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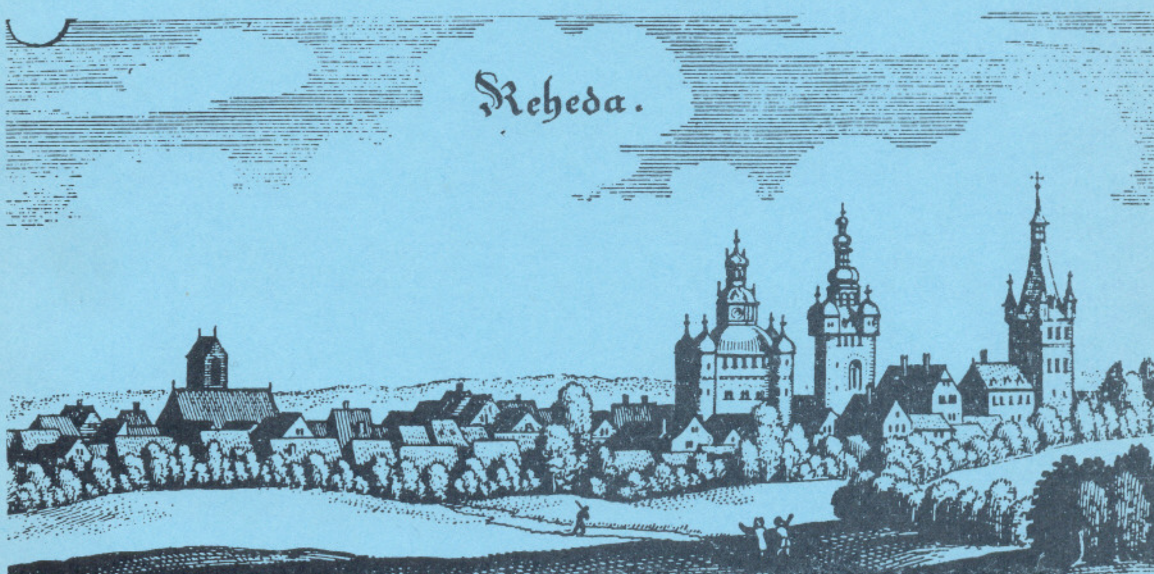

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The Chain Store Paradox

by

Reinhard Selten

It is the purpose of this paper to present the example of a simple game in extensive form where the actual behavior of well informed players cannot be expected to agree with the clear results of game theoretical reasoning. A story about a fictitious chaim store and its potential competitors is a convenient way to describe the game. This expositionary device should not be misunderstood as a model of a real situation.[1)] In view of the story the game will be called "the chain store game". The disturbing disagreement between plausible game behavior and game theoretical reasoning constitutes the "chain store paradox".

The chain store paradox throws new light on the well known difficulties which arise in connection with the finite supergame of the prisoners' dilemma game. A limited rationality approach seems to be needed in order to explain human strategic behavior. An attempt shall be made to discuss the possibility of a "three-level theory of decision making" as an explanation of discrepancies between game theoretic analysis and human behavior.

1. The chain store game

Consider the following fictitious market situation:

A chain store, also called player A, has branches in 20 towns, numbered from 1 to 20. In each of these towns there is a potential competitor, a small business man who might raise money at the local bank in order to establish a second shop of the same kind. The potential competitor at town k is called player k. Thus the game has 21 players : the chain store, player A and

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[1)] Nevertheless the industrial organization flavor of the story is not purely fortitious. I became aware of the problem in the course of a conversation about the theory of entry preventing prices. I am grateful to professor A. Gutowsky of the university of Frankfurt am Main with whom I had this very interesting interchange of ideas.

its 20 potential competitors the players k with k = 1, ...,20. Apart from these 20 players the chain store does not face any other competition, neither now nor in the future.

Just now none of the 20 small business men has enough owned capital to be able to get a sufficient credit from the local bank but as time goes on, one after the other will have saved enough to increase his owned capital to the required amount. This will happen first to player 1, then to player 2, etc. As soon as this time comes for player k, he must decide whether he wants to establish a second shop in his town or whether he wants to use his owned capital in a different way. If he chooses the latter possibility, he stops to be a potential competitor of player A.

If a second shop is established in town k, then player A has to choose between two price policies for town k. His response may be "cooperative" or "aggressive". The cooperative response yields higher profits in town k, both for player A and for player k, but the profits of player A in town k are even higher if player k does not establish a second shop. Player k's profits in case of an aggressive response are such that it is better for him not to establish a second shop if player A responds in this way.

After this description of the fictitious market situation which yields a convenient economic interpretation of the chain store game, a more abstract and more precise description of the rules must be supplied in order to remove possible sources of misunderstandings. In this section we consider a first version of the chain store game. For some purposes it is convenient to introduce a somewhat different second version of the game. This will be done in a later section. In both cases it will be useful to assume that there are m potential competitors, where m may be any positive integer. Nevertheless it is convenient to focus attention on m = 20, since the game changes its character if m becomes too small. The extensive form of the first version with m potential competitors will be denoted by Γ_m^1.

<u>Rules for Γ_m^1, the first version of the chain store game:</u>

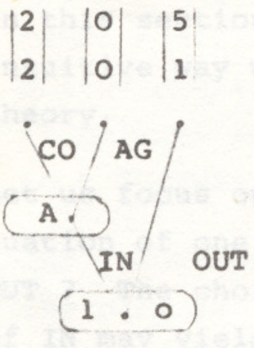
The game has m + 1 players, player A and Players 1, ..., m. The game is played over a sequence of m consecutive periods 1, ..., m. At the beginning of period k player k must decide between IN and OUT. (The decision IN means that a second shop is established by player k.) Player k's decision is immediately made known to all players. No further decisions are made in period k if player k's decision was OUT. If his decision was IN, then player A has to choose between COOPERATIVE and AGGRESSIVE (both words stand for possible price policies of player A in town k). This decision is immediately made known to all players, too. Then for k = 1, ..., m-1 the period k + 1 begins and is played according to the same rules. The game ends after period m.

Player A's payoff is the sum of m partial payoffs for the periods 1,...,m. Player A's partial payoffs and the payoffs of the players 1,...,m are given by table 1.

player k's decision	player A's decision in period k	player k's payoff	player A's partial payoff for period k
IN	COOPERATIVE	2	2
IN	AGGRESSIVE	0	0
OUT	-	1	5

<u>Table 1</u>: player A's partial payoffs and player k's payoff.

The game is played in a non-cooperative way. The players cannot commit themselves to threats or promises. No binding contracts are possible. Side payments are not permissible. The players are not allowed to talk during the game.



	IN	OUT
COOPERATIVE	2 / 2	5 / 1
AGGRESSIVE	0 / 0	5 / 1

Figure 1: The extensive form Γ_1^1 and the normal form of Γ_1^1.

Player A's payoffs are above and player 1's payoffs are below. "CO" and "AG" stand for "COOPERATIVE" and "AGGRESSIVE". The game begins at the origin o. Information sets are indicated by lines which encircle vertices belonging to the same information set. The player who has to make a choice at a given information set is indicated by the appropriate symbol. - In the representation of the normal form, player A's payoff is given in the upper left corner and player 1's payoff is given in the lower right corner.

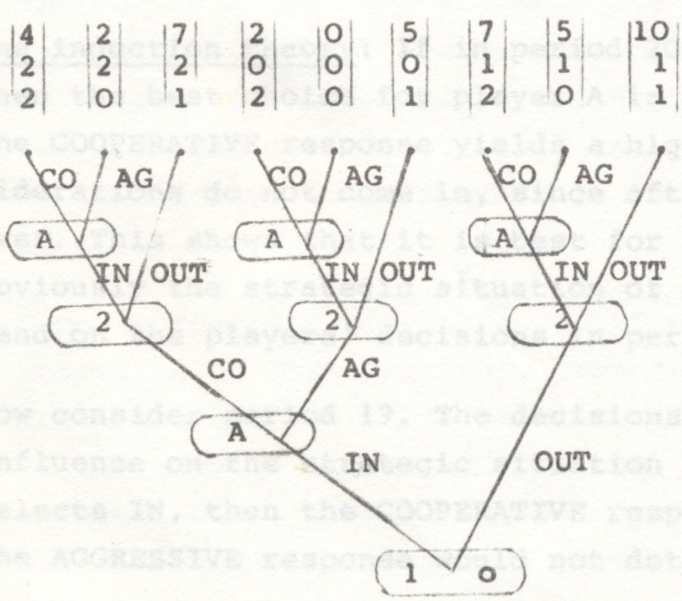


Figure 2: The extensive form Γ_2^1. The components of the payoff vectors above the endpoints refer to the payoffs of players A,1 and 2 in that order from above to below. (For further explanations of the graphical representation see figure 1).

