A Positive Theory of Network Connectivity

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Abstract

This paper develops a positive theory of network connectivity, seeking to explain the micro-foundations of alternative network topologies as the result of self-interested actors. By building roads, landowners hope to increase their parcels' accessibility and economic value. A simulation model is performed on a grid-like land use layer with a downtown in the center, whose structure resembles the early form of many Midwestern and Western (US) cities. The topological attributes for the networks are evaluated. This research posits that road networks experience an evolutionary process where a tree-like structure first emerges around the centered parcel before the network pushes outward to the periphery. In addition, road network topology undergoes clear phase changes as the economic values of parcels vary. The results demonstrate that even without a centralized authority, road networks have the property of self-organization and evolution, and, that in the absence of intervention, the tree-like or web-like nature of networks is a result of the underlying economics.

Keywords: road network, land parcel, network evolution, network growth, phase change, centrality measures, degree centrality, closeness centrality, betweenness centrality, network structure, treeness, circuitness, topology

1 Introduction

The design of street patterns in cities is contentious. Traditional engineering design guidelines argue for a hierarchical, even tree-like, street and highway patterns (American Association of State Highway and Transportation Officials, 2001), particularly in new suburban environments. Many urban designers propose a flatter, web-like pattern developing organically (Alexander, 1965) or from plans (Grammenos et al., 2008; Martin, 2008). Aspects of network form have been codified in design principles both historically (Ben-Joseph, 2005; Ben-Joseph and Szold, 2005; Southworth and Ben-Joseph, 2003) and for transit oriented development (Jacobson and Forsyth, 2008). This debate has been held at a normative level, with arguments about which pattern satisfies different values (e.g. efficiency for cars vs. efficiency for pedestrians, how to deal with through traffic, and what is the best allocation of land between transportation and other uses).

Road networks, as artifacts of human activities, display interesting patterns of organization. While top-down organization appears predominant (for instance, since its inception in 1921, federal financial aid has funded improvements of the most important roads in the US (Rae, 1971)), order can also emerge from completely decentralized and spontaneous interactions of individuals. Many of the modern roads in their earliest incarnation, for example, were constructed by individuals. As Powers (1910) indicated:

> "Our public roads are an evolution from the primary paths made by animals and by men. Of the identity of the first beings who made paths in the wilder-

> ness we are uncertain. Whatever their character and origin, we may be reasonably certain that they had roads of some sort."

The emergence of the "roads of some sort", without a centralized plan, must involve numerous discrete decisions. So what incentives beget the interactions of individuals and ultimately produce the road patterns we see today?

Although a spectrum of sources can lead to the birth of roads, we desire to understand the economic incentives for road network growth from a microscopic view. This research aims to model the impact of individual land owners' behavior on road network patterns. The idea accords with Powers's review of the history of road building in the US. First, according to Powers (1910), the early roads were built due to a call for communication and navigation. In this research, we assume that roads are built by self-interested land developers who aim to increase their own land parcels' accessibility. Second, "road building began at centers and spread out with the spread of population" (Powers, 1910). So in this paper a center with the highest economic value of accessibility is presumed to exist (and therefore other land owners want to connect to it). This represents for instance the location of a port or railroad station that allows access to the outside world. Third, as anecdotal evidence about early roads in Massachusetts Bay Colony depicts, "in 1636 a measure was passed in the Massachusetts Bay Colony which provided that two or three men from adjacent towns get together and lay out proper roads...provided they did not necessitate pulling down a man's house or going through his garden or orchard" (Powers, 1910).

This paper seeks to develop a positive theory of network connectivity which explains the patterns we observe based on the motivations of individual agents, the developers and the residential and commercial locators they serve, rather than a normative theory of what they should be, based on ideals of planners, architects, or engineers. We then seek to model the growth of roads comporting with this underlying theory. Examining dynamics rather the statics is about understanding the evolution of the system rather than just its end-state. There have been a number of attempts to model road growth, which can be cataloged into three streams distinguished by modeling perspective.

First, in *probabilistic network growth models*, each link is presumably born with a probability. There have been many attempts to describe urban form, developing mathematical models of connectivity (Salingaros, 1998, 2000; Salingaros et al., 2000) and urban growth (Batty, 2006; Batty and Longley, 1986; Longley et al., 1991). A notable example is the random graph model, arguably the first application of modern graph theory to explain real-world networks (Erdös and Rényi, 1959). Other approaches include the exponential model (Dorogovtsev and Mendes, 2002), preferential attachment model(Barabási and Albert, 1999; Price, 1965), Markov graph (Frank and Strauss, 1986; Wasserman and Pattison, 1996), and Newman-Gastern model (Gastner and Newman, 2006).

Second, in *network design models*, a link is built to optimize a centralized objective, such as minimizing the Euclidean distance(Gastner and Newman, 2006), minimizing detour

(Schweitzer et al., 1998), or maximize transportation potential between two locations (Yamins et al., 2003).

Third, in *agent-based discrete choice models*, agents construct links with local objectives. For example, Helbing et al. (1998, 1997) adopts an active walker model to model the evolution of trails in urban green spaces. Yerra and Levinson (2005) models network growth with localized investment rules. Levinson and Yerra (2006) investigates the self-organization of road networks using a travel demand model coupled with revenue, cost, and investment models. Xie and Levinson (2009*b*) adopts the approach of iterative process of network loading, traffic demand dynamics, investment, and disinvestment. Such decentralized agent-based approaches provide a down-to-top perspective to examine phase changes of network growth, path dependency (Arthur, 1989) and multiple equilibria (Correa et al., 2004; Yang, 1998). Models of network evolution have been reviewed in Xie and Levinson (2009*a*).

The next section describes land use affinity, the attractiveness between key activities that shapes how networks will be constructed. A discussion of network configuration and accessibility, which describes alternative street configurations, follows. Local and metropolitan dynamics of network and land use co-evolution for different facilities and land uses illustrate the model. A simple model of whether a link would be constructed by a self-interested developer is presented, and this is applied in a simulation in a subsequent section to illustrate the different types of connections that might emerge under different circumstances. The paper concludes with discussion.

2 Land use affinity

To understand the relationship between activity types, assume there are two major types of places: homes and firms. People (who in this simple model are all workers) live at homes and are employed at workplaces (firms).

Firms want to be near other firms because *economies of agglomeration* emerge when different firms co-locate. These economies lower costs of production (or increase quality). Agglomeration economies, sometimes referred to as external economies of scale, extend traditional internal, or firm-level, economies of scale, which as a firm increases production reduces its per unit average cost (Rosenthal and Strange, 2004). Sources of agglomeration economies include:

- labor market pooling and other input sharing (sharing),
- access to specialized goods and services (matching),
- technological spillovers (learning),
- natural advantage,

- home market effects,
- consumption opportunities, and
- rent-seeking

Marshall (1920) developed and Duranton and Puga (2004) describes the first three points. Natural advantage has to do with some inherent ability to have lower production costs in a particular location, due for example to the presence of an input (milling companies locating near waterfalls before electric transmission networks are an example). Home market effects result when local demand for a good (such as snowmobiles in northern Minnesota) exceeds the average. Economies of consumption have several sources, but the ability to comparison shop for both substitutes and complements may make a district advantageous to consumers (who can economize on travel costs by trip chaining, and create demand to be near competitors, who may be more likely to shop at a store in a cluster than one away from either competitors or complements (Huang and Levinson, 2009, 2010). Further the joint location of complementary firms means that the "complement of my complement" will also be co-located, which may be a direct competitor. When firms do desire to co-locate, this must outweigh any negative effects of being near other employers, such as having to pay more for labor.

Firms want to be near workers, because workers constitute a major input into the production process, and nearby firms can pay less for those workers as travel costs need not be compensated.

Workers want to be near firms which provide jobs, since being near more jobs means there is additional demand, and thus higher wages.

Workers want to live at homes far from other workers, since this reduces both competition for jobs (and means higher wages) and competition for land (and thus means lower rents and more space). This last "centrifugal" factor keeps cities from converging onto a single point.

Table 1 summarizes these relationships.

Table 1: Affinity between types of places

	Firms	Homes
Firms	+	+
Homes	+	-

Plus (+) indicates attraction, Minus (-) indicates aversion.

An analysis of residential prices as a function of accessibility to jobs and to workers (El-Geneidy and Levinson, 2006) corroborates the observation that workers want proximity to jobs and separation from other workers. The affinity of workers for jobs applies to subcenters as well as CBDs (Cho et al., 2008). Similarly, opportunities seem to have an affinity for individuals, in that they make themselves accessible to all socio-economic groups (Scott and Horner, 2008). This logic bears some resemblance to Krugman (1996), though that model fixed workers rather than modeling their dynamics, and underlies Levinson et al. (2007), which models locational dynamics and network speed and capacity, but keeps network topology fixed.

3 Network configurations and accessibility

In this paper we model the road network as an undirected graph on a land-use layer comprising a grid of land parcels which roads cannot cross. While the gridiron pattern is idealized, it has been widely adopted in many places of the US [1] and elsewhere (Ben-Joseph, 2005; Marcuse, 1987; Stanislawski, 1946). To illustrate, the central cities of the Minneapolis-St. Paul (Twin Cities), Minnesota region have a typical grid-like pattern (see the 1906 map in Figure 1). Given the historical accounts, we endeavor to match our model to the real environment yet also simple enough to convey the results and implications most clearly. In this research, we explore the topological properties of road networks from our model and the dynamic process which generates them.

US cities typically possess a highly integrated network of streets in central business districts and to a lesser, but still significant extent, in older residential areas. However in newer, post-World War II residential areas, networks have a lower level of connectivity. Figure 2 illustrates by comparing Golden Valley, a largely post-World War II, first ring suburb of Minneapolis on the west of Theodore Wirth Park (and north of I-394), with a largely residential area of North Minneapolis, which is to the east of the park. Downtown Minneapolis is at the very east edge of the figure. Minneapolis is grid-like and highly connected. Golden Valley has a much lower share of through streets, more cul-de-sacs, and larger block sizes (and thus fewer km of roads per km^2 of area). The very southwest corner of the map is an office park and retail center at the interchange of I-394 and Mn 100.

Based on these observations, assume there are two basic types of transportation configurations, trees and circuits.

A treelike configuration is strictly hierarchical, to go to any point from a point, one can proceed down a path toward the center. There is only one path between an origin and destination. If your destination is on the same branch as the origin, you can reach your destination directly, but if it is on a different branch, you must go in to the first common point on the network between the various points and the center, and then proceed out-

[1] The wide adoption of the grid-like pattern mainly results from the history of land survey practice in the US in the 18th century, although whether its benefit outweighs its cost remains in question.

ward. In contrast, a weblike configuration presents multiple paths between origins and destinations.

Many-to-one and one-to-many networks are often well-served by tree-like structures; examples include water and sewer networks, broadcasting, and natural gas. Communication and transportation networks, which need to support many-to-many transactions, may not be able to scale as well with tree-like structures, and thus often develop more cross-connections. There are reasons for even one-to-many type networks having multiple paths, including variable demands requiring distributed capacity, and network reliability, e.g. on an electric grid, if a wire goes down, will electricity still flow? There are also examples of efficient transportation networks (e.g. freight-logistics, FedEx being the most illustrative) where all shipments are sent to a single national center (e.g. Memphis) for sorting before being redistributed, even if the shipment is local.

Both transportation networks and places are organized hierarchically. Road networks have local streets at the base of the hierarchy, serving primarily to enable land access at low speeds and low levels of traffics, and interstate highways at the top, serving high volumes of fast, longer distance movements, as shown in 4. Places range from individual buildings (e.g. a condo association) through neighborhoods, towns or cities, counties, states, and countries.

