence. In the more recent history, situations similar to these ones happened in regions such as Latvia and Estonia. According to Shafir (1995), linguistic assimilation is particularly encouraged when the host region language is the language of a prosperous country. In the cases of Latvia and Estonia, immigrants from the former USSR had not this undertanding given supremacy in terms of language was always associated to Moscow. Shafir (1995) adds stating that, when after several generations, the indigenous population still considers the political authority exercised over it as illegitimate and as a result only due to the anexation of their territory, an empire overcomes the concept of state. In this kind of multicultural context where, in fact, the immigrants are backed by the central power of the empire and do not see themselves as minorities, in the long run, political stability may be a critical factor.

In the Baltic republics of the former USSR, two paralell societies emerged, one native, the other an alien one, both of them with their own elites, which created a difficult barrier for integration. In 1985, only 52.6% of the population of Latvia had Latvian origin. The corresponding figure in Estonia was 63.1%. These figures become more dramatic to the host populations if we consider the fact that the populations of Latvia and Estonia at that time were 2.6 and 1.5 million, respectively, demasiated small compared to the size of the former USSR.

Proposition 4: In countries where the cost of learning is neither low nor high and nationalism can not be prevented, a fully polymorphic country will always hold over time, in which the proportions of nationalists and learners will oscilate over generations.

Proof: This case is similar to the Buyer-Seller game of Friedman (1991) which was later used by Cressman, Morrison and Wen (1998) in their model to study the dynamics of crime. We have a dynamics similar to the predator-prey model of Lotka-Volterra in which both species of animals compete against each other but not against themselves (no-overcrowding). In the country case, we have $\theta^{NE} = (\overline{p}, \overline{q})$; $\theta^{NE} \bigcap \theta^{ESS} = \emptyset$ and an evolutionary equilibrium does not exist. In the interior of the square $[0,1]^2$ we have closed orbits which spiral around the Nash equilibrium counter clockwise$_{\bullet}$

If we assume an initial state in which welcome and non-leaner types are numerous within their populations, eg. $(0.1, 0.1)$, nationalism starts to rise among the nationals population, achieves a very high proportion rising at the same time the incentives to favour assimilated immigrants. When the proportion of nationalists starts to outnumber that of welcome-types, the proportion of non-learners starts to decrease dramatically in the population of immigrants. This pattern in the immigrants population acts over time on the nationals population reducing the incentives for nationalistic behaviour. Welcome types start to outnumber the proportion of nationalists and as a consequence, over time, the proportion of non-learners grows. The cycle then re-starts. This evolutionary pattern is presented in figure 4. An important piece of information is that the Nash equilibrium represents the average behaviour of the populations over a long interval of time [Cressman, Morrison and Wen (1998)].

2.1.2 Host countries implementing anti-nationalist laws

In this section, I analyze the cases in which the government enforces policies to prevent nationalistic behaviour in the essential services.

Proposition 5: In countries where there are official measures to prevent nationalistic behaviour, unless the cost of learning is low, over time immigrants will not become culturally assimilated while nationalism disappears.

Proof: We have $n_1 < 0$; $n_2 > 0$; $\dot{p} < 0$. The dominant strategy for the population of nationals is to welcome immigrants while the latter best respond not learning the host country culture unless $f(c) < f(K)$. $\theta^{NE} = \theta^{ESS} = \theta^{EE} = (0,0)$ for $f(c) > f(K)$, which is located at the southwest corner of the square $[0,1]^2$ and $\theta^{NE} = \theta^{ESS} = \theta^{EE} = (0,1)$ for $f(c) < f(K)$•

Thus, comparing propositions 2, 3 and 5, we can see that complete assimilation or not independs of the policy adopted by the government with respect to nationalism prevention and is more related to the degree of cultural distance. But, on the other hand, the pattern of multiculturalism a country develops over time is closely related to the way policy makers deal with nationalism. The multiculturalism pattern found in proposition 5 under $\theta^{EE} = (0,0)$ is very different from the one of proposition 3 and the former matches with policies found in countries like the UK, Canada, Australia and Sweden.

2.2 Effect of immigrants' enclaves

In this section, I introduce the effect of the existence of enclaves of immigrants in the host country. In this case, immigrants are more likely to interact among themselves and the evolution pattern for their population depends not only on the evolution of the population of nationals but also on the so called own population effects in the EGT literature. Similar to the analysis of own population effects in Marmefelt (2004), I assume when enclaves exist the immigrants population interact with the population of nationals with probability 1 and with their own population with probability ρ. When $\rho = 0$, we fall back to the case without enclaves.

With respect to the payoffs immigrants get from the own population effect, I assume that the benefit from this interaction accrues basically to the non-learner-type immigrants. A straightfoward example of the kind of gain the non-learner-types obtain from this interaction is the increased ability to acquire new information about the host country. This becomes possible only when they network with their own people due to their lack of knowledge of the host country language. The acquisition of information about the "new" society in which they are inserted is higher when they meet with a learner than when they meet another non-learner. On the other hand, learners benefit residually from this own population effect due to the fact that they are perfectly able to acquire all information by themselves.

Based on these assumptions the matrix M' corresponding to the own population effect for the immigrants is given by:

$$M' = \begin{pmatrix} 0 & m'_1 \\ m'_2 & 0 \end{pmatrix}$$

where $m'_1 = m'_{12} - m'_{22}$, $m'_2 = m'_{21} - m'_{11}$ and:

$$\begin{aligned} m'_{11} &= u^m(L,L) = 0 \\ m'_{12} &= u^m(L,H) = 0 \\ m'_{21} &= u^m(H,L) = \beta \cdot f(K) \\ m'_{22} &= u^m(H,H) = \gamma \cdot f(K) \end{aligned}$$

with $1 > \beta > \gamma > 0$ and $f(K)$ is the utility learners obtain from their perfect ability to interact with the environment.

The dynamics governing both population states is now given by equation (3) and by:[13]

$$\dot{q} = (u(x,e^1) - u(x,y) + \rho(u(y,e^1) - u(y,y)))q$$

$$\dot{q} = (m_1(1-p) - m_2 p + \rho m'_1(1-q) - \rho m'_2 q)(1-q)q \tag{10}$$

$$\dot{q} = ((f(K) - f(c))(1-p) - (f(c) - (1-\alpha)f(P) - f(K))p - \rho f(K)(\gamma(1-q) + \beta q))q(1-q) \tag{11}$$

[13]The population of nationals continues to have no own population effect, hence eq.(3) is not affected by the existence of enclaves.

The analysis of the non-linear system of ODE now is not as straightforward as in section 2.1. I will address it using the technique of linearization of the system at the critical points. In the case of a non-hyperbolic[14] system, I employ a Liapunov function to identify the stability of the critical point.

Rewriting eqs. (1) and (10) as:

$$\dot{p} = F^1(p,q)$$
$$\dot{q} = F^2(p,q)$$

The Jacobian matrix of the system at a given critical point (p,q) is given by:

$$\Omega(p,q) = \begin{pmatrix} \frac{\partial F^1(p,q)}{\partial p} & \frac{\partial F^1(p,q)}{\partial q} \\ \frac{\partial F^2(p,q)}{\partial p} & \frac{\partial F^2(p,q)}{\partial q} \end{pmatrix}$$

where:

$$\frac{\partial F^1(p,q)}{\partial p} = (n_1(1-q) - n_2 q)(1-2p)$$

$$\frac{\partial F^1(p,q)}{\partial q} = -(p-p^2)(n_1+n_2)$$

$$\frac{\partial F^2(p,q)}{\partial p} = -(q-q^2)(m_1+m_2)$$

$$\frac{\partial F^2(p,q)}{\partial q} = (1-2q)(m_1(1-p) - m_2 p + \rho m_1'(1-q) - \rho m_2' q) - (q-q^2)\rho(m_1'+m_2')$$

The new non-linear system has seven possible stationary points which are candidates for an evolutionary equilibrium. These points are $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, as before, and $(\overline{p_1} = \frac{f(c)-f(k)(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})}{(1-\alpha)f(P)}$, $\overline{q_1} = \frac{(1-\alpha)f(P)-f(X)}{(1-\alpha)f(P)})$, the latter when $f(c) \in [f(K)(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})+\varepsilon; f(K)(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})+(1-\alpha)f(P)-\varepsilon]$ and $(1-\alpha)f(P) > f(X)$; $(\overline{p_2} = 0, \overline{q_2} = \frac{f(K)(1-\rho\gamma)-f(c)}{\rho f(K)(\beta-\gamma)})$, the latter when $f(c) \in [f(K)(1-\beta\rho)+\varepsilon; f(K)(1-\gamma\rho)-\varepsilon]$; $(\overline{p_3} = 1, \overline{q_3} = \frac{f(K)(1-\rho\gamma)+(1-\alpha)f(P)-f(c)}{\rho f(K)(\beta-\gamma)})$, the latter when $f(c) \in [f(K)(1-\beta\rho)+(1-\alpha)f(P)+\varepsilon; f(K)(1-\gamma\rho)+(1-\alpha)f(P)-\varepsilon]$.