2. A first view of the paradox

In this section the chain store paradox will be introduced in an intuitive way without making use of the formal tools of game theory.

Let us focus our attention on the case m = 20. Consider the situation of one of the player 1,...,20. Should he choose IN or OUT ? The choice of OUT guarantees a payoff of 1. The choice of IN may yield a payoff of 2 if player A's response is COOPERATIVE but if the response is AGGRESSIVE, then the payoff is 0.

Consider the situation of player A.How should he respond to a choice of IN ? the COOPERATIVE response yields a partial payoff of 2 and the AGGRESSIVE response yield a partial payoff of 0. In the short run the COOPERATIVE response is more advantageous but in the long run it may pay to choose the AGGRESSIVE response in order to discourage the choice of IN.

There are two different theories about the adequate behavior in the game. One will be called the "INDUCTION THEORY" and the other will be called the " deterrence theory ".

<u>The induction theory</u>: If in period 20 player 20 selects IN, then the best choice for player A is the COOPERATIVE response. The COOPERATIVE response yields a higher payoff. Long run considerations do not come in, since after period 20 the game is over. This shows that it is best for player 20 to choose IN. Obviously the strategic situation of period 20 does not depend on the players' decisions in period 1,...,19.

Now consider period 19. The decisions in period 19 have no influence on the strategic situation in period 20. If player 19 selects IN, then the COOPERATIVE response is best for player A. The AGGRESSIVE response would not deter player 20.

It is clear that in this way we can go on to conclude by induction that each player k should choose IN and each time player A should use the COOPERATIVE response. The strategic situation in the remainder of the game does not depend on the decisions up to period k. If it is already known that in periods k + 1,...,20 players k + 1,...,20 will choose IN and player A will always select the COOPERATIVE choice, then it follows that also in period k a choice of IN should lead to a COOPERATIVE response.

The induction theory comes to the conclusion that each of the players 1,...,20 should choose IN and player A should always react with his COOPERATIVE response to the choice of IN. If the game is played in this way, then each of the players 1,...,20 receives a payoff of 2 and player A receives a total payoff of 40.

The deterrence theory: Player A should not follow the reasoning of the induction theory. He can get more than 40. It is true that the reasoning of the induction theory is very compelling for the last periods of the game. Therefore player A cannot neglect these arguments completely. He should decide on the basis of his intuition for how many of the last periods he wants to accept the induction argument. Suppose he decides to accept the argument for the last 3 periods 18,19 and 20, but not for the periods 1,...,17. Then, on the basis of this decision he should act according to the following strategy: In the periods 1,...,17 the response to a choice of IN is AGGRESSIVE, in periods 18,19 and 20 the response to a choice of IN is COOPERATIVE.

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Suppose that the players 1,...,20 expect that player A behaves according to this strategy. Then it is best for players 1,...,17 to choose OUT and it is best for players 18,19,20 to choose IN. If the game is played in this way, players 1,...17 will receive a payoff of 1, players 18,19 and 20 will receive a payoff of 2 and player A will receive a payoff of 91.

Even if some of the players 1,...20 have a different view of the force of the induction argument, player A will still be better off than the induction theory suggests. Suppose that not only the 3 players 18,19 and 20, but also 10 of the players 1,...,17 choose IN, whereas the others choose OUT. In this case player A's payoff will be 41 which is still more than 40.

Suppose that early in the game 2 or 3 of the players 1,...,17 choose IN. If they are punished by Player A's AGGRESSIVE response, then most of the others will have learnt their lesson. It may still be true that player 17 feels that the induction argument applies to him,too,and the same may be true for player 16, but on the whole, it seems to be very improbable that more than 5 of the players 1,...,17 will choose IN. This means that it is very probable that player A will have a payoff of at least 66.

It may also happen that in spite of the fact that player A does not plan to react by his AGGRESSIVE response to choices of IN by players 18,19 and 20, player 18 and maybe even player 19 will still be deterred by this threat.

Since the players 1,...,20 can expect that player A will follow the deterrence theory, they should behave accordingly. If up to period k - 1 not very many of the players 1,...,k-1 selected IN and player A's response was always AGGRESSIVE, then player k should select OUT unless he feels that period k is sufficiently near to the end of the game, to make it probable, that player A will accept the induction argument for period k.

The deterrence theory does not yield precise rules of behavior, since some details are left to the intuition of the players but this does not impair the practical applicability of the theory.

Comparison of the two theories: As we shall see in section 8,only the induction theory is game theoretically correct. Logically, the induction argument cannot be restricted to the last periods of the game. There is no way to avoid the conclusion that it applies to all periods of the game.

Nevertheless the deterrence theory is much more convincing. If I had to play the game in the role player A, I would follow the deterrence theory. I would be very surprised if it failed to work. From my discussions with friends and colleagues I get the impression that most people share this inclination. In fact, up to now I met nobody who said that he would behave according to the induction theory. My experience suggests that mathematically trained persons recognize the logical validity of the induction argument but they refuse to accept it as a guide to practical behavior.

It seems to be safe to conjecture that even in a situation where all players know that all players understand the induction argument very well, player A will adopt a deterrence policy and the other players will expect him to do so.

The fact that the logical inescapability of the induction theory fails to destroy the plausibility of the deterrence theory is a serious phenomenon which merits the name of a paradox. We call it the "chain store paradox".

3. The second version of the chain store game

Consider a fictitious market situation similar to that described in section 1. Again the chain store, player A, has 20 branches in 20 towns and there is one potential competitor, player k in each town k. But now we assume that already at the beginning of the game every potential competitor has a sufficient amount of owned capital but there is only one bank where they all have to apply for credit if they want to establish a second shop. As long as there are any applicants, in every period the bank gives a credit to one of them who is selected randomly. Thus in every period exactly one of the players 1,...,20 establishes a second shop until a period arrives where there are no applicants. If this happens the game ends. Before the end of the game a player k who did not yet establish a shop may or may not apply for a credit in every period; he may change his decision in the next period. In order to avoid misunderstandings a more precisely formulated set of rules is given below. The extensive form of the second version of the game with m competitores will be denoted by Γ_m^2.

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Rules for Γ_m^2, the second version of the chain store game:

The game has m + 1 players, player A and players 1,...,m. The game is played over a sequence of periods t = 1,...,T, where T is determined by the decisions of the players. In every period t some of the players 1,...,m are called "outside" and others are called "inside". At the beginning, in period 1 all of them are outside. Let M_t be the set of outside players in period t.