There are advantages to a hierarchy of streets, such a hierarchy: aggregates traffic so there can be economies of scale in construction and operation; separates access and movement functions to reduces conflict, making both safer; keeps residential neighborhoods quiet; reduces network redundancy; allows exclusion of higher levels and separates of layers to enable financing by different agencies or levels of government. (Levinson and Krizek, 2008). That however does not imply there is a necessary advantage to a particular network topology. In the demerits column, a hierarchy: increases travel distance (backtrack costs); increases criticality of specific points (less redundancy means greater vulnerability); creates potential correspondence problems (mismatches) between level of government and level of road network for which it is responsible; and is perhaps less legible (Lynch, 1960) and thus increases difficulty in navigation compared to flat networks.

How these hierarchies are organized is related to, but not identical to, the topology. Some topological structures (e.g. trees) map directly to hierarchies; others (e.g. circuits) may be hierarchical in function (e.g. with some links operating with faster traffic or carrying more flow than others), but the hierarchy is not obvious from a simple graph representation. These topological configurations are defined in Xie and Levinson (2007) and used below.

The advantages of a tree are the lower construction cost (it has a shorter center-line length of network). Its disadvantage is the higher travel time compared with a web, on which it is easier to travel from and to non-central locations (less backtracking is required). A web thus has higher accessibility due to its high connectivity. It also has greater redundancy in case of failure, while a closed link on a tree will eliminate all access to particular points on the network (Jenelius, 2010). Real street networks are combinations of trees and webs, some are more tree-like, others more web-like.

Accessibility measures the ease of reaching destinations. The higher the travel cost, the lower the accessibility. It also measures the value of destinations: the more activities at the destination, the more valuable it is. A variety of measures of accessibility have been suggested that track this from different perspectives (Geurs and van Wee, 2004; Handy and Niemeier, 1997; Kwan, 1998; Miller, 1991, 1999; Ottensmann and Lindsey, 2008; Weber and Kwan, 2002)

An accessibility increase does two things. First it increases total wealth. Agglomeration economies caused by new infrastructure enlarge aggregate output. But second, it redistributes wealth, as the locations where the accessibility gains are larger gain more of that aggregate wealth. Places which do not increase accessibility at least as much as average may find themselves losing economic opportunities which will relocate to take advantage of the accessibility benefits.

A simple accessibility model is constructed between five activity centers, a centrally located hub and cities on each of four spokes. In the first case, it is a strict hub and spoke (tree) network, so that to go between any two spokes, one must travel through the hub. It is assumed that otherwise all centers (cities) are of equal size (and thus value), and the four spokes are symmetrically placed. In the second case, direct routes between the spokes are constructed, so to go from, e.g., the east spoke to the south spoke there is a direct route (at a distance of $\sqrt{2}$ times the distance between the spoke and the hub), but to go from the east to west spoke cities still requires passage through the hub. Schedules are assumed indifferent.

The accessibility model follows the classic Hansen model (Hansen, 1959) in which impedance is a negative exponential function of time. The results are shown in 5.

As can be seen, as willingness to travel decreases, and as time increases, the advantage of the hub over the spoke increases from 1 (no difference) to 4 (the hub has four times the accessibility as a spoke). This is because if the time is great enough (or willingness to travel low enough), people can travel from a spoke to the hub, but the cost of reaching a second spoke through the hub is too great to be valuable, while the hub, due to it centrality, can reach all four spokes. In the second case, with direct routes, the same pattern emerges, but the spokes are relatively stronger (though still not as strong as the hub). Different geometries will have different numerical answers, but the general logic remains, a tree-like structure reinforces the center relative to a web-like network.

4 Economics

The economic theory of network connectivity we are tracing out is one of individual land owners behaving in a self-interested way building links in order to maximize profit, with the result that under different circumstances, different network topologies emerge. We can formulate the individual objective as below.

The marginal profit for parcel owner i (which also indicates parcel i) to build segment k in road set R ($\Delta\Pi_i(k)$) equals the extra value with link k compared with value without link k minus the construction expenditures of link k.

$$\Delta\Pi_i(k) = A_i(R[0...k]) - A_i(R[0...k)) - e \cdot l_k \tag{1}$$

where

$$A_i(R) = \sum_{j=1}^{J} w_j \cdot f(c_{ij}) \tag{2}$$

$$f(c_{ij}) = c_{ij}^{-\delta} \tag{3}$$

The first part of the function reflects the change in accessibility A_i associated with a new link. In this specification, c_{ij} is the shortest path travel cost between parcel i and parcel j; $f(c_{ij})$ is the impedance function, in this case a gravity relationship; δ represents the decay parameter; w_j refers to the value of accessing parcel j, which takes on a pre-determined value.

The value of accessibility depends on location and, accessibility may be comprised of multiple land use types. In this research we define two types of land use. In this paper, w_c, the value of accessing j if it is a *commercial* parcel (or an important locale such as a CBD or port or airport) is higher than w_n, representing *non-commercial* parcels (e.g. residential land use).

The second part of this function represents the total expenditures associated with building roads The length of the newly-built link is l_k is multiplied by the unit cost e.

If the net benefit is positive to the land owners, that on-site link will be constructed. There may even be cases where off-site links would be valuable, but that is much more complicated case as they require the permission of other parties. However, since they are off-site, they may be of value to other parties as well.

5 Local networks

To illustrate this model, consider a monocentric city, with a downtown, comprising 1 zone, and suburbs comprising the 4 surrounding zones to the east, west, north and south. Downtown is strictly commercial, the suburbs are strictly residential. Each zone is controlled by a different developer. Each zone has access to an extant backbone network (there are no stranded zones). However after subdivision, it is the developer's responsibility to connect newly created properties to the network. The developer has many options for doing so.

Each zone is to be subdivided into a grid of 16 parcels, which must have at least a corner on the network (there can be no stranded parcels). Each parcel must have access to the road network, so any parcels that are created that are internal to the zone must be connected by road links internal to the site, which are constructed at the time of subdivision.

There are many possible configurations even with this level of detail. Assume downtown is at a crossroads. Each of the four suburban zones are bifurcated by a road leading to the crossroads, as shown in Figure 6.

In this case, each suburban zone has potentially 8 stranded parcels. Downtown has 4 stranded parcels (shaded). Furthermore, 8 parcels in the suburban zone front on the road, as do 12 in the Downtown. There may be a desire for "access control" to ensure there are not too many driveways on major roads (American Association of State Highway and Transportation Officials, 2001). There are a variety of network topologies that could be used to connect these parcels.

Potential connections are drawn in Figure 7. The East suburb parcel shows two such connections, on the north half (EN), there are two cul-de-sacs from East-West Highway, on the south half (ES) there is one T-shaped dual cul-de-sac from East-West Highway. The north half requires less pavement, the south half fewer access points. Other configurations are also shown on the diagram, some of which tie into downtown streets.

There are three symmetric classes of parcels in the example, central (fronting the crossroads), side (fronting one road), and stranded (fronting neither road). Consider several networks, which are increasingly cumulative:

1. No construction: each parcel loads onto the innermost node (so the central parcels load directly onto the crossroads, the side parcels onto the innermost node). The stranded parcels remain stranded.
2. Minimal construction, each parcel loads onto the most useful node for connectivity, there is a cul-de-sac connecting the stranded parcels to the North-South Highway. Four links are constructed.
3. Full grid construction within the downtown parcel. Eight link segments are built.
4. Full grid construction plus eight links to the suburban parcels. Sixteen link segments are built.

Any new link will only be constructed by developers if the net benefit of that additional accessibility to their own parcel outweighs the increased cost. For Network 2, this is more likely to be true because the additional links add potential development (formerly stranded parcels) to the network which would be otherwise inaccessible. Network 3 adds no development, and merely shortens travel times in the downtown area, so while may be, in the net, beneficial, likely has diminishing marginal benefits compared to Network 2. Network 4 likely has smaller marginal benefits still, shortening travel distances for about half of suburban-downtown trips (those which no longer have to backtrack on the highway).

6 Metropolitan dynamics

The local dynamics suggest through streets may be uncommon in the absence of an outside force in residential areas. At a higher level however, connectivity is still important. A city growing from a small seed (e.g. a port) that is connected to a hinterland, is in the upper left of Table 2. A fully dispersed city on a web is in the lower left. This section suggests that monocentric cities with tree-like transportation networks at the highest levels of the hierarchy, are just an initial phase, and over time as cities grow, more weblike networks emerge with a more polycentric distribution of jobs. However just because a more weblike network emerges at the highest level of the street hierarchy, that does not require that local streets also be weblike, rather than treelike, the arguments from the previous section hold.

Table 2: Alternative urban forms

		Network Configuration	
		Tree	Web
Land Use	Monocentric	(mt)	(mw)
	Polycentric	(pt)	(pw)

If a commercial activity emerges in more than one hinterland (that are separated on the network by a degree closer to 90 than 180 degrees), there may be a desire to connect those two hinterland points directly. This occurs if the benefits of such a connection (due to increased productivity and trade) outweigh the costs of building and operating such a link. If such a link is built, a web-like structure begins to form. If new links intersect existing links, they may create new nodes, which themselves may emerge as centers of activity (a cross-roads). These links are usually sufficiently important they are built by government agencies or by private firms charging tolls for use, rather than by individual developers.

Examples of cross-roads emerging as centers are widespread, perhaps most notably Tysons Corner in Virginia, which is the largest job center in metropolitan Washington, DC and 12th largest business district in the United States (Meyer, 2008). Tysons Corner was at the intersection of Route 7 - Leesburg Pike (running from Alexandria, Virginia to Leesburg (county seat of Loudoun County) and Route 123 (Chain Bridge Road/Dolley Madison Boulevard), running from the Chain Bridge (in Washington DC, just north of Georgetown) to the City of Fairfax, Virginia (county seat of Fairfax County). The value of this once rural crossroads was enhanced with the construction of the Capital Beltway (I-495) and the Dulles Airport road in the 1960s, which intersect near the crossroads. First a shopping mall, and later offices located here. The site is mapped in Figure 3. In this case the nucleus occurs because Alexandria and Georgetown, both towns annexed into Washington, DC when it was created (and before Alexandria was retro-ceded to Virginia) were small centers in their own rights as ports on either side of the Potomac River. Since cities are not points, routes which travel from a hinterland to one point in a city may cross routes with another route connecting a different hinterland point with a different point in the city.

Because the web-like structure increases accessibility, the ease of reaching destinations, there begins an auto-catalyzing process. By increasing absolute accessibility at outer nodes (and increasing their relative share of total regional accessibility), those areas become more attractive, and thus makes the location of jobs more polycentric. The more centers there are, and the bigger they are, the greater the desire to connect them directly. This development path translates trees to webs (mt) → (pt) → (pw). The alternative development path (mt) → (mw) → (pw) may occur if the monocentric land use begets a web configuration as beltways are built before there are multiple centers. Evidence suggests that transportation may either lead, lag, or both lead and lag land development, depending on the context (Levinson, 2008; Xie and Levinson, 2010). This process is likely difficult to reverse.

The claim is that imposing a web network with a monocentric land use as a starting point will evolve to a polycentric land use. Similarly, organizing a polycentric land use pattern on a tree-like transportation system will drive the network to evolve to a web. In short, the hypothesis is that a polycentric land use with a web-like arterial (though not necessarily local street) network is the stable equilibrium of an organic (market-driven) network growth or deployment process.

A new disruptive network (i.e. a faster or otherwise better network) will create a new centralizing tendency, as it cannot be everywhere at once, and will begin in the most valuable place (typically connecting the center to edges in a tree-like fashion). That reorients and recenters land development for a time, until the webs of the new network are established, and the cycle repeats.

That said, how does this play out locally, in neighborhoods or subdistricts within cities. If there is a desire for within-neighborhood connectivity, then web-like street systems should emerge, but if there is no desire for that connectivity, a tree-like street system may suffice for local streets.