Proposition 6: As in the case where enclaves do not exist, a stationary state $\theta^0 = (1,1)$, in which all nationals behave nationalisticaly and all immigrants are learners is never an evolutionary equilibrium.

Proof: At $\theta^0 = (1,1)$, the trace of the Jacobian, $tr(\Omega)$, is $n_2 + m_2 + \rho m_2'$ and its determinant, $det(\Omega)$, is $n_2(m_2 + \rho m_2')$, where n_2 is always positive. Hence, if $det(\Omega) > 0 \Rightarrow tr(\Omega) > 0$ and both

[14] A dynamical system is hyperbolic at a fixed point x^* if every eigenvalue of the Jacobian matrix has nonzero real part [Gintis (2000)].

distinct eigenvalues are positive leading to an unstable node. On the other hand, if $det(\Omega) < 0$ then we have two real distinct eigenvalues,[15] one positive and the other negative, for which the corresponding critical point is a saddle point$_\bullet$

Proposition 7: When the immigrants community is very large making interaction among themselves very likely and the gain for non-learner immigrants from networking is very high, $\beta \to 1 \wedge \rho \to 1$, an evolutionary equilibrium involving complete assimilation of immigrants never exists.

Proof: As before, when the cost of learning is very low, $f(c) \in [0, f(k)(1-\rho\beta) - \varepsilon]$, independently of the existence or not of laws against nationalistic behaviour, immigrants become assimilated and nationalism disappears over time, that is, $\theta^{EE} = (0,1)$. At this critical point, $tr(\Omega) = -n_2 - (m_1 - \rho m_2')$ and $det(\Omega) = n_2(m_1 - \rho m_2')$. Given $n_2 > 0$, $det(\Omega) > 0$ only if $(m_1 - \rho m_2') > 0$, implying also $tr(\Omega) < 0$. Hence, we have a stable node and an evolutionary equilibrium whenever $f(c) < S = f(k)(1-\rho\beta)$. On the other hand, when $\beta \to 1 \wedge \rho \to 1$, $S \to 0$ and complete assimilation is not possible$_\bullet$

Note that when $S > 0$ and complete assimilation is possible, the interval for $f(c)$ is now narrower than when there was no enclave (see proposition 2) and is in line with the empirical literature on migration. The effect of an enclave of immigrants is to make assimilation a less competitive strategy. Only individuals with very low switching language costs find profitable to become assimilated [Lazear (1999)].

2.2.1 Host countries implementing anti-nationalist laws

In this section, I analyze the specific equilibria for the case when enclaves exist and nationalism is prevented. When nationalism is prevented, the point $(\overline{p_1}, \overline{q_1})$ is never a stationary point, given it requires $(1-\alpha)f(P) > f(X)$. For this case, we always have a minimum of four stationary points (the corners of the square) and a maximum of six stationary points (the corners plus $(\overline{p_2}, \overline{q_2})$ and $(\overline{p_3}, \overline{q_3})$). The latter happens when $f(K) > \frac{(1-\alpha)f(P)}{\rho(\beta-\gamma)} \wedge (1-\alpha)f(P) > 0$, in which case $f(K)(1-\beta\rho) + (1-\alpha)f(P) \in [f(K)(1-\beta\rho) + \varepsilon; f(K)(1-\gamma\rho) - \varepsilon]$.

Proposition 8: In countries where enclaves exist and there are enough official measures to prevent nationalistic behaviour, as before, when the cost of learning is not low enough, over time immigrants will not become completely culturally assimilated while nationalism disappears. But now, it may be possible to have an equilibrium in which the population of immigrants is polymorphic, with part of them still learning the language.

Proof: Putting propositions 7 and 8 together, we have:

[Table 1 - see Appendix III]

[15]For critical points at which the off-main diagonal elements of the Jacobian matrix are all zero, $(tr(\Omega))^2 \geq 4det(\Omega)$ and the eigenvalues are necessarily real numbers. Hence, stability only requires $tr(\Omega) < 0 \wedge det(\Omega) > 0$.

Case A was already proven in Proposition 7. To proof case B, at $(0,\overline{q_2})$, $tr(\Omega) = n_1(1-\overline{q_2}) - n_2\overline{q_2} - (\overline{q_2} - \overline{q_2}^2)\rho(m'_1 + m'_2) < 0$, given $n_1 < 0$ for the nationalism-prevented case and $n_2 > 0$. Also $det(\Omega) = -[n_1(1-\overline{q_2}) - n_2\overline{q_2}][(\overline{q_2} - \overline{q_2}^2)\rho(m'_1 + m'_2)] > 0$. Hence, $(0,\overline{q_2})$ is a stable node. For case C, at $(0,0)$, $det(\Omega) = n_1(m_1 + \rho m'_1)$ and $tr(\Omega) = n_1 + m_1 + \rho m'_1$. For stability, we require $(1-\alpha)f(P) < f(X)$, that is, nationalism prevention and $f(c) > f(K)(1-\rho\gamma)$, i.e., the cost of learning is not low enough. To complete the proof, I show that when $(1,\overline{q_3})$ is a stationary point, it is always a saddle point and that the corner $(1,0)$ is not stable. At the former, $det(\Omega) = [n_1(1-\overline{q_3}) - n_2\overline{q_3}][(\overline{q_3} - \overline{q_3}^2)\rho(m'_1 + m'_2)] < 0$. At the latter, $det(\Omega) = n_1(m_2 - \rho m'_1)$ and $tr(\Omega) = -n_1 - (m_2 - \rho m'_1)$, requiring $n_1 > 0 \Rightarrow$ free nationalism for stability$_\bullet$

Hence, I summarize in the following table all the possible cases we may have with respect to equilibria and stationary points for the case of enclaves together with nationalism prevention:

[Table 2 - see Appendix III]

In section 3.2, I present a numerical example for the case when all six stationary points are present. Comparing propositions 5, 7 and 8, the existence of enclaves has two impacts. The first, as stated before, is that it reduces the cost boundary for which an immigrant is indifferent between learning or not the host country language. The second impact of the enclave is the existence of a set of learning cost values for which polymorphism is an equilibrium and part of the population of immigrants learns while the other part does not. The width of this interval, as well as the minimum learning cost below which all immigrants become assimilated over time, depends on the enclaves' importance, ρ, and on the parameters β and γ, that is, the interaction gain non-learner types get when they live in enclaves. If enclaves are very large as well as the utility gain non-learner-types obtain, then the multiculturalism pattern found in proposition 5 holds independently of the cost of learning and, after some generations, no immigrant becomes assimilated at all:

$$\rho \to 1; \beta \to 1; \gamma \to 1 \wedge f(k)(1-\rho\gamma) \to f(k)(1-\rho\beta) \to 0$$

Deepening the discussion a bit further with respect to the results of this section and those of section 2.1.2 (nationalism prevented and no enclaves), when enclaves exist, to have no assimilation at all requires a lower cost/effort of learning than before. Without enclaves this required $f(c) > f(K)$ while now it requires a lower cost, $f(c) > f(K)(1-\rho\gamma)$. On the other extreme, with the existence of enclaves, to have complete assimilation of immigrants over generations requires an even lower cost of learning than before. Without enclaves, complete assimilation required $f(c) < f(K)$ while now it requires $f(c) < f(K)(1-\rho\beta) < f(K)(1-\rho\gamma) < f(K)$. For $f(K)(1-\rho\beta) < f(c) < f(K)(1-\rho\gamma)$, there exists a set of values for the cost of learning leading to a transition state, i.e., a state in which some immigrants learn while others do not (polymorphism). Because now immigrants live in enclaves, non-learners can benefit from interacting with other immigrants, especially learners, in order to get information about the host country. Without enclaves, polymorphism could not be an equilibrium because non-learner immigrants could not benefit from such interaction. Immigrants then had an one-shot decision over generations, either complete assimilation or no assimilation at all. This one-shot decision was naturally selected through the evolutionary mechanism behind the replicator dynamics.

Based on the above discussion, starting from $f(c) = f(K)$ and decreasing the cost of learning, without enclaves, all non-learner immigrants would disappear over generations because the behaviour of becoming a learner would be naturally selected given the extra utility $f(K)$, only obtained by learners, more than offsets the cost of learning $f(c)$. With enclaves, things change. In order to have some assimilation, the natural selection mechanism behind the replicator dynamics equations requires a lower cost of learning. Also, if this cost of learning is below $f(K)(1-\rho\gamma)$ but still above $f(K)(1-\rho\beta)$, natural selection does not lead to complete assimilation over time. Some individuals in the population of immigrants will end up learning given the extra utility $f(K)$ more than offsets the incurred cost of learning $f(c)$. On the other hand, natural selection will allow for the co-existence of non-learners which do not face the cost $f(c)$ but still get an extra expected utility $\rho f(K)\left[\beta q+\gamma(1-q)\right]$ (lower than $f(K)$), due to the existence of the enclave.