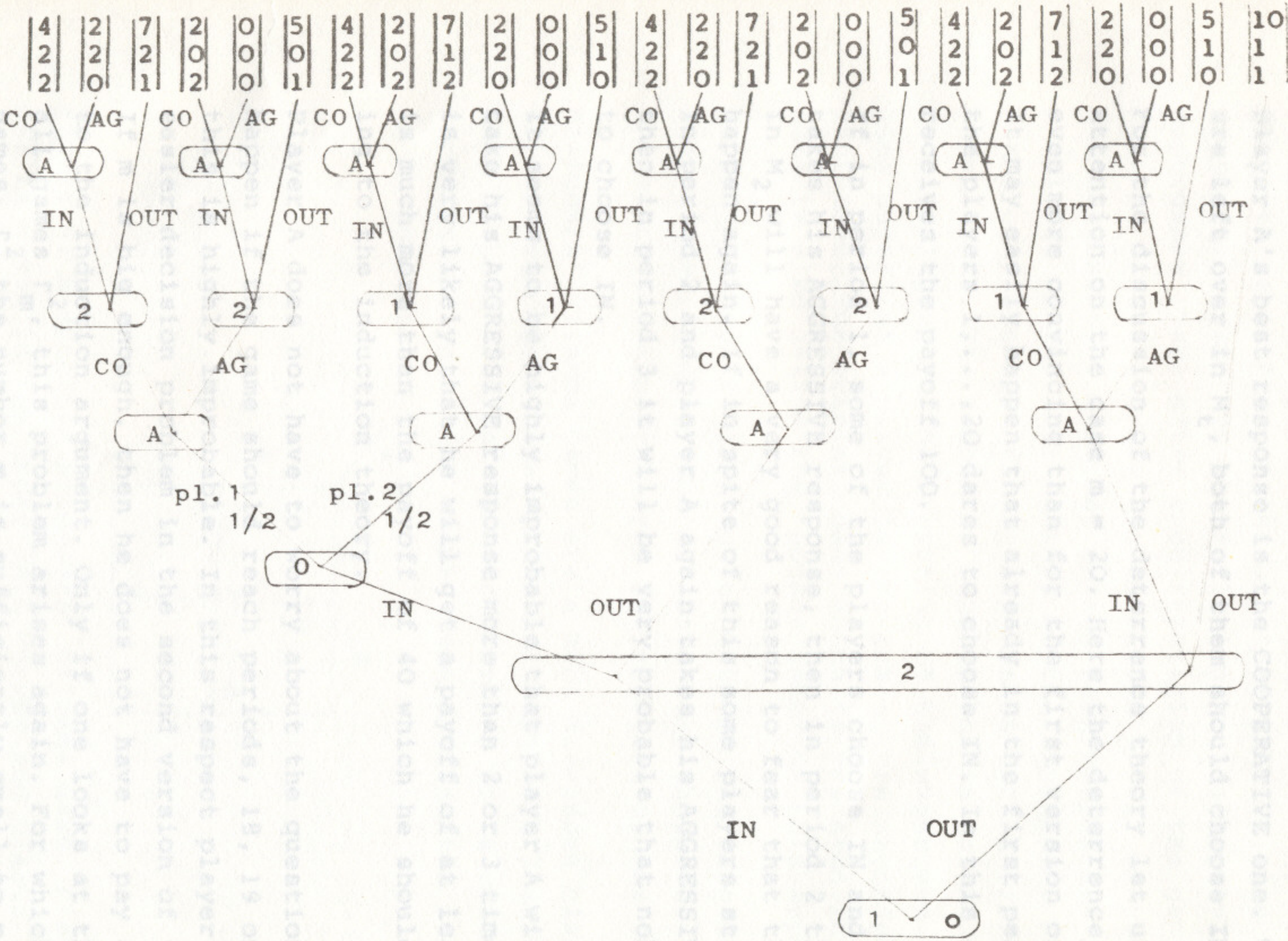
In every period t each player in M_t has to decide between IN and OUT. These decisions are made secretly. Let I_t be the set of all players in M_t who choose IN in period t. A random mechanism selects a player $j_t \epsilon I_t$. Each of the players in I_t has the same probability to become the selected player j_t. In period t + 1 the player j_t becomes an inside player. M_{t+1} is the set $M_t - \{j_t\}$.

The players in M_t must make their decisions for period t without knowing the decisions of the other members of M_t for period t. Immediately after these decisions have been made they are made known to all players.

If in period t the set I_t is empty, then period t is the last period T and player A does not have to make a decision for this period. This is not the only way in which the end of the game can be reached. It may happen that M_{t+1} is empty; then t is the last period T. (In this case we must have T = m.)

If I_t is not empty, then player A has to choose between a "COOPERATIVE" and an "AGGRESSIVE" response in period t. This decision is immediately made known to all players. Player A has full knowledge of all past decisions when he makes his choice.

Player j_t receives the payoff 2 if player A's choice in period t is COOPERATIVE, he receives 0 if player A's choice in period t is AGGRESSIVE . Let m_c be the number of periods where player A's decision was COOPERATIVE. Player A receives the payoff $2m_c + 5(m+1-T)$, if I_T is empty and $2m_c$ otherwise.



<u>Figure 3</u>: The extensive Form Γ_2^2. (See explanations below figures 1 and 2.) The symbol O refers to the random player who selects either player 1 or player 2 with probabilities 1/2.

<u>The Paradox in the second version of the game:</u> Game theoretically the induction theory holds for the second version of the game, too. If all players 1,...,m with the exception of one already have chosen IN, then the last one can do so, too, since player A's best response is the COOPERATIVE one. Therefore, if two are left over in M_t, both of them should choose IN, etc.

For the discussion of the deterrence theory let us focus our attention on the case m = 20. Here the deterrence theory is even more convincing than for the first version of the game. It may easily happen that already in the first period none of the players 1,...,20 dares to choose IN. In this case player A receives the payoff 100.

If in period 1, some of the players choose IN and player A takes his AGGRESSIVE response, then in period 2 the players in M_2 will have a very good reason to fear that the same will happen again. If in spite of this some players still choose IN in period 2 and player A again takes his AGGRESSIVE response, then in period 3 it will be very probable that nobody dares [illegible] way
to choose IN.

It seems to be highly improbable that player A will have to take his AGGRESSIVE response more than 2 or 3 times. Thus it is very likely that he will get a payoff of at least 85 which is much more than the payoff of 40 which he should get according to the induction theory.

Player A does not have to worry about the question what will happen if the game should reach periods, 18, 19 or 20, since this is highly improbable. In this respect player A has an easier decision problem in the second version of the game. If m is big enough, then he does not have to pay any attention to the induction argument. Only if one looks at the set of all games Γ^2_m, this problem arises again. For which of the games Γ^2_m the number m is sufficiently small to make the induction theory acceptable ?

4. A look at the finite supergame of the prisoners' dilemma game

If the same game in normal form is played again and again for a finite or infinite number of times by the same set of players, then a supergame of the original game in normal form results. The k-th repetition of the original game is also called period k of the supergame. In the following we shall only consider such supergames where after each repetition of the game, the strategy choices of all players are announced to all players; thus at the beginning of each new period each player has a complete knowledge of the past history of the supergame. Moreover, we shall only consider finite supergames with a finite number of repetitions. The number of repetitions is assumed to be known to all players at the beginning of the supergame. The supergame payoff of a player is the sum of his payoffs for all repetitions. The original game which is repeated in a given supergame is also called the "source game" of this supergame.

It is important to distinguish between the supergame and its source game. A supergame may have game theoretical properties which are not apparent from the analysis of the source game.

Prisoners' dilemma games are a much discussed class of symmetrical two-person games in normal form with 2 strategies for each of both players. For our purposes it is convenient to focus attention on the normal form represented in figure 4 which is in this class.

1 \ 2	A	B
A	3 / 3	0 / 4
B	4 / 0	1 / 1

Figure 4: A specific prisoners' dilemma game. Player 1's payoff is given in the upper left corner and player 2's payoff is given in the lower right corner of the fields representing the strategy combinations.

Let Γ_m^3 be the extensive form of the supergame which results from the m-fold repetition of the normal form of figure 4. The graphical representation of Γ_2^3 is given in figure 5.

It is well known that for sufficiently large m, say $m = 100$, the analysis of Γ_m^3 leads to a result which is very similar to the chain store paradox.[2] In the same way as in the case of the chain store game we are faced with a conflict between two theories, an "induction theory" and a "cooperation theory". (The cooperation theory corresponds to the deterrence theory.) The induction theory is the game theoretically correct one but the cooperation theory seems to be the better guide to practical behavior.

The conflict between the two theories is less serious than for the case of the chain store game. Practical recommendations for a laboratory situation, where the payoffs are money payoffs, can be given on the basis of a third kind of theoretical reasoning. It is plausible to assume that the utility payoffs of the players are different from the money payoffs. The "benevolence theory" which will be presented below is a theory of this type.[3] The practical conclusions of this theory are similar to those of the cooperation theory but contrary to the cooperation theory the benevolence theory does not face any logical difficulties. It is not necessary to reject the induction argument, since it does not apply.[4]

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2) The book "Games and Decisions" by Luce and Raiffa contains a thorough discussion of the finite supergame of the prisoners' dilemma game (Luce-Raiffa 1957, pp. 97 - 102)

3) Luce and Raiffa are aware of the possibility of such theories but their view is that of the cooperation theory. I think that they underemphasize the paradoxical nature of their recommendations. (Luce-Raiffa 1957, pp. 97 - 102)

4) Luce and Raiffa suggest that this is not a solution of the problem, since it is possible to imagine a laboratory situation, where the psychological effects are compensated by appropriate changes of the money payoffs (Luce-Raiffa 1957, p.98n). This argument is not conclusive. It is very hard to imagine a laboratory situation of this kind. Therefore one might argue that one would be inclined to behave according to the induction theory if one were confronted with such a situation.