If we accept that firms want to be connected to other firms, then neighborhoods that are primarily commercial should have web networks, a local street grid. This would occur in classic central business districts and even in emerging centers to the extent there is inter-firm interaction. However if there is no local interaction among firms, and they are co-located only because of proximity to regional transportation facilities, then tree-like networks may be sustained.

On the other hand, consider a residential neighborhood. If residences don't value proximity to other residences, then a minimalist local network, which connects to a higher level network is acceptable, implying a local tree-like network as described above. So mono-culture residential suburbs would be tree-like, which is what we find in post-World War II suburbs built by developers (Southworth and Ben-Joseph, 2003).

Mixed use neighborhoods are more complicated, but likely to be web-like as the firm-firm and residence-firm connections may outweigh the isolation preferred by residence-residence interactions.

7 Topological attributes

7.1 Centrality measures

Some centrality measures are used to evaluate the importance of nodes: betweenness centrality (B), closeness centrality (C), and degree centrality (D). While these concepts are originally proposed to measure certain properties for each node, here we calculate their mean values for all nodes/roads to assess the collective structural feature.

Let's assume undirected graph G of J nodes (potential junctions) and K links; the graph can be represented by $J \times J$ matrix, where an element, if equaling 1, indicates the existence of a link and zero otherwise. This is a sparse matrix because links can only be constructed parallel to the x or y axis.

Betweenness, a measure of centrality of a node in a network, is the fraction of shortest paths between node pairs that pass through the node of interest. Nodes that occur on more shortest paths between other nodes have higher betweenness centrality. The betweenness centrality of node i is:

$$B_i = \frac{1}{(J-1)(J-2)} \sum_{j,h \in G, j \neq h \neq i} n_{jh}(i)/n_{jh} \tag{4}$$

where n_{jh} is the number of shortest paths between j and h, and $n_{jh}(i)$ represents the shortest paths between j and h which contain node i. In this research, the multiple centrality measures are calculated through the UCINET software (Borgatti et al., 2002).

Closeness centrality, C, is used to measure to which extent a node i is near to all the other nodes along the shortest path (Sabidussi, 1966). The closeness centrality of node i

is calculated as:

$$C_i = \frac{J-1}{\sum_{j \in G, j \neq i} d_{ij}} \tag{5}$$

where d_{ij} is the shortest path length between i and j, the smallest sum of the edges length throughout all the possible paths in the graph between i and j.

Degree centrality is based on the idea that important nodes have the largest number of ties to other nodes in the graph. Based on Wasserman and Faust (1994), the degree centrality of node i is defined as:

$$D_i = \frac{\sum_{j=1}^{J} a_{ij}}{J-1} = \frac{p_i}{J-1} \tag{6}$$

where p_i is the degree of node i, i.e., the number of nodes adjacent to i.

7.2 Network topology measures

Planar transportation networks have two basic forms: branching networks and circuit networks (Haggett and Chorley, 1969). Circuit networks have closed circuits, whereas branching networks display tree-like structures. Xie and Levinson (2007) further formally defines structural elements such as branch, ring, circuit, and beltway, and develops an algorithm to measure the degree to which a network is treelike (or circuit-like). According to their definition, the *ring* (ϕ_{ring}) or *web* (ϕ_{web}) nature of a network can be measured by the total length of arterials on rings or on webs divided by the total length of arterials (refer to Xie and Levinson (2007) for details about how to find rings and webs in a network).

Therefore, the *treeness* and *circuitness* measure of a network are defined as:

$$1 - \phi_{tree} = \phi_{circuit} = \phi_{ring} + \phi_{web} \tag{7}$$

Both measures range from 0 to 1. A high treeness ratio means a network is more tree-like; a high circuitness ratio suggests a circuit network. These measures are used to examine the topological connectivity of road networks given different parameter values.

8 Simulation Model

This section models network growth using economic micro-foundations, adopting the theory described above, and applying it in a simulation framework.

8.1 Assumptions

In this research, we define a road (link) as a physical connection between two adjacent parcels. The road network to be built overlays a grid-like land layer of N land parcels, respectively owned by N land owners. The value of a land parcel is determined by its accessibility to other land parcels. Land owners build roads to increase the accessibility of their own parcels (and thus increase parcel values). Roads (links) can only run parallel to the x-axis or y-axis, with no overpasses. In addition, road construction is irreversible; once a road is built, it cannot be severed. Multiple iterations are run until a stable road pattern emerges (i.e., no new links are built).

The agent model is programmed on the Netlogo platform (Wilensky, 1999). In programming, we adopt a square-like region as the basic layer with N land parcels. In our outputs, a non-commercial parcel is symbolized by a green circle, and a commercial parcel is marked by a red circle.

Parcel owners take turns to build roads; the sequence is randomly decided. Each parcel owner can make two choices at one time: (1) building one link between two adjacent land parcels which are not yet connected, or (2) building no links. This is a locally selfish, myopic optimization, maximizing short term benefit for the agent itself, similar to the greedy algorithm.

8.2 Results and analysis

Our basic experiment is performed in the context of a 9×9 evenly-spaced grid in the form of a square, each point of which stands for a parcel (or land owner), where 50 iterations are tested. The parameter values used in the basic simulation are shown in Table3. In the beginning there is no road. The road network reaches equilibrium at the end of the fourth iteration. Our results find multiple stable road network patterns given different sequences of decision-making for parcel owners. Some exemplary patterns are illustrated in Figure 8. All the networks consist of tree-like and circuit-like structures. The networks display star-like shapes with a few redundant links among certain branches. On the periphery, the link connectivity is similar to a cul-de-sac pattern, Different sequences of decision-making for parcel owners lead to different network topologies in both temporal and spatial terms, exhibiting path dependency Arthur (1988). Since the center has the highest value of access, nearer parcels connect to the center first.

As more parcels connect to the network, the value of connecting to the whole network ascends, and ultimately all parcels are connected to the network. Figure 9 displays network patterns in each iteration until equilibrium. We can see that at the end of the second iteration, the network pattern is still tree-like. At the end of the fourth iteration, the network expands to parcels on the periphery; some redundant links are added to the tree-like structure. After four iterations the network stops changing, reaching stability. This evolutionary path reveals that road network growth is a dynamic process where new roads are first built to connect to important parcels before they expand to less important parcels. The tree-like structure emerges first; redundant links are later added to the networks, which render multiple traveling paths from one parcel to another.

What then are the impacts of different values of the key parameters on road network patterns? First of all, we perform a sensitivity test by changing the value of w_n (the value of non-central parcels) from 0 to 1000 (with step size 50), while keeping other parameters fixed. Our hypothesis is that as w_n becomes larger, the network contains more links and nodes have better connectivity. As expected, the simulation reveals phase changes for road networks given different values of w_n. The different networks are compared via centrality measures and connectivity measures. As can be seen in Fig. 10, the increase of average degree centrality and closeness centrality attest to this fact; however, the betweenness centrality exhibits a slight decreasing trend in that a star-like network entails only a limited number of hubs for other paths to go through. Fig. 11 further reveals that as w_n is lower, the $treeness$ equals 1 (and $circuitness$ equals 0). As w_n increases, $treeness$ ratio generally descends, implying a topological evolution from tree-like to circuit-like.

The second sensitivity test focuses on the construction cost per unit length of a new link (indicated by e). As its value changes from 0 to 1000, the network evolves from fully-connected grid-like to less connected circuit-like, and finally reduces to a binary tree; Fig. 11 reflects this phenomenon as the treeness ratio diminishes. When the cost is low, more redundant or parallel links are built. When the cost is high, only the links around the center are built and is complete tree-like; the the degree centrality and closeness centrality fall off accordingly (see Fig. 10). When e exceeds 900, there are no links in the network. It shows that construction cost is also an important factor in influencing network structure.

The absolute values of these thresholds depend deeply on modeling assumptions. The existence of thresholds which result in the differentiation between partially connected and fully connected, and between tree-like and web-like networks however is an important insight. As values of accessibility change (places become more or less highly valued) or the cost structure changes, market-built networks will take on different topologies.

8.3 The cases of multiple centers and mega centers

In reality, multiple centers exist within metropolitan areas; examples include the San Francisco-Oakland-San Jose MSA and the Minneapolis-St. Paul-Bloomington MSA. To understand the road network growth under different geographical patterns, we examine two scenarios, each with nine centers. In the first case, those centers are concentrated, in the second they are dispersed.

In the case of nine concentrated centers of activity (see Fig. 14), the central area can be seen as a big cluster of activity nodes, with each other fully connected, acting as the hub of the network; the grid-like structure is similar to the layout of downtown. Displaying a tree-like structure, other nodes are connected on the spokes of these central nodes, which bears resemblance to cul-de-sacs in a suburb. Our results reveal that areas with high accessibility (such as downtown) tend to be fully connected on the grid network, whereas areas with low accessibility (such as suburban neighborhoods which desire more personal space) will naturally generate cul-de-sac-like road structure.

We further concentrate on the scenario of nine dispersed centers (see Fig. 15). Compared with Fig. 14, the link density tends to be more even. The roads around the nine centers are fully connected; each center is a local hub. The density of road networks rises first and then decreases as the distance gets farther away from the center. This is similar to the bid-rent curve.

9 Discussion and conclusions

In this paper, an agent model is developed to illustrate the dynamics of road network growth. The model is based on the assumption that self-interested land parcel owners build roads to increase the accessibility of each's parcel and thereby to enhance parcel value. After reviewing the development of early roads in Minneapolis-St. Paul since the late 19th century, we simulate network growth on a grid-like land use layer with a downtown (the central parcel with high value of accessibility) in the center. The network topologies are evaluated by three centrality measures (degree centrality, closeness centrality, and betweenness centrality). We first find that networks evolve from a simple tree-like structure to a more connected network which provides multiple paths from one destination to another. Our simulation results also support that the development of road network experiences an evolutionary process, and that when the economic or social conditions of places reach certain thresholds, network patterns could go through spectacular phase changes.

This research on topology (with undifferentiated links) provides insights into the formation of early roads, the foundation of today's hierarchical transportation systems, and complements research on hierarchical links (varying in speed and capacity) given fixed web-like topology in Yerra and Levinson (2005). While fully recognizing that central authorities have played an important role in advancing current road networks, we study the dynamics of roads out of individuals' spontaneous behavior.

Large US cities today have road networks whose per capita capacity is independent of spatial extent, " road networks are built as though traffic is completely decentralized" (Samaniego and Moses, 2008), but have flows which reflect a mix of centralized and decentralized land use. This suggests that networks are leading land use in the move toward decentralization, supporting previous findings on transit systems (Levinson, 2008; Xie and Levinson, 2010). That argument is also supported by evidence from Switzerland since 1950, where network investment changed the distribution of population and employment, effectively shrinking the country (Axhausen, 2008).

Although the growth of road networks in the real world are impacted by numerous factors, this research sheds light on the effect of a possible economic incentive—the value of accessibility. Our simulation results replicate the dynamic growth of road networks and their phase changes in different economic conditions. Two major implications can be derived.

First, road networks have the property of self-organization and evolution. Even without a central authority or following an optimal design, interesting road network patterns emerge out of individual parcel owners' road-building behavior. When certain economic conditions are met, roads are first built around the central parcel, and then gradually cover the parcels on the periphery. The tree-like (non-redundant) structure is the emergent topological characteristic in the first stage; as time progresses, the network not only reaches other parcels farther from the center, but also provides multiple paths for already-connected parcels. Meanwhile the value of the whole network for each parcel increases.

Second, the growth of road network also features path dependency and phase changes. Regarding path dependency, we find different sequences of decisions lead to different network topologies; moreover, the degrees of connectivity differ for individual, but otherwise similar parcels. For phase changes, as the values of some parameters in the model exceed certain threshold, road network topology experiences a clear-cut transformation. This implies that even a small variation of certain economic conditions for places may trigger fundamental changes for road networks in the long run.