2.2.2 Free nationalism

Now, I analyze the context in which nationalism is not officialy prevented and enclaves exist in the host country. When this is the case, depending on the cost of learning, the game state space $\theta = [0,1]^2$ has a minimum of four (the corners of the square) and a maximum of seven stationary states. These points are the same as in the last section plus the point $(\overline{p_1};\overline{q_1})$. Depending on the values of the parameters, and depending on the value of the cost of learning, there are five possible cases with respect to the presence or not of the stationary points in the model. These five cases are displayed in the figure below:

[Figure 1 - see Appendix III]

Proposition 9: As in all cases presented so far, when nationalism is not prevented and enclaves exist, multiple stationary points can co-exist in the state space but they are never simultaneous evolutionary equilibria, i.e., the state space has at most one evolutionary equilibrium. Also, depending on the cost of learning, an evolutionary equilibrium in which both populations continue to be polymorphic may exist.

Proof: From the proof of proposition 7, for $f(c) \in [0, f(K)(1-\rho\beta)-\varepsilon]$, $\theta^{EE} = (0,1)$. From the proof of proposition 8, for the corner $(1,0)$, all stability requires is free nationalism and $f(c) > (1-\alpha)f(P) + f(K)(1-\rho\gamma)$. When the point $(\overline{p_1};\overline{q_1})$ is stationary, we have $tr(\Omega) = -(m_1' + m_2')\rho(\overline{q_1} - \overline{q_1}^2)$, which is always negative. Also, $det(\Omega) = -(\overline{p_1} - \overline{p_1}^2)(\overline{q_1} - \overline{q_1}^2)(m_1 + m_2)(n_1 + n_2)$, which is always positive. Also, for the specific case of this stationary point, we have $[tr(\Omega)]^2 - 4det(\Omega) < 0$ and the stationary point is a stable focus. Hence, for any range of the cost of learning for which $(\overline{p_1};\overline{q_1})$ is a stationary point, it is indeed an evolutionary equilibrium in which both populations continue to be polymorphic. When $(\overline{p_2};\overline{q_2})$ is a stationary point, we have at this point $tr(\Omega) = [n_1(1-\overline{q_2}) - n_2\overline{q_2}] - [(\overline{q_2} - \overline{q_2}^2)\rho(m_1' + m_2')]$ and $det(\Omega) = -[n_1(1-\overline{q_2}) - n_2\overline{q_2}][(\overline{q_2} - \overline{q_2}^2)\rho(m_1' + m_2')]$. In order to have $tr(\Omega) < 0$ and $det(\Omega) > 0$, we need $f(c) < f(K)(1-\rho\gamma(1-\overline{q_1}) - \rho\beta\overline{q_1})$. Hence, whenever the point $(\overline{p_2};\overline{q_2})$ is a stationary point, it is an evolutionary equilibrium for all ranges of the cost of learning from $f(c) = f(K)(1-\rho\beta)$ up to the inferior boundary of $f(c)$ for which the point $(\overline{p_1};\overline{q_1})$ starts to become stationary. Whenever $(\overline{p_3};\overline{q_3})$ is a stationary point, we have $tr(\Omega) = -[n_1(1-\overline{q_3}) - n_2\overline{q_3}] - [(\overline{q_3} - \overline{q_3}^2)\rho(m_1' + m_2')]$ and $det(\Omega) = [n_1(1-\overline{q_3}) - n_2\overline{q_3}][(\overline{q_3} - \overline{q_3}^2)\rho(m_1' + m_2')]$. For being a stable critical point and, hence, an evolutionary equilibrium, we require $f(c) > (1-\alpha)f(P) + f(K)(1-\rho\gamma(1-\overline{q_1}) - \rho\beta\overline{q_1})$. Hence, this point is an evolutionary equilibrium for all ranges of the cost of learning from the upper boundary for which the point $(\overline{p_1};\overline{q_1})$ is a stationary point up to the upper boundary for which the point $(\overline{p_3};\overline{q_3})$ is a stationary point$\bullet$

Summarizing the results of the above proposition:

[Table 3 - see Appendix III]

In section 2.2.1, when nationalism was prevented and enclaves existed, independently of the value of the cost of learning $f(c)$, there was the possibility of ending up with no assimilation at all over generations when $\rho \to 1$, $\beta \to 1$, $\gamma \to 1$. When nationalism is not prevented, from table 3, it can be seen that this possibility can not hold anymore and, independently of ρ, β and γ, there exists a range of values for $f(c) > 0$ such that some assimilation will hold in the evolutionary equilibrium. For example, the evolutionary equilibrium $(\overline{p_1},\overline{q_1})$ holds for some $f(c) < (1-\alpha)f(P) + f(K)(1-\rho\gamma(1-\overline{q_1}) - \rho\beta\overline{q_1})$ and, therefore, $\exists f(c) > 0$ leading to this evolutionary equilibrium even if $\rho \to 1$, $\beta \to 1$ and $\gamma \to 1$, given that $(1-\alpha)f(P) > 0$.

Also, comparing the results found in section 2.2.1 and in this section, the main difference is that, when nationalism is not prevented, both populations can end-up in a polymorphic state. So, for an intermediate level of the cost of learning the host country language, in a country where the population of immigrants departs from an initial state containing both learners and non-learners and the population of nationals have both nationalists and welcome-type individuals, none of the types will become extinct over generations. The populations will reach an equilibrium in which all types continue to coexist. Also, as when enclaves did not exist, when the cost of learning is high enough, multiculturalism, $\theta^{EE} = (1,0)$, is an equilibrium which may de-stabilyze the country politically.

3 Numerical simulation

3.1 No enclaves

In this section, I present a short numerical example for the case in which the effort of learning is neither low nor high, eq.(8), which is the only one whose outcome with respect to assimilation depends on the existence or not of government policies to prevent nationalism. I assume $f(c) = 0.55$; $(1-\alpha)f(P) = 1.5$; $f(K) = 0.2$; $f(X) = 0.6$, for the case with $(1-\alpha)f(P) > f(X)$ and $f(X) = 2$, for $(1-\alpha)f(P) < f(X)$, together with the initial conditions $p_0 = 0.1 \wedge q_0 = 0.1$ and $p_0 = 0.5 \wedge q_0 = 0.5$. For the case without nationalism prevention, equations (3) and (4) become, respectively:

$$\dot{p} = (0.9 - 1.5q)p(1-p)$$

$$\dot{q} = (-0.35 + 1.5p)q(1-q)$$

and $\overline{p} = 0.2333$, $\overline{q} = 0.6$. The vector field corresponding to such dynamics is:

[Figure 2 - see Appendix III]

From the vector field, clearly the Nash equilibrium is Liapunov stable (but not assymptotically stable and, hence, not evolutionary stable under replicator dynamics). I use the following Liapunov function[16] to proof this formally:

$$V(p,q) = -0.35\ln p - 1.15\ln(1-p) - 0.9\ln q - 0.6\ln(1-q)$$

This function has a strict minimum at $(\overline{p} = 0.2333, \overline{q} = 0.6)$ because:

[16]More precisely, a Liapunov function for the above system of ODE is $V(p,q) - V(\overline{p},\overline{q})$, but the use of $V(p,q)$ instead does not affect the proof. The only difference is that $V(p,q) - V(\overline{p},\overline{q}) = 0$ and is a strict minimum at $(\overline{p} = 0.2333, \overline{q} = 0.6)$, while $V(p,q)$ is also a strict minimum but is different from zero at $(\overline{p},\overline{q})$. To find this Liapunov function, I employed separation of variables in a procedure similar to the one in section 11.2 of Hirsch, Smale and Devaney (2004) for the Lotka-Volterra system. Many books present a formal definition of Liapunov stability, but one I found particularly nice can be seen in the same book on pages 194-195.

$$\frac{\partial V(p,q)}{\partial p} = \frac{-0.35}{p} + \frac{1.15}{1-p} = 0 \Rightarrow p = \overline{p}$$

$$\frac{\partial V(p,q)}{\partial q} = \frac{-0.9}{q} + \frac{0.6}{1-q} = 0 \Rightarrow q = \overline{q}$$

and

$$\frac{\partial^2 V(p,q)}{\partial p^2} = \frac{0.35}{p^2} + \frac{1.15}{(1-p)^2} > 0; \forall p$$

$$\frac{\partial^2 V(p,q)}{\partial p^2}\frac{\partial^2 V(p,q)}{\partial q^2} = \left(\frac{0.35}{p^2} + \frac{1.15}{(1-p)^2}\right)\left(\frac{0.9}{q^2} + \frac{0.6}{(1-q)^2}\right) > 0; \forall p; \forall q$$

Let B be an open ball about (p,q) in the plane. For neutral stability of (p,q), all we require is $\frac{dV(p,q)}{dt} = 0$ for all $(p,q) \in B$:

$$\frac{dV(p,q)}{dt} = \left(\frac{-0.35}{p} + \frac{1.15}{1-p}\right)(0.9-1.5q)p(1-p) + \left(\frac{-0.9}{q} + \frac{0.6}{1-q}\right)(-0.35+1.5p)q(1-q)$$

$$\Rightarrow \frac{dV(p,q)}{dt} = 0; \forall p; \forall q \in B$$

When policies to prevent nationalism are available, equations (3) and (4) become, respectively:

$$\dot{p} = (-0.5-1.5q)p(1-p)$$

$$\dot{q} = (-0.35+1.5p)q(1-q)$$

and the corresponding vector field is:

[Figure 3 - see Appendix III]

For both systems of non-linear differential equations, the solutions were obtained numerically in MATLAB using the classic fourth order four-stage Runge-Kutta method (see Appendix II) using a constant step size equal to $\Delta t = 0.02$. The solution paths are presented in figure 4, in which the state space is presented by the unit square. I present two orbits for the free nationalism case for the initial conditions $p_0 = 0.1 \wedge q_0 = 0.1$ and $p_0 = 0.5 \wedge q_0 = 0.5$. In the figure, it can be seen the points corresponding to these sets of initial conditions as well as the center of the orbits, which is the Nash equilibrium in mixed strategies but not an evolutionary equilibrium. The figure also shows the trajectory corresponding to the case with nationalism prevention corresponding to the latter set of initial conditions as well as the state $\theta^{EE} = (0,0)$. I also present a table with some values for p and q along the trajectories, obtained from the Runge-Kutta algorithm:

[Figure 4 - see Appendix III]

[Table 4 - see Appendix III]

From the table, we can see the oscillatory polymorphic pattern of both nationals and immigrants for the nationalism-free case. Both start with a very small proportion of nationalists and learners among their respective populations (10% each). Then, nationalists rise quickly (after 300 steps, they reached 82%), creating an incentive for the growth of learners among the immigrants' population. At step 500, learners achieved 96%, almost the totallity of the immigrants, and this led to a sharp reduction in nationalistic behaviour among the population of nationals. Nationalists achieved a level as low as 1.5% at step 900, leading then to a subsequent sharp reduction of learners over time and to the re-start of the cycle.

For the case when nationalism is prevented, we can clearly see the number of nationalists drops very quickly, becoming already residual at step 200. This fall leads to the drop in the proportion of learners over time, converging both populations to monomorphic states with 100% of welcome-types and non-learners.

3.2 Influence of enclaves

I start this section presenting an example for the case in which nationalism is prevented and all six possible stationary states exist. For this, I select $f(c) = 0.12$; $(1-\alpha)f(P) = 0.05$; $f(K) = 0.2$; $f(X) = 0.1$, $\rho = 1$, $\gamma = 0.2$ and $\beta = 0.85$. For these values of the parameters, the equations governing the system dynamics are:

$$\dot{p} = (-0.05 - 0.05q)p(1-p)$$

$$\dot{q} = (0.04 + 0.05p - 0.13q)q(1-q)$$

The six stationary points are the four corners of the square plus $(\overline{p_2} = 0, \overline{q_2} = 0.308)$ and $(\overline{p_3} = 1, \overline{q_3} = 0.692)$. The vector field in the figure below clearly shows that the corners corresponding to $p = 1$ are unstable nodes, the points $(1, 0.692)$, $(0,0)$ and $(0,1)$ are saddle points and the only attractor is indeed the stable node $(0, 0.308)$, which was the expected outcome given $0.03 = f(K)(1-\rho\beta) < f(c) < f(K)(1-\rho\gamma) = 0.16$.

[Figure 5 - see Appendix III]

Now, I present a numerical simulation for all five possible cases presented in table 3 (free nationalism and enclaves). I used $(1-\alpha)f(P)=0.1$; $f(K)=0.2$; $f(X)=0.08$, $\rho=1$, $\gamma=0.2$ and $\beta=0.85$. The corresponding equation governing the share of nationalists among the population of nationals is:

$$\dot{p}=(0.02-0.1q)p(1-p)$$

The equation governing the share of learners has the form:

$$\dot{q}=((0.16-f(c))+0.1p-0.13q)q(1-q)$$

I used the following values for $f(c)$ in the simulations carried: 0.020; 0.100; 0.145; 0.200; 0.235; 0.400 and the corresponding evolutionary equilibria were, respectively, $\theta^{EE}=(0,1)$, corner; $\theta^{EE}=(0,0.462)$, edge of the square with an isomorphic population of welcome-type nationals and a polymorphic population of immigrants; $\theta^{EE}=(0.11,0.2)$, center, with both populations evolving to a polymorphic equilibrium; $\theta^{EE}=(0.66,0.2)$, same case as the previous one; $\theta^{EE}=(1,0.192)$, edge of the square with an isomorphic population of nationalists and a polymorphic population of immigrants; $\theta^{EE}=(1,0)$, corner.

The following two figures present the solution paths and the corresponding vector fields leading to each of the above evolutionary equilibria for different sets of initial conditions. As in section 3.1, the solution paths were obtained using the Runge-Kutta algorithm and the simulations were carried in MATLAB. For the case with $f(c)=0.145$, it can be seen that all seven possible stationary points are present in the state space although only the stable focus is an evolutionary equilibrium:

[Figure 6 - see Appendix III]

[Figure 7 - see Appendix III]

4 Further Research

The framework presented in this paper suggests several directions for future research. In this section, I start exploring briefly two possibilities which are the existence of integration policies by the government and the endogeneization of the government role according to which nationalism prevention or not would depend endogenously on the proportion of nationalists in the population of nationals given that the national citizens would be the ones electing the government.

With respect to the first possibility, I relax the assumption that only nationals can work in the public services. I assume the host country government has some policy according to which additional working posts are created in the public sector in order to be filled by assimilated immigrants. That means some learners may now achieve complete integration into the national collective. Notice that, differently from affirmative policies such as the one implemented in South Africa after the end of apartheid, my framework assumes a policy of quotas which is very pro-immigration once I assume immigrants do not steal any jobs from the already employed nationals, i.e., the public employment level is simply raised through public expenditure and the new posts are destinated to assimilated non-national individuals. I assume there is a proportion Π of integrated (hired) learners. Hence, whenever an immigrant looks for a public service, she has a probability of meeting with a non-national employee given by:

$$\frac{\Pi x_L^t}{\Pi x_L^t + x_N^t + x_W^t}$$

where x_i^t is the number of individuals of i-type at time t ($N =$ nationalists, $W =$ welcome and $L =$ learners). With respect to the original payoffs in section 2.1, now, when a learner seeking for a service meets a learner-type employee, the only change is that she does not feel discriminated (same payoff as when meeting with a welcome-type national):

$$u^m(L,L) = f(P,K) - f(c)$$

Also, when a non-learner seeks for a service and meets a learner-type employee, she gets the complete service, does not feel discriminated and gets $\beta f(K)$ [this last term is the same corresponding to the utility gain whenever this pairwise meeting happens in the enclaves case, see section 2.2]:

$$u^m(H,L) = f(P) + \beta f(K)$$

In summary, whenever an immigrant seeks for a public service, she faces the following game:

$$\begin{array}{cc} & \begin{array}{ccc} N & W & L \end{array} \\ \begin{array}{c} L \\ H \end{array} & \begin{pmatrix} 0,0 & m_1,n_2 & 0,0 \\ m_2,n_1 & 0,0 & m',0 \end{pmatrix} \end{array}$$

where $m' = u^m(H,L) - u^m(L,L) = f(c) - (1-\beta)f(K)$. The corresponding probabilities of a given type of player being called at time t to play the above stage game are given below:

$$\begin{array}{c} \\ \frac{x_L^t}{x_L^t+x_H^t} \\ \frac{x_H^t}{x_L^t+x_H^t} \end{array} \begin{array}{c} \begin{array}{ccc} \frac{x_N^t}{\Pi x_L^t+x_N^t+x_W^t} & \frac{x_W^t}{\Pi x_L^t+x_N^t+x_W^t} & \frac{\Pi x_L^t}{\Pi x_L^t+x_N^t+x_W^t} \end{array} \\ \begin{pmatrix} 0,0 & m_1,n_2 & 0,0 \\ m_2,n_1 & 0,0 & m',0 \end{pmatrix} \end{array}$$

The new system of ordinary differencial equations would be then derived from the above payoff matrix but now, differently from sections 2.1 and 2.2, the probabilites of who an immigrant will face at the public services office depends on both populations making the derivation of the system more complicated.

With respect to the second idea for future research, I assume that the parameter α accounting for how accessible the public services are to non-learners would be endogenous. The parameter would depend on the ideology of the elected government, which would depend on the proportion of nationalists in the population. The idea of government here would be associated with a bi-party parliament where the proportions of nationalist and welcome-type deputies would be identical to the ones present in the population of nationals. Hence, a population of nationals composed only by nationalists would elect a 100% nationalist parliament so that $\alpha = 0$. On the other extreme, when nationalism becomes extinct, the parliament would be completely composed by welcome-type deputies and $\alpha = 1$. A possible way of trying to make α endogenous would be to assume $\alpha = 1-p$ and $(1-\alpha) = p$ in the utility functions of sections 2.1 and 2.2. The analysis of the new non-linear system would be in principle more complicated than the ones presented in this paper.

5 Conclusion

In this paper I analyzed the evolutionary pattern over generations of the behaviours of immigrants and nationals living in a country where both cultural barriers and nationalism co-exist. At the initial conditions of the model, both populations are necessarily polymorphic and an immigrant may be either a learner or a non-learner while nationals may welcome immigrants or adopt nationalistic attitudes against them. I also took into account the role of the government in preventing or not these nationalistic behaviours as well as the effect of the existence of an immigrants' enclave in the host country.