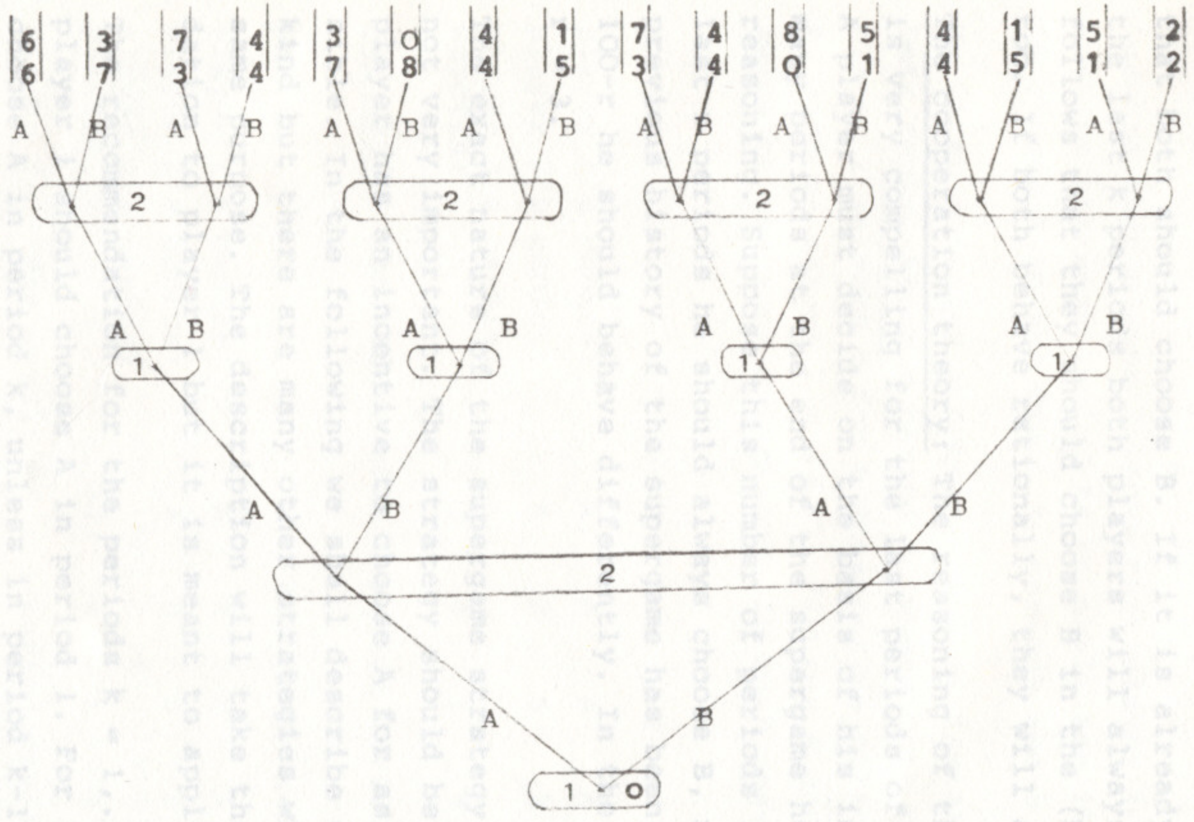

Figure 5: The supergame Γ_2^3 which results from a 2-fold repetition of the specific prisoners' dilemma game of figure 4.

In the following we shall outline the three theories for Γ_m^3. We shall focus our attention on the case m = 100

The induction theory: Clearly, in the last repetition of the game it is better to choose B, whatever the other player does. This determines the last period of Γ_{100}^3. Both players will choose B. Therefore the situation in the second last period is not different from that of the last one. Again it is clear that both should choose B. If it is already clear that for the last k periods both players will always choose B, then it follows that they should choose B in the (k+1)-th last period, too. If both behave rationally, they will always choose B.

The cooperation theory: The reasoning of the induction theory zwingend is very compelling for the last periods of the supergame . A player must decide on the basis of his intuition for how many periods at the end of the supergame he should follow this reasoning. Suppose this number of periods is r. Then in the last r periods he should always choose B, no matter what the previous history of the supergame has been but for the first 100-r he should behave differently. In the following we assume r = 3.

The exact nature of the supergame strategy up to period 97 is not very important. The strategy should be such that the other player has an incentive to choose A for as many periods as possible. In the following we shall describe one strategy of this kind but there are many other strategies which would serve the same purpose. The description will take the form of a recommendation to player 1 but it is meant to apply to player 2, too.

The recommendation for the periods k = 1,...,97 is as follows: player 1 should choose A in period 1. For k=2,...,97 he should choose A in period k, unless in period k-1 player 1 selected A and player 2 selected B; in this case player 1's choice in period k should be B.

This kind of behavior has the following interpretation: With the exception of the last 3 periods, player 1 is willing to use A as long as he observes that player 2 has chosen A. If player 2 deviates to B then player 1 punishes this deviation by a selection of B in the next period but immediately afterwards he returns to A, in the expectation that player 2 will return to A, too. If this expectation is disappointed, a new punishment will follow. Each punishment lasts for one period only.

Suppose that player 2 knows that player 1 behaves in this way. What is his best reply? As we shall see it is best for player 2 to choose A in the first 96 periods and B in the 4 last ones.

We first consider a special kind of deviation from the proposed best reply. After a period k, where both players have chosen A, player 2 selects B for s consecutive periods and then returns to A. Here we assume $k + s < 97$. Figures 6 and 7 describe the results of two such deviations. In both cases the deviation does

period:	k	k+1	k+2	k+3	k+4	k+5
player 1's choice:	A	A	B	A	B	A
player 2's choice:	A	B	B	B	B	A
player 2's partial payoffs:	3	4	1	4	1	3

<u>Figure 6</u>: The result of a 4-period deviation.

period:	k	k+1	k+2	k+3	k+4	k+5
player 1's choice:	A	A	B	A	B	A
player 2's choice:	A	B	B	B	A	A
player 2's partial payoffs:	3	4	1	4	0	3

<u>Figure 7</u>: The result of a 3-period deviation.

not pay. Obviously this is true for all deviations of the same kind. The deviation yields a payoff of at most 5/2 per deviation period whereas a choice of A yields 3 per period.

It can be seen easily that other kinds of deviations do not pay either. One can restrict ones attention to deviations for s consecutive periods $k + 1,\ldots,k + s$, which are such that both players never choose A at the same time and where both players select A in periods k and $k + s + 1$. The situation is essentially the same for $k = 0$ or $k + s + 1 = 97$. After the end of a deviation of this kind player 1 will behave as if no deviation occurred. Whenever player 1 chooses A in the periods $k + 1,\ldots,k + s$ player 2 chooses B. Therefore in these periods player 1 alternates between A and B. The best payoff per deviation period which player 2 can get under these conditions is 5/2. In a period where player 1 chooses A, player 2 can get at most 4 and in a period where player 1 chooses B, player 2 can get at most 1. Moreover s must be an even number, since player 1's choice in period $k + 1$ is A and A is always followed by B as long as the play does not return to a situation where both select A.

Suppose that both players follow the recommendations of the cooperation theory. Each player i selects a number r_i of periods at the end of the supergame where he plans to use B under all circumstances but in earlier periods he follows the pattern of one period punishments described above. Suppose that we have $r_1 = 3$ and $r_2 = 4$. As we have seen, in this case player 2 uses a best reply to player 1's strategy. More generally, we can say that player i uses a best reply to player j's strategy if $r_i = r_j + 1$. It is impossible that both players choose their numbers r_i in such a way that a game theoretical equilibrium results, where both players use best replies against the other player's strategy but it is possible that one of them uses a best reply to the other player's behavior. Therefore the cooperation theory does not recommend a specific number r_i. Each player i must decide on the basis of his intuition which r_i he wants to select. In order to do this he may try to optimize with respect to his subjective expectations about the other player. It is true that at least one of them must have wrong subjective expectations. Nevertheless both can try to do their best.