The model assumes selfish actors. Introducing a welfare, rather than profit maximizing objective is likely to change the trade-off points, perhaps leading to a more web-like network earlier, but not to change the underlying analysis. While regulation may have become more strict in the late 20th century than previously, developments also became larger on average, so that on-site activities, which remained more in the purview of the original developer or land-owner, would become a larger share of the network. This may help explain the more tree-like, hierarchical aspects of late 20th century development compared with earlier decades and centuries.

We observe that networks are likely to be dense in the center and become sparser over space as (1) commercial activities transition to residential activities because of economies of agglomeration and relative affinities between land uses, and (2) land densities diminish. A natural consequence is that residential areas where the shortened time does not outweigh the additional cost of construction will require another mechanism in order to obtain through links. Some mechanisms are outlined below:

- public provision of local roads,
- rules embedded in sub-division ordinances,
- large developments that can internalize the otherwise external benefits of through-connectivity, or

- side payments from other developers in order to obtain through links.

In the absence of such a mechanism, the local street network will retain the lollipop-like cul-de-sac pattern found in modern suburbs.

Acknowledgments

All opinions and errors are those of the authors.

References

Alexander, C. (1965), "A city is not a tree", Vol. 122(1), pp. 58–61.

American Association of State Highway and Transportation Officials (2001), "Policy on Geometric Design of Highways and Streets", *Washington, DC* .

Arthur, W. B. (1988), "Urban systems and historical path dependence", *Cities and Their Vital Systems: Infrastructure Past, Present, and Future* , pp. 85–97.

Arthur, W. B. (1989), "Competing technologies, increasing returns, and lock-in by historical events", *The Economic Journal* , pp. 116–131.

Axhausen, K. (2008), "Accessibility: long-term perspectives", *Journal of Transport and Land Use* , Vol. 1(2), pp. 5–22.

Barabási, A. L. and Albert, R. (1999), "Emergence of scaling in random networks", *Science* , Vol. 286(5439), p. 509.

Batty, M. (2006), "Hierarchy in cities and city systems", *Hierarchy in Natural and Social Sciences* , Springer, pp. 143–168.

Batty, M. and Longley, P. (1986), "The fractal simulation of urban structure", *Environment and Planning A* , Vol. 18(9), Pion Ltd, London, pp. 1143–1179.

Ben-Joseph, E. (2005), *The Code of the City: Standards and the Hidden Language of Place Making*, Oxford Univ. Press, New York.

Ben-Joseph, E. and Szold, T. (2005), *Regulating place: standards and the shaping of urban America*, Routledge.

Borgatti, S., Everett, M. and Freeman, L. (2002), "UCINET for Windows: Software for Social Network Analysis (v. 6)", *Harvard, MA: Analytic Technologies* .

Cho, E., Rodriguez, D. and Song, Y. (2008), "The role of employment subcenters in residential location decisions", *Journal of Transport and Land Use* , Vol. 1(2), pp. 121–151.

Correa, J., Schulz, A. and Stier-Moses, N. (2004), "Selfish routing in capacitated networks", *Mathematics of Operations Research* , pp. 961–976.

Dorogovtsev, S. and Mendes, J. (2002), "Evolution of networks", *Advances in Physics* , Vol. 51(4), pp. 1079–1187.

Duranton, G. and Puga, D. (2004), "Micro-foundations of urban agglomeration economies", *Handbook of Regional and Urban Economics* , Vol. 4, Elsevier, pp. 2063–2117.

El-Geneidy, A. and Levinson, D. (2006), Access to destinations: Development of accessibility measures, Technical report, Minnesota: Minnesota Department of Transportation.

Erdös, P. and Rényi, A. (1959), "On random graphs", *Publicationes Mathematicae* , Vol. 6, pp. 290–297.

Frank, O. and Strauss, D. (1986), "Markov graphs", *Journal of the American Statistical Association* , pp. 832–842.

Gastner, M. and Newman, M. (2006), "Shape and efficiency in spatial distribution networks", *Journal of Statistical Mechanics* , Vol. 1.

Geurs, K. T. and van Wee, B. (2004), "Accessibility evaluation of land-use and transport strategies: Review and research directions", *Journal of Transport Geography* , Vol. 12, pp. 127–140.

Grammenos, F., Craig, B., Pollard, D. and Guerrera, C. (2008), "Hippodamus Rides to Radburn: A New Model for the 21st Century", *Journal of Urban Design* , Vol. 13(2), Routledge, part of the Taylor & Francis Group, pp. 163–176.

Haggett, P. and Chorley, R. (1969), *Network Analysis in Geography*, Edward Arnold London.

Handy, S. L. and Niemeier, D. A. (1997), "Measuring accessibility: An exploration of issues and alternatives", *Environment and Planning A* , Vol. 29(7), pp. 1175–1194.

Hansen, W. (1959), "How accessibility shapes land use", *Journal of the American Planning Association* , Vol. 25(2), Routledge, pp. 73–76.

Helbing, D., Keltsch, J. and Moln(á)r, P. (1998), "Modelling the evolution of human trail systems", *Nature* , Vol. 388, pp. 47–50.

Helbing, D., Schweitzer, F., Keltsch, J. and Molnar, P. (1997), "Active walker model for the formation of human and animal trail systems", *Physical Review E* , Vol. 56(3), pp. 2527–2539.

Huang, A. and Levinson, D. (2009), "Retail Location Choice with Complementary Goods: An Agent-Based Model", *Complex Sciences* , Springer, pp. 175–187.

Huang, A. and Levinson, D. (2010), "Why retailers cluster: An agent model of location choice on supply chains", *Environment and Planning B* , in press.

Jacobson, J. and Forsyth, A. (2008), "Seven American TODs: Good practices for urban design in transit-oriented development projects", *Journal of Transport and Land Use* , Vol. 1(2), pp. 51–88.

Jenelius, E. (2010), "User inequity implications of road network vulnerability", *Journal of Transport and Land Use* , Vol. 2(3/4), pp. 57–73.

Krugman, P. (1996), *The Self Organizing Economy (Mitsui Lectures in Economics)*, Blackwell Pub.

Kwan, M.-P. (1998), "Space-time and integral measures of individual accessibility: A comparative analysis using a point-based framework", *Geographical Analysis* , Vol. 30(3), pp. 191–216.

Levinson, D. (2008), "Density and dispersion: the co-development of land use and rail in London", *Journal of Economic Geography* , Vol. 8(1), Oxford Univ Press, p. 55.

Levinson, D. and Krizek, K. (2008), *Planning for Place and Plexus: Metropolitan Land Use and Transport*, Routledge.

Levinson, D., Xie, F. and Zhu, S. (2007), The co-evolution of land use and road networks, *in* R. E. Allsop, M. G. H. Bell and B. Heydecker, eds, 'Seventeenth International Symposium on Transportation and Traffic Theory at Imperial College, London', Emerald Group Pub Ltd., pp. 839–859.

Levinson, D. and Yerra, B. (2006), "Self-organization of surface transportation networks", *Transportation Science* , Vol. 40(2), pp. 179–188.

Longley, P., Batty, M. and Shepherd, J. (1991), "The size, shape and dimension of urban settlements", *Transactions of the Institute of British Geographers* , Vol. 16(1), Institute of British Geographers, pp. 75–94.

Lynch, K. (1960), *The image of the city*, the MIT Press.

Marcuse, P. (1987), "The grid as city plan: New York City and Laissez-Faire planning in the nineteenth century", *Planning Perspectives* , Vol. 2, Routledge, pp. 287–310.

Marshall, A. (1920), *Principles of economics: an introductory volume*, Macmillan London.

Martin, L. (2008), "The grid as generator", *arq: Architectural Research Quarterly* , Vol. 4(4), Cambridge Univ Press, pp. 309–322.

Meyer, E. (2008), 'A Shopping Nexus Outside Washington Plots a Future as an Urban Center', http://www.nytimes.com/2008/12/17/business/17tysons.html?_r=2. published December 16, 2008, accessed March 25, 2010.

Miller, H. J. (1991), "Modelling accessibility using space-time prism concepts within geographic information systems", *International Journal of Geographic Information Systems* , Vol. 5(3), pp. 287–301.

Miller, H. J. (1999), "Measuring space-time accessibility benefits within transportation networks: basic theory and computational procedures", *Geographical Analysis* , Vol. 31(1), pp. 1–26.

Ottensmann, J. and Lindsey, G. (2008), "A use-based measure of accessibility to linear features to predict urban trail use", *Journal of Transport and Land Use* , Vol. 1(1), pp. 41–63.

Powers, E. L. (1910), 'History of Road Building: First Law to Improve United States Highways Passed in 1639', Jan. 2, The New York Times.

Price, D. J. (1965), "Networks of Scientific Papers", *Science* , Vol. 149, p. 510.

Rae, J. B. (1971), *The Road and the Car in American Life*, Cambridge, MA: MIT press.

Rosenthal, S. and Strange, W. (2004), "Evidence on the nature and sources of agglomeration economies", *Handbook of regional and urban economics* , Vol. 4, Elsevier, pp. 2119–2171.

Sabidussi, G. (1966), "The centrality index of a graph", *Psychometrika* , Vol. 31(4), pp. 581–603.

Salingaros, N. (1998), "Theory of the urban web", *Journal of Urban Design* , Vol. 3(1), Routledge, pp. 53–71.

Salingaros, N. (2000), "Complexity and urban coherence", *Journal of Urban Design* , Vol. 5(3), Routledge, pp. 291–316.

Salingaros, N., Coward, L., West, B. and van Bilsen, A. (2000), 'Principles of urban structure'.

Samaniego, H. and Moses, M. (2008), "Cities as organisms: Allometric scaling of urban road networks", *Journal of Transport and Land Use* , Vol. 1(1), pp. 21–39.

Schweitzer, F., Ebeling, W., Rose, H. and Weiss, O. (1998), "Optimization of road networks using evolutionary strategies", *Evolutionary Computation* , Vol. 5(4), pp. 419–438.

Scott, D. and Horner, M. (2008), "Examining The Role of Urban Form In Shaping People's Accessibility to Opportunities: An Exploratory Spatial Data Analysis", *Journal of Transport and Land Use* , Vol. 1(2), pp. 89–119.

Southworth, M. and Ben-Joseph, E. (2003), *Streets and the Shaping of Towns and Cities*, Island Press.

Stanislawski, D. (1946), "The origin and spread of the grid-pattern town", *Geographical Review* , Vol. 36, JSTOR, pp. 105–120.

Wasserman, S. and Faust, K. (1994), *Social Network Analysis: Methods and Applications*, Cambridge University Press.

Wasserman, S. and Pattison, P. (1996), "Logit models and logistic regressions for social networks: I. An introduction to Markov graphs and p", *Psychometrika* , Vol. 61(3), pp. 401–425.

Weber, J. and Kwan, M.-P. (2002), "Bringing time back in: A study on the influence of travel time variations and facility opening hours on individual accessibility", *The Professional Geographer* , Vol. 54(2), pp. 226–240.

Wilensky, U. (1999), 'NetLogo', http://ccl.northwestern.edu/netlogo/. Center for Connected Learning and Computer-Based Modeling, Northwestern University. Evanston, IL.

Xie, F. and Levinson, D. (2007), "Measuring the structure of road networks", *Geographical Analysis* , Vol. 39(3), Citeseer, p. 336.

Xie, F. and Levinson, D. (2009*a*), "Modeling the Growth of Transportation Networks: A comprehensive review", *Networks and Spatial Economics* , Vol. 9(3), Springer, pp. 291–307.

Xie, F. and Levinson, D. (2009*b*), "The topological evolution of road networks", *Computers, Environment, and Urban Systems* , Vol. 33(3), pp. 211–223.