Two central ideas in the paper are the use of language learning as a proxy for cultural assimilation and that immigrants have to meet nationals whenever the individuals in the former group look for public services, in which employment is restricted to national citizens. The dynamics of the evolution process was modelled using replicator dynamics, initially without own population effects and then extending the differential equation related to the immigrants' dynamics in order to take into account the own population effects derived from the interactions among the latter group taking place in the enclave.

When there is no enclave, the dynamical analysis of the system of ordinary differential equations showed that, independently of the existence or not of free nationalism, a monomorphic population of learner immigrants together with the extinction of nationalism is an evolutionary equilibrium (EE) for a low cost of learning the host country language. This outcome can be seen in Latin American countries such as Brazil where a large amount of Portuguese, Spanish and Italian speakers arrived during the 20^{th} century becoming completely assimilated without the appearance of enclaves or nationalism. On the other hand, when the effort for becoming assimilated is high, multiculturalism is always an EE because immigrants evolve to a monomorphic population in which learners become extinct over time. But in this case, the pattern of multiculturalism is directly linked to the government policy given that nationalism prevention brings nationalism to extinction while free nationalism leads to the extinction of any national citizen welcoming immigrants. The former pattern of multiculturalism is more likely to be seen in the UK, Australia, Sweden and Canada while the latter resembles the former colonies in Africa and Asia during the last century and also some former Soviet republics such as Latvia and Estonia.

More interesting is the case when the effort of learning falls in an intermediate level and nationalism is not prevented. In this case, no EE exists at all and the dynamics is very similar to the Lotka-Volterra predator-prey model for competition between two animal species with no-overcrowding. Over generations, both populations of immigrants and nationals continue to be polymorphic in such way that the shares of their sub-populations keep changing over time. In the state space, a complete evolutionary cycle is charactetized by a closed orbit about the Nash equilibrium, departing and always re-starting at the initial conditions.

The existence of an enclave in the host country makes meetings among immigrants more likely to occur. In such a context, non-learners would be able to obtain an extra-utility due to the acquisition of information from leaner immigrants about the host country society, places and daily environment given their lack of knowledge of the local language prevents them from doing this. In line with the empirical literature on migration, comparing the case of host countries with and without an enclave, the upper boundary of the cost of learning still leading to the complete assimilation of immigrants over time is lower in the former case. Complete assimilation now requires individuals with lower switching language costs.

In extreme cases when an enclave is more likely a ghetto, that is, the likelihood of meetings among immigrants is one and the extra utility gain to non-learners approaches the utility level learners are able to get themselves from interacting with the environment, complete assimilation of immigrants cannot be an EE. Part of the learners become extinct over time when there is free nationalism while under nationalism prevention, no assimilation at all takes place. These latter results are linked with another difference in the evolutionary pattern when an enclave exists. Differently from the case without enclaves, we may have EEs characterized by polymorphism in both populations for free nationalism and polymorphism in the population of immigrants when nationalism is prevented. Finally, in all cases studied in the paper, no multiple EE exists, although there are always multiple stationary points at the boundary and, in some cases, in the interior of the state space.

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Appendix I

In this appendix, I analyze the derivation and the stability at the five stationary points of the non-linear system of ordinary differential equations composed of equations (1) and (2). Assume at the beginning of the time frame, there are two very large populations, one of nationals and the other of immigrants, both of size x^t, such that:

$$x^t = x^t_N + x^t_W$$
$$x^t = x^t_L + x^t_H$$

Where the subscripts N, W, L and H account respectively for the sizes of the sub-populations of nationalist, welcome, learner and non-learner types. Assume n contests (stage-games) take place during the time interval ε such that the proportion of players from each population taking part in the contests equals $\varepsilon = n/x^t$. Assume also that $p^t = x^t_N/x^t$ and $q^t = x^t_L/x^t$ are the proportions of nationalists and learners, respectively, at time zero. Relying on the law of large numbers and, based on the stage-game payoff matrix of section 2.1 which is repeated below:

$$\begin{array}{cc} & \begin{array}{cc} L & H \end{array} \\ \begin{array}{c} N \\ W \end{array} & \begin{pmatrix} 0,0 & n_1,m_2 \\ n_2,m_1 & 0,0 \end{pmatrix} \end{array}$$

In one given contest, the expected payoff accruing to the sub-population of nationalists is given by:[17]

$$p^t(1-q^t)n_1$$

And in n contests:

$$np^t(1-q^t)n_1$$

[17] Originally, in biology, the outcome of each stage-game defines the number of offspring produced by an animal adopting behaviour x-type when contesting with another animal adopting behaviour y-type. Animals only produce offspring after taking part in the contests during which they may escalate and fight (or simply refrain from fighting) for nesting sites. A good nesting site may require fighting for it depending on the opponent. Some authors also assume that the animals contesting and giving birth die after the process and are therefore not restored back to the original population [see Weibull (1997) and Miekisz, J. (undated)]. Others do not assume the animals die and they are replaced back to the population together with their offspring [see Gintis (2000)]. The dynamics does not change using either approaches. In the approach presented in this appendix, I assume that individuals taking part in the contests (stage-games) are restored back to the population after the pairwise meeting.

Hence, the subpopulation of nationalists at time $t+\varepsilon$ is:

$$x_N^{t+\varepsilon} = x_N^t + np^t(1-q^t)n_1 \Rightarrow x_N^{t+\varepsilon} = x_N^t + \varepsilon x_N^t(1-q^t)n_1$$

Using the same reasoning, the subpopulation of welcome-type nationals at time $t+\varepsilon$ is:

$$x_W^{t+\varepsilon} = x_W^t + n(1-p^t)q^t n_2$$

Such that the population of nationals at time $t+\varepsilon$ is:

$$x^{t+\varepsilon} = x_N^t + x_W^t + np^t(1-q^t)n_1 + n(1-p^t)q^t n_2$$

$$\Rightarrow x^{t+\varepsilon} = x^t + \varepsilon x^t p^t(1-q^t)n_1 + \varepsilon x^t(1-p^t)q^t n_2$$

Based on the above equations, the proportion of nationalists at time $t+\varepsilon$, $p^{t+\varepsilon}$, becomes:

$$p^{t+\varepsilon} = \frac{x_N^{t+\varepsilon}}{x^{t+\varepsilon}} = \frac{x_N^t + \varepsilon x_N^t(1-q^t)n_1}{x^t + \varepsilon x^t p^t(1-q^t)n_1 + \varepsilon x^t(1-p^t)q^t n_2}$$

And the change in the proportion p from time t to time $t+\varepsilon$ is:

$$p^{t+\varepsilon} - p^t = \frac{p^t\varepsilon[(1-q^t)(1-p^t)n_1 - (1-p^t)q^t n_2]}{1+\varepsilon p^t(1-q^t)n_1 + \varepsilon(1-p^t)q^t n_2}$$

Dividing both sides by ε and taking the limit of $\varepsilon \to 0$, gives equation (1):

$$\dot{p} = (n_1(1-q) - n_2 q)p(1-p)$$

Using the population of immigrants, the same type of reasoning can be used to derive equation (2). Now, turning to the stability analysis, the Jacobian matrix at a given stationary point (p,q) in the interior or boundary of the square $[0,1]^2$ is:

$$\Omega(p,q) = \begin{pmatrix} \frac{\partial F^1(p,q)}{\partial p} & \frac{\partial F^1(p,q)}{\partial q} \\ \frac{\partial F^2(p,q)}{\partial p} & \frac{\partial F^2(p,q)}{\partial q} \end{pmatrix}$$

Analyzing the stability at each stationary point presented in proposition 1:

At point $(0,0)$: $tr[\Omega(0,0)] = n_1 + m_1$; $det[\Omega(0,0)] = n_1 m_1$. For stability, requires: $m_1 < 0 \Rightarrow f(c) > f(K)$ and $n_1 < 0 \Rightarrow (1-\alpha)f(P) < f(X)$, i.e., nationalism prevented as in proposition 5. At point $(1,1)$: $tr[\Omega(1,1)] = n_2 + m_2$; $det[\Omega(1,1)] = n_2 m_2$. For stability, requires: $n_2 < 0 \Rightarrow f(X) < 0$, which is never satisfied, hence, never an EE. At point $(1,0)$: $tr[\Omega(1,0)] = -n_1 - m_2$; $det[\Omega(1,0)] = n_1 m_2$. For stability, requires: $n_1 > 0 \Rightarrow (1-\alpha)f(P) > f(X)$, i.e., free nationalism and $m_2 > 0 \Rightarrow f(c) > (1-\alpha)f(P) + f(K)$ as in proposition 3. At point $(0,1)$: $tr[\Omega(0,1)] = -n_2 - m_1$; $det[\Omega(0,1)] = n_2 m_1$. For stability, requires: $n_2 > 0$, i.e., either free or prevented nationalism and $m_1 > 0 \Rightarrow f(c) < f(K)$, such as in propositions 2 and 5.