Suppose that player 1 selects $r_1 = 3$ and player 2 succeeds to <u>"outguess"</u> him by the selection of $r_2 = 4$. Then Player 1 receives a supergame payoff of 291. Player 2 receives 295. If both players would always choose B as the induction theory suggests, each of them gets 100. Clearly, for reasonably small r_i it is much more advantageous to be outguessed in the cooperation theory than to use a best reply in the induction theory.

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<u>The benevolence theory:</u> Strictly speaking this theory is not a theory for Γ^3_{100}. It is a theory for a laboratory situation, where the payoffs are money payoffs. The utility payoffs are assumed to be the sum of two components, a "primary" utility which depends linearly on the money payoffs and a "secondary" utility which depends on the player's perception of his social relations with the other player. The perceived character of the social relationship is determined by the past history of the supergame and by the way in which the decisions influence the primary utilities. In the light of the primary utilities past and future choices are interpreted as friendly or unfriendly acts. A friendly atmosphere is preferred to an unfriendly one. The benevolence theory is only one specific theory which can be constructed on the basis of these general ideas. One should not overemphasize the details of the psychological mechanism which yields the secondary utilities. The exact nature of this mechanism cannot be clarified without empirical research. The assumptions which will be made below are purely speculative. They exemplify a certain type of explanations for a plausible pattern of behavior in the finite prisoners' dilemma supergame.

The secondary utilities are assumed to reflect the following tendencies: (1) A friendly social relationship is preferred to an unfriendly one. (2) A player does not want to be "mean" in the sense that he disappoints the other player's trust.

The simplest way to model tendency (1) is the assumption that each of both players receives a secondary utility a for every period, where both players choose A. The constant a reflects the strength of the tendency. Since the selection of A by both players is the obvious cooperative solution of the game of figure 4, it is reasonable to suppose that no other combination of choices creates the impression of a friendly relationship.

In order to make an assumption about the secondary utilities resulting from tendency (2), the notion of trust must be made more precise. Imagine that in period t - s - 1 at least one player selected B, but then for the s following periods up to period t - 1 both players selected A. In this situation a player shows "trust" if he selects A in period t. If he selects B and the other selects A, then he "disappoints the trust" of the other player. There is no disappointment of trust if both of them choose B, since in this case there was no trust in the first place.

Obviously there is more reason to expect trust and to extend trust after a long sequence of choice combinations (A,A) than after a short one. (The first symbol refers to the choice of player 1 and the second to that of player 2). Therefore it is more objectionable to disappoint the trust of the other player after a long sequence of this kind than after a short one.

In view of these considerations we assume that a player who selects B in a period t where the other player selects A, experiences a negative secondary utility $-b_s$ where s is the number of periods with choice combinations (A,A) after the latest period $k < t$ such that in period k the choice combination was different from (A,A). (Obviously we have $s = t - 1 - k$). It is assumed that b_s is an increasing function of s.

1 \ 2	A	B
A	a , a	0 , $-b_s$
B	$-b_s$, 0	0 , 0

Figur 8: Bimatrix of secondary utilities.

If the secondary utilites are added to the primary utilities than the original supergame Γ_m^3 is transformed into a new game which we denote by Γ_m^4. Player i's payoff in Γ_m^4 is the sum of all his primary and secondary payoffs for all m periods. Obviously Γ_m^4 does not have the structure of a supergame, since b_s depends on the past history of the play.

Suppose that in Γ_{100}^4 both players have chosen A for 99 periods. Then, up to the irrelevant additive constant 99(3+a) the last period payoffs are given by the bimatrix in figure 9.

	A	B
A	$3+a$, $3+a$	$4-b_{99}$, 0
B	$4-b_{99}$, 0	1 , 1

Figure 9: Last period payoffs after 99 periods with (A,A).

For $a+b_{99} \geq 1$ the bimatrix game of figure 9 has two equilibrium points in pure strategies (A,A) and (B,B). The benevolence theory does not give a different result from the induction theory unless this is the case. Therefore in the following we shall always assume $a+b_{99} \geq 1$.

As we shall see under this assumption the game Γ_{100}^4 has equilibrium points which are such that both players always choose A if both of them stick to their equilibrium strategies. An example is supplied by the following simple rule of behavior: In period t choose A if A was always selected by both players in periods 1,...,t - 1; otherwise choose B.

If this rule is followed by the other player, then it does not pay to deviate to B in a period where B was not chosen before by at least one of the players. For $t < 100$ already the sum of the primary utilities is reduced by a deviation of this kind and the secondary utilities make it even more disadvantageous. For $t = 100$ a deviation of this kind does not pay because of $a + b_{99} \geq 1$. The situation is that of figure 9.

Let $\bar{s}$ be the smallest integer with $a + b_{\bar{s}} \geq 1$. If $\bar{s}$ is sufficiently small, then it is possible to change the rule in the direction of a pattern of one period punishments and returns to A as described in our discussion of the cooperation theory, without destroying the equilibrium character of the strategy pair. Such possibilities will not be explored here. It is sufficient to demonstrate that the induction argument does not apply to Γ_m^4 if the influence of the secondary utilities is big enough.

Another way in which the strategy pair may be changed without destroying its equilibrium character is as follows: Suppose that we have $\bar{s} < 99$. Consider the following rule of behavior: For $t < 100$ choose A if A was chosen by both players in periods $1,\ldots,t-1$; otherwise choose B. In period 100 choose B.

Obviously for $\bar{s} < 99$ the players use an equilibrium pair of strategies for Γ_{100}^4 if both apply this rule. In the equilibrium play both choose A up to period 99 but in period 100 an "end effect" takes place and both choose B. We may say that in period 100 the mutual trust breaks down.

Note that the strategy pair where both choose B under all circumstances is an equilibrium pair for Γ_{100}^4, too. The benevolence theory permits that trust is established between the players but it does not exclude the possibility that no trust is established.

Comparison of the three theories: The logical conclusions of the induction theory are inescapable if no secondary utilities are introduced and the game Γ_m^3 is taken literally. Nevertheless the recommendations of the cooperation theory are much more plausible. This does not necessarily mean that the induction argument fails to be behaviorally convincing. Probably one cannot form a gesunden sound intuitive judgement about the practical usefulness of different strategical recommendations without thinking about a concrete situation like a laboratory experiment where the payoffs are money payoffs. Therefore it may be impossible to avoid that one's intuition is influenced by the presence of secondary utilities. As soon as secondary utilities enter the picture, theories of the type of the benevolence theory provide rational reasons not to accept the induction argument. Intuitive judgement and game theoretical analysis are brought into agreement. Unfortunately, in the light of the chain store paradox this easy escape from the problems posed by the induction argument is less convincing than one may think if one looks at the finite prisoners' dilemma supergame in isolation.

A remark on the evidence from prisoners' dilemma experiments: Many experimental studies have been based on prisoners' dilemma supergames. Unfortunately in most cases the number of repetitions was not made known to the players at the beginning of the game. If the number of periods is not revealed, then the experimental situation is more like an infinite supergame. The infinite supergame has equilibrium points in pure strategies where the equilibrium play is such that the players always take the cooperative choice (in our case A). Suppose that the game of figure 4 is repeated an infinite number of times and that the long run average payoff or more precisely the limes inferior of the average payoff is taken as the supergame payoff. (The ordinary limes may not exist). It can be seen easily that an equilibrium point for this game Γ_{∞}^3 is obtained if the players always behave as recommended by the cooperation theory for the first 100 - r periods.

There are some experiments where the laboratory situation did correspond to the finite supergame with money payoffs. (See for example Lave 1962, Lave 1965, Morehous 1973, Rapoport and Dale 1973.)

The results do not show any obvious disagreement with the cooperation theory or the benevolence theory, at least, if one is willing to make adjustments for the possibility that a sizable proportion of the subjects did not understand the strategic situation very well. In many cases the players manage to achieve cooperation in the sense that both of them take the cooperative choice for a long sequence of periods. It happens quite often that the cooperation breaks down in the last periods. Such end effects are predicted by the cooperation theory and not excluded by the benevolence theory.