Xie, F. and Levinson, D. (2010), "How streetcars shaped suburbanization: a Granger causality analysis of land use and transit in the Twin Cities", *Journal of Economic Geography* , Vol. 10(3), Oxford Univ Press, pp. 453–470.
URL: *http://dx.doi.org/doi:10.1093/jeg/lbp031*

Yamins, D., Rasmussen, S. and Fogel, D. (2003), "Growing urban roads", *Networks and Spatial Economics* , Vol. 3(1), pp. 69–85.

Yang, H. (1998), "Multiple equilibrium behaviors and advanced traveler information systems with endogenous market penetration", *Transportation Research Part B* , Vol. 32(3), pp. 205–218.

Yerra, B. and Levinson, D. (2005), "The emergence of hierarchy in transportation networks", *The Annals of Regional Science* , Vol. 39(3), pp. 541–553.

Table 3: Values of parameters in the basic simulation

Variable	Description	Value
δ	distance decay parameter	0.3
d	length of a new link (mi)	4
e	cost of building a new link between two adjacent nodes ($)	500
w_c	value of connecting to the center land parcel ($)	1500
w_n	value of connecting a non-center land parcel ($)	400
N	total land parcels (owners)	81

Note: d also equals the distance between two adjacent parcels.

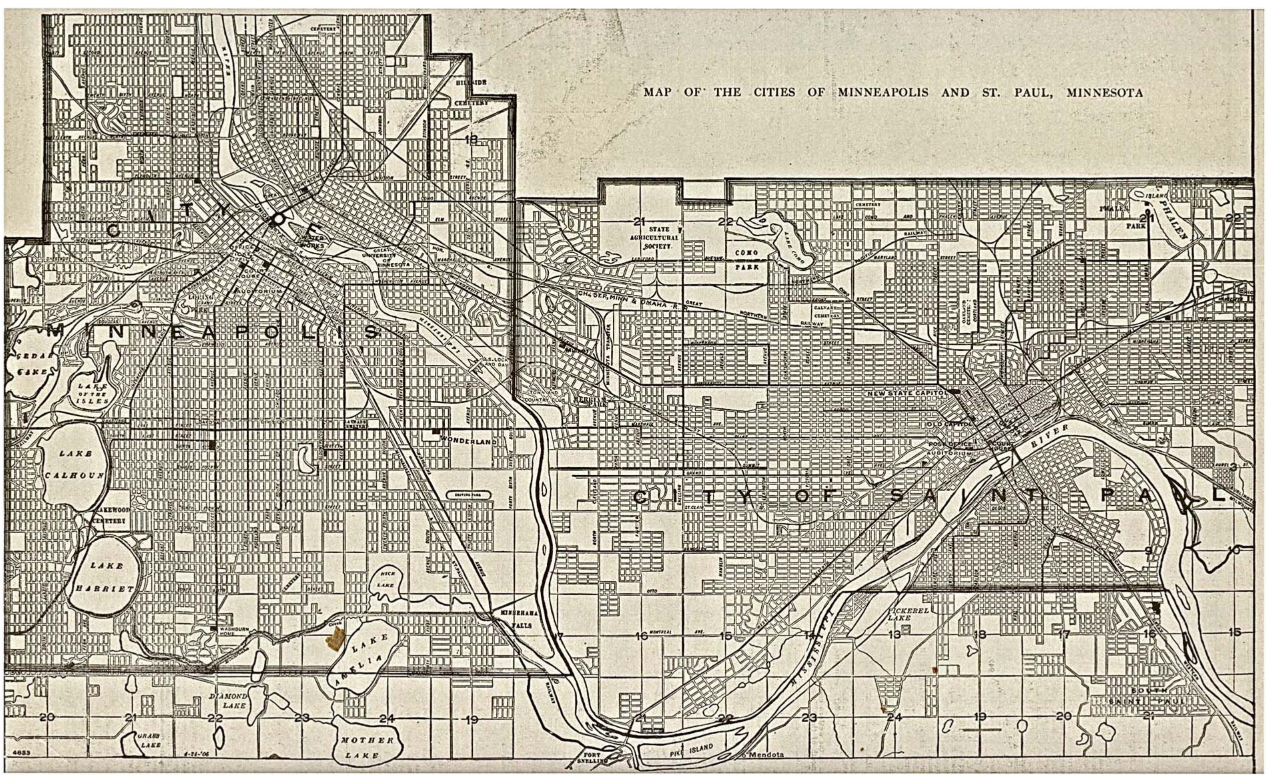
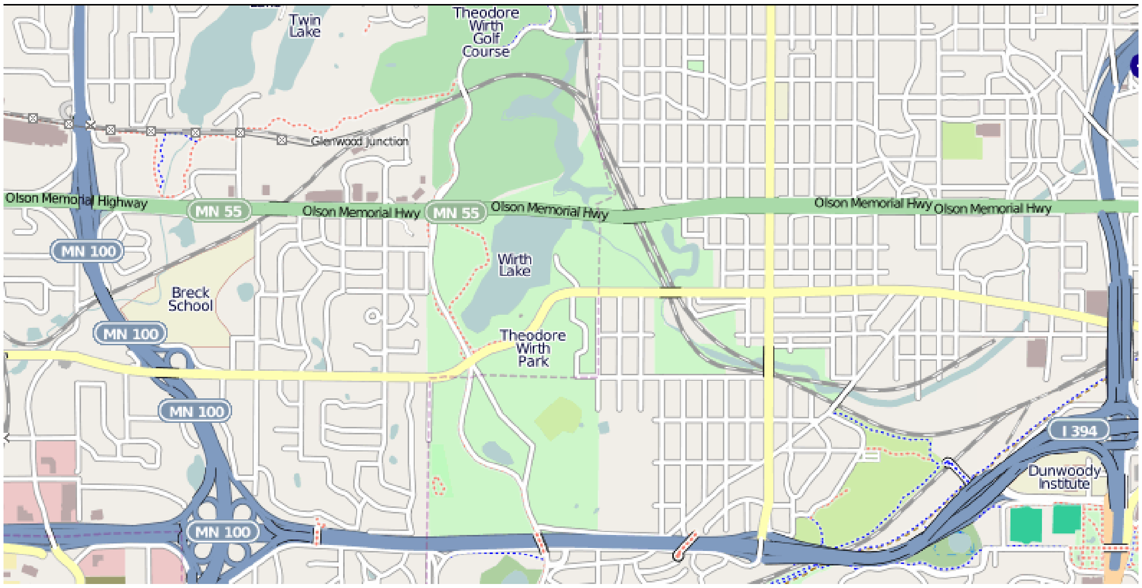


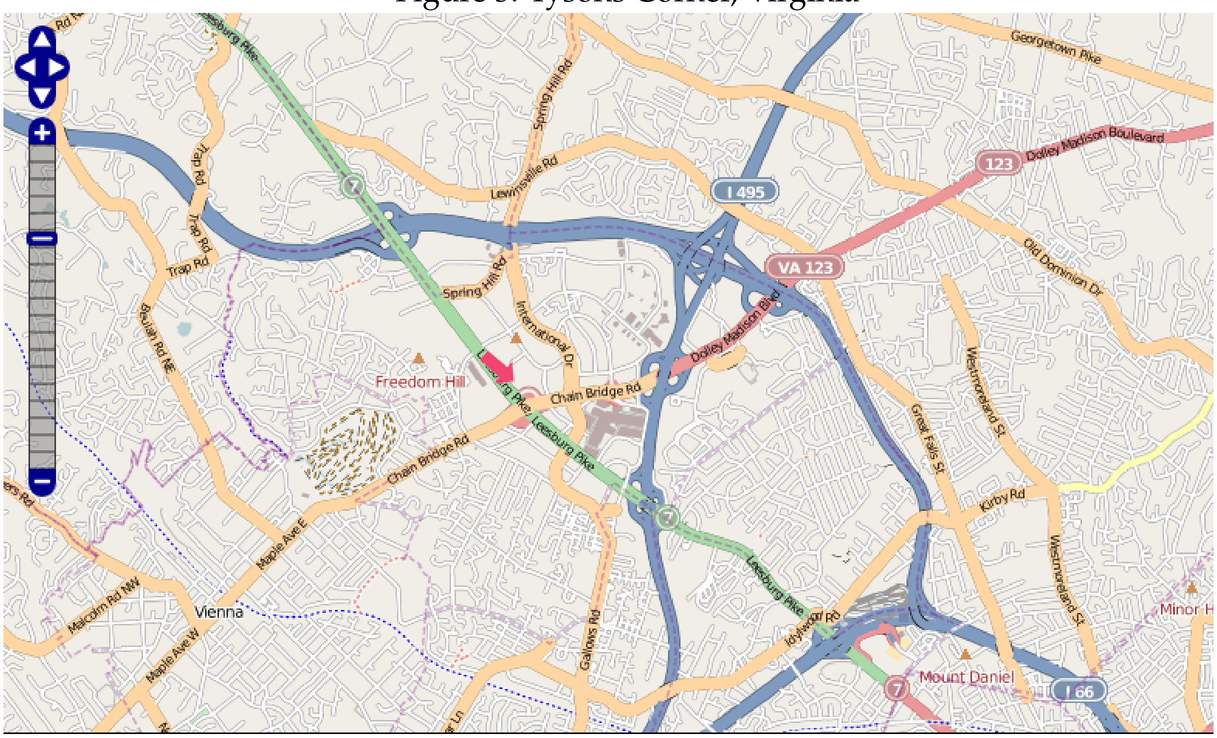
Figure 1: Road map of Minneapolis-St.Paul, MN, 1906 (The New Encyclopedic Atlas and Gazetteer of the World. Edited and Revised by Francis J. Reynolds, 1917) Source: http://www.lib.utexas.edu/maps/historical/minneapolis_1906.jpg.

Figure 2: Golden Valley and North Minneapolis.



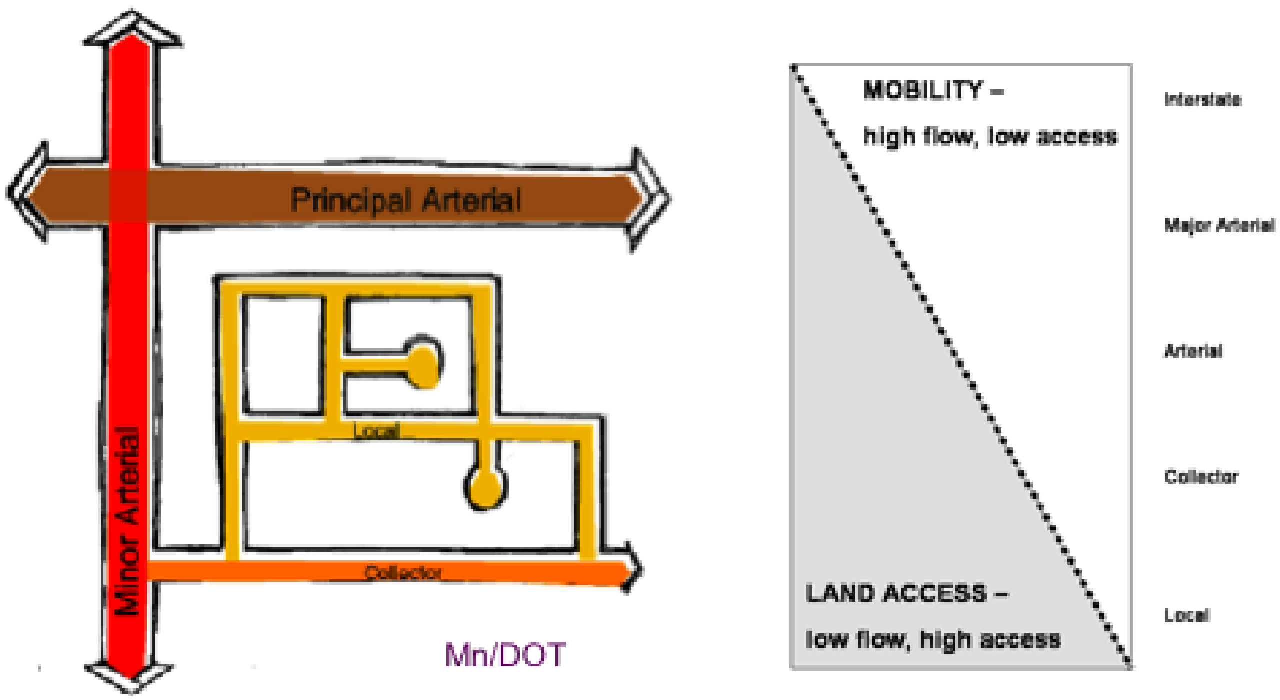
Golden Valley is to the west of Theodore Wirth Park, Minneapolis is to the east. Source: http://OpenStreetMap.org