At point $(\overline{p},\overline{q})$: $tr[\Omega(\overline{p},\overline{q})] = 0$; $det[\Omega(\overline{p},\overline{q})] = -(\overline{p}-\overline{p}^2)(\overline{q}-\overline{q}^2)(n_1+n_2)(m_1+m_2) > 0$. The dynamical system is non-hyperbolic at this stationary point given the eigenvalues (pure imaginary) have zero real part. I can only conclude from the nullclines in the squared state space that the vector field winds in the counter-clockwise direction about the stationary point. But I can not use linearization to identify the stability of the system at this stationary point. For this, I employ the following Liapunov function:

$$V(p,q) = m_1 \ln p + m_2 \ln(1-p) - n_1 \ln q - n_2 \ln(1-q) - c \text{ such that } V(\overline{p},\overline{q}) = 0$$

Proposition: $V(p,q)$ is a non-strict Liapunov function for $(\overline{p},\overline{q})$ and $(\overline{p},\overline{q})$ is Liapunov stable but not asymptotically stable.

Proof: $V(p,q)$ has a strict minimum at $(\overline{p},\overline{q})$:

$$\frac{\partial V(p,q)}{\partial p} = \frac{m_1}{p} - \frac{m_2}{1-p} = 0 \Rightarrow p = \overline{p}$$

$$\frac{\partial V(p,q)}{\partial q} = \frac{-n_1}{q} + \frac{n_2}{1-q} = 0 \Rightarrow q = \overline{q}$$

2^{nd} order conditions of the Hessian at $(\overline{p},\overline{q})$:

$$\frac{\partial^2 V(p,q)}{\partial p^2} = \frac{-m_1}{p^2} + \frac{-m_2}{(1-p)^2} > 0$$

$$\frac{\partial^2 V(p,q)}{\partial p^2}\frac{\partial^2 V(p,q)}{\partial q^2} = \left(\frac{-m_1}{p^2} + \frac{-m_2}{(1-p)^2}\right)\left(\frac{n_1}{q^2} + \frac{n_2}{(1-q)^2}\right) > 0$$

Hence, $V(\overline{p},\overline{q}) = 0 \wedge V(p,q) > 0 \Leftrightarrow (p,q) \neq (\overline{p},\overline{q})$.

Let B be an open ball about $(\overline{p},\overline{q})$ in the plane. For neutral stability of $(\overline{p},\overline{q})$, all we require is $\frac{dV(p,q)}{dt} \leq 0$ for all $(p,q) \in \{B - (\overline{p},\overline{q})\}$ and $\frac{dV(p,q)}{dt} = 0$ for $(\overline{p},\overline{q})$:

$$\frac{dV(p,q)}{dt} = \frac{m_1(1-p)-m_2p}{p(1-p)}[n_1(1-q)-n_2q]p(1-p) + \frac{-n_1(1-q)+n_2q}{q(1-q)}[m_1(1-p)-m_2p]q(1-q) = 0;$$

$\forall p; \forall q \in B$

Hence, $V(p,q)$ is indeed a non-strict Liapunov function for $(\overline{p},\overline{q})$; $(\overline{p},\overline{q})$ is Liapunov stable but not asymptotically stable. Hence, $(\overline{p},\overline{q})$ is a Nash equilibrium but not an EE.

Appendix II

In this appendix, I show the Runge-Kutta algorithm used to solve the two systems of ODE in section 3.1. For the step from t_n to t_{n+1}, we have:

$$\begin{pmatrix} p_{n+1} \\ q_{n+1} \end{pmatrix} = \begin{pmatrix} p_n \\ q_n \end{pmatrix} + \frac{h}{6} \cdot \begin{pmatrix} p_{n1} + 2 \cdot p_{n2} + 2 \cdot p_{n3} + p_{n4} \\ q_{n1} + 2 \cdot q_{n2} + 2 \cdot q_{n3} + q_{n4} \end{pmatrix}$$

where $h = t_{n+1} - t_n = \Delta t$ and:

$$\begin{array}{ll}
p_{n1} = \frac{\partial p}{\partial t}(p_n, q_n) & q_{n1} = \frac{\partial q}{\partial t}(p_n, q_n) \\
p_{n2} = \frac{\partial p}{\partial t}(p_n + (h/2) \cdot p_{n1}, q_n + (h/2) \cdot q_{n1}) & q_{n2} = \frac{\partial q}{\partial t}(p_n + (h/2) \cdot p_{n1}, q_n + (h/2) \cdot q_{n1}) \\
p_{n3} = \frac{\partial p}{\partial t}(p_n + (h/2) \cdot p_{n2}, q_n + (h/2) \cdot q_{n2}) & q_{n3} = \frac{\partial q}{\partial t}(p_n + (h/2) \cdot p_{n2}, q_n + (h/2) \cdot q_{n2}) \\
p_{n4} = \frac{\partial p}{\partial t}(p_n + h \cdot p_{n3}, q_n + h \cdot q_{n3}) & q_{n4} = \frac{\partial q}{\partial t}(p_n + h \cdot p_{n3}, q_n + h \cdot q_{n3})
\end{array}$$

For the nationalism-free case, the number of iteractions until one orbit was completed equalled 1600 and 1125 steps for initial conditions $p = 0.1$; $q = 0.1$ and $p = 0.5$; $q = 0.5$, respectively. For the case when nationalism is officialy prevented, the number of steps equalled 1103 until the evolutionary equilibrium was achieved.

Appendix III - Tables and Figures

Case	Value of $f(c)$	Equilibrium
A	$[0; f(K)(1-\rho\beta)-\varepsilon]$	$(0;1)$
B	$[f(K)(1-\rho\beta)+\varepsilon; f(K)(1-\rho\gamma)-\varepsilon]$	$(0,\overline{q_2})$
C	$[f(K)(1-\rho\gamma)+\varepsilon; +\infty)$	$(0;0)$

Table 1: Possible evolutionary equilibria given the cost of learning the language (nationalism prevented and presence of enclaves).

Case with $f(K) < (1-\alpha)f(P)/(\rho(\beta-\gamma))$		
Value of $f(c)$[18]	Equilibrium	Stationary points
$[0;A)$	$(0,1)$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$
$(A;B)$	$(0,\overline{q_2})$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$; $(0,\overline{q_2})$
$(B;C)$	$(0,0)$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$
$(C;D)$	$(0,0)$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$; $(1,\overline{q_3})$
$(D;+\infty)$	$(0,0)$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$
Case with $f(K) > (1-\alpha)f(P)/(\rho(\beta-\gamma))$		
Value of $f(c)$	Equilibrium	Stationary points
$[0;A)$	$(0,1)$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$
$(A;C)$	$(0,\overline{q_2})$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$; $(0,\overline{q_2})$
$(C;B)$	$(0,\overline{q_2})$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$; $(0,\overline{q_2})$; $(1,\overline{q_3})$
$(B;D)$	$(0,0)$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$; $(1,\overline{q_3})$
$(D;+\infty)$	$(0,0)$	$(0,0)$; $(0,1)$; $(1,0)$; $(1,1)$

Table 2: Possible equilibria and stationary points for the case of enclaves together with nationalism prevention.

[18] Where $A = f(K)(1-\rho\beta)$; $B = f(K)(1-\rho\gamma)$; $C = (1-\alpha)f(P) + f(K)(1-\rho\beta)$; $D = (1-\alpha)f(P) + f(K)(1-\rho\gamma)$.

Value of $f(c)$	Equil.
$[0; f(K)(1-\rho\beta)-\varepsilon]$	$(0;1)$
$[f(K)(1-\rho\beta); f(K)(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})-\varepsilon]$	$(0,\overline{q_2})$
$[f(K)(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1}); (1-\alpha)f(P)+f(K)(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})]$	$(\overline{p_1},\overline{q_1})$
$[(1-\alpha)f(P)+f(K)(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})+\varepsilon; (1-\alpha)f(P)+f(K)(1-\rho\gamma)]$	$(1,\overline{q_3})$
$[(1-\alpha)f(P)+f(K)(1-\rho\gamma)+\varepsilon; +\infty)$	$(1;0)$

Table 3: Possible evolutionary equilibria given the cost of learning the host country language (free nationalism and presence of enclaves).

	Free nationalism		Nationalism prevented	
	Nationalists	Learners	Nationalists	Learners
initial conditions	10.00%	10.00%	50.00%	50.00%
step 100	33.86%	9.17%	6.98%	50.50%
step 200	67.95%	19.06%	0.75%	35.48%
step 300	82.02%	54.10%	0.12%	21.62%
step 400	75.70%	86.68%	0.03%	12.06%
step 500	54.05%	95.89%	0.01%	6.38%
step 600	27.96%	97.51%	0.00%	3.27%
step 700	11.21%	97.15%	0.00%	1.65%
step 800	4.06%	95.43%	0.00%	0.83%
step 900	1.51%	91.80%	0.00%	0.41%
step 1000	0.64%	85.15%	0.00%	0.21%
step 1200	0.29%	59.13%	*****	*****
step 1600	9.96%	10.01%	*****	*****

Table 4: Proportion of Nationalists and Learners along the solution trajectories.