The way in which the game is described to the players strongly influences the behavior of the subjects (Evans and Crumbaugh 1966, Pruitt 1967, Pruitt 1970, Guyer, Fox and Hamburger 1973). According to these experiments one must expect that it makes a difference whether Γ^3_m is described by figure 4 or by a table of the following kind:

	I take for myself	and I give to him
A	0	3
B	1	0

<u>Figure 10</u>: Alternative description of the game of figure 4.

Here both players have the same table and each of the players receives as his payoff for one period the sum of what he "takes for himself" and what the other player "gives to him". The representation of the game seems to influence the interpretation of the other player's choices in terms of his intentions. In figure 10 choice A looks more "cooperative" than in figure 4. Looking at figure 4, a subject may think: "he has selected A because he wanted to receive the payoff of 3", whereas figure 10 suggests another kind of interpretation: "he has given 3 to me and has taken nothing for himself in order to show his good will." Presentation effects of this kind point in the direction of secondary utilities. Probably the benevolence theory does not provide the best explanation in terms of secondary utilities but some psychological effects do come in.

Comparison with the chain store paradox: Since secondary utilities seem to be important for the prisoners' dilemma supergame, one may be tempted to try to apply the same kind of reasoning to the chain store game. What kind of assumptions about secondary utilities can be made in order to avoid the chain store paradox ?

One could assume that human beings have some kind of "internal commitment power". Once somebody has made a plan, a negative utility will be attached to any change of the plan. This idea is in agreement with the theory of cognitive dissonance (Festinger 1957).

Suppose that player A in the first version of the chain store game makes an internal plan to react by his AGGRESSIVE response to a choice of IN up to period 17. Assume that the negative utility for a change of his plan is -3. Then he has a good reason to stick to his plan, since in period 17 it will be better to react by the AGGRESSIVE response. (As before primary and secondary utilities are assumed to combine additively.)

If player A has this internal commitment power, it would be even better and just as feasible to make an internal commitment to take the AGGRESSIVE response up to the last period of the game. This is not very plausible. Therefore the "internal commitment theory" which, by the way, would be applicable to the finite prisoners' dilemma game, too, does not seem to be a reasonable theory for the chain store game.

Another possibility of introducing a secondary utility is as follows: if player A follows the behavior prescribed by the deterrence theory and nevertheless many of the players 1,..., 20 select IN, then player A will become very angry. As an angry person he will have a positive secondary utility for aggressive behavior. This is in agreement with the frustration aggression hypothesis. (Dollard,Doob,Miller, Mowres and Sears 1939).The "anger theory" has similar implications as the internal commitment theory. Player A should be able to deter all players 1,...,20. The deterrence should not break down for some of the last players. Therefore the "anger theory" is as implausible as the "internal commitment theory ".

The game Γ^3_{100} is a 2-person game where both players interact for a considerable number of periods, some interpersonal relationship can be expected to develop. Contrary to this the chain store game Γ^1_{20} is a 21-person game where player A interacts with each of the players 1,...,20 at most once; there is no occasion for the development of interpersonal relationships. This is an important difference between both games which is partly responsible for the fact that plausible theories based on secondary utilities are much more difficult to construct for the chain store game.

On the basis of these considerations it seems to be justified to draw the following conclusion: Theories based on secondary utilities do not provide a satisfactory explanation for the fact that rational players refuse to accept the conclusions of the induction theory as a guide to practical behavior. It is necessary to look for a different explanation.

5. Sketch of a three-level theory of decision making

In this section an attempt shall be made to develop an informal model of some aspects of the human decision process. The general approach is based on the idea that a decision may be reached on three different levels, the levels of routine, imagination and reasoning. The theory is speculative rather than based on empirical facts other than circumstantial evidence.

It is of course an oversimplification to assume that there are exactly three levels of decision making, neatly seperable from each other. There cannot be any doubt about the fact that the decision process is much more complicated than the simplistic picture which we are going to paint. The three-level theory canno claim to be more than a heuristic tool for the investigation of problems of limited rationality.

<u>The level of routine:</u> The level of routine may be thought of as a simple mathematical learning model where the possibilities with which one of k alternatives 1,...,k in a given decision problem is selected, depends on the experience with similar decision problems in the past.[5] On the routine level decisions are made without any conscious effort. The underlying criteria of similarity between decision situations are crude and sometime inadequate.

<u>The level of imagination:</u> On the level of imagination the decision maker tries to visualize how the selection of different alternatives may influence the probable course of future events. The result of this process of imagination is the selection of one alternative which appears to be preferable to other alternatives. The decision maker does not know why he imagines one scenario rather than another. The imagination process is governe by a multitude of procedural decisions which are made on the routine level. We may say that the imagination level employs the routine level. The imagination process is similar to a computer simulation. The program of this simulation is determined on the routine level.

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5) Since the appearance of the classical work by Bush and Mosteller (Bush and Mosteller 1955) many mathematical learning models of this kind have been explored in the literature (See e.g. Restle and Greeno 1970)

The level of reasoning: The level of reasoning is characterized by a conscious effort to analyse the situation in a rational way on the basis of explicit assumptions whose validity is examined in the light of past experience and logical thinking. The result of the reasoning process is the selection of an optimal alternative. The level of reasoning needs the help of the lower levels of imagination and routine. Ordinarily logical analysis is based on some kind of simplified model whose assumptions are products of imagination. Moreover, the results of the imagination process are used as heuristic hints which guide the process of reasoning.

The predecision: Suppose that a decision maker is confronted with a decision problem where he has to select between k alternatives 1,...k. Which of the three levels are activated by this situation? Since the higher levels need the help of the lower levels there are only three possibilities. (1) Only the routine level is activated. We may say that the decision maker does not stop to think. (2) The routine level and the imagination level are activated. The decision maker visualizes the consequences of different alternatives but he does not transcend the level of imagination. (3) All three levels are activated. A conscious effort is made to analyse the situation in a rational way.

Obviously, a decision has to be made which of the three possibilities (1), (2) and (3) is selected. This decision will be called the "predecision". The predecision is made on the routine level.

The final decision: After the predicision has been made those levels which have been activated will begin to operate. Normally each of these levels will produce one alternative which will be called a "level decision". We assume that the routine level always reaches a level decision but we do not exclude the possibility that the imagination process or the reasoning process are employed without reaching any conclusion. Time may be too short or the decision problem may be too difficult.

Suppose that several level decisions have been reached. Generally these level decisions will be different from each other. Obviously a decision has to be made which selects one of the level decisions. This decision is called the "final decision". The final decision determines the actual behavior. It is made on the routine level.

Note that we do not assume that a decision on a higher level automatically supercedes a decision on a lower level. No final decision would be needed if this were the case. It is an important feature of the three level theory that a decision maker who has found the rational way of behavior may make the final decision to do something else.

<u>The influence of past experience on predecision and final decision</u>: predecision and final decision are the results of learning processes which operate on the routine level. In both cases the decision is a decision between levels. The tendency to select one level rather than another will be influenced by the consequences of similar decisions in the past.

Let us first look at the final decision. If the final decision was made in favor of one level, e.g. the level of reasoning and it turns out that the behavior in the decision situation is rewarded by a success, then this will strengthen the tendency to make a final decision in favor of this level in case of a similar decision situation in the future. The tendency is weakened by the experience of a failure.

The tendency to make one predecision rather than another will also be influenced by the successes and failures experienced in similar decision situations in the past. If a final decision in favor of a certain level was successful then the probability of a predicision which activates this level and the lower ones is increased. The probability is decreased by the experience of a failure.

It may happen that after the decision has been made,it turns out that it would have been better to take another level decision as the final decision. This will also influence the tendenciesto select one level rather than another.

<u>The short run character of success and failure:</u> The way in which a learning process operates depends on the criteria which define what constitutes a success or a failure. The process cannot function well if there is a lack of feedback, successes and failures must be experienced sufficiently often. Therefore, the definition of success and failure must be based on short run criteria; within a reasonably short time after a decision has been made it must be possible to determine whether the consequences of the decision are favorable or unfavorable.

The short run character of success and failure does not exclude the pursuit of long run goals. Long run goals may be approached by short run measures of achievement. Each step in the right direction is experienced as a success.

There is no reason to suppose that the substitution of short run measures of achievement for the pursuit of long run goals will work in a similar way as a long run optimization in the sense of modern decision theory. Therefore one cannot expect that learning processes have the tendency to produce a way of behavior which approximates long run utility maximization.