Figure 3: Tysons Corner, Virginia



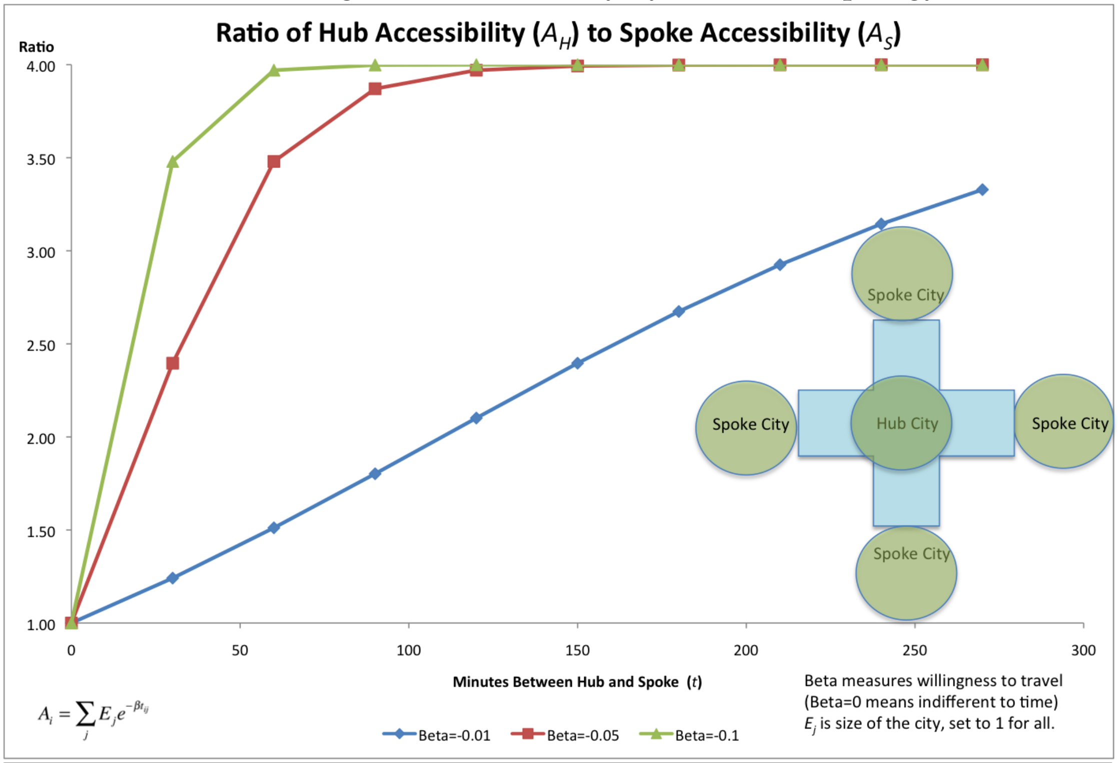
Tysons Corner, Virginia, Washington, DC is to the east. Source: http://OpenStreetMap.org.

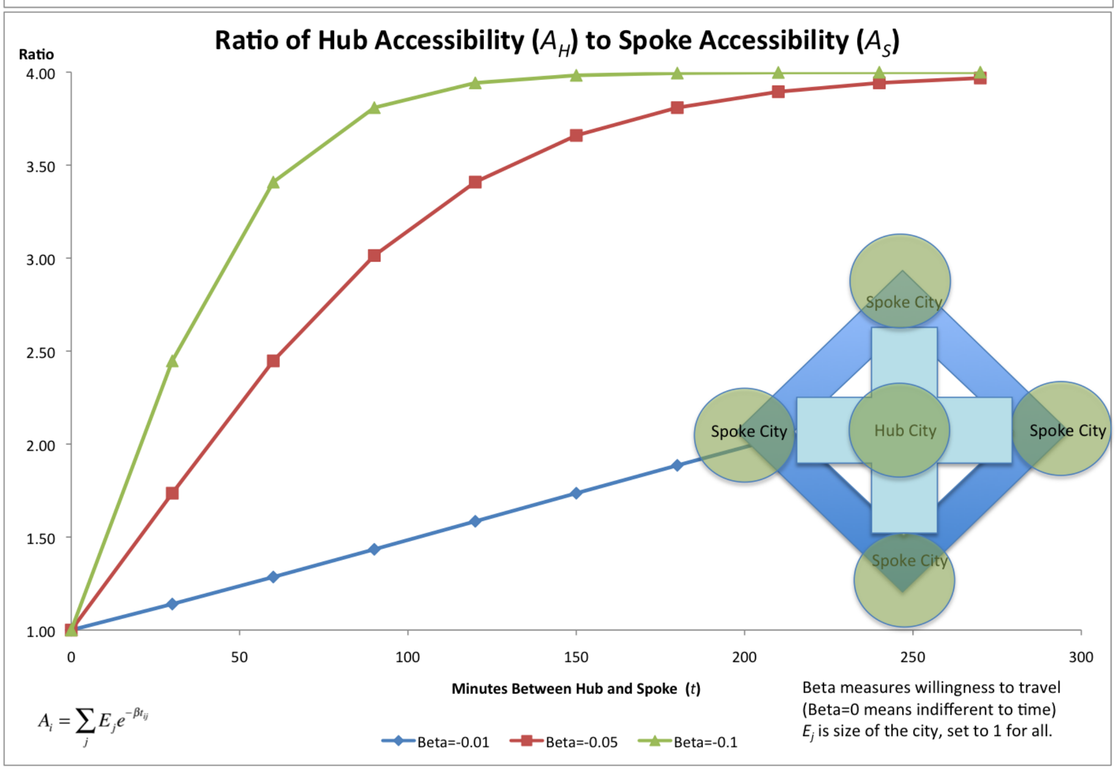
Figure 4: Hierarchy of Roads



Source: (Levinson and Krizek, 2008)