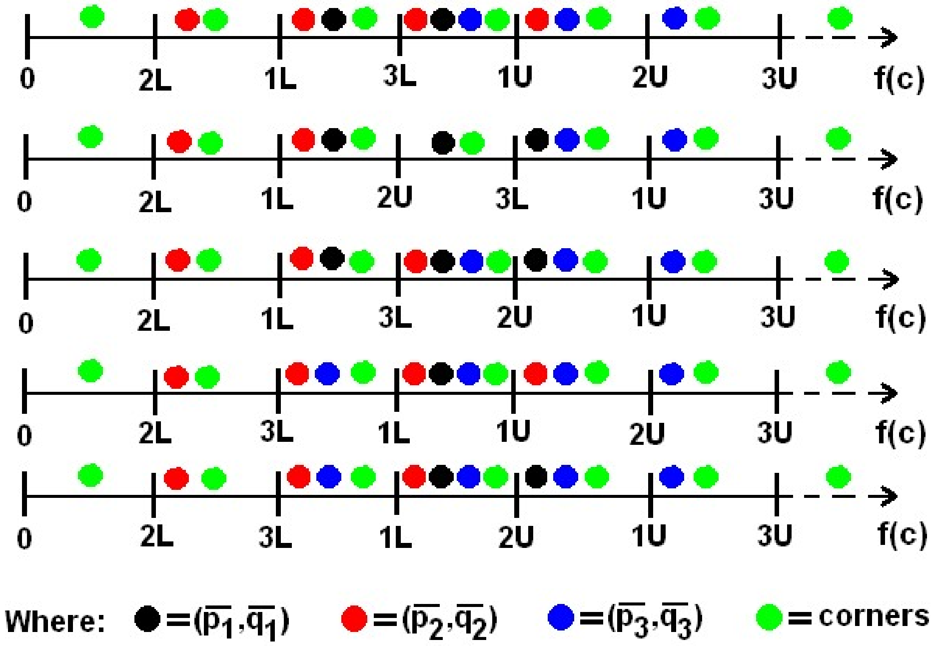

Figure 1: Stationary points present in the state space according to the cost of learning, where 1L= $f(K)(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})$; 1U= $f(K)(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})+(1-\alpha)f(P)$; 2L= $f(K)(1-\rho\beta)$; 2U= $f(K)(1-\rho\gamma)$; 3L=$(1-\alpha)f(P)+f(K)(1-\rho\beta)$; 3U=$(1-\alpha)f(P)+f(K)(1-\rho\gamma)$.

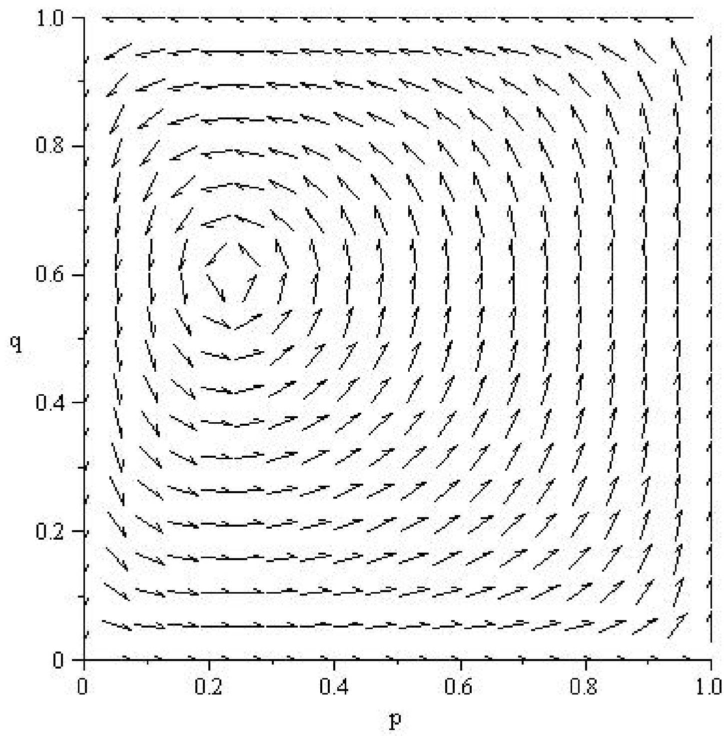

Figure 2: Vector field for countries where host language is neither close nor far from the origin country one and nationalism can not be prevented.

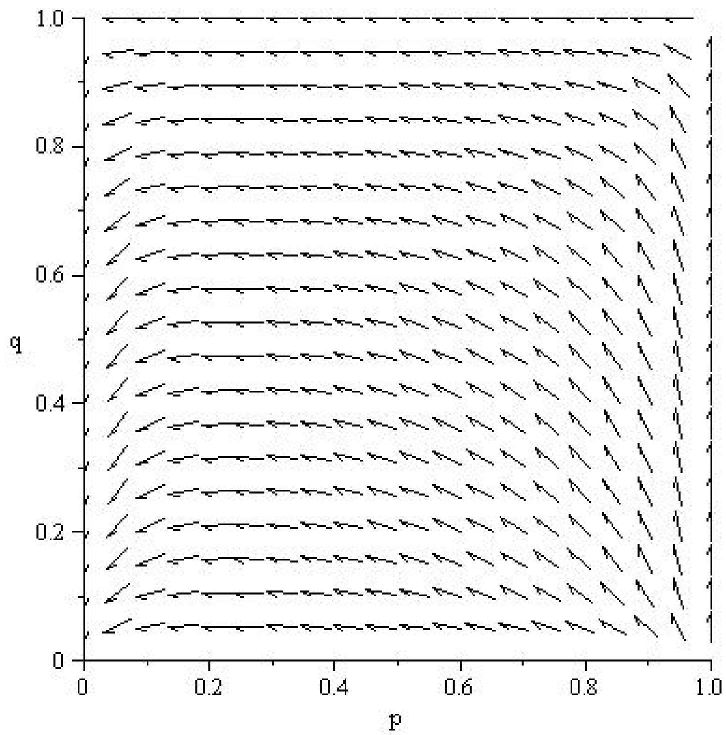

Figure 3: Vector field for countries where there exist measures preventing nationalism and the cost of learning is neither low nor high.

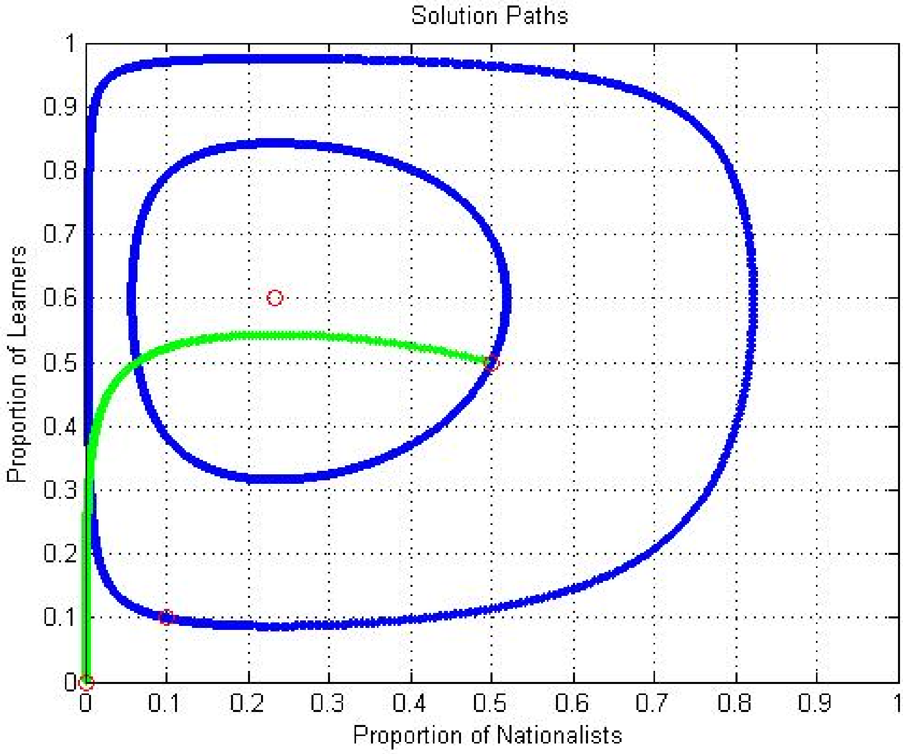

Figure 4: Trajectories for both cases of nationalism-prevention policies and free nationalism.

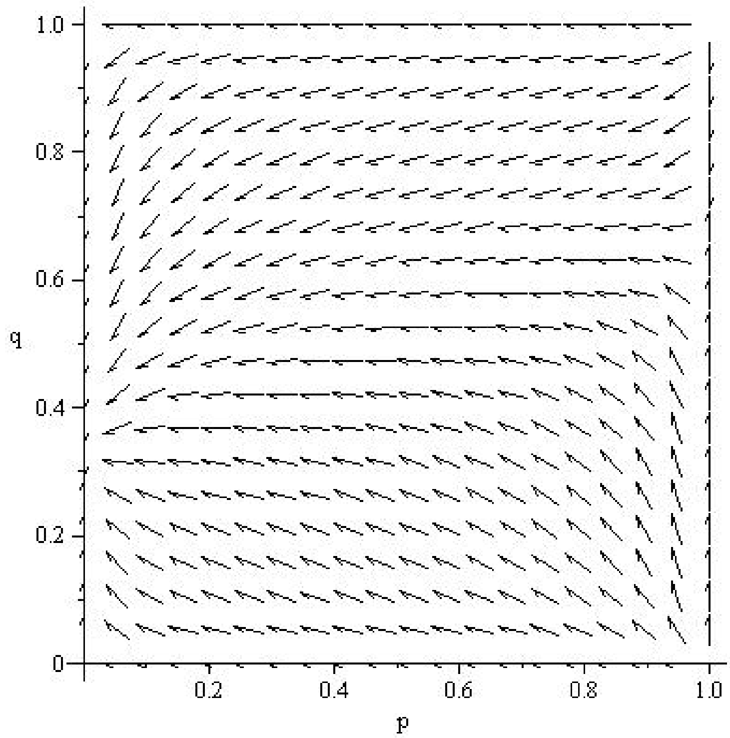

Figure 5: Vector field when nationalism is prevented and an enclave exists - case with six stationary points.

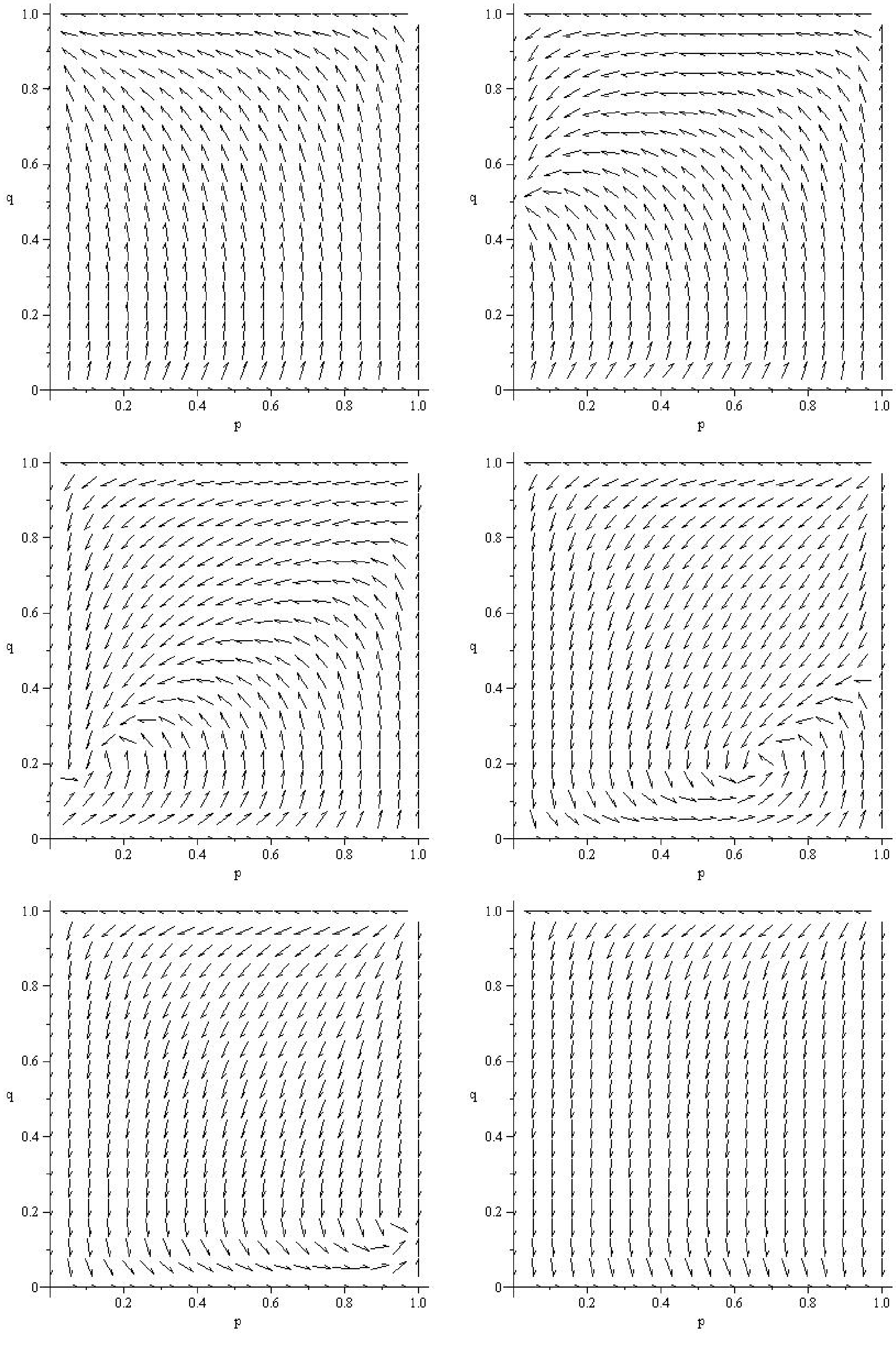

Figure 6: From top left to bottom right, vector fields for $f(c) = 0.020$; 0.100; 0.145; 0.200; 0.235 and 0.400

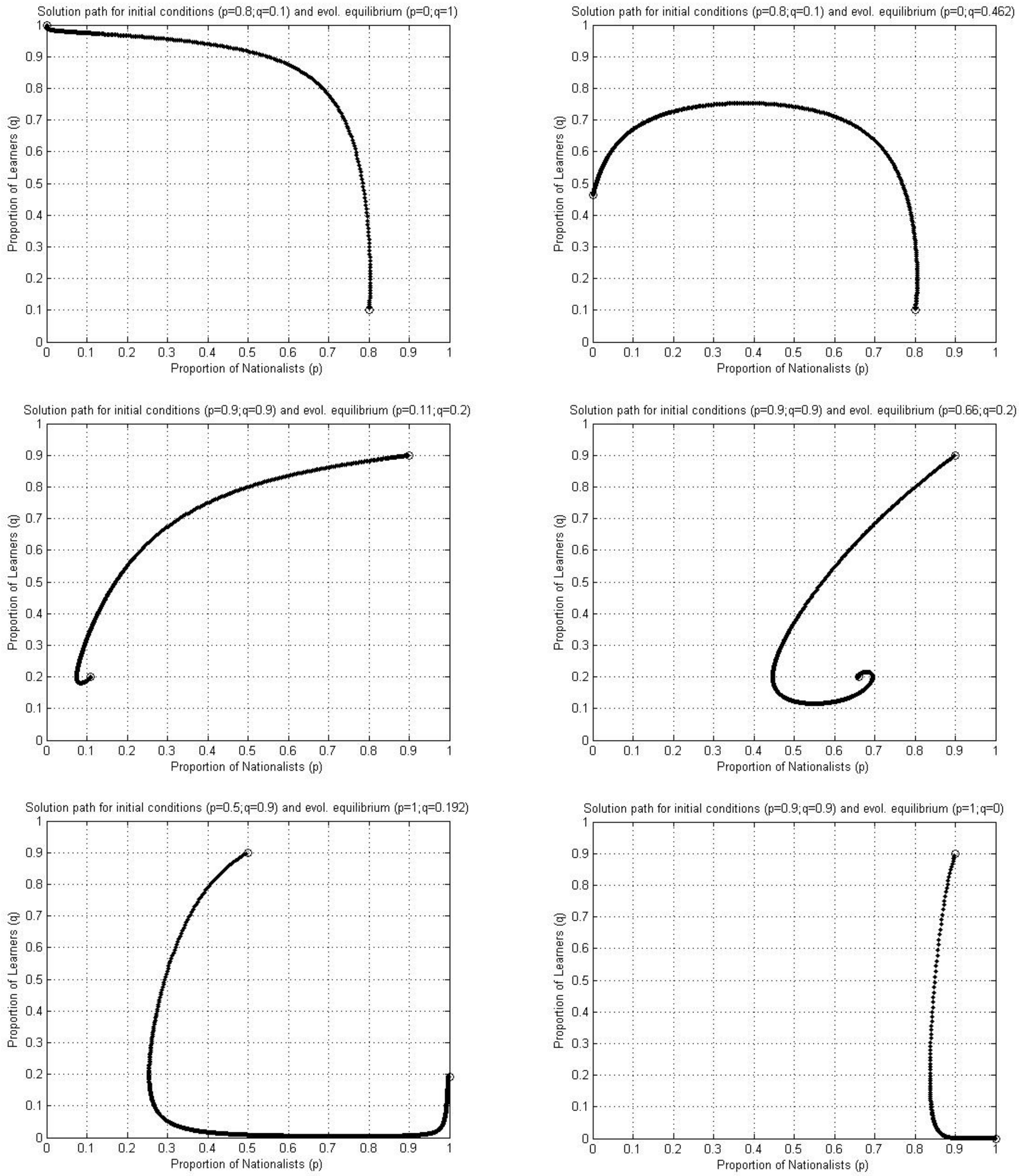

Figure 7: From top left to bottom right, solution paths for $f(c) = 0.020$; 0.100; 0.145; 0.200; 0.235; 0.400 and different sets of initial conditions.