<u>Economy of decision effort:</u> Decision time and decision effort are scarce commodities. In terms of these commodities the imagination process is more costly than the routine process and the reasoning process is more costly than the imagination process. The predicision serves the purpose to allocate decision time and effort in a reasonable way.

In view of these considerations one may ask the question why the final decision sometimes does not select the level decision produced by the highest activated level. After all, the decision effor has been spent already.

The answer is quite simple. It is not true that the higher level always yields the better decision. The reasoning process is not infallible. It is subject to logical and computational mistakes. The imagination process has its shortcomings, too. Which level has the best chance to produce a successful decision will depend on the nature of the decision problem. Therefore it is necessary to gather experiences about the comparative merits of the decisions made on different levels. For such purposes it may be useful to produce a higher level decision in a situation where the final decision will select a lower level decision with a very high probability. The selection of the lower level decision does nc mean that the decision effort spent on the higher level is wasted.

Why rational behavior cannot be learnt completely: Suppose that a decision maker is repeatedly confronted with the same kind of decision problem under uncertainty; assume that on the level of reasoning he is able to find the rational solution of a problem of this kind. In order to have something specific in mind, we may think of a sequence of investment decision situation where some amount of money can be invested in several different ways; the goal is the maximization of profit.

Since the decision is made under uncertainty the rational solution in the sense of modern decision theory will involve the maximization of expected utility; in our specific example we may assume that this expected utility can be represented by expected profit.

If the decision has long run consequences then the utility maximization will be long run; in our specific example the expected profit to be maximized will be a discounted sum of a stream of expected future profits or something similar. From our remarks on the short run character of success and failure it is clear that in this case it is not very probable that a long process of learning will lead to a decision behavior which approaches the rational solution. In the following we shall assume that there are no such problems. The decision situation is supposed to be such that it is rational to maximize short run expected profit.As we shall see, even in this case a long process of learning may fail to approach the rational solution.

The learning process which determines the probabilities with which the final decision selects one level decision or another operates on the routine level. Since expected profit is not observed,the experience of actual profits will supply the criteria of success and failure which guide this learning process. Because of the uncertainty of the decision situation it is unavoidable that sometimes the rational decision produced on the level of reasoning appears to be a failure whereas the routine process or the imagination process may seem to be more successful. This will weaken the tendency to take the rational choice. Even if the rational choice has a much higher rate of success than the decisions produced on the other levels, failures will occur with some probability and the decision maker will never trust his reasoning process completely. From time to time he will not take his rational choice.

Consider a situation where in our specific example the decision maker had some very bad experiences with a certain kind of investment, say the investment in common stock. On the reasoning level he comes to the conclusion that this was due to some unforeseen events which had a very low probability when the decision was made and that under the present circumstances the investment in common stock is the most profitable one. Nevertheless, he cannot help to be impressed by his bad experience. He feels less inclined to trust his reasoning process. On the lower levels an investment in common stock does not seem to be advisable. On the routine level he has learnt to fear the repetition of his bad experiences. On the level of imagination he vividly vizualizes the repetition of the unforeseen events which reduced the price of stock in spite of the fact that on the level of reasoning he knows that now such events are even less probable than before. In the end he makes the final decision to choose another investment possibility.

6. The induction problem in the light of the three-level theory of decision making

For the sake of shortness we use the term "induction problem" in order to refer to the difficulties which arise in connection with the induction theories for the two versions of the chain store game and for the finite prisoners' dilemma supergame. In the following the ideas of the preceding section will be applied to this problem.

Why strategic decisions are likely to come from the level of imagination. Most of the strategic decision problems which occur in human life are quite complicated. Usually rational solutions are not easily available. Even in the case of relatively simple parlor games it is rarely possible to compute optimal strategies. Strategic decision problems of business and war are subject to the additional difficulty that the unstructured nature of such situations makes it very hard to analyse them in a rigorous way.

It is plausible to assume that under such circumstances the level of imagination has the best chance to produce a successful decision. Usually the visualization of the possible consequences of different choices will reveal some important structural details of the strategic situation which are not obvious enough to be recognized on the routine level. Therefore the imagination level is likely to produce better decisions than the routine level. In a game situation it is often important to put oneself into the situation of the other player in order to form an expectation about his behavior. This can be done on the level of imagination. A player who does not stop to think and makes his decision on the routine level is likely to make some mistakes which can be easily avoided by imagining oneself to be in the other player's position.

If a player tries to analyse the game situation in a rigorous way, then he will often find that the process of reasoning does not lead to any clear conclusion. This will weaken his tendency to activate the level of reasoning in later occasions of the same kind. It may also happen that the process of reasoning yields an inadequate decision which is the result of rigorous thinking about an oversimplified model of the situation. The decision situation itself is often not sufficiently well structured to permit the direct application of rigorous analysis. The analysis must be applied to a model of the situation. The level of reasoning needs the help of the level of imagination in order to construct the model.The imagination process is not unlikely to be more reliable as a generator of scenarios than as a generator of assumptions for a model of the situation.

On the basis of these considerations one must expect that the final decision shows a strong tendency in favor of the level of imagination even in such cases where the situation is well structured and the application of rigorous thinking is not too difficult.

Application to the induction problem. Obviously the induction argument is a result of abstract thinking which is done on the level of reasoning. On the level of imagination a clear and detailed visualization of a sequence of two, three or four periods is possible - the exact number is not important. A similarily clear and detailed visualization of a sequence of 20 periods is not possible. For a small number of periods the conclusions of the induction argument can be obtained by the visualization of scenarios. For a large number of periods the scenarios will either be restricted to several periods, e.g. at the end of the game or the visualization will be vague in the sense that the individual periods are not seen in detail. A player may imagine that "in the beginning" something else will happen than "towards the end" without having any clear view of the extension of these vaguely defined parts of the game.

On the level of imagination one cannot find anything wrong with the deterrence theory for the two versions of the chain store game and with the cooperation theory for the finite prisoners' dilemma supergame. On the contrary, the scenarios which support these theories appear to be very convincing.

The fact that the last periods appear to be different from the earlier ones is easy to understand with the help of the three-level theory. Theories based on **secondary utilities** do not exclude end effects but they do not really explain them. (See our discussion of the benevolence theory in section 4.) The three level theory seems to be a very natural way to look at the induction problem.

7. Perfect equilibrium points

In the following some game theoretical concepts are introduced which are needed in order to make the induction argument precise. For the purposes of this section a game will always be a finite n-person game in extensive form with perfect recall.[6] Games with perfect recall can be analyzed with the help of behavior strategies. There is no need to consider other kinds of strategies.[7]

Definitions[8] : Let $\mathcal{U}_i$ be the set of all information sets of player i in a game Γ. A behavior strategy

$$(1) \qquad q_i = \left\{ q_U \right\}_{U \in \mathcal{U}_i}$$

of player i in Γ assigns a probability distribution q_U over the choices at U to every information set of player i. If γ is a choice at U, then $q_U(\gamma)$ is the probability with which γ is chosen by q_i.

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6) See Kuhn 1953 and Aumann 1964

7) Kuhn has proved that in a game with perfect recall a payoff equivalent behavior strategy can be found for every ordinary mixed strategy. (Kuhn,1953 , p. 213)

8) It will be assumed that the reader is familar with the notion of a game in extensive form (See Kuhn 1953 or Selten 1960 and Selten 1964)

Let Q_i be the set of all behavior strategies q_i of player i in Γ. An n-tuple $q = (q_1, \dots, q_n)$ with $q_i \varepsilon Q_i$ is called a <u>strategy combination for Γ</u>. The set of all strategy combinations for Γ is denoted by Q.

Since no other strategies are considered, in the following behavior strategies often will be simply called strategies. For every strategy combination $q \varepsilon Q$ an <u>expected payoff vector</u> $H(q) = (H_1(q), \dots, H_n(q))$ is defined in the usual way. $H_i(q)$ is player i's expected payoff under the condition that the strategies in q are used by the players.

If q is a strategy combination and q_i' is a strategy for player i, then the notation q/q_i' is used for the strategy combination $(q_1, \dots, q_i', \dots, q_n)$ which results from q, if in q player i's strategy q_i is replaced by q_i', whereas all other strategies in q remain unchanged.