Figure 5: Accessibility by Network Topology





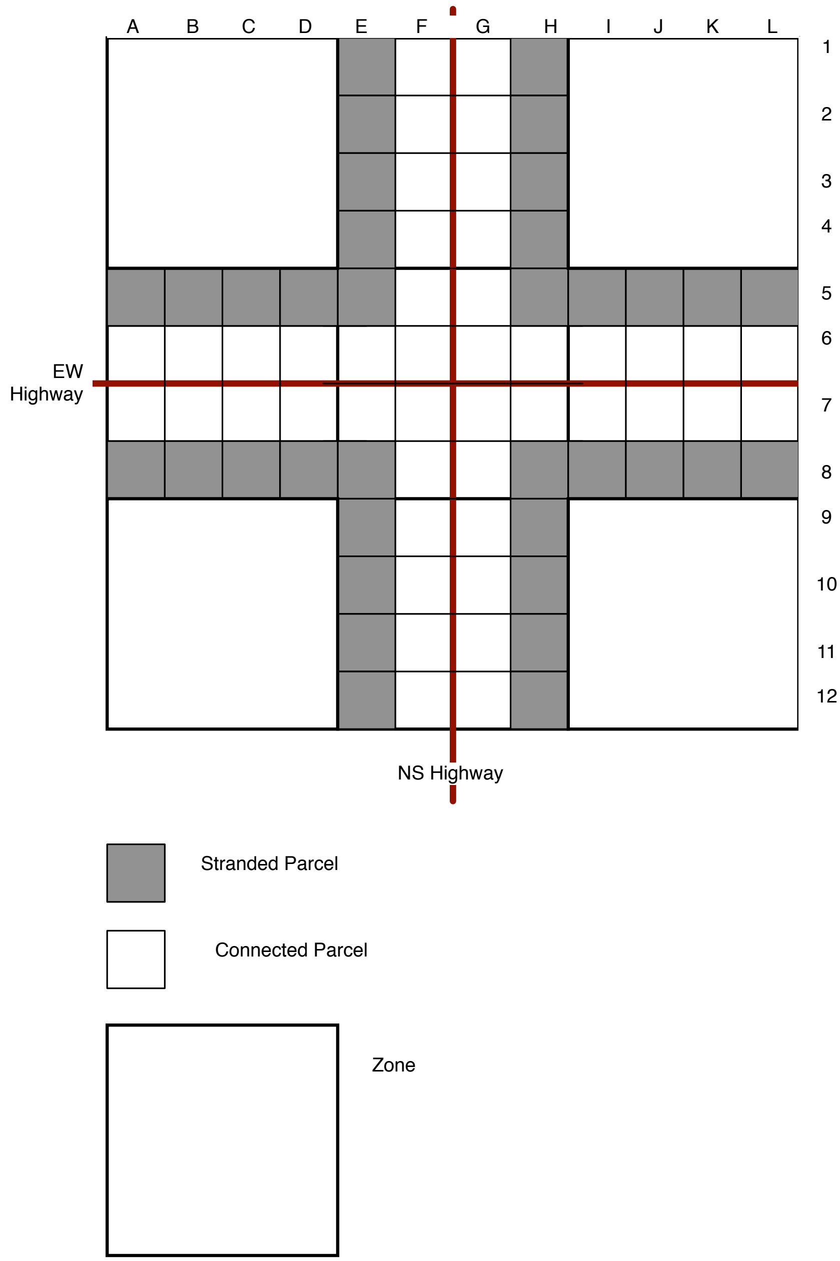


Figure 6: Zones, Parcels, and Backbone Networks

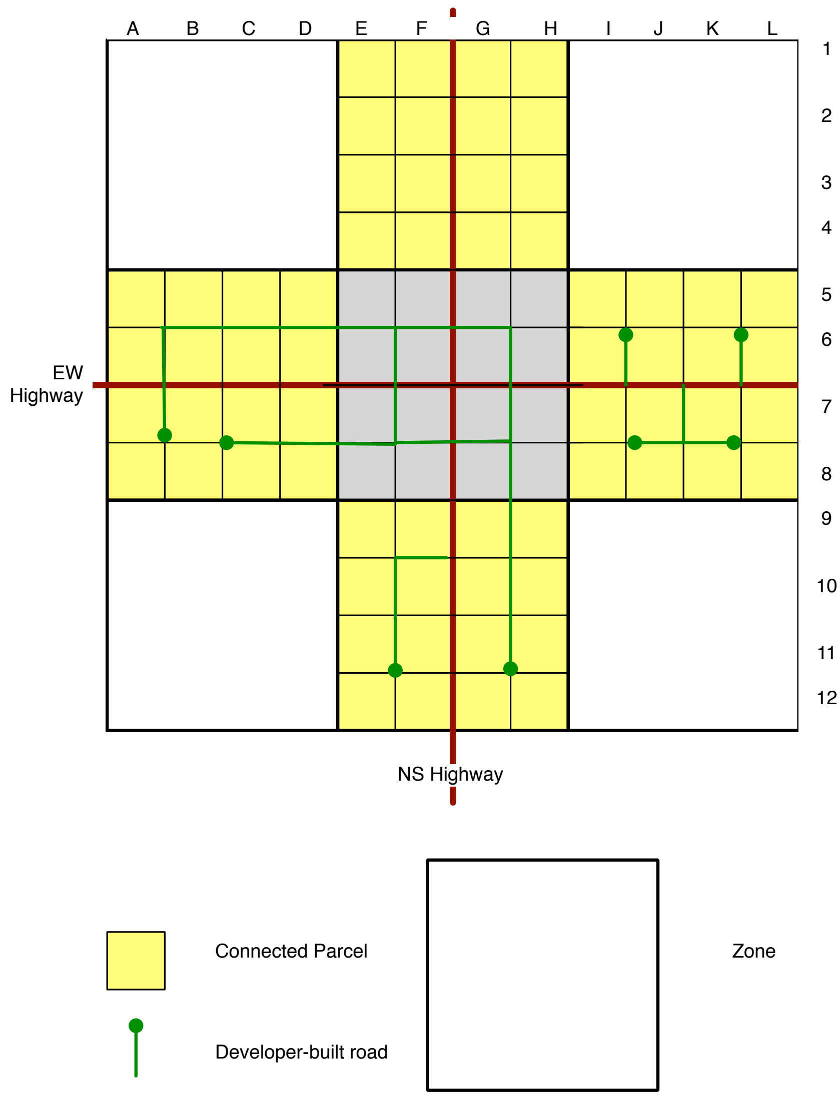


Figure 7: Types of Link Connections

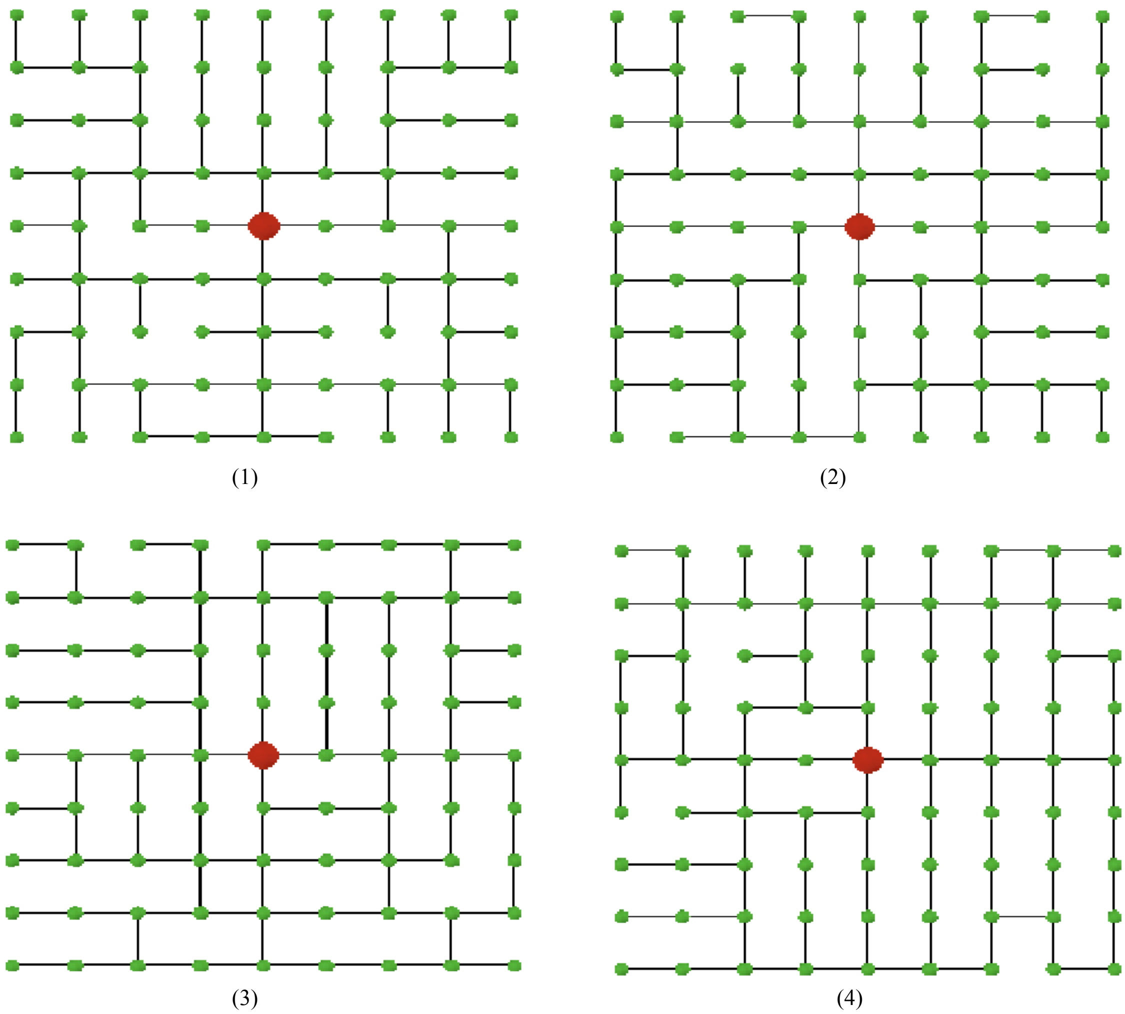
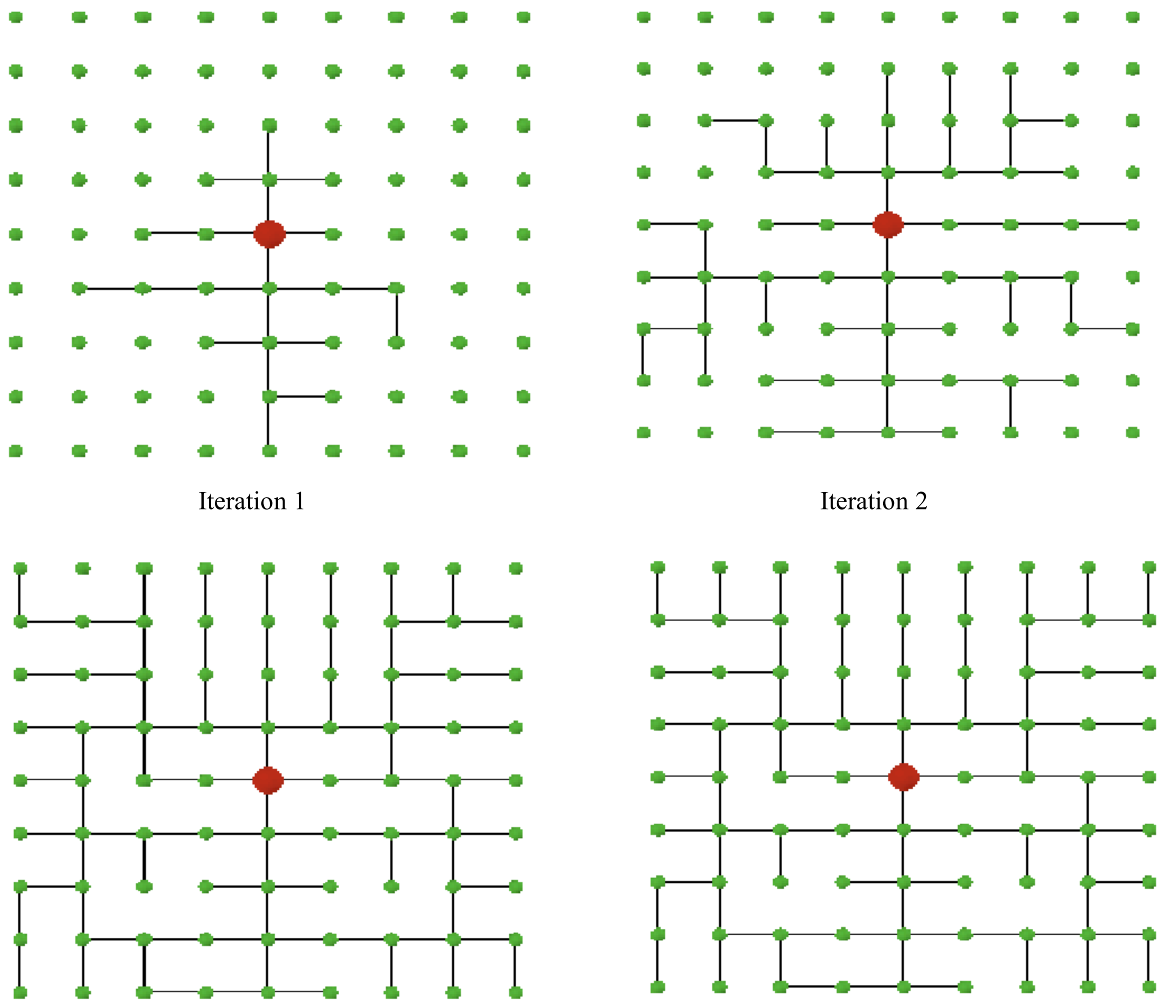


Figure 8: Exemplary resultant road network patterns given different sequences of decision making for parcel owners, with β = -0.3, c = 500, w_n = 400, $w_c = 1500$, N=81, and $d = 4$.



Iteration 1 Iteration 2

Iteration 3 Iteration 4

Figure 9: The evolution of road networks from Iteration 0 to Iteration 4 (the network structure), with β = -0.3, c = 500, w_n = 400, w_c = 1500, N=81, and $d = 4$. There is no road in the beginning. A tree-like structure emerges at the end of first iteration. At the end of Iteration 6, the network is fully-connected. The green links indicate the roads generated in the current iteration; the dark links stand for the roads emerged in previous iterations.

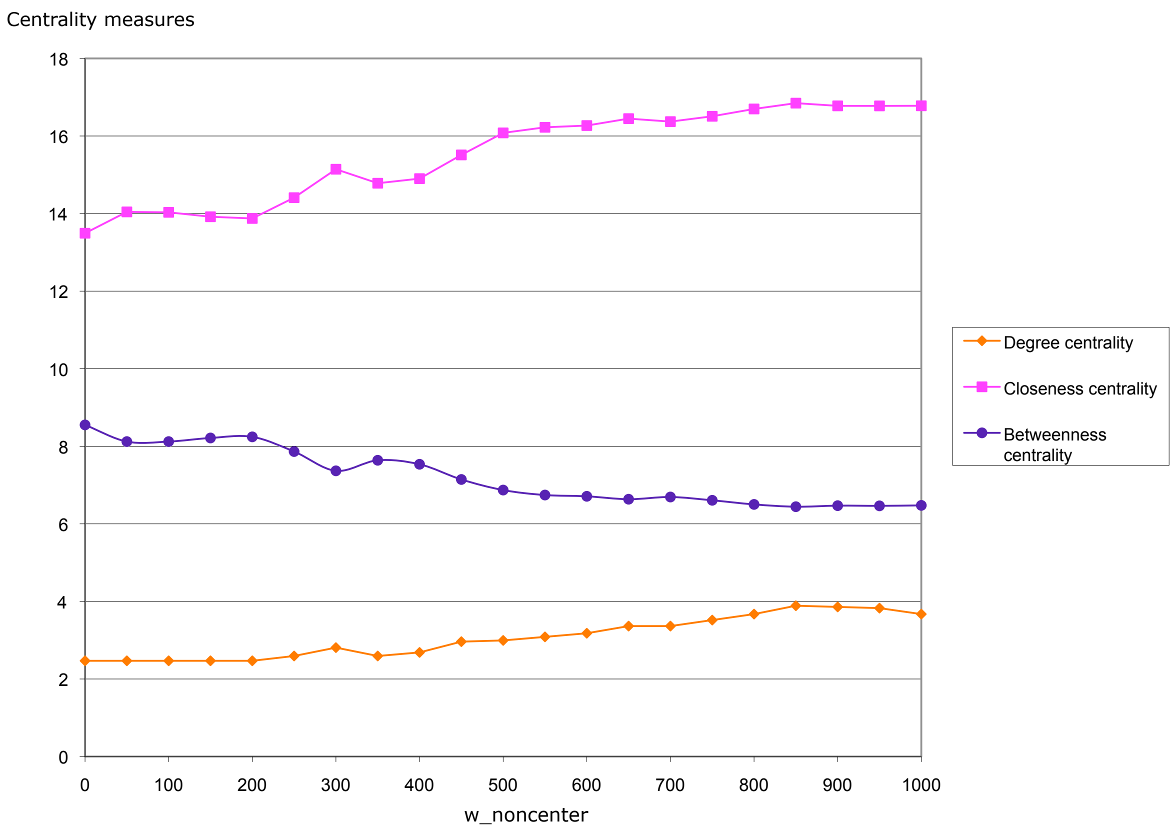


Figure 10: Centrality measures change as w_n changes from 0 to 1000, with β = -0.3, c = 500, w_c = 1500, N=81, and $d = 4$.

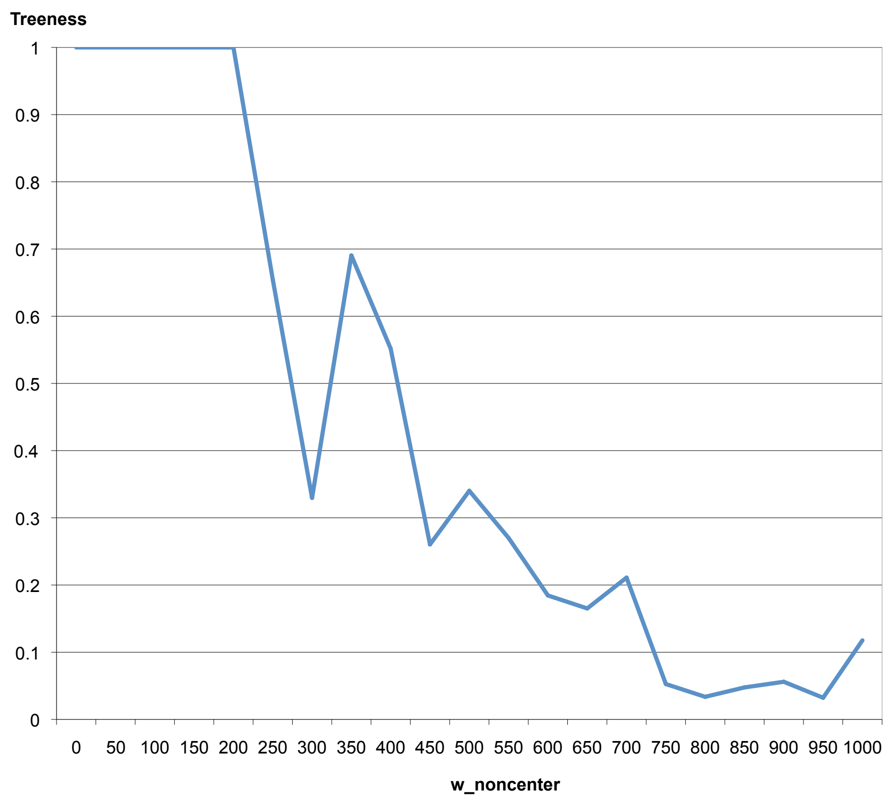


Figure 11: The treeness ratio changes as w_n changes from 0 to 1000, with β = -0.3, c = 500, w_c = 1500, N=81, and $d = 4$. When the value of w_n is low, the network is more tree-like; when w_n grows, the network becomes more circuit-like as more redundant or parallel links are constructed.

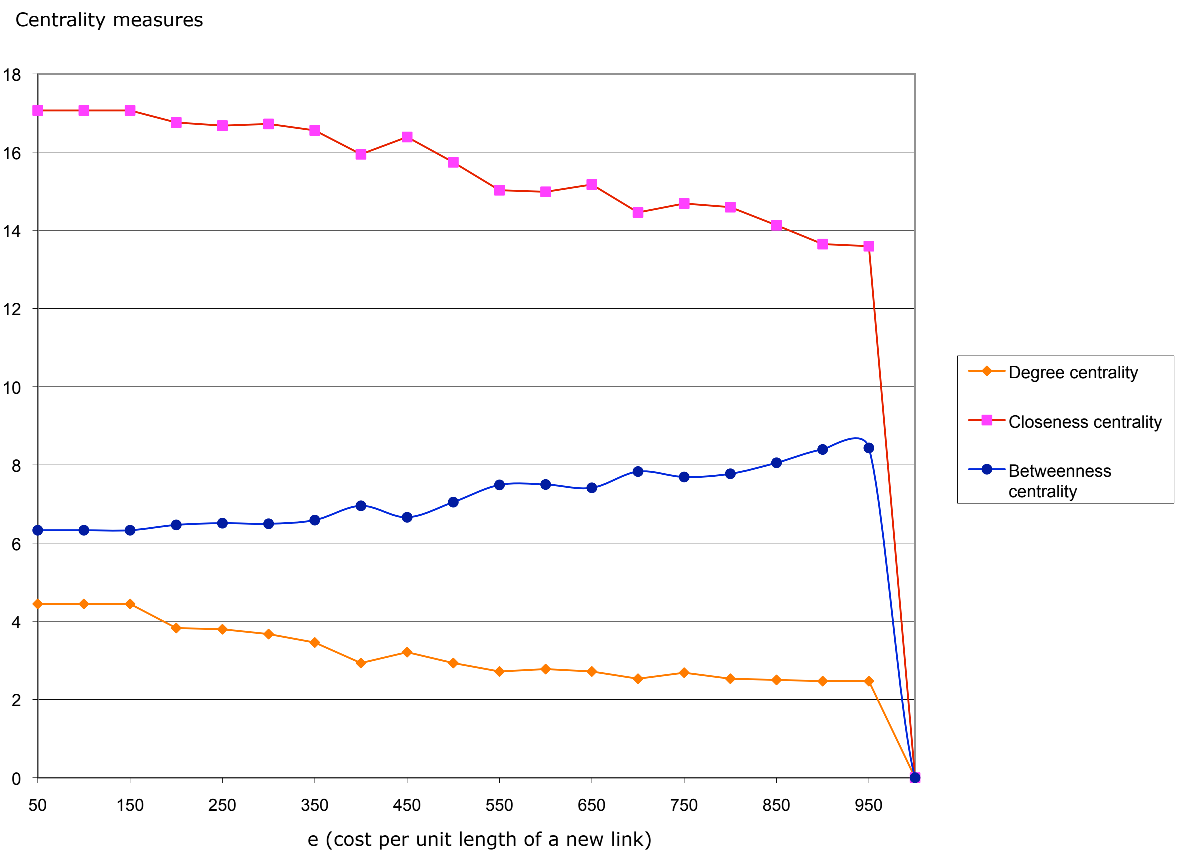


Figure 12: Centrality measures change as unit cost of construction e changes from 50 to 1000, with β = -0.3, c = 500, w_c = 1500, w_n = 400, N=81, and $d = 4$.

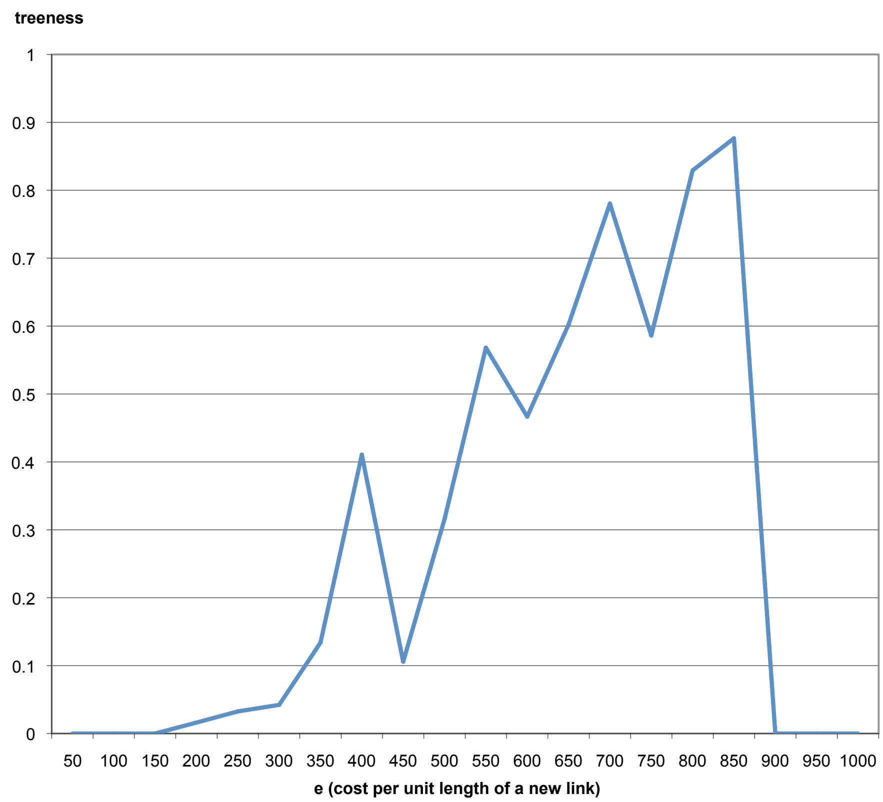


Figure 13: The treeness ratio changes as cost of construction e changes from 50 to 1000. When the value of e is low, the network is fully-connected. As the cost increases, the network is less connected as a mix of tree-like and circuit-like structures. As the cost increases, the network shrinks to tree-like. When $e > 950$, no links are built and treeness is undefined.

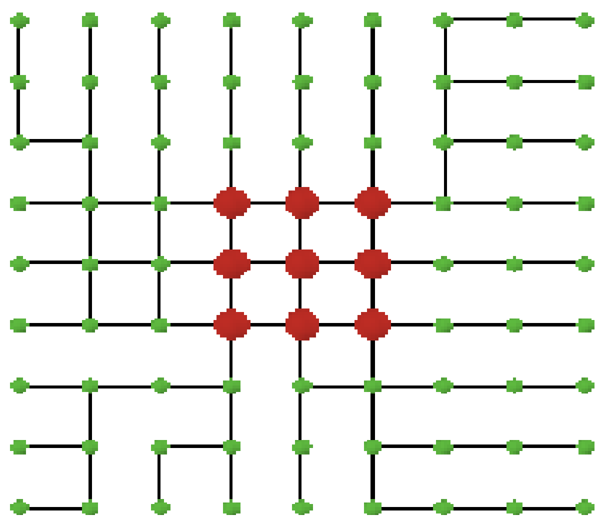


Figure 14: Road network structure in equilibrium in a situation of nine concentrated activity nodes. The network has both tree-like and circuit-like structures. The network density is the highest for the cluster (marked by red dots); it decreases as one moves farther away from the center.

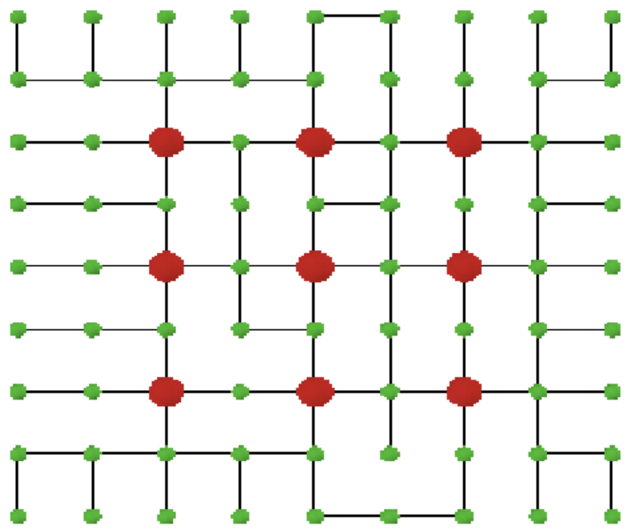


Figure 15: Road network structure in equilibrium in a situation of nine dispersed. The centers all have full connections. On the periphery and between centers, there are more tree-like structures.