A strategy $\tilde{q}_i$ is called a <u>best reply</u> to the strategy combination $q = (q_1, \dots, q_n)$ if we have

(2) $$H_i(q/\tilde{q}_i) = \max_{q_i' \varepsilon Q_i} H_i(q/q_i')$$

An <u>equilibrium point</u> is a strategy combination $q^* = (q_1^*, \dots, q_n^*)$ where for $i = 1, \dots, n$ the strategy q_i^* is a best reply to q^*.

<u>Subgames</u>: Let x be a vertex of the tree K of a game Γ. Let K_x be the subtree which contains x and all those parts of K which come after x in the sense that they can be reached by a play after x has been reached. K_x is the tree of a <u>subgame</u> Γ_x if and only if K_x has the following property: if an information set U contains at least one vertex of K_x then every vertex in U belongs to K_x. - The subgame Γ_x results from Γ by restricting the rules of Γ to K_x: On K_x the information sets and the choices of the players, the probabilities of random choices and the payoffs are the same as in Γ.

A strategy q_{xi} of player i for a subgame Γ_x of Γ is called induced by a strategy q_i for Γ if on Γ_x the strategies q_{xi} and q_i prescribe the same behavior. A strategy combination $q_x=(q_{x1},\ldots,q_{xn})$ for Γ_x is <u>induced</u> by a strategy combination $q=(q_1,\ldots,q_n)$ if for l=1,...,n the strategy q_{xi} is induced by q_i.

<u>Perfect equilibrium points.</u> A <u>perfect equilibrium point</u> $q^* = (q_1^*, \ldots, q_n^*)$ for a game Γ is an equilibrium point for Γ which induces an equilibrium point $q_x^* = (q_{x1}^*, \ldots, q_{xn}^*)$ on every subgame Γ_x of Γ.

It has been argued elsewhere that a strictly non-cooperative solution of a game in extensive form must be a perfect equilibrium point. [10)] A rational theory which specifies complete strategic recommendations for all players in a game Γ must prescribe a perfect equilibrium point for Γ. The theory must prescribe an equilibrium point,since otherwise at least one of the players can improve his payoff by a deviation from the theoretical recommendations,if the other players behave in the prescribed way. A situation of this kind should not only be excluded in the game as a whole but also in the subgames of the game. This is not automatically true for every equilibrium point since an equilibrium point for the whole game may induce a disequilibrium strategy combination on a subgame which is not reached if the equilibrium point for the whole game is played. It is clear that a rational theory should prescribe rational behavior in all parts of the game, even in those parts which cannot be reached if the game is played rationally.

The difference between a perfect equilibrium point and an imperfect one can be exemplified with the help of the game Γ_1^1 in figure 1. As we can see from the bimatrix, this game has two equilibrium points in pure strategies; the equilibrium point COOPERATIVE/IN is perfect and the equilibrium point AGGRESSIVE/OUT is imperfect. After player 1 has selected IN, a subgame begins;

10) See Selten 1965, Selten 1968 or Selten 1973

this subgame has only one equilibrium point, namely the COOPERATIVE response of player A. It follows immediately that Γ_2^1 has one and only one perfect equilibrium point, the strategy combination COOPERATIVE/IN.

The imperfect equilibrium point AGGRESSIVE/OUT has an interesting interpretation: player A **threatens** to take the AGGRESSIVE respon to the choice of IN. If this threat is believed by player 1, then it is better for him to choose OUT. Player A does not have to execute his threat if player 1 chooses OUT. The subgame after the choice of IN is not reached by AGGRESSIVE/OUT.

Player A's threat is not credible. Player 1 knows that it is not in the interest of player A to take the AGGRESSIVE response after a choice of IN. Therefore it is better for player 1 to choose IN. The imperfect equilibrium point is no rational alternative to the perfect one. Player 1 cannot be deterred.

8. Precise statement of the induction theory

A precise statement of the induction theory for the two versions of the chain store game and for the finite prisoners' dilemma supergame requires the concept of a perfect equilibrium point. The deterrence theory for the two versions of the chain store game is not incompatible with the idea of an equilibrium point. As we have seen in the preceding section even in Γ_1^1 an imperfect equilibrium point is available, where player 1 is deterred from choosing IN. The deterrence theory fails to be game theoretically correct since it is incompatible with the concept of a perfect equilibrium point.

It is well known that in the case of the finite prisoners' dilemma supergame the cooperation theory is already incompatible with the equilibrium point concept.[11] Nevertheless, it is more adequate to apply the notion of a perfect equilibrium point.

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[11] See Luce-Raiffa 1957, pp. 99 - 100

The finite prisoners' dilemma supergame is a game in extensive form. Moreover, the natural way of reasoning from behind, first looking at the last period, then on the second last, etc. is closely connected to the requirement of perfectness. Among the imperfect equilibrium points of the finite prisoners' dilemma supergame there are many which in some unreached subgames prescribe the cooperative choice A in the last period.

The following theorem contains a precise statement of the induction theory.

<u>Theorem:</u> For m = 1,2, ... each of the games Γ_m^1 , Γ_m^2 and Γ_m^3 has one and only one perfect equilibrium point. In the case of the two versions of the chain store game Γ_m^1 and Γ_m^2 the uniquely determined perfect equilibrium point requires that whenever one of the players 1, ..., m has to make a choice, he chooses IN and whenever player A has to make a choice, he chooses COOPERATIVE. In the case of the finite prisoners' dilemma supergame Γ_m^3 , the uniquely determined perfect equilibrium point requires that each of both players selects B under all circumstances in every period.

<u>Proof:</u> Let us first look at the two versions of the chain store game. There is no difference between Γ_1^1 and Γ_1^2. Our discussion in the preceding section has shown that as far as Γ_1^1 is concerned, the assertion of the theorem is correct. Assume that the *theorem holds for* $\Gamma_1^1, \ldots, \Gamma_{m-1}^1$ *and for* $\Gamma_1^2, \ldots, \Gamma_{m-1}^2$. *Up to* the numbering of the players and up to some strategically irrelevant constants in the payoff function the subgames of Γ_m^1 at the beginning of period 2 have the same structure as Γ_{m-1}^1. Analogously the subgames at the beginning of period 2 of Γ_m^2 have essentially the same structure as Γ_{m-1}^2. In view of the definition of the perfectness requirement it is clear that a perfect equilibrium point is induced on every subgame by a perfect equilibrium point. It follows from the induction assumption that the subgames at the beginning of period 2 have exactly one perfect equilibrium point each. The perfect equilibrium point of the whole game must induce these equilibrium points which prescribe

the behavior required by the theorem. Since the behavior in the subgames does not depend on the outcome of period 1, there is only one way in which this behavior can be completed in order to construct a perfect equilibrium point for the whole game by adding prescriptions for period 1. In Γ_m^1 player 1 has to choose IN and player A has to take his COOPERATIVE response. In Γ_m^2 the players 1,...,m must choose IN and player A has to take his COOPERATIVE response. It is clear that player A must behave in this way. He cannot influence the other players' behavior in later periods. If he would behave differently, the perfect equilibrium point would fail to induce an equilibrium point in the subgame which begins with player A's response in period 1. It follows that in period 1 it is better for player 1 in Γ_m^1 and for the players 1, ..., m in Γ_m^2 to choose IN. This shows that the theorem holds for Γ_m^1 and Γ_m^2.

Let us now look at Γ_m^3. The assertion of the theorem holds for Γ_1^3. This game has only one equilibrium point, namely (B,B). Assume that the theorem is correct for Γ_1^3, ..., Γ_{m-1}^3. Up to a strategically irrelevant additive constants in the payoff functions, the subgames of Γ_m^3 at the beginning of period 2 have the same structure as Γ_{m-1}^3. Therefore each of these subgames has exactly one perfect equilibrium point which prescribes the choice of B under all circumstances. A perfect equilibrium point Γ_m^3 must prescribe the same behavior for periods 2, ..., m. There is only one way in which this behavior can be completed by a prescription of choices for period 1, if one wants to construct an equilibrium point for the whole game: both players must choose B in period 1. Given the behavior in the subgames, in period 1 the choice of B yields a better payoff than the choice of A, independently of the other player's choice in period 1. This completes the proof of the theorem.